7.1 The Ideal Op-Amp (Operational Amplifier)

The ideal op-amp model is a key building block of designing analog filters, amplifiers, oscillators, sources, and more.

Operational amplifiers, usually shortened to just "op-amps", are an essential building block of analog electronic systems. In different configurations with a few other components, op-amps can be used to process and manipulate an analog voltage signal in many different ways. This includes many kinds of filters (low-pass, high-pass, band-pass, integrator, differentiator), amplifiers (buffer, inverting, non-inverting, differential, summing, instrumentation), oscillators, comparators, sources (voltage, current), converters (voltage-to-current, current-to-voltage), and even some nonlinear applications.

These applications are *tremendously* useful, and we'll look at ieach one individually in the upcoming sections, but first let's understand the ideal op-amp on its own.

Ideal vs. Non-Ideal

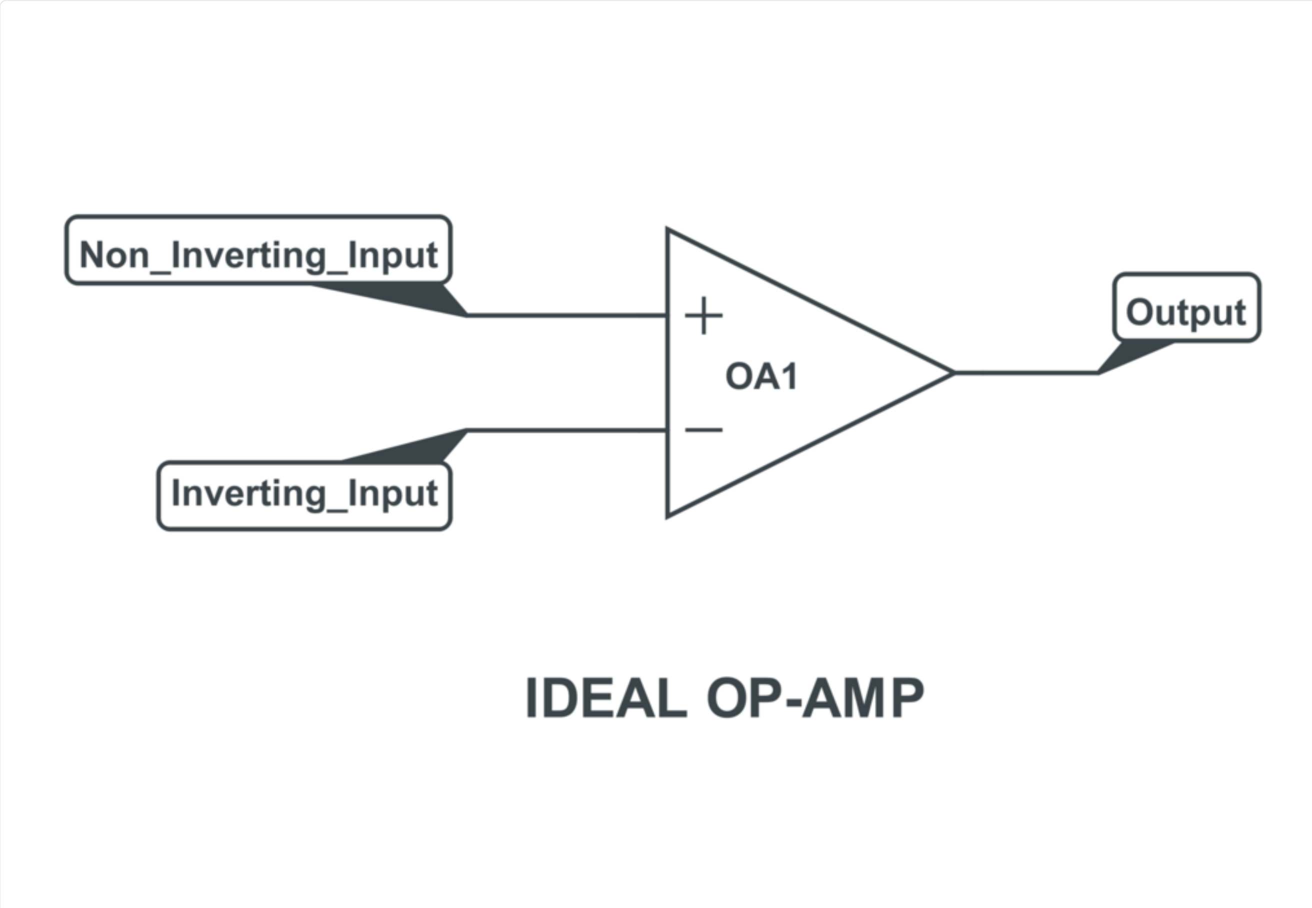
Today, an op-amp is an integrated circuit (IC) containing a few dozen individual transistors and passive components. Historically, before the age of ICs (1960s-1970s), most amplifiers or analog signal processing stages would be purpose-designed for a specific application to avoid the op-amp's relatively high complexity and cost. But now that IC op-amps have only a few pins and cost just a few pennies, it usually makes sense to take advantage of their enormous potential for making analog designs simpler.

Most op-amps aspire to perform like the **ideal op-amp**, a theoretical model that both works well in simulation and makes it easy to solve circuits by hand. As a result, most design and analysis will treat the op-amp as being ideal, and that's how we'll begin.

