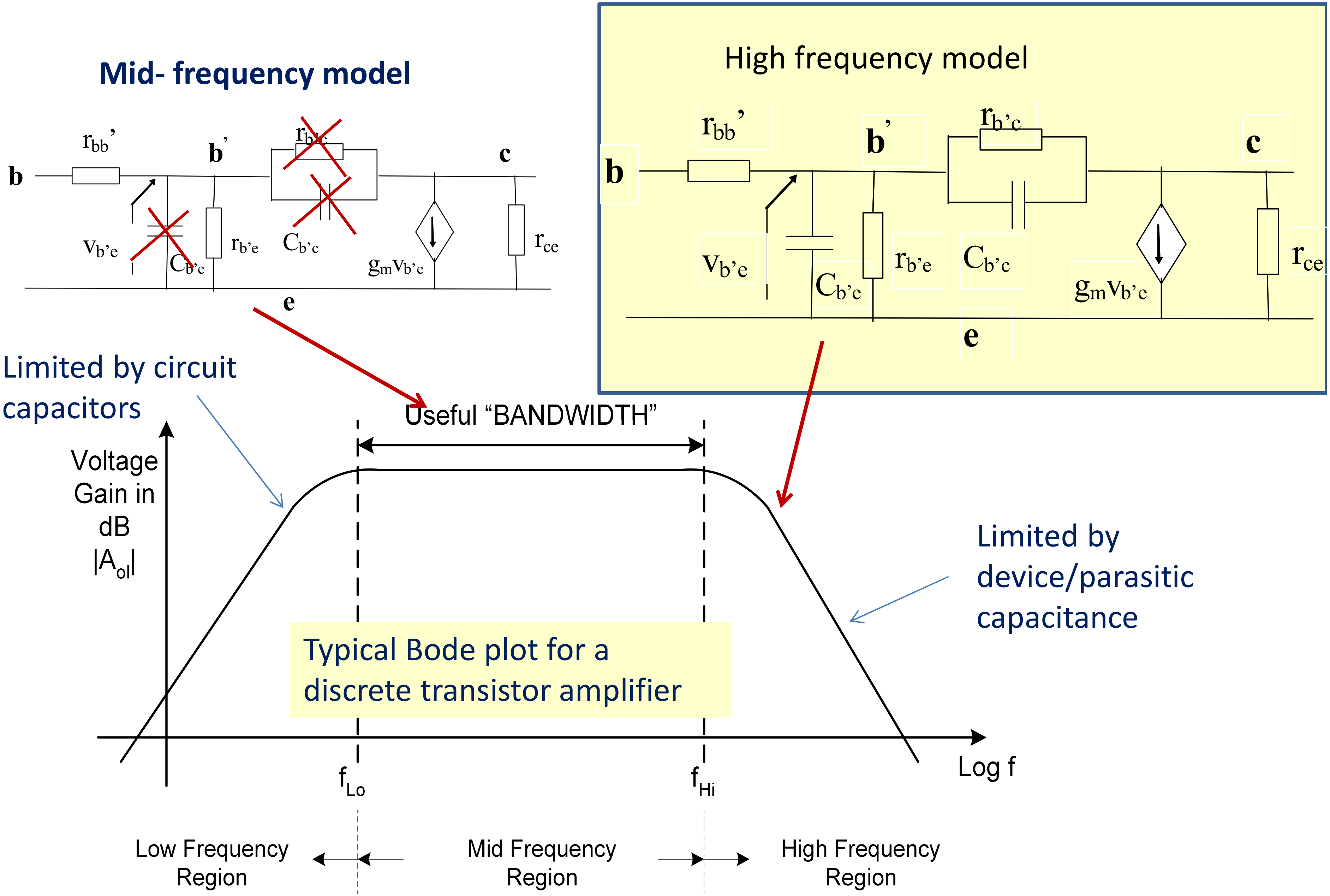
Last time: a linear, small-signal BJT model



Part 2: Bipolar transistor amplifier configurations

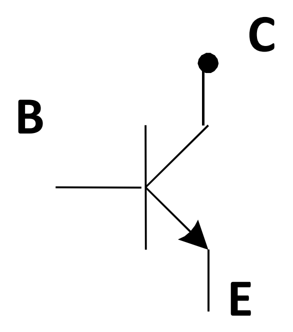
- established an ac model for the bipolar transistor,
- now analyse the basic amplifier configurations.

1. **Common emitter (CE)**
2. **Common emitter with emitter degradation (CE-ED)**
3. **Common collector (CC) also known as emitter follower (EF)**
4. **Common base (CB)**

- voltage and current gains, input and output resistance etc.
- each configuration has particular properties that we can usefully employ when building up complex electronic circuits....

