SELECTED SYMBOLS AND ABBREVIATIONS

Symbol or Abbreviation	Description
$\mathbf{a}$	acceleration
$\mathbf{a}_t$	tangential component of acceleration
$\mathbf{e}_t$	unit vector tangent to path
$\mathbf{e}_n$	unit vector in principal normal direction
$\mathbf{e}_i$	unit vector in direction indicated by the specific value of i
$\mathbf{f}$	resultant force
g	gravitational field intensity
$\mathbf{g}$	impulse resultant force
$\mathbf{H}_o$	angular momentum about O
I_P	moment of inertia about P
κ	local curvature of path
k_P	radius of gyration about P
M_i	mass of ith particle
M	total mass
$\mathbf{M}_P$	moment of forces about P
N	coefficient of kinetic fraction
ρ	radius of curvature of path
$\mathbf{r}$	position vector
s	distance along path
t	time
T	kinetic energy
$\mathbf{v}$	velocity
W	work
α	angular acceleration
μ	coefficient of sliding friction
ω	angular velocity

PROBLEMS

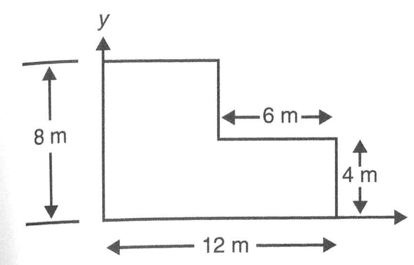

Exhibit 9.1

9.1 The uniform density flat plate shown in Exhibit 9.1 has a mass of 480 kg. The density of the body is most nearly:

a. 10 kg/m^2 c. 6.7 kg/m^2
b. 4.6 kg/m^2 d. 11.8 kg/m^2

9.2 Assuming the density of the plate in Exhibit 9.1 is 9 kg/m^2, the mass moment of inertia of the body is most nearly:

a. 10,400 kg-m^2 c. 9,600 kg-m^2
b. 18,300 kg-m^2 d. 7,400 kg-m^2

9.3 A particle is thrown vertically upward from the edge A of the ditch shown in Exhibit 9.3. If the initial velocity is 4 m/s, and the particle is known to hit the bottom, B, of the ditch exactly 6 seconds after it was released at A, determine the depth of this ditch. Neglect air resistance.

a. 24.0 m c. 200 m
b. 152.6 m d. 176.6 m

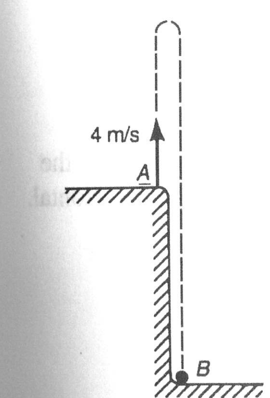

Exhibit 9.3

9.4 The slider P in Exhibit 9.4 is driven by a complex mechanism in such a way that (i) it remains on a straight path throughout; (ii) at the instant $t = 0$, the slider is located at A, (iii) at any general instant of time, the velocity of P is given by $v = (3t2 - t + 2)$ m/s. Determine the distance of P from point O when t = 2 s.

a. 26 m c. 6 m
b. 10 m d. 12 m

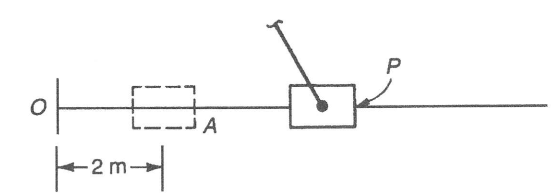

Exhibit 9.4

9.5 A particle in rectilinear motion starts from rest and maintains the acceleration profile shown in Exhibit 9.5. The displacement of the particle in the first 8 seconds is:

a. 4 m
b. 28 m
c. 24 m
d. 20 m

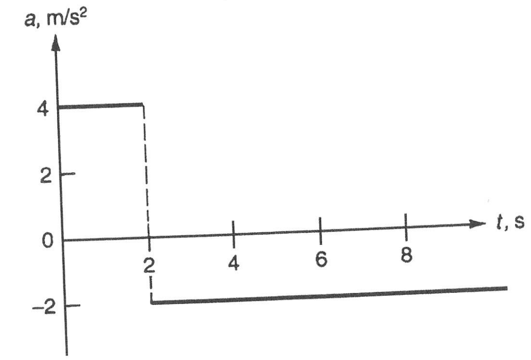

Exhibit 9.5

9.6 A ball is thrown by a player (Exhibit 9.6) from a position 2 m above the ground surface with a velocity of 40 m/s inclined at 60° to the horizontal. Determine the maximum height, H, the ball will attain.

a. 63.2 m
b. 61.2 m
c. 30.6 m
d. 31 m

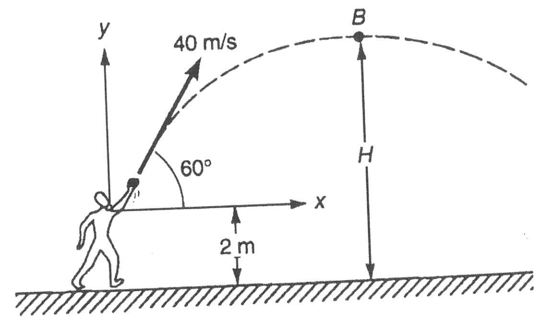

Exhibit 9.6

9.7 A golf ball (Exhibit 9.7) is struck horizontally from point A of an elevated fairway. Determine the initial speed that must be imparted to the ball if the ball is to strike the base of the flag stick on the green 140 meters away. Neglect air friction.

a. 34.3 m/s
b. 103 m/s
c. 90 m/s
d. 19.2 m/s

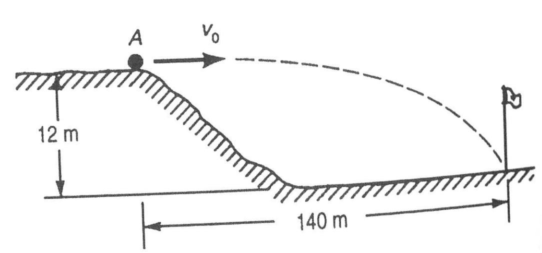

Exhibit 9.7

9.8 In Exhibit 9.8, the rod R rotates about a fixed axis at O. A small collar B is forced down the rod (toward O) at a constant speed of 3 m/s relative to the rod. If the value of θ at any given instant is $\theta = (t^2 + t - 2)$ rad, find the magnitude of the acceleration of B at time $t = 1$ second, when B is known the be 1 meter away from O.

a. 8.0 m/s²
b. 20.2 m/s²
c. 18.4 m/s²
d. 3.0 m/s²

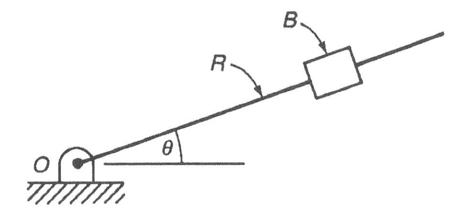

Exhibit 9.8

9.9 A rocket (Exhibit 9.9) is fired vertically upward from a launching pad at B, and its flight is tracked by radar from point A. Find the magnitude of the velocity of the rocket when $\theta = 45°$ if $\dot{\theta} = 0.1$ rad/s.

Q1

a. 36 m/s
b. 180 m/s
c. 90 m/s
d. 360 m/s

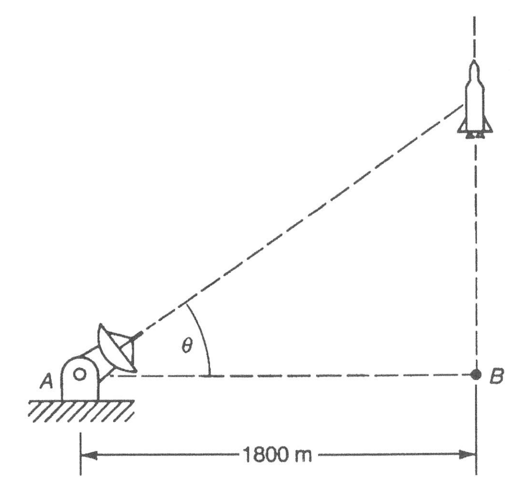

Exhibit 9.9

9.10 A particle is given an initial velocity of 50 m/s at an angle of 30° with the horizontal as shown in Exhibit 9.10. What is the radius of curvature of its path at the highest point, C?

a. 19.5 m
b. 255 m
c. 221 m
d. 191 m

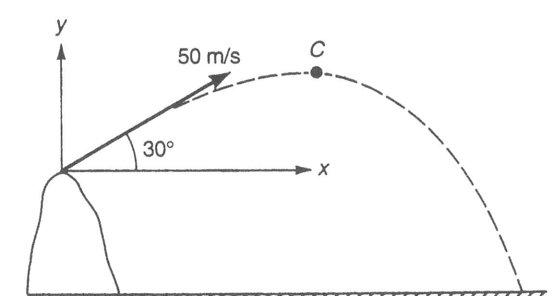

Exhibit 9.10

9.11 An automobile moves along a curved path that can be approximated by a circular arc of radius 110 meters. The driver keeps his foot on the accelerator pedal in such a way that the speed increases at the constant rate of 3 m/s². What is the total acceleration of the vehicle at the instant when its speed is 20 m/s?

a. 22.0 m/s² c. 3.0 m/s²
b. 3.6 m/s² d. 4.7 m/s²

9.12 A pilot testing an airplane at 800 kph wishes to subject the aircraft to a normal acceleration of 5 *gs* in order to fulfill the requirements of an on-board experiment. Find the radius of the circular path that would allow the pilot to do this.

