Robust Passivity-Based Control of Underactuated Systems via Neural Approximators and Bayesian Inference

Boise State University

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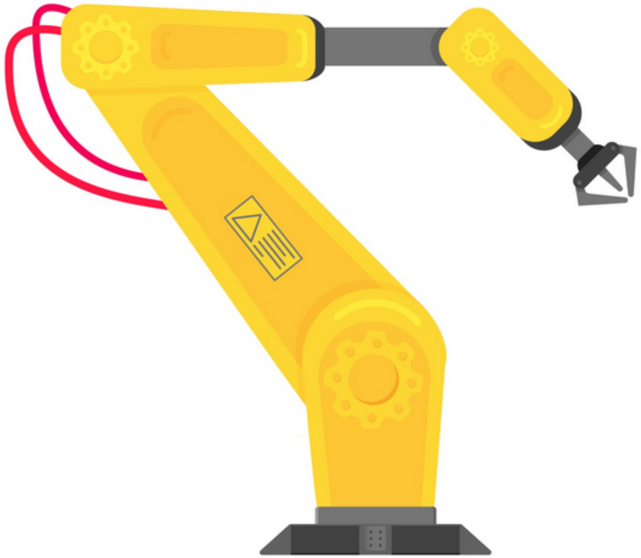
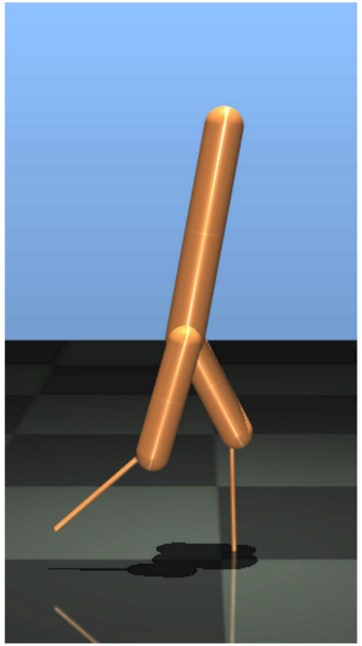
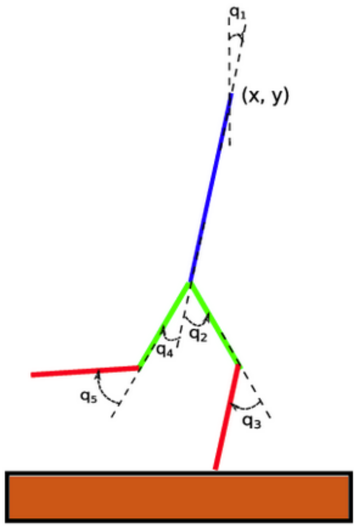
Underactuated Robots



Torque-limited manipulators

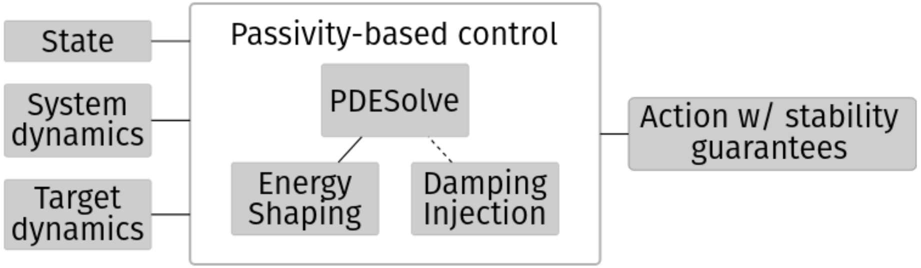




Walking robots

Existing Methods

Passivity-based Control

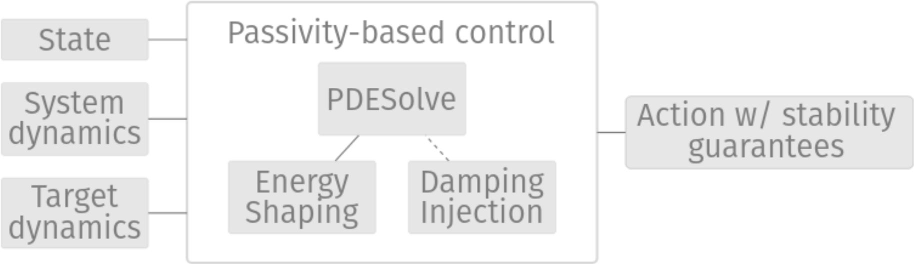


Strengths
- Stability guarantees
- Closed-form policy

Weaknesses
- Model uncertainties
- Need to solve PDEs

Existing Methods

Passivity-based Control



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- Stability guarantees
- Closed-form policy

