ECE 697 Modeling and High-Performance Control of Electric Machines
HW 2 Solutions
Spring 2022

Problem 15 *A Three Phase Generator*
(a) The flux in stator loop $1-1'$ due to the rotor's magnetic field is

$$\begin{aligned}\phi_{1-1'} = \int_{S_1} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \int_0^{\ell_1} \int_{-\pi/2}^{\pi/2} B_{R\max} \cos(\theta-\theta_R)\hat{\mathbf{r}} \cdot (r_S d\theta d\ell \hat{\mathbf{r}}) &= r_S \ell_1 B_{R\max} \sin(\theta-\theta_R)\Big|_{-\pi/2}^{\pi/2} \\ &= 2 r_S \ell_1 B_{R\max} \cos(\theta_R).\end{aligned}$$

(b) Similarly, the fluxes in stator loops $2-2'$ and $3-3'$ are

$$\begin{aligned}\phi_{2-2'} = \int_{S_2} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \int_0^{\ell_1} \int_{\pi/6}^{7\pi/6} B_{R\max} \cos(\theta-\theta_R)\hat{\mathbf{r}} \cdot (r_S d\theta d\ell \hat{\mathbf{r}}) &= r_S \ell_1 B_{R\max} \sin(\theta-\theta_R)\Big|_{\pi/6}^{7\pi/6} \\ &= 2 r_S \ell_1 B_{R\max} \cos(\theta_R - 2\pi/3) \\ \phi_{3-3'} = \int_{S_3} \vec{\mathbf{B}} \cdot d\vec{\mathbf{S}} = \int_0^{\ell_1} \int_{5\pi/6}^{11\pi/6} B_{R\max} \cos(\theta-\theta_R)\hat{\mathbf{r}} \cdot (r_S d\theta d\ell \hat{\mathbf{r}}) &= r_S \ell_1 B_{R\max} \sin(\theta-\theta_R)\Big|_{5\pi/6}^{11\pi/6} \\ &= 2 r_S \ell_1 B_{R\max} \cos(\theta_R - 4\pi/3).\end{aligned}$$

(c) The induced emfs are then

$$\begin{aligned}\xi_{1-1'} &= -\frac{d\phi_{1-1'}}{dt} = 2 r_S \ell_1 B_{R\max} \omega_R \sin(\omega_R t) \\ \xi_{2-2'} &= -\frac{d\phi_{2-2'}}{dt} = 2 r_S \ell_1 B_{R\max} \omega_R \sin(\omega_R t - 2\pi/3) \\ \xi_{3-3'} &= -\frac{d\phi_{3-3'}}{dt} = 2 r_S \ell_1 B_{R\max} \omega_R \sin(\omega_R t - 4\pi/3).\end{aligned}$$

$$\xi_{1-1'} + \xi_{2-2'} + \xi_{3-3'} = 2 r_S \ell_1 B_{R\max} \omega_R \Big(\sin(\omega_R t) + \sin(\omega_R t - 2\pi/3) + \sin(\omega_R t - 4\pi/3)\Big) \equiv 0.$$

(d) With the electric field in the air gap given by

$$\vec{\mathbf{E}}_R(\theta-\theta_R) = \omega_R B_{R\max} r_S \cos(\theta-\theta_R)\,\hat{\mathbf{z}}$$

it follows that

$$\begin{aligned}\xi_{1-1'} &= \int_{1'}^{1} \vec{\mathbf{E}}_R(\theta-\theta_R) \cdot d\vec{\boldsymbol{\ell}} \\ &= \int_{side1} (\omega_R B_{R\max} r_S \cos(\pi/2-\theta_R)\,\hat{\mathbf{z}}) \cdot (d\ell \hat{\mathbf{z}}) + \int_{side1'} (\omega_R B_{R\max} r_S \cos(-\pi/2-\theta_R)\,\hat{\mathbf{z}}) \cdot (-d\ell \hat{\mathbf{z}}) \\ &= 2\omega_R B_{R\max} r_S \ell_1 \sin(\omega_R t)\end{aligned}$$

where $\theta_R = \omega_R t$ was used.