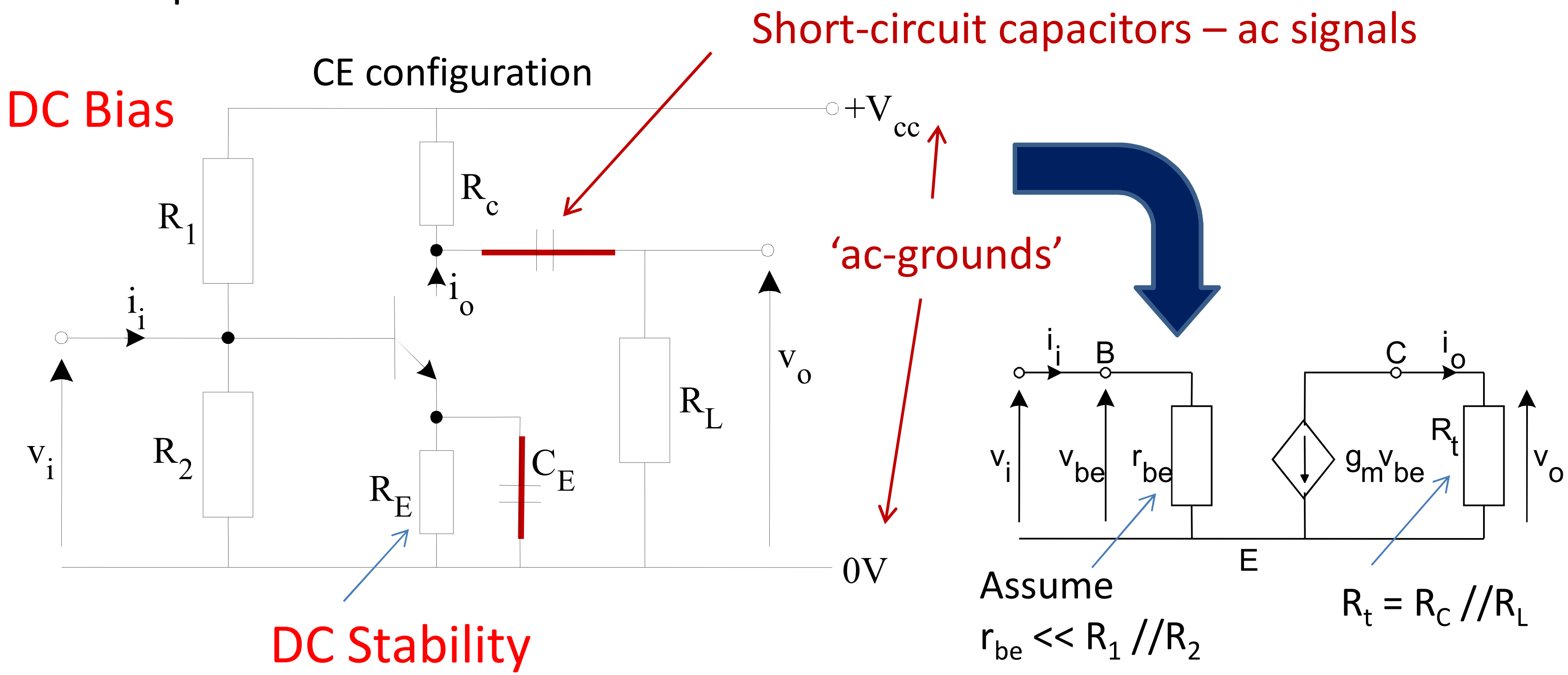
.. by cascading stages!

So look at examples of two and three stage amplifiers



Golden Rules for drawing equivalent circuits

1. Draw the equivalent circuit of the transistor first
2. Identify the a.c. grounds (voltage sources look like short circuits for a.c. currents!)
3. Convert capacitors to short circuits (we are interested in ‘mid-frequency’ regimes where the impedance of coupling/de-coupling capacitors is zero
4. Add the other circuit components (resistors)to obtain the complete equivalent circuit.



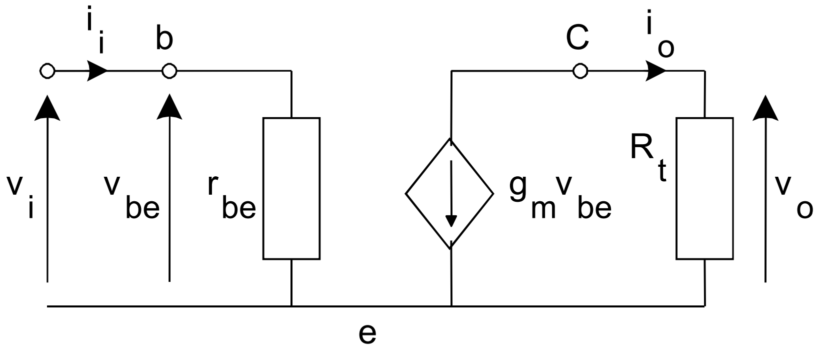
The common emitter amplifier

Voltage Gain, A_V:

