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3. For what value(s) of m will the straight line with equation $y = mx - 6$

(a) touch (b) intersect (c) never meet

the parabola with equation $y = x^2$.

7. For what value(s) of k is the line $2x = 3y + k$ a tangent to the parabola $y = x^2 - 3x + 4$?

9. For what values of k do $\begin{cases} y = kx \\ y = kx^2 + 3x + k \end{cases}$ have no solution?

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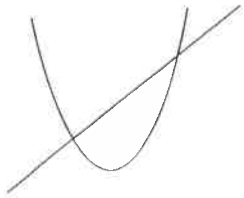
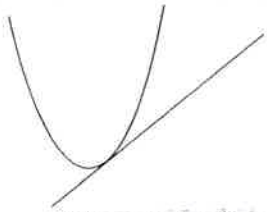
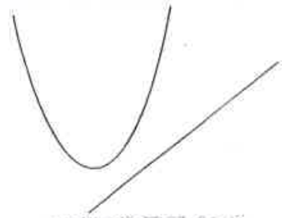
Again, we see that the discriminant can be used to determine the geometrical relationship between the parabola and the straight line.

When solving the simultaneous system of equations

$$y = px^2 + qx + r \quad -(1)$$
$$y = mx + k \quad -(2)$$

which results in solving the quadratic $ax^2 + bx + c = 0$ (after equating (1) to (2)) we have three possible outcomes:

Case 1	Case 2	Case 3
		
i.e., $\Delta = b^2 - 4ac > 0$ The straight line cuts the parabola twice.	i.e., $\Delta = b^2 - 4ac = 0$ The straight line touches the parabola at one point. We say the straight line is a **tangent** to the parabola.	i.e., $\Delta = b^2 - 4ac < 0$ The straight line neither touches nor cuts the parabola anywhere.

EXAMPLE 2.34 Find the value(s) of m for which the straight line with equation $y = mx - 2$ is a tangent to the parabola with equation $y = x^2 - 3x + 7$.

We start by solving the system of equations as we have done previously:

$$y = x^2 - 3x + 7 \quad -(1)$$
$$y = mx - 2 \quad -(2)$$

equating (1) to (2): $\quad x^2 - 3x + 7 = mx - 2$

$$\Leftrightarrow x^2 - (m+3)x + 9 = 0$$

Then, for the straight line to be a tangent, it means that the line and the parabola touch. This in turn implies that the discriminant is zero.

That is, $\Delta = [-(m+3)]^2 - 4 \times 1 \times 9 = (m+3)^2 - 36$
$\qquad = (m + 3 + 6)(m + 3 - 6)$ [using the diff. of two squares]
$\qquad = (m+9)(m-3)$

Then, setting $\Delta = 0$ we have $(m+9)(m-3) = 0 \Leftrightarrow m = -9$ or $m = 3$.

Geometrically we have: