Proceedings of the 27th Conference
on Decision and Control
Austin, Texas • December 1988

DYNAMIC FEEDBACK LINEARIZATION WITH APPLICATION TO AIRCRAFT CONTROL

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ABSTRACT

We study the problem of linearization of a nonlinear system by dynamic state feedback. A necessary condition and a sufficient condition are presented and are shown to apply to a general aircraft model.

Keywords:

Aircraft control, Nonlinear systems, Feedback linearization, Dynamic state feedback.

1. INTRODUCTION

A natural generalization of static feedback transformations for system (Σ)

$$\dot{z} = f(z) + \sum_{i=1}^{m} u_i(t) g_i(t) \quad z \in M, \ u \in \mathbf{R}^m \qquad (\Sigma)$$

is given by dynamic feedback transformations of type

$$\begin{aligned} \dot{w} &= a(x,w) + b(x,w)v \quad w \in \mathbf{R}^q \\ u &= \alpha(x,w) + \beta(x,w)v \end{aligned} \qquad (1.1)$$

where the matrix $\begin{pmatrix} b(x,w) \\ \alpha(x,w) \end{pmatrix}$ has rank m.

Note that if state measurements are available, a dynamic feedback (1.1) is not necessarily more difficult to implement than a static one.

Dynamic compensation (1.1) was recently introduced in the study of nonlinear systems with outputs: it was used in [4] to achieve input-output decoupling and in [5] to solve the linear model matching problem. In [8], [9] sufficient conditions are given under which a system with outputs is simultaneously input-output decoupled and linearized by a dynamic compensator. Our approach is totally different than the one of [6] and [9] since we consider systems without output: even if the states are viewed as outputs, available results, which require right-invertibility, do not apply in general. In fact the transformations obtained by our state-space approach provide the right output functions that ensure input-output dynamic feedback linearization without zero-dynamics, and consequently with stability.

In this paper we investigate the transformation of system (Σ) into linear controllable system (L)

$$\dot{x} = Ax + Bv \quad x \in \mathbf{R}^{n'}, \ v \in \mathbf{R}^m \qquad (L)$$

via dynamic compensation of type (1.1) and extended state diffeomorphism

$$x = \phi(z,w) \qquad (1.2)$$

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Two results are presented without proof:

- A necessary condition for complete linearization by a dynamic feedback with given simple particular structure (see (2.3)).
- A sufficient condition for complete dynamic feedback linearization.

Both conditions are given in terms of distributions directly computable from system (Σ). The necessary condition and the sufficient condition for dynamic feedback linearization are both generalizations of the well known necessary and sufficient condition of Jakubzyk-Respondek [10] and Hunt-Su-Meyer [7] for static feedback linearization.

We introduce two sets of distributions which play a crucial role in the analysis of dynamic feedback. They are obtained by an algorithm in a finite number of steps, with a bound depending only on the dimension of the state manifold.

We conclude with an example of aircraft control where we show that the above techniques work.

2. PRELIMINARIES, DEFINITIONS AND BASIC RESULTS

Consider the nonlinear system on a connected, paracompact analytic manifold of dimension n

$$\dot{z} = f(z) + \sum_{i=1}^{m} g_i(z) u_i(t) = f(z) + G(z)u \quad z \in M \qquad (\Sigma)$$

where f, $g_1, \ldots, g_m$ are real analytic vector fields on U_o, a neighborhood of z_o, equilibrium of f, $g_1(0), \ldots, g_m(0)$ being linearly independent.

If we define the distribution

$$\Gamma = span\{ g_1, \ldots, g_m \},$$

system (Σ) can be described in a coordinate free way by the couple (f, Γ). On the other hand, (Σ) can be obtained from (f, Γ) by suitably choosing a basis of Γ, $g_1, \ldots, g_m$, and local coordinates $(z_1, \ldots, z_n)$. Thus, (Σ) is a local coordinatization for (f, Γ).

We recall the classical intrisic definition of feedback equivalence ([8],[11]):

Definition 1.

Two systems (f, Γ) and $(\tilde{f}, \tilde{\Gamma})$ are said to be feedback equivalent if

(i) $\Gamma = \tilde{\Gamma}$;

(ii) $\exists\ g \in \Gamma$ such that $g(z_o) = 0$ and $f + g = \tilde{f}$.

