**ECE 697 Modeling and High-Performance Control of Electric Machines**
**HW 3 Solutions**
**Spring 2022**

**Problem 1** 1

**Problem 2** 2

**Problem 3** 3

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**Problem 6** *High-Gain Current Command Control*

**(a)** Redraw the block diagram to obtain the equivalent block diagram shown in Figure 1. The inner loop

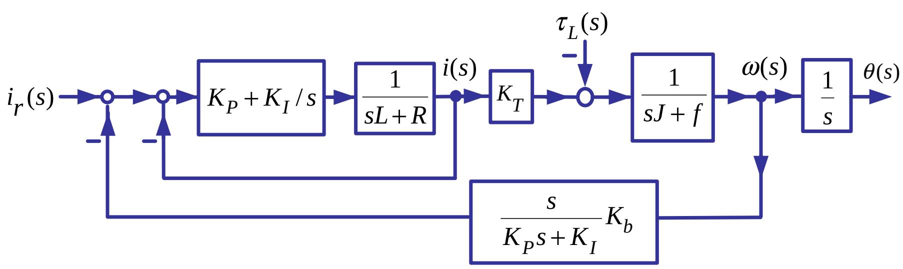


Figure 1: Equivalent block diagram for the PI current controller.

simplifies to

$$\frac{\dfrac{K_P s + K_I}{s}\dfrac{1}{sL+R}}{1+\dfrac{K_P s + K_I}{s}\dfrac{1}{sL+R}} = \frac{K_P s + K_I}{s(sL+R)+K_P s + K_I}.$$

Then

$$\begin{aligned}
\frac{i(s)}{i_r(s)} &= \frac{\dfrac{K_P s + K_I}{s(sL+R)+K_P s+K_I}}{1+\dfrac{s}{K_P s+K_I}K_b\dfrac{K_T}{sJ+f}\dfrac{K_P s+K_I}{s(sL+R)+K_P s+K_I}} \\
&= \frac{(sJ+f)(K_P s+K_I)}{(sJ+f)(s^2L+(R+K_P)s+K_I)+sK_bK_T} \\
&= \frac{(sJ+f)(K_P s+K_I)}{JLs^3+(J(R+K_P)+Lf)s^2+(JK_I+K_TK_b+f(R+K_P))s+fK_I}
\end{aligned}$$

**(b)** Let $K_P = k$, $K_I = k^2$ and divide through by $k^2$ to obtain

$$\frac{i(s)}{i_r(s)} = \frac{(sJ+f)(s/k+1)}{\dfrac{JL}{k^2}s^3+\left(\dfrac{J}{k}+\dfrac{RJ+Lf}{k^2}\right)s^2+\left(J+\dfrac{K_TK_b+Rf}{k^2}+\dfrac{f}{k}\right)s+f}.$$

Let $k \to \infty$ to finally get

$$\frac{i(s)}{i_r(s)} = \frac{(sJ+f)(s/k+1)}{\frac{JL}{k^2}s^3 + \left(\frac{J}{k} + \frac{RJ+Lf}{k^2}\right)s^2 + \left(J + \frac{K_T K_b + Rf}{k^2} + \frac{f}{k}\right)s + f} \to 1.$$

The Routh-Hurwitz criterion can be used to show this is stable for all $k$ large enough. To do so the Routh table for $a_3 s^3 + a_2 s^2 + a_1 s + a_0$ is constructed as follows.

$$\begin{array}{ccc} s^3 & a_3 & a_1 \\ s^2 & a_2 & a_0 \\ s & (a_2 a_1 - a_0 a_3)/a_2 & \\ s^0 & a_0 & \end{array}$$

For large $k$ we have

$$a_3 = \frac{JL}{k^2}, a_2 \approx \frac{J}{k}, a_1 \approx J, a_0 = f$$

Then $a_3 > 0, a_2 > 0, a_1 > 0, a_0 > 0$ and

$$a_2 a_1 - a_0 a_3 \geq \frac{J^2}{k} - \frac{JLf}{k^2} = \frac{J^2}{k}(1 - \frac{Lf/J}{k}).$$

Thus the PI current controller is stable for all $k > \dfrac{J}{Lf}$ or

$$kR > \frac{R/L}{f/J}.$$

Typically the electrical time constant $L/R$ is much less than the mechanical time constant $J/f$ making the ratio $\dfrac{R/L}{f/J}$ large.

**(c)** The transfer function from $i_r$ to $\omega$ is

$$\begin{aligned}
\frac{\omega(s)}{i_r(s)} &= \frac{\frac{K_P s + K_I}{s(sL+R) + K_P s + K_I} \frac{K_T}{sJ+f}}{1 + \frac{K_P s + K_I}{s(sL+R) + K_P s + K_I} \frac{K_T}{sJ+f} \frac{K_b s}{K_P s + K_I}} \\
&= \frac{K_T\left(K_P s + K_I\right)}{(s(sL+R) + K_P s + K_I)(sJ+f) + sK_T K_b} \\
&= \frac{K_T\left(K_P s + K_I\right)}{JLs^3 + \left(JK_P + RJ + Lf\right)s^2 + \left(JK_I + K_T K_b + Rf + K_P f\right)s + fK_I}.
\end{aligned}$$

Setting $K_P = k$, $K_I = k^2$ and dividing through by $k^2$ results in

$$\frac{\omega(s)}{i_r(s)} = \frac{K_T\left(ks+1\right)}{\frac{JL}{k^2}s^3 + \left(\frac{J}{k} + \frac{RJ+Lf}{k^2}\right)s^2 + \left(J + \frac{K_T K_b + Rf}{k^2} + \frac{f}{k}\right)s + f} \to \frac{K_T}{sJ+f} \quad \text{as} \quad k \to \infty.$$

**(d)** See the simulation files.