so $$A_V \equiv \frac{v_o}{v_i} = \frac{v_o}{v_{be}}$$

$$v_o = i_o R_t = -g_m v_{be} R_t$$ Ohm's Law

$$A_V = -g_m R_t$$



Use the simplest model!
Even omit $r_{bb'}$ and r_{ce}

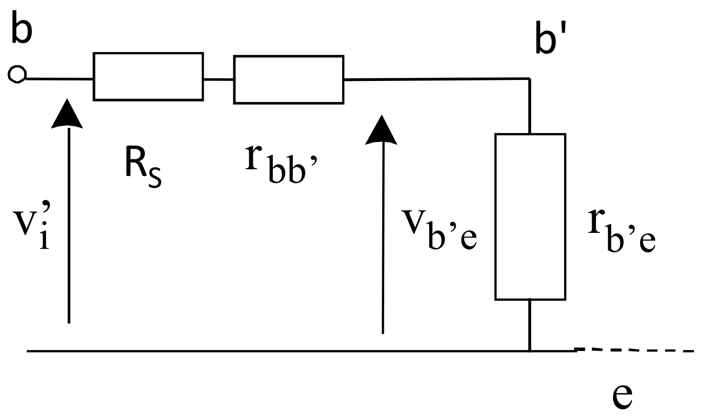
Assume
$r_{be} << R_1 // R_2$

Include effect of r_{bb}' and R_S :

$$A_V = \frac{v_o}{v_i'} = \frac{v_o}{v_{b'e}} \frac{v_{b'e}}{v_i'}$$ Chain rule

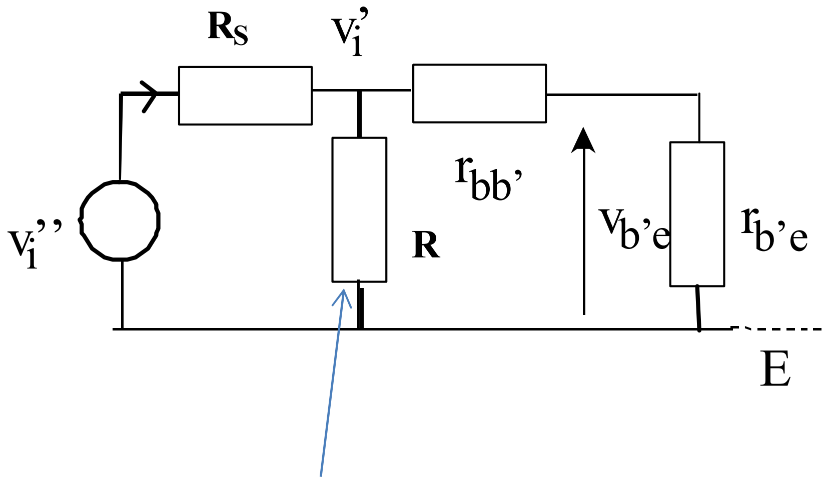
$$v_{b'e} = v_i' \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s}$$ Potential division

$$A_V' = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s}$$



Note the degradation of voltage gain by 'coupling term'

Full treatment – include effect of bias resistors



$R = R_1 \,//\, R_2$

R and R_s 'load' the front end of the amplifier – reduce the gain

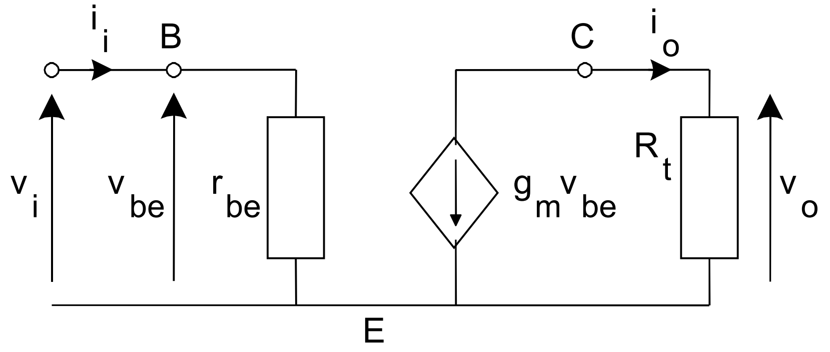
$$A_V'' = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'}} \frac{R // \left(r_{bb}' + r_{b'e}\right)}{R_S + R // \left(r_{b'e} + r_{bb'}\right)}$$

Ex: Prove this equation....

Input resistance

Ohm's Law gives

$$R_i = \frac{v_i}{i_i} = r_{be}$$



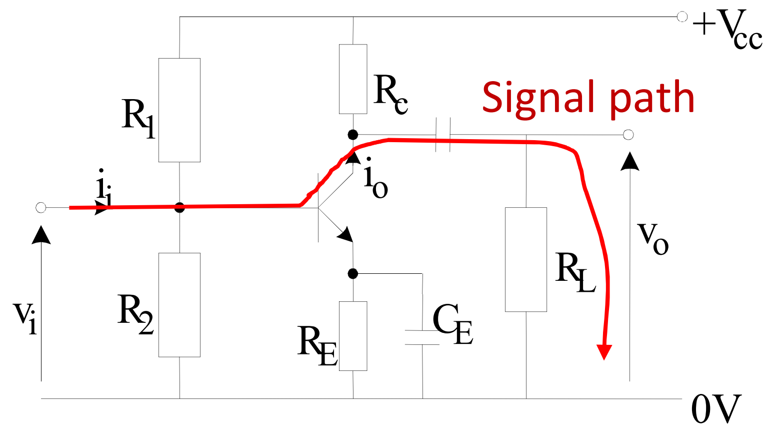
include r_{bb}': $R_i = r_{b'e} + r_{bb}'$

$r_{bb'}$ small; typically 50-100 (see data sheets)

easy to include the effect of R (bias resistors)

$$R_i' = \frac{v_i'}{i_i'} = R // \left(r_{b'e} + r_{bb'}\right)$$

usually make $R >> (r_{b'e} + r_{bb}')$



- so signal goes into the transistor not through the bias resistors!!