The transformation $(f, \Gamma) \rightarrow (f + g, \Gamma)$, $g \in \Gamma$, is called a feedback transformation.

Consider the distributions

$$Q^{o} = span\{ g_1, \ldots, g_m \}$$
$$Q^{i+1} = span\{ \overline{Q}_i , ad_f^{i+1} Q^{o} \}$$

where $\overline{Q}$ denotes the involutive closure of the distribution Q.

We recall the static feedback linearization result ([7],[10]).

Theorem 2.1 ([7],[10])

System (f, Γ) is static feedback linearizable if and only if

(i) Q^i is involutive of constant rank for $i = 0,\ldots,n-1$.

(ii) $\dim Q^{n-1} = n$.

Let us now focus attention on dynamic feedbacks:

Definition 2.

- Let f be an analytic vector field on M. We say that an analytic vector field f^e on $M \times \mathbf{R}^q$ is an extension of f if and only if $T\pi_1(f^e) = f$ where π_1 is the first projection: $M \times \mathbf{R}^q \to M$, and where $T\pi_1$ denotes its tangent application. More generally T will be used as the tangent operator for differentiable mappings or manifolds.

- A feedback extension of system (f, Γ) is a couple $(f^e + g^e, \Gamma^e)$ satisfying:

  — Γ^e is a subdistribution of rank m of $\Gamma \times T\mathbf{R}^q$

  — f^e is an extension of f on $M \times \mathbf{R}^q$

  — $g_e \in \Gamma \times T\mathbf{R}^q$ and $T\pi_2(g_e) = 0$ where π_2 is the second projection: $M \times \mathbf{R}^q \to \mathbf{R}^q$.

In local coordinates a feedback extension of system (Σ) is given by

$$\begin{aligned} \dot{z} &= f(z) + G(z)\alpha(z,w) + G(z)\beta(z,w)v \\ \dot{w} &= a(z,w) + B(z,w)v \end{aligned} \tag{2.1}$$

where

$$\begin{aligned} \dot{w} &= a(z,w) + \sum_{i=1}^{m} b_i(z,w)v_i \quad w \in \mathbf{R}^q \\ u &= \alpha(z,w) + \beta(z,w)v \end{aligned} \tag{2.2}$$

is a (nonsingular) dynamic state feedback compensator of order q with $rank \begin{pmatrix} B(z,w) \\ \beta(z,w) \end{pmatrix} = m$. Obviously (2.2) generalizes the notion of static state feedback. The composition of such transformations is associative but not commutative. Since the inverse transformation is not generally defined, the set of feedback extensions does not form a group.

We now define a new set of distributions related to a special kind of compensator (2.2).

Let $\mathbf{N}_m = \{0, \cdots, m\}$ and $\mathbf{N}_m^* = \{1, \cdots, m\}$ Let P be a subset of $\mathbf{N}_m^*$, $J_P = \mathbf{N}_m - P$ and $K_P = \mathbf{N}_m^* - P$. If $P = \{p\}$ we shall write, with obvious abuse of notation, p instead of P. Let $\mu = card\ K_P$, let us assume that we have constructed a compensator of order $j\mu$ where all the inputs g_i, $i \in K_P$, are extended by a j^{th} order integrator, namely:

$$\begin{cases} \dot{w}_1^i &= w_2^i \\ &\vdots \\ \dot{w}_{j-1}^i &= w_j^i \quad i \in K_P \\ \dot{w}_j^i &= v_i \end{cases}$$
$$\begin{cases} u_i &= w_1^i \quad i \in K_P \\ u_k &= v_k \quad k \in P \end{cases} \tag{2.3}$$

The extended system has the form

$$\dot{\hat{x}} = \hat{f}(\hat{x}) + \sum_{i=1}^{m} v_i \hat{g}_i(\hat{x})$$

with $\hat{x} = (x,w) \in M \times \mathbf{R}^{j\mu}$ and, with an obvious abuse of notation,

$$\hat{f}(\hat{x}) = f(x) + \sum_{i \in K_P} w_1^i g_i(x) + \sum_{i \in J_P} \sum_{r=1}^{j-1} w_{r+1}^i \frac{\partial}{\partial w_r^i}$$

