ECE 697 Modeling and High-Performance Control of Electric Machines
HW 11 Solutions
Spring 2022

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Problem 16 i_{dref}

With

$$\psi_{dref} = \begin{cases} \psi_{d0} & \text{for} \quad |\omega| < \omega_{base} \\ \psi_{d0}\omega_{base}/\left|\omega\right| & \text{for} \quad |\omega| \geq \omega_{base} \end{cases}$$

it follows that

$$\frac{\partial \psi_{dref}}{\partial \omega} = \begin{cases} 0 & \text{for} \quad |\omega| < \omega_{base} \\ -\psi_{d0}\omega_{base}/\left|\omega\right|^2 & \text{for} \quad |\omega| \geq \omega_{base}. \end{cases}$$

i_{dref} must be chosen to satisfy

$$\frac{d\psi_{dref}}{dt} = -\frac{1}{T_R}\psi_{dref} + \frac{M}{T_R} i_{dref}$$

or

$$\frac{\partial \psi_{dref}}{\partial \omega_{ref}} \frac{d\omega_{ref}}{dt} = -\frac{1}{T_R}\psi_{dref} + \frac{M}{T_R} i_{dref}$$

so that

$$i_{dref} \triangleq \frac{T_R}{M}\left(\frac{\partial \psi_{dref}}{\partial \omega_{ref}} \alpha_{ref} + \frac{1}{T_R}\psi_{dref} \right).$$

Problem 17 *Nested Loop Control Structure*

Field Energy and Torque

Problem 18 *Torque from Conservation of Energy*

(a) Multiplying the fourth equation by $\sigma L_S i_d$, the fifth equation by $\sigma L_S i_q$, and adding gives

$$\frac{1}{2}\sigma L_S \frac{d}{dt}\left(i_d^2 + i_q^2\right) + \sigma L_S \gamma\left(i_d^2 + i_q^2\right) - (\eta M/L_R)\psi_d i_d + (n_p M/L_R)\omega\psi_d i_q = i_d u_d + i_q u_q. \tag{1}$$

Substituting $\gamma = \dfrac{M^2 R_R}{\sigma L_R^2 L_S} + \dfrac{R_S}{\sigma L_S}$, this becomes

$$\frac{1}{2}\sigma L_S \frac{d}{dt}\left(i_d^2 + i_q^2\right) + \left(\frac{M^2 R_R}{L_R^2} + R_S\right)\left(i_d^2 + i_q^2\right) - (R_R M/L_R^2)\psi_d i_d + (n_p M/L_R)\omega\psi_d i_q = i_d u_d + i_q u_q. \tag{2}$$

(b) The field energy is given by

$$\begin{aligned} W_{\text{field}}(i_{Sa}, i_{Sb}, i_{Ra}, i_{Rb}, \theta_R) &\triangleq \frac{1}{2}L_S(i_{Sa}^2 + i_{Sb}^2) + \frac{1}{2}L_R(i_{Ra}^2 + i_{Rb}^2) + M i_{Sa}\Big(+i_{Ra}\cos(\theta_R) - i_{Rb}\sin(\theta_R)\Big) \\ &\quad + M i_{Sb}\Big(+i_{Ra}\sin(\theta_R) + i_{Rb}\cos(\theta_R)\Big) \end{aligned}$$

Using

$$\begin{bmatrix} \psi_{Ra} \\ \psi_{Rb} \end{bmatrix} = L_R \begin{bmatrix} \cos(n_p\theta) & -\sin(n_p\theta) \\ \sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix} \begin{bmatrix} i_{Ra} \\ i_{Rb} \end{bmatrix} + M \begin{bmatrix} i_{Sa} \\ i_{Sb} \end{bmatrix}$$

or, equivalently,

$$\frac{1}{L_R}\begin{bmatrix} \psi_{Ra} - M i_{Sa} \\ \psi_{Rb} - M i_{Sb} \end{bmatrix} = \begin{bmatrix} \cos(n_p\theta) & -\sin(n_p\theta) \\ \sin(n_p\theta) & \cos(n_p\theta) \end{bmatrix} \begin{bmatrix} i_{Ra} \\ i_{Rb} \end{bmatrix}$$