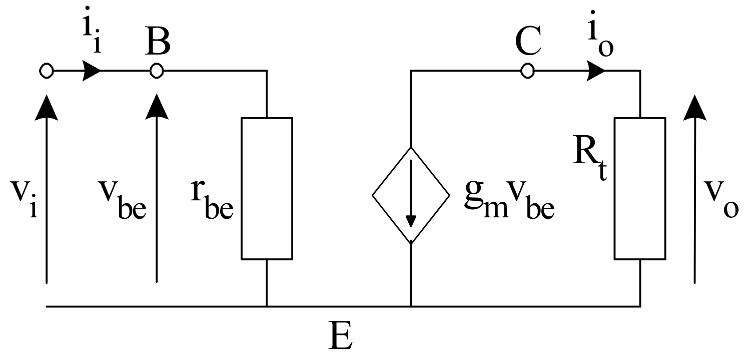
Current Gain

$$A_i \equiv \frac{i_o}{i_i}$$

We can write: $$i_o = -g_m v_{be}$$

$$v_{be} = i_i r_{be}$$

so $$A_i = \frac{i_o}{i_i} = -g_m r_{be} = -\beta_o$$



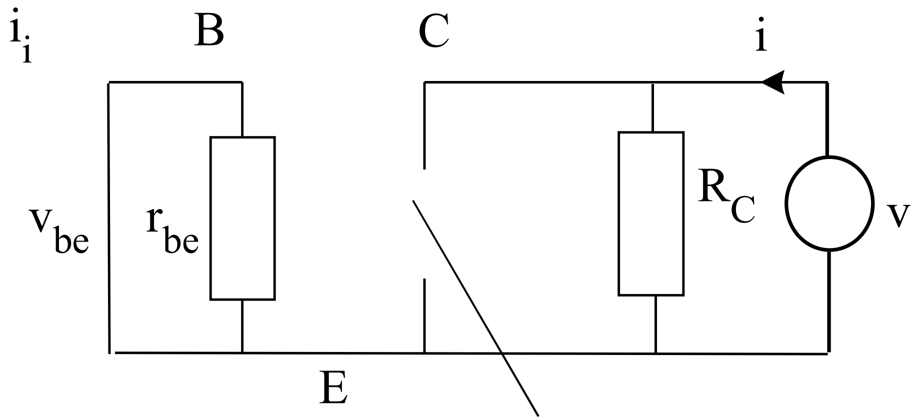
(**Q:** what is the effect of including r_{bb}'??)

Output resistance

Technique:
1. Set i/p voltage = 0
2. Let $R_L \rightarrow$ infinity
3. Drive o/p terminal by a volt source, *v*