$$\hat{g}_i(\hat{x}) = \frac{\partial}{\partial w_j^i} \qquad i \in K_P$$

$$\hat{g}_k(\hat{x}) = g_k(x) \qquad k \in P$$

Let $\hat{Q}^{o} = span\{\hat{g}_1,\ldots,\hat{g}_m\}$ and $\hat{Q}^{i+1} = inv.\ clos.\ \hat{Q}^i + ad_f^{i+1}\hat{Q}^{o}$. According to Theorem 2.1 the distributions $\hat{Q}^i$ are the right tools to analyze the action of feedback transformations. However they are not directly computable in terms of the given vector fields $f, g_1, \cdots, g_m$. For this purpose we introduce the distributions $Q_P^{i,j}(u)$ which are directly computable in terms of $f, g_1, \cdots, g_m$ and are related to the distributions $\hat{Q}^i$.

Definition 3.

Let $A_1, \ldots, A_n, A$ be n + 1 vector fields, we define

$$Sad_{A_1 \ldots A_n} A = \sum_{\sigma \in \Sigma_n} ad_{A_{\sigma(1)}} \ldots ad_{A_{\sigma(n)}} A$$

where Σ_n is the set of all permutations of $\{1,\ldots,n\}$. $Sad_{A_1 \ldots A_n} A$ is called the symmetric adjoint of order n of A with respect to $A_1, \ldots, A_n$. If n = 0, we set $Sad_{\emptyset} A = A$.

The symmetric adjoints have the following elementary properties:

$$Sad_{A_{\sigma(1)} \ldots A_{\sigma(n)}} A = Sad_{A_1 \ldots A_n} A \text{ for every permutation } \sigma \in \Sigma_n. \tag{2.4}$$

$$Sad_{A_1 \ldots A_{n+1}} A = \sum_{k=1}^{n+1} ad_{A_k} Sad_{A_1, \ldots, A_{k-1}, A_{k+1}, \ldots, A_{n+1}} A \tag{2.5}$$

$$Sad_{A_1} A = ad_{A_1} A \tag{2.6}$$

$$Sad_{\underbrace{A \ldots A}_{n \text{ times } A}} B = n!\, ad_A^n B \tag{2.7}$$

For homogeneity of notations we set $g_o = f$ and $u_o = 1$.

Definition 4.

For each subset P of $\mathbf{N}_m^*$ we define two sets of distributions, namely:

$$Q_P^{0,1}(u) = Q_P^{0,1} = span\{ g_k, k \in P \}$$

$$Q_P^{0,j+1}(u) = \overline{Q}_P^{0,j}(u)$$
$$+ span\{ \sum_{r_1, \ldots, r_j \in J_P} u_{r_1} \ldots u_{r_j} Sad_{g_{r_1} \ldots g_{r_j}} g_k, k \in P \}$$

$$Q_P^{i+1,j}(u) = \overline{Q}_P^{i,j}$$

$$+\ span\ \{ \sum_{\tau_1,\ldots,\tau_i \in J_P} u_{\tau_1}\ldots u_{\tau_i} Sad_{g_{\tau_1}\ldots g_{\tau_i}} g_\tau\ ,\ \tau \in K_P\ ,$$
$$\sum_{\tau_1,\ldots,\tau_{i+j} \in J_P} u_{\tau_1}\ldots u_{\tau_{i+j}} Sad_{g_{\tau_1}\ldots g_{\tau_{i+j}}} g_k\ ,\ k \in P\ \}$$

$$Q_P^{i,j} = \bigcup_{u \in \mathbf{R}^m} Q_P^{i,j}(u)$$

We also define the distribution $Q_P^{i,0}$ as $Q_P^{0,0} = Q_P^{0,0}(u) = \{ 0 \}$, $Q_P^{i,0} = Q_P^{i,0}(u) = Q^{i-1}\ \ \forall i \geq 1$.

Remark.

