Design of the ISS-Bioreactor

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This is a technical report for ISS-Bioreactor project at Boise State University.

1. INTRODUCTION

We design an input PWM filter to create the desired 0−10V sine wave at 90Hz that the controller expects from a carrier PWM signal at a carrier frequency 36.6kHz.

2. THEORETICAL ANALYSIS

We perform basic PWM filter design to generate the driving $0-10$V sine wave for the Physik Instrumente (PI)'s piezoelectric actuator [1].

2.1. Basic Lowpass Filter Design

Consider the first-order filter sketched in Figure 1 with a sinusoidal driving voltage. The current i over the capacitor is $i = C\frac{\mathrm{d}v_c}{\mathrm{d}t}$. KVL around the circuit gives

$$RC\frac{\mathrm{d}v_c}{\mathrm{d}t} + v_c = v_{\mathrm{sig}} = A\cos(\omega t) \tag{1}$$

This differential equation has the transfer function

$$G(s) = \frac{1}{RCs+1}.$$

The steady-state solution of the differential equation (1) is obtained as

$$\begin{aligned} v_c(t) &= A\,|G(j\omega)|\cos(\omega t + \angle G(j\omega)) \\ &= \frac{A}{\sqrt{1+\omega^2R^2C^2}}\cos(\omega t - \arctan(\omega RC)) \end{aligned} \tag{2}$$

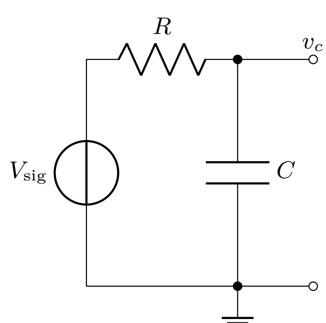

FIG. 1: A first-order low-pass filter circuit.

Since we do not want our signal to be attenuated by the low-pass filter, we must choose the values of R and C such that $\omega RC \ll 1$ or $2\pi RC \ll 1/f$.

Unfortunately, our actual input from the microcontroller is not a pure sine wave, rather a PWM signal. Therefore, we also need the value of $2\pi RC$ to be large so that it attenuates the high frequencies present in the PWM signal. This is difficult to achieve with just a single low-pass filter. Hence, we design a second-order low-pass filter in the next subsection.

2.2. Second-Order LPF Design

The second-order filter is implemented in the first part of the circuit presented in Figure 2. The second part of this circuit amplifies the sine wave extracted from its PWM modulation from $0-3.3$V to $0-10$V. We analyze the circuit so as to figure out the values of the various resistances and capacitances.

We start by analyzing how the input voltage, v_i, to the non-inverting amplifier, responds to Teensy-generated PWM voltage, v_t. This is a $0-3.3$V PWM signal to be filtered to extract the modulated sine wave. Thanks to the buffer op-amp, the transfer function from the Teensy input v_t to the input v_i to the non-inverting amplifier op-amp is given by

$$\frac{V_i(s)}{V_t(s)} = H(s) = \frac{1}{R^2C^2s^2 + 2RCs + 1}. \tag{3}$$

This is a fully-damped transfer function with poles at $s_{1,2} = -1/RC$. In other words, the cut-off frequency of this filter is at $f = \frac{1}{2\pi RC}$ with a roll-off of 40dB per decade. Contrast this to the first-order filter of the previous subsection where the roll-off was 20dB per decade. The greater roll-off rate allows us be able to select $2\pi RC \ll 1/f$ while simultaneously achieving excellent high-frequency attenuation.

Our desired signal is a 90Hz sinusoidal. We do not want to lose this signal, i.e., we want our transfer function $H(s)$ to approximately have a unity gain at this frequency. We set $2\pi RC \leq 1/900$ or $RC \leq 1.768\times10^{-4}\Omega\,\mathrm{F}$ to satisfy this requirement. Using some standard values of the resistance $R = 1\mathrm{k}\Omega$ and the capacitance $C = 104\mathrm{nF}$, we obtain $RC = 1.04\times10^{-4}$, achieving the desired goal. The attenuation at Teensy's PWM carrier frequency of 36.6kHz is found by

$$20\log_{10}\{|H(j2\pi 36600)|\} = -55.163\,\mathrm{dB}.$$

Lastly, we want to amplify the input voltage v_i thrice in order to hit the $0-10$V mark. The gain of the

