1) $U_2 = 24 = U_1 \cdot r \Rightarrow U_1 = \frac{24}{r}$ I

$U_3 = 12(p-1) = U_1 \cdot r^2$

$S_3 = 76 = \frac{U_1(r^2-1)}{r-1}$ II

I in II

$\Rightarrow 76 = \frac{24(r^2-1)}{r(r-1)}$

$= \frac{24(r+1)}{r}$

$\Rightarrow 76r = 24r + 24$

$\Rightarrow 52r = 24$

$\Rightarrow r = \frac{6}{13} \approx 0.462$

when $r = \frac{6}{13}$, $24 = U_1 \cdot \frac{6}{13}$

$\Rightarrow U_1 = \frac{144}{13} \approx 11.08$

when $U_1 = \frac{144}{13}$, $r = \frac{6}{13}$

then $12(p-1) = U_1 \cdot r^2$

$= \frac{144}{13} \cdot \left(\frac{6}{13}\right)^2$

$\Rightarrow p-1 = \frac{432}{2197}$

$\Rightarrow p = \frac{2629}{2197} \approx 2.2$

3) $\frac{20}{(2x-3)(x+1)}$

$\frac{a}{2x-3} + \frac{b}{x+1}$

$a(x+1) + b(2x-3) = 20$

$\Rightarrow ax + a + 2bx - 3b = 20$

$\Rightarrow x(a+2b) + a - 3b = 20$

$\Rightarrow a + 2b = 0$ I and, $a - 3b = 20$ II

I - II

$\Rightarrow 5b = -20$

$\Rightarrow b = -4$

when $b=-4$, $a=8$

$$\Rightarrow \frac{20}{(2x-3)(x+1)} = \frac{8}{2x-3} - \frac{4}{x+1}$$

4) $2e^{x} = 9 + 5e^{-x}$

$\Rightarrow 2e^{2x} = 9e^{x} + 5$, let $y = e^{x}$

$\Rightarrow 2y^{2} = 9y + 5$

$\Rightarrow 2y^{2} - 9y - 5 = 0$

$\Rightarrow (y-5)(2y+1) = 0$

$\Rightarrow y = 5$ or $y = -\frac{1}{2}$

$\Rightarrow e^{x} = 5$ $\Rightarrow e^{x} = -\frac{1}{2}$, $e^{x} > 0$

$\Rightarrow x = \ln 5$ $\Rightarrow x = \ln -\frac{1}{2}$ $\therefore x = \ln 5$

5) $x^{3} + 8x^{2} + ax + b = f(x)$

$\Rightarrow f(2) = 0$

$f(3) = 15$

$\Rightarrow 8 + 8 \cdot 4 + 2a + b = 0$

$\Rightarrow 2a + b = -40$ I

$27 + 8 \cdot 9 + 3a + b = 15$

$\Rightarrow 3a + b = -84$ II

I − II

$\Rightarrow -a = -44$

$\Rightarrow a = 44$

$\Rightarrow 2a + b = -40$

$\Rightarrow 88 + b = -40$

$\Rightarrow b = 48$