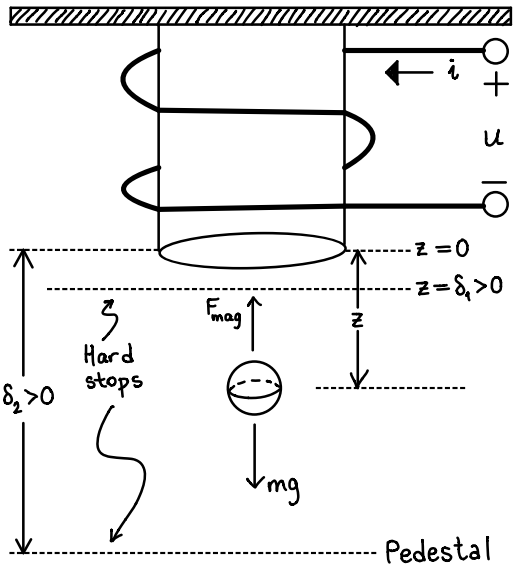

$$F_{mag} = -C\left(\frac{i}{z}\right)^2$$

States

$$\mathbb{R}^3 \ni x = (x_1, x_2, x_3) = (z, \dot{z}, i)$$

Equations of motion

percussion

$$\dot{x}_1 = x_2$$

$$dx_2 = \left(g - \frac{C}{m}\frac{x_3^2}{x_1^2}\right)dt + d\lambda$$

$$\dot{x}_3 = -\frac{R}{L}x_3 + \frac{2C}{L}\left(\frac{x_2 x_3}{x_1}\right) + \frac{1}{L}u$$

$$d\lambda = \begin{cases} \lambda_1 & \text{if } z = \delta_1 \\ -\lambda_2 & \text{if } z = \delta_2 \end{cases}.$$

$$0 \le \lambda_1 \perp -\dot{z} \ge 0$$

$$0 \le \lambda_2 \perp \dot{z} \ge 0$$

Simple controller

This controller will not reason about the plausible contacts, i.e., will assume $d\lambda \equiv 0$. It will also assume knowledge of state.

Consider the state transformation

$x \longmapsto \xi$, where (desired levitation height)

$$\begin{aligned} x_1 \longmapsto \xi_1 &= x_1 - x_{1d} \\ x_2 \longmapsto \xi_2 &= x_2 \\ x_3 \longmapsto \xi_3 &= g - \frac{C}{m}\frac{x_3^2}{x_1^2}. \end{aligned}$$

In the ξ-coordinates,

$$\dot{\xi}_1 = \xi_2,$$

$$\dot{\xi}_2 = \xi_3,$$

$$\dot{\xi}_3 = f(x) + g(x)u,$$

where

$$f(x) = -\frac{4C^2}{mL}\frac{x_2 x_3^2}{x_1^4} + \frac{2RC}{mL}\frac{x_3^2}{x_1^2} + \frac{2C}{m}\frac{x_2 x_3^2}{x_1^3},$$

$$g(x) = -\frac{2C}{mL}\frac{x_3}{x_1^2}.$$

Let $u = \frac{1}{g(x)}\left(w - f(x)\right)$ to get the system

$$\begin{aligned}\dot{\xi}_1 &= \xi_2\\ \dot{\xi}_2 &= \xi_3\\ \dot{\xi}_3 &= w \overset{\text{set}}{=} -\langle k, \xi\rangle, \qquad k \in \mathbb{R}^3_{++}\end{aligned}$$

Characteristic polynomial

$$k_1 + k_2\lambda + k_3\lambda^2 + \lambda^3 \overset{\text{set}}{=} (\lambda + p_1)(\lambda + p_2)(\lambda + p_3); \quad p_i \in \mathbb{R}_+$$
$$= p_1p_2p_3 + (p_1p_2 + p_1p_3 + p_2p_3)\lambda + (p_1 + p_2 + p_3)\lambda^2 + \lambda^3,$$

i.e., $k_1 = p_1p_2p_3$, $k_2 = p_1p_2 + p_1p_3 + p_2p_3$, $k_3 = p_1 + p_2 + p_3$.

Suppose $p_1 = p_2 = p_3 = p$, then $k_1 = p^3$, $k_2 = 3p^2$, $k_3 = 3p$.

Some sample values are given by

$p \rightarrow$	1	2	5	10	20	50	100
k_1	1	8	125	1000	8000	125×10^3	10^6
k_2	3	12	75	300	1200	7500	3×10^4
k_3	3	6	15	30	60	150	300