Later, we'll discuss the ways in which this ideality breaks down in real-world non-ideal op-amps. These limitations are crucial for knowing when you can approximate your analysis as an ideal op-amp, and when you can't. They can also help you choose the correct op-amp to implement your design.

Ideal Inputs, Outputs, and Gain

The ideal op-amp is a voltage amplifier with two inputs and one output:



Ideal Op-Amp Symbol
circuitlab.com/c42kpkxv8a43k
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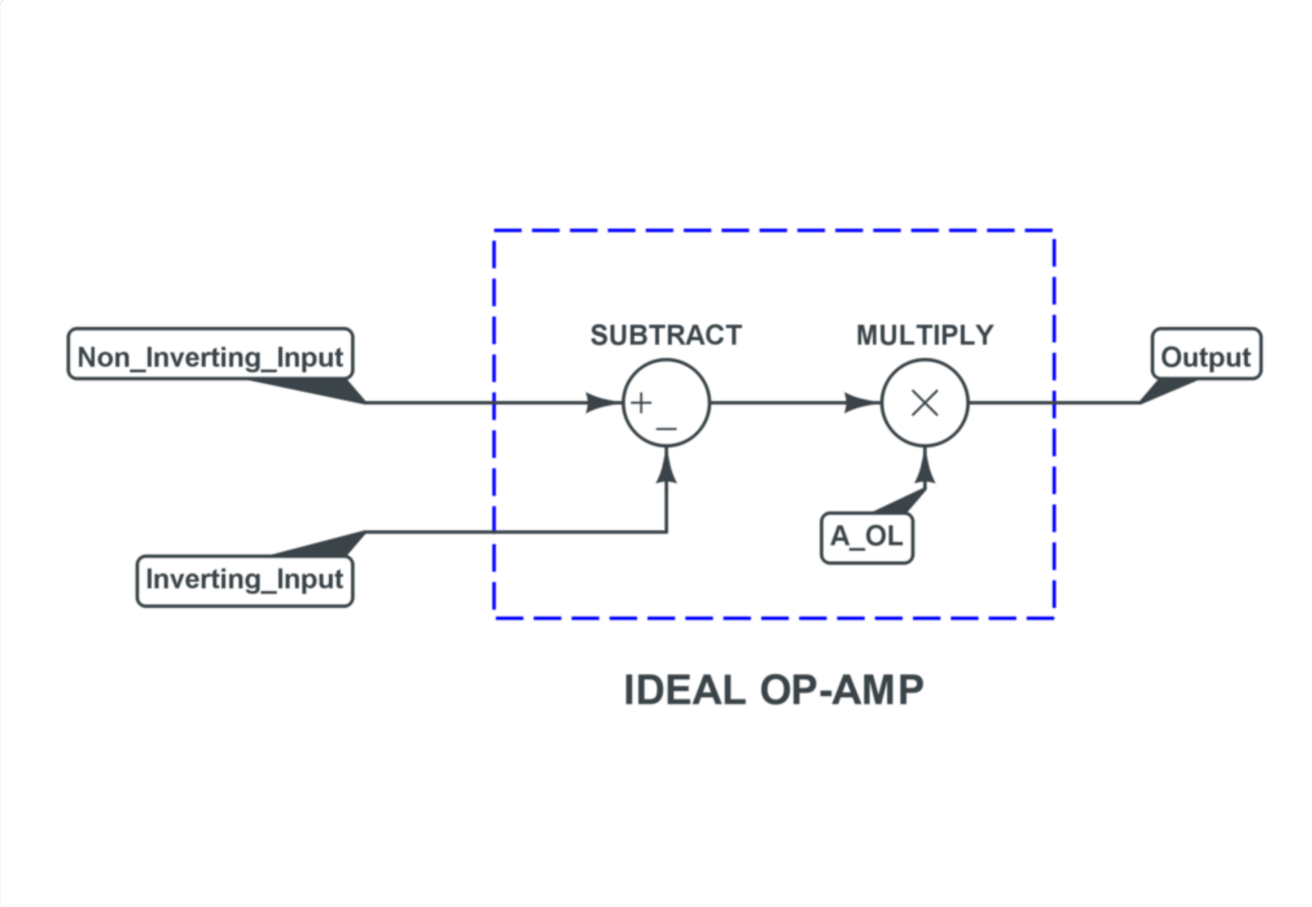
The two inputs are called the **non-inverting input (+)** and the **inverting input (-)**.

Keep a close eye on the + and - signs labeled within the triangle! The op-amp is commonly drawn either way, with + on top or on bottom, whatever makes the rest of the schematic easiest to draw. (In CircuitLab, select the op-amp and press “V” to flip the symbol vertically.) If you unintentionally swap the two inputs, your design won’t work, both on paper and in the real world!

