Modeling and High-Performance Control of Electric Machines

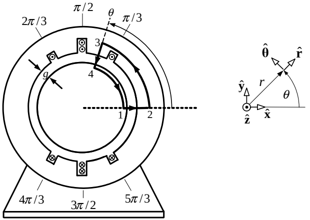
Chapter 4 Problems

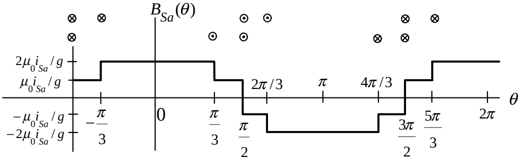
John Chiasson

Wiley-IEEE Press 2005

The following slides go over problem 6 of Chapter 4.

Three-Phase Machine and Space Harmonics

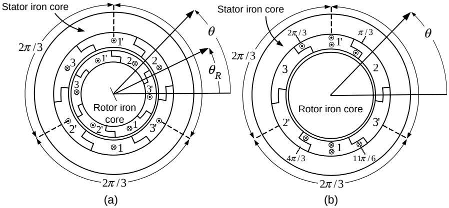




$$
\begin{aligned}
B_{Sa}(\theta) &= \mu_0 \frac{i_{Sa}}{g}\frac{4}{\pi} \times \sum_{k=1,3,5,\ldots}^{\infty} \frac{\left(1+\cos\left(\frac{k\pi}{6}\right)\right)}{k} \sin\Big(k(\theta+\frac{\pi}{2})\Big) \\
&= \mu_0 \frac{i_{Sa}}{g}\frac{4}{\pi}\left(\frac{2+\sqrt{3}}{2}\sin\Big(\theta+\frac{\pi}{2}\Big)+\frac{1}{3}\sin\Big(3(\theta+\frac{\pi}{2})\Big)+\frac{1-\sqrt{3}/2}{5}\sin\Big(5(\theta+\frac{\pi}{2})\Big)+\cdots\right. \\
&= \mu_0 \frac{i_{Sa}}{g}\frac{4}{\pi}\Big(1.866\cos(\theta) - 0.333\cos(3\theta) + 0.0268\cos(5\theta) \mp \cdots \qquad (0.333/1.866 = 0.18)
\end{aligned}
$$

Three-Phase Machine and Space Harmonics

- The conductors have an infinitesimal cross section.
- The iron has infinite permeability, i.e., $\vec{\mathbf{H}} = 0$ in the iron.
- The air gap is small.



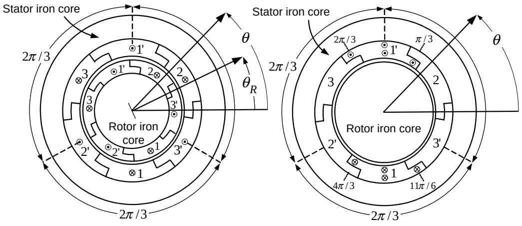
Using Ampère's law it follows that

$$B_{S1}(r,\theta) = \mu_0 \frac{i_{S1}}{g}\frac{4}{\pi}\frac{r_R}{r} \times \sum_{k=1,3,5,\cdots}^{\infty} \frac{1+\cos(\frac{k\pi}{6})}{k} \sin\left(k(\theta + \frac{\pi}{2})\right)$$

$$B_{S2}(r,\theta) = \mu_0 \frac{i_{S2}}{g}\frac{4}{\pi}\frac{r_R}{r} \times \sum_{k=1,3,5,\cdots}^{\infty} \frac{1+\cos(\frac{k\pi}{6})}{k} \sin\left(k(\theta - \frac{2\pi}{3} + \frac{\pi}{2})\right)$$

$$B_{S3}(r,\theta) = \mu_0 \frac{i_{S3}}{g}\frac{4}{\pi}\frac{r_R}{r} \times \sum_{k=1,3,5,\cdots}^{\infty} \frac{1+\cos(\frac{k\pi}{6})}{k} \sin\left(k(\theta - \frac{4\pi}{3} + \frac{\pi}{2})\right).$$

Three-Phase Machine and Space Harmonics



(a) (b)

The first harmonic is

$$\vec{\mathbf{B}}_{S,1} = \vec{\mathbf{B}}_{S_1,1} + \vec{\mathbf{B}}_{S_2,1} + \vec{\mathbf{B}}_{S_3,1}$$

$$= \frac{\mu_0}{g}\frac{4}{\pi}\frac{r_R}{r}(1+\cos(\frac{\pi}{6}))\Big(i_{S1}\sin(\theta+\frac{\pi}{2}) + i_{S2}\sin(\theta-\frac{2\pi}{3}+\frac{\pi}{2}) + i_{S3}\sin(\theta-\frac{4\pi}{3}+\frac{\pi}{2})\Big).$$

