

Answers

For questions that require a proof (normally indicated by any of the following instructions: prove, show, verify or explain) no answer has been supplied and the question number or subpart has been omitted. So, if you discover a question with a missing answer, this is because the solution is in the question and you have to derive that solution for yourself.

Chapter 1

Exercise 1A

1. (a) (i) 30 (ii) 57
(b) (i) 13 (ii) 29
2. (a) 140 (b) 16
3. (a) 6 (b) 2 (c) 12
4. 48
5. (a) 90 (b) 48 (c) 63
6. 15
7. (a) 15 637 960
(b) 10 104 528
8. 115 316 136
9. 8
10. (a) 720
(b) 648
11. (a) 60
(b) 125
12. (a) 81
(b) 125

Exercise 1B

1. (a) (i) 120 (ii) 720
(b) (i) 48 (ii) 360
(c) (i) 600 (ii) 240
2. (a) (i) 40 320 (ii) 39 916 800
(b) (i) 1080 (ii) 1 814 400
(c) (i) 475 372 800 (ii) 357 840
3. (a) 720 (b) 40 320
(c) $26! = 4.03 \times 10^{26}$ (3SF)
4. (a) 5040
(b) 3600
5. (a) 120
(b) 24

6. (a) $17! = 3.56 \times 10^{14}$ (3SF)
(b) $16! = 2.09 \times 10^{13}$ (3SF)
7. 2880
8. (a) 720
(b) 240
9. $30! = 2.65 \times 10^{32}$ (3SF)
10. $6 \times 5! \times 4! = 17280$

Exercise 1C

1. (a) (i) 7 (ii) 12
(b) (i) 56 (ii) 990
(c) (i) 9900 (ii) 2550
(d) (i) n (ii) $a+1$
(e) (i) $a^2 - a$ (ii) $b^2 + b$
(f) (i) $\frac{1}{(x+8)(x+7)}$
(ii) $\frac{1}{(x-4)(x-5)(x-6)}$
2. (a) (i) $\frac{9!}{6!}$ (ii) $\frac{6!}{2!}$
(b) (i) $\frac{1016!}{1012!}$ (ii) $\frac{309!}{306!}$
(c) (i) $\frac{(n+5)!}{(n+1)!}$ (ii) $\frac{(n+2)!}{(n-2)!}$
3. (a) (i) $9 \times 9!$ (ii) $131 \times 10!$
(b) (i) $11 \times 13!$ (ii) $21 \times 15!$
(c) (i) $121 \times 9!$ (ii) $110 \times 10!$
(d) (i) $(n-1) \times (n-1)!$
(ii) $(n^2 + 3n + 1) \times n!$
4. $n = 5$
5. $n = 10$
6. $n = 5$

ANSWER HINT (4, 5, 6)

Did you think about turning this into a quadratic?

Exercise 1D

1. (a) (i) 21 (ii) 792
(b) (i) 60 (ii) 60
(c) (i) 126 (ii) 135
(d) (i) 136 (ii) 36
2. (a) (i) 28 (ii) 126
(b) (i) 912 (ii) 14
(c) (i) 1176 (ii) 980

3. (a) (i) 14 (ii) 27
 (b) (i) 21 (ii) 17
4. 5005
5. (a) 35 (b) 15
6. 15 380 937
7. 36960
8. 31 500
9. $\binom{140}{12} \binom{128}{10} \binom{118}{10} = 1.62 \times 10^{45}$ (3SF)
10. (a) 43 680
 (b) 65 520
11. (a) 35 (b) 35
 (c) 31 (d) 33
12. (a) 126
 (b) 120
13. 105
14. (a) 120
 (b) 210
15. 24
16. $\binom{45}{15} \binom{30}{15} = 5.35 \times 10^{19}$ (3SF)

Exercise 1E

1. 560
2. 600
3. (a) 120
 (b) 1320
4. (a) 4920
 (b) 4800
5. 19557
6. 270200
7. 65 559
8. (a) 11082
 (b) 48387
9. 696

Exercise 1F

1. (a) (i) 6 (ii) 5
 (b) (i) 56 (ii) 110
 (c) (i) 720 (ii) 1320

2. (i) 5040 (ii) 5040
3. (i) 60 (ii) 210
4. (a) (i) $n = 7$
 (b) (i) $n = 11$ (ii) $n = 14$
5. 7.75×10^{10} (3SF)
6. 255 024
7. 504
8. 336
9. 3 276 000
11. 186
12. 84
13. 4624
14. $n = 3$

Exercise 1G

1. $13! \times 2 = 1.25 \times 10^{10}$
2. 2 488 320
3. 30 240
4. 150×10^{14}
5. (a) 32 432 400
 (b) 45 360
6. (a) 17280
 (b) 5760
 (c) 43200
 (d) 2880

Mixed examination practice 1

Short questions

1. 210
2. 120
3. 30 240
4. 729
5. 55
6. $n = 5$
7. 8640
8. $n = 15$
9. 2947
10. 480

11. 672
 12. 921 164 400
 13. 25 200
 14. 112

Long questions

1. (a) 48
 (b) 72
 (c) 42
2. (a) 20
 (b) 22
 (c) 30
3. (a) 121 080 960
 (b) 3 991 680
 (c) 27 941 760
4. (a) We select 2 out of 4 places to put R's in.
 (b) $\binom{2n}{n}$
 (c) 20
 (d) $\binom{n+m-2}{n-1}$
5. (b) 2047
 (c) 5775
6. (a) 4495
 (b) 22
 (c) 26

Chapter 2

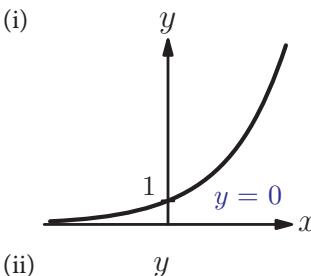
Exercise 2A

1. (a) (i) 6^7 (ii) 5^8
 (b) (i) a^8 (ii) x^9
 (c) (i) 7^{-3} (ii) 5^5
 (d) (i) x^2 (ii) x^5
 (e) (i) g^{-12} (ii) k^{-8}
2. (a) (i) 6^1 (ii) 5^{-2}
 (b) (i) a^{-2} (ii) x^3
 (c) (i) 5^9 (ii) 7^{15}
 (d) (i) x^6 (ii) x^{-11}

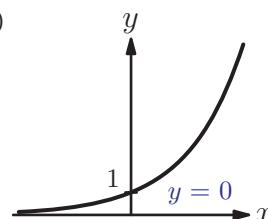
- | | |
|--|-------------------------------|
| (e) (i) 2^2 | (ii) 3^{-14} |
| (f) (i) g^6 | (ii) k^{-8} |
| 3. (a) (i) 2^{12} (ii) 3^{14} | |
| (b) (i) 5^{-4} | (ii) 7^{-6} |
| (c) (i) 11^2 | (ii) 13^{15} |
| (d) (i) 2^{17} | (ii) 3^3 |
| (e) (i) 6^{12} | (ii) 3^6 |
| 4. (a) (i) 2^{10} (ii) 3^{14} | |
| (b) (i) 2^9 | (ii) 2^{20} |
| (c) (i) 2^{13} | (ii) 3^4 |
| (d) (i) 2^9 | (ii) 3^{11} |
| (e) (i) 2^{-6} | (ii) 3^{-6} |
| (f) (i) 2^2 | (ii) 3^{10} |
| 5. (a) (i) $8x^6$ (ii) $9x^8$ | |
| (b) (i) $2x^6$ | (ii) $3x^8$ |
| (c) (i) $9a^{10}$ | (ii) 16 |
| (d) (i) $\frac{1}{2x}$ | (ii) $\frac{y^2}{9}$ |
| (e) (i) $\frac{2}{x}$ | (ii) $3y^2$ |
| (f) (i) $\frac{5x^2y^4}{9}$ | (ii) $\frac{ab^5}{8}$ |
| (g) (i) $\frac{p^3}{2q^2}$ | (ii) $\frac{2^7 3^{10}}{x^7}$ |
| 6. (a) (i) x^3 (ii) x^{12} | |
| (b) (i) $2x^5$ | (ii) $\frac{1}{2x^4}$ |
| (c) (i) $\frac{4}{3x^3}$ | (ii) $\frac{y^{12}}{x^6}$ |
| 7. (a) (i) $\frac{5}{3}$ (ii) $-\frac{3}{2}$ | |
| (b) (i) $-\frac{1}{2}$ | (ii) $-\frac{3}{4}$ |
| (c) (i) 4 | (ii) 2 |
| (d) (i) 4 | (ii) 0 |
| (e) (i) 4 | (ii) 11 |
| (f) (i) 3 | (ii) 3 |
| 8. 5×10^{-4} | |
| 9. 8cm | |
| 10. (a) $k = \frac{1}{3}$ | |
| (b) $A = 16 \text{ cm}^2$ | |
| 11. $2^{350} = (2^7)^{50} = (128)^{50}$ | |
| $5^{150} = (5^3)^{50} = (125)^{50}$ | |
| 12. $b = 1, a = \frac{3}{2}$ | |

Exercise 2B

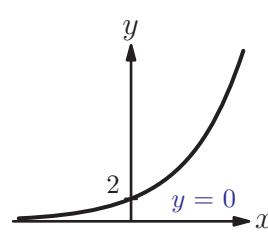
1. (a) (i)



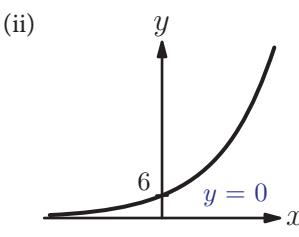
(ii)



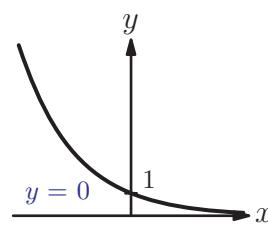
(b) (i)



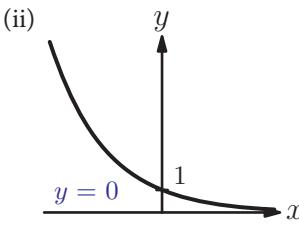
(ii)



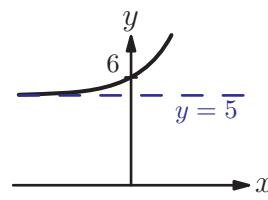
(c) (i)



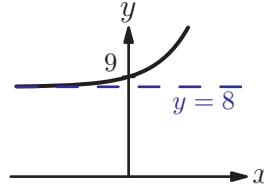
(ii)



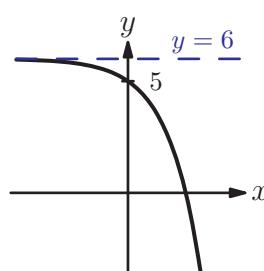
(d) (i)



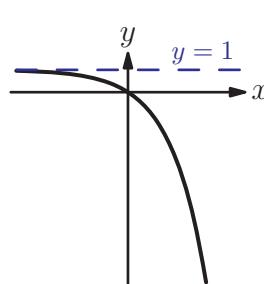
(ii)



(e) (i)



(ii)

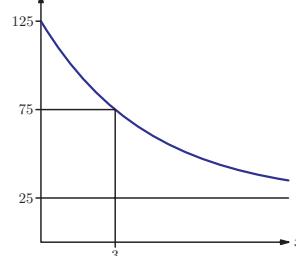


2. 13.31 m^2

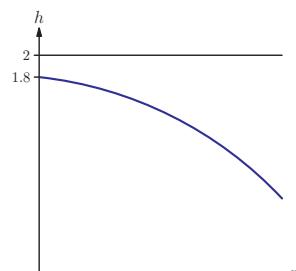
3. (a) $A = 25, B = 100, k = 3$

(b) 26°C

(c)



4. (a)

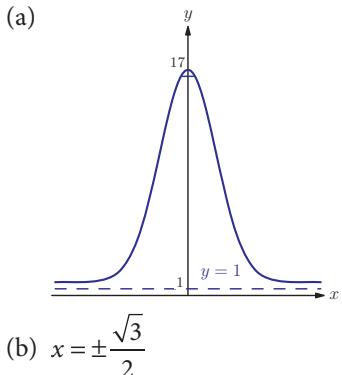


(b) 1.8 m

(c) 1.60 m

(d) The branch may not be long enough to reach the ground, or it might break before it reaches the ground.

5. (a)



(b) $x = \pm \frac{\sqrt{3}}{2}$

6. 41.2°C

7. (a) 0 m^{-1}
(b) 40 m^{-1}

Exercise 2C

1. (a) (i) 3.72 (ii) -0.283
(b) (i) 8.15 (ii) 1.36
(c) (i) 7.39 (ii) 0.0498
(d) (i) 8.24 (ii) 0.00274
2. $\sqrt[6]{\pi^4 + \pi^5} \approx e$

Exercise 2D

1. (a) (i) 3 (ii) 2
(b) (i) 1 (ii) 1
(c) (i) 0 (ii) 0
(d) (i) -1 (ii) -3
(e) (i) $\frac{1}{2}$ (ii) $\frac{1}{3}$
(f) (i) $\frac{1}{2}$ (ii) $\frac{1}{2}$
(g) (i) $\frac{2}{3}$ (ii) $\frac{3}{4}$
(h) (i) $\frac{3}{2}$ (ii) $\frac{5}{4}$
(i) (i) $\frac{3}{4}$ (ii) 2.25
(j) (i) $-\frac{1}{2}$ (ii) $-\frac{1}{2}$

2. (a) (i) 1.70 (ii) -0.602
(b) (i) -2.30 (ii) 2.30

3. (a) (i) $5 \log x$ (ii) $5 \log x$
(b) (i) $\log x \log y - \log y + 3 \log x - 3$
(ii) $(\log x)^2 + 4 \log x + 4$

- (c) (i) $\frac{1}{\log a} + \frac{1}{\log b}$
(ii) $\log a + 1$

4. (a) (i) $x = 3^y$ (ii) $x = 16^y$

- (b) (i) $x = a^{y+1}$ (ii) $x = a^{y^2}$

- (c) (i) $x = \sqrt[3]{3y}$ (ii) $x = \sqrt{y}$

5. (a) (i) $x = 5$ (ii) $x = 2$

- (b) (i) $x = 0.4$ (ii) $x = 0.25$

- (c) (i) $x = 6$ (ii) $x = 100$

6. $x = 111$

7. $x = -3$

8. $x = \frac{e^2 + 1}{3}$

9. $x = 9, \frac{1}{9}$

10. $x = 81, y = 25$

11. $x = 10^{1.5} = 31.6$

12. $x = \sqrt[9]{4} = 1.17$

13. 5.50

Exercise 2E

1. (a) (i) 4 (ii) 1/2
(b) (i) 6 (ii) 3/2

2. (a) (i) $7y$ (ii) $2x + y$

- (b) (i) $x + 2y - z$
(ii) $2x - y - 3z$

- (c) (i) $2 - y - 5z$
(ii) $1 + y + 2z$

- (d) (i) $x - 4y$
(ii) $2 + 2x + y + 2z$

- (e) (i) $2 + \frac{y}{x}$ (ii) $\frac{x-z}{y} - 1$

- (f) (i) $\frac{y}{x} \times 10^{x-y}$ (ii) $\frac{x+2z}{x+y}$

3. (a) (i) $x = 2$ (ii) $x = 4$

- (b) (i) $x = 9$ (ii) $x = 2$

- (c) (i) $x = \frac{1}{4}$ (ii) $x = 8$

- (d) (i) $x = 2^{\frac{12}{5}} = 5.28$
(ii) $x = 2^{10} = 1024$

- (e) (i) $x = 8$ (ii) $x = 4$

- (f) (i) $x = \frac{1}{3}$ (ii) $x = 8$

4. $\frac{1}{3}e^{\frac{3}{2}}$

5. (a) $a+2b$ (b) $2(a-b)$

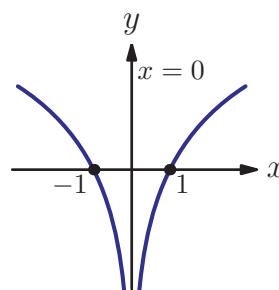
6. $x=2, \frac{1}{2}$

8. -1

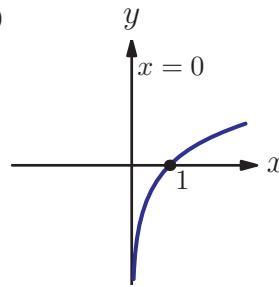
9. $a=\frac{1}{b}$

Exercise 2F

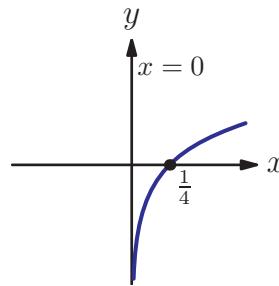
1. (a) (i)



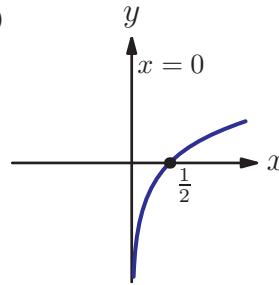
(ii)



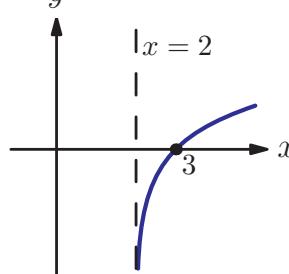
(b) (i)