Conceptually, the ideal op-amp subtracts the two inputs, and then multiplies that difference by a huge number called the **open-loop gain** A_{OL} :

$$V_{\mathrm{out}} = A_{\mathrm{OL}} \left(V_{+} - V_{-} \right)$$

As signal processing steps, this subtraction and multiplication looks like:



Ideal Op-Amp Subtraction and Multiplication
circuitlab.com/cv47d3w6nc9cs

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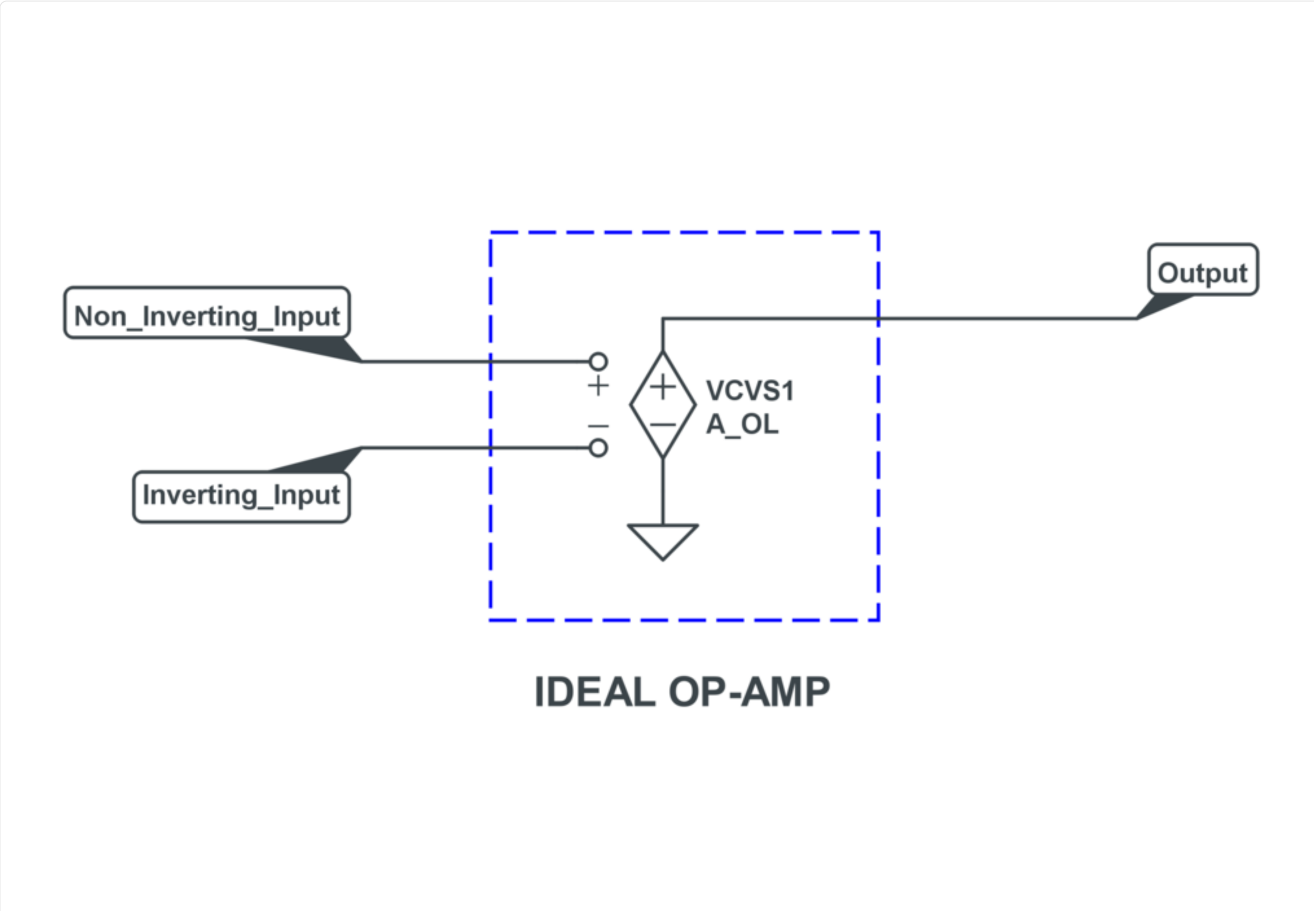
Alternatively, the ideal op-amp can be modeled as a Voltage Controlled Voltage Source (VCVS):



Ideal Op-Amp as VCVS
circuitlab.com/cs79ppnt7dr8f

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If you look carefully, the VCVS model above raises a new question: why did a ground suddenly appear within the op-amp? Since voltages are always relative, this is implying that $V_{\text{offset}} = 0$ in the more complete and correct equation:

$$(V_{\text{out}} - V_{\text{offset}}) = A_{\text{OL}}\left(V_{+} - V_{-}\right)$$
$$V_{\text{out}} = A_{\text{OL}}\left(V_{+} - V_{-}\right) + V_{\text{offset}}$$

If we take an op-amp and we short together the input terminals so that $V_{+} - V_{-} = 0$, the output will be $V_{\text{out}} = V_{\text{offset}}$. In the real world, in a real op-amp with the inputs shorted together, the output will not necessarily be any particular voltage, and whatever voltage it is will certainly be relative to whatever else we're measuring. However, in ideal op-amp circuit analysis, we usually assume $V_{\text{offset}} = 0$ as a simplifying assumption because either:

- The op-amp is being used in a **closed-loop feedback configuration**, where a static offset becomes irrelevant after applying feedback rules (especially since the gain A_{OL} is so large), or
- The op-amp is being used in an **open-loop configuration** with no feedback, in which case we saturate the output into non-linear, non-ideal behavior quickly anyway.

How big is the gain? In real-world non-ideal op-amps, typical values of the open-loop gain are from the hundreds of thousands to the tens of millions:

$$A_{\text{OL,non-ideal,typ}} = 10^5 \text{ to } 10^7$$

That's really big! A millivolt difference in the inputs becomes hundreds or thousands of volts at the output! It's so big that in *ideal* op-amp analysis, we make another simplifying assumption, taking the limit assuming that the gain goes to infinity:

$$V_{\text{out}} = A_{\text{OL}} \left(V_+ - V_- \right)$$
$$A_{\text{OL,ideal}} \rightarrow \infty$$

That's the **algebraic model of the ideal op-amp**: it subtracts the voltage at the inverting input from the non-inverting input, and then multiplies the difference by a very large gain that approaches infinity.

