Problem 2 Chapter 5

Torque Versus Slip for the Induction Motor

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We now assume rotor loops have **nonzero** inductance L_R.
The equations for the rotor currents are $(\theta_S - \theta_R = (\omega_S - \omega_R)\, t\ )$

$$L_R \frac{di_{Ra}}{dt} + R_R i_{Ra} = \xi_{Ra}, \quad \xi_{Ra} = +\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g}(\omega_S - \omega_R) \sin\left((\omega_S - \omega_R)t\right)$$

$$L_R \frac{di_{Rb}}{dt} + R_R i_{Rb} = \xi_{Rb}, \quad \xi_{Rb} = -\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g}(\omega_S - \omega_R) \cos\left((\omega_S - \omega_R)t\right).$$

The **stable linear time-invariant** system

$$L\frac{di}{dt} + Ri = A\cos(\omega t + \phi)$$

has the **steady-state** solution

$$i_{SS}(t) = |G(j\omega)|\, A \cos\Big(\omega t + \phi + \angle G(j\omega)\Big), \quad G(j\omega) = \frac{1}{R + j\omega L}.$$

The **steady-state** solution for the currents are then

$$i_{RaSS} = \frac{(\omega_S - \omega_R)}{\sqrt{R_R^2 + (\omega_S - \omega_R)^2 L_R^2}} \frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g} \times \sin\left((\omega_S - \omega_R)t - \tan^{-1}\left(\frac{(\omega_S - \omega_R)L_R}{R_R}\right)\right)$$

$$i_{RbSS} = -\frac{(\omega_S - \omega_R)}{\sqrt{R_R^2 + (\omega_S - \omega_R)^2 L_R^2}} \frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g} \times \cos\left((\omega_S - \omega_R)t - \tan^{-1}\left(\frac{(\omega_S - \omega_R)L_R}{R_R}\right)\right).$$

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The torque on rotor phase a is given by

$$\begin{aligned}\tau_{Ra} &= \frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g} i_{RaSS} \sin\Big((\omega_S - \omega_R)t\Big)\\ &= \frac{(\omega_S - \omega_R)}{\sqrt{R_R^2 + (\omega_S - \omega_R)^2 L_R^2}} \left(\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g}\right)^2 \times \\ &\quad \sin\left((\omega_S - \omega_R)t - \tan^{-1}\left(\frac{(\omega_S - \omega_R)L_R}{R_R}\right)\right) \sin\Big((\omega_S - \omega_R)t\Big).\end{aligned}$$

Similarly, the torque on rotor phase b is then

$$\begin{aligned}\tau_{Rb} &= -\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g} i_{RbSS} \cos\Big((\omega_S - \omega_R)t\Big)\\ &= \frac{(\omega_S - \omega_R)}{\sqrt{R_R^2 + (\omega_S - \omega_R)^2 L_R^2}} \left(\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g}\right)^2 \times \\ &\quad \cos\left((\omega_S - \omega_R)t - \tan^{-1}\left(\frac{(\omega_S - \omega_R)L_R}{R_R}\right)\right) \cos\Big((\omega_S - \omega_R)t\Big).\end{aligned}$$

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Combining the above results, the **total torque** is given by

$$
\begin{aligned}
\tau &= \tau_{Ra} + \tau_{Rb} \\
&= \left(\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g}\right)^2 \frac{(\omega_S - \omega_R)}{\sqrt{R_R^2 + (\omega_S - \omega_R)^2 L_R^2}} \cos\left(-\tan^{-1}\left(\frac{(\omega_S - \omega_R) L_R}{R_R}\right)\right) \\
&= \left(\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g}\right)^2 \frac{(\omega_S - \omega_R)}{\sqrt{R_R^2 + (\omega_S - \omega_R)^2 L_R^2}} \frac{1}{\sqrt{\left(\frac{(\omega_S - \omega_R) L_R}{R_R}\right)^2 + 1}} \\
&= \left(\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g}\right)^2 \frac{1}{L_R} \frac{(\omega_S - \omega_R) L_R / R_R}{((\omega_S - \omega_R) L_R / R_R)^2 + 1} \\
&= \left(\frac{\mu_0 \ell_1 \ell_2 N_S I_S}{2g}\right)^2 \frac{1}{L_R} \frac{1}{2} \frac{2}{S/s_p + s_p/S}
\end{aligned}
$$

where

$$
S \triangleq \frac{\omega_S - \omega_R}{\omega_S}, \quad s_p \triangleq \frac{R_R}{\omega_S L_R}.
$$

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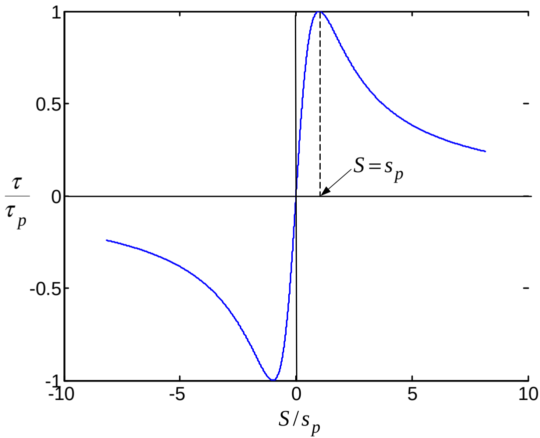

- $S \triangleq \frac{\omega_S - \omega_R}{\omega_S}$ is the **normalized slip.** $S_p \triangleq \dfrac{R_R}{\sigma \omega_S L_R}$ is the **pull out slip**.
- $s_p \triangleq \dfrac{R_R}{\omega_S L_R} = \sigma S_p$ where σ is the **leakage factor**.