Then output resistance,

$$R_o \equiv \frac{v}{i} = R_C$$

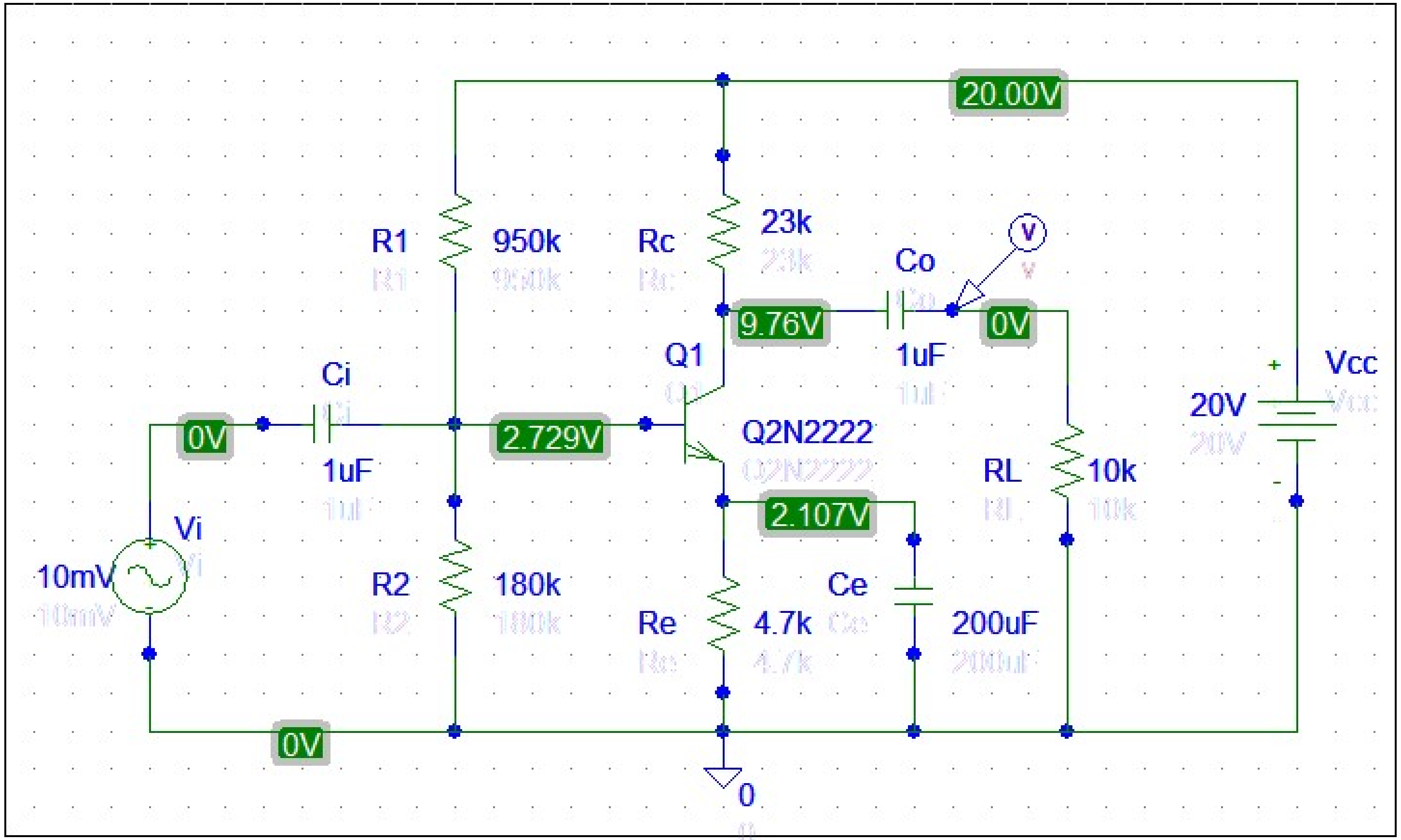


v_{be} = 0, so no current

So here, $R_o = R_C$

If we include effect of r_{ce} : $R_o = R_C \,//\, r_{ce}$

PSPICE simulation(DC voltages)



V_{BE} = 2.73 – 2.11 = 0.62 V V_{CE} = 9.76 – 2.11 = 7.65 V

PSPICE simulation(DC currents)

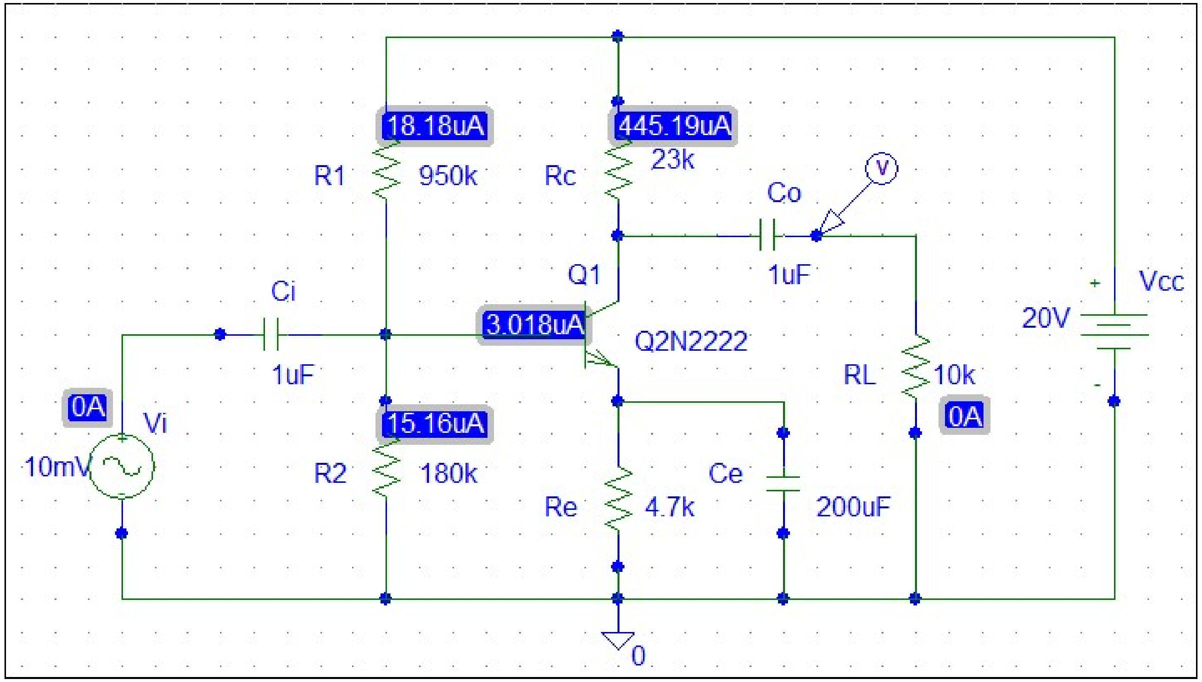
$I_C = 0.45\ mA$

$g_m = 40 I_C = 40 \times 0.45m$
$= 18\ mA/V$

$r_{be} = \dfrac{\beta_o}{g_m} = \dfrac{104}{18}\ \text{k}$
$= 13\ \text{k}\Omega$

$A_V = g_m \times ac\ load$

$ac\ load = \dfrac{R_C \times R_L}{R_C + R_L}$
$= 7\ \text{k}\Omega$



Small-signal values: $A_V = 126$, $R_{in} = r_{be} = 13\ \text{k}\Omega$, $R_o \sim R_C = 23\ \text{k}\Omega$