Even in real op-amps, the datasheet often guarantees only a *minimum* open-loop gain, but not a maximum. You can't and shouldn't design a circuit relying on knowing the exact value of the open-loop gain of an op-amp.

It can be hard to think about infinities! One helpful mental trick is to pause time and imagine what's happening dynamically: instead of jumping immediately to infinity, imagine that when given a slight difference in inputs, the ideal op-amp's output voltage just starts rising, rising, rising toward infinity! As we introduce different closed-loop feedback configurations later, you'll see that this rapid rising of the output voltage eventually finds its way back to affect one or both of the same op-amp's inputs, so don't be alarmed: the infinities won't last very long.

It can be hard to do algebra with infinities, too. A suggestion is to keep A_{OL} in place as a variable, and only at the end, take the limit $A_{\text{OL}} \rightarrow \infty$.

Intuitive Model

The ideal op-amp continuously measures the voltages at its inputs, and adjusts its output voltage:

- If the non-inverting (+) input is at a **higher** voltage than the inverting (-) input, the op-amp will **increase** its output voltage.
- If the non-inverting (+) input is at a **lower** voltage than the inverting (-) input, the op-amp will **decrease** its output voltage.

In equation form:

$$\begin{array}{ll} V_{\text{out}} \text{ increases} & \text{if } V_+ > V_- \\ V_{\text{out}} \text{ decreases} & \text{if } V_+ < V_- \\ V_{\text{out}} \text{ steady} & \text{if } V_+ = V_- \end{array}$$

If feedback is present and in the correct direction, then the op-amp will continuously make adjustments to its output voltage until the two input voltages are the same.

Ideal Op-Amp Assumptions

There are a number of other assumptions engineers make about ideal op-amps. All of these assumptions will break for real (non-ideal) op-amps, so keep an eye out for how they might affect your circuit.

By learning about these ideality assumptions, we can decide when we can design a circuit assuming the op-amp is ideal (and thus much easier to analyze), and when this simplified model is likely to collide with reality. We'll explore these issues in more depth in later sections.

Infinite Input Impedance

No current can flow into or out of the input terminals of an ideal op-amp. The input terminals can only measure their voltages. From Thevenin Equivalent Circuits, this is like saying that the input impedance looking into the input terminals is infinite: $Z_{\text{in}} = \infty$

Zero Output Impedance

The output of an ideal op-amp can hold its V_{out} and supply any amount of current, in or out, without that voltage changing. In the Thevenin equivalent model looking into the output terminal (and ground), it appears like a voltage source with zero resistance – therefore zero output impedance: $Z_{\text{out}} = 0$

Zero Input Offset

In ideal op-amps we assume that the non-inverting and inverting inputs are perfectly balanced so that $V_{\text{out}} = A_{\text{OL}}(V_+ - V_-)$. In the real world, due to manufacturing processes, there's some input offset voltage such that $V_{\text{out}} = A_{\text{OL}}(V_+ - V_- + V_{\text{input offset}})$. You can think of this conceptually by simply adding a small voltage source in series with one of the inputs. If DC accuracy matters, this input offset (even just a few millivolts!) can be a big deal, especially because it can drift while the circuit is operating. But in an ideal op-amp, we assume: $V_{\text{input offset}} = 0$

Zero Power Consumption

The schematic symbol for the ideal op-amp omits connections to the power supply, but a real op-amp has to get power from somewhere and deliver power to the schematic. On a datasheet, this starts with the op-amp's **quiescent current** I_Q . (See Power for a discussion of power and energy bookkeeping in circuits.) In ideal op-amps, we treat this like a VCVS: it's an active source and can supply power to the circuit.

Infinite Slew Rate

The rate at which an op-amp can change its output voltage is called the **slew rate**. In real op-amps, there's a limit to how fast the output can rise or fall, measured in $\frac{\mathrm{V}}{\mathrm{s}}$. (This is similar to the mental trick about thinking about infinite open-loop gain discussed above.) In ideal op-amps, we allow an infinite slew rate: the output can move infinitely fast.

Infinite Bandwidth

In addition to the slew rate limit (which is a nonlinear limit), there's also a bandwidth limit in real op-amps: they are not responsive to all frequencies. Real op-amps have an open-loop gain which is a function of frequency, $A_{\mathrm{OL}}(f)$, and it declines at high frequencies. In particular, the **gain-bandwidth product (GBW)** is the frequency at which the op-amp's open-loop gain drops to 1. Notably, the gain starts declining far before that frequency. But in ideal op-amps, we assume the open-loop gain is constant and large (approaching infinity) for all frequencies.

Infinite Gain

As discussed extensively above, we assume ideal op-amps have gain approaching infinity. Real op-amps have finite open-loop gain, which can limit the amount of amplification we can get from a single op-amp stage.

Perfect Linearity

In ideal op-amps, we assume that if we double the input voltage difference, we'll double the output voltage. Real op-amps are made of nonlinear components and this isn't true. However, because op-amps are used in closed-loop feedback configurations, the feedback keeps the input voltage difference extremely small, inside the range where we do see basically linear behavior. It's safe to assume linearity in the ideal op-amp.