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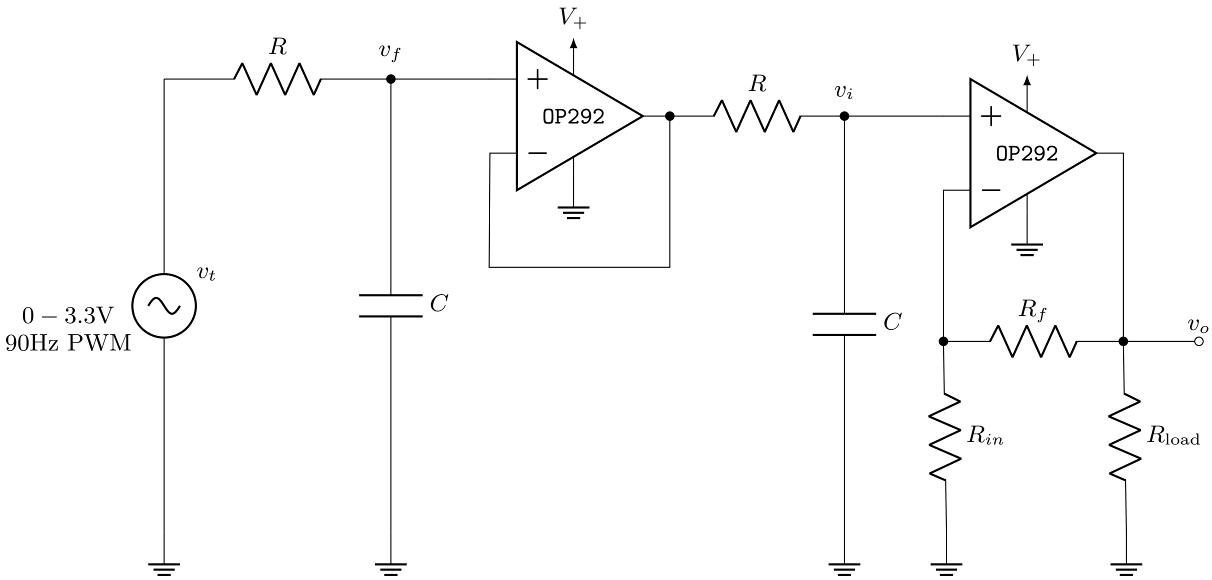

FIG. 2: The signal generator circuit.

non-inverting amplifier is $k = 1 + R_f/R_{in}$. We choose $R_f = 2\text{k}\Omega$ and $R_{in} = 1\text{k}\Omega$ to achieve $k = 3$. The high gain bandwidth product of the `OP292` op-amp is read from its datasheet to be GBP = 4MHz. Hence the transfer function from the input voltage v_i to the output voltage v_o that will be applied to the PI controller is approximately given by

$$\frac{V_o(s)}{V_i(s)} = \frac{k}{\frac{k}{2\pi\text{GBP}}s + 1} \approx \frac{3}{1.194 \times 10^{-7}s + 1},$$

which will have a firm unity gain at our desired oscillation frequency of 90Hz.

2.3. Sallen-Key Architecture

Another well-known architecture that works well for this sort of problem is the Sallen-Key low-pass filter. This setup replaces the two RC+op-amp combination whose output enters the amplifier as depicted in Figure 3.

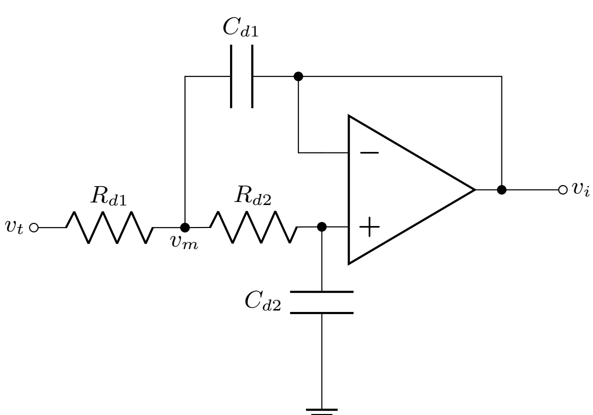

FIG. 3: Sallen Key (second-order) low-pass filter.