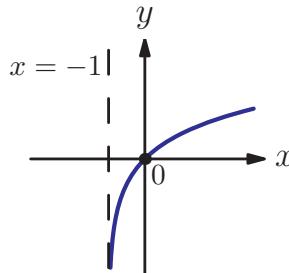
(ii)



(c) (i)



(ii)



Exercise 2G

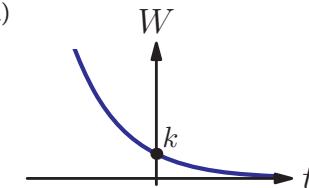
1. (a) (i) 2.45 (ii) 116
 (b) (i) -0.609 (ii) 4.62
 (c) (i) -1.71 (ii) 0.527
 (d) (i) 1.11 (ii) -2.98

2. (a) 100
 (b) 48 299
 (c) 2.24 h

3. (a) 64
 (b) 4.96 h

4. (a) 4.96 units
 (b) 138.8 mins

5. (a)



- (b) 2.3 mins

6.
$$\frac{\ln\left(\frac{5}{4}\right)}{\ln\left(\frac{1}{36}\right)}$$

7. $x = 10 + \log_7 3$

8. 11.3 min

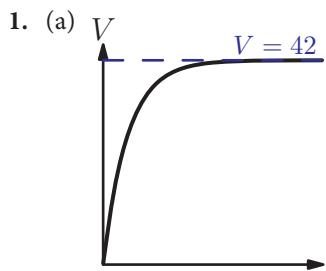
9. (b) 0.742

Mixed examination practice 2

Short questions

1. $x = \pm 24$
2. (a) $2a + \frac{b}{2} - c$
 (b) $\frac{a-1}{2}$
 (c) $\frac{b-c}{2}$
3. $x = e^{\frac{4}{3}} = 3.79, y = e^{\frac{10}{3}} = 28.0$
4. $x = 1 \pm \sqrt{1 - e^y}$
5. $x = \frac{\ln 3}{\ln 2}$
6. $a = b^{-2}$
7. $x = 5^{\frac{5}{3}}$ or $5^{\frac{-5}{3}}$
8. $x = e^2$ or e^{-2}

Long questions



- (b) 0 ms⁻¹
 (c) 42 ms⁻¹
 (d) 3.71s

2. (a) $k = 37000, a = \left(\frac{22}{37}\right)^{0.1} = 0.949$
 (b) 2750
 (c) 2039
 (d) $k = 7778, a = \left(\frac{10000}{7778}\right)^{0.1} = 1.025$
 (e) 2.5%

3. (a) $y = 3x^2$
 (b) $y = e^6 x^4$
 (c) $y = 2e^{3x-3}$
 (d) 2

Chapter 3

Exercise 3A

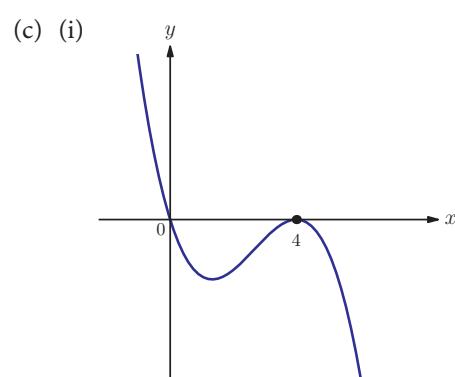
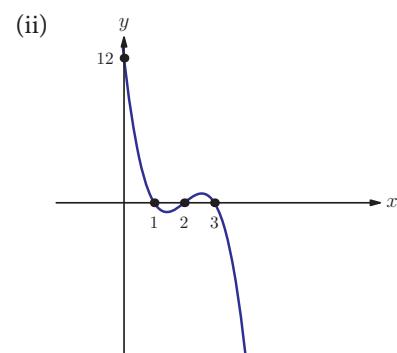
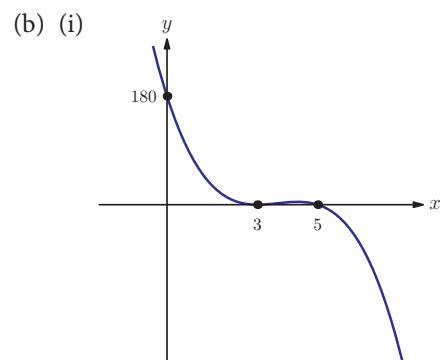
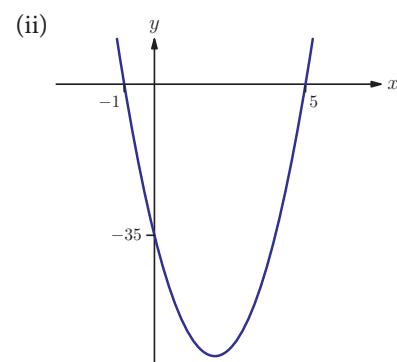
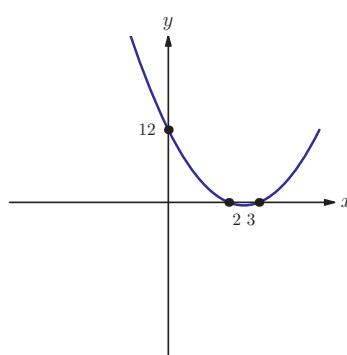
1. (a) Order 3, lead coefficient 3
 (b) Order 5, lead coefficient -1
 (c) No
 (d) No
 (e) No
 (f) No
 (g) Order 7, lead coefficient 2
 (h) Order 0, lead coefficient 1
2. (a) (i) $6x^3 + 8x^2 - 29x + 14$
 (ii) $3x^3 + 16x^2 + 23x + 6$
 (b) (i) $2x^4 - 15x^3 + 4x^2 + 4x - 1$
 (ii) $2x^4 - 7x^3 - 30x^2 + 6x + 15$
 (c) (i) $b^4 + b^3 - 3b^2 + 14b - 4$
 (ii) $r^4 - 11r^3 + 33r^2 - 62r + 14$
 (d) (i) $-x^6 + 2x^5 + 5x^4 - 10x^3 - x^2 + 5$
 (ii) $-x^6 + 2x^4 + x^3 - x^2 - x$
3. (a) (i) $x^2 + 5x - 1$
 (ii) $x^2 + x - 6$
 (b) (i) $3x^2 + 2x - 2$
 (ii) $5x^2 - 2$
 (c) (i) $x^3 - 2x^2 + 3x + 7$
 (ii) $x^3 - x^2 + x + 7$
 (d) (i) $x^2 + 5$
 (ii) $x - 2$
4. (a) (i) $x^3 + x^2 + 3$
 (ii) $x^3 + x^2 + 2$
 (b) (i) $2x^2 + 3$
 (ii) $x - 3$
5. (a) (i) $a = 4, b = -6$
 (ii) $a = 3, b = 1$
 (b) (i) $a = b = 2$
 (ii) $a = 0, b = -3$
 (c) (i) $a = 2, b = -2$
 (ii) $a = 2, b = 5$
 (d) (i) $a = -4, b = -6$
 (ii) $a = 10, b = 3$
 (e) (i) $a = \pm 2, b = 2$
 (ii) $a = \pm 2, b = \mp 5$
7. (a) Yes
 (b) No

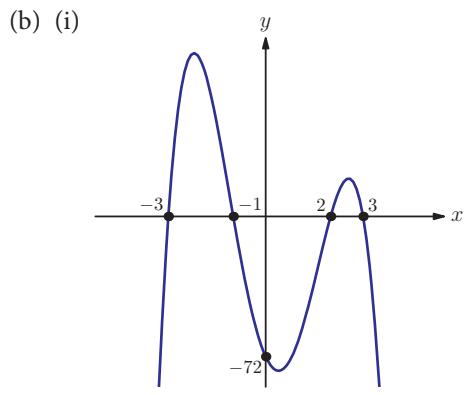
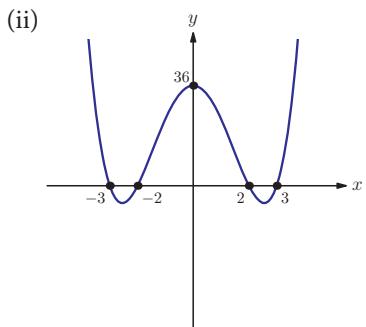
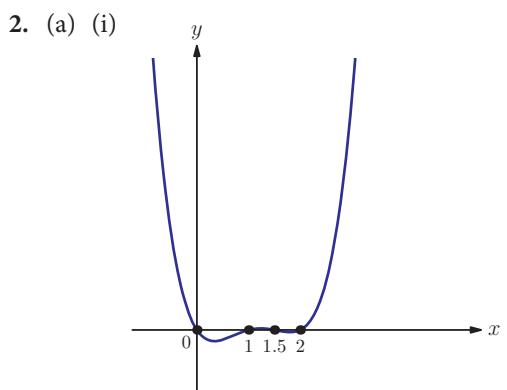
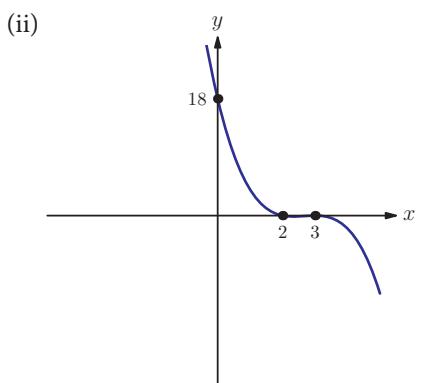
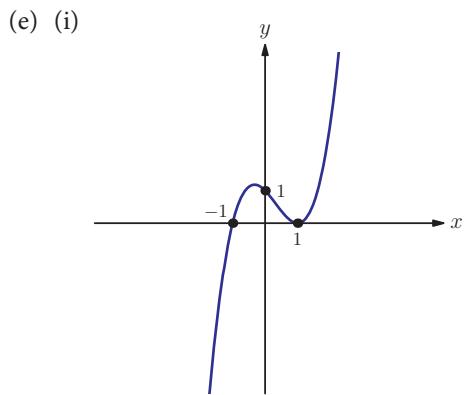
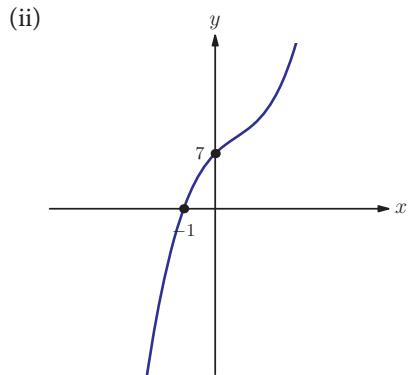
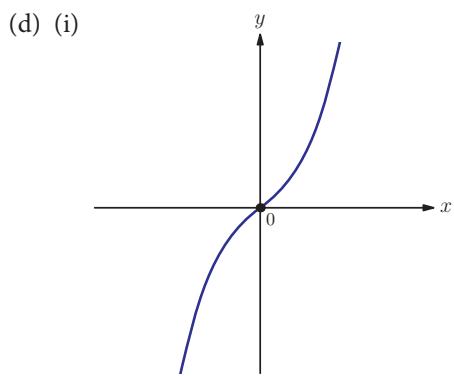
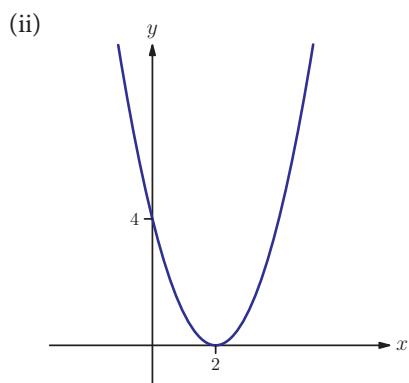
Exercise 3B

1. (a) (i) 3 (ii) -2
 (b) (i) -7 (ii) 5
 (c) (i) -2 (ii) -4
 (d) (i) -8 (ii) 3
2. (a) No (b) No
 (c) Yes (d) Yes
 (e) No (f) Yes
 (g) No (h) Yes
 (i) No (j) No
3. (a) (i) $(x+1)(x-1)(x+2)$
 (ii) $(x+1)(x-2)(x+2)$
 (b) (i) $(x-2)^2(x-3)$
 (ii) $(x+2)^3$
 (c) (i) $(x-1)(x^2-2x+10)$
 (ii) $(x-3)(x^2+x+5)$
 (d) (i) $(x-1)(2x-1)(3x-1)$
 (ii) $(x+2)(4x+3)(3x-5)$
4. $a=1, b=-18$
5. $a=-44, b=48$
6. $k=0, 4$
7. $k=-\frac{1}{2}$
8. (a) $a=2, b=59$ (b) $(x+8)$
9. (a) $a=-12, b=22$ (b) 0
10. 14
11. $a=37, b=-30$

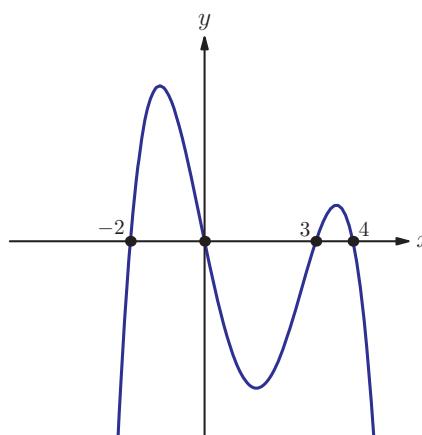
Exercise 3C

1. (a) (i)

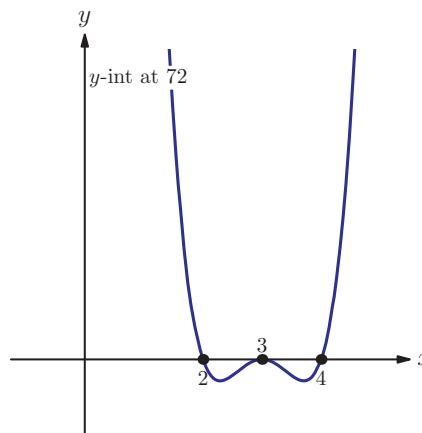




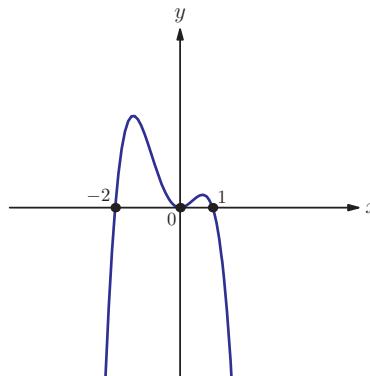
(ii)



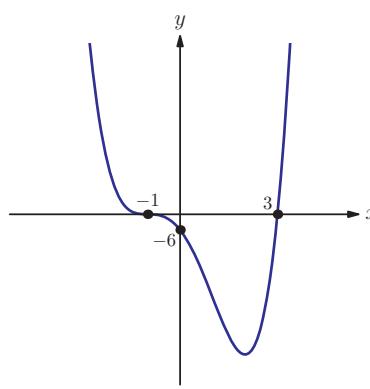
(c) (i)



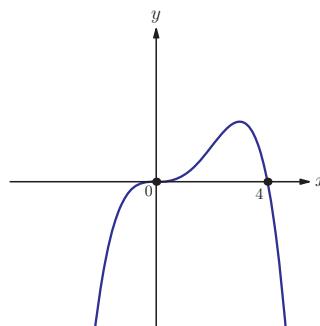
(ii)



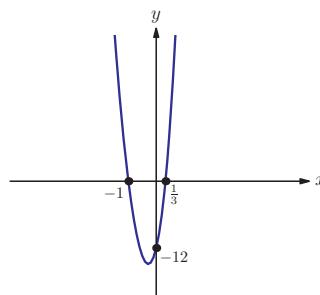
(d) (i)



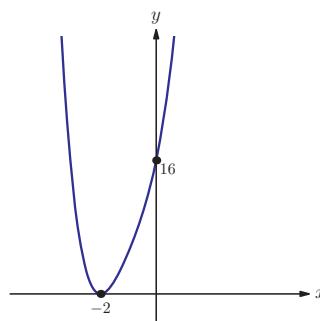
(ii)



(e) (i)



(ii)



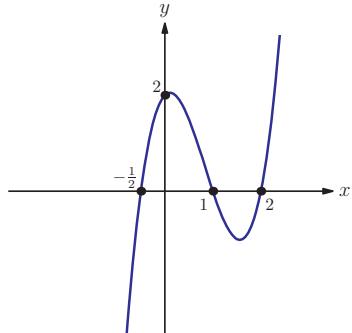
3. (i) A1, B3, C2 (ii) A3, B1, C2

4. (a) (i) $a = 2, b = -8, c = 6$ (ii) $a = 5, b = 5, c = -10$ (b) (i) $a = -\frac{1}{4}, b = \frac{3}{4}, c = 1$ (ii) $a = -1, b = 0, c = 1$ 5. (a) (i) $y = 2(x-1)(x-4)(x+2)$ (ii) $y = 6x(x+2)(x-3)$ (b) (i) $y = -5x(x-1)(x+2)$ (ii) $y = -(x-1)(x+2)(x+4)$ (c) (i) $y = -(x+1)(x-2)^2$ (ii) $y = (x+1)^2(x-2)$ (d) (i) $y = x(x+2)(x+3)(2x-1)$ (ii) $y = -2(x-3)(x-4)(x+1)(x-2)$ (e) (i) $y = -3x^2(x-1)(x-3)$ (ii) $y = 5x(x-2)^2(2x+1)$

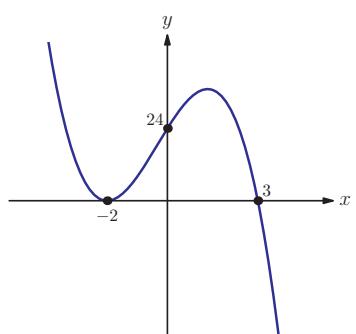
(f) (i) $y = 2(x-3)^2(x+1)^2$

(ii) $y = -(x-3)^2(x-1)^2$

6. (b) $(x-2)(x-1)(2x+1)$
(c)



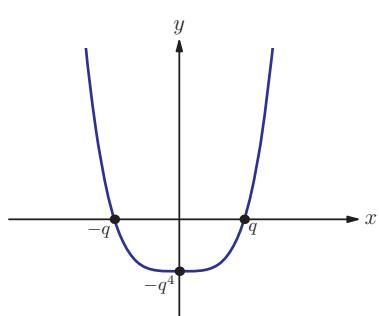
7.



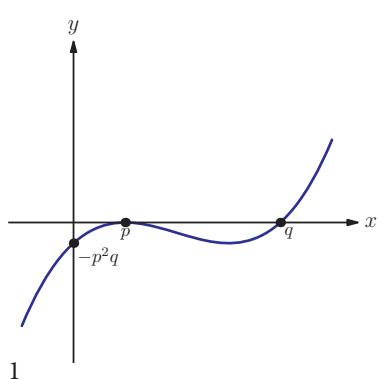
8. (a) $p=2, q=-8, r=-6, s=36$
(b) $p=-1, q=3, r=0, s=0$

9. (a) $(x-q)(x+q)(x^2+q^2)$

(b)



10. (a)



(b) 1

Exercise 3D

1. (a) (i) 36
(b) (i) -47
(c) (i) 0
(d) (i) 49
(ii) 68
(ii) -119
(ii) 0
(ii) 49

2. (a) Two
(b) None
(c) One
(d) Two

3. (a) (i) $x = \frac{3 \pm \sqrt{5}}{2}$

(ii) $x = \frac{1 \pm \sqrt{5}}{2}$

(b) (i) $x = -1, \frac{2}{3}$

(ii) $x = \frac{3 \pm \sqrt{7}}{2}$

(c) (i) $x = \frac{4}{3}, -1$

(ii) $x = \frac{1}{2}, -1$

(d) (i) $x = 2 \pm \sqrt{7}$

(ii) $x = 1, -\frac{3}{2}$

4. (a) (i) $k < \frac{1}{4}$

(ii) $k < \frac{25}{12}$

(b) (i) $k = \frac{3}{5}$

(ii) $k = -\frac{1}{24}$

(c) (i) $k \geq -\frac{5}{4}$ (ii) $k \leq \frac{1}{16}$

(d) (i) $k > \frac{3}{8}$ (ii) $k > -\frac{25}{12}$

(e) (i) $k = \frac{17}{4}$ (ii) $k = \frac{55}{32}$

(f) (i) $k = 1$ (ii) $k = \frac{1}{32}$

(g) (i) $k < 0$ (ii) $k < 0$

5. $m = \pm \sqrt{2}$

6. $k = \frac{11}{2} \pm \sqrt{30}$

7. $c \geq \frac{17}{16}$

8. $0 < k < 6$

9. $-9 < k < -1$

10. $m \leq -8$ or $m \geq 0$

11. $m < -\frac{9}{16}$

12. $k = \pm 9$

Mixed examination practice 3

Short questions

1. $k+2$

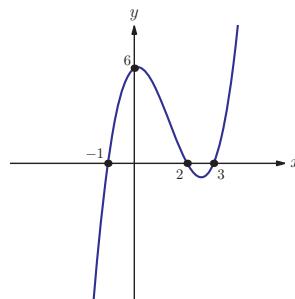
2. a, c negative, b positive, $b^2 - 4ac = 0$

3. $a = 1, b = 2, c = -12, d = -18, e = 27$

4. $a = 1, b = 0$

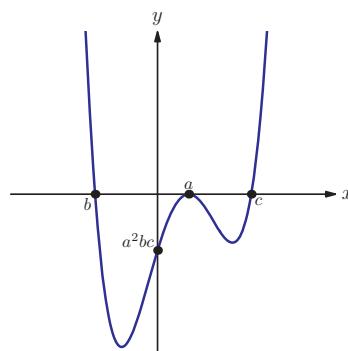
5. (b) $f(x) = (x-2)(x+1)(x-3)$

(c)



6. $(a, b) = \pm \left(\frac{5}{3}, -\frac{4}{3} \right), \pm \left(-\frac{1}{3}, \frac{8}{3} \right)$

7.



8. $3 \pm 2\sqrt{2}$

9. $-4\sqrt{3} < k < 4\sqrt{3}$

10. $k \leq -\sqrt{5} - \frac{1}{2}$ or $k \geq \sqrt{5} - \frac{1}{2}$

11. $a = -10, b = -18$

12. (a) $k-1, 1$
(b) $-3, 5$

14. $-2\sqrt{2} \leq k \leq 2\sqrt{2}$

Long questions

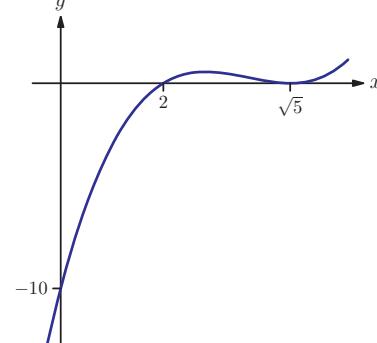
1. (a) $(0, -a)$

(b) $x = -\frac{b}{2}$

(d) $b > 7$ or $b < -5$

2. (b) $p = \pm 2\sqrt{5}$

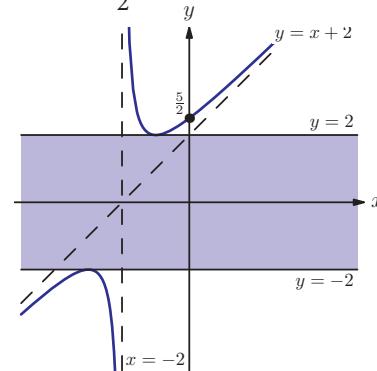
(c)



3. (b) $x = -2, y = x+2$

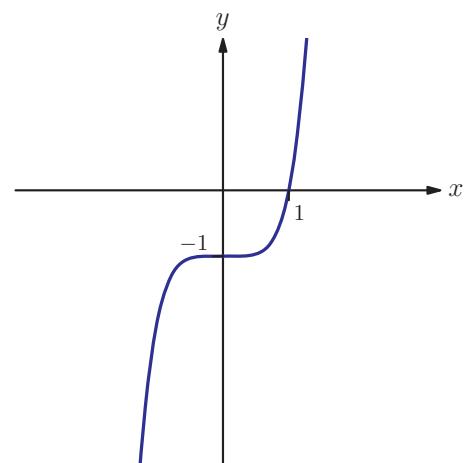
(c) $x = \frac{y \pm \sqrt{y^2 - 4}}{2} - 2$

(e)



4. (a) 5

(c)





Chapter 4

Exercise 4A

1. (a) (i) $x = 3$ (ii) $x = -1$

(b) (i) $x = \frac{1}{2}, -\frac{3}{5}$
(ii) $x = 3, -3$

(c) (i) $x = 27, 1$

(ii) $x = 16, \frac{1}{81}$

(d) (i) $x = 7, \pm\sqrt{3}$

(ii) $x = \frac{6}{5}, 1, 4$

(e) (i) $x = 2$

(ii) $x = 1 - \log_2 7$

2. (a) (i) $x = -3, 1, 4$

(ii) $x = -1, -3, 5$

(b) (i) $x = 2, \frac{3 \pm \sqrt{5}}{2}$

(ii) $x = 1, \frac{5 \pm \sqrt{17}}{2}$

3. (a) (i) $x = 1, 2, 3$

(ii) $x = -2, 1, 3$

(b) (i) $x = 1, -1$

(ii) $x = -3, 2, 4$

4. $x = \frac{2}{3} \pm 2$.

5. $x = 0, 4, -4$

Exercise 4B

1. (a) (i) $a = \pm\sqrt{3}, \pm\sqrt{7}$

(ii) $x = \pm 2, \pm\sqrt{3}$

(b) (i) $x = -\sqrt[3]{5}, \sqrt[3]{1.5}$

(ii) $a = 1, -2$

(c) (i) $x = \pm\sqrt{2 + \sqrt{6}}$

(ii) $x = \pm\sqrt{6}$

(d) (i) $x = 4, 16$

(ii) $x = 16, 36$

(e) (i) $x = \ln 4$

(ii) $x = \ln 4, \ln 5$

(f) (i) $x = 1, \log_5 10$

(ii) $x = 2, \log_2 3$

(g) (i) $x = 1, \sqrt{2}$ (ii) $x = 3, 9$

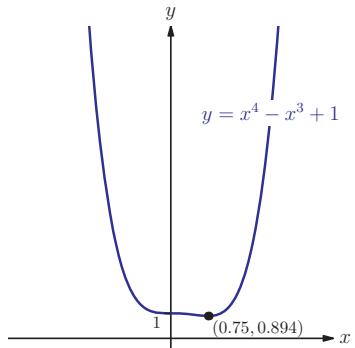
2. $x = 0, 2$

3. $x = 0, \log_a 5$

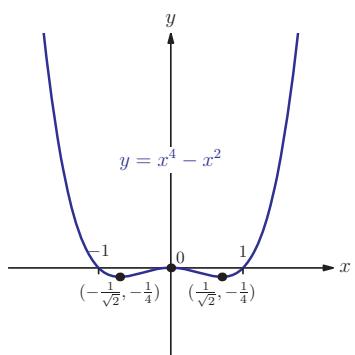
4. $x = 2, 32$

Exercise 4C

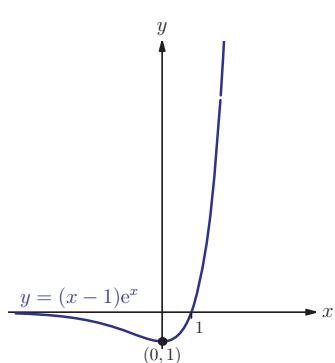
1. (a) (i)



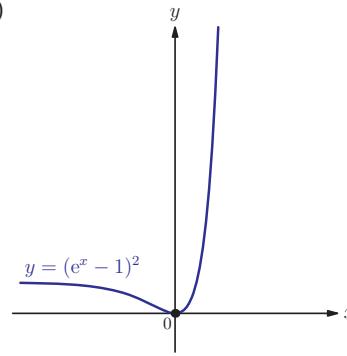
(ii)



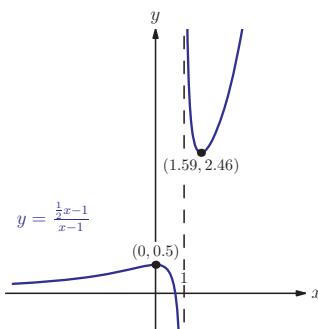
(b) (i)



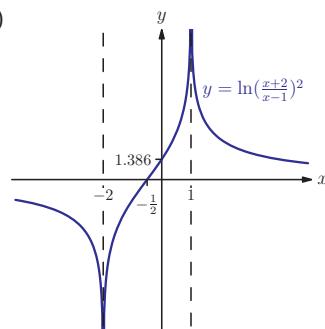
(ii)



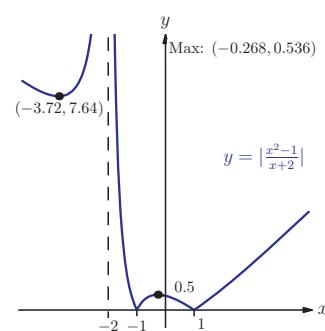
(c) (i)



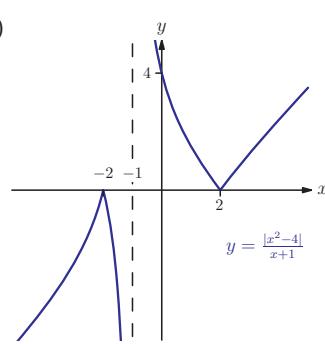
(ii)



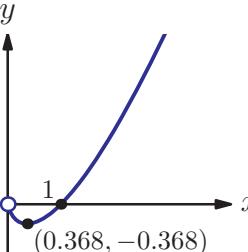
(d) (i)



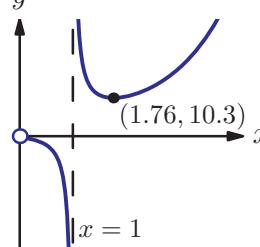
(ii)



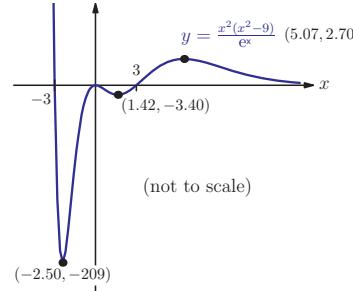
2.



3.



4.



Exercise 4D

1. (a) (i) $x = -1.88, 0.347, 1.53$
 (ii) $x = -4.49$
- (b) (i) $x = 0$
 (ii) -1.74
- (c) (i) No solution
 (ii) $x = 3.59$

2. $x = 1.53$

3. (a) 3 (b) 1 (c) 1

Exercise 4E

1. (a) (i) $(-2, -3), (1, 0)$
 (ii) $(3, 0)$
- (b) (i) $(-3, -9), (4, 5)$
 (ii) No intersection

2. (a) (i) $(3, 1), \left(\frac{-11}{5}, \frac{-8}{5}\right)$
 (ii) $(-3, 3), (5, -1)$

- (b) (i) $(1,3), (3,1)$
(ii) $(-3,-5), (-5,-3)$
- (c) (i) $(-1,6), (2,3)$
(ii) $(1,-3), (-1,-5)$

4. $-1 \pm 2\sqrt{6}$

5. $\pm 6\sqrt{2}$

7. $x = 3, y = 1$ or $y = 3, x = 1$

8. $x = \sqrt{3}, y = 9\sqrt{3}$

Exercise 4F

1. (a) (i) $x = 3, y = -1, z = 1$
(ii) $x = -2, y = 2, z = 1$
- (b) (i) $x = 3, y = -3, z = 0$
(ii) $x = -2, y = 1, z = 0$
- (c) (i) $x = 3, y = -1, z = 2$
(ii) $x = -1, y = 0, z = 3$

2. (a) Unique solution

(b) No solutions

(c) Infinitely many solutions

(d) No solutions

(e) Unique solution

3. (a) (i) $x = 2-t, y = 1+t, z = t$
(ii) $x = 2t+1, y = t-1, z = t$
- (b) $x = 2t+1, y = t, z = 0$
- (c) $x = 5-t+2s, y = s, z = t$

4. -2

5. -9

6. $x = 1, y = z = \frac{2-a}{3}$

7. (a) $a = 1$ or 2

(b) $x = 1 + \frac{t}{5}, y = \frac{3t}{5}, z = t$

8. (a) -2

(b) 9

(c) $x = t+5, y = t-1, z = t$

Exercise 4G

1. (a) (i) $x < 3.08$ (ii) $x > 1.58$
- (b) (i) $x \leq 4.11$ (ii) $x \geq 7.54$
- (c) (i) $x < 1.91$ (ii) $x < 0.743$
- (d) (i) $x \geq 1.36$ (ii) $x < 1.08$
- (e) (i) $x > -1.77$ (ii) $x \leq 4.49$
- (f) (i) $-2 < x < 0$ or $x > 2$
(ii) $x < -\sqrt{6}$ or $0 < x < \sqrt{6}$

2. (a) $0 < x < 1.30$ or $x > 12.7$
(b) $1 < x < 1.30$ or $x > 12.7$

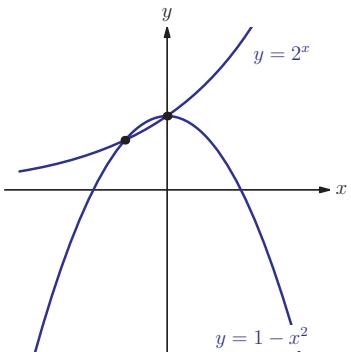
Exercise 4H

2. $4p^3 + 27q^2 = 0$

Mixed examination practice 4

Short questions

1. (a)



(b) 2

2. 0.541

3. $-1.12 < x < -0.379$ or $x > -2.02$

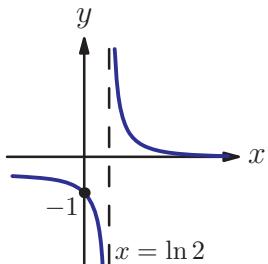
4. $x = 2, y = -2, z = -3$

5. $x = e^3$

6. (a) $x = \pm 2, \pm 3$

(b) $-3 \leq x \leq -2$ or $2 \leq x \leq 3$

7. (a)



