Switched Reluctance Motors

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1.1 Operation of the Switched Reluctance Motor

Figure 1.1 shows a perspective view of a three phase switched reluctance (SR) motor. Figure 1.1(a) shows the stator where the winding of phase a is drawn. Figure 1.1(b) shows the rotor which has no windings. The stator and rotor are both made of laminated sheets of soft iron. As the figure shows, the SR motor has teeth both on the rotor and stator making it a doubly salient machine. The particular machine illustrated in Figure 1.1 has six stator teeth and four rotor teeth and is referred to as a $S6 - R4$ SR motor. Each phase consists of two coils connected in series and wound on stator teeth which are on diametrically opposite sides of the stator. The two coils can be considered a coupling of two phases to make a single phase.

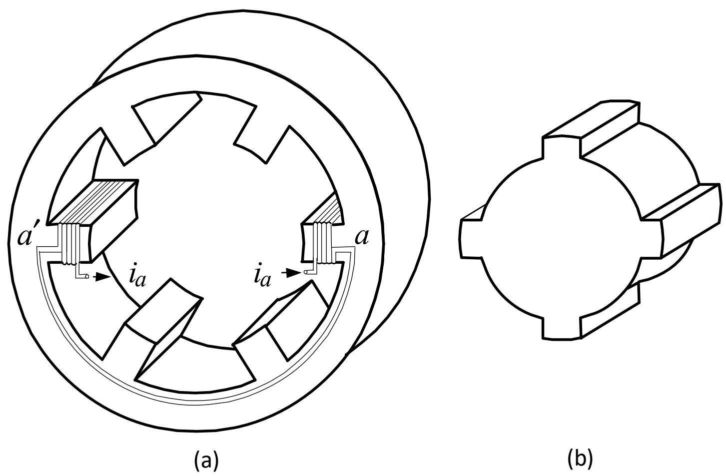

FIGURE 1.1. Perspective view of an SR machine. (a) The stator with the winding of phase a shown. (b) The rotor.

A zero position of the rotor is taken to be the position in which the rotor and stator teeth are completely *unaligned*. That is, as shown in the left-hand side of Figure 1.2(a), at the zero position, the midpoint between two rotor teeth is lined up with the midpoint of the teeth of stator phase a. To understand the operation of this motor, consider the simple case where current is only in phase a so that $i_a = i_0$ and $i_b = i_c = 0$. Then the current in phase a magnetizes the stator iron creating a magnetic field. This field goes across the air gap and magnetizes the rotor to attract the rotor iron to the stator iron. If the rotor is at $\theta_R = 30°$ as shown in Figure 1.2(b), the magnetic attractive force between the stator and rotor iron has a component tangent to the rotor as well as normal (perpendicular) to the rotor. This tangential component produces torque to cause the rotor to rotate in the *counterclockwise* direction.

If the rotor is at $\theta_R = 0°$ as in Figure 1.2(a), the symmetry of this position with respect to stator phase a shows that the attractive force is only normal to the rotor and thus no torque is produced. Figures 1.2(c)(d) shows the rotor with $\theta_R = 45°$ and $\theta_R = 60°$. At $\theta_R = 45°$, a set of rotor teeth are completely *aligned* with a set of stator teeth and the symmetry of this position shows there is no tangential force and therefore no torque. When $\theta_R = 60°$, the torque has the same magnitude as when $\theta_R = 30°$, but it is in the *clockwise* direction. Note that this mechanism of producing torque by magnetizing the rotor's soft iron is such that $i_a = \pm i_0$ produces the same torque.

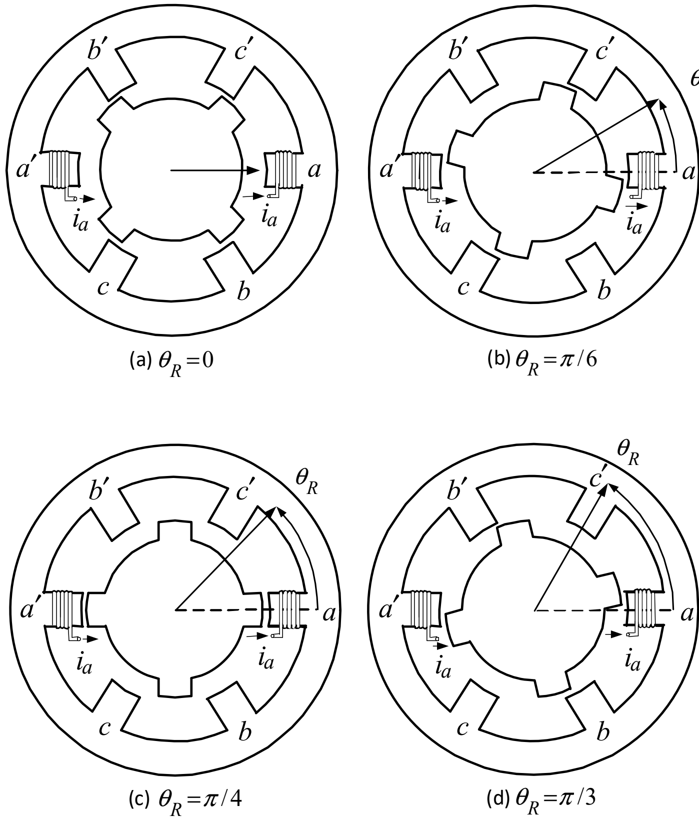

FIGURE 1.2. Switched reluctance motor with the rotor at (a) $\theta = 0°$ (b) $\theta = 30°$ (c) $\theta = 45°$ (d) $\theta = 60°$.

The torque produced by phase a as a function of rotor position θ_R with i_a constant is given in Figure 1.3. This data is collected experimentally.

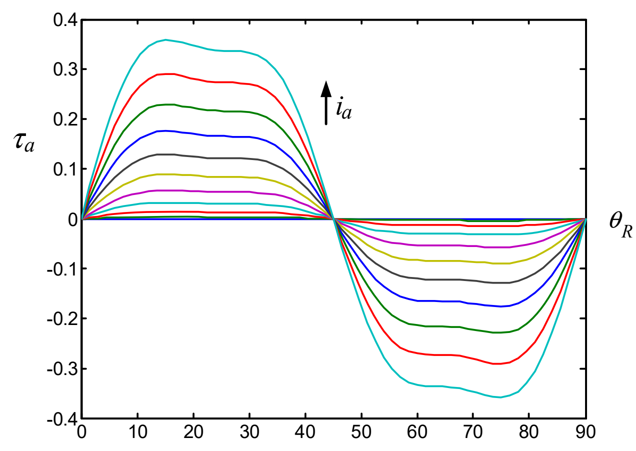

FIGURE 1.3. Phase a torque as a function of θ (from [1]).

The data of Figure 1.3 is used to solve for i_a as a function of θ_R and τ_a as displayed in Figure 1.4.

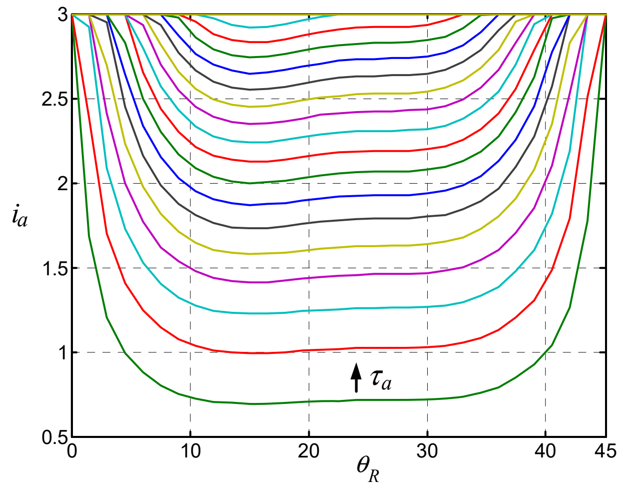

FIGURE 1.4. Current required in phase a to produce a given torque at a given rotor position.

We write

$$i_a = g(n_R\theta_R, \tau_a)$$

to denote the current in phase a as a function of the rotor position and the required torque. This function would typically be implemented as a look up table for real time use.

Stepping Operation

Figure 1.5 shows the torque for each of the three phases with the current in each phase held constant at i_0.

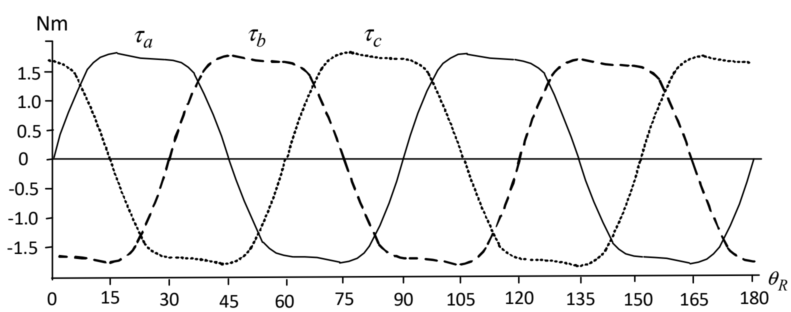

FIGURE 1.5. Torque vs rotor angle for each phase with $i_a = i_b = i_c = i_0 > 0$.

With n_{ph} the number of stator phases and n_R the number of rotor teeth, define an angle step size as $\theta_s \triangleq \dfrac{2\pi}{n_{ph}n_R}$ ($= 30°$ in Figure 1.5). Then with $i_a = i_0, i_b = i_c = 0$, at the rotor positions $\theta = \dfrac{\pi}{n_R} \pm n\dfrac{2\pi}{n_R}$($=$ $45° \pm n90°$ in Figure 1.5) the torque τ is zero *and* $\partial\tau/\partial\theta < 0$. That is, these angular positions are *stable* equilibrium points[1] and these points are exactly when the stator teeth of phase a are completely aligned with a set of rotor teeth. Similarly, with $i_a = 0, i_b = i_0, i_c = 0$ the angles $\theta = \dfrac{\pi}{n_R} + \theta_s \pm n\dfrac{2\pi}{n_R}$($= 75° \pm n90°$

[1]Simply meaning that if the rotor is slightly perturbed from these points, the torque acting on the rotor pushes to restore it back to this position.

in Figure 1.5) are stable equilibrium points and correspond to the stator teeth of phase b being completely aligned with a set of rotor teeth. Finally, Figure 1.5 shows that with $i_a = 0, i_b = 0, i_c = i_0$ the stable equilibrium points are $\theta = \dfrac{\pi}{n_R} + 2\theta_s \pm n\dfrac{2\pi}{n_R}$ ($= 105° \pm n90°$ in Figure 1.5) and correspond to the stator teeth of phase c being completely aligned with a set of rotor teeth.

Consider at time $t = 0$ we have $\theta_R(0) = 45°$ with the currents are given by $i_a(0) = i_0, i_b(0) = 0, i_c(0) = 0$ and the rotor is aligned with phase a as shown in Figure 1.2(c). Then set $i_a(t) = 0, i_b(t) = i_0, i_c(t) = 0$ for $0 < t < T$ and the rotor angle $\theta_R(t) \to \theta_R(0)+\theta_s$ where T is chosen long enough so that the transient oscillations die out. Then set $i_a(t) = 0, i_b(t) = 0, i_c(t) = i_0$ for $T < t < 2T$ and the rotor angle $\theta_R(t) \to \theta_R(0) + 2\theta_s$. Repeating this sequencing of the phase currents (i_a, i_b, i_c) in the order $\{(i_0, 0, 0), (0, i_0, 0), (0, 0, i_0)\}$ every T seconds results in the rotor "stepping" an angular distance θ_s every T seconds.

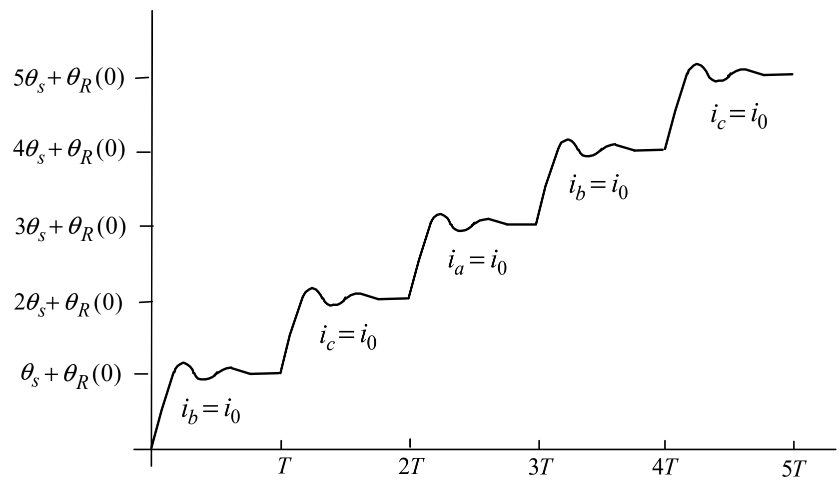

FIGURE 1.6. Stepping action of a SR motor. Only the labeled current is energized; the other two are set to 0.

1.2 Phase Flux Linkage

The flux linkage in phase a is a function of both the current i_a in the phase and the rotor position θ_R. To measure the flux linkage in the phase winding, one holds the rotor angle fixed (locked) at some $\theta_R = \theta_{R0}$ and applies a voltage u_a to it to bring the current up from $i_a(0) = 0$ to $i_a(t)$. By Faraday's law the voltage induced in the phase winding is $-d\lambda_a/dt$ and Kirchhoff's voltage law gives $-d\lambda_a/dt + u_a - Ri_a = 0$ so the flux linkage is obtained from

$$\lambda_a(t) = \int_0^t e_a(t')dt'$$

where $e_a \triangleq u_a - Ri_a$. At the same time, the current $i_a(t)$ is measured so that the data $(\lambda_a(t), i_a(t))$ may be used to plot λ_a versus i_a as shown in Figure 1.7. This is referred to as a *magnetization* curve. As the current in the phase winding increases, the magnetized iron saturates. This is reflected in the measured flux linkage curve shown in Figure 1.7. Specifically, it is seen at first that the flux linkage increases rapidly as a function of the current until it gets above above some value (denoted as i_{sat} in the figure) where the rate of increase of the flux linkage decreases dramatically.

If the flux linkage versus phase current is measured for a set of rotor angles θ_R in the range $0 \le \theta_R \le \pi/4$, then the set of flux curves shown in Figure 1.8 results. The flux in phase a is a function of θ_R and i_a, being denoted as $\lambda_a = \lambda(n_R\theta_R, i_a)$ where n_R is the number of rotor teeth. By symmetry, this flux linkage is seen to be periodic in θ_R with period $2\pi/n_R$. Figure 1.8 shows that at $\theta_R = 0$ (the rotor and stator are completely unaligned), the flux linkage is essentially linear as a function of i_a. In contrast, at $\theta_R = \pi/4$ (the rotor and stator are completely aligned), the flux linkage saturates rapidly as a function of current and is a nonlinear function i_a.

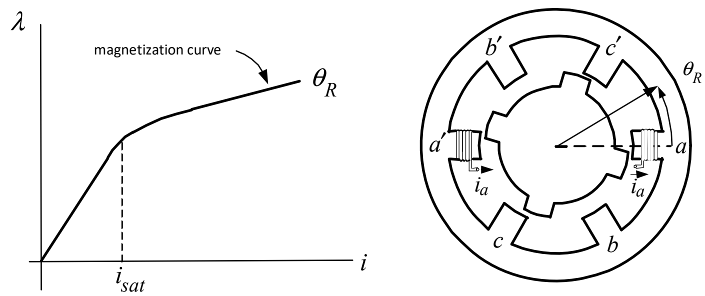

FIGURE 1.7. Flux linkage λ_a plotted versus i_a for $\theta = \theta_0$ (adapted from Miller [2]).

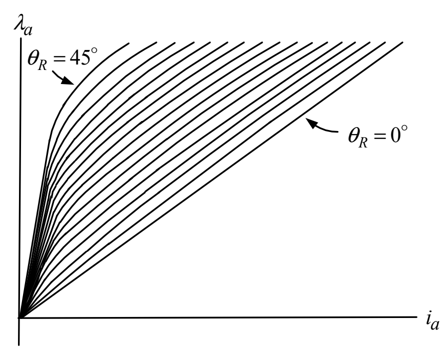

FIGURE 1.8. Flux linkage $\lambda_a(n_R\theta_R, i_a)$ as a function of i_a for various rotor angles θ_R.

As described above the flux linkage function $\lambda(n_R\theta_R, i_a)$ is experimentally determined. Work has been done to find a parametric representation of this function [1][3]. This experimentally determined flux linkage function is found with the other phase currents set to zero, i.e., $i_b = i_c = 0$. It turns out that the currents i_b, i_c in these two phases produce only a small amount of flux linkage in phase a. Consequently, for modeling purposes, the mutual flux between the phases is taken to be zero. With n_{ph} denoting the number of stator phases, stator phase b is offset from phase a by $2\pi/n_{ph}$ radians and phase c is offset by $2\pi/n_{ph}$ from phase b. In addition the rotor is invariant under a rotation of π/n_R radians so that the flux functions are offset by only $\theta_s \triangleq \dfrac{2\pi}{n_{ph}n_R}$(= 30° in Figure 1.2). Specifically,

$$\begin{aligned}
\lambda_a(n_R\theta_R, i_a) &= \lambda\left(n_R\theta_R, i_a\right) \\
\lambda_b(n_R\theta_R, i_b) &= \lambda\left(n_R(\theta_R - \theta_s), i_b\right) \\
\lambda_c(n_R\theta_R, i_c) &= \lambda\left(n_R(\theta_R - 2\theta_s), i_c\right).
\end{aligned} \tag{1.1}$$

The electrical equations of the motor are then

$$\begin{aligned}
\frac{d}{dt}\lambda_a(n_R\theta_R, i_a) &= -R_S i_a + u_a \\
\frac{d}{dt}\lambda_b(n_R\theta_R, i_b) &= -R_S i_b + u_b \\
\frac{d}{dt}\lambda_c(n_R\theta_R, i_c) &= -R_S i_c + u_c
\end{aligned}$$

1.3 Phase Torque

To complete the model an expression for the torque produced is now developed. The air gap between the rotor and stator is not uniform as is the case for an induction motor (or a synchronous motor with a sinusoidally wound rotor). Further an SR motor has no windings (or permanent magnet) on the rotor and the normal operation of the motor results in the stator iron being saturated. These facts result in there not being a simple analytical expression for the magnetic field produced by the phase windings[2]. In order to model the torque a general conservation of energy approach is used (see [2]). This approach is quite general and can accommodate the magnetic saturation in the phases. In order to develop a mathematical model of the SR torque from conservation of energy, the concepts of (magnetic) *field energy* and *co-energy* are required.

Field Energy

Consider the rotor locked at some angle θ_R, e.g., as shown in the right-hand side of Figure 1.9. The corresponding magnetization curve λ versus i with the rotor in this position is shown in the left-hand side of Figure 1.9. With the rotor locked at θ_R, the current in phase a is increased from $i_a = 0$ to $i_a = i$. During this time the induced emf at the terminals of the phase windings is $-d\lambda_a/dt$ so that the instantaneous power in the phase winding is

$$P_{\text{phase } a \text{ winding}} = -\frac{d\lambda_a}{dt} i_a.$$

As the current increases from 0 to i, the flux linkage also increases (i.e., $d\lambda_a/dt > 0$) so that the power in the phase is negative. This simply means the phase is absorbing power from the voltage source. Consequently, the electrical power provided by the voltage source is the just the negative of this power given by

$$P_{\text{electrical}} = e_a i_a = \frac{d\lambda_a}{dt} i_a \tag{1.2}$$

where $e_a = u_a - Ri_a$.

