# Reply to Reviewers

May 18, 2022

**Submission Number: 22-0227**
**Paper title:** Robust Passivity-Based Control of Underactuated Systems via Neural Approximators and Bayesian Inference

We thank the Editor and the Reviewers for the thorough review of our work and the insightful comments. We have revised our manuscript according to the Editor's and Reviewers' recommendations. In the following, we report the original comments from each Reviewer (*italic fonts*) and our replies (normal fonts). We recorded the changes made to the main manuscript in this document for quick reference. In addition, we submitted the revised manuscript highlighting the revisions in magenta color.

# ASSOCIATE EDITOR

We thank the Editor and each of the Reviewers for the suggestions that contribute to improving the quality of this manuscript. In the sequel, we strive to address each issue pointed in the recommendations.

The common recommendations among both referees include 1) discussing major contributions of this work compared to our previous work titled, “Data-Driven Design of Energy-Shaping Controllers for Swing-up Control of Underactuated Robots”, 2) providing theoretical justification for the robustness properties of Bayesian-inference based control design, 3) establishing a baseline control design technique to compare against the Bayesian approach, and 4) fixing any typos and grammatical mistakes.

For each of the remaining comments from the reviewers, we followed the recommendations and have summarized the revisions made to the manuscript one-by-one in this document.

We again reiterate our gratitude to the reviewers for the detailed recommendations. We truly feel that they led to revisions that greatly improved the quality and clarity of the paper.

## REFEREE #1

*This paper investigated the data-driven approach to learn storage functions parameterized neural network for underactuated mechanical systems. Two practical algorithms were proposed to deal with the point estimate and probabilistic inference of optimal parameters. The method was evaluated in both simulation and real-world on an inverted pendulum. The paper is well organized and clear, and the results show that the Bayesian-based algorithm is more robust to parametric uncertainties. However, the reviewer has some concerns over the technical contribution of this work. Following are some detailed comments.*

Q. **Ref-1.1** — *What is the major contribution of this work compared to [12]? From the reviewer's understanding, these two papers are both focused on data-driven control of underactuated systems.*

A. **Ref-1.1** — The major contributions of this work include the formulation of the passivity-based control problem as a stochastic optimization problem, and the use of Bayesian learning to automatically find the probability distributions over the control law parameters. These contributions amount to a control design framework for underactuated robotic systems that rigorously addresses the effects of system parameter and measurement uncertainties. This is in contrast to [12], wherein the solution to the optimization problem is deterministic in nature, obscuring the ability to reason about any uncertainties in the nominal dynamical model used during training.

Per the suggestion of the reviewer, we summarize the major contribution of our work at the end of Section I: Introduction as follows.

> The specific contributions of this work are threefold: (i) Motivated by [12], we incorporate uncertainties into the dynamics and cast the passivity-based control synthesis problem as a stochastic optimization problem. The closed-loop (energy-like) storage function, from which the control law is derived, is not restricted to a certain form and instead represented by a neural network whose parameters are random variables. (ii) We apply Bayesian learning and develop an algorithm that finds a suitable probability distribution of the neural net parameters automatically. In contrast to deterministic optimization in [12], this approach provides a probability distribution over the neural net parameters instead of a point estimate, providing a way to reason about model uncertainties and measurement noise during the learning process. (iii) We demonstrate the efficacy and the improved robustness of the proposed Bayesian framework, using the method presented in [12] as a baseline. The comparison is performed on a benchmark underactuated control problem–the inertia wheel pendulum–both in simulation and real-world experiments.

Q. **Ref-1.2** — *Why could the Bayesian inference-based approach be more robust to uncertainties?*

A. **Ref-1.2** — Due to the limited space available in the manuscript, we present separately a theoretical justification for the robustness properties of the Bayesian approach in our arXiv paper titled "Robustness of Control Design via Bayesian Learning". We have added a brief summary of this work as a remark under Section II: Methods as follows.

> *Remark 1*: We provide a thorough comparison of the deterministic and Bayesian solutions to the optimal control of a one-dimensional linear dynamical system that is subject to parameter and measurement uncertainties in [17]. The simplicity of the underlying system facilitates the computation of exact solutions, revealing the fact that the Bayesian solution improves the robustness of the closed-loop system against both type of uncertainties. This comparison serves as a theoretical justification for the improved robustness properties of controllers obtained via Bayesian learning techniques.