To solve this circuit, assume an ideal op-amp model so that the inputs of the op-amp both has potential v_i. Let the current i flowing over R_{d1} split into i_1 and i_2, the former flowing into C_{d1} and the latter into C_{d2} through R_{d2}. KCL gives

$$\begin{aligned} i &= i_1 + i_2 = \frac{1}{R_{d1}}(v_t - v_m), \\ i_1 &= C_1\frac{\mathrm{d}(v_m - v_i)}{\mathrm{d}t}, \\ i_2 &= C_2\frac{\mathrm{d}v_i}{\mathrm{d}t}. \end{aligned}$$

KVL around the bottom loop gives $v_m = R_{d2}i_2 + v_i = R_{d2}C_{d2}\frac{\mathrm{d}v_i}{\mathrm{d}t} + v_i$. Plugging this into the second equation gives $i_1 = R_{d2}C_{d1}C_{d2}\frac{\mathrm{d}^2v_i}{\mathrm{d}t^2}$. The first and third equations above combine with this expression for i_1 to yield

$$R_{d1}R_{d2}C_{d1}C_{d2}\ddot{v}_i + (R_{d1} + R_{d2})C_{d2}\dot{v}_i + v_i = v_t. \quad (4)$$

We will take $R_{d1} = R_{d2} = R_d$ and $C_{d1} = C_{d2} = C_d$ so that the transfer function for this linear differential equation is observed to coincide with that of equation (3) of Section 2.2.

$$H(s) = \frac{V_i(s)}{V_t(s)} = \frac{1}{R_d^2C_d^2s^2 + 2R_dC_ds + 1}.$$

Both theory and simulation shows that some good values for the resistances R_d and the capacitances C_d are $R_d = 47\text{k}\Omega$ and $C_d = 1\text{nF}$, with a cutoff frequency of $f \approx 3386\text{Hz}$. There is a range of resistance and capacitance values that work well around these nominal values.

Remark 1. *This is the filter to be implemented in the final design.*

3. RESULTS

We provide extensive simulation and experimental data and their interpretation, supporting that the proposed analog signal generator works as intended.

3.1. Simulation

We perform a realistic simulation with a second-order low-pass filter we came up with in Section 2.2. One of the important aspects of this design is the selection of the op-amp. In order to keep the common-mode voltage at 0V we choose the op-amps as CMOS type. One such op-amp is `OP292` [2], which is used in this simulation.

The PWM signal generated by Teensy [3] is simulated exactly with a carrier frequency of 36.6kHz modulating the signal

$$V_{\text{pwm}} = {}^{3.3}/_{2} + {}^{3.3}/_{2} \sin(2\pi 90 t).$$

Finally, the impedance of the load (PI's controller) is read off from its datasheet and inserted as a 100kΩ resistance. The circuit that is simulated using LTSpice [4] is presented in Figure 4.

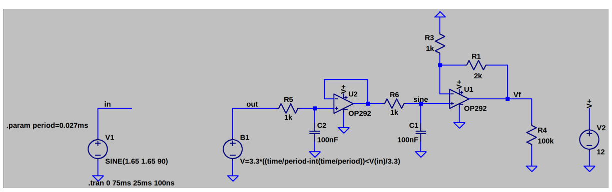

FIG. 4: The signal generator circuit in LTSpice

The simulation generates the relevant voltage responses, provided in Figure 5. The top plot shows the PWM signal generated by Teensy modulating a sine-wave at 90Hz frequency. The individual plots in the middle show the output of the first (cyan) and the second (purple) RC low-pass filters (v_f and v_i, respectively) that extract the modulated signal from its PWM representation. Finally, the last plot shows the amplified signal (gain = 3) through the op-amp `OP292`. This signal is ready to be sent to the PI controller.

The performance of the Sallen-Key architecture from Section 2.3 is shown in Figure 6, where the values for the resistances were taken to be $R_d = 68\text{k}\Omega$ and capacitances to be $C_d = 1\text{nF}$.