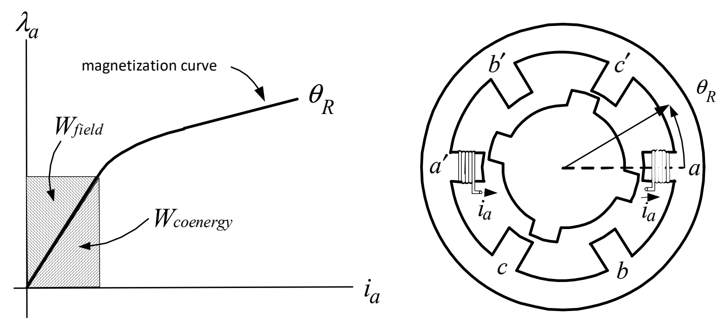

FIGURE 1.9. Flux λ_a plotted versus i_a with θ_R constant. The stored magnetic energy is the area under the i_a vs. λ_a magnetization curve (adapted from Miller [2]).

Let t denote the time that the current reaches i and let λ denote the corresponding flux linkage (see Figure 1.9). The total electrical energy $W_{\text{electrical}}$ supplied to the phase by the voltage source is

$$W_{\text{electrical}} = \int_0^t \frac{d\lambda_a(t')}{dt} i_a(t') dt'.$$

[2]Even if one had an analytical expression for the stator magnetic field, there are no windings on the rotor (as for an induction motor) to compute the torque. In the case of the pm synchronous motor, the magnetic field produced by the rotor magnet was sinusoidally distributed in θ and did *not* depend on the stator magnetic field. Consequently, the torque could be computed by finding the induced emf in the stator phases by the rotor magnet and then using conservation of energy. In the case of the SR motor, the magnetic field produced by the rotor came from it being magnetized by the stator magnetic field. As a result, there is not a simple analytical expression for the rotor magnetic field.

Further, with the rotor fixed during this procedure, no mechanical power is produced. Consequently, all of this energy goes into (is stored in) the magnetic field established in the stator/rotor iron and air gap. In other words, with the rotor locked at θ_R, $W_{\text{electrical}} = W_{\text{field}}$ so that

$$W_{\text{field}}(n_R\theta_R, \lambda) = \int_0^t i_a(t')\frac{d\lambda_a}{dt'}dt'\bigg|_{\theta_R \text{ held constant}} = \int_0^\lambda i_a d\lambda\bigg|_{\theta_R \text{ held constant}}$$

where a change of variables was used to get the final expression.[3] The energy W_{field} is therefore equal to the area under i versus λ as shown in Figure 1.9.

This derivation required the rotor locked at θ_R in order to conclude that $W_{\text{electrical}} = W_{\text{field}}$, i.e., this energy supplied by the power source was stored in the field energy as no mechanical work was done. Now consider the case in which the rotor is not locked and moves under the action of the magnetic torque. This is illustrated in Figure 1.10a where the left-hand side shows the rotor initially at θ_{R0} and the right-hand side side shows the rotor at θ_R. To make the rotor move from θ_{R0} to θ_R, the current in phase a is brought from 0 to some value i. That is, in the time interval from 0 to some t, the current in phase a is brought from 0 to i, the rotor goes from θ_{R0} to θ_R and the flux linkage goes from 0 to $\lambda_a = \lambda(n_R\theta_R, i)$. The electrical power put into the phase by the power source is still given by $P_{\text{electrical}} = i_a d\lambda_a/dt$ making the electrical energy supplied by the source during this move

$$W_{\text{electrical}} = \int_0^t i_a(t')\frac{d\lambda_a(t')}{dt}dt' = \int_0^\lambda i_a d\lambda_a.$$

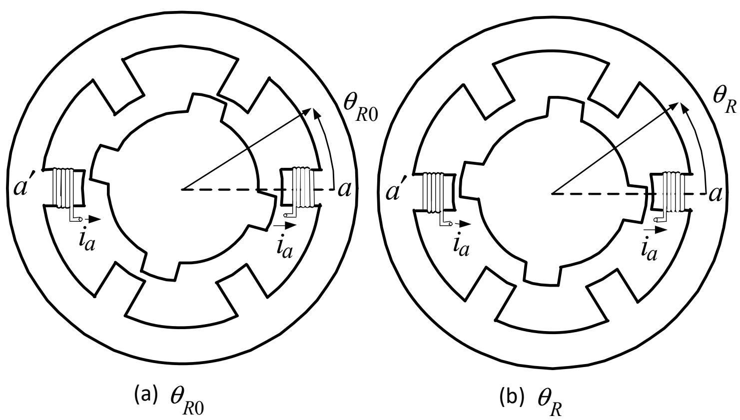

FIGURE 1.10. The rotor moves from θ_{R0} to θ_R as i_a is increased from 0 to i (adapted from [2]).

The energy $W_{\text{electrical}}$ is equal to the area under the i versus λ curve as the rotor moves which is the *sum* of the two cross hatched areas in Figure 1.11. With the rotor at θ_R, the area under the *magnetization curve* corresponding to θ_R (see Figure 1.11) is now the energy W_{field} stored in the magnetic field. In other words, if the rotor is at θ_R with the current i_a in the stator phase, then the magnetic field distribution in the SR motor (i.e., the rotor/stator iron and air gap) is the *same* as in Figure 1.9. Consequently, the stored energy W_{field} in the magnetic field in Figure 1.11 is the same as stored energy in Figure 1.9. To reiterate, if the rotor is at the angle θ_R with the current i_a in the phase a, then the magnetic energy W_{field} stored is the *same*

[3] The change of variables is computed as follows: Let $\lambda = f(t')$ denote the flux linkage as a function of time in the phase as the current is built up. Write the inverse function as $t' = g(\lambda)$ so that $\lambda = f(g(\lambda))$ and $\frac{df(t')}{dt'}\frac{dg(\lambda)}{d\lambda} = \frac{df(g(\lambda))}{d\lambda} = 1$. Make the change of variables $t' = g(\lambda), dt' = \frac{dg(\lambda)}{d\lambda}d\lambda$ where $\lambda_0 = f(t_0)$ to get $\int_0^{t_0}\frac{d\lambda}{dt'}i(t')dt' = \int_0^{t_0}\frac{df(t')}{dt'}i(t')dt' = \int_0^{\lambda_0}\frac{df(t')}{dt'}i(g(\lambda))\frac{dg(\lambda)}{d\lambda}d\lambda = \int_0^{\lambda_0}i(g(\lambda))d\lambda = \int_0^{\lambda_0}id\lambda$

independent of how the motor got to the angular position θ_R with the current i_a in the phase a winding, i.e., with or without movement of the rotor.

By conservation of energy, the difference between the two areas $W_{\text{electrical}} - W_{\text{field}}$ is the mechanical energy delivered.

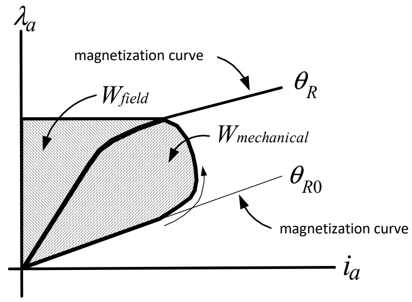

FIGURE 1.11. i vs λ as the phase current goes from 0 to i and the rotor goes from θ_{R0} to θ_R (adapted from [2]).

Co-energy and Torque

Consider the situation where the phase current is held *constant* with the rotor unlocked and it moves a small angular distance from θ_{R0} to θ_R $(> \theta_{R0})$ by the attractive magnetic force as indicated in Figure 1.12. The goal here is to compute the torque produced by this magnetic force. The change in electrical energy supplied $\Delta W_{\text{electrical}}$ by the power supply connected to the phase as the rotor moves from $\theta_{R0} = \theta_R(t_0)$ to $\theta_R = \theta_R(t)$ with the phase current held constant at i_a, is

$$\int_{t_0}^{t} i_a \frac{d\lambda_a}{dt'} dt' = \int_{\lambda_0}^{\lambda} i_a d\lambda_a$$

which is the area *abcd* in Figure 1.13. That is, $\Delta W_{\text{electrical}} = abcda$. Figure 1.13 shows that the change in the energy stored in the magnetic field ΔW_{field} is given by[4]

$$\Delta W_{\text{field}} = \int_{\substack{0 \\ \text{Along the } i \text{ vs. } \lambda \\ \text{curve for } \theta_R}}^{\lambda} i_a d\lambda_a - \int_{\substack{0 \\ \text{Along the } i \text{ vs. } \lambda \\ \text{curve for } \theta_{R0}}}^{\lambda_0} i_a d\lambda_a = obco - oado$$

By conservation of energy the change in energy $\Delta W_{\text{electrical}}$ supplied by the source goes into the change in magnetic field energy ΔW_{field} and into the mechanical energy produced $\Delta W_{\text{mechanical}} = \int_{\theta_{R0}}^{\theta_R} \tau d\omega_R = \tau \Delta\theta$ where we assume $\Delta\theta_R \triangleq \theta_R - \theta_{R0}$ is small enough that τ doesn't change as the rotor goes from θ_{R0} to θ_R. Energy conservation requires

$$\Delta W_{\text{electrical}} = \Delta W_{\text{field}} + \Delta W_{\text{mechanical}} = \Delta W_{\text{field}} + \tau\Delta\theta_R.$$

This mechanical energy produced is therefore

$$\Delta W_{\text{mechanical}} = \tau\Delta\theta_R = \Delta W_{\text{electrical}} - \Delta W_{\text{field}} = abcda - (obco - oado) = (abcda + oado) - obco = oabo.$$

[4] This expression is valid only if the phase current is held constant as the rotor moves from θ_{R0} to θ_R.