(b) $x = \ln 2$

8. (a) -7

(b) $x = 5-t, y = t, z = 9$

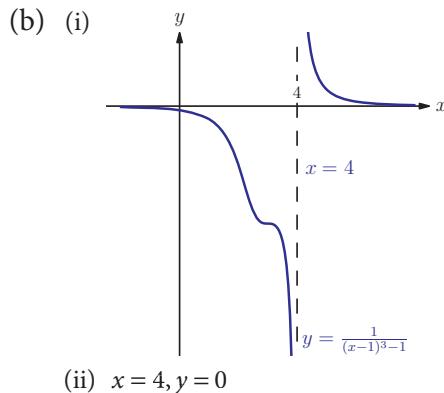
9. ± 5

10. $0 < x < 1$ or $1.43 < x < 8.61$

11. $x = 1$

Long questions

1. (a) (i) $k = 27$
 (ii) $x = 4$



(ii) $x = 4, y = 0$

(c) $(1.44, -1.38)$

2. (a) $x = \pm\sqrt{2}$
 (b) (i) $k = -1$ (ii) $x = 1 \pm \sqrt{2}$
 (c) $x < 1 - \sqrt{2}$ or $x > 1 + \sqrt{2}$

3. (a) 0
 (b) 5
 (c) $x = 4 - t, y = -1, z = t$

Chapter 5

Exercise 5A

1. (a) Function: many-to-one
 (b) Relation
 (c) Relation
 (d) Function: one-to-one
 (e) Function: one-to-one
 (f) Function: one-to-one
2. (a) (i) Yes (ii) Yes
 (b) (i) Yes (ii) Yes
 (c) (i) No (ii) No

Exercise 5B

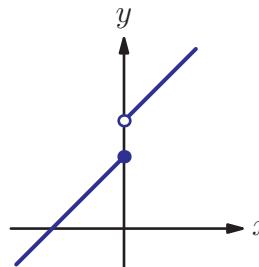
1. (a) (i) 24 (ii) 140
 (b) (i) 14 (ii) 4
 (c) (i) $3z^2 - z$ (ii) $3a^2 - a$
 (d) (i) $3x^2 + 5x + 2$
 (ii) $3x^2 - 13x + 14$
 (e) (i) $-x$ (ii) $57x^2 - 11x$
 (f) (i) $\frac{3-x}{x^2}$ (ii) $3x - \sqrt{x}$

2. (a) (i) 3 (ii) 7
 (b) (i) 0 (ii) 1
 (c) (i) $1 + \log_{10} y$ (ii) $1 + \log_{10} z$
 (d) (i) $2 + \log_{10} x$ (ii) $3 + \log_{10} y$
 (e) (i) $4 + 5\log_{10} x$ (ii) 1
3. (a) (i) 4 (ii) -8
 (b) (i) $3x + 1 - \sqrt{y}$ (ii) $6x + 4 - 2\sqrt{x}$
 (c) (i) $24x + 2 - 6\sqrt{x}$ (ii) $6x^2 + x - \sqrt{x^2 + 1}$

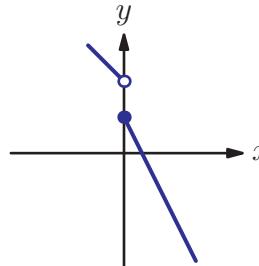
Exercise 5C

1. (a) Domain: \mathbb{R} Range: $[0, \infty[$
 (b) Domain: \mathbb{R} Range: $[0, \infty[$
 (c) Domain: $[0, \infty[$ Range: \mathbb{R}
 (d) Domain: $[0, \infty[$ Range: \mathbb{R}
2. (a) (i) $x \neq -2$ (ii) $x \neq 7$
 (b) (i) $x \neq 2$ or -4 (ii) $x \neq \pm 3$
 (c) (i) $y \geq 1$ (ii) $x \geq -3$
 (d) (i) $a > 1$ (ii) $x < \frac{2}{5}$
 (e) (i) $x \neq 0$ or -1 (ii) $x \geq -1$
 (f) (i) $x \leq -\sqrt{5}$ or $x \geq \sqrt{5}$ (ii) $x \leq -3$ or $x \geq 1$
 (g) (i) $x \geq 0$ (ii) $x \geq -\frac{3}{2}$
3. (a) (i) $y \leq 7$ (ii) $y \geq 3$
 (b) (i) $y \geq 12$
 (ii) $y \geq 5, y \in \mathbb{Z}$
 (c) (i) $y \geq 0$ (ii) $y \geq 0$
 (d) (i) $y \leq -1$ or $y > 0$
 (ii) $y > 0$

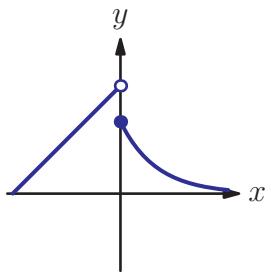
4. (a) (i) Not continuous, range $]-\infty, 2] \cup]3, \infty[$



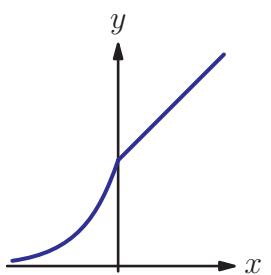
- (ii) Not continuous, range $]-\infty, 1] \cup]2, \infty[$



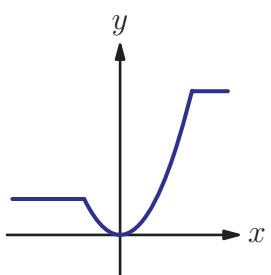
- (b) (i) Not continuous, range $]-\infty, 3[$



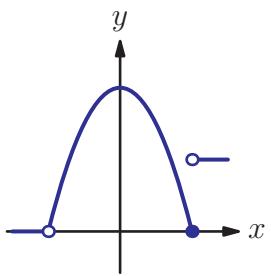
- (ii) Continuous, range $]0, \infty[$



- (c) (i) Continuous, range $[0, 4]$



- (ii) Not continuous, range $[0, 2]$



5. $x \geq 5$

6. $x \geq 1, x \neq 2, x \neq 3$

7. $a = 15$

8. $x < -2$ or $x > -1$

9. $x \leq \frac{1}{2}$ or $x > 12$

10. (a) (i) $a \leq x < b$ (ii) \emptyset

(b) $f(a) = \begin{cases} \ln(b-a) & \text{for } a < b \\ \text{undefined} & \text{for } a \geq b \end{cases}$

Exercise 5D

1. (a) (i) 5 (ii) 26
 (b) (i) $9x+8$ (ii) $9x^2+12x+5$
 (c) (i) $9\sqrt{a}+17$
 (ii) $y^4-4y^3+8y^2-8y+5$
 (d) (i) $9x^2+17$
 (ii) $27z^2+36z+17$

2. (a) (i) x^2 (ii) x^3
 (b) (i) $3x-5$ (ii) x^2+5x+6
 (c) (i) $x+4$ (ii) $x^{\frac{2}{3}}$
 (d) (i) $\ln(\ln x)$

$$\text{(ii)} \ln\left(\frac{x+1}{3}\right)$$

3. $x = 0, -2$

4. $x = -\frac{1}{3}$

5. (a) $y \neq 2$ (b) $x = 1.5$

(c) Domain: $x < -1$ or $x \geq 1.5$

Range: $y > 0$ and $y \neq \sqrt{2}$

6. (a) $\sqrt[3]{2x+3}$ (b) $2\sqrt[3]{x}+3$

7. (a) $a = \frac{-4}{3}, b = \frac{-2}{3}$ (b) $y \geq 0$

8. (a) x^2 is not always > 3

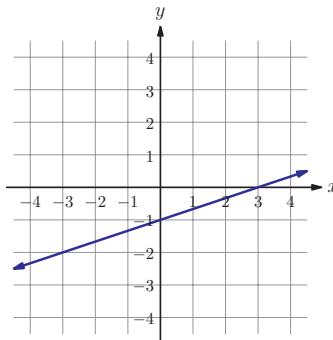
(b) $x \in]-\infty, -\sqrt{3}[\cup]\sqrt{3}, \infty[$

9. $\frac{x-2}{6}$

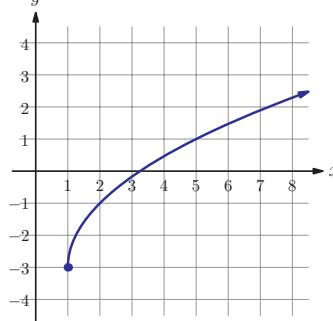
Exercise 5E

1. (a) (i) $\frac{x-1}{3}$ (ii) $\frac{x+3}{7}$
 (b) (i) $\frac{2x}{3x-2}, x \neq \frac{2}{3}$ (ii) $\frac{x}{1-2x}, x \neq \frac{1}{2}$
 (c) (i) $\frac{xb-a}{x-1}, x \neq 1$ (ii) $\frac{x-1}{bx-a}, x \neq \frac{a}{b}$
 (d) (i) $1-x$ (ii) $\frac{x-2}{3}$
 (e) (i) $\frac{x^2+2}{3}, x \geq 0$ (ii) $\frac{2-x^2}{5}, x \geq 0$
 (f) (i) $\frac{1-e^x}{5}$ (ii) $\frac{e^x-2}{2}$
 (g) (i) $2\ln\left(\frac{x}{7}\right), x > 0$ (ii) $\frac{1}{10}\ln\left(\frac{x}{9}\right), x > 0$
 (h) (i) $5-\sqrt{y+19}$ (ii) $\sqrt{y+10}-3$

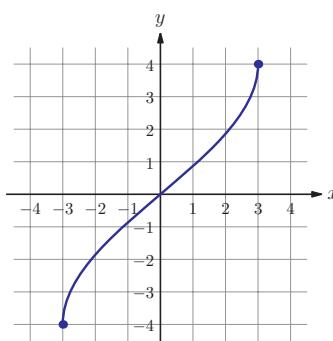
2. (a)



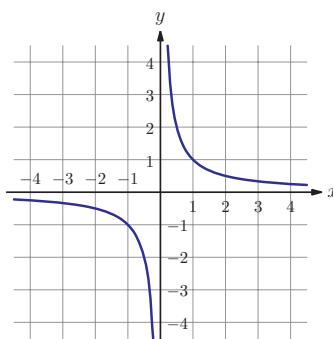
(b)



(c)



(d)



3.

(a) (i) x , yes(ii) x , yes(b) (i) $4x$, no(ii) $\frac{x}{16}$, no(c) (i) $20 + x$, no(ii) $2 + x$, no(d) (i) x , yes(ii) x , yes(e) (i) x , yes(ii) x , yes(f) (i) $\frac{x^4}{3}$, no(ii) x^9 , no(g) (i) $\sqrt[4]{x}$, no(ii) $\sqrt[9]{x}$, no(h) (i) $\frac{x-1}{x}$, no(ii) $\frac{1+x}{2+x}$, no(i) (i) x , yes(ii) x , yes4. $x \geq 0$

5. (a) -1 (b) 1

6. -23

$$7. f^{-1}(x) = \frac{1}{2} \ln\left(\frac{x}{3}\right)$$

$$8. (f \circ g)^{-1}(x) = \sqrt[3]{\frac{x-3}{2}}$$

9. (a) $\ln 3$ 10. $x = -1$

$$11. f^{-1}(x) = -\sqrt{\frac{4x+4}{1-x}}, x \neq 1$$

$$12. (a) k=0, f^{-1}(x) = -\sqrt{x}, x \geq 0$$

$$(b) k=-1, f^{-1}(x) = \sqrt{x-2} - 1, x \geq 2$$

$$(c) k=0, f^{-1}(x) = -x, x \geq 0$$

$$13. (a) f^{-1}(x) = \frac{e^x}{3} + 1$$

$$(b) g \circ f(x) = 3x - 3, x > 1$$

14. (a) $k=2$ (b) (i) $[-\infty, 0] \cup]2, \infty[$

$$(ii) f^{-1}(x) = \begin{cases} 1 + \sqrt{-x}, & x \leq 0 \\ 1 - \sqrt{x-2}, & x > 2 \end{cases}$$

15. (b) $k = -3$

Exercise 5F

$$1. (a) (i) \left(-\frac{1}{3}, 0\right), \left(0, \frac{1}{3}\right) \quad (ii) \left(-\frac{5}{2}, 0\right), (0, 5)$$

$$(b) (i) \left(\frac{3}{2}, 0\right), \left(0, -\frac{3}{7}\right) \quad (ii) \left(\frac{5}{3}, 0\right), \left(0, -\frac{5}{2}\right)$$

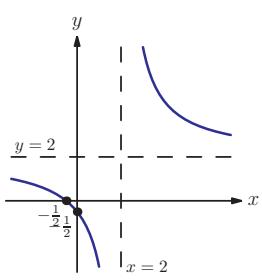
$$2. (a) (i) x=1, y=4 \quad (ii) x=7, y=2$$

$$(b) (i) x=\frac{1}{2}, y=\frac{3}{2} \quad (ii) x=\frac{5}{3}, y=\frac{4}{3}$$

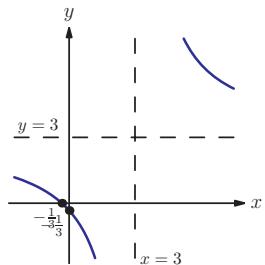
$$(c) (i) x=\frac{-5}{2}, y=\frac{-1}{2} \quad (ii) x=\frac{2}{3}, y=\frac{-2}{3}$$

$$(d) (i) x=2, y=0 \quad (ii) x=\frac{-1}{2}, y=0$$

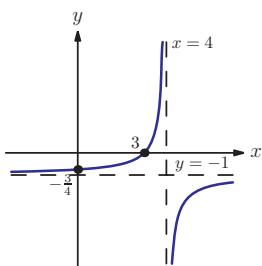
3. (a) (i)



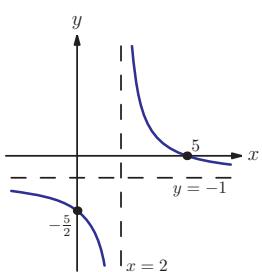
(ii)



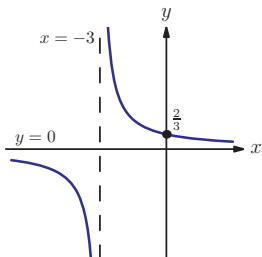
(b) (i)



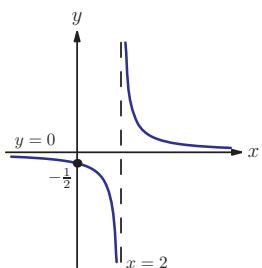
(ii)



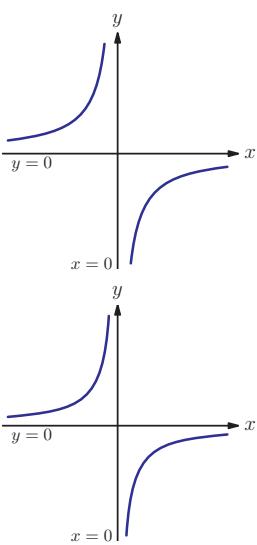
(c) (i)



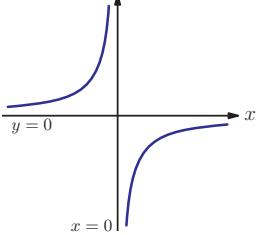
(ii)



(d) (i)



(ii)



4. (a) (i) Domain $x \neq 0$, Range $y \neq 0$, $f^{-1}(x) = \frac{3}{x}$

(ii) Domain $x \neq 0$, Range $y \neq 0$, $f^{-1}(x) = \frac{7}{x}$

(b) (i) Domain $x \neq 3$, Range $y \neq 0$, $f(x) = \frac{2+3x}{x}$

(ii) Domain $x \neq 0$, Range $y \neq 0$, $f^{-1}(x) = \frac{5^x - x}{x}$

(c) (i) Domain $x \neq \frac{1}{3}$, Range $y \neq \frac{2}{3}$,

$$f^{-1}(x) = \frac{x+1}{3x-2}, x \neq \frac{2}{3}$$

(ii) Domain $x \neq \frac{-1}{2}$, Range

$$y \neq 2, f^{-1}(x) = \frac{-x-5}{2x-4}, x \neq 2$$

(d) (i) $x \neq -1$, Range

$$y \neq 2, f^{-1}(x) = \frac{5-2x}{x+2}, x \neq -2$$

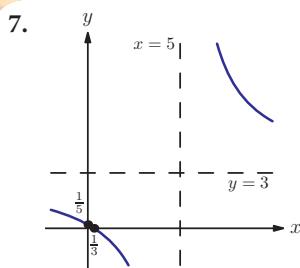
(ii) Domain $x \neq \frac{3}{4}$, Range

$$y \neq \frac{3}{4}, f^{-1}(x) = \frac{3x-1}{4x-3}, x \neq \frac{3}{4}$$

$$5. x = \frac{4}{5}, y = \frac{-3}{5}$$

6. (a) Domain $x \neq -3$, Range $y \neq 0$

$$(b) f^{-1}(x) = \frac{1-3x}{x}$$

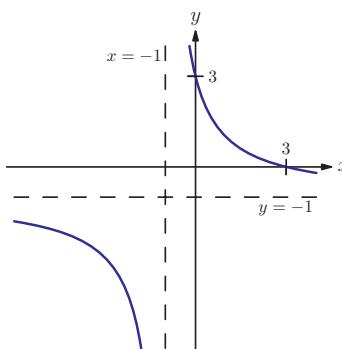


8. (a) $y \neq \frac{a}{2}$
 (b) $f^{-1}(x) = \frac{8x+3}{2x-a}, x \neq \frac{a}{2}$
 (c) 8

Mixed examination practice 5

Short questions

1. (a) $3^x - 3$
 (b) $\sqrt[3]{\ln\left(\frac{x}{3}\right) + 1}$
2. (a) $y = \log_2 x$ (b) $(1, 0)$
3. (a) $x = 5, y = -4$
 (b) $f^{-1}(x) = \frac{5x+3}{x+4}$
4. (a) $(x-3)^2 + 1$ (b) $\sqrt{x-1} + 3$
 (c) $x \geq 1$
5. (a) $(x-3)^2 - 7$ (b) $y \geq -7$ (c) $\sqrt{x+7} + 3$
6. (a) $y \in \mathbb{R}, y \neq -1$
 (b)



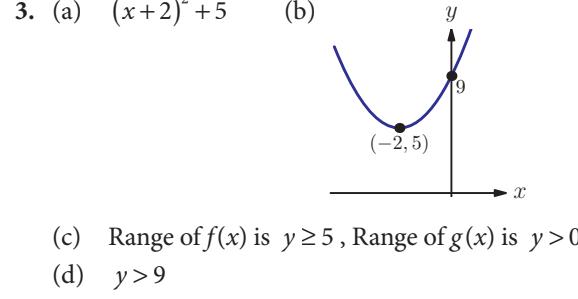
- (c) $f^{-1}(x) = \frac{3-x}{x+1}, x \neq -1, y \neq -1$
7. (a) (i) $[0, 3] \cup]5, \infty[$
 (ii) $f^{-1}(x) = \begin{cases} \ln\left(\frac{3}{x}\right), & 0 < x \leq 3 \\ 5-x, & x > 5 \end{cases}$

Domain: $]0, 3] \cup]5, \infty[$
 (b) $p = 5$

8. (a) $a = -2, b = 1$ (b) $y \geq 0$

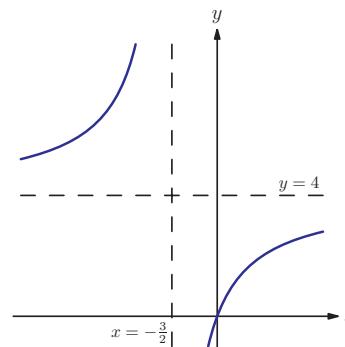
Long questions

1. (a) 10
 (b) $4 - x^2$
 (c) Reflection in the line $y = x$
 (d) (i) $\sqrt{x-1}$ (ii) $y > 3$
 (iii) $x > 10$
 (e) $x = -4, 1$
2. (a) (i) 15 (ii) $y \in \mathbb{R}$
 (iii) $\frac{3x+5}{x-1}$ (iv) $4x+3$
 (b) $f(x)$ can be 1, which is not in the domain of g .
 (c) (i) $\frac{x+3}{x-1}$ (ii) $x \neq 1$ (iii) $y \neq 1$
3. (a) $(x+2)^2 + 5$ (b)



- (c) Range of $f(x)$ is $y \geq 5$, Range of $g(x)$ is $y > 0$
 (d) $y > 9$

4. (a) $y = \frac{8x}{2x+3}$
 (b)



- (c) (i) $\frac{16x+8k}{4x+2k+3}$ (ii) $x = -\frac{2k+3}{4}, y = 4$

(iii) $f(x) = f^{-1}(x) = \frac{16x-76}{4x-16}$

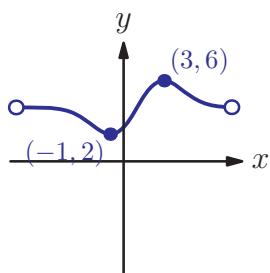
5. (b) $f\left(\frac{1}{x}\right) + 2f(x) = \frac{2}{x} + 1$ (c) $\frac{1}{3}\left(\frac{4}{x} - 2x + 1\right)$



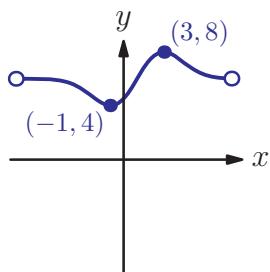
Chapter 6

Exercise 6A

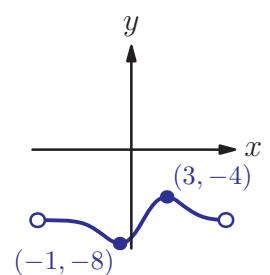
1. (a) (i)



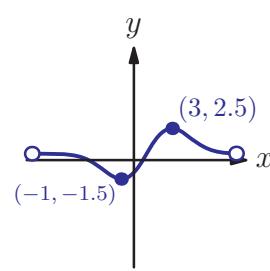
(ii)



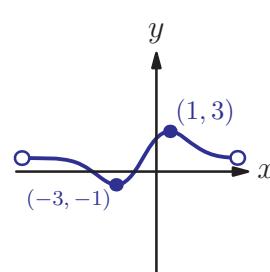
(b) (i)



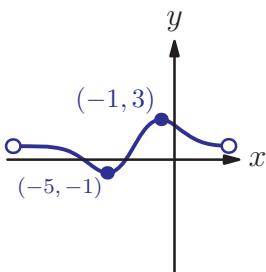
(ii)



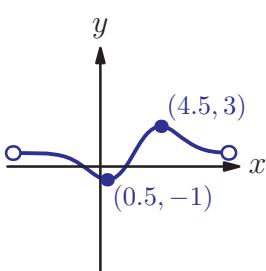
(c) (i)



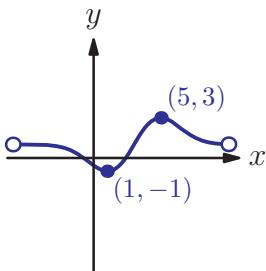
(ii)



(d) (i)



(ii)



2. (a) (i) $y = 3x^2 + 3$

(ii) $y = 9x^3 - 7$

(b) (i) $y = 7x^3 - 3x + 4$

(ii) $y = 8x^2 - 7x + 6$

(c) (i) $y = 4(x - 5)^2$

(ii) $y = 7(x + 3)^2$

(d) (i) $y = 3(x + 4)^3 - 5(x + 4)^2 + 4$

(ii) $y = (x - 3)^3 + 6(x - 3) + 2$

3. (a) (i) Vertically down 5 units

(ii) Vertically down 4 units

(b) (i) Left 1 unit

(ii) Left 5 units

(c) (i) Right 4 units

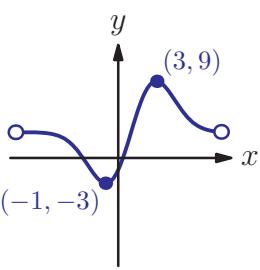
(ii) Right 5 units

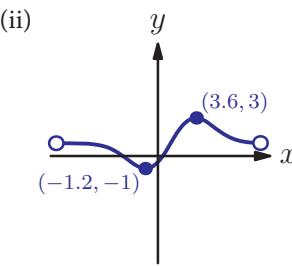
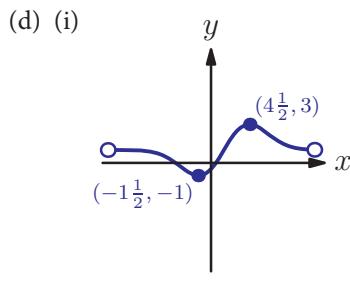
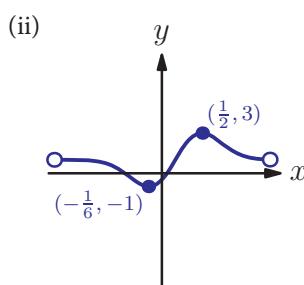
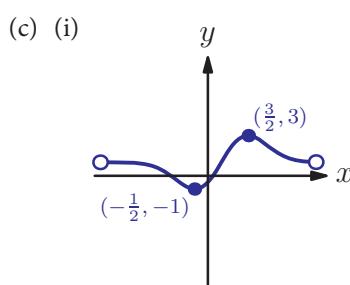
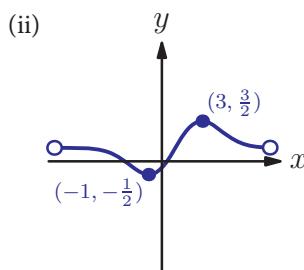
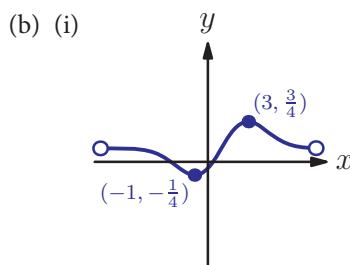
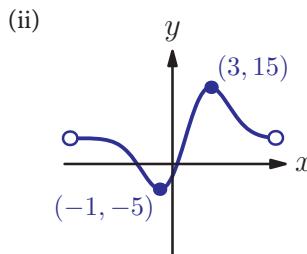
(d) (i) Left 3 units

(ii) Right 2 units

Exercise 6B

1. (a) (i)





2. (a) (i) $y = 21x^2$

(ii) $y = 18x^3$

(b) (i) $y = \frac{1}{3}(7x^3 - 3x + 6)$

(ii) $y = \frac{4}{5}(8x^2 - 7x + 1)$

(c) (i) $y = x^2$

(ii) $y = 7\left(\frac{x}{5}\right)^2$

(d) (i) $y = 3(2x)^3 - 5(2x)^2 + 4$

(ii) $y = \left(\frac{3x}{2}\right)^3 + 6\left(\frac{3x}{2}\right) + 2$

3. (a) (i) Vertical stretch, scale factor 4

(ii) Vertical stretch, scale factor 6

(b) (i) Horizontal stretch, scale factor $\frac{1}{3}$

(ii) Horizontal stretch, scale factor $\frac{1}{4}$

(c) (i) Horizontal stretch, scale factor 2

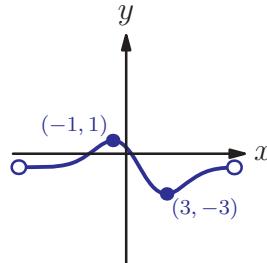
(ii) Horizontal stretch, scale factor 5

(d) (i) Horizontal stretch, scale factor $\frac{1}{3}$

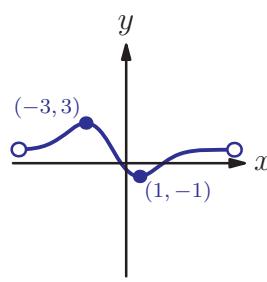
(ii) Horizontal stretch, scale factor 2

Exercise 6C

1. (a)



(b)



2. (a) (i) $y = -3x^2$

(ii) $y = -9x^3$

(b) (i) $y = -7x^3 + 3x - 6$

(ii) $y = -8x^2 + 7x - 1$

(c) (i) $y = 4x^2$

(ii) $y = -7x^3$

(d) (i) $y = -3x^3 - 5x^2 + 4$

(ii) $y = -x^3 - 6x + 2$

3. (a) (i) Reflection in the x -axis

(ii) Reflection in the x -axis

(b) (i) Reflection in the y -axis

(ii) Reflection in the y -axis

(c) (i) Reflection in the x -axis

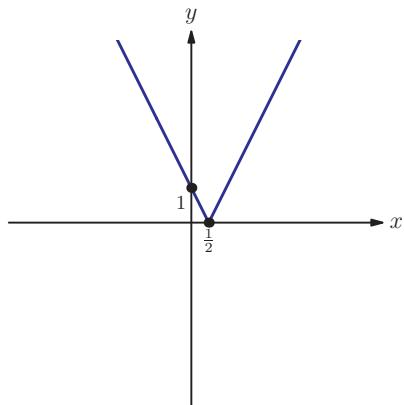
(ii) Reflection in the x -axis

(d) (i) Reflection in the y -axis

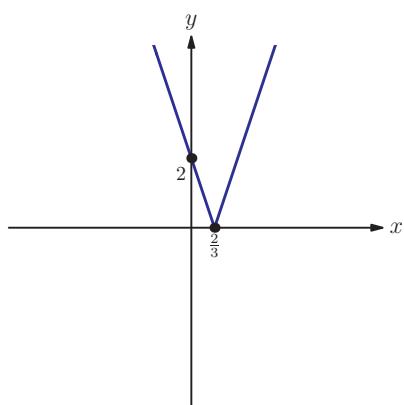
(ii) Reflection in the y -axis

Exercise 6D

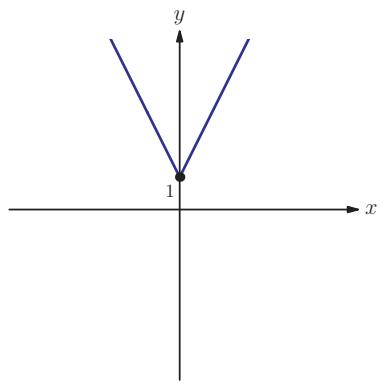
1. (a) (i)



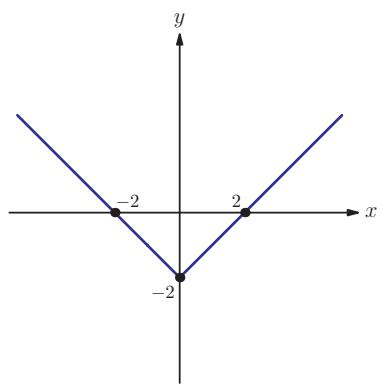
(ii)



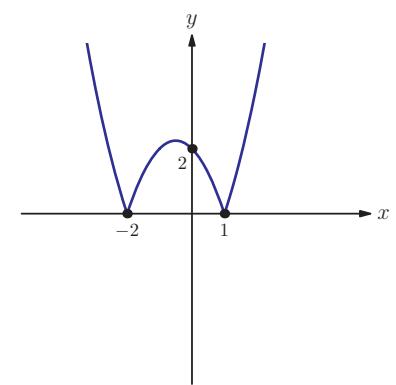
(b) (i)



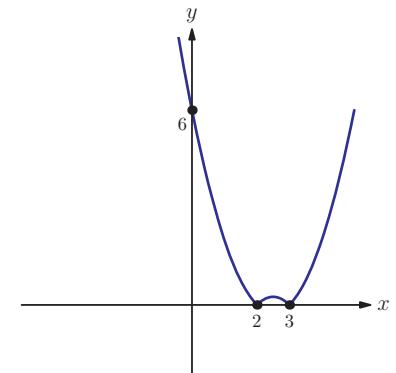
(ii)



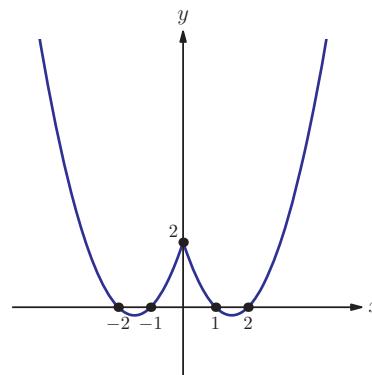
(c) (i)



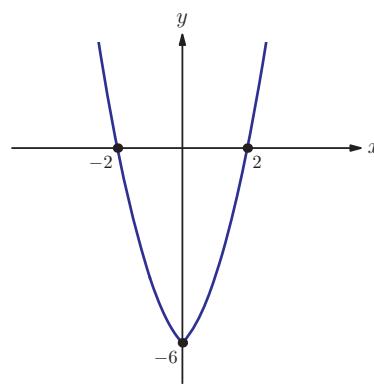
(ii)



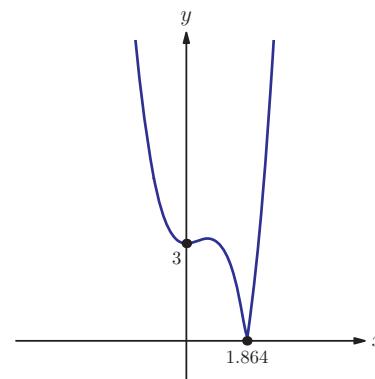
(d) (i)



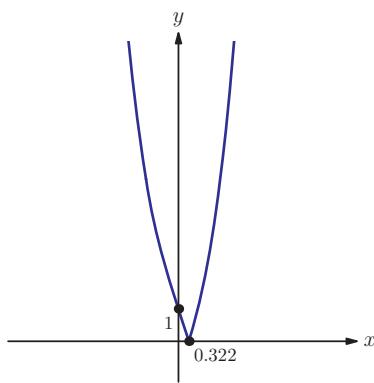
(ii)



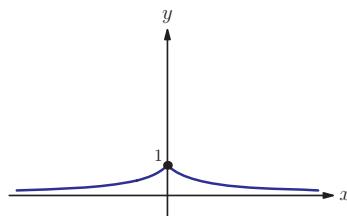
2. (a) (i)



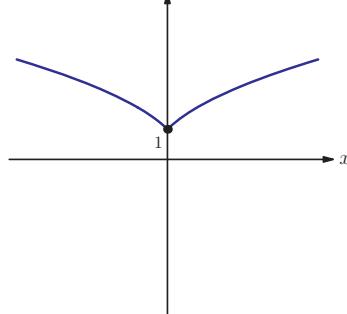
(ii)



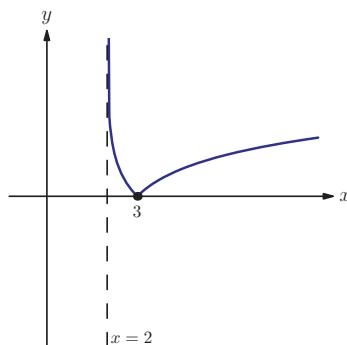
(b) (i)