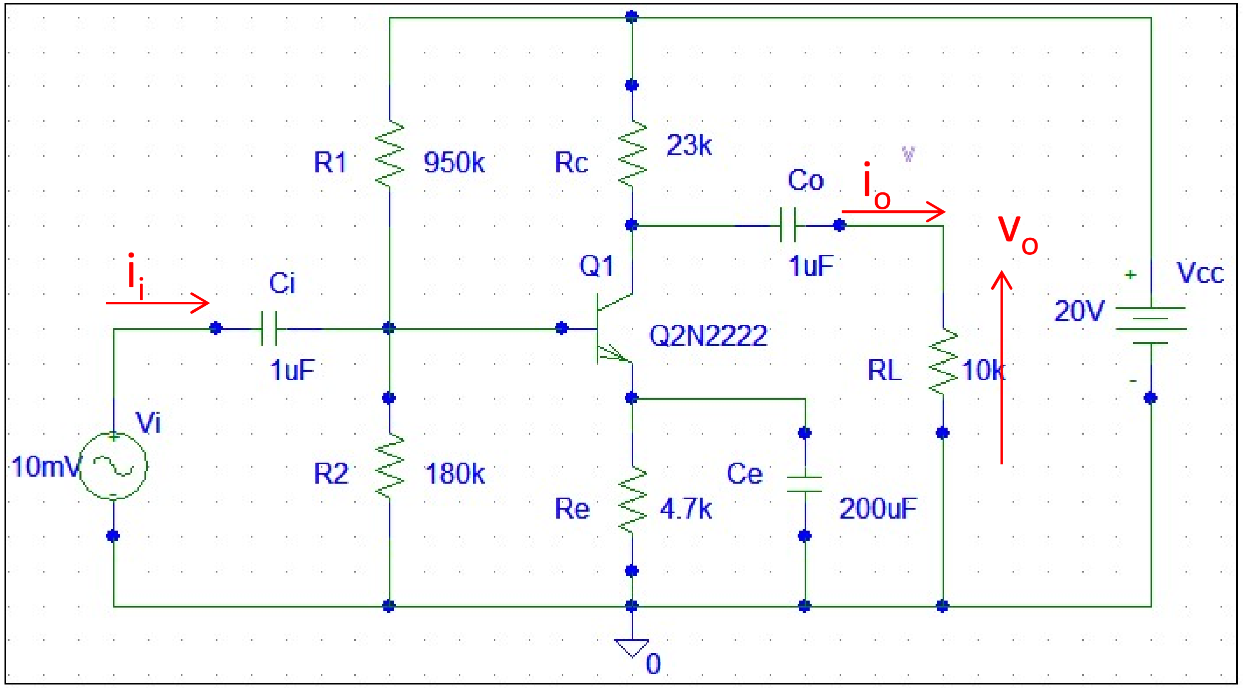
PSPICE simulation: R_{in}, A_V, A_i

Apply ac sweep (v_{in} = 10 mV)