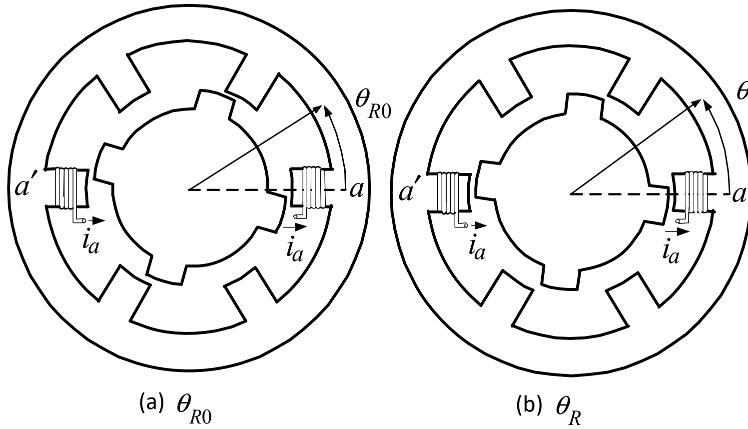

FIGURE 1.12. The motor shaft rotates from θ_{R0} to θ_R with the phase current held constant.

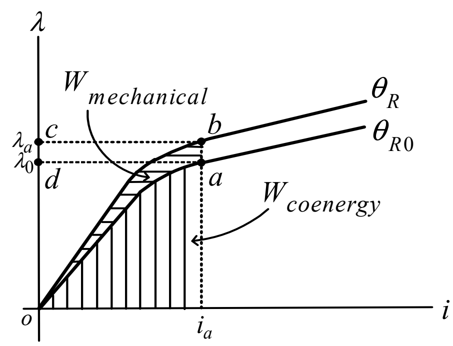

FIGURE 1.13. Mechanical energy and co-energy (adapted from [2]). The current i is constant as the rotor moves.

If one defines the *co-energy* as (see Figure 1.9)

$$W_{\text{coenergy}}(n_R\theta_R, i_a) \triangleq \underset{\substack{\text{Along the } \lambda \text{ vs. } i \\ \text{curve for } \theta_R}}{\int_0^{i_a}} \lambda di$$

then

$$\tau\Delta\theta_R = \Delta W_{\text{mechanical}} = oabo = W_{\text{coenergy}}(n_R\theta_R, i_a) - W_{\text{coenergy}}(n_R\theta_{R0}, i_a). \tag{1.3}$$

That is, the area *oabo* is simply the change in co-energy $\Delta W_{\text{coenergy}}$ as the rotor moves from θ_{R0} to θ_R with the phase current held constant at i_a. Dividing equation (1.3) by $\Delta\theta_R$ and taking the limit $\Delta\theta_R \to 0$ gives the expression for torque as

$$\tau = \frac{\partial W_{\text{coenergy}}(n_R\theta_R, i_a)}{\partial\theta_R} = \frac{\partial}{\partial\theta_R}\int_0^{i_a} \lambda(n_R\theta_R, i)di.$$

The co-energy is simply a convenient mathematical quantity used to find the torque in the presence of magnetic saturation. It does not have any physical significance itself. Further, from Figure 1.9 we see

$$W_{\text{field}}(n_R\theta_R, i_a) + W_{\text{coenergy}}(n_R\theta_R, i_a) = \underset{\substack{\text{Along the} \\ i \text{ vs. } \lambda \\ \text{curve for } \theta_R}}{\int_0^{\lambda_a} i d\lambda} + \underset{\substack{\text{Along the} \\ \lambda \text{ vs. } i \\ \text{curve for } \theta_R}}{\int_0^{i_a} \lambda di} = \lambda_a(n_R\theta_R, i_a)i_a$$

or

$$W_{\text{coenergy}}(n_R\theta_R, i_a) = \lambda(n_R\theta_R, i_a)i_a - W_{\text{field}}(n_R\theta_R, i_a).$$

Example 1 *Linear Magnetization Curve*

With $L_2 > L_1 > 0$, consider an SR motor whose flux linkage is given by

$$\lambda(n_R\theta_R, i) \triangleq \Big(L_2 - L_1 \cos(n_R\theta_R)\Big)i. \tag{1.4}$$

The magnetization curves for this flux linkage model are shown in Figure (1.14). These curves are all linear in the current, that is, the flux linkage does not saturate. Consequently, this is not a realistic model for an actual SR motor. However, analyzing such a simple model does provide insight.

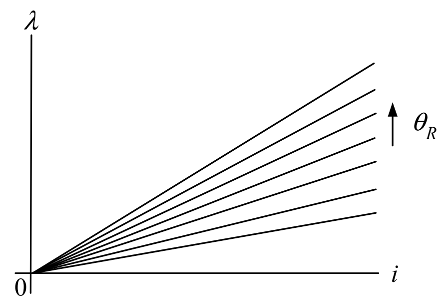

FIGURE 1.14. $\lambda(n_R\theta_R, i)$ for an SR motor without magnetic saturation.

The phase flux linkages are

$$\begin{aligned} \lambda_a(n_R\theta_R, i_a) &= \lambda(n_R\theta_R, i) \\ \lambda_b(n_R\theta_R, i_b) &= \lambda(n_R(\theta_R - \theta_s), i) \\ \lambda_c(n_R\theta_R, i_c) &= \lambda(n_R(\theta_R - 2\theta_s), i). \end{aligned} \tag{1.5}$$

The inductance function for phase a is $L(\theta_R) = L_2 - L_1 \cos(n_R\theta_R)$. The function $L(\theta_R)$ has a maximum value of $L_2 + L_1$ when $\theta_R = \pi/n_R \pm 2\pi/n_R$ which corresponds to stator teeth of phase a being completely aligned with rotor teeth (see the left-hand side of Figure 1.2). It has a minimum value of $L_2 - L_1$ when $\theta_R = 0 \pm 2\pi/n_R$ corresponding to the stator teeth of phase a being completely unaligned with the rotor teeth as in 1.2(a). Let

$$\begin{aligned} W_{\text{coenergy}} &= \int_0^i \lambda(n_R\theta_R, i)di = L(\theta_R)i^2/2 \\ \tau(n_R\theta_R, i) &= \frac{\partial W_{\text{coenergy}}}{\partial\theta_R} = \frac{\partial L(\theta_R)}{\partial\theta_R}\frac{i^2}{2} = \frac{n_R L_1 i^2 \sin(n_R\theta_R)}{2} \end{aligned}$$

so that the torque functions are then

$$\begin{aligned} \tau_a(n_R\theta_R, i) &= \tau(n_R\theta_R, i_a) \\ \tau_b(n_R\theta_R, i) &= \tau(n_R(\theta_R - \theta_s), i_b) \\ \tau_c(n_R\theta_R, i) &= \tau(n_R(\theta_R - 2\theta_s), i_c) \end{aligned}$$

1.4 Mathematical Model

The dynamic equations of a three phase switched reluctance motor are

$$\begin{aligned}
\frac{d}{dt}\lambda_a(n_R\theta_R, i_a) &= -R_S i_a + u_a \\
\frac{d}{dt}\lambda_b(n_R\theta_R, i_b) &= -R_S i_b + u_b \\
\frac{d}{dt}\lambda_c(n_R\theta_R, i_c) &= -R_S i_c + u_c \\
J\frac{d\omega_R}{dt} &= \tau(n_R\theta_R, i_a, i_b, i_c) - \tau_L
\end{aligned}$$

where

$$\begin{aligned}
\tau(n_R\theta_R, i_a, i_b, i_c) &= \tau_a(n_R\theta_R, i_a) + \tau_b(n_R\theta_R, i_b) + \tau_c(n_R\theta_R, i_c) \\
\tau_a(n_R\theta_R, i_a) &= \frac{\partial}{\partial\theta_R}\int_0^{i_a}\lambda_a(n_R\theta_R, i_a)di_a \\
\tau_b(n_R\theta_R, i_b) &= \frac{\partial}{\partial\theta_R}\int_0^{i_b}\lambda_b(n_R\theta_R, i_b)di_b \\
\tau_c(n_R\theta_R, i_c) &= \frac{\partial}{\partial\theta_R}\int_0^{i_c}\lambda_c(n_R\theta_R, i_c)di_c
\end{aligned}$$

with the flux linkages given by (1.1).

Expanding these equations gives

$$\begin{aligned}
\frac{di_a}{dt} &= \left(-R_S i_a - \frac{\partial\lambda_a(n_R\theta_R, i_a)}{\partial\theta_R}n_R\omega_R + u_a\right)\Big/\frac{\partial\lambda_a(n_R\theta_R, i_a)}{\partial i_a} \\
\frac{di_b}{dt} &= \left(-R_S i_b - \frac{\partial\lambda_b(n_R\theta_R, i_b)}{\partial\theta_R}n_R\omega_R + u_b\right)\Big/\frac{\partial\lambda_b(n_R\theta_R, i_b)}{\partial i_b} \\
\frac{di_c}{dt} &= \left(-R_S i_c - \frac{\partial\lambda_c(n_R\theta_R, i_c)}{\partial\theta_R}n_R\omega_R + u_c\right)\Big/\frac{\partial\lambda_c(n_R\theta_R, i_c)}{\partial i_c} \\
J\frac{d\omega_R}{dt} &= \tau(n_R\theta_R, i_a, i_b, i_c) - \tau_L
\end{aligned} \tag{1.6}$$

These expressions show that a model of the SR motor can be developed by using the experimentally determined $\lambda(n_R\theta_R, i)$ magnetization curves.