3.2. Experiment

The design is implemented on a simple setup on my table top by lighting two red LEDs, first one using the filtered PWM and the second using the amplified signal, both obtained using the circuit in Section 2.3.

Figure 7 shows the response of the second-order Sallen-Key LPF observed through an oscilloscope. This is the output of the signal generator in response to the Teensy generated 90Hz $0-3.3$V PWM signal modulating the desired sine wave. This response is satisfactory and should drive the PI controller without any issues.

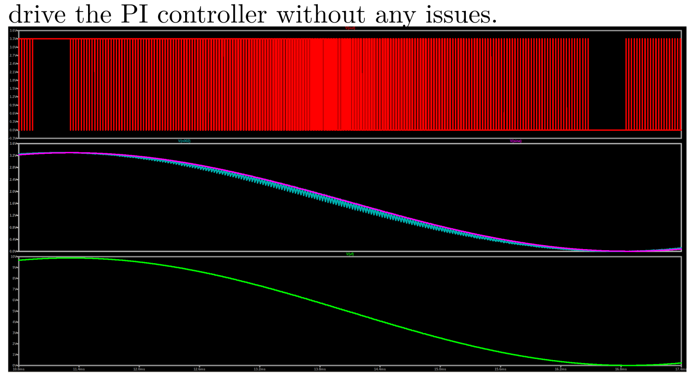

FIG. 5: The response from the simulation for one full period.

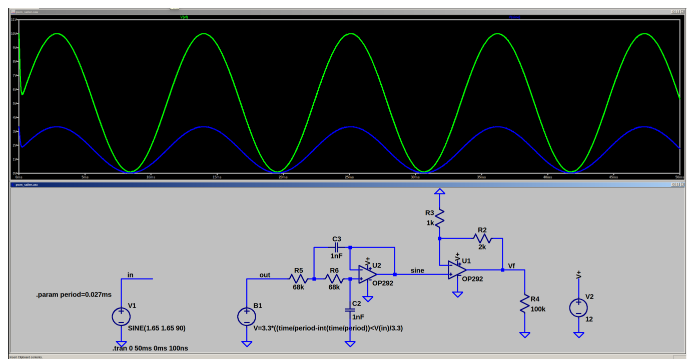

FIG. 6: The response of the Sallen-Key architecture.

4. DISCUSSION AND CONCLUSIONS

Let's go to ISS!

5. ACKNOWLEDGMENT

The author thanks Drs. John Chiasson and Vishal Saxena of Electrical and Computer Engineering at Boise State University and University of Delaware, respectively, for their invaluable insights into the practical implementation of the signal generator.

[1] Physik Instrumente, *E-610*, URL https://www.physikinstrumente.com/en/products/controllers-and-drivers/nanopositioning-piezo-controllers/e-610-piezo-amplifier-controller-601000.
[2] Analog Devices, *OP 292*, URL https://www.analog.com/en/products/op292.html.
[3] PJRC, *Teensy*, URL https://www.pjrc.com/teensy/.

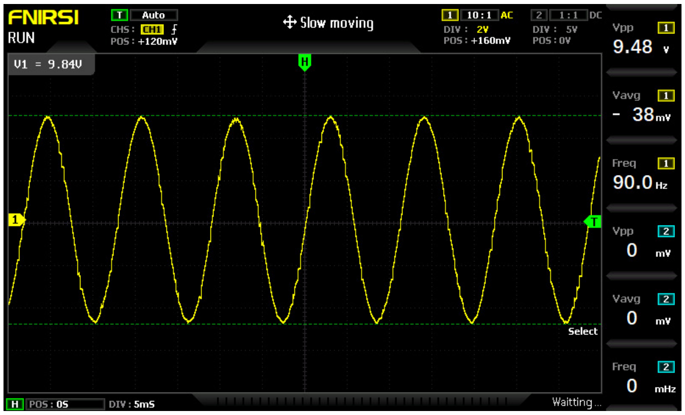

FIG. 7: The response of the second-order Sallen-Key filter

[4] Analog Devices, *LTSpice*, URL https://www.analog.com/en/design-center/design-tools-and-calculators/ltspice-simulator.html.