$$R_{in} = \frac{v_i}{i_i}$$

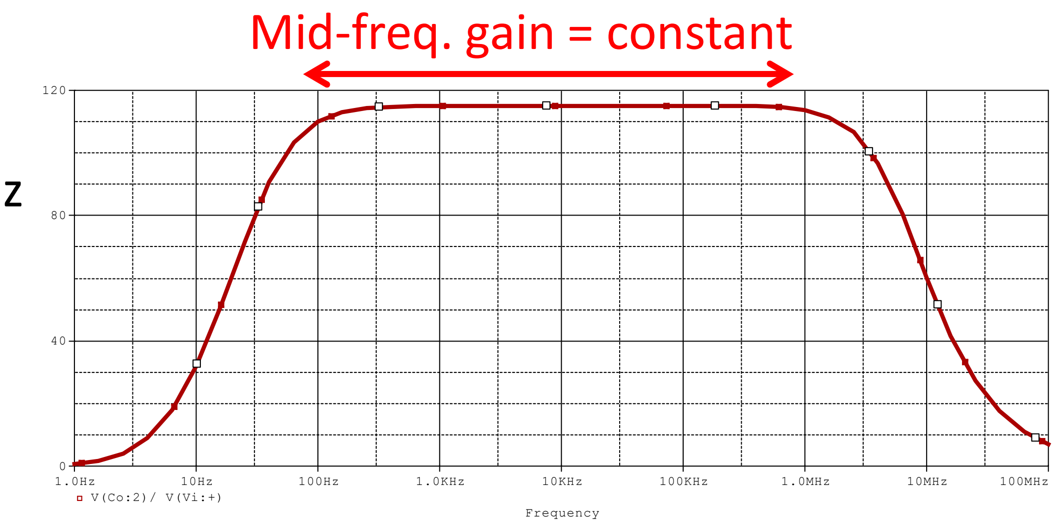
$$A_v = \frac{v_o}{v_i}$$

$$A_i = \frac{i_o}{i_i}$$



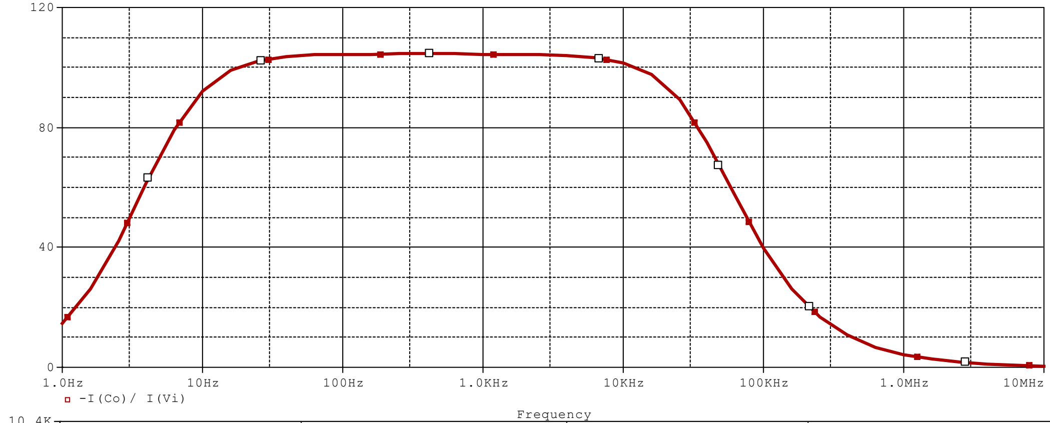
Ac sweep from 1 Hz to 100 MHz

V-Gain is 115 cf 126 (theory)



