CONTINUED FRACTIONS

Consider the continued fraction below.

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}$$

We can consider this "infinite fraction" as a sequence of terms, t_n, where

$$t_1 = 1+1$$

$$t_2 = 1+\frac{1}{1+1}$$

$$t_3 = 1+\cfrac{1}{1+\cfrac{1}{1+1}}$$

1. Determine a generalized formula for t_{n+1} in terms of t_n.

2. Compute the decimal equivalents of the first 10 terms. Enter the terms into a data table and plot the relation between *n* and t_n using spreadsheet. Include a printout of your graph. What do you notice? What does this suggest about the value of $t_n - t_{n+1}$ as *n* gets very large?

3. What problems arise when you try to determine the 200th term?

4. Use the results of step 1 and step 2 to establish an **exact** value for the continued fraction.

5. Now consider another continued fraction.

$$2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{\cdots}}}}}}}$$

Repeat steps 1 to 4 using this continued fraction.

6. **Now consider the general continued fraction.**

$$k+\cfrac{1}{k+\cfrac{1}{k+\cfrac{1}{k+\cfrac{1}{k+\cfrac{1}{k+\cfrac{1}{k+\cfrac{1}{\cdots}}}}}}}$$

By considering other values of *k*, determine a generalized statement for the exact value of any such continued fraction.

Conjecture and justify your generalized statement for the following cases:

(a) $k>0$
(b) $k=0$
(c) $k<-1$
(d) $k=-1$
(e) $0>k>-1$