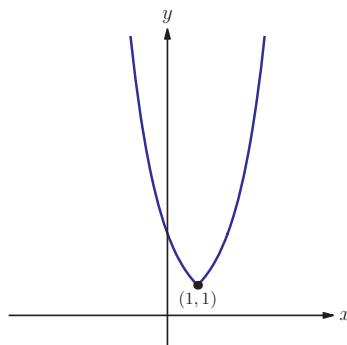
(ii)



(c) (i)



(ii)



3. (a) (i) $x = \pm 4$

(b) (i) $x = 0, 4$

(c) (i) $x = 0, -8$

(ii) $x = \pm 18$

(ii) $x = -1, \frac{1}{3}$

(ii) $x = -\frac{2}{3}, 8$

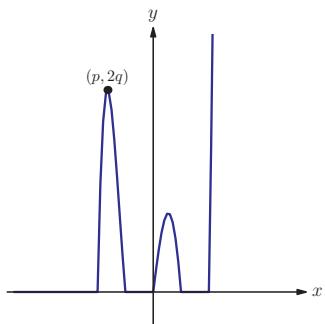
- (d) (i) $x = \frac{2}{3}, 4$ (ii) $x = \frac{1}{4}, \frac{9}{2}$
4. (a) (i) $x = -2, 3$ (ii) $x = -\frac{1}{2}$
- (b) (i) $x = \frac{1}{2}$ (ii) $x = -1, -2$
- (c) (i) $x = 4, -\frac{4}{3}$ (ii) $x = 1, -\frac{1}{3}$
- (d) (i) No solutions (ii) No solutions
5. (a) (i) $x \in]-\infty, -5[\cup]5, \infty[$
(ii) $x \in]-\infty, -2[\cup]2, \infty[$
- (b) (i) $-3 < x < 3$ (ii) $-10 < x < 10$
- (c) (i) $x \in]-\infty, -\frac{5}{2}[\cup]\frac{3}{2}, \infty[$
(ii) $-\frac{1}{3} < x < \frac{5}{3}$
- (d) (i) $\frac{4}{3} < x < 6$ (ii) $x \in]-\infty, 1[\cup]5, \infty[$
- (e) (i) $-\frac{4}{3} < x < 4$ (ii) $-1 < x < 1$

6. $x \in \mathbb{R}$ 7. $0.472, 8.47$

8. $2 \leq x \leq 5$

10. $\frac{q^2}{2}$

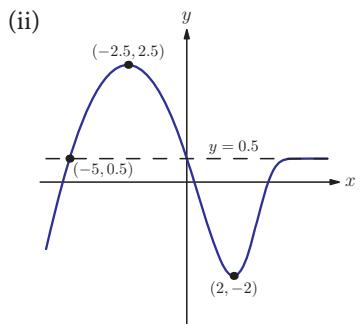
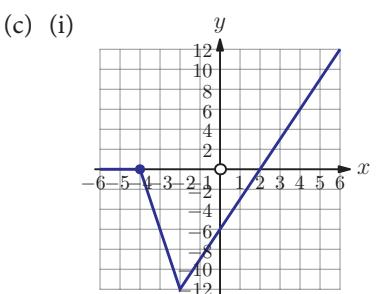
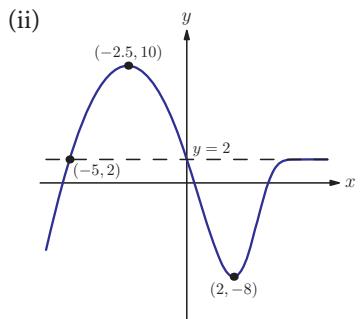
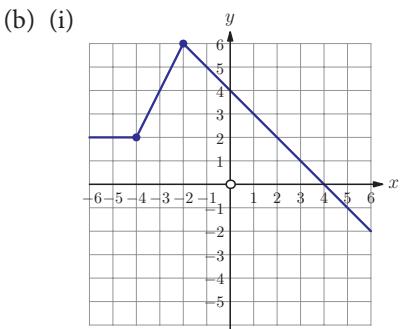
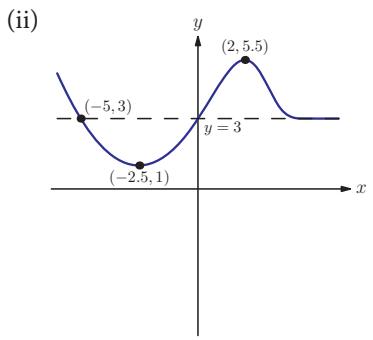
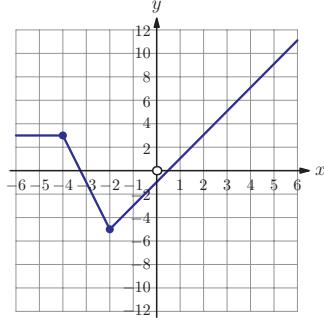
11.



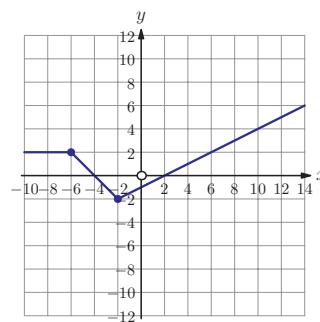
Exercise 6E

1. (a) $y = p(f(x) + c)$ (b) $y = f\left(\frac{x}{q} + d\right)$

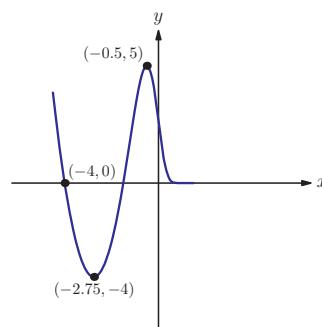
2. (a) (i)



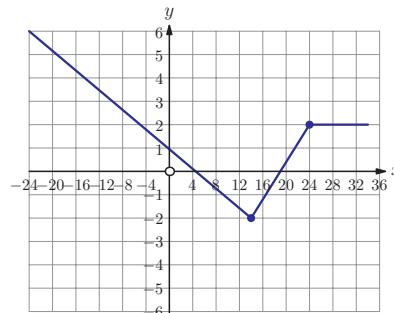
(d) (i)



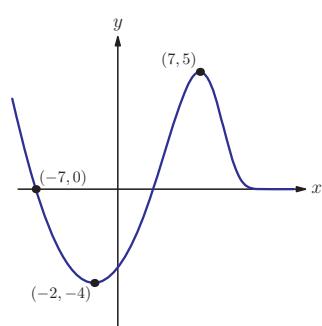
(ii)



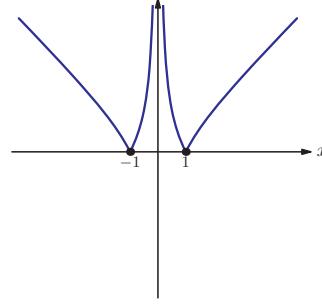
(e) (i)



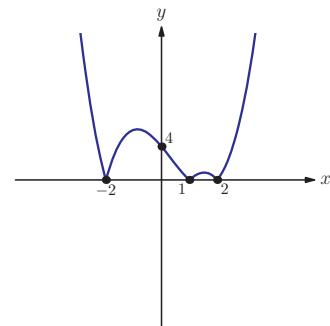
(ii)



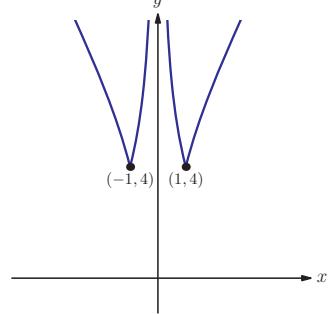
3. (a) (i)



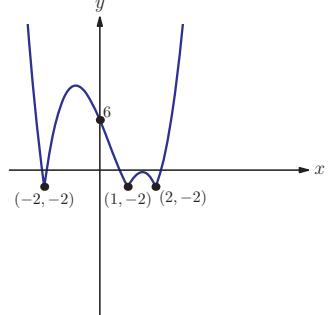
(ii)



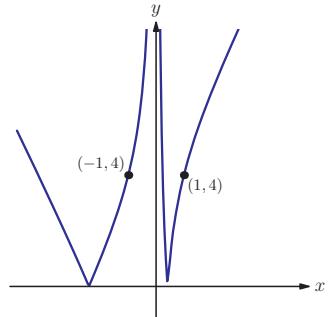
(b) (i)



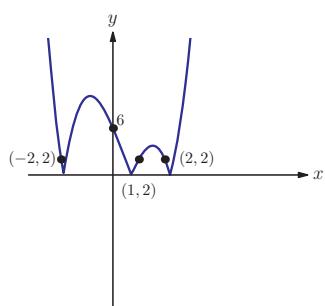
(ii)



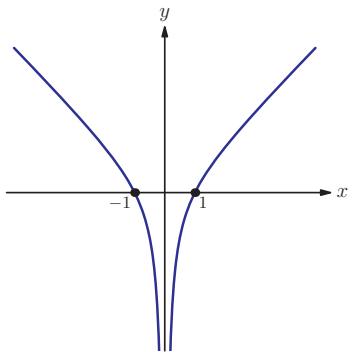
(c) (i)



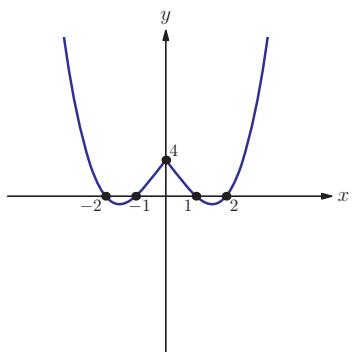
(ii)



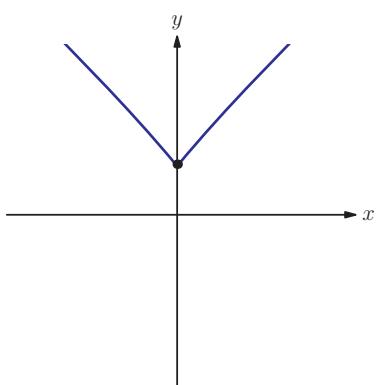
4. (a) (i)



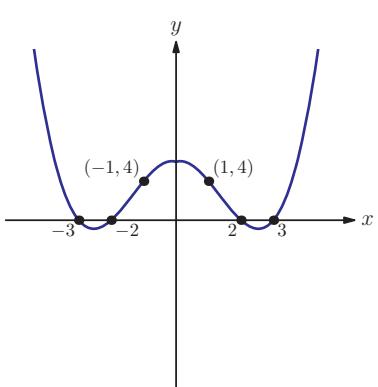
(ii)



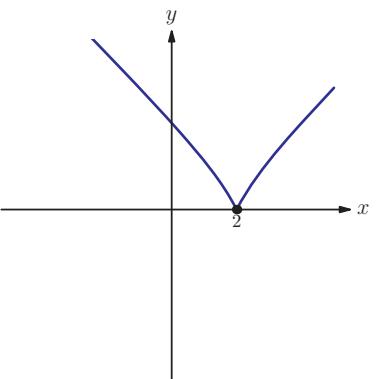
(b) (i)



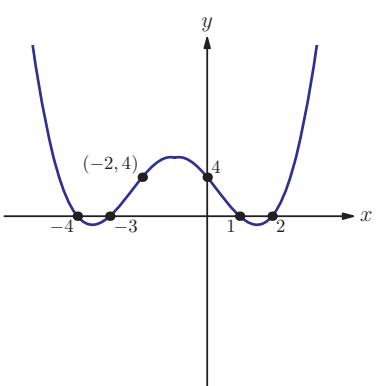
(ii)



(c) (i)



(ii)



5. (a) (i) $k(x) = 2f(x) - 6$ Stretch sf 2 relative to

$$y=0 \text{ and translation } \begin{pmatrix} 0 \\ -6 \end{pmatrix}$$

(ii) $k(x) = 5f(x) + 4$ Stretch sf 5 relative to $y=0$ and translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

(b) (i) $h(x) = 5 - 3f(x)$ Stretch sf 3 relative to $y=0$, reflection in $y=0$ and translation $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$

(ii) $h(x) = 4 - 8f(x)$ Stretch sf 8 relative to $y=0$, reflection in $y=0$ and translation $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

6. (a) (i) $g(x) = 6x^2 - 6$

$$(ii) g(x) = x^2 + 1$$

(b) (i) $g(x) = x^2 + 4$

$$(ii) g(x) = 7x^2 - 4$$

(c) (i) $g(x) = 4 - 2x^2$

(ii) $g(x) = 6 - 2x^2$

(d) (i) $g(x) = 5 - x^2$

(ii) $g(x) = -3 - 3x^2$

7. (a) (i) $g(x) = f(x+1)$ translation $\begin{pmatrix} -1 \\ 0 \end{pmatrix}$.

(ii) $g(x) = f(x-3)$ Or reflection in $x=0$
then translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

(b) (i) $h(x) = f(2x)$ Horizontal stretch sf $\frac{1}{2}$

(ii) $h(x) = f\left(\frac{x}{3}\right)$: Stretch sf 3 relative to $x=0$

(c) (i) $k(x) = f(2x+2)$: Translation $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$ then
stretch sf $\frac{1}{2}$ relative to $x=0$

(ii) $k(x) = f(3x-1)$: Translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ then
stretch sf $\frac{1}{3}$ relative to $x=0$

8. (a) (i) $g(x) = 32x^2 - 16x - 2$

(ii) $g(x) = 8x^2 + 16x + 4$

(b) (i) $g(x) = 8x^2 + 64x + 124$

(ii) $g(x) = \frac{9x^2}{2} - 9x + \frac{1}{2}$

(c) (i) $g(x) = 2x^2 - 12x + 14$

(ii) $g(x) = 2x^2 + 12x + 14$

9. (a) (i) $g(x) = 2f(x+1) - 2$: Vertical stretch

sf 2 then translation by $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

(ii) $g(x) = 3f(x-4) - 40$: Vertical stretch

sf 3 then translation by $\begin{pmatrix} 4 \\ -40 \end{pmatrix}$

(b) (i) $g(x) = f(x-3) - 4$: translation by $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$

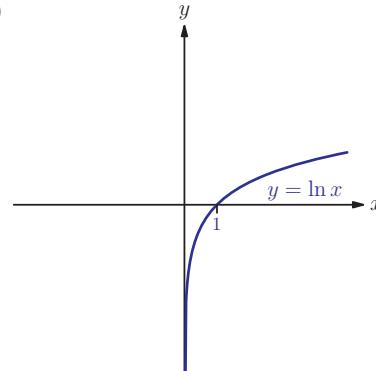
(ii) $g(x) = -4f(x-1) + 8$: Vertical stretch
sf 4 and reflection in x -axis then
translation by $\begin{pmatrix} 8 \\ 1 \end{pmatrix}$

10. (i) $a = 5, b = 7$

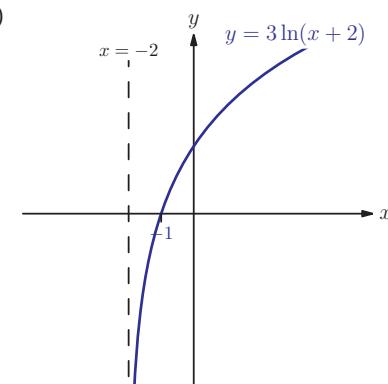
(ii) $a = 2, b = 3, c = -5$

11. $h(x) = 4^{x+1} + 16x - 4$

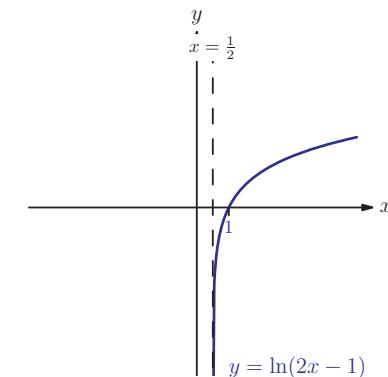
12. (a)



(b)

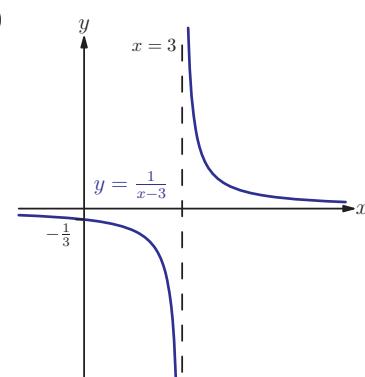


(c)

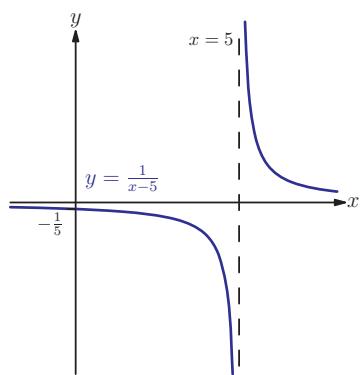


Exercise 6F

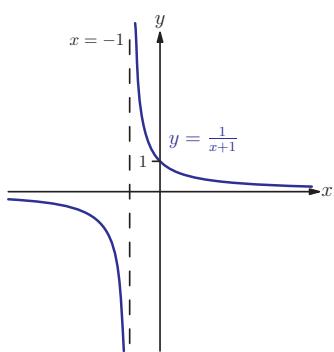
1. (a) (i)