Example 2 *Linear Magnetization Curve*

Using the expressions (1.4) and (1.5) for the flux linkages along with setting $K_m = n_R L_1/2$ gives the nonlinear state-space model

$$\begin{aligned}
\frac{di_a}{dt} &= \frac{-R_S i_a - n_R L_1 \sin(n_R\theta_R)\, n_R\omega_R + u_a}{L_2 - L_1\cos(n_R\theta_R)} \\
\frac{di_b}{dt} &= \frac{-R_S i_b - n_R L_1 \sin(n_R(\theta_R - \theta_s))\, n_R\omega_R + u_b}{L_2 - L_1\cos(n_R(\theta_R - \theta_s))} \\
\frac{di_c}{dt} &= \frac{-R_S i_c - n_R L_1 \sin(n_R(\theta_R - 2\theta_s))\, n_R\omega_R + u_c}{L_2 - L_1\cos(n_R(\theta_R - 2\theta_s))} \\
J\frac{d\omega_R}{dt} &= K_m\Big(i_a^2\sin(n_R\theta_R) + i_b^2\sin(n_R(\theta_R - \theta_s)) + i_c^2\sin(n_R(\theta_R - 2\theta_s))\Big) - \tau_L.
\end{aligned}$$

1.5 Current Control of the SR Motor

The process of building up the current in one phase and taking it down in another is referred to as *current commutation.* Let $i_a^d(n_R\theta_R,\tau_d), i_b^d(n_R\theta_R,\tau_d), i_c^d(n_R\theta_R,\tau_d)$ denote the desired currents for each phase to achieve current commutation. The phase voltages u_a, u_b, u_c must be chosen so that $i_a \to i_a^d, i_b \to i_b^d, i_c \to i_c^d$. One might consider a first simplification by choosing the feedback voltages in (1.6) as

$$\begin{aligned}
u_a &= R_S i_a + \frac{\partial\lambda_a(n_R\theta_R,i_a)}{\partial\theta_R}n_R\omega_R + v_a\frac{\partial\lambda_a(n_R\theta_R,i_a)}{\partial i_a} \\
u_b &= R_S i_b + \frac{\partial\lambda_b(n_R\theta_R,i_b)}{\partial\theta_R}n_R\omega_R + v_b\frac{\partial\lambda_b(n_R\theta_R,i_b)}{\partial i_b} \\
u_c &= R_S i_c + \frac{\partial\lambda_c(n_R\theta_R,i_c)}{\partial\theta_R}n_R\omega_R + v_c\frac{\partial\lambda_c(n_R\theta_R,i_c)}{\partial i_c}
\end{aligned} \tag{1.7}$$

where v_a,v_b, v_c are new inputs. This results in the model (1.6) becoming

$$\begin{aligned}
\frac{di_a}{dt} &= v_a \\
\frac{di_b}{dt} &= v_b \\
\frac{di_c}{dt} &= v_c \\
J\frac{d\omega_R}{dt} &= \tau(n_R\theta_R,i_a,i_b,i_c) - \tau_L
\end{aligned}$$

Then, for an appropriate choice of $K > 0$, choosing v_a,v_b, v_c as

$$\begin{aligned}
v_a &= K(i_a^d - i_a) \\
v_b &= K(i_b^d - i_b) \\
v_c &= K(i_c^d - i_c)
\end{aligned} \tag{1.8}$$

will ensure $i_a \to i_a^d, i_b \to i_b^d, i_c \to i_c^d$. However, simply using the high gain feedback

$$\begin{aligned}
u_a &= K(i_a^d - i_a) \\
u_b &= K(i_b^d - i_b) \\
u_c &= K(i_c^d - i_c)
\end{aligned} \tag{1.9}$$

without using the cancellation feedback (1.7) will often suffice assuming the currents are not changing too rapidly.

1.6 Specifying the Torque - Current Commutation

The torque in the motor is given by

$$\begin{aligned}
\tau &= \tau_a(n_R\theta_R,i_a) + \tau_b(n_R\theta_R,i_b) + \tau_c(n_R\theta_R,i_c) \\
&= \tau(n_R\theta_R,i_a) + \tau(n_R\left(\theta_R-\theta_s\right),i_b) + \tau(n_R\left(\theta_R-2\theta_s\right),i_c)
\end{aligned}$$

Example 3 *Linear Magnetization Curve*
If the SR motor has a linear magnetization curve then we may write

$$\tau = \frac{n_R L_1}{2}\Big(i_a^2\sin\left(n_R\theta_R\right) + i_b^2\sin\left(n_R(\theta_R-\theta_s)\right) + i_c^2\sin\left(n_R(\theta_R-2\theta_s)\right)\Big).$$

This is a complicated nonlinear function of the phase currents and the rotor position. The fundamental problem to controlling the SR motor is as follows: Given a desired torque τ_d, at any rotor position θ_R find the currents i_a, i_b, i_c required in each phase so that torque τ produced by the motor equals this desired torque. With $i_a = i_b = i_c = i_0 > 0$, the individual torque versus rotor angle for each of the three phases are plotted in Figure 1.15 below.

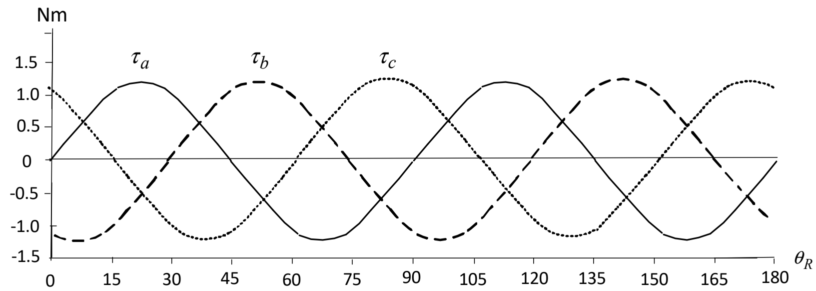

FIGURE 1.15. Torque vs rotor angle for each phase with $i_a = i_b = i_c = i_0 > 0$.

In order to find the currents in each phase, recall the torque function in (1.15) for phase a given by

$$\tau(n_R\theta_R, i) = \frac{n_R L_1 i^2 \sin(n_R\theta_R)}{2}.$$

This torque function does not depend on the sign of the current so the current will always be assumed positive. This torque function gives positive torque for $0 < \theta_R < \pi/n_R$ and negative torque for $\pi/n_R < \theta_R < 2\pi/n_R$. Inverting this expression gives current function

$$i_a = g(n_R\theta_R, \tau_a) = K_m\sqrt{\frac{\tau_a}{|\sin(n_R\theta_R)|}}, \quad K_m = \sqrt{\frac{2}{n_R L_1}}$$

so that for given τ and a given angular position θ_R, this is the phase current i required in that phase to obtain the torque τ.

More generally (not assuming linear magnetization), let's write the current functions for each phase as

$$\begin{aligned} i_a &= g(n_R\theta_R, \tau_a) \\ i_b &= g(n_R(\theta_R - \theta_s)), \tau_b) \\ i_c &= g(n_R(\theta_R - 2\theta_s)), \tau_c) \end{aligned}$$

To fix ideas, let's assume for the time being that the desired torque τ_d is positive. Then the phase torques τ_a, τ_b, τ_c must be chosen at each position θ_R such that

$$\tau_d = \tau_a + \tau_b + \tau_c$$

and such that the current constraints are not violated, that is,

$$|i_k| \leq I_{\max}, \quad k = a, b, c.$$

Great care is needed to build up and bring down the currents in the individual phases in order to produce a smooth torque. To illustrate, consider the SR motor at constant speed producing a constant torque. Figure 1.16 shows a typical phase current reference (commutation profiles) for each phase needed to produce a constant total torque. That is, using the currents in Figure 1.16, we have the total torque

$$\tau_d = \tau_a(n_R\theta_R, i_a) + \tau_b(n_R\theta_R, i_b) + \tau_c(n_R\theta_R, i_c)$$

is constant.

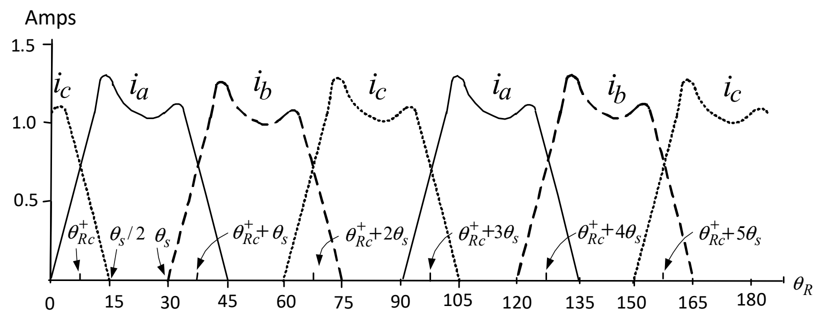

FIGURE 1.16. Phase currents vs. θ_R so the total torque $\tau_d = \tau_a(n_R\theta_R, i_a) + \tau_b(n_R\theta_R, i_b) + \tau_c(n_R\theta_R, i_c)$ is constant

Current Commutation with Positive Torque

The commutation of the current from phase c to phase a in the interval $0 \leq \theta_R \leq \theta_s$ ($\theta_s = 30°$ in Figure 1.16 is now described following the procedure in [4]. At $\theta_R = 0$ only phase c can produce positive torque. As the rotor position increases, current is put into phase a as it can now produce positive torque, but most of the torque is produced by phase c. As illustrated in Figure 1.16 we use θ_{Rc}^+ to denote the angular position where two phases can produce the *same* torque $\tau_d/2 > 0$ (for a total motor torque of τ_d) with the *same* current in each their phases. For example, at θ_{Rc}^+ we have $i_a = g(n_R\theta_{Rc}^+, \tau_d/2) = g(n_R(\theta_{Rc}^+ + 2\theta_s), \tau_d/2) = i_c$. We refer to θ_{Rc}^+ as the *current commutation angle*[5] θ_{Rc}^+. This is the angle where the two phases c and a can each produce half the required torque $\tau_d/2$ so that the total torque is τ_d (phase b is not producing any torque at this instant). As all the functions are periodic (see Figure 1.16), the commutation angles are $\theta_{Rc}^+ \pm n\theta_s$.

To characterize the commutation procedure, the interval $[0, \theta_s]$ is broken down into the three subintervals $[0, \theta_{Rc}^+][\theta_{Rc}^+, \theta_s/2][\theta_s/2, \theta_s]$ and the phase currents are specified for each of these subintervals.