Weaknesses

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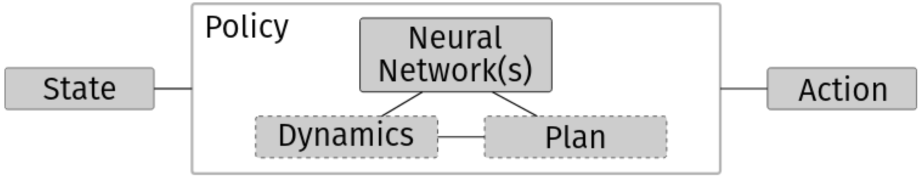
Reinforcement learning



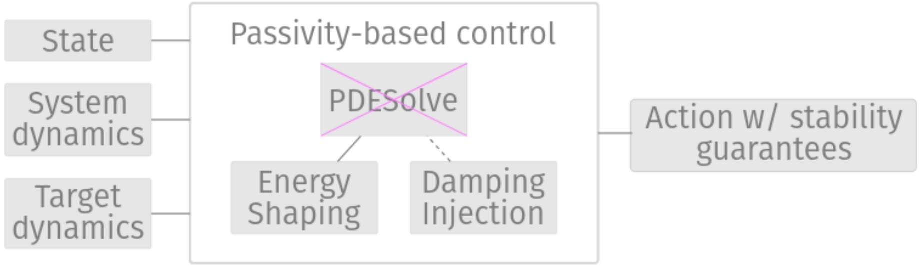
Strengths

- More general
- Unknown dynamics OK

Weaknesses

- Sample complexity
- Stability guarantees?

Data-Driven Passivity-Based Control



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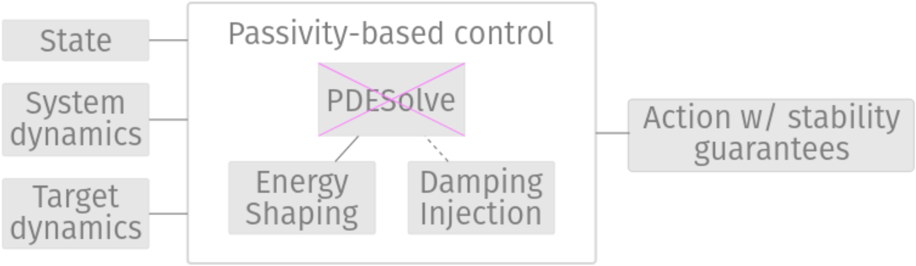

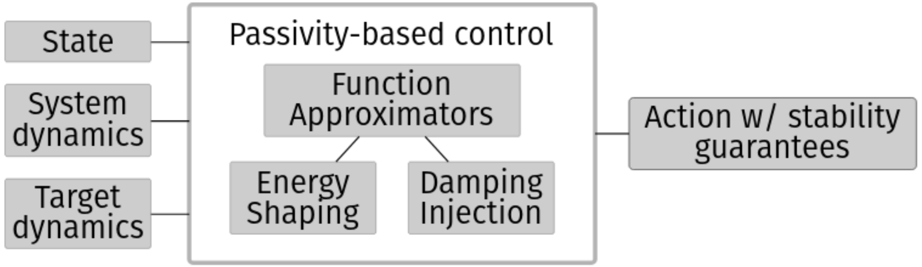

- Systematic approach
- Prior domain knowledge
- Stability is *intrinsic*
- Model uncertainty considerations via Bayesian learning

Background

Passive System Theory and Passivity-Based Control (PBC)

Passivity

A dynamical system

$$\Sigma\colon\quad \begin{cases} \dot{x} &= f(x,u) \\ y &= h(x,u) \end{cases} \qquad x \in \mathcal{X} \subset \mathbb{R}^{2n},\ u \in \mathcal{U} \subset \mathbb{R}^m$$