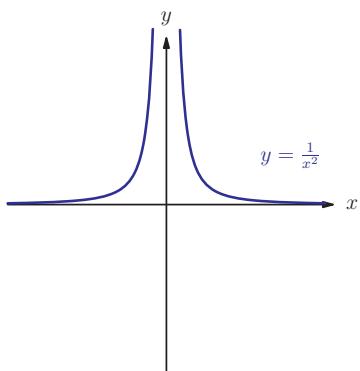
(ii)



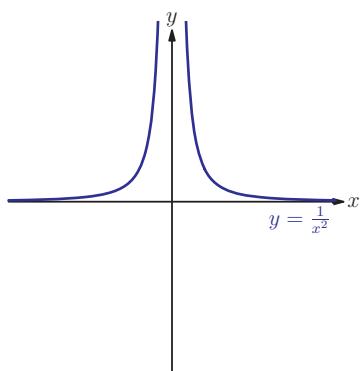
(b) (i)



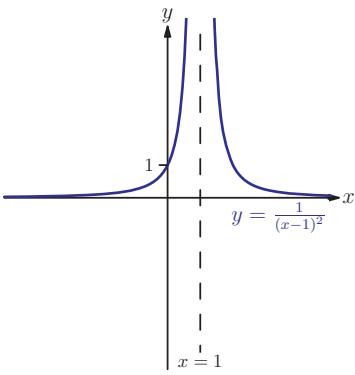
(ii)



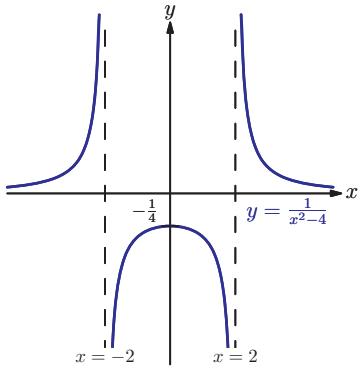
(c) (i)



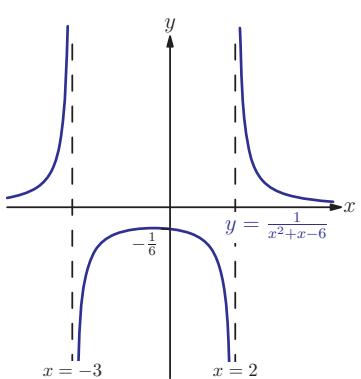
(ii)



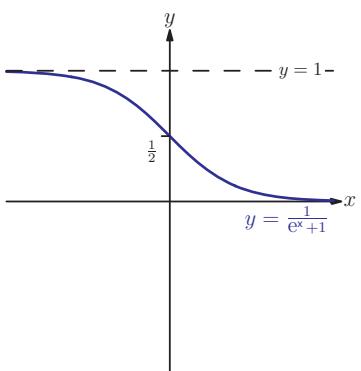
(d) (i)

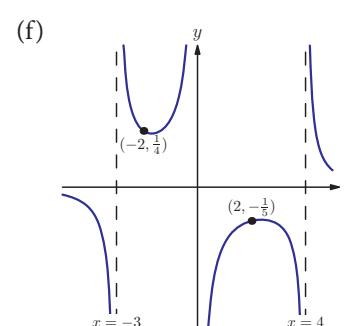
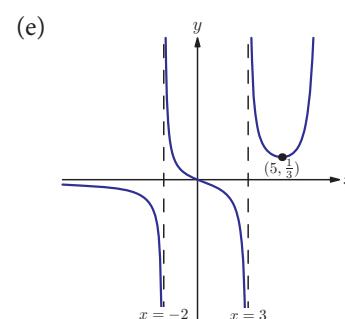
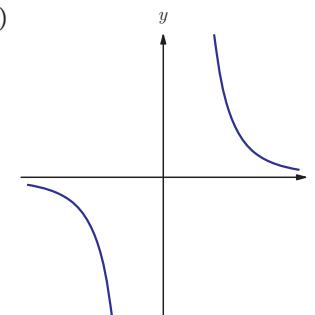
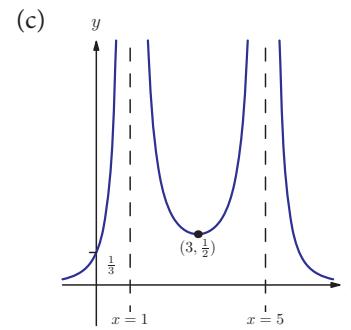
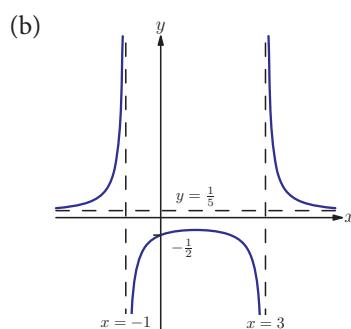
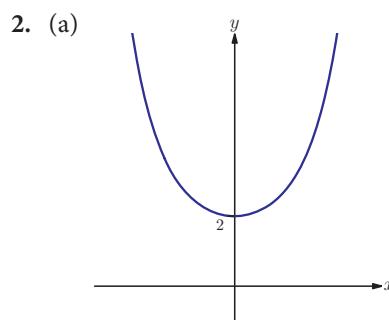
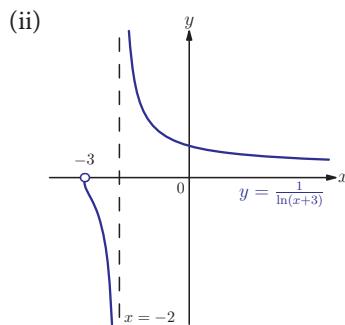
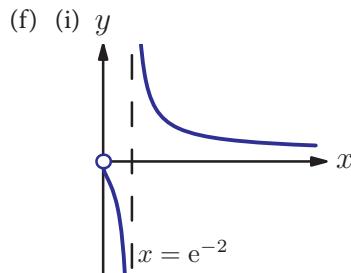
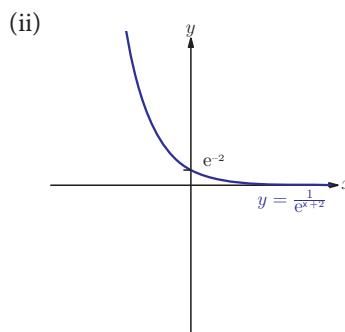


(ii)



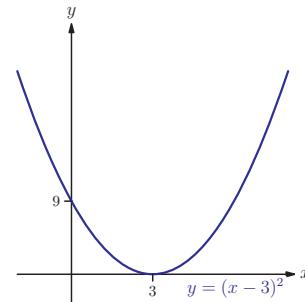
(e) (i)

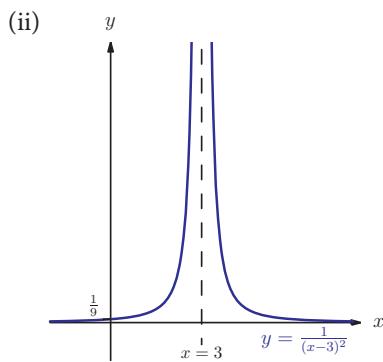




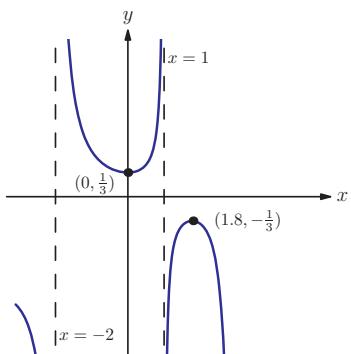
3. (a) $p = 3, q = 0$

(b) (i)

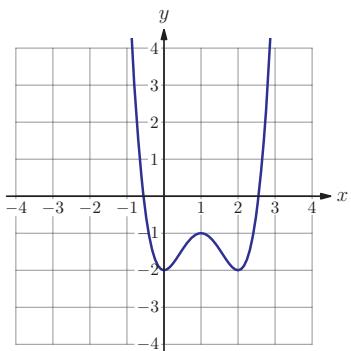




4.

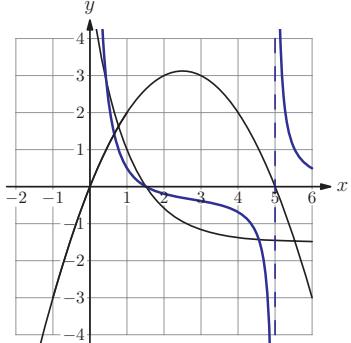


5.



6. (a) $x = 0, x = \pm\sqrt{\ln 4}$

7.



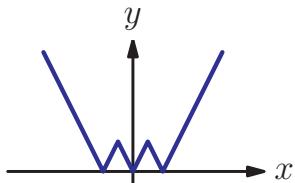
Exercise 6G

- 1 (a) (i) even (ii) even
 (b) (i) odd (ii) odd

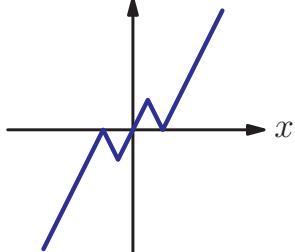
- (c) (i) neither (ii) neither
 (d) (i) odd (ii) even
 (e) (i) neither (ii) neither

2. (a) Two-fold rotational symmetry about the origin

(b) (i)



(ii)



9. (a) $x = -3$

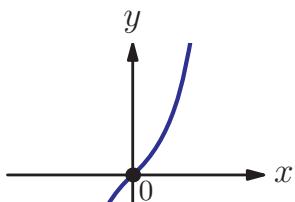
(b) $a = 3$

10. Reflective symmetry through $x = 5$

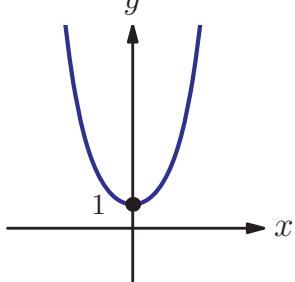
11. $f \circ f(4) = 4$: f is self-inverse if reflective in $y=x$

12. (b) $\frac{1}{2}(f(x) + f(-x))$

(d) (i)



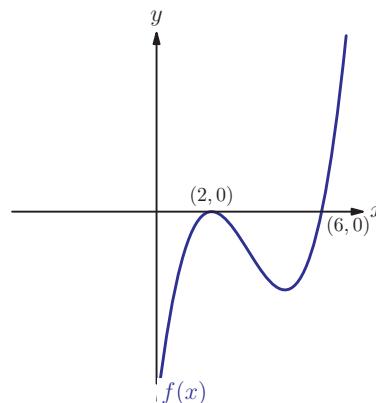
(ii)



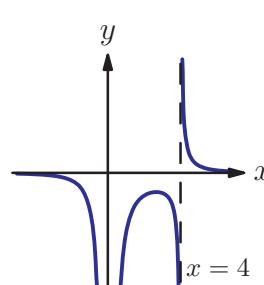
Mixed examination practice 6

Short questions

1. (a)



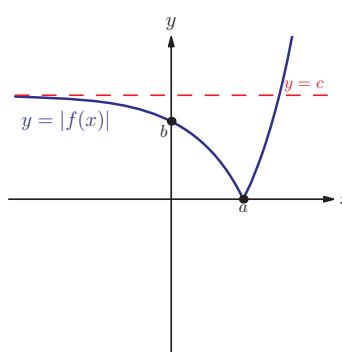
(b)



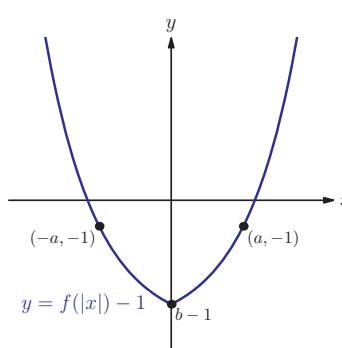
2. $y = 2x^2 - 12x^2 + 24x - 18$

3. $\frac{1}{3} < x < 1$

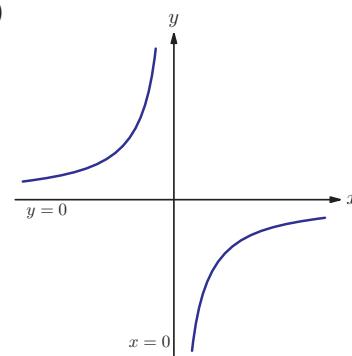
4. (a)



(b)



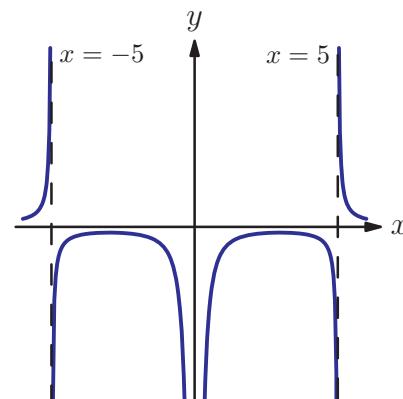
5. (a)



(b) Vertical stretch with scale factor 3 and reflection in the x-axis (or y-axis)

(c) $f^{-1}(x) = -\frac{3}{x}$

6. (a)

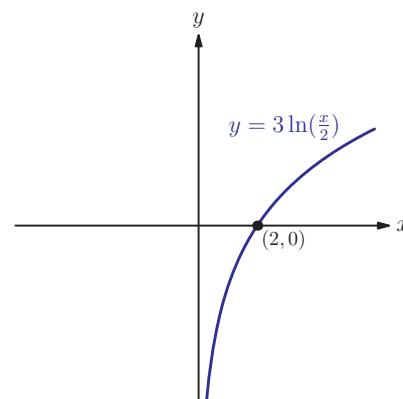


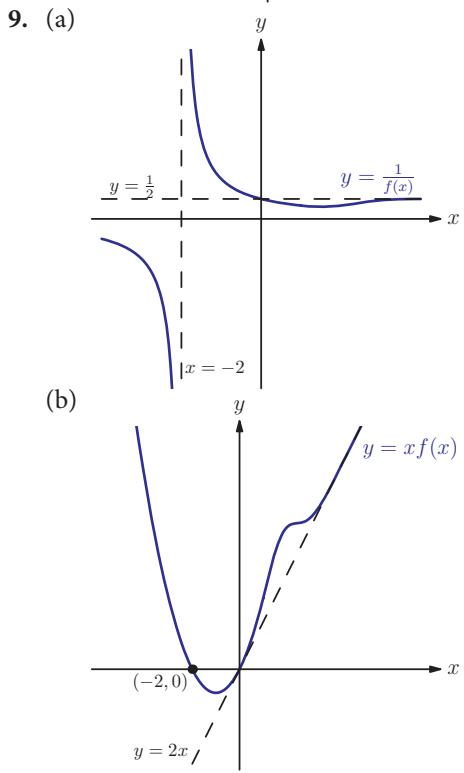
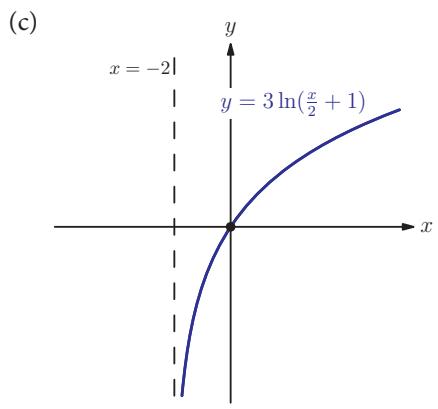
(b) $\left(-3, -\frac{1}{5}\right), \left(3, -\frac{1}{5}\right)$

7. Translation by $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ and vertical stretch with scale factor (sf)3

8. (a) Horizontal stretch with sf 2; vertical stretch with sf 3

(b)

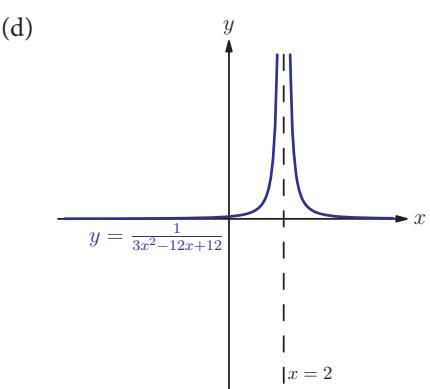




10. $x \geq 0$

Long questions

1. (a) Translation by $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and vertical stretch with sf 3.
- (b) Translation by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ and translation by $\begin{pmatrix} 0 \\ 10 \end{pmatrix}$
- (c) Translation by $\begin{pmatrix} 5 \\ 10 \end{pmatrix}$ and vertical stretch with scale factor 3.