Unlimited Input Voltage Range

An ideal op-amp can have inputs of any value; only their difference matters. But in a real op-amp, there will be limits on the allowed input voltages to prevent damaging the input transistors. The subtraction won't work properly if your inputs exceed these limits, and your circuit won't work as designed. (More subtly, you'll get nonlinear distortion before you reach the hard limits.) In most cases, the limits are right around the positive and negative power supply voltages, but you should check the datasheet to be sure.

Unlimited Output Voltage Range

An ideal op-amp can output any voltage. But in a real op-amp, you're limited to what the output transistors can deliver. These limits are usually right around the positive and negative power supply voltages, but you should check the datasheet.

Perfect Power Supply Rejection

An ideal op-amp only responds to changing voltages on its non-inverting and inverting input pins. But a real op-amp may "leak" some of the variation from its power supply pins into the output. (This is captured as the Power Supply Rejection Ratio [PSRR] spec on a datasheet.) This lets a noisy power supply contaminate a signal.

Zero Noise

An ideal op-amp doesn't add any noise to the signal. But in a real op-amp, noise is added and possibly even amplified.

Real-World Op-Amp Tradeoffs

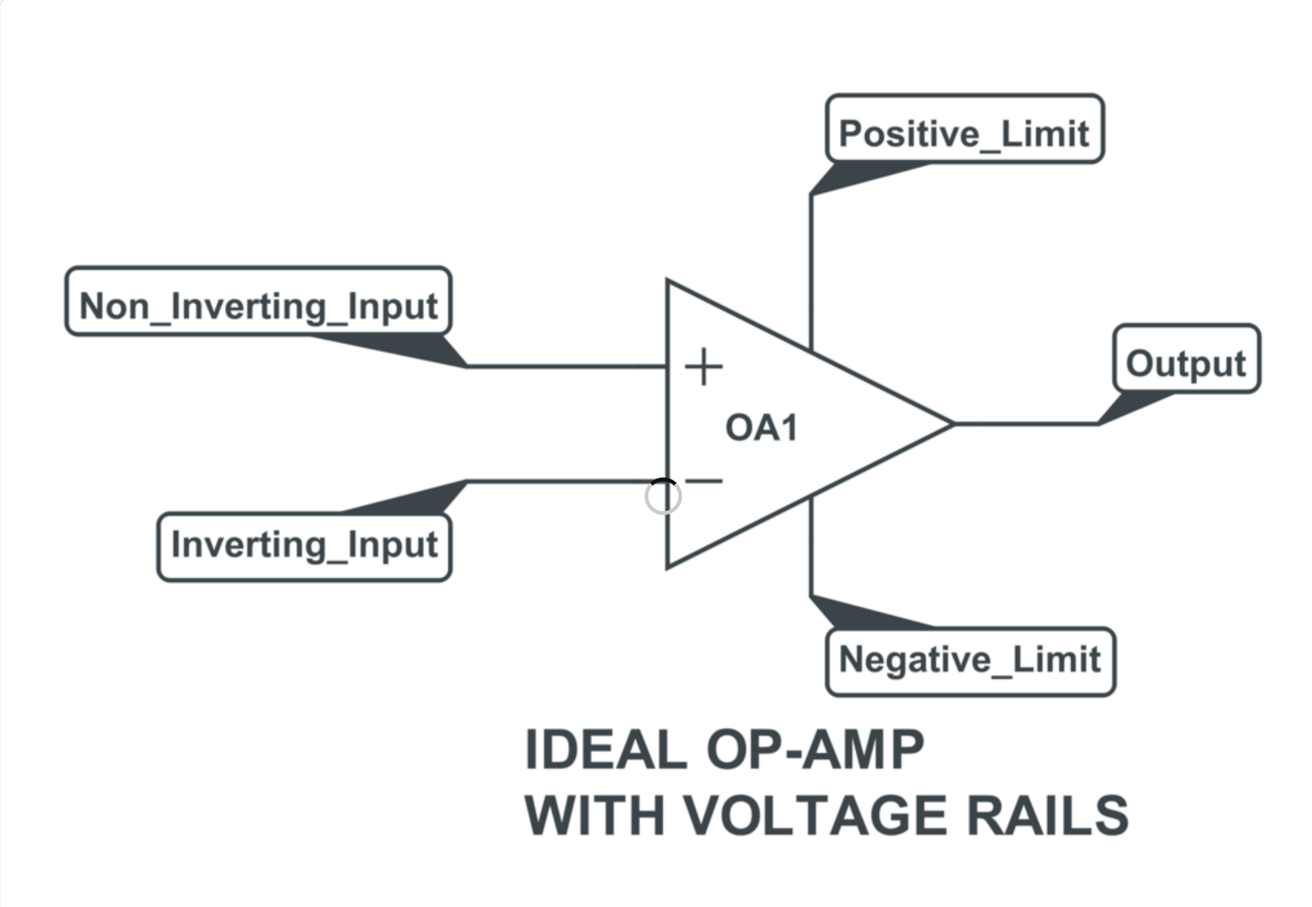
The ideal op-amp is pretty fantastic! Unfortunately, they're all sold out. The real IC op-amps you can buy are non-ideal in all of the ways described above, and semiconductor manufacturers have to make their own tradeoffs to hit their target specs and price point.

As a result, if the analog design problem you're trying to solve is especially demanding in any direction, you might not want to use an op-amp. For example, if you need to design an amplifier stage with the absolutely highest frequency performance, or one with the absolutely lowest power consumption, you probably aren't going to use an op-amp.