it follows that

$$\begin{aligned} W_{\text{field}} &\triangleq \frac{1}{2}L_S(i_{Sa}^2 + i_{Sb}^2) + \frac{1}{2}L_R(i_{Ra}^2 + i_{Rb}^2) + \frac{M}{L_R} i_{Sa}\Big(\psi_{Ra} - M i_{Sa}\Big) + \frac{M}{L_R} i_{Sb}\Big(\psi_{Rb} - M i_{Sb}\Big) \\ &= \frac{1}{2}L_S(i_{Sa}^2 + i_{Sb}^2) + \frac{1}{2}\frac{1}{L_R}\Big((\psi_{Ra} - M i_{Sa})^2 + (\psi_{Rb} - M i_{Sb})^2\Big) + \frac{M}{L_R} i_{Sa}\Big(\psi_{Ra} - M i_{Sa}\Big) \\ &\quad + \frac{M}{L_R} i_{Sb}\Big(\psi_{Rb} - M i_{Sb}\Big) \\ &= \frac{1}{2}L_S(i_{Sa}^2 + i_{Sb}^2) + \frac{1}{2}\frac{1}{L_R}\left(\psi_{Ra}^2 + \psi_{Rb}^2\right) + \frac{1}{2}\frac{M^2}{L_R}\left(i_{Sa}^2 + i_{Sb}^2\right) - \frac{M}{L_R}\left(\psi_{Ra} i_{Sa} + \psi_{Rb} i_{Sb}\right) \\ &\quad + \frac{M}{L_R} i_{Sa}\Big(\psi_{Ra} - M i_{Sa}\Big) + \frac{M}{L_R} i_{Sb}\Big(\psi_{Rb} - M i_{Sb}\Big) \\ &= \frac{1}{2}L_S(i_{Sa}^2 + i_{Sb}^2) + \frac{1}{2}\frac{1}{L_R}\left(\psi_{Ra}^2 + \psi_{Rb}^2\right) - \frac{1}{2}\frac{M^2}{L_R}\left(i_{Sa}^2 + i_{Sb}^2\right) \\ &= \frac{1}{2}\sigma L_S(i_{Sa}^2 + i_{Sb}^2) + \frac{1}{2}\frac{1}{L_R}\left(\psi_{Ra}^2 + \psi_{Rb}^2\right) \\ &= \frac{1}{2}\sigma L_S(i_d^2 + i_q^2) + \frac{1}{2}\frac{1}{L_R}\psi_d^2. \end{aligned} \tag{3}$$

(c) By conservation of energy (power) it must be that

$$\frac{d}{dt}W_{\text{field}} + R_S\left(i_{Sa}^2 + i_{Sb}^2\right) + R_R(i_{Ra}^2 + i_{Rb}^2) + \tau\omega = i_{Sa}u_{Sa} + i_{Sb}u_{Sb} \tag{4}$$

where $\tau\omega$ is the mechanical power produced.

Note that by (3),

$$\frac{d}{dt}W_{\text{field}} = \frac{1}{2}\sigma L_S \frac{d}{dt}(i_d^2 + i_q^2) + \frac{1}{2}\frac{1}{L_R}\frac{d}{dt}\psi_d^2 = \frac{1}{2}\sigma L_S \frac{d}{dt}(i_d^2 + i_q^2) - \frac{R_R}{L_R^2}\psi_d^2 + R_R\frac{M}{L_R^2}\psi_d i_d.$$

Also, the power lost in the rotor windings can be written in the field-oriented coordinates as

$$\begin{aligned}
R_R(i_{Ra}^2 + i_{Rb}^2) &= \frac{1}{L_R^2}R_R\left((\psi_{Ra} - Mi_{Sa})^2 + (\psi_{Rb} - Mi_{Sb})^2\right) \\
&= \frac{1}{L_R^2}R_R\left(\psi_{Ra}^2 + \psi_{Rb}^2\right) + \frac{M^2}{L_R^2}R_R(i_{Sa}^2 + i_{Sb}^2) - 2\frac{M}{L_R^2}R_R\left(\psi_{Ra}i_{Sa} + \psi_{Rb}i_{Sb}\right) \\
&= \frac{1}{L_R^2}R_R\psi_d^2 + \frac{M^2}{L_R^2}R_R(i_d^2 + i_q^2) - 2\frac{M}{L_R^2}R_R\psi_d i_d.
\end{aligned}$$