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is **dissipative** with respect to some supply rate s if there exists a *storage function* $\mathcal{H}\colon \mathcal{X} \to \mathbb{R}^{+}$ such that

$$\mathcal{H}(x(t_1)) \le \mathcal{H}(x(t_0)) + \int_{t_0}^{t_1} s(u(t), y(t))\,\mathrm{d}t$$

for all $x(t_0) = x_0$, all input u, and all $t_1 \ge t_0$

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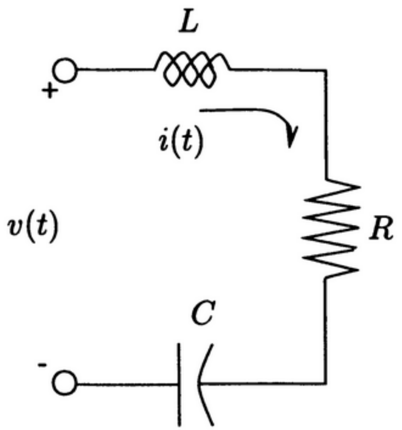
The system Σ is **passive** if it is dissipative with supply rate

$$s = u^\top y.$$

Passive System Example



Kirchoff's law

$$v = Ri + \frac{1}{C}\int_0^t i(\tau)\,\mathrm{d}\tau + L\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$vi - Ri^2 = \frac{\mathrm{d}}{\mathrm{d}t}\left(\underbrace{\frac{1}{2C}\left(\int_0^t i(\tau)\,\mathrm{d}\tau\right)^2}_{\mathcal{V}} + \underbrace{\frac{1}{2}Li^2}_{\mathcal{T}}\right)$$

Let $\mathcal{H} = \mathcal{V} + \mathcal{T}$, integrate to obtain

$$\underbrace{\mathcal{H}(t)}_{\text{available}} - \underbrace{\mathcal{H}(0)}_{\text{initial}} = \underbrace{\int_0^t v(\tau)i(\tau)\,\mathrm{d}\tau}_{\text{supplied}} - \underbrace{\int_0^t Ri^2(\tau)\,\mathrm{d}\tau}_{\text{dissipated}} < \int_0^t v(\tau)i(\tau)\,\mathrm{d}\tau$$

Stability of Passive Systems

$$\Sigma\colon \quad \begin{cases} \dot{x} & = f(x,u), \qquad f(0,0)=0, \\ y & = h(x,u), \qquad h(0,0)=0, \end{cases}$$

Lemma (Khalil, 2002)

The origin of Σ is *stable*, i.e

$$y \equiv 0 \implies x \equiv 0$$

if Σ is passive, i.e

$$\mathcal{H} \geq 0, \quad \dot{\mathcal{H}} = \frac{\partial \mathcal{H}}{\partial x} f(x,u) \leq u^\top y$$

Passivity-Based Control (PBC)

$$\Sigma_o\colon \quad \begin{cases} \dot{x} &= f(x) + g(x)u, \\ y &= h(x) \end{cases}$$

Main idea — Select $u(x) = u_{es} + u_{di}$ that renders the closed-loop system passive.

$$\Sigma_d\colon \quad \begin{cases} \dot{x} &= f_d(x) + g(x)u_{di}, \quad f_d := f(x) + g(x)u_{es}(x) \\ y_d &= h_d(x) \end{cases}$$

Control problem is cast as a search for H_d and h_d s.t. $\dot{H}_d \leq y_d^\top u_{di}$

PBC Example - Simple Pendulum

System Dynamics

$H(q,p) = \frac{1}{2}J^{-1}p^2 + mgl(1 - \cos q)$

$$\begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \nabla_q H \\ \nabla_p H \end{bmatrix} + \begin{bmatrix} 0 \\ G \end{bmatrix} u \quad , \qquad y = \dot{q}$$

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Choose $u = u_{es} + u_{di}$ that transforms system into a passive one with $x^\star = (q^\star, 0)$

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$Gu_{es} = \nabla_q H - \nabla_q H_d, \quad Gu_{di} = -GK_D G^\top \nabla_p H_d$

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$\boxed{u = -mgl \sin(x) - K_P(q - q^\star) - K_D \dot{q}}$

High-dimensional Problem: Ballbot