The list of references is updated to include the arXiv paper, [17]:


> [17] N. A. Ashenafi, W. Sirichotiyakul, and A. C. Satici, "Robustness of control design via bayesian learning," 2022. [Online]. Available: https://arxiv.org/abs/2205.06896

Q. **Ref-1.3** — *It seems to me that the major reason [that the Bayesian approach is more robust] is that the authors injected noises during collecting the training data, while this trick was not used in the first approach. This is a common trick in RL called domain randomization for improving robustness. It would make sense to make a comparison between the two algorithms with domain randomization being applied to both of them.*

A. **Ref-1.3** — Introducing domain randomization in the first approach (Algorithm 1) would lead to a set of deterministic control law parameters, referred to as "point estimates" hereafter. This is in contrast to our Bayesian framework (Algorithm 2), where the control law parameters are characterized by the posterior probability distributions. The point estimates may be interpreted as the *maximum a posteriori* (MAP) estimate of the probability distribution that the Bayesian framework provides.

During the development of this work, we investigated the efficacy of the MAP approach through experiments. We found that the point estimates are prone to be biased under large uncertainties in system parameters or measurement [13]. For example, if the uncertainty in system parameters is large, the optimal controller parameter $\theta$ for the true system may be quite far from the deterministic solution. Hence, we do not include a direct comparison in the manuscript. Instead, the proposed framework alleviates this problem by marginalizing over the learned posterior distribution of the control law parameters.

We clarify this point by adding the following remark under Section III-D of the manuscript.

> *Remark 2*: Introducing uncertainties to the deterministic training finds a point estimate of the optimal controller parameter, which may be interpreted as the mean of the optimal posterior distribution that Bayesian learning provides. A point estimate of the learned parameters is prone to be biased (for example, if the uncertainty in system parameters is large, the optimal parameter $\theta$ for the true system parameter may be quite far from the deterministic solution). This bias-variance trade-off problem is alleviated by Bayesian inference which allows one to marginalize over the posterior parameter distribution [13].

Q. **Ref-1.4** — *The experiment part only included the results of the two proposed methods, while a proper baseline method is missing. It would make sense to include an RL baseline or the method from [12] if the proposed method is different from [12].*

A. **Ref-1.4** — The results presented in Section IV are direct comparisons between the proposed Bayesian method and the baseline method from [12]. To clarify this, we add the following sentence to the first paragraph of Section IV: Case Study: Inertia-Wheel Pendulum.

> In this section, we validate the proposed control design framework on the problem of swinging-up and stabilizing the inverted position of an inertia wheel pendulum (IWP), shown in Fig.1. We provide experimental results from simulation and real-world hardware in order to thoroughly demonstrate the efficacy and robustness claims of Bayesian inference. We use the deterministic solution for NeuralPbc as the baseline on which we compare the performance of the Bayesian solution.

Q. **Ref-1.5** — *Both the control input matrix and observation matrix are noted by $G$, can you change one of them to be a different letter?*

A. **Ref-1.5** — In passivity-based control, the observation matrix is sometimes chosen the same as the transpose of the control input matrix as in reference [1] to facilitate control synthesis and stability analysis. For clarity, we state the dependency of $G$ on $q$ explicitly in the second equation of (2) as follows:

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G(q) \end{bmatrix} u, \\ y &= G(q)^\top \dot{q}, \end{aligned}$$

## REFEREE #2

*The paper deals with robust passivity-Based control of underactuated systems via neural approximators and Bayesian inference. Authors exploited the Bayesian theory to improve the robustness of the controller. They proposed an interesting Bayesian learning algorithm, and the case study is very well presented.*

Q. **Ref-2.1** — *The paper is well-written and organized, and easy to follow. There some typos and grammatical mistakes. Then the authors are invited to proofread their paper before resubmission.*

A. **Ref-2.1** — Thank you for the opportunity to improve the submission. We have thoroughly read through the manuscript and fixed any typos and grammatical mistakes.