a. 502 m c. 1007 m
b. 3308 m d. 1453 m

9.13 At the instant $t = 0$, the disk *D* in Exhibit 9.13 is spinning about a fixed axis through *O* at an angular speed of 300 rpm. Bearing friction and other effects are known to slow the disk at a rate that is *k* times its instantaneous angular speed, where *k* is a constant with the value $k = 1.2\ \mathrm{s}^{-1}$. Determine when (from $t = 0$) the disk's spin rate is cut in half.

a. 6.5 s c. 0.8 s
b. 13.1 s d. 0.6 s

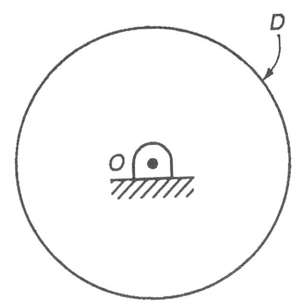

Exhibit 9.13

9.14 In Exhibit 9.14, a flywheel 2 m in radius is brought uniformly from rest up to an angular speed of 300 rpm in 30 s. Find the speed of a point P on the periphery 5 seconds after the wheel started from rest.

a. 10.5 m/s c. 62.8 m/s
b. 5.2 m/s d. 100.0 m/s

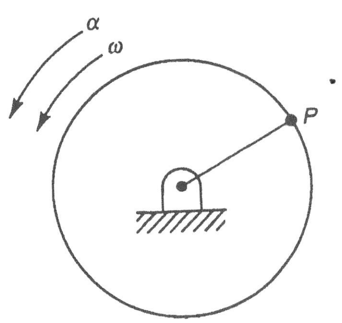

Exhibit 9.14

9.15 The block *B* (Exhibit 9.15) slides along a straight path on a horizontal floor with a constant velocity of 2 m/s to the right. At the same time, the disk, *D*, of 3-m diameter rolls without slip on the block. If the velocity of the center, *O*, of the disk is directed to the left and remains constant at 1 m/s, determine the angular velocity of the disk.

a. 0.3 rad/s counterclockwise
b. 2.0 rad/s counterclockwise
c. 0.7 rad/s counterclockwise
d. 0.7 rad/s clockwise

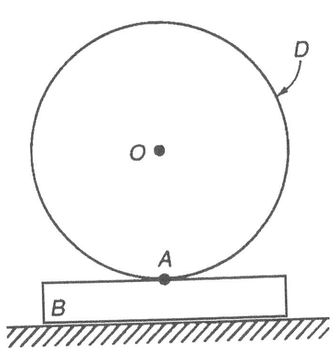

Exhibit 9.15

9.16 In Exhibit 9.16, the disk, *D*, rolls without slipping on a horizontal floor with a constant clockwise angular velocity of 3 rad/s. The rod, *R*, is hinged to *D* at *A*, and the end, *B*, of the rod touches the floor at all times. Determine the angular velocity of *R* when the line *OA* joining the center of the disk to the hinge at *A* is horizontal as shown.

a. 0.6 rad/s counterclockwise
b. 0.6 rad/s clockwise
c. 3.0 rad/s counterclockwise
d. 3.0 rad/s clockwise

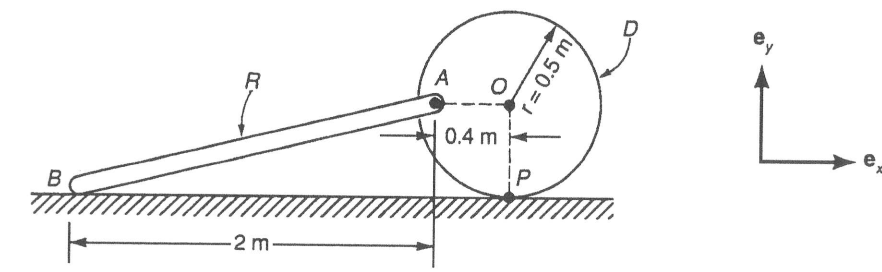

Exhibit 9.16

9.17 The fire truck in Exhibit 9.17 is moving forward along a straight path at the constant speed of 50 km/hr. At the same time, its 2-meter ladder *OA* is being raised so that the angle θ is given as a function of time by $\theta = (0.5t^2 - t)$ rad, where *t* is in seconds. The magnitude of the acceleration of the tip of the ladder when $t = 2$ seconds is:

a. 0 c. 2.0 m/s²
b. 4.0 m/s² d. 2.8 m/s²

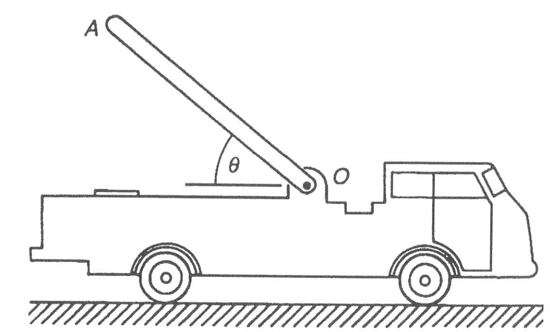

Exhibit 9.17

9.18 Q2 In Exhibit 9.18, the block B is constrained to move along a horizontal rectilinear path with a constant acceleration of 2 m/s² to the right. The slender rod, R, of length 2 m is pinned to B at O and can swing freely in the vertical plane. At the instant when $\theta = 0°$ (rod is vertical), the angular velocity of the rod is zero but its angular acceleration is 2.5 m/s² clockwise. Find the acceleration of the midpoint G of the rod at this instant ($\theta = 0°$).

a. 3.0 m/s² ← c. 2.5 m/s² ←
b. 0.5 m/s² → d. 2.5 m/s² →

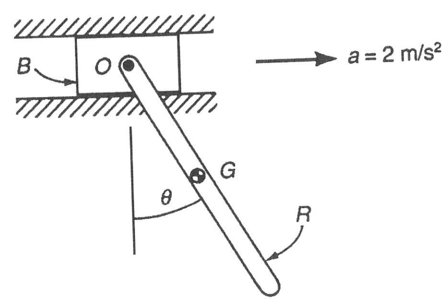

Exhibit 9.18

9.19 The block, B, in Exhibit 9.19, contains a square-cut circular groove. A particle, P, moves in this groove in the clockwise direction and maintains a constant speed of 6 m/s relative to the block. At the same time, the block slides to the right on a straight path at the constant speed of 10 m/s. Find the magnitude of the absolute velocity of P at the instant when $\theta = 30°$.

a. 8.7 m/s c. 4 m/s
b. 16 m/s d. 14 m/s

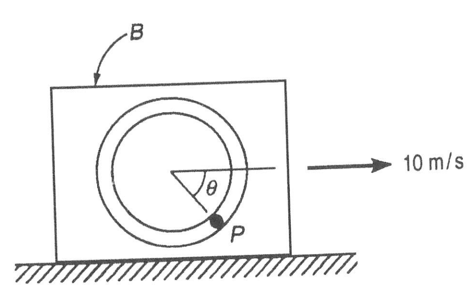

Exhibit 9.19

9.20 In Exhibit 9.20, a pin moves with a constant speed of 2 m/s along a slot in a disk that is rotating with a constant clockwise angular velocity of 5 rad/s. Calculate the absolute acceleration of this pin when it reaches the position C (directly above O). The unit vectors $\mathbf{e}_x$ and $\mathbf{e}_y$ are fixed to the disk.

a. $17.5\mathbf{e}_y$ m/s² c. $-2.5\mathbf{e}_y$ m/s²
b. $-17.5\mathbf{e}_y$ m/s² d. $-22.5\mathbf{e}_y$ m/s²

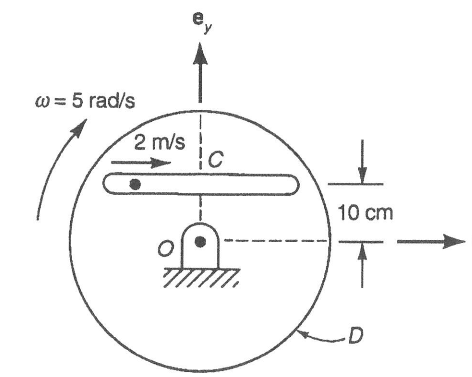

Exhibit 9.20

9.21 In Exhibit 9.21, a particle P of mass 5 kg is launched vertically upward from the ground with an initial velocity of 10 m/s. A constant upward thrust $T = 100$ newtons is applied continuously to P, and a downward resistive force $R = 2z$ newtons also acts on the particle, where z is the height of the particle above the ground. Determine the maximum height attained by P.

a. 6.0 m c. 15.8 m
b. 45.5 m d. 55.5 m

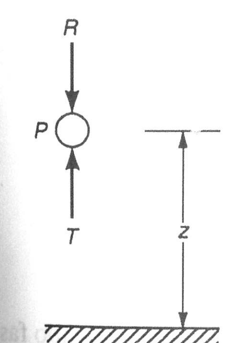

Exhibit 9.21

9.22 Determine the force P required to give the block shown in Exhibit 9.22 an acceleration of 2 m/s² up the incline. The coefficient of kinetic friction between the block and the incline is 0.2.

a. 39.2 N c. 44.6 N
b. 21.9 N d. 49.8 N

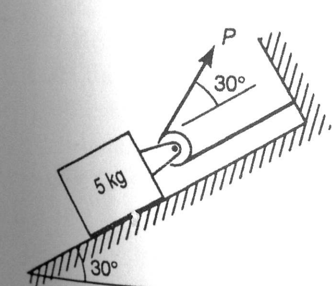

Exhibit 9.22

9.23 In Exhibit 9.23 the rod R rotates in the vertical plane about a fixed axis through the point O with a constant counterclockwise angular velocity of 5 rad/s. A collar B of mass 2 kg slides down the rod (toward O) so that the distance between B and O decreases at the constant rate of 1 m/s. At the instant when $\theta = 30°$ and $r = 400$ mm, determine the magnitude of the applied force P. The coefficient of kinetic friction between B and R is 0.1.

Q3

a. 9.9 N c. 10.5 N
b. 11.9 N d. 0.3 N

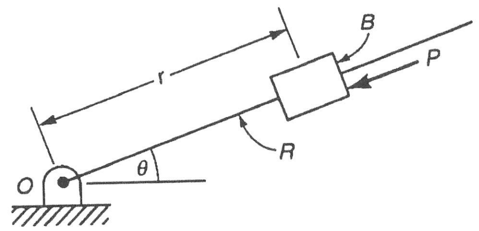

Exhibit 9.23

9.24 The 3-kg collar in Exhibit 9.24 slides down the smooth circular rod. In the position shown, its velocity is 1.5 m/s. Find the normal force (contact force) the rod exerts on the collar.

a. 12.2 N c. 19.2 N
b. 24.4 N d. 12.2 N

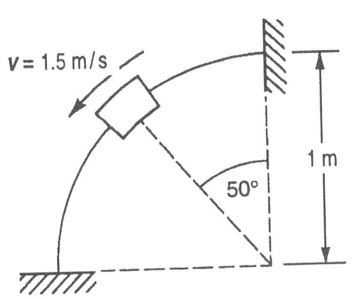

Exhibit 9.24

9.25 Forklift vehicles, Exhibit 9.25, tend to roll over if they are driven too fast while turning. For a vehicle of mass m with a mass center that describes a circle of radius R, find the relationship between the forward speed u and the vehicle dimensions and path radius at the onset of tipping.

a. $u = (Rgb/H)^{0.5}$ c. $u = (gh)^{0.5}$
b. $u = (RgH/b)^{0.5}$ d. $u = (bg)^{0.5}$

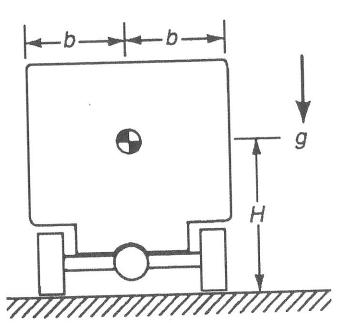

Exhibit 9.25

9.26 A toy rocket of mass 1 kg is placed on a horizontal surface, and the engine is ignited (Exhibit 9.26). The engine delivers a force equal to $(0.25 + 0.5t)$ N, where t is time in seconds, and the coefficient of friction between the rocket and the surface is 0.01. Determine the velocity of the rocket 7 seconds after ignition.

a. 14.0 m/s c. 13.3 m/s
b. 3.7 m/s d. 26.3 m/s

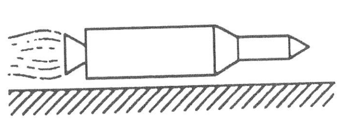

Exhibit 9.26

9.27 A 2000-kg pickup truck is traveling backward down a 10° incline at 80 km/hr when the driver notices through his rearview mirror an object on the roadway. He applies the brakes, and this results in a constant braking (retarding) force of 4000 N. How long does it take the truck to stop?

a. 11.1 s c. 2.3 s
b. 74.9 s d. 13.0 s

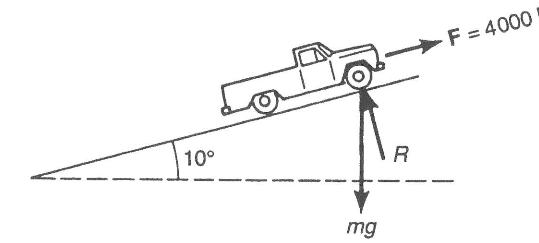

Exhibit 9.27

9.28 In Exhibit 9.28, a particle C of mass 2 kg is sliding down a smooth incline with a velocity of 3 m/s when a horizontal force $P = 15$ N is applied to it. What is the distance traveled by C between the instant when P is first applied and the instant when the velocity of C becomes zero?

a. 1.7 m c. 2.8 m
b. 5.6 m d. 0.9 m

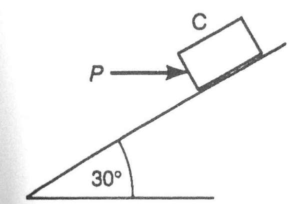

Exhibit 9.28

9.29 A particle moves in a vertical plane along the path ABC shown in Exhibit 9.29. The portion AB of the path is a quarter-circle of radius r and is smooth. The portion BC is horizontal and has a coefficient of friction μ. If the particle has mass m and is released from rest at A, determine the horizontal distance H that the particle will travel along BC before coming to rest.

a. μr c. r/μ
b. $2r$ d. $2r/\mu$

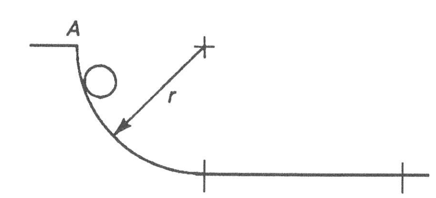

Exhibit 9.29

9.30 In Exhibit 9.30, a block of mass 2 kg is pressed against a linear spring of constant $k = 200$ N/m through a distance Δ on a horizontal surface. When the block is released at A, it travels along the straight horizontal path ADB and traverses point B with a velocity of 1 m/s. If the coefficient of kinetic friction between the block and the floor is 0.2, find Δ.

Q4

a. 0.22 m c. 0.26 m
b. 0.12 m d. 0.08 m

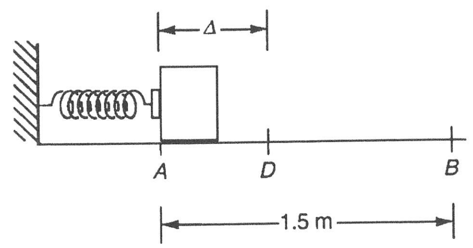

Exhibit 9.30

9.31 In Exhibit 9.31, a 6-kg block is released from rest on a smooth inclined plane as shown. If the spring constant $k = 1000$ N/m, determine how far the spring is compressed. Assume the acceleration of gravity, $g = 10$ m/s².

a. 0.40 m c. 0.83 m
b. 0.45 m d. 3.96 m

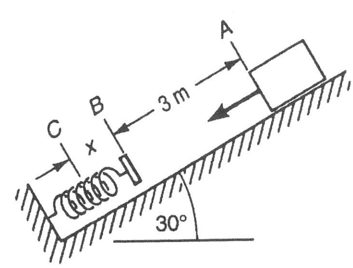

Exhibit 9.31

9.32 A train of joyride cars full of children in an amusement park is pulled by an engine along a straight-level track. It then begins to climb up a 5° slope. At a point B, 50 m up the grade when the velocity is 32 km/h, the last car uncouples without the driver noticing (Exhibit 9.32). If the total mass of the car with its passengers is 500 kg and the track resistance is 2% of the total vehicle weight, calculate the total distance up the grade where the car stops at point C.

a. 260 m c. 48.7 m
b. 37.6 m d. 87.6 m

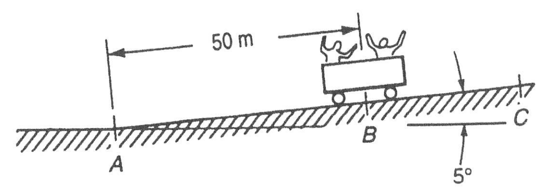

Exhibit 9.32

9.33 Two identical rods, each of mass 4 kg and length 3 m, are rigidly connected as shown in Exhibit 9.33. Determine the moment of inertia of the rigid assembly about an axis through the point A and perpendicular to the plane of the paper.

Q5

a. 19 kg-m² c. 18 kg-m²
b. 23 kg-m² d. 15 kg-m²

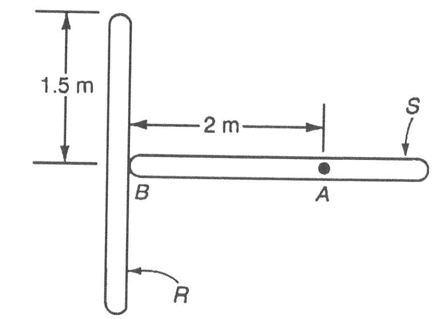

Exhibit 9.33

9.34 A torque motor, represented by the box in Exhibit 9.34, is to drive a thin steel disk of radius 2 m and mass 1.5 kg around its shaft axis. Ignoring the bearing friction about the shaft and the shaft mass, find the angular speed of the disk after applying a constant motor torque of 5 N-m for 5 seconds. The initial angular velocity of the shaft is 1 rad/s.

a. 8.3 rad/s c. 5.2 rad/s
b. 7.3 rad/s d. 9.3 rad/s

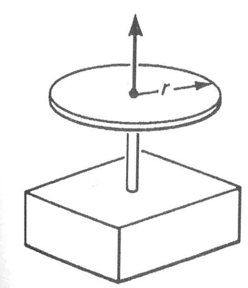

Exhibit 9.34

9.35 In Exhibit 9.35, the uniform slender rod R is hinged to a block B that can slide horizontally. Determine the horizontal acceleration α that must be given to B in order to keep the angle θ constant at 10°, balancing the rod in a tilted position.

a. 1.73 m/s² c. 9.81 m/s²
b. 0 d. 9.66 m/s²

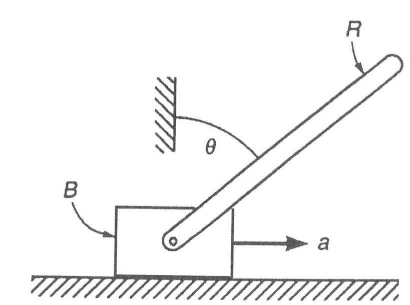

Exhibit 9.35

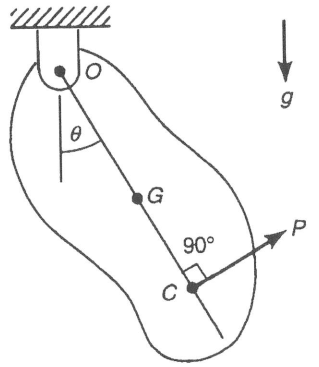

Exhibit 9.36

9.36 In Exhibit 9.36, a force P of constant magnitude is applied to the physical pendulum at point C and remains perpendicular to OC at all times. The pendulum moves in the vertical plane and has mass 3 kg; its mass center is located at G, and the distances are $OG = 1.5$ m, $OC = 2$ m. Also, $P = 10$ N and the radius of gyration of the pendulum about an axis through C and perpendicular to the plane of motion is 0.8 m. Determine the angular acceleration of the pendulum when $\theta = 30°$.
a. 5.31 rad/s^2 counterclockwise
b. 5.31 rad/s^2 clockwise
c. 0.26 rad/s^2 counterclockwise
d. 0.26 rad/s^2 clockwise