Fortunately, there are thousands of different models of op-amps available for sale, and they all make different tradeoffs among these non-idealities. In many cases, by understanding your design problem and how it maps to these non-idealities, you'll be able to find one that meets your needs out of the box!

Ideal Op-Amp with Supply Rail Voltage Limits

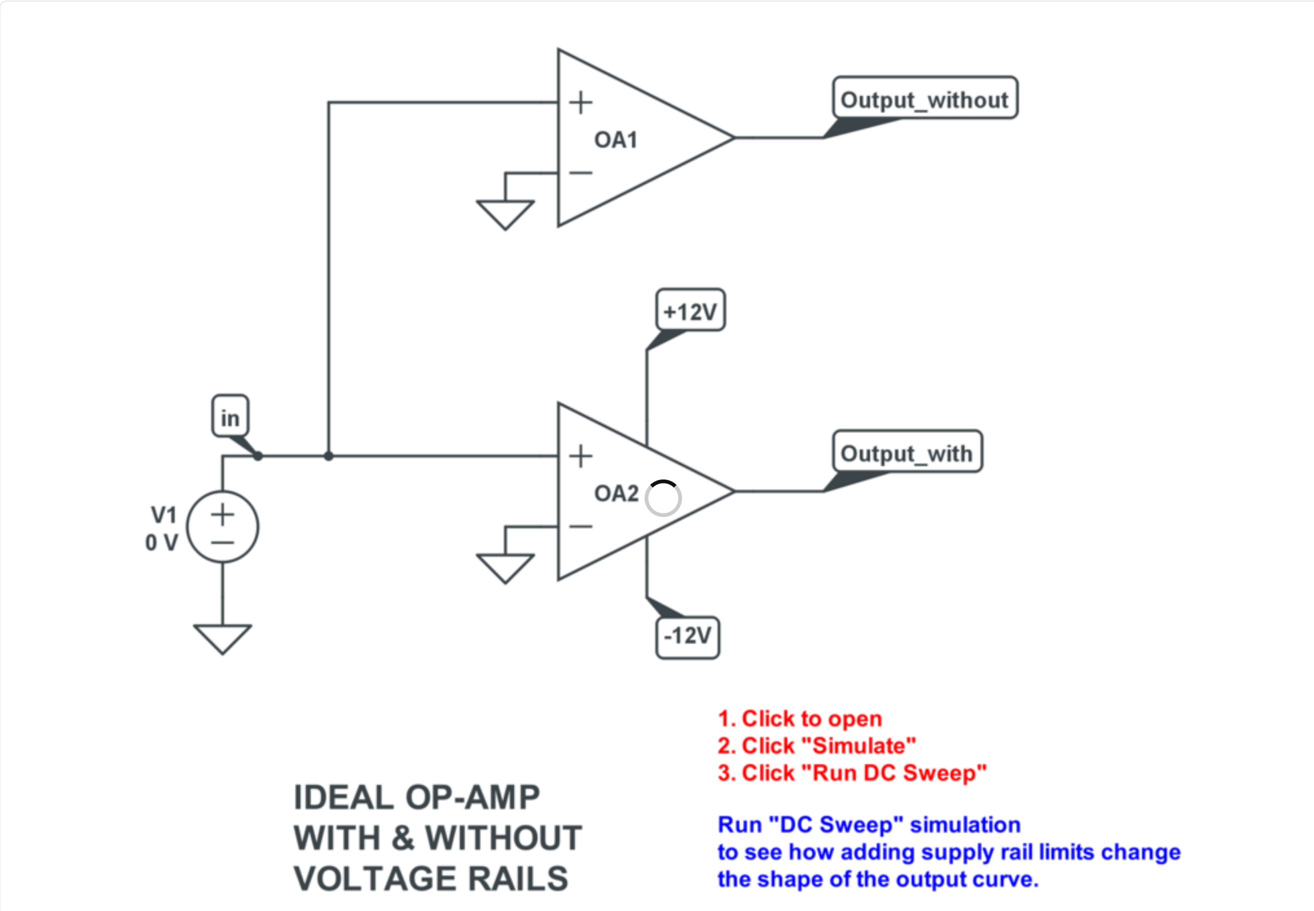
It's often useful to relax the "Unlimited Output Voltage Range" assumption above and instead, model an **ideal op-amp with voltage rails**, where the output is contrained to lie within the provided range.



Ideal Op-Amp with Voltage Rails Symbol
circuitlab.com/c3nhuzcp8d3zc

Edit - Simulate

It's useful to run a DC Sweep simulation to see what the output of the ideal op-amp looks like, open-loop, with and without voltage rails. The two output curves overlap in the middle, when the limits aren't exceeded. But with voltage rails, the `V(Output_with)` line is chopped off to be flat and horizontal once the limits are exceeded:



Op-Amp With and Without Voltage Rails DC Sweep Comparison
circuitlab.com/cy6wk2cksasa6

Edit - Simulate

Exercise Click to open and simulate the circuit above and observe how one output appears clipped as the input varies.