With $i_{S1} = I_S\cos(\omega_S t)$, $i_{S2} = I_S\cos(\omega_S t - 2\pi/3)$, and $i_{S3} = I_S\cos(\omega_S t - 4\pi/3)$

$$\vec{\mathbf{B}}_{S,1} = \vec{\mathbf{B}}_{S_1,1} + \vec{\mathbf{B}}_{S_2,1} + \vec{\mathbf{B}}_{S_3,1}$$

$$= \frac{\mu_0}{g}\frac{4}{\pi}\frac{r_R}{r}(1+\cos(\pi/6))I_S\Big(\cos(\omega_S t)\sin(\theta+\frac{\pi}{2}) + \cos(\omega_S t - 2\pi/3)\sin(\theta-\frac{2\pi}{3}+\frac{\pi}{2}) +$$

$$\cos(\omega_S t - 4\pi/3)\sin(\theta-\frac{4\pi}{3}+\frac{\pi}{2})\Big)$$

$$= \frac{\mu_0}{g}\frac{4}{\pi}\frac{r_R}{r}(1+\cos(\pi/6))\frac{3}{2}I_S\cos(\theta-\omega_S t) \quad \text{where } 1+\cos(\pi/6) = 1.866$$

Three-Phase Machine and Space Harmonics

- Triplen harmonics $\quad k = 3m, \;\; m = 1, 2, 3, \ldots$

With balanced currents, i.e., $i_{S1} + i_{S2} + i_{S3} \equiv 0$, the triplen harmonics are zero!

$$\vec{\mathbf{B}}_{S,3m} = \vec{\mathbf{B}}_{S_1,3m} + \vec{\mathbf{B}}_{S_2,3m} + \vec{\mathbf{B}}_{S_3,3m}$$

$$= \frac{\mu_0}{g}\frac{4}{\pi}\frac{r_R}{r}\frac{1+\cos\left(\frac{3m\pi}{6}\right)}{3m}\Big(i_{S1}\sin(3m(\theta+\pi/2)) + i_{S2}\sin(3m(\theta - 2\pi/3 + \pi/2)) + i_{S3}\sin(3m(\theta - 4\pi/3 + \pi/2))\Big)$$

$$= \frac{\mu_0}{g}\frac{4}{\pi}\frac{r_R}{r}\frac{1+\cos\left(\frac{3m\pi}{6}\right)}{3m}\Big(i_{S1}\sin(3m(\theta+\pi/2)) + i_{S2}\sin(3m(\theta + \pi/2)) + i_{S3}\sin(3m(\theta + \pi/2))\Big)$$

$$= \frac{\mu_0}{g}\frac{4}{\pi}\frac{r_R}{r}\frac{1+\cos\left(\frac{3m\pi}{6}\right)}{3m}(i_{S1} + i_{S2} + i_{S3})\sin(3m(\theta+\pi/2)) \equiv 0$$

Three-Phase Machine and Space Harmonics

- $\vec{\mathbf{B}}_{S5}$ **- The** $\mathbf{5}^{th}$ **harmonic**
- $i_{S1} = I_S\cos(\omega_S t), i_{S2} = I_S\cos(\omega_S t - 2\pi/3)$, and $i_{S3} = I_S\cos(\omega_S t - 4\pi/3)$

$$\begin{aligned}
\vec{\mathbf{B}}_{S5}(r,\theta,t) &= \vec{\mathbf{B}}_{S_15}(r,\theta,t) + \vec{\mathbf{B}}_{S_25}(r,\theta,t) + \vec{\mathbf{B}}_{S_35}(r,\theta,t) \\
&= \frac{\mu_0 I_S}{g}\frac{4}{\pi}\frac{r_R}{r} \times \frac{\left(1+\cos\left(\frac{5\pi}{6}\right)\right)}{5} \times \left(\sin\left(5(\theta + \frac{\pi}{2})\right)\cos(\omega_S t) + \right. \\
&\quad \left. \sin\left(5(\theta - \frac{2\pi}{3} + \frac{\pi}{2})\right)\cos(\omega_S t - \frac{2\pi}{3}) + \sin\left(5(\theta - \frac{4\pi}{3} + \frac{\pi}{2})\right)\cos(\omega_S t - \frac{4\pi}{3})\right) \\
&= \frac{\mu_0 I_S}{g}\frac{4}{\pi}\frac{r_R}{r} \times \frac{\left(1+\cos\left(\frac{5\pi}{6}\right)\right)}{5}\frac{3}{2}\cos\left(5\theta + \omega_S t\right) \\
&= \frac{3}{2}\frac{4}{\pi}\frac{\mu_0 I_S}{g}\frac{r_R}{r} \times \frac{\left(1+\cos\left(\frac{5\pi}{6}\right)\right)}{5}\cos\left(5\left(\theta + \frac{\omega_S}{5}t\right)\right)
\end{aligned}$$

- $\dfrac{\left(1+\cos\left(\frac{5\pi}{6}\right)\right)}{5} = 0.027$ and $0.027/1.866 = 0.015$
- $\vec{\mathbf{B}}_{S5}(r,\theta,t)$ has a fixed (symmetric) spatial distribution with respect to its magnetic axis located by $\theta + (\omega_S/5)t = 0$.
- Its magnetic axis rotates an the angular speed of $d\theta/dt = -(\omega_S/5)\,t$, which is in the negative (clockwise) direction.

