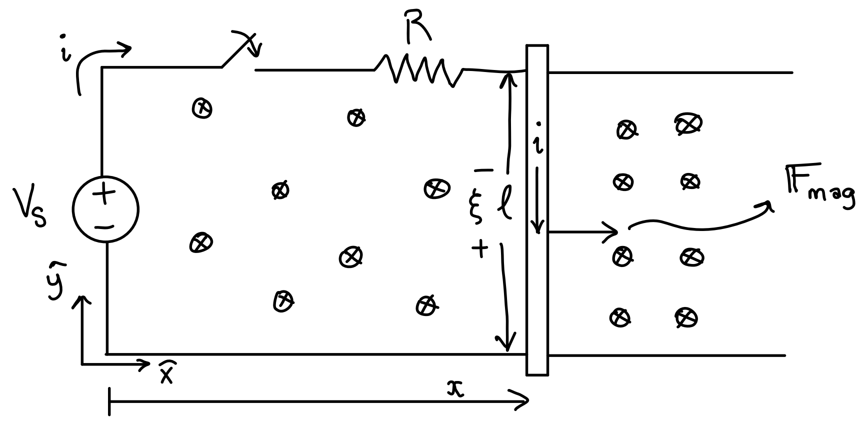
③



$\mathbb{B} = -B\hat{z}$

$\hat{n} = -\hat{z}$

(a) What is the flux through the surface?

$d\vec{S} = -dx\,dy\,\hat{z}$

$$\phi = \int_S \vec{B}\cdot d\vec{S} = \int_0^{\ell}\int_0^{x} (-B\hat{z})\cdot(-dx\,dy\,\hat{z}) = \int_0^{\ell}\int_0^{x} B\,dx\,dy = B\ell x.$$

(b) What is the direction of positive travel around this flux surface?

The positive direction of travel around the flux surface is clockwise.

(c) What is the induced electomotive force ξ in terms of B, ℓ and the speed v of the bar?

$$\xi = -\frac{d\phi}{dt} = -\frac{d}{dt}(B\ell x) = -B\ell v.$$

(d) Do V_S and ξ have the same sign convention?

When V_S and ξ are both positive, they both want to force current around the loop in the clockwise direction. In this sense they have the same sign convention.

Explain why ξ is now negative.

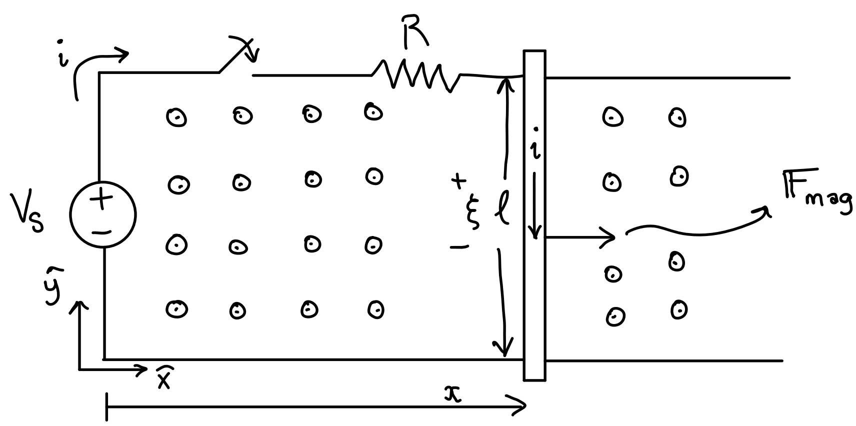
Since energy (or instantaneously power) needs to be conserved, ξ had better act to force current in the ccw direction. Hence the negative value.

Write down the equations for i and v if $L=0$.

$$\left.\begin{aligned} V_S - B\ell v &= Ri \\ m\dot{v} &= i\ell B - fv \end{aligned}\right\} \quad m\dot{v} + \left(\frac{\ell^2 B^2}{R} + f\right) v = \frac{\ell B}{R} V_S ,$$

where m is the mass of the moving rod and f is the coefficient of viscous friction.