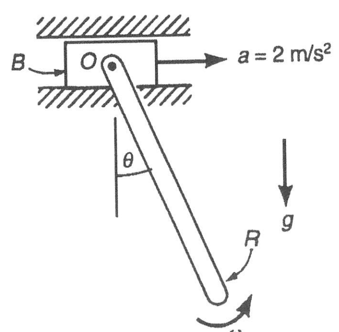

Exhibit 9.37

9.37 In Exhibit 9.37, the block B moves along a straight horizontal path with a constant acceleration of 2 m/s^2 to the right. The uniform slender rod R of mass 1 kg and length 2 m is connected to B through a frictionless hinge and swings freely about O as B moves. Determine the horizontal component of the reaction force at O on the rod when $\theta = 30°$ and $\omega = 2$ rad/s counterclockwise.
a. 4.31 N →
b. 4.31 N ←
c. 6.31 N →
d. 6.31 N ←

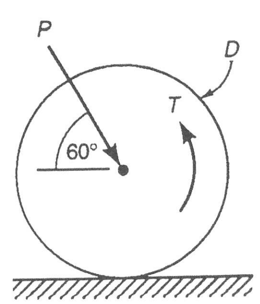

Exhibit 9.38

9.38 Q6 In Exhibit 9.38, a homogeneous cylinder rolls without slipping on a horizontal floor under the influence of a force $P = 6$ N and a torque $T = 0.5$ N-m. The cylinder has radius 1 m and mass 2 kg. If the cylinder started from rest, what is its angular velocity after 10 seconds?
a. 8.3 rad/s
b. 6.8 rad/s
c. 1.7 rad/s
d. 0.68 rad/s

Exhibit 9.39

9.39 A slender rod of length 2 m and mass 3 kg is released from rest in the horizontal position (Exhibit 9.39) and swings freely (no hinge friction). Find the angular velocity of the rod when it passes a vertical position.
a. 4.43 rad/s
b. 3.84 rad/s
c. 7.68 rad/s
d. 5.43 rad/s

9.40 The uniform slender bar of mass 2 kg and length 3 m is released from rest in the near-vertical position as shown in Exhibit 9.40, where the torsional spring is undeformed. The rod is to rotate clockwise about O and come gently to rest in the horizontal position. Determine the stiffness k of the torsional spring that would make this possible. The hinge is smooth.
a. 47.8 N-m/rad
b. 37.5 N-m/rad
c. 0.7 N-m/rad
d. 23.8 N-m/rad

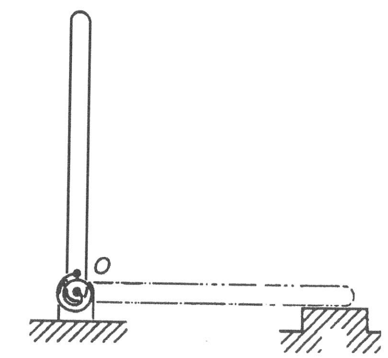

Exhibit 9.40

9.41 Q7 A solid homogeneous cylinder is released from rest in the position shown in Exhibit 9.41 and rolls without slip on a horizontal floor. The cylinder has a mass of 12 kg. The spring constant is 2 N/m, and the unstretched length of the spring is 3 m. What is the angular velocity of the cylinder when its center is directly below the point O?
a. 1.33 rad/s
b. 1.63 rad/s
c. 1.78 rad/s
d. 2.31 rad/s

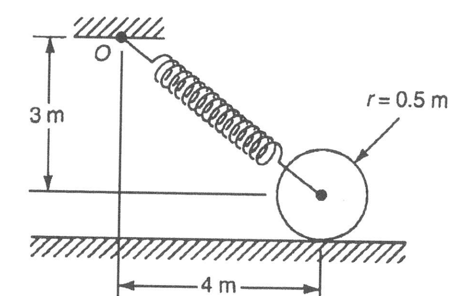

Exhibit 9.41

SOLUTIONS

9.1 c. The density of the plate is:

$$\rho = \frac{\text{mass}}{\text{area}} = \frac{480\text{ kg}}{\left[(8\times 6)+(4\times 6)\right]\text{m}^2} = \frac{480\text{ kg}}{72\text{ m}^2} = 6.67\text{ kg/m}^2$$

9.2 a. Assuming that the density of the plate is 9 kg/m², the mass moment of inertia of the flat plate may be found by:

$$I_x = \frac{1}{3}mh^2 = \frac{1}{3}(\rho Ah^2) = \frac{1}{3}\rho(6)(8)^3 + \frac{1}{3}\rho(6)(4)^3$$

$$= \frac{1}{3}(9\text{ kg/m}^2)(6\text{ m})(8\text{ m})^3 + \frac{1}{3}(9\text{ kg/m}^2)(6\text{ m})(4\text{ m})^3$$

$$= 9216 + 1152 = 10{,}368\text{ kg}-\text{m}^2$$

9.3 b. Let the depth of the ditch be h, and set up a vertical s-axis, positive upwards with the origin at A (Exhibit 9.3a). Then, for motion between A and B, $s_0 = 0$, $s = -h$, $v_0 = 4$ m/s, $a = -9.81$ m/s², and $t = 6$ s. Substituting these values in the relationship $s = s_0 + v_0t + (at^2)/2$ yields $-h = 0 + 4(6) - 9.81(6)^2/2$ or $h = 152.6$ m.

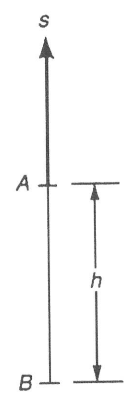

Exhibit 9.3a

9.4 d. Set up a horizontal s-axis with origin at O, and positive to the right (Exhibit 9.4a). Then, $sO = 2$ m, and $v = ds/dt$, so

$$\int_{s_0}^{s} ds = \int_0^t v\,dt = \int_0^t (3t^2 - t + 2)\,dt$$

or $s = s_0 + t^3 - t^2/2 + 2t$. Substituting values into the equation yields $s(2\text{s}) = 12$ m.

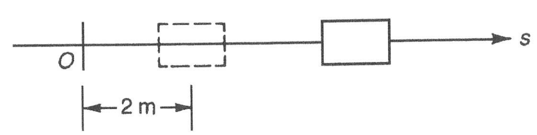

Exhibit 9.4a

9.5 d. The velocity-time curve for this particle is shown below the acceleration curve in Exhibit 9.5a. (Velocity at any given instant t_1 equals the area under the acceleration curve from time 0 to the time t_1, plus the initial velocity). Any area above the $a = 0$ line is counted as positive, and any area below is counted as negative.

The displacement D is the total area under the velocity curve between $t = 0$ and $t = 8$ s. The area of a triangle is $0.5(b)(h)$. Hence, $D = 0.5(6)(8) - 0.5(2)(4) = 20$ m.

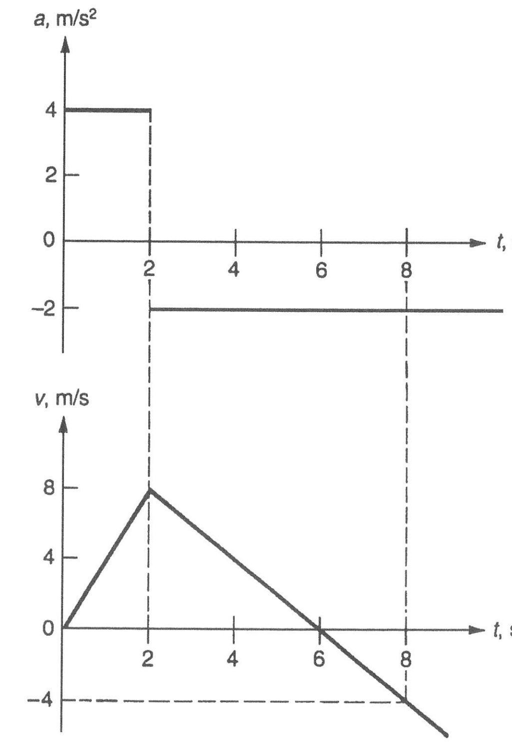

Exhibit 9.5a

9.6 a. When the ball reaches its highest position, B, the vertical component of the velocity of the ball is zero. Applying Equation (9.12) vertically,

$$v_y^2 = v_{yo}^2 + 2a_y(y - y_0)$$
$$0 = (40\ \sin\ 60°)^2 - 2(9.81)(y - y_0)$$

Thus, $y - y_0 = 61.2$ m and $H = 63.2$ m.

9.7 c. Refer to Exhibit 9.7a.