The preceding distributions are related by the following obvious inclusions:

$$\overline{Q}_P^{i,j}(u) \subset Q_P^{i+1,j}(u)\ ,\quad \overline{Q}_P^{i,j} \subset Q_P^{i+1,j} \tag{2.8}$$

$$\overline{Q}_P^{i,j}(u) \subset Q_P^{i,j+1}(u)\ ,\quad \overline{Q}_P^{i,j} \subset Q_P^{i,j+1} \tag{2.9}$$

These distributions can be computed in a finite number of steps as shown in the following:

Theorem 2.2

Let $\overline{j}$ be the smallest integer such that, for every u, $Q_P^{0,\overline{j}+1}(u) = Q_P^{0,\overline{j}}(u)$; we have

$$\overline{j} \leq n. \tag{2.10}$$

For every integers i, j, $i \geq 0$, $j \geq \overline{j}$, we have

$$\begin{aligned} Q_P^{i,j}(u) &= Q_P^{i,\overline{j}}(u) \\ Q_P^{i,j} &= Q_P^{i,\overline{j}} \end{aligned} \tag{2.11}$$

For every i, j such that $i + j \geq n$, $0 \leq j \leq \overline{j}$, we have

$$Q_P^{i,j} = Q^{n-1} \tag{2.12}$$

Therefore for every P, we need to compute at most $\frac{(n+1)(n+2)}{2}$ distributions $Q_P^{i,j}$, namely

$$\begin{aligned} 0 \leq j \leq \overline{j} \leq n \\ 0 \leq i + j \leq n \end{aligned} \tag{2.13}$$

The relationships between the distributions $Q_P^{i,j}(u)$ and particular dynamic extensions are displayed in the next lemma.

Lemma 2.3

We have

$$\begin{array}{llcll}
(i) & Q_p^{0,i+1}|_x & = & \bigcup_{w \in \mathbf{R}^{j\mu}} T\pi_1(\hat{Q}^i|_{(x,w)}) & 0 \leq i \leq j-1 \\
(ii) & Q_p^{i,j}|_x & = & \bigcup_{w \in \mathbf{R}^{j\mu}} T\pi_1(\hat{Q}^{i+j-1}|_{(x,w)}) & i \geq 1 \\
(iii) & \hat{Q}^i & \subset & Q_p^{0,i+1} \times T\mathbf{R}^{(i+1)\mu} & 0 \leq i \leq j-1 \\
(iv) & \hat{Q}^{i+j-1} & = & Q_p^{i,j}(u) \times T\mathbf{R}^{j\mu} & i \geq 1 \\
(v) & inv.\ clos.\ \hat{Q}^{i+j-1} & = & \overline{Q}_p^{i,j} \times T\mathbf{R}^{j\mu} & i \geq 0
\end{array}$$

where π_1 is the first projection $M \times \mathbf{R}^{j\mu} \rightarrow M$ and $T\mathbf{R}^{k\mu}$ stands for

$$span\ \{ \frac{\partial}{\partial w_\nu^\tau}\ ,\ \tau \in K_P\ ,\ j-k \leq \nu \leq j\ \}.$$

The proofs can be found in [3].

3. COMPLETE DYNAMIC FEEDBACK LINEARIZATION

We say that system (Σ) is dynamic feedback linearizable by compensator (2.3) if there exist a static feedback and a diffeomorphism on the extended space defined by compensator (2.3) such that system (Σ) is transformed into a linear controllable system (L).

We state the following necessary and sufficient results without proof. The interested reader can find detailed proofs in [3].

Theorem 3.1

Let j be a given integer and P a subset of $\mathbf{N}_m^*$. A necessary condition for system (Σ) to be dynamic feedback linearizable by compensator (2.3) is:

$$Q_P^{i,j}(u) = Q_P^{i,j}\ ,\quad Q_P^{i,j} = \overline{Q}_P^{i,j}\quad \forall i \geq 0. \tag{3.1}$$

Theorem 3.2

Assume that there exist an integer j and a subset P of $\mathbf{N}_m^*$ such that:

$$Q_p^{0,i}(u) = Q_P^{0,i}\ ,\quad Q_P^{0,i} = \overline{Q}_P^{0,i}\quad \text{for every } i,\ 1 \leq i \leq j \tag{3.2}$$

$$Q_P^{i,j}(u) = Q_P^{i,j}\ ,\quad Q_P^{i,j} = \overline{Q}_P^{i,j}\quad \text{for every } i,\ i \geq 0 \tag{3.3}$$

then system (Σ) is dynamic feedback linearizable by compensator (2.3). The extended space being $M \times \mathbf{R}^{j\mu}$.

Remark.

If we specialize to the case $j = 0$, Theorems 3.1 and 3.2 become equivalent and we recover the necessary and sufficient condition of static feedback linearization (see Theorem 2.1).