Three-Phase Machine and Space Harmonics

- $\vec{\mathbf{B}}_{S7}$ - **The 7^{th} harmonic**
- $i_{S1} = I_S \cos(\omega_S t), i_{S2} = I_S \cos(\omega_S t - 2\pi/3)$, and $i_{S3} = I_S \cos(\omega_S t - 4\pi/3)$

$$\vec{\mathbf{B}}_{S7}(r,\theta,t) = \vec{\mathbf{B}}_{S_17}(r,\theta,t) + \vec{\mathbf{B}}_{S_27}(r,\theta,t) + \vec{\mathbf{B}}_{S_37}(r,\theta,t)$$

$$= \frac{\mu_0 I_S}{g}\frac{4}{\pi}\frac{r_R}{r} \times \frac{1+\cos(\frac{7\pi}{6})}{7} \times \Bigg(\sin\left(7(\theta + \frac{\pi}{2})\right)\cos(\omega_S t) +$$

$$\sin\left(7(\theta - \frac{2\pi}{3} + \frac{\pi}{2})\right)\cos(\omega_S t - 2\pi/3) + \sin\left(7(\theta - \frac{4\pi}{3} + \frac{\pi}{2})\right)\cos(\omega_S t - 4\pi/3)\Bigg)$$

$$= \frac{\mu_0 I_S}{g}\frac{4}{\pi}\frac{r_R}{r} \times \frac{1+\cos(\frac{7\pi}{6})}{7}\left(-\frac{3}{2}\cos\left(7\theta - \omega_S t\right)\right)$$

$$= -\frac{3}{2}\frac{4}{\pi}\frac{\mu_0 I_S}{g}\frac{r_R}{r} \times \frac{1+\cos(\frac{7\pi}{6})}{7}\cos\left(7\left(\theta - \frac{\omega_S}{7}t\right)\right).$$

- $\dfrac{1+\cos(\frac{7\pi}{6})}{7} = 0.02$ and $0.02/1.866 = 0.01072$
- $\vec{\mathbf{B}}_{S7}(r,\theta,t)$ has a fixed (symmetric) spatial distribution with respect to its magnetic axis located by $\theta - (\omega_S/7)t = 0$
- Its magnetic axis rotates an the angular speed of $d\theta/dt = (\omega_S/7)t$, which is in the positive (counterclockwise) direction.

Trigonometric Identity

Consider the expression

$$\cos(\omega_S t)\sin(\theta+\pi/2)+\cos(\omega_S t-2\pi/3)\sin(\theta-2\pi/3+\pi/2)+\cos(\omega_S t-4\pi/3)\sin(\theta-4\pi/3+\pi/2)$$

This is the same as

$$\cos(\omega_S t)\cos(\theta)+\cos(\omega_S t-2\pi/3)\cos(\theta-2\pi/3)+\cos(\omega_S t-4\pi/3)\cos(\theta-4\pi/3).$$

Using $\cos(\theta_1)\cos(\theta_2)=\frac{1}{2}\cos(\theta_1+\theta_2)+\frac{1}{2}\cos(\theta_1-\theta_2)$ the expression becomes

$$\underbrace{\frac{1}{2}\cos(\omega_S t+\theta)+\frac{1}{2}\cos(\omega_S t+\theta-4\pi/3)+\frac{1}{2}\cos(\omega_S t+\theta-8\pi/3)}_{0}+\frac{3}{2}\cos(\omega_S t-\theta).$$

This follows because

$$\begin{aligned}
&\frac{1}{2}\cos(\omega_S t+\theta)+\frac{1}{2}\cos(\omega_S t+\theta-4\pi/3)+\frac{1}{2}\cos(\omega_S t+\theta-8\pi/3)\\
&\quad=\quad\frac{1}{2}\cos(\omega_S t+\theta)+\frac{1}{2}\cos(\omega_S t+\theta-2\pi/3)+\frac{1}{2}\cos(\omega_S t+\theta-4\pi/3)\\
&\quad=\quad\frac{1}{2}\mathrm{Re}\{e^{j(\omega_S t+\theta)}+e^{j(\omega_S t+\theta-2\pi/3)}+e^{j(\omega_S t+\theta-4\pi/3)}\}\\
&\quad=\quad\frac{1}{2}\mathrm{Re}\{e^{j(\omega_S t+\theta)}(1+e^{-j2\pi/3)}+e^{-j4\pi/3})\}\\
&\quad=\quad 0
\end{aligned}$$

and $1+e^{-j2\pi/3)}+e^{-j4\pi/3}=1+(-1/2+j\sqrt{3}/2)+(-1/2-j\sqrt{3}/2)=0.$