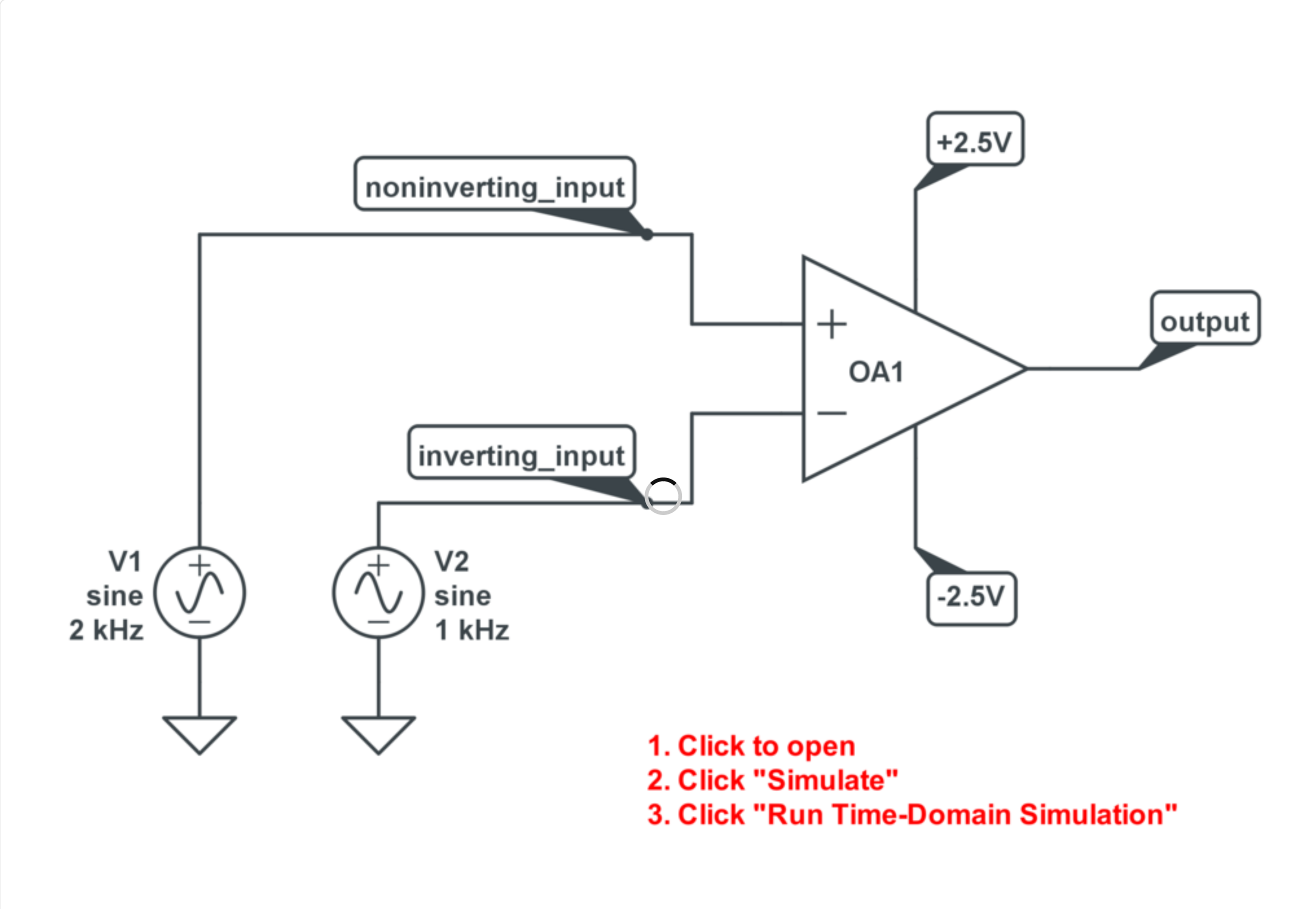
(Note that for many real op-amps, its output can't swing all the way to the positive supply rail, and can't swing all the way down to the negative one.)

Now that we have an ideal op-amp with voltage rails, we can use it open-loop as a voltage comparator. The ideal op-amp's infinite gain is effectively *overruled* by having voltage output limits, so in effect:

$$\begin{aligned} V_{\text{out}} &= V_{\text{limit,pos}} \qquad \text{for } V_+ > V_- + \epsilon \\ V_{\text{out}} &= V_{\text{limit,neg}} \qquad \text{for } V_+ < V_- - \epsilon \end{aligned}$$

for some very small ϵ .

This can be demonstrated by connecting two sinusoidal function generators with different frequencies to the op-amp's two inputs:



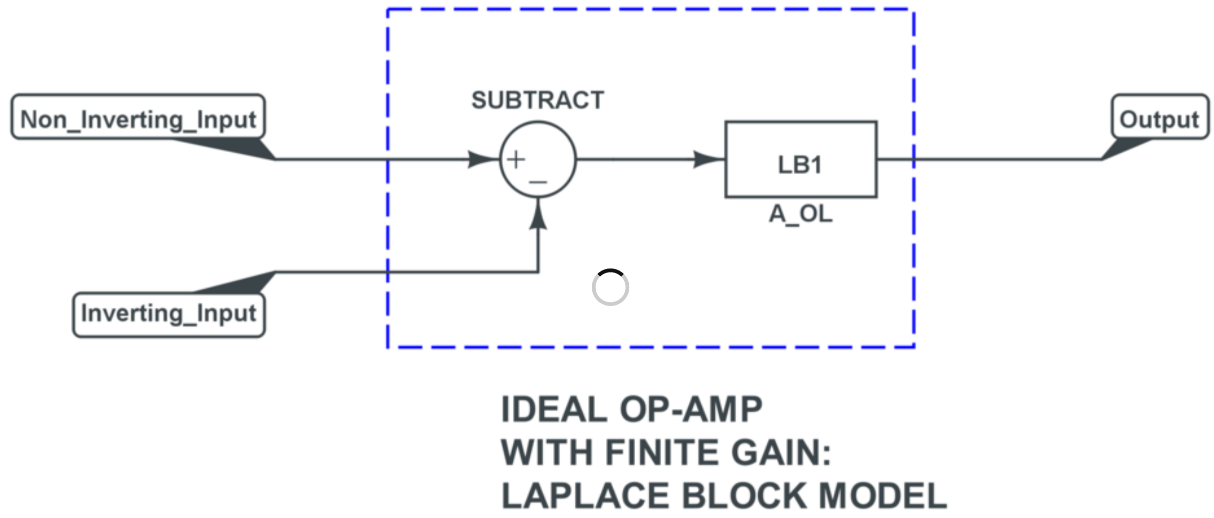
Op-Amp with Voltage Rails as Analog Comparator
circuitlab.com/c39z7cwks2hrz
Edit - Simulate