4. APPLICATION TO AIRCRAFT CONTROL

We consider the following nonlinear aircraft model.

- (x,y,z) are the coordinates of the centre of mass in an absolute frame, the vertical z-axis is oriented downward.
- (u,v,w) are the velocity components in a relative frame linked to the plane.
- (p,q,r) are the components of the kinetic moment in the relative frame.
- (Ψ,Θ,Φ) are the yaw, pitch and roll angles respectively.

We consider p, q, r and ρ as control variables where ρ is the thrust. We have

$$\begin{aligned} \dot{x} &= u\ \cos\Psi \cos\Theta + v\,(\cos\Psi \sin\Theta \sin\Phi - \sin\Psi \cos\Phi) \\ &\quad + w\,(\cos\Psi \sin\Theta \cos\Phi + \sin\Psi \sin\Phi) \end{aligned}$$

$$\begin{aligned} \dot{y} &= u\ \sin\Psi \cos\Theta + v\,(\sin\Psi \sin\Theta \sin\Phi + \cos\Psi \cos\Phi) \\ &\quad + w\,(\sin\Psi \sin\Theta \cos\Phi - \cos\Psi \sin\Phi) \end{aligned}$$

$$\dot{z} = u\ \sin\Theta - v\ \cos\Theta \sin\Phi - w\ \cos\Theta \cos\Phi$$

$$\dot{u} = -g\ \sin\Theta + r\ v - q\ w + \frac{X}{m}$$

$$\dot{v} = g\ \cos\Theta\ \sin\Phi - r\ u + p\ w + \frac{Y}{m} \tag{4.1}$$

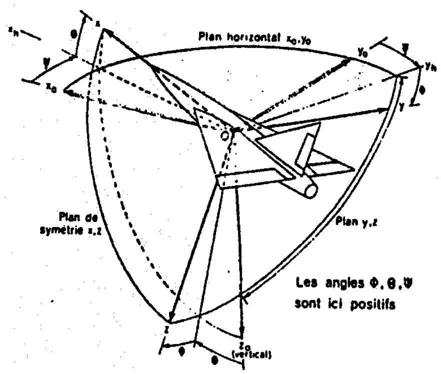
$$\dot{w} = g\ \cos\Theta\ \cos\Phi + q\ u - p\ v + \frac{Z}{m}$$

$$\dot{\Phi} = p + \mathrm{tg}\Theta(q\ \sin\Phi + r\ \cos\Phi)$$

$$\dot{\Theta} = q\ \cos\Phi - r\ \sin\Phi$$

$$\dot{\Psi} = \frac{q\ \sin\Phi + r\ \cos\Phi}{\cos\Theta}$$

where X, Y, Z are the components of the resultant of the forces, gravity excepted. We assume that $Y = Y(x, y, z, u, v, w, \Psi, \Theta, \Phi)$, $Z = Z(x, y, z, u, v, w, \Psi, \Theta, \Phi)$ and $X = X_o(x, y, z, u, v, w, \Psi, \Theta, \Phi) + J\rho$.



Let $\xi = \begin{pmatrix} x \\ y \\ z \\ u \\ v \\ w \\ \Phi \\ \Theta \\ \Psi \end{pmatrix}$, the system takes the form

$$\dot{\xi} = f(\xi) + p g_1(\xi) + q g_2(\xi) + r g_3(\xi) + \rho g_4(\xi) \tag{4.2}$$

with

$$\begin{aligned} f &= f_x \frac{\partial}{\partial x} + f_y \frac{\partial}{\partial y} + f_z \frac{\partial}{\partial z} \\ &\quad + f_u \frac{\partial}{\partial u} + f_v \frac{\partial}{\partial v} + f_w \frac{\partial}{\partial w} \\ g_1 &= w \frac{\partial}{\partial v} - v \frac{\partial}{\partial w} + \frac{\partial}{\partial \Phi} \\ g_2 &= -w \frac{\partial}{\partial u} + u \frac{\partial}{\partial w} + \sin\Phi\ \mathrm{tg}\Theta \frac{\partial}{\partial \Phi} \\ &\quad + \cos\Phi \frac{\partial}{\partial \Theta} + \frac{\sin\Phi}{\cos\Theta} \frac{\partial}{\partial \Psi} \\ g_3 &= v \frac{\partial}{\partial u} - u \frac{\partial}{\partial v} + \cos\Phi\ \mathrm{tg}\Theta \frac{\partial}{\partial \Phi} \\ &\quad - \sin\Phi \frac{\partial}{\partial \Theta} + \frac{\cos\Phi}{\cos\Theta} \frac{\partial}{\partial \Psi} \\ g_4 &= \frac{J}{m} \frac{\partial}{\partial u} \end{aligned} \tag{4.3}$$