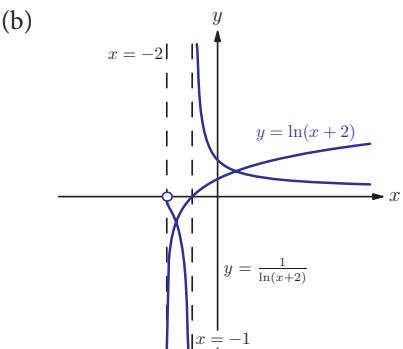


2. (a) $y = 3$
 (b) $p = 3, q = 1$
 (c) Translation with vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

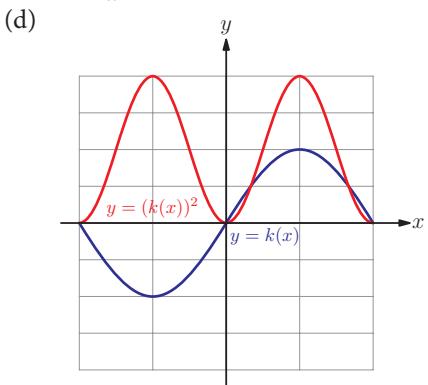
(d) $f^{-1}(x) = \frac{2x - 5}{x - 3}, x \neq 3$

(e) Reflection in the line $y = x$

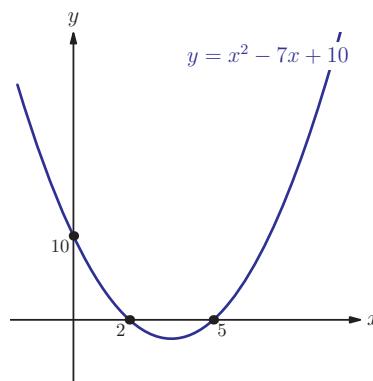
3. (a) Translation by $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$



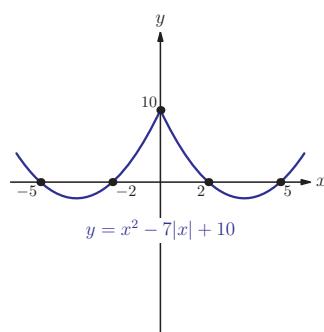
- (c) (i) $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$
 (ii) $a = -1, b = 6, c = -10, d = -1$



4. (a)



(c)



(d) $x = \pm \frac{10}{7}$

(e) $x = \pm 3, \pm 4$

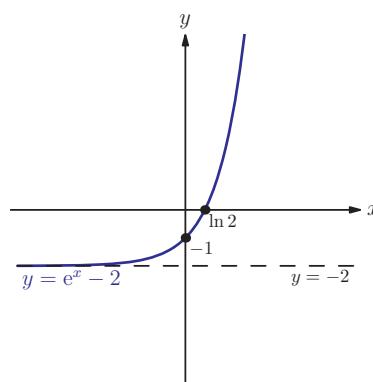
5. (a) -18

(b) 6

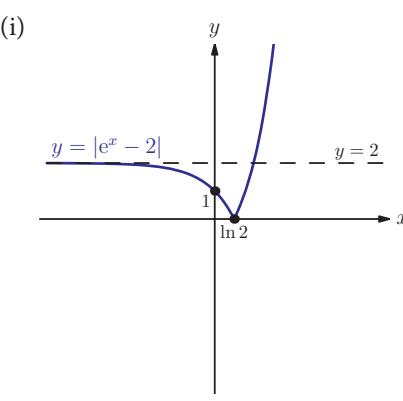
(c) $p = 3, q = 17$

(d) $x \in \mathbb{R}$

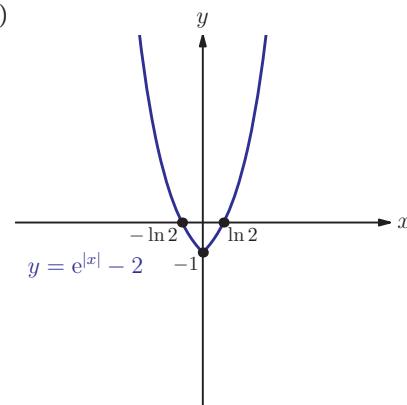
6. (a)



(b) (i)



(ii)



(c) $x = \ln(2 - \sqrt{3}), x \geq \ln 2$

Chapter 7

Exercise 7A

1. (a) (i) 3.1, 8.1, 13.1, 18.1, 23.1
(ii) 10, 6.2, 2.4, -1.4, -5.2

(b) (i) 0, 1, 4, 13, 40
(ii) 1, -1, -19, -181, -1639

(c) (i) 2, 3, 6, 18, 108
(ii) 2, 1, $\frac{1}{2}, \frac{1}{2}, 1$

(d) (i) 3, 4, 8, 9, 13
(ii) -3, 3, -5, 7, -9

(e) (i) 0, 4, 8, 12, 16

(ii) 13, 11, 9, 7, 5

2. (a) (i) 5, 8, 11, 14, 17

(ii) -4.5, -3, -1.5, 0, 1.5

(b) (i) 0, 7, 26, 63, 124

(ii) 5, 20, 45, 80, 125

(c) (i) 3, 9, 27, 81, 243

(ii) 4, 2, 1, $\frac{1}{2}, \frac{1}{4}$

(d) (i) 1, 4, 27, 256, 3125

(ii) 1, 0, -1, 0, 1

3. (a) (i) $u_{n+1} = u_n + 3, u_1 = 7$

(ii) $u_{n+1} = u_n - 0.8, u_1 = 1$

(b) (i) $u_{n+1} = 2u_n, u_1 = 3$

(ii) $u_{n+1} = 1.5u_n, u_1 = 12$

(c) (i) $u_{n+1} = u_n + n + 1$, $u_1 = 1$

(ii) $u_{n+2} = 2(u_{n+1} + u_n)$,
 $u_1 = 1, u_2 = 2$

4. (a) (i) $u_n = 2n$

(ii) $u_n = 2n - 1$

(b) (i) $u_n = 2^n$ (ii) $u_n = 5^n$

(c) (i) $u_n = n^2$ (ii) $u_n = n^3$

(d) (i) $u_n = \frac{n}{n+1}$

(ii) $u_n = \frac{2n-1}{2^n}$

5. (a) $u_2 = 4, u_3 = 8, u_4 = 16$

(b) (i) $u_n = 2^{n-1}$

Exercise 7B

1. (a) (i) 27 (ii) 39

(b) (i) 120 (ii) $\frac{665}{48}$

(c) (i) $14b$ (ii) $19p$

2. (a) (i) $\sum_2^{43} r$ (ii) $\sum_3^{30} 2r$

(b) (i) $\sum_2^7 \frac{1}{2^r}$ (ii) $\sum_0^5 \frac{2}{3^r}$

(c) (i) $\sum_{r=2}^{10} 7ra$ (ii) $\sum_{r=0}^{19} r^b$

Exercise 7C

1. (a) (i) $u_n = 9 + 3(n-1)$

(ii) $u_n = 57 + 0.2(n-1)$

(b) (i) $u_n = 12 - (n-1)$

(ii) $u_n = 18 - \frac{1}{2}(n-1)$

(c) (i) $u_n = 1 + 3(n-1)$

(ii) $u_n = 9 + 10(n-1)$

(d) (i) $u_n = 4 - 4(n-1)$

(ii) $u_n = 27 - 7(n-1)$

(e) (i) $u_n = -17 + 11(n-1)$

(ii) $u_n = -32 + 10(n-1)$

2. (a) (i) 33 (ii) 29
(b) (i) 100 (ii) 226

3. (a) $a_n = 5 + 8(n-1)$ (b) 50

4. 121

5. 25

6. 17

7. $a = 2, b = -3$

8. (b) 456 pages

Exercise 7D

1. (a) (i) 3060 (ii) 1495
(b) (i) 9009 (ii) 23 798
(c) (i) -204 (ii) 1470
(d) (i) 667.5 (ii) 14.25

2. (a) (i) 13 (ii) 32 (iii) 53

(b) $\frac{x}{2}$

3. $a = 15, d = -8$

4. (a) $S_n = \frac{n}{2}(3n+1)$ (b) 30

5. 559

6. $a = 14, d = -8$

7. 55

8. $u_n = 6n - 5$

9. $\theta = 20^\circ$

10. 10 300

11. 23 926

Exercise 7E

1. (a) (i) $u_n = 6 \times 2^{n-1}$
(ii) $u_n = 12 \times \left(\frac{3}{2}\right)^{n-1}$

(b) (i) $u_n = 20 \times \left(\frac{1}{4}\right)^{n-1}$
(ii) $u_n = \left(\frac{1}{2}\right)^{n-1}$

(c) (i) $u_n = (-2)^{n-1}$
(ii) $u_n = 5 \times (-1)^{n-1}$

- (d) (i) $u_n = ax^{n-1}$
(ii) $u_n = 3 \times (2x)^{n-1}$
2. (a) (i) 13 (ii) 7
(b) (i) 10 (ii) 10
(c) (i) 10 (ii) 8
3. (a) (i) 15 (ii) 31
(b) (i) 33 (ii) 17
4. 39366
5. 10
6. 16
7. 2.5 or -1.82
8. ± 384
9. $a = 7$ or -3.52
10. $a = -2$, $b = 4$
11. $m = 7$

Exercise 7F

1. (a) (i) 17 089 842 (ii) 2303.4375
(b) (i) 514.75 (ii) 9.487 171
(c) (i) 39 368 (ii) 9840
(d) (i) 191.953 125 or 63.984 375
(ii) 24 414 062.5 or 16 276 041.67
2. (a) (i) $r = 3$ (ii) $r = 0.2$
(b) (i) $r = -6$ (ii) $r = -0.947$
3. (a) 5 (b) $S_n = \frac{375(5^n - 1)}{4}$
4. $a = 5$, $r = \frac{3}{2}$
5. (a) $1 + x + x^2 + x^3$
(b) $(x-1)(x^5 + x^4 + x^3 + x^2 + x + 1)$

Exercise 7G

1. (a) (i) $\frac{27}{2}$ (ii) $\frac{196}{3}$
(b) (i) $\frac{1}{3}$ (ii) $\frac{26}{33}$
(c) (i) Divergent
(ii) Divergent
(d) (i) $\frac{25}{3}$ (ii) $\frac{18}{5}$
(e) (i) Divergent (ii) $\frac{7}{3}$
2. (a) (i) $|x| < 1$ (ii) $|x| < 1$
(b) (i) $|x| < \frac{1}{3}$ (ii) $|x| < \frac{1}{10}$

- (c) (i) $|x| < \frac{1}{5}$ (ii) $|x| < \frac{1}{3}$
(d) (i) $|x| < 4$ (ii) $|x| < 12$
(e) (i) $|x| < 3$ (ii) $|x| < \frac{4}{5}$
(f) (i) $|x| > 2$ (ii) $|x| > \frac{1}{2}$
(g) (i) $1 < x < 2$ (ii) $0 < x < 4$
(h) (i) $\frac{1}{2} < x < 1$ (ii) $x < -\frac{1}{2}$
(i) (i) $|x| < 1$ (ii) $|x| < \frac{1}{\sqrt[3]{4}}$
3. $-\frac{54}{3}$
4. (a) $27 = \left(\frac{1 - \left(\frac{-1}{3} \right)^n}{2} \right)$
(b) $S_\infty = \frac{27}{2}$
5. (a) $\frac{2}{3}$ (b) 9
6. $\frac{1}{8}$
7. (a) $|x| < \frac{3}{2}$ (b) 5
8. 9
9. (a) $1 < x < \frac{5}{3}$ (b) 7
10. (a) $x < 0$ (b) $x = -3$
11. (a) 3 (b) ∞

Exercise 7H

1. (a) £34.78
(b) £1194.05
2. (a) \$60 500
(b) 22 years
3. (a) 5000×1.063^n
(b) \$6786.35
(c) (i) $5000 \times 1.063^n > 10000$
(ii) 12 years
4. (a) 10 (b) 23.7%
5. (a) \$265.33

(b) 235 months

6. (a) 12 days

(b) Day 102

7. (a) 0.8192 m

(b) 15.85 m

8. (b) $25000(1.04^n - 1)$

(c) Year 29

Mixed examination practice 7

Short questions

1. 97.2

2. (a) 1, 5, 9

(b) $4n - 3$

3. 13th

4. 2

5. $d = 0, -\frac{1}{4}$

6. 4.5

7. 19 264

8. $\ln\left(\frac{a^{69}}{b^{138}}\right)$

Long questions

1. (a) $10000 + 800n$

(b) 10000×1.05^n

(c) $n < 19$ years

2. (a) $2n - 1$ (b) 6

(c) 64

3. (a) n (b) $\frac{n(n+1)}{2}$

(c) $\frac{n(n-1)}{2} + 1$

(e) 32

4. (b) $150000 \times 1.06^n - \frac{500000(1.06^n - 1)}{3}$

(c) 40 years

Chapter 8

Exercise 8A

1. (a) 4

(b) 35

(c) 7

(d) 56

2. (a) $792x^5y^7$ (b) $11440a^7b^9$

(c) $10c^3d^2$ (d) $36a^2b^7$

(e) $15x^2y^4$

Exercise 8B

1. (a) (i) 216 (ii) 20

(b) (i) $560x^3y^4$ (ii) $-280x^3y^4$

(c) (i) -5 (ii) 78 030

2. (a) (i) 56 (ii) 80

(b) (i) -672 (ii) -32

3. (a) (i) $32 - 80x + 80x^2 - 40x^3 + 10x^4 - x^5$

(ii) $729 + 1458x + 1215x^2 + 540x^3 + 135x^4 + 18x^5 + x^6$

(b) (i) $243x^5 + 405x^4y + 270x^3y^2$

(ii) $16c^4 - 32c^3d + 24c^2d^2$

(c) (i) $8x^6 - 36x^5 + 54x^4 - 27x^3$

(ii) $8x^{-3} + 60x^{-2}y + 150x^{-1}y^2 + 125y^3$

(d) (i) $16z^8 + 96z^5 + 216z^2 + 216z^{-1} + 81z^{-4}$

(ii) $27x^3y^3 + 135x^3y + 225x^3y^{-1} + 125x^3y^{-3}$

4. (a) n (b) $\frac{1}{2}n^2 - \frac{1}{2}n$

(c) $\frac{1}{6}n^3 - 6n^2 + \frac{1}{3}n$

5. (a) $80x y^4$ (b) $-80x^2 y^3$

6. 720

7. $-945x^5$

8. 79 200 000

9. 14

10. 12

11. 9

12. 7

Exercise 8C

1. (a) (i) -4 (ii) 126
(b) (i) -5 (ii) -28
2. (i) 15 (ii) 40
3. (i) 5733 (ii) -272
4. (a) (i) $3x^7 - 17x^8 + 16x^7$
(ii) $-x^7 + 16x^6 - 105x^5$
(b) (i) $1+x-4x^2$
(ii) $128+64x-96x^2$

ANSWER HINT (5,6,7)

In questions 5,6 and 7 there are algebraic tricks that make the expansions much easier.

5. $y^6 + 18y^7 + 135y^8 + 540y^9$
6. $1 - 10x^2 + 45x^4$
7. $1 - 20x + 190x^2 - 1140x^3$
8. $m = 3, n = 15$ and $m = -5, n = -17$
9. $n = 5, k = 2$ and $n = 17, k = -1$

Exercise 8D

1. (a) $1 + 35x + 525x^2 + 4375x^3$; 1.407
(b) $64 + 576x + 2160x^2$; 64.5782
2. (a) $81 - 540x + 1350x^2$
(b) 80.4614
3. (a) $128 + 2240x + 16800x^2$
(b) 130.257
4. (a) $128 + 1344x + 6048x^2$
(b) (i) 322.28 (ii) 142.0448
(c) part (ii) Smaller value of x means higher order terms much smaller and therefore less important.

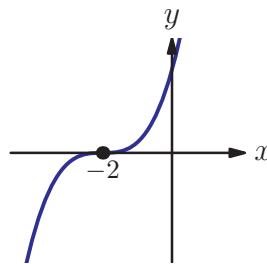
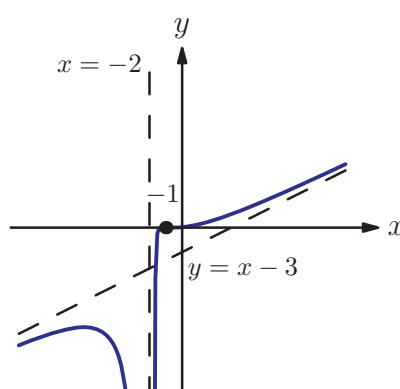
Mixed examination practice 8

Short questions

1. -101376
2. $232 - 164\sqrt{2}$

3. (a) $32 + 80x + 80x^2 + 40x^3 + 10x^4 + x^5$
(b) 32.808 040 1001
4. $243 + 162x - 2484x^2$
5. $x^8 - 8x^5 + 24x^2 - 32x^{-1} + 16x^{-4}$
6. 3 or -3
7. $m = -8, n = -34$ or $m = 5, n = 31$

Long questions

1. (a)
 
 A Cartesian coordinate system showing a curve with a sharp corner (cusp) at the point (-2, 0). The curve passes through the origin (0, 0) and continues upwards and to the right, increasing rapidly.
2. (a) $x = -2, \left(0, \frac{1}{16}\right), (-1, 0)$
 (b) $f(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$,
 $g(x) = x^4 + 8x^3 + 24x^2 + 32x + 16$
 (c) (i) $k = 3, a = 10$ (ii) $y = x - 3$
 (d)
 
 A Cartesian coordinate system showing three lines and a curve. A vertical dashed line is at $x = -2$. A horizontal dashed line is at $y = 1$. A solid blue curve passes through the point $(-2, 1)$ and continues upwards and to the right. A solid black line passes through the point $(-2, 1)$ and has a positive slope. A dashed black line is tangent to the blue curve at the point $(-2, 1)$.

3. (a) $7 + 5\sqrt{2}$
 (b) $\binom{n}{k}(\sqrt{2})^k$ (d) 24
4. (c) $\frac{r+2}{n-r-1}$