Horizontal Motion:

$$x = x_0 + v_{x0}t + (a_x t^2)/2$$
$$140 = 0 + v_0 t + 0$$
$$t = 1.56 \text{ s}$$

Vertical Motion:

$$y = y_0 + y_{y0}t + (a_y t^2)/2$$
$$-12 = 0 + 0 - 0.5(9.81)t^2$$
$$v_o = 140/\text{t} = 140/1.56 = 89.7 \text{ m/s}$$

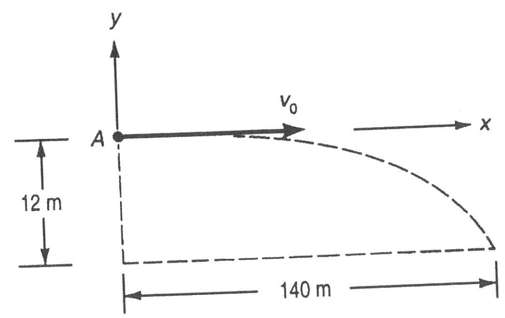

Exhibit 9.7a

9.8 c. In Exhibit 9.8a, we have

$$r = 1 \text{ m}; \qquad \dot{r} = -3 \text{ m/s}; \qquad \ddot{r} = 0$$

Since $\theta = (t^2 + t - 2)$ rad, differentiation gives

$$\dot{\theta} = (2t+1) \text{ rad/s; at } t = 1 \text{ s, } \dot{\theta} = 2(1) + 1 = 3 \text{ rad/s}$$

Also, $\ddot{\theta} = 2 \text{ rad/s}^2$.

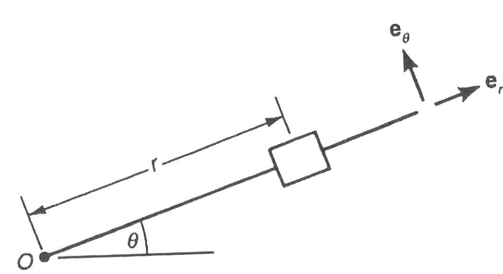

Exhibit 9.8a

Acceleration of the collar is therefore given by

$$\begin{aligned}\mathbf{a} &= (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \\ &= [0 - 1(3)^2]\mathbf{e}_r + [1(2) + 2(-3)(3)]\mathbf{e}_\theta \ \mathbf{m/s^2} \\ &= [-9\mathbf{e}_r - 16\mathbf{e}_\theta]\ \mathbf{m/s^2}\end{aligned}$$

Hence, $|a| = [(-9)^2 + (-16)^2]^{0.5} = 18.4 \text{ m/s}^2$.

9.9 d. Refer to Exhibit 9.9a. The velocity is

$$v = \left(v_r^2 + v_\theta^2\right)^{0.5} \quad \text{where } v_r = \dot{r} \text{ and } v_\theta = r\dot{\theta}$$

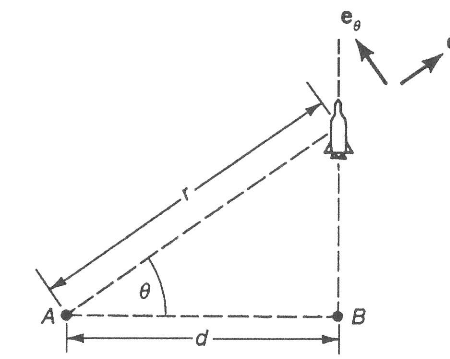

Exhibit 9.9a

Now, $r = d/\cos\theta$ so

$$v_r = \dot{r} = \frac{d\dot{\theta}\sin\theta}{\cos^2\theta} = \frac{d\dot{\theta}\tan\theta}{\cos\theta}$$

and

$$v_\theta r\dot{\theta} = \frac{d\dot{\theta}}{\cos\theta}$$

Thus

$$v^2 = v_r^2 + v_\theta^2 = \frac{d^2\dot{\theta}^2(\tan^2\theta + 1)}{\cos^2\theta} = \frac{d^2\dot{\theta}^2}{\cos^4\theta}; \quad \ddot{r} = 0$$

and

$$v = \frac{d\dot{\theta}}{\cos^2\theta} = \frac{1800(0.1)(2)}{1} \text{ m/s} = 360 \text{ m/s}$$

9.10 d. At the highest point, the vertical component of velocity is zero, and the acceleration is normal to the path. Thus, $v = v_x = v_{x0} + a_x t = 50$ cos 30° + 0. Also, the normal component of the acceleration is $a_n = -a_y = 9.81 \text{ m/s}^2$. But $a_n = v^2/\rho$. Hence $\rho = v^2/a_n$ = (50 cos 30°)²/9.81 = 191 m.

9.11 d. The tangential acceleration is given as $a_t = 3 \text{ m/s}^2$; the normal acceleration is $a_n = v^2/\rho = [(20)^2/110] \text{ m/s}^2 = 3.6 \text{ m/s}^2$. Hence, the total acceleration is $a = [(3)^2 + (3.6)^2]^{0.5} \text{ m/s}^2 = 4.7 \text{ m/s}^2$.

9.12 c. The normal acceleration is given by $a_n = v^2/\rho$. So, $\rho = v^2/a_n$. Substituting values (converted to consistent units), we obtain

$$\rho = \frac{[800(1000)]^2}{[60(60)]^2(5)(9.81)} = 1007 \text{ m}$$

9.13 **d.** At any given instant, the angular acceleration of the disk is $\alpha = -k\omega = d\omega/dt$. So,

$$\int_{\omega_0}^{\omega} \frac{d\omega}{\omega} = -k\int_0^t dt$$

$$\ln \omega|_{\omega_0}^{\omega} = \ln(\omega/\omega_0) = -kt \text{ and } t = -(1/k)\ln(\omega/\omega_0) = -(1/1.2)\ln(0.5) = 0.6\,s$$

9.14 **a.** Initially, $\omega_0 = 0$. At $t = 30$ s, $\omega = 300$ rpm $= [300\,(2\pi)/60]$ rad/s $= 10\pi$ rad/s. For uniformly accelerated rotational motion,

$$\omega = \omega_0 + \alpha t$$
$$\alpha = (\omega - \omega_0)/t = (10\pi - 0)/30 \text{ rad/s}^2 = \pi/3 \text{ rad/s}^2$$
$$\omega(5) = \omega_0 + \alpha t = 0 + (\pi/3)(5) \text{ rad/s} = 5\pi/3 \text{ rad/s}$$
$$v_P(5) = \omega(5)r = (5\pi/3)\,(2) \text{ m/s} = 10.5 \text{ m/s}$$

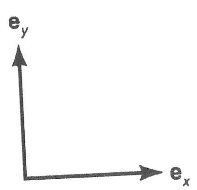

Exhibit 9.15a

9.15 **b.** Adopt the coordinate system of Exhibit 9.15a. Rolling of the disk without slip on B implies that the velocity of the point A, viewed as a point on D, equals the velocity of A viewed as a point on B. Hence, $\mathbf{v}_A = 2\mathbf{e}_x$ m/s. Since A and O are points of the same rigid body D, $\mathbf{v}_O = \mathbf{v}_A + \omega \times \mathbf{r}_{AO}$, or $-1\mathbf{e}_x = 2\mathbf{e}_x + \omega\mathbf{e}_z \times (1.5)\mathbf{e}_y = (2 - 1.5\omega)\mathbf{e}_x$. Hence, $-1 = 2 - 1.5\omega$ or $\omega = 2$ rad/s. The positive sign indicates a counterclockwise rotation.

9.16 **a.** In the current configuration, $\mathbf{v}_P = 0$. P and A are points on D, so

$$\mathbf{v}_A = \mathbf{v}_P + \omega_D \times \mathbf{r}_{PA} = 0 - 3\mathbf{e}_z \times (0.5\mathbf{e}_y - 0.4\mathbf{e}_x)$$
$$= (1.5\mathbf{e}_x + 1.2\mathbf{e}_y) \text{ m/s}$$

Similarly, because B and A are points on R, $\mathbf{v}_B = \mathbf{v}_A + \omega_R \times \mathbf{r}_{AB}$. Therefore,

$$v_B\mathbf{e}_x = 1.5\mathbf{e}_x + 1.2\mathbf{e}_y + \omega_R\mathbf{e}_z \times (-2\mathbf{e}_x - 0.5\mathbf{e}_y)$$
$$= (1.5 + 0.5\omega_R)\mathbf{e}_x + (1.2 - 2\omega_R)\mathbf{e}_y$$

Equating the coefficients of $\mathbf{e}_y$ yields $0 = 1.2 - 2\omega_R$, or $\omega_R = 0.6$ rad/s. The positive sign indicates that ω_R is in the positive $\mathbf{e}_z$ direction, so the rotation is counterclockwise.

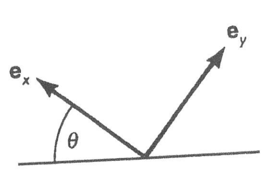

Exhibit 9.17a

9.17 **d.** Referring to the coordinate system in Exhibit 9.17a, the accelerations are

$$\mathbf{a}_A = \mathbf{a}_O + \alpha \times \mathbf{r}_{OA} + \omega \times (\omega \times \mathbf{r}_{OA})$$
$$\mathbf{a}_O = 0 \text{ (constant velocity)}$$

Since $\theta = (0.5t^2 - t)$ rad, $\dot{\theta} = (t-1)$rad/s and $\ddot{\theta} = 1$ rad/s^2.

So, at $t = 2$ s, $\omega = \dot{\theta}\mathbf{e}_z = 1\mathbf{e}_z$ rad/s. Since $a = \ddot{\theta}\,\mathbf{e}_z = 1\mathbf{e}_z$ rad/s^2,

$$\mathbf{a}_A = 0 + \mathbf{e}_z \times (2\mathbf{e}_x) + \mathbf{e}_z \times [\mathbf{e}_z \times (2\mathbf{e}_x)] = (2\mathbf{e}_y - 2\mathbf{e}_x) \text{ m/s}^2$$

Hence, $|\mathbf{a}_A| = (2^2 + 2^2)^{0.5}$ m/s^2 = 2.8 m/s^2.