where

$$\begin{aligned} f_x &= u\ \cos\Psi\ \cos\Theta + v\,(\cos\Psi\ \sin\Theta\ \sin\Phi - \sin\Psi\ \cos\Phi) \\ &\quad + w\,(\cos\Psi\ \sin\Theta\ \cos\Phi + \sin\Psi\ \sin\Phi) \\ f_y &= u\ \sin\Psi\ \cos\Theta + v\,(\sin\Psi\ \sin\Theta\ \sin\Phi + \cos\Psi\ \cos\Phi) \\ &\quad + w\,(\sin\Psi\ \sin\Theta\ \cos\Phi - \cos\Psi\ \sin\Phi) \\ f_z &= u\ \sin\Theta - v\ \cos\Theta\ \sin\Phi - w\ \cos\Theta\ \cos\Phi \end{aligned} \tag{4.4}$$

We remark that

$$L_{g_i} f_x = L_{g_i} f_y = L_{g_i} f_z = 0 \quad i = 1, 2, 3, \tag{4.5}$$

$$\begin{aligned} &span\ \{ g_1, g_2, g_3, g_4, ad_{g_4} g_2, ad_{g_4} g_3 \} \\ &= inv\ .clos\ \{ g_1, g_2, g_3, g_4 \} \\ &= span\ \{ \frac{\partial}{\partial u}, \frac{\partial}{\partial v}, \frac{\partial}{\partial w}, \frac{\partial}{\partial \Phi}, \frac{\partial}{\partial \Theta}, \frac{\partial}{\partial \Psi} \} \end{aligned} \tag{4.6}$$

and that $span\ \{ g_1, g_2, g_3 \}$ is involutive. We deduce that Q^0 is not involutive, $dim\ Q^0 = 4$ and $dim\ \overline{Q}^0 = 6$. The reader can check that Q^2 is the whole tangent space. According to Theorem 2.1, the system is not static feedback linearizable.

Let $P = \{ 1, 2, 3 \}$, we have

$$Q_P^{0,1}(u) = Q_P^{0,1} = span\ \{ g_1, g_2, g_3 \} \tag{4.7}$$

$Q_P^{0,1}$ is involutive.

$$\begin{aligned} Q_P^{1,1}(u) = Q_P^{0,1} + span\ \{ & g_4, ad_f\ g_1 + u_4 ad_{g_4} g_1, \\ & ad_f\ g_2 + u_4 ad_{g_4} g_2, ad_f\ g_3 + u_4 ad_{g_4} g_3 \} \end{aligned} \tag{4.8}$$

The equalities (4.5) imply that the vector fields $ad_f\ g_i$, $i = 1, 2, 3$, according to (4.6), belong to $\overline{Q}^0$, we thus deduce that for almost every u

$$Q_P^{1,1}(u) = Q_P^{1,1} = \overline{Q}^0. \tag{4.9}$$

In particular, $Q_P^{1,1}$ is involutive. The reader can check that

$$Q_P^{2,1}(u) = Q_P^{2,1} = Q^1 \tag{4.10}$$

i.e. $Q_P^{2,1}$ is the whole tangent space.

We can see that Theorem 3.2 applies, system (4.2) is dynamic feedback linearizable, more precisely we consider the extended system

$$\begin{aligned} \dot{\xi} &= f(\xi) + w g_4(\xi) + v_1 g_1(\xi) + v_2 g_2(\xi) + v_3 g_3(\xi) \\ \dot{w} &= v_4 \end{aligned} \tag{4.11}$$

where

$$\begin{aligned} \rho &= w \\ p &= v_1 \\ q &= v_2 \\ r &= v_3 \end{aligned} \tag{4.12}$$

System (4.11) is static feedback linearizable. The diffeomorphism and static state feedback on the extended system (4.11) are finally obtained by the classical method of [7],[10].

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