The interval $0 \leq \theta_R \leq \theta_{Rc}^+$

In this interval only phases a and c can produce positive torque and therefore i_b is set to 0. Linearly ramp up the current i_a in phase a from 0 to $g(n_R\theta_{Rc}^+, \tau_d/2)$, that is, set

$$i_{\text{Rising}}(n_R\theta_R) = \theta_R \frac{g(\theta_{Rc}^+, \tau_d/2)}{\theta_{Rc}^+} \text{ for } 0 \leq \theta_R \leq \theta_{Rc}^+.$$

The torque produced in phase a is then (See Figure 1.16 for $0 \leq \theta_R \leq \theta_{Rc}^+$)

$$\tau_{\text{Rising}} \triangleq \tau(n_R\theta_R, i_{\text{Rising}}(n_R\theta_R)) \text{ for } 0 \leq \theta_R \leq \theta_{Rc}^+.$$

Set the current in phase c as

$$i_{\text{Strong}} = g(n_R(\theta_R - 2\theta_s), \tau_d - \tau_{\text{Rising}})$$

so that phase c produces the torque $\tau_d - \tau_{\text{Rising}}$. As phase a is producing the current τ_{Rising}, the total total torque for $0 \leq \theta_R \leq \theta_{Rc}^+$ is τ_d. (See Figure 1.16 for $0 \leq \theta_R \leq \theta_{Rc}^+$)

Note that when the rotor is at θ_{Rc}^+, both phases have the same current $g(n_R\theta_{Rc}^+, \tau_d/2)$ and are producing the same torque $\tau_d/2$ for a total torque τ_d.

[5] In general the angle θ_{Rc}^+ will depend on the desired torque τ_d.

The interval $\theta_{Rc}^{+} \le \theta_R \le \theta_s/2$

In the interval $\theta_{Rc}^{+} \le \theta_R \le \theta_s/2$ ($\theta_s/2 = 15°$ in Figure 1.16) the current in phase c is ramped down from $g(n_R\theta_{Rc}^{+}, \tau_d/2)$ to 0 by setting it equal to[6]

$$i_{\text{Falling}}(n_R\theta_R) = g(\theta_{Rc}^{+}, \tau_d/2) - (\theta_R - \theta_{Rc}^{+})\frac{g(\theta_{Rc}^{+}, \tau_d/2)}{\theta_s/2 - \theta_{Rc}^{+}}$$

The torque produced by phase c is then

$$\tau_{\text{Falling}} \triangleq \tau(n_R(\theta_R - 2\theta_s), i_{\text{Falling}})$$

The current in phase a is set as

$$i_{\text{Strong}} = g(n_R\theta_R, \tau_d - \tau_{\text{Falling}})$$

so that phase a is producing the torque $\tau_d - \tau_{\text{Falling}}$. As phase c is producing the torque τ_{Falling}, the total torque is τ_d. (See Figure 1.16 for $\theta_{Rc}^{+} < \theta_R < \theta_s/2$)

The interval $\theta_s/2 \le \theta_R \le \theta_s$

For the interval $\theta_s/2 \le \theta_R \le \theta_s$, only phase a can produce positive torque. Consequently, the current in phase a is set as $i_a = g(n_R\theta_R, \tau_d)$ with the other phase currents zero. In this case phase a produces the full torque τ_d. (See Figure 1.16 for $15° = \theta_s/2 \le \theta_R \le \theta_s = 30°$.)

This current commutation procedure is then repeated between phases a and b for $\theta_s \le \theta_R \le 2\theta_s$, then between phases b and c for $2\theta_s \le \theta_R \le 3\theta_s$, and so on. Given the desired torque τ_d and the rotor position θ_R, this current commutation algorithm provides the reference current for the individual phases as illustrated in Figure 1.16.

A general procedure for $\tau_d > 0$ can be formulated as follows. Recall that $3\theta_s = \pi/n_R$ is the period of the SR motor. Let

$$\begin{aligned} \theta_R^{+} &= \theta_R - \theta_{Rc}^{+} \\ \theta_{3\theta_s} &\triangleq \theta_R^{+} \bmod 3\theta_s \text{ for } \tau_d > 0 \end{aligned}$$

and

$$\begin{aligned} \Theta_1 &= \{\theta : 0 \le \theta_{3\theta_s} \le \theta_s\} \\ \Theta_2 &= \{\theta : \theta_s \le \theta_{3\theta_s} \le 2\theta_s\} \\ \Theta_3 &= \{\theta : 2\theta_s \le \theta_{3\theta_s} \le 3\theta_s\} \end{aligned}$$

Using θ_{Rc}^{+} as a new reference angle, the period $3\theta_s = \pi/n_R$ of the SR motor is broken into three subintervals. Note that in each of these three subintervals of length θ_s, one phase current is rising, one is falling, and one is the strong current (i.e., can produce the most torque for a given current) and this is summarized in the table below.

Interval	Phase a	Phase b	Phase c
Θ_1	Strong	Rising	Falling
Θ_2	Falling	Strong	Rising
Θ_3	Rising	Falling	Strong

To describe these currents let

$$\theta_{\theta_s} \triangleq \theta_{3\theta_s} \bmod \theta_s,$$

[6] The current in phase c must be brought to zero at $\theta_s/2$ because as θ moves past $\theta_s/2$, phase c will produce negative torque.

which locates the angle in the interval $[0, \theta_s]$. With reference to Figure 1.17 define

$$i_{\text{Rising}}(\theta_{\theta_s}, \tau_d) = \begin{cases} \left(\theta_{\theta_s} - \ (\theta_s - \theta^+_{Rc})\right) \dfrac{g(\theta^+_{Rc}, \tau_d/2)}{\theta^+_{Rc}} & \theta_s - \theta^+_{Rc} \le \theta_{\theta_s} \le \theta_s \\ 0 & 0 \le \theta_{\theta_s} \le \theta_s - \theta^+_{Rc} \end{cases} \tag{1.10}$$

$$i_{\text{Falling}}(\theta_{\theta_s}, \tau_d) = \begin{cases} g(\theta^+_{Rc}, \tau_d/2) - \theta_{\theta_s} \dfrac{g(\theta^+_{Rc}, \tau_d/2)}{\theta_s/2 - \theta^+_{Rc}} & 0 \le \theta_{\theta_s} \le \theta_s/2 - \theta^+_{Rc} \\ 0 & \theta_s/2 - \theta^+_{Rc} \le \theta_{\theta_s} \le \theta_s \end{cases} \tag{1.11}$$

$$i_{\text{Strong}}(\theta_{\theta_s}, \tau_d) = g(\theta_{\theta_s}, \tau_d - \tau_{\text{Rising}} - \tau_{\text{Falling}}) \tag{1.12}$$

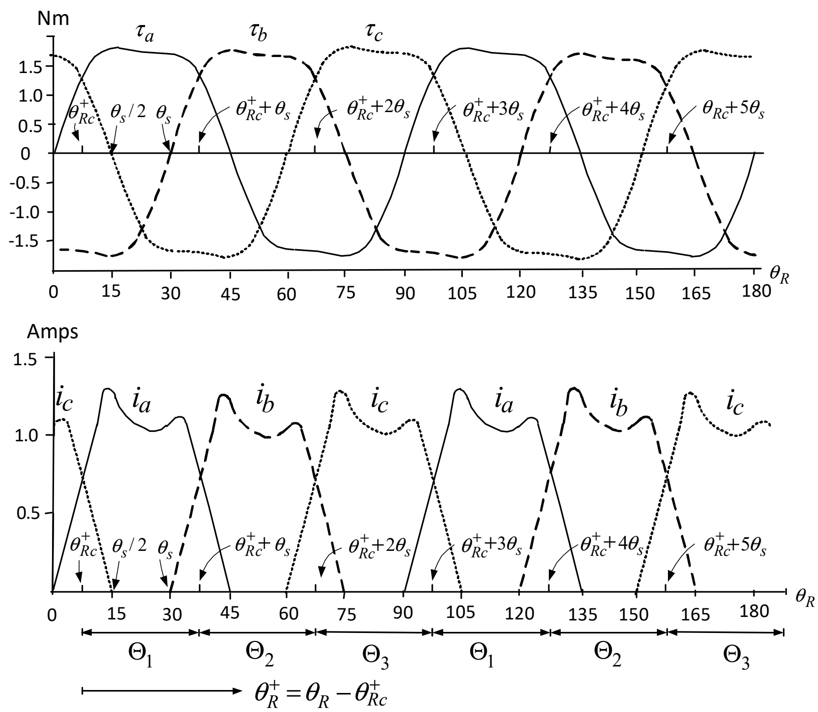

FIGURE 1.17. Top: Torque vs. θ_R for constant i. Bottom: Phase currents vs. θ_R to achieve constant positive torque.

Given a rotor angle θ_R the angle $\theta_{3\theta_s} \triangleq \theta^+_R \bmod 3\theta_s = (\theta_R - \theta^+_{Rc}) \bmod 3\theta_s$ is computed to determine which subinterval $\Theta_1, \Theta_2, \Theta_3$ it lies. After this is determined the above table specifies which phase should have the rising current given by (1.10), which phase should have the falling current given by (1.11), and finally which phase has the strong current.

$$\begin{aligned} \theta_{\theta_s} &\in \Theta_1 \text{ set } i_a(\theta_{\theta_s}, \tau_d) = i_{\text{Strong}}, i_b(\theta_{\theta_s}, \tau_d) = i_{\text{Rising}}, i_c(\theta_{\theta_s}, \tau_d) = i_{\text{Falling}} \\ \theta_{\theta_s} &\in \Theta_2 \text{ set } i_a(\theta_{\theta_s}, \tau_d) = i_{\text{Falling}}, i_b(\theta_{\theta_s}, \tau_d) = i_{\text{Strong}}, i_c(\theta_{\theta_s}, \tau_d) = i_{\text{Rising}} \\ \theta_{\theta_s} &\in \Theta_3 \text{ set } i_a(\theta_{\theta_s}, \tau_d) = i_{\text{Rising}}, i_b(\theta_{\theta_s}, \tau_d) = i_{\text{Falling}}, i_c(\theta_{\theta_s}, \tau_d) = i_{\text{Strong}} \end{aligned}$$

Figure 1.18 is a plot from a simulation showing the total torque obtained from such a current commutation scheme.