Exercise Click to open and simulate the circuit above. Watch how the output swings to either extreme as the inputs cross.

In the real world, an op-amp is not a great analog voltage comparator: there are far better purpose-built parts. However, it's one of the few applications of op-amps without feedback, so you can actually build and test this in your lab.

Laplace Transfer Function

It's useful to model an op-amp circuits in the Laplace domain because we can solve feedback systems algebraically. In particular, a useful model for the ideal op-amp involves having a finite open-loop gain A_{OL} :



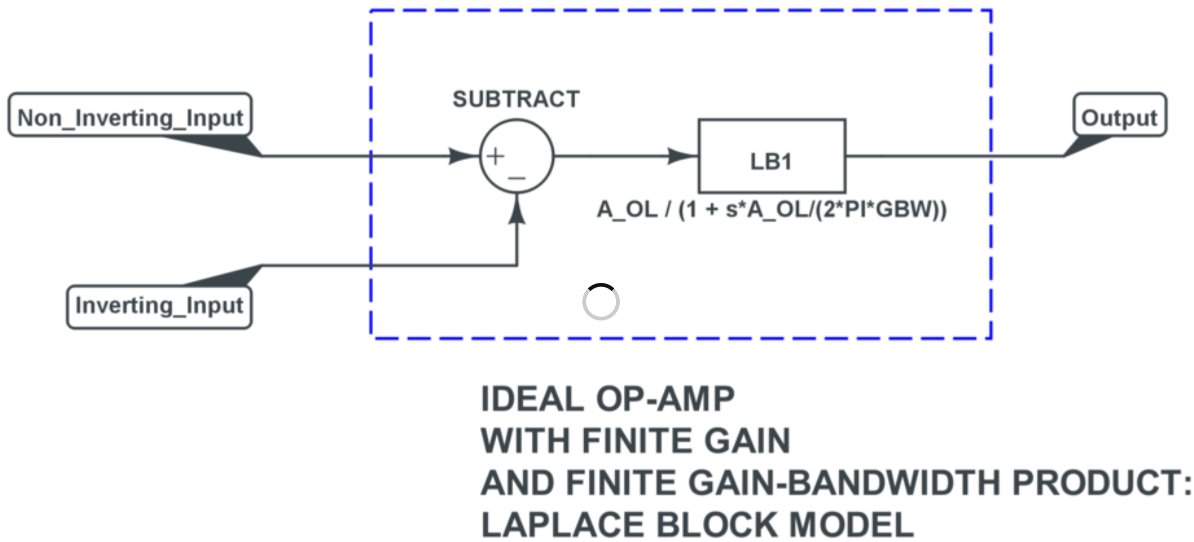
Ideal Op-Amp with Finite Gain: Laplace Block Model
circuitlab.com/ctq3jgk8rqewh

Edit - Simulate

An even more useful model involves having a finite gain-bandwidth product GBW . This is modeled as having a finite gain A_{OL} at DC, with a one-pole low-pass filter with a corner frequency $f_c = \frac{\mathrm{GBW}}{A_{\mathrm{OL}}}$. The low-pass component has transfer function $G_{\mathrm{lpf}}(s) = \frac{1}{1+\frac{s}{\omega}}$, where $\omega = 2\pi f_c$. Combining the gain and low-pass gives us:

$$G(s) = \frac{A_{\mathrm{OL}}}{1 + s\left(\frac{A_{\mathrm{OL}}}{2\pi \mathrm{GBW}}\right)}$$

and can be implemented in CircuitLab as shown:



Ideal Op-Amp with Finite Gain and Gain-Bandwidth Product: Laplace Block Model
circuitlab.com/can8pyj7dxf2b
Edit - Simulate

We'll use this model in the upcoming application sections to algebraically solve closed-loop feedback examples.

What's Next

How useful is it to have an amplifier with really enormous (ideally infinite!) gain? By itself, not so much. In this section, we've examined the open-loop behavior, and the most useful outcome is a mediocre analog voltage comparator.

But once we build a circuit around the ideal op-amp, we can "close the loop" and tame the wildly huge amplification into something we can design and control with **closed-loop feedback**. It turns out that having a subtract-and-multiply-by-infinity component is an almost magically useful building block for a wide range of analog signal processing needs. We'll explore these in the next several sections, starting with one of the simplest: the Op-Amp Voltage Buffer.