④



$\vec{B} = B\hat{z}$, $B>0$

(a) What is the magnetic force F_{mag} on the sliding bar?

$$F_{mag} = i\vec{\ell} \times \vec{B} = i(-\ell\hat{y}) \times B\hat{z} = -i\ell B\hat{x}.$$

(b) Take the normal to the surface to be $\hat{n} = \hat{z}$. What is the flux through the surface?

$$\phi = \int_S \vec{B} \cdot d\vec{S} = \int_0^{\ell}\int_0^{x} B\hat{z} \cdot (dx\,dy\,\hat{z}) = \int_0^{\ell}\int_0^{x} B\,dx\,dy = B\ell x.$$

(c) What is the induced emf ξ?

$$\xi = -\frac{d\phi}{dt} = -B\ell v.$$

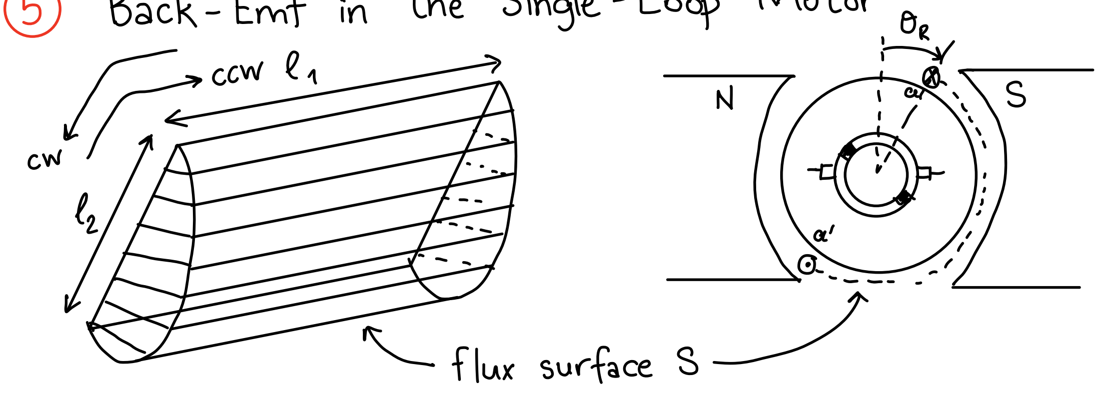
(d) What is the sign convention for the induced emf ξ drop around the loop?

If $\xi > 0$, it would act to push the current in the ccw direction.

(e) Do V_s and ξ have the same sign convention?

No, they have the opposite sign convention.

5 Back-Emf in the Single-Loop Motor



(a) With $0 < \theta_R < \pi$, and using the inward normal $\hat{n} = -\hat{r}$, compute the flux through the surface in terms of the magnitude B of the radial magnetic field in the air gap, the axial length ℓ_1, the diameter ℓ_2 of the motor, and the angle θ_R of the rotor?

$$\vec{B} = B\hat{r}, \qquad d\vec{S} = -\frac{\ell_2}{2}\, d\theta\, dz\, \hat{r}$$

$$\phi = \int_S \vec{B}\cdot d\vec{S} = \int_0^{\ell_1}\int_{\theta_R}^{\pi} (B\hat{r})\cdot\left(-\frac{\ell_2}{2}\, d\theta\, dz\, \hat{r}\right) + \int_0^{\ell_1}\int_{\pi}^{\pi+\theta_R} (-B\hat{r})\cdot\left(-\frac{\ell_2}{2}\, d\theta\, dz\, \hat{r}\right)$$

$$= \int_0^{\ell_1}\int_{\theta_R}^{\pi} -\frac{B\ell_2}{2}\, d\theta\, dz + \int_0^{\ell_1}\int_{\pi}^{\pi+\theta_R} \frac{B\ell_2}{2}\, d\theta\, dz = \frac{\ell_1\ell_2 B}{2}(\theta_R - \pi) + \frac{\ell_1\ell_2 B}{2}\theta_R$$

$$= \ell_1\ell_2 B\left(\theta_R - \pi/2\right)$$

(b) What is the positive direction of travel around the flux surface S?

The positive direction of travel is ccw.

(c) What is the emf induced in the rotor loop? What is the sign convention? Do V_S and ξ have the same convention? Explain why ξ is negative. Draw an equivalent circuit for the rotor loop current.

$$\xi = -\frac{d\phi}{dt} = -\ell_1 \ell_2 B \omega_R$$

If $\xi > 0$, then it would act to push current in the ccw-direction.

V_S and ξ have the same sign convention.

ξ is negative b/c it needs to resist V_S so as to satisfy conservation of energy.

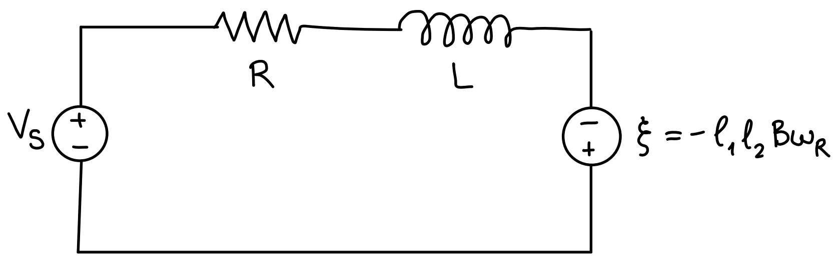


Fig. Equivalent circuit for the rotor loop current.