Using these two expressions, equation (4) can be written in the field-oriented coordinate system as

$$\begin{aligned}
&\frac{d}{dt}W_{\text{field}} + R_S\left(i_d^2 + i_q^2\right) + \frac{1}{L_R^2}R_R\psi_d^2 + \frac{M^2}{L_R^2}R_R(i_d^2 + i_q^2) - 2\frac{M}{L_R^2}R_R\psi_d i_d + \tau\omega = i_d u_d + i_q u_q \\
&\implies \frac{1}{2}\sigma L_S\frac{d}{dt}(i_d^2 + i_q^2) + R_S\left(i_d^2 + i_q^2\right) + \frac{M^2}{L_R^2}R_R(i_d^2 + i_q^2) - \frac{M}{L_R^2}R_R\psi_d i_d + \tau\omega = i_d u_d + i_q u_q \\
&\implies \frac{1}{2}\sigma L_S\frac{d}{dt}\left(i_d^2 + i_q^2\right) + \left(\frac{M^2 R_R}{L_R^2} + R_S\right)\left(i_d^2 + i_q^2\right) - \frac{M}{L_R^2}R_R\psi_d i_d + \tau\omega = i_d u_d + i_q u_q.
\end{aligned}$$

Comparing this last expression with (2) gives

$$\tau\omega = (n_p M/L_R)\omega\psi_d i_q$$

so that the mechanical torque is given by

$$\tau = \frac{n_p M}{L_R}\psi_d i_q$$

and the back-emf voltage is

$$\frac{n_p M}{L_R}\psi_d\omega.$$

Observers

Problem 19 *Discretization of the Flux Observer*
Simulation problem

Problem 20 *Stability of a Discrete Flux Observer*
The solution is essentially in the statement of the problem.

Problem 21 *Discretization of the Flux Observer in the Field-Oriented Coordinate System*
Simulation problem.

Problem 22 *Discretization of the Flux Observer Equations*
The solution is essentially in the statement of the problem.

Problem 23 *Flux Observer Based on Position Measurements*

Consider the flux observer (5)

$$\frac{d\hat{\rho}}{dt} = n_p\omega + \eta M \left(-i_{Sa}\sin(\hat{\rho}) + i_{Sb}\cos(\hat{\rho}) \right) / \hat{\psi}_d$$
$$\frac{d\hat{\psi}_d}{dt} = -\eta\hat{\psi}_d + \eta M \left(i_{Sa}\cos(\hat{\rho}) + i_{Sb}\sin(\hat{\rho}) \right). \tag{5}$$

Let

$$s \triangleq \rho - n_p\theta$$

so that

$$\begin{aligned} \frac{ds}{dt} &= \eta M \left(-i_{Sa}\sin(\rho) + i_{Sb}\cos(\rho) \right) / \psi_d \\ &= \eta M \left(-i_{Sa}\sin\left(s(t) + n_p\theta(t) \right) + i_{Sb}\cos\left(s(t) + n_p\theta(t) \right)\right) / \psi_d \end{aligned}$$

and

$$\frac{d\psi_d}{dt} = -\eta\psi_d + \eta M \left(i_{Sa}\cos\left(s(t) + n_p\theta(t) \right) + i_{Sb}\sin\left(s(t) + n_p\theta(t) \right)\right)$$

Define an estimator by

$$\frac{d\hat{s}}{dt} = \eta M \left(-i_{Sa}\sin(\hat{\rho}) + i_{Sb}\cos(\hat{\rho}) \right) / \hat{\psi}_d$$

$$\frac{d\hat{\psi}_d}{dt} = -\eta\hat{\psi}_d + \eta M \left(i_{Sa}\cos(\hat{\rho}) + i_{Sb}\sin(\hat{\rho}) \right)$$

where

$$\hat{\rho}(t) \triangleq n_p\theta(t) + \hat{s}(t).$$

Note that this flux estimator requires the stator currents and rotor position, but not the rotor speed.