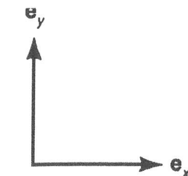

Exhibit 9.18a

9.18 **b.** With the coordinate system in Exhibit 9.18a, we can write

$$\mathbf{a}_O = 2\mathbf{e}_x \text{ m/s}^2;\ \omega = 0;\ \alpha = -2.5\mathbf{e}_z \text{ rad/s}^2$$

Now, $\mathbf{a}_G = \mathbf{a}_O + \mathbf{a} \times \mathbf{r}_{OG} + \omega \times (\omega \times \mathbf{r}_{OG})$ where $\mathbf{r}_{OG} = -1\mathbf{e}_y$ m. Hence

$$\mathbf{a}_G = [2\mathbf{e}_x - 2.5\mathbf{e}_z \times (-1)\mathbf{e}_y + 0] \text{ m/s}^2 = -0.5\mathbf{e}_x \text{ m/s}^2$$

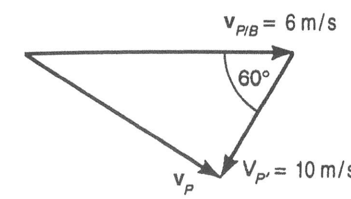

Exhibit 9.19a

9.19 **a.** We know that $\mathbf{v}_P = \mathbf{v}_{P/B} + \mathbf{v}_{P'}$ where $\mathbf{v}_{P/B}$ is the velocity of P relative to B and $\mathbf{v}_{P'}$ is the velocity of the point P' of the block that coincides with P at the instant under consideration (coincident point velocity). Here, $|\mathbf{v}_{P/B}| = 6$ m/s and $|\mathbf{v}_{P'}| = 10$ m/s (velocity of block). Because $\mathbf{v}_P$ is the vector sum of $\mathbf{v}_{P/B}$ and $\mathbf{v}_{P'}$, as shown in Exhibit 9.19a, we can use the law of cosines,

$$(\mathbf{v}_P)^2 = 10^2 + 6^2 - 2(10)(6)\cos 60° = 76$$
$$\mathbf{v}_P = 8.7 \text{ m/s}$$

9.20 **d.**

$$\mathbf{a}_P = \mathbf{a}_{P/D} + \mathbf{a}_{P_e}\rho' + \mathbf{a}_C$$

Here,
$\mathbf{a}_{P/D}$ = relative acceleration = 0
$\mathbf{a}_{P'}$ = acceleration of the point of D that is coincident with P at the instant under consideration:

$$\mathbf{a}_{P'} = \mathbf{a}_O + \alpha \times \mathbf{r}_{OC} + \omega \times (\omega \times \mathbf{r}_{OC})$$
$$= 0 + 0 + (-5\mathbf{e}_z) \times [(-5\mathbf{e}_z) \times (0.1\mathbf{e}_y)]$$
$$= -2.5\mathbf{e}_y \text{ m/s}^2$$

$\mathbf{a}_C$ = Coriolis acceleration = $2\omega \times \mathbf{v}_{P/D}$
$\mathbf{a}_C = 2(-5\mathbf{e}_z) \times 2\mathbf{e}_x = -20\mathbf{e}_y$ m/s^2

Finally,

$$\mathbf{a}_P = [-2.5\mathbf{e}_y - 20\mathbf{e}_y] \text{ m/s}^2 = -22.5\mathbf{e}_y \text{ m/s}^2$$

9.21 d. The free-body diagram is shown in Exhibit 9.21a. Apply Newton's second law:

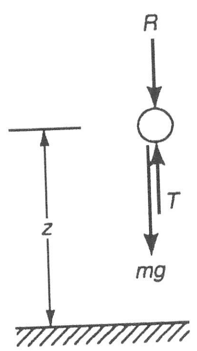

Exhibit 9.21a

$$\sum F_z = ma_z$$
$$T - R - mg = ma$$

$$a = [(T-R)/m] - g = [(100-2z)N/5\text{ kg}] - 9.81\text{ m/s}^2$$

so

$$a = 10.2 - 0.4z = \frac{v\,dv}{dz}$$

and

$$\int_0^H (10.2 - 0.4z)dz = \int_{10}^0 v\,dv$$

where H is the highest height attained. Note also that $v = 0$ at this height. Integration yields,

$$10.2H - 0.2H^2 = [v^2/2]_{10}^0 = -50$$

or

$$0.2H^2 - 10.2H - 50 = 0$$

Solving this quadratic (and discarding the negative value) gives the maximum height,

$$H = 55.5\text{ m}$$

9.22 b. Refer to Exhibit 9.22a. Apply Newton's second law in the x and y directions.

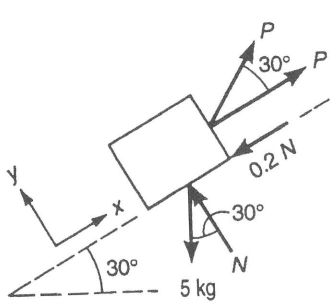

Exhibit 9.22a

$$\sum F = ma_x \qquad \text{(i)}$$

$$P + P\cos 30° - 0.2N - 5(9.81)\sin 30° = 5(2)$$

$$\sum F_y = 0 \qquad \text{(ii)}$$
$$N + P\sin 30° - 5(9.81)\cos 30° = 0$$

Solving Equations (i) and (ii) simultaneously yields $P = 21.9$ N.

9.23 a. In Exhibit 9.23a, apply Newton's second law in the radial and transverse directions.

$$\sum F_\theta = ma_\theta$$
$$N - mg\cos\theta = m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) \qquad \textbf{(i)}$$
$$\sum F_r = ma_r$$

$$\mu N - P - mg\sin\theta = m(\ddot{r} - r\dot{\theta}^2) \qquad \textbf{(ii)}$$

Substitute values into Equations (i) and (ii):

$$N - 2(9.81)\cos 30° = 2[0 + 2(-1)(5)] \qquad \textbf{(iii)}$$
$$0.1N - P - 2(9.81)\sin 30° = 2[0 - 0.4(5)^2] \qquad \textbf{(iv)}$$

From Equation (iii), $N = -3.0$ newtons. Substituting this value into Equation (iv) gives $P = 9.9$N.

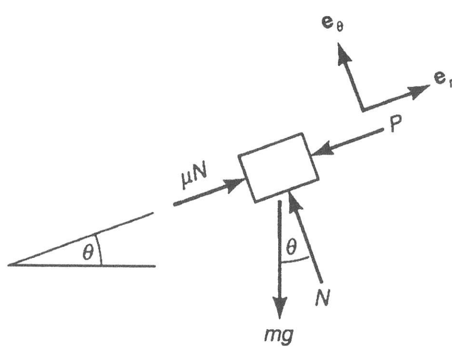

Exhibit 9.23a

9.24 a. Apply Newton's second law to the diagrams in Exhibit 9.24:

$$\sum F_n = ma_n$$

$$mg\cos\theta - N = mv^2/\rho$$

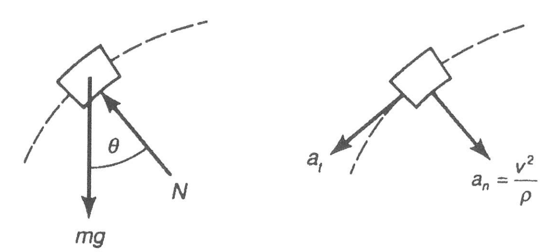

Exhibit 9.24a

or

$$\begin{aligned} N &= m[g\cos\theta - v^2/\rho] \\ &= (3)[9.81\cos 50° - 1.5^2/1] \\ &= 12.2\text{ N} \end{aligned}$$

The positive sign indicates that N is directed as shown in the free-body diagram, Exhibit 9.24a.

9.25 **a.** At the onset of tipping, the free-body diagram and the inertia force diagram are as shown in Exhibit 9.25a. Take moments about point A:

$$mgb = m(u^2/R)H$$

Hence,

$$u = (Rgb/H)^{0.5}$$

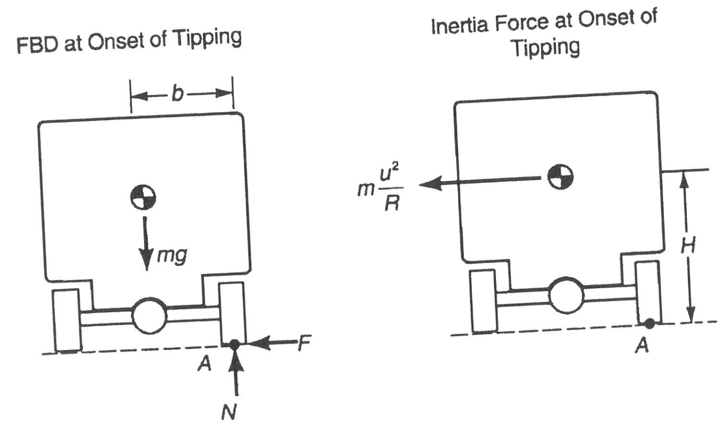

Exhibit 9.25a

9.26 **c.** In Exhibit 9.26a, the sum of the forces in the vertical direction yields

$$N = mg$$

Apply the impulse-momentum principle in the horizontal direction [Equation (9.32)]:

$$\int_{t_1}^{t_2} F_{\text{horiz}}\, dt = mv_2 - mv_1$$

$$F_{\text{horiz}} = (0.25 + 0.5t) - 0.1N$$

Thus

$$\int_0^7 [0.25 + 0.5t - (0.01)(9.81)]dt = (1)v_2 - 0$$

or

$$0.25t + 0.25t^2 - 0.098t\Big|_0^7 = v_2 = 13.3 \text{ m/s}$$

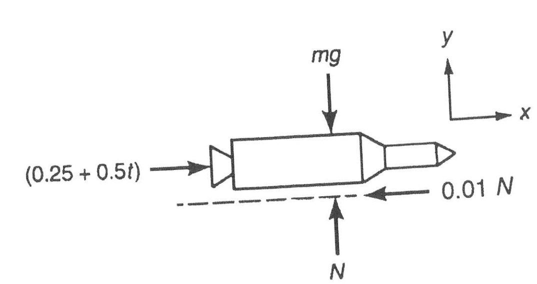

Exhibit 9.26a

9.27 **b.** Apply the impulse-momentum principle between the instant t_1 when the brakes are applied and the instant t_2 when the truck comes to a stop:

$$\int_{t_1}^{t_2} \mathbf{F}dt = m\mathbf{v}_2 - m\mathbf{v}_1$$

In the direction tangent to the road surface,

$$\int_0^t (mg \sin 10° - F)dt = 0 - mv_1$$

or

$$[2000(9.81)\sin 10° - 4000]t = -2000\frac{80(1000)}{60(60)}$$

so that $t = 74.9$ s.

