Proof of 2nd law

let $\log_c x = p$, $\log_c y = q$

$\Rightarrow c^p = x$, $c^q = y$

$$\frac{x}{y} = \frac{c^p}{c^q} = c^{p-q}$$

$\Rightarrow \log_c \frac{x}{y} = p - q$

$\Rightarrow \log_c \frac{x}{y} = \log_c x - \log_c y$