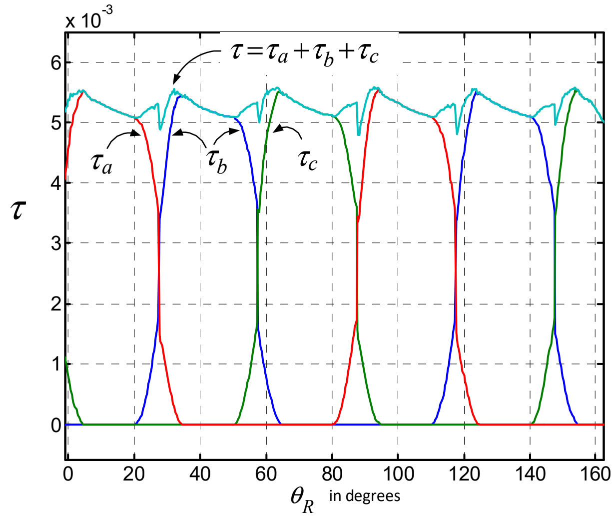

FIGURE 1.18. Phase torque and total torque vs θ_R.

Current Commutation with Negative Torque

We now consider the case that the desired torque $\tau_d < 0$. The top of Figure 1.19 below shows the currents in each phase required to achieve a constant negative torque. As shown on the bottom of Figure 1.19 the angular positions $\theta_{Rc}^- \pm n\theta_s$ are where two phases carrying the same current produce the same negative torque. Also note in Figure 1.19 that $\theta_{Rc}^- < 0$ and by the symmetry of the τ versus θ_R curves we also have $\theta_{Rc}^- = -\theta_{Rc}^+$.

Define

$$\begin{aligned}\theta_R^- &= \theta_R - (\theta_{Rc}^- - \theta_s) = \theta_R + \theta_{Rc}^+ + \theta_s \\ \theta_{3\theta_s} &\triangleq \theta_R^- \bmod 3\theta_s \text{ for } \tau_d < 0\end{aligned}$$

and

$$\begin{aligned}\Theta_1 &= \{\theta : 0 \le \theta_{3\theta_s} \le \theta_s\} \\ \Theta_2 &= \{\theta : \theta_s \le \theta_{3\theta_s} \le 2\theta_s\} \\ \Theta_3 &= \{\theta : 2\theta_s \le \theta_{3\theta_s} \le 3\theta_s\}.\end{aligned}$$

With a new reference angle of $\theta_{Rc}^- - \theta_s = -\theta_{Rc}^+ - \theta_s$, the period $3\theta_s = \pi/n_R$ of the SR motor is broken into these three subintervals. In each of these three subintervals of length θ_s, one phase current is rising, one is falling, and one is the strong current. Note the table above is still valid for $\tau_d < 0$. However, the rising and falling currents now occupy a different portion of the interval $[0, \theta_s]$ than when $\tau_d > 0$. To characterize these currents let

$$\theta_{\theta_s} \triangleq \theta_{3\theta_s} \bmod \theta_s$$

which locates the angle in the interval $[0, \theta_s]$.

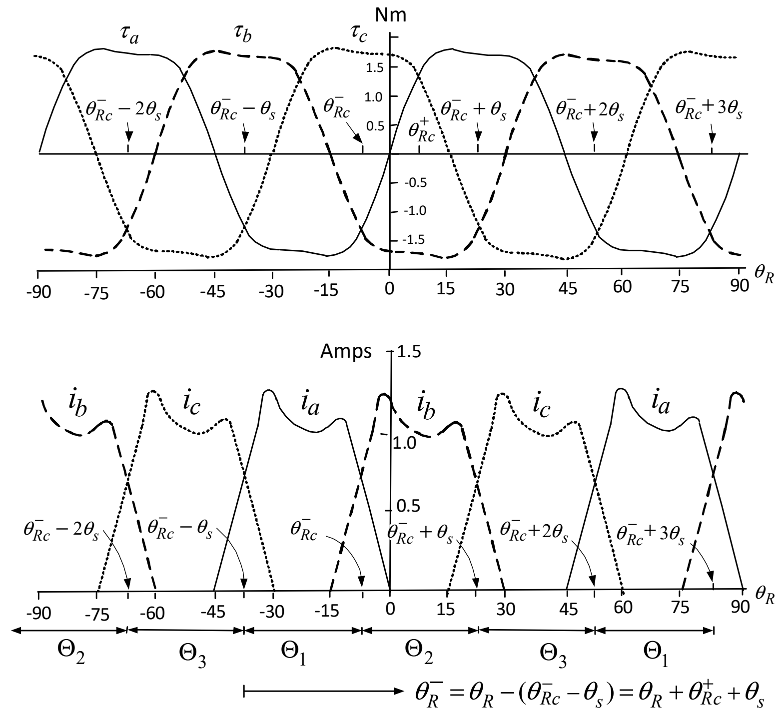

FIGURE 1.19. Top: Torque vs. θ_R for constant i. Bottom: Phase currents vs. θ_R to achieve constant negative torque.

Define

$$i_{\text{Rising2}}(\theta_{\theta_s}, \tau_d) = \begin{cases} \left(\theta_{\theta_s} - (\theta_s + \theta_{Rc}^-)\right) \dfrac{g(\theta_{Rc}^-, \tau_d/2)}{-\theta_{Rc}^-} & \theta_s + \theta_{Rc}^- \le \theta_{\theta_s} \le \theta_s \\ 0 & 0 \le \theta_{\theta_s} \le \theta_s + \theta_{Rc}^- \end{cases} \tag{1.13}$$

$$i_{\text{Falling2}}(\theta_{\theta_s}, \tau_d) = \begin{cases} g(\theta_{Rc}^-, \tau_d/2) - \theta_{\theta_s} \dfrac{g(\theta_{Rc}^-, \tau_d/2)}{-\theta_{Rc}^-} & 0 \le \theta_{\theta_s} \le -\theta_{Rc}^- \\ 0 & -\theta_{Rc}^- \le \theta_{\theta_s} \le \theta_s \end{cases} \tag{1.14}$$

$$i_{\text{Strong}}(\theta_{\theta_s}, \tau_d) = g(\theta_{\theta_s}, \tau_d - \tau_{\text{Rising2}} - \tau_{\text{Falling2}}) \tag{1.15}$$

Given a rotor angle θ_R, the angle $\theta_{3\theta_s} \triangleq \theta_R^- \bmod 3\theta_s$ is computed to determine within which of the subintervals $\Theta_1, \Theta_2, \Theta_3$ it lies. After this is determined, the above table specifies which phase should have the rising current given by (1.13), which phase should have the falling current given by (1.14), and which is the strong current.

$$\begin{aligned} \theta_{\theta_s} &\in \Theta_1 \text{ set } i_a(\theta_{\theta_s}, \tau_d) = i_{\text{Strong}}, i_b(\theta_{\theta_s}, \tau_d) = i_{\text{Rising}}, i_c(\theta_{\theta_s}, \tau_d) = i_{\text{Falling}} \\ \theta_{\theta_s} &\in \Theta_2 \text{ set } i_a(\theta_{\theta_s}, \tau_d) = i_{\text{Falling}}, i_b(\theta_{\theta_s}, \tau_d) = i_{\text{Strong}}, i_c(\theta_{\theta_s}, \tau_d) = i_{\text{Rising}} \\ \theta_{\theta_s} &\in \Theta_3 \text{ set } i_a(\theta_{\theta_s}, \tau_d) = i_{\text{Rising}}, i_b(\theta_{\theta_s}, \tau_d) = i_{\text{Falling}}, i_c(\theta_{\theta_s}, \tau_d) = i_{\text{Strong}} \end{aligned}$$

Remarks The papers [4][3][5][6][7] have all addressed the issue of current commutation in detail. These methods are based on a model of the motor that assumes the stator phases are magnetically decoupled from each other (i.e., $\lambda_{12}(n_R\theta, i_b) = \lambda_{13}(n_R\theta, i_c) \equiv 0$). Wallace and Taylor [4] have pointed out that both of these assumptions do not completely hold and have shown that due to this coupling, the total torque is not simply the sum of the torques produced by the individual phases (see Figure 13 of [4]) resulting in some residual torque ripple. In fact, [8] and [9] point out that the mutual coupling can be of enough significance that it can be used to estimate the motor position. In [10] the mutual coupling between phases is analyzed showing its effect on position estimation can be around 3°. This would suggest that a more complicated model of the motor be used that does include the mutual coupling between phases.

1.7 Identification of the Model

The model for the controller [4] depends on knowing the flux functions $\lambda_k(n_R\theta, i)$ and torque functions $\tau_k(n_R\theta, i)$ which are nonlinear in both θ and i. The *functional form* of the flux and torque functions $\lambda_k(n_R\theta, i)$, $\tau_k(n_R\theta, i)$ are not known a priori (and are non-parametric) and consequently must be determined experimentally. The model (1.6) shows that the torque can be computed from the flux linkages. However, as the torque is the output of interest, it would reduce a possible source inaccuracy to do a direct measurement of the torque as is now described. As explained above, the torque produced by phase a with the rotor at θ is the negative of the torque produced at $\dfrac{2\pi}{n_R} - \theta$. Consequently, the torque need only be measured for $\theta = 0$ to $\dfrac{\pi}{n_R}$. With n_θ the number of discrete position measurements, n_I the number of discrete phase current measurements, the position and current measurements can be determined for $\theta_k = \dfrac{(k-1)}{n_\theta}\dfrac{2\pi}{n_R}$ with $k = 1 : n_\theta + 1$ and for $i_m = \dfrac{m-1}{n_I} I_{\max}$ with $m = 1 : n_I + 1$.

To find the torque versus phase current and rotor position, a DC motor is coupled to the shaft of the SR motor and used to control the shaft position. A known current i is applied to the SR motor phase and the DC motor is commanded to hold the rotor shaft at a fixed position θ. Measuring the DC armature current, the DC motor torque is simply $K_T i_{\text{armature}}$ and this equals the SR motor torque. That is, the SR motor torque is $K_T i_{\text{armature}}$ when the rotor is at θ and the SR phase has the current i in the winding. A systematic procedure to identify the torque model is as follows [1] (see also [11]).