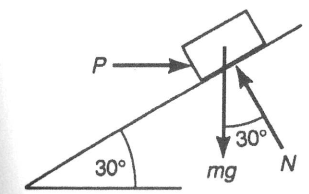

Exhibit 9.28a

9.28 **c.** Refer to Exhibit 9.28a. The subscript 1 is used for the instant when the force P is used first applied, and the subscript 2 is used for the instant when the block comes to rest. Apply the work-energy principle between 1 and 2:

$$W_{1-2} = T_2 - T_1$$
$$(mg\sin 30° - P\cos 30°)\Delta x = 0 - \tfrac{1}{2}mv_1^2$$

which yields

$$\Delta x = \frac{\frac{1}{2}mv_1^2}{P\cos 30° - mg\sin 30°} = \frac{0.5(2)3^2}{15\cos 30° - 2(9.81)\sin 30°} = 2.83 \text{ m}$$

9.29 **c.** Consult Exhibit 9.29a. Apply the work-energy principle between A and B, and then between B and C.

$$A \to B: \quad W_{A-B} = T_B - T_A$$

$$mgr = \frac{1}{2} m v_B^2 - 0 \qquad \text{(i)}$$

$$B \to C: \quad W_{B-C} = T_C - T_B$$

$$N = mg \qquad \text{(ii)}$$

$$-\mu N H = 0 - \frac{1}{2} m v_B^2 \qquad \text{(iii)}$$

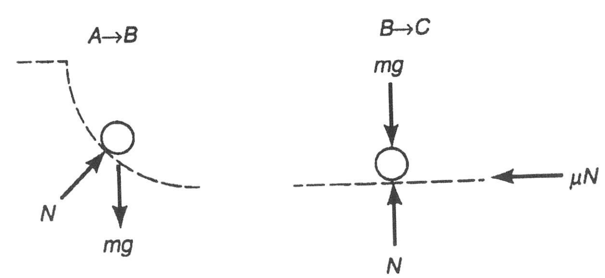

Exhibit 9.29a

Substituting Equations (i) and (ii) into Equation (iii),

$$-\mu mgH = -mgr, \quad \text{or} \quad H = r/\mu$$

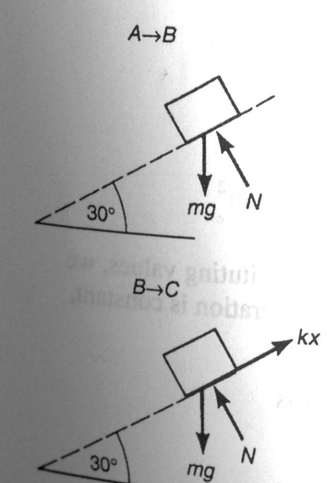

Exhibit 9.31a

9.30 **c.** Let D be the position at which the spring has its natural (unstretched) length. Apply the work-energy principle (Exhibit 9.30a) from A to D:

$$W_{A-D} = T_D - T_A$$

$$\frac{1}{2} k\Delta^2 - \mu N \Delta = \frac{1}{2} m v_D^2 - 0$$

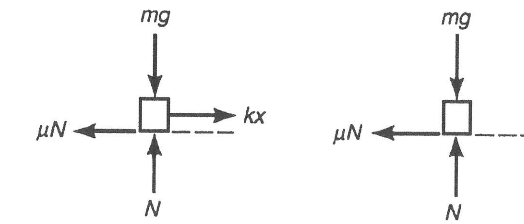

Exhibit 9.30a

Since $N = mg$, we have

$$\frac{1}{2} m v_D^2 = \frac{1}{2} k\Delta^2 - \mu mg\Delta \qquad \text{(i)}$$

Now apply the work-energy principle from D to B:

$$W_{D-B} = T_B - T_D$$

$$-\mu N(1.5 - \Delta) = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_D^2$$

Again, since $N = mg$, and $\frac{1}{2} m v_D^2$ is given by Equation (i), we have

$$-\mu mg(1.5 - \Delta) = \frac{1}{2} m v_B^2 - \frac{1}{2} k\Delta^2 + \mu mg\Delta$$

or

$$\Delta = \sqrt{\frac{2\left(1.5\mu mg + 0.5 m v_B^2\right)}{k}} = 0.26 \text{ m}$$

9.31 **b.** Apply the work-energy principle between A and B, and then between B and C.

$$W_{A-B} = T_B - T_A$$

With the forces shown in Exhibit 9.31a,

$$mg \sin 30°(3) = \frac{1}{2} m v_B^2 - 0$$

$$W_{B-C} = T_C - T_B$$

$$-\frac{1}{2} kx^2 + mg \sin 30°(x) = 0 - \frac{1}{2} m v_B^2 = -mg \sin 30°(3)$$

Substituting values, we obtain the quadratic equation $500x^2 - 30x - 90 = 0$, which can be solved to yield $x = 0.45$ m.

9.32 d. Using the diagram in Exhibit 9.32a, apply the work-energy principle between B and C:

$$W_{B-C} = T_c - T_B$$

$$-mg \sin 5°(x) - 0.02\, mgx = 0 - \frac{1}{2}mv_B^2$$

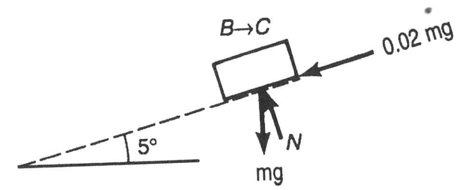

Exhibit 9.32a

or

$$x = \frac{\frac{1}{2}mv_B^2}{mg\sin 5° + 0.02\, mg} = \frac{0.5(500)\left[\frac{(32\times 1000)}{60\times 60}\right]^2}{500(9.81)\sin 5° + 0.02(500)9.81} = 37.6 \text{ m}$$

Total distance up the grade is $(50 + 37.6)$ m = 87.6 m.

9.33 b. Consult Exhibit 9.33a. The moment of inertia of each rod about its mass center is

$$I_{R/B} = I_{S/O} = \frac{1}{12}ml^2 = \frac{1}{12}(4)(3)^2 = 3 \text{ kg–m}^2$$

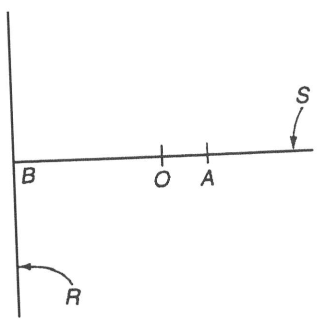

Exhibit 9.33a

Here, O is the mass center of S, and B is the mass center of R. Apply the parallel axes theorem:

$$I_{R/A} = I_{R/B} + m(2)^2 = 3 + 4(2)^2 = 19 \text{ kg–m}^2$$

$$I_{S/A} = I_{S/O} + m(2-1.5)^2 = 3 + 4(0.5)^2 = 4 \text{ kg–m}^2$$

And, for the assemblage,

$$I_A = I_{R/A} + I_{S/A} = (19+4)\text{kg–m}^2 = 23 \text{ kg–m}^2$$

9.34 d. Since $M = I\alpha = (1/2)mr^2\alpha$, $\alpha = M/(0.5mr^2)$. Substituting values, we have $\alpha = 1.67$ rad/s^2. Because this angular acceleration is constant, the final angular velocity is given by

$$\omega = \omega_0 + \alpha t = 1 + 1.67(5) = 9.3 \text{ rad/s}$$

9.35 a. When the desired configuration is achieved, the rod is in translation. The free-body diagram and the inertia force diagram for the rod are shown in Exhibit 9.35a. Taking moments about point O,

$$mg(l/2)\sin\theta = ma(l/2)\cos\theta$$

where l is the length of the rod. Thus,

$$a = g\tan\theta = 9.81 \tan 10° = 1.73 \text{ m/s}^2$$

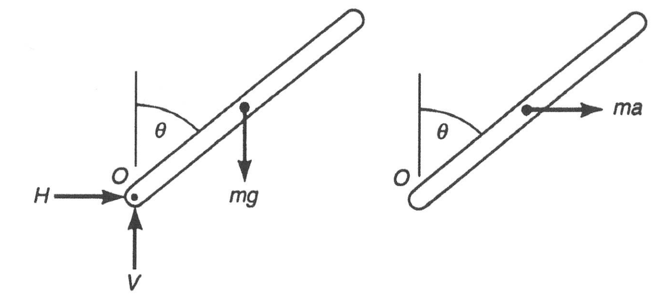

Exhibit 9.35a

9.36 d. From the diagrams in Exhibit 9.36a, and taking moments about O,

$$PH - mgl\ \sin\theta = I_G\alpha + ml^2\alpha$$

so that

$$\alpha = (PH - mgl\ \sin\theta)/(I_G + ml^2)$$

Now,

$$I_C = I_G + m(H-l)^2$$

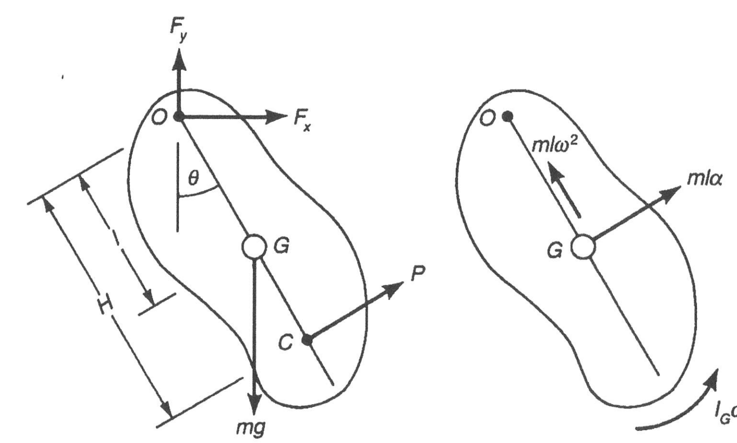

Exhibit 9.36a

from the parallel axis theorem. Thus,

$$I_G = I_C - m(H-l)^2 = mk^2 - m(H-l)^2$$

and

$$\alpha = \frac{PH - mgl\ \sin\theta}{m[k^2 - (H-l)^2] + ml^2}$$

Substitute values to get

$$\alpha = \frac{10(2) - 3(9.81)(1.5)(0.5)}{3[(0.8)^2 - (0.5)^2 + (1.5)^2]} = 0.26 \text{ rad/s}^2$$