1. For each $m = 1 : n_I + 1$, apply the constant current $\dfrac{m-1}{n_I} I_{\max}$ to phase a of the SR motor. With current only in phase a, the rotor position goes to the stable equilibrium point $\theta = \dfrac{\pi}{n_R} \pm \dfrac{2\pi}{n_R}$. Consequently, the initial rotor position is $\theta = \dfrac{\pi}{n_R}$.

2. With the constant current in the SR motor phase, starting with $k = n_\theta$ and going down to $k = 2$, command and hold the position of the DC motor at $\theta_k = \dfrac{(k-1)}{n_\theta}\dfrac{2\pi}{n_R}$ (note that $k = n_\theta + 1$ and $k = 1$ result in zero SR torque regardless of phase current). At each of these discrete positions, with the shaft stationary, measure the DC armature current. The DC motor torque is then $K_T i_{\text{armature}}$ which equals the SR motor torque.

3. Repeat the above for $m = 1, \cdots, n_I + 1$.

Defining $\theta_s \triangleq \dfrac{2\pi}{n_{ph} n_R}$ the other two phase torques are given by $\tau_b(n_R\theta, i_b) = \tau_a(n_R(\theta - \theta_s), i_b)$ and $\tau_c(n_R\theta, i_c) = \tau_a(n_R(\theta - 2\theta_s), i_c)$.

With $n_\theta = 30, n_I = 10, n_R = 4, I_{\max} = 3$, this identification procedure gives the data points for the function $\tau_a = \tau_a(n_R\theta, i)$ in Figure 1.20 below[12].

From this data, the function $\tau_a = \tau_a(n_R\theta, i_a)$ may be inverted to find $i_a = g_a(n_R\theta, \tau_a)$. That is, given a requested torque τ_a produced by phase a, $i_a = g_a(n_R\theta, \tau_a)$ is the current required in this phase to produce this torque at the given angle. The function $\tau_a(n_R\theta, i_a)$ was inverted using the approach in [5] resulting in Figure 1.21.

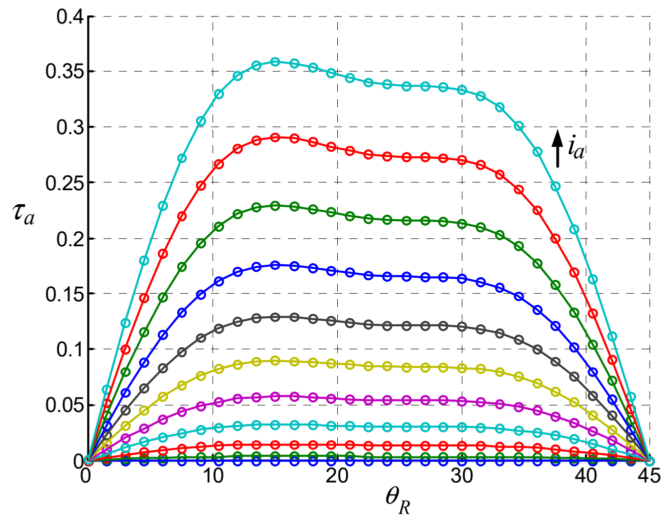

FIGURE 1.20. Torque as a function of rotor position and phase current.

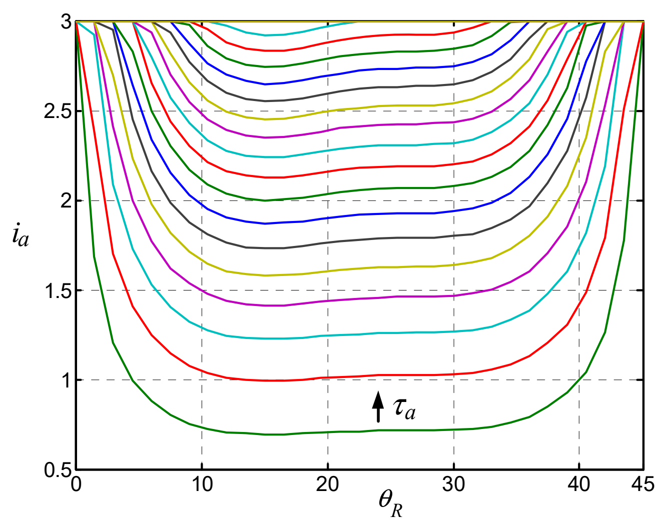

FIGURE 1.21. Current in phase a required to produce a given torque at a given rotor position.

1.8 Position and Speed Control

With J the rotor inertia and $(\theta_{Rd}(t), \omega_{Rd}(t), \alpha_{Rd}(t)) = (\theta_{Rd}(t), d\theta_{Rd}(t)/dt, d^2\theta_{Rd}(t)dt^2)$ the desired angular position, speed, and acceleration, we have $\theta_{Rd} \to \theta_R, \omega_{Rd} \to \omega_R$ using the feedback control

$$\tau_d = J\left(\int_0^t K_I(\theta_{Rd} - \theta_R)dt + K_p(\theta_{Rd} - \theta_R) + K_d(\omega_{Rd} - \omega_R) + \alpha_{Rd}\right). \tag{1.16}$$

Simulations

Results from a SIMULINK simulation of the above controller are given below. The SR motor that was simulated had the following parameters: $J = 0.0001$ Kg-m^2, $f = 0.002$ Nm/rad/sec, $R_S = 5$ Ohms, $n_R = 4$, $n_{ph} = 3$, $\theta_s = 2\pi/(n_R n_{ph}) = 30°$ [12]. The feedback gain for the current controller (1.9) was chosen to be 10^4 while the gains for the torque controller (1.16) were chosen to put the all closed-loop poles at -10^3. The torque curves given in Figure 1.20 were used for the mechanical torque model of the motor while the data in Figure 1.21 was used to implement the commutation strategy outlined in the previous section. Figure 1.22 shows position and speed for the chosen trajectory profile.

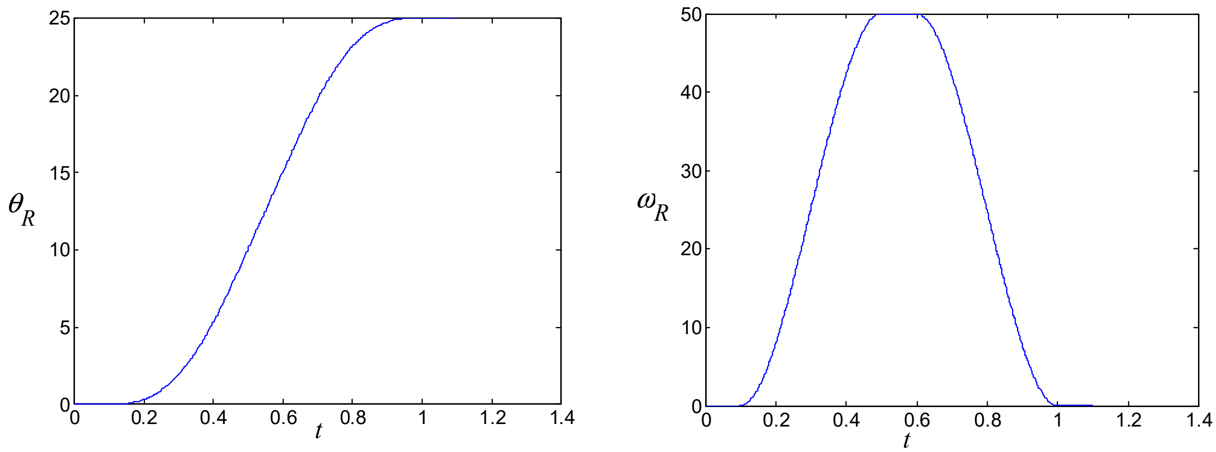

FIGURE 1.22. Left: θ_R in radians versus t. Right: ω_R in radians/sec versus t.

The corresponding current in phase a of the motor is shown on the left side of Figure 1.23. The currents in the phases b & c are similar except shifted by θ_s and $2\theta_s$, respectively. The right side of Figure 1.23 shows the total torque in Newton-meters produced by the current in all three phases. Even under these "ideal" conditions, there is torque ripple as is shown above in Figure 1.18.

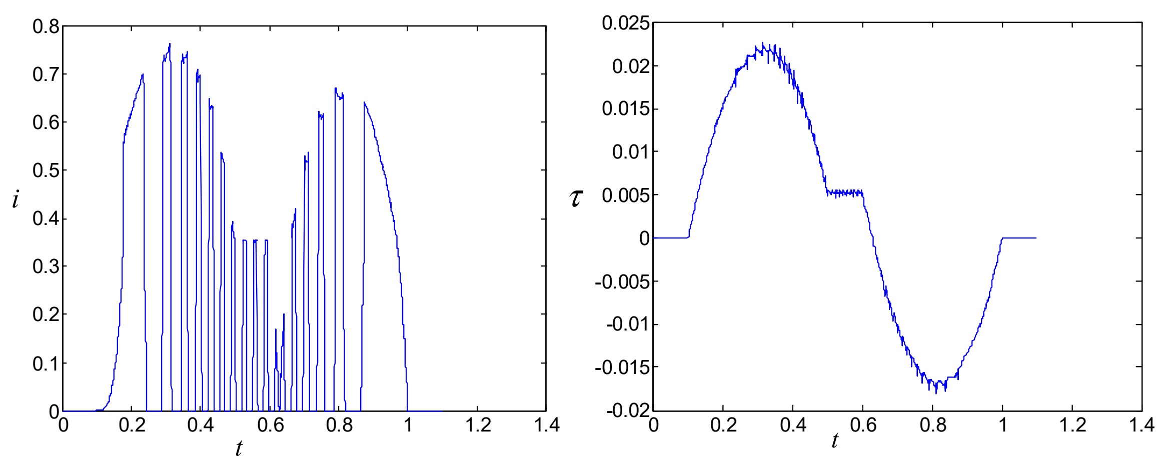

FIGURE 1.23. Left: i_a versus t. Right: τ versus t.

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