9.37 b. C is the center of mass of the rod in Exhibit 9.37a. Summing moments about O gives

$$-mg(l/2)\sin\theta = (1/12)ml^2\alpha + m(l/2)^2\alpha + ma(l/2)\cos\theta \qquad \text{(i)}$$

Summing forces along the horizontal, gives

$$H = ma + m(l/2)\,\alpha\cos\theta - m\omega^2(l/2)\sin\theta \qquad \text{(ii)}$$

From Equation (i),

$$\alpha = -\frac{3}{2}\frac{(g\sin\theta\cos\theta)}{l}$$

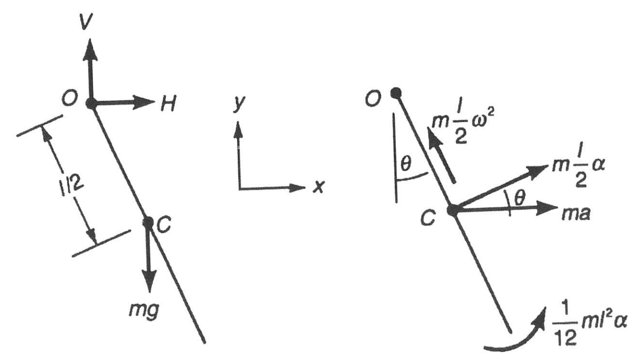

Exhibit 9.37a

Substituting values,

$$\alpha = -4.98\ \text{rad/s}^2 \qquad \text{(iii)}$$

Substituting Equation (iii) and the given values into Equation (ii) yields

$$H = (1)(2) + (1)(1)(-4.98)\cos 30° - (1)(2)^2(1)\sin 30° = -4.31\ \text{N}$$

9.38 a.

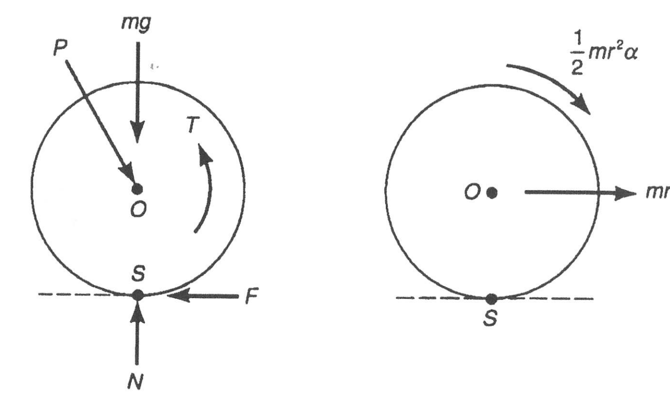

Exhibit 9.38a

Using the free-body diagram shown in Exhibit 9.38a, take moments about the contact point S:

$$rP\cos 60° - T = mr^2\alpha + (1/2)\,mr^2\alpha = (3/2)\,mr^2\alpha$$

or

$$\alpha = (rP\cos 60° - T)/(1.5mr^2) = \text{constant}$$

Substituting values, we find

$$\alpha = 0.83\ \text{rad/s}^2$$

With a constant angular acceleration, the angular velocity is

$$\omega = \omega_0 + \alpha t = 0 + 0.83(10) = 8.33\ \text{rad/s}$$

9.39 **b.** Exhibit 9.39a shows the forces acting on the rod as it swings from position 1 (horizontal) to position 2 (vertical). The work-energy principle gives

$$W_{1\to 2} = T_2 - T_1$$

That is,

$$mg\frac{l}{2} = \frac{1}{2}I_A\omega_2^2 - 0 = \frac{1}{2}\times\frac{1}{3}ml^2\omega_2^2$$

and

$$\omega^2 = \sqrt{\frac{3g}{l}} = \sqrt{\frac{(3)(9.81)}{2}} = 3.84 \text{ rad/s}$$

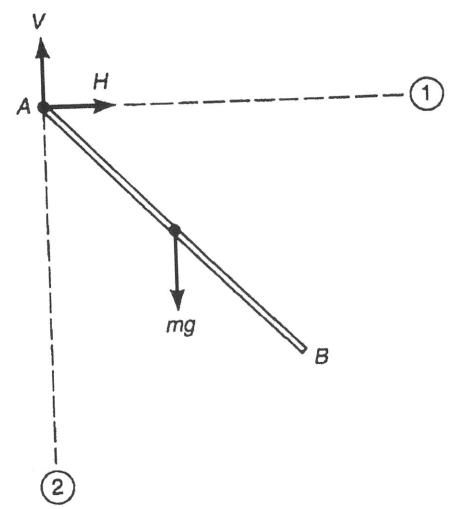

Exhibit 9.39a

9.40 **d.** Exhibit 9.40a shows the forces and torque acting on the rod as it rotates from position 1 (vertical) to position 2 (horizontal). The work-energy relation gives

$$W_{1-2} = T_2 - T_1$$

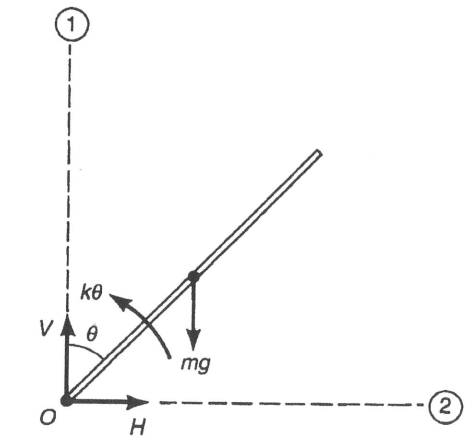

Exhibit 9.40a

That is,

$$mg\frac{l}{2} + \frac{l}{2}k(\theta_1^2 - \theta_2^2) = \frac{1}{2}I_O\omega_2^2 - \frac{1}{2}I_O\omega_1^2$$

Now, $\theta_1 = 0$, $\theta_2 = \pi/2k$, and $\omega_2 = \omega_2 = 0$. Thus,

$$k = \frac{mgl}{\theta_2^2 - \theta_2^2} = \frac{2(9.8)(3)}{(\pi/2)^2} = 23.8 \text{ N-m/rad}$$

9.41 **a.** The forces acting on the cylinder during this motion are shown in Exhibit 9.41a. Applying the work-energy principle,

$$W_{1-2} = T_2 - T_1$$

F and *R* do no work because their point of application has zero velocity (rolling without slip); *mg* does no work because its point of application moves perpendicular to the force. Work done by the spring force is

$$W_{sp}\frac{1}{2}k\left(\Delta_1^2 - \Delta_2^2\right) = W_{1-2}$$

where

$$\Delta_1 = [(3^2 + 4^2)^{0.5} - 3]\,\text{m} = 2\text{ m}, \quad \text{and} \quad \Delta_2 = 0$$

$$T_1 = 0, \text{ and } T_2 = \frac{1}{2}m(v_P)^2 + \frac{1}{2}I_P\omega^2 = \frac{1}{2}m(\omega r)^2 + \frac{1}{2}\times\frac{1}{2}mr^2\omega^2 = \frac{3}{4}mr^2\omega^2$$

Substituting into the work-energy principle yields

$$W_{1-2} = \frac{1}{2}k\Delta_i^2 = \frac{3}{4}mr^2\omega^2$$

and

$$\omega = \left(\frac{2}{3}\frac{k}{m}\right)^{0.5}\frac{\Delta_1}{r} = \left(\frac{2(2)}{3(12)}\right)^{0.5} \times \frac{2}{0.5} = 1.33 \text{ rad/s}$$

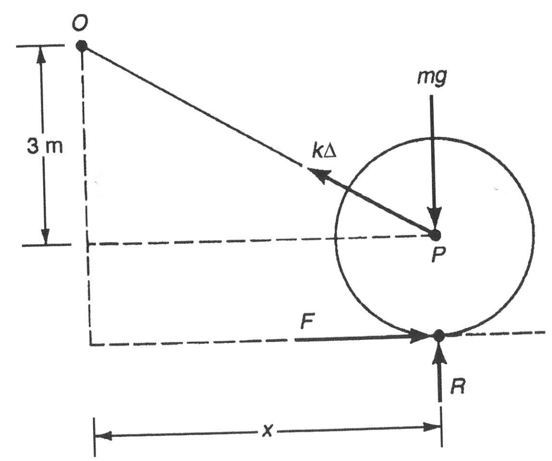

Exhibit 9.41a

CHAPTER 10

Mechanics of Materials

James R. Hutchinson and Jerry H. Hamelink

OUTLINE