Ac sweep from 1 Hz to 100 MHz

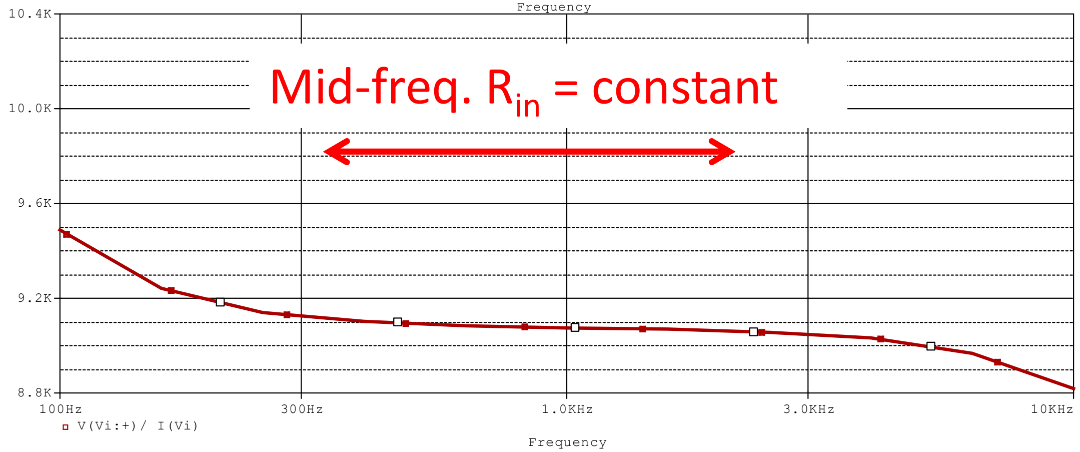
I-Gain is 104, giving β_o = 104



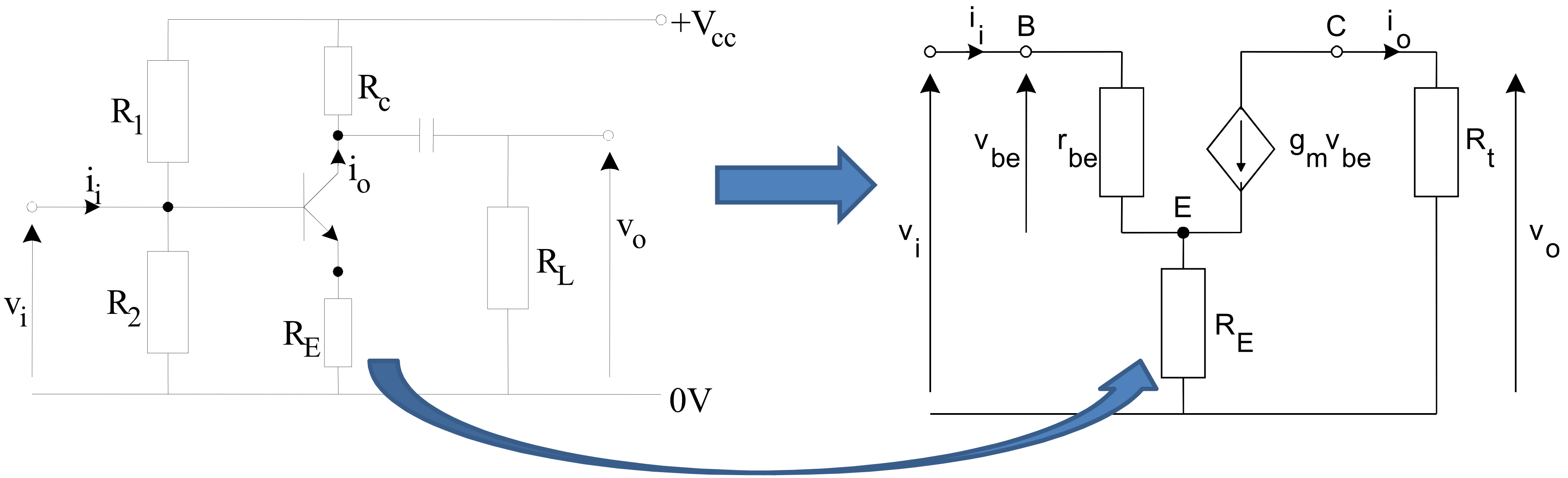
Ac sweep from 100 Hz to 10 kHz

R_{in} = 9.1 kΩ (at 1 kHz)
cf 13 kΩ (theory, r_{be})

(take account of base resistors, get 12 kΩ)



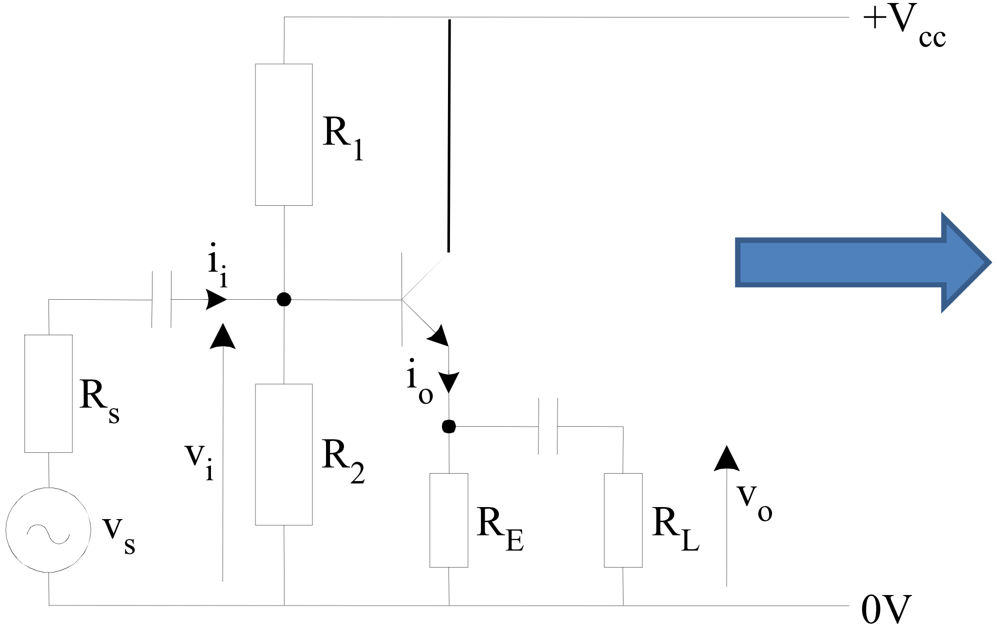
Common-emitter with emitter degradation

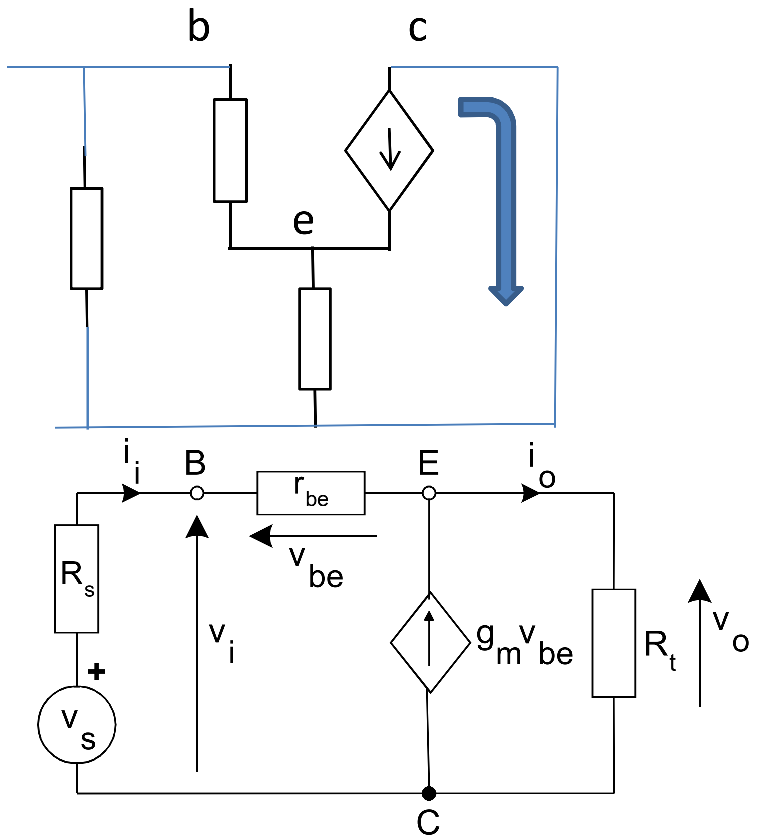


Remove C_E in CE circuit

(for analysis see exercise 2 – you do this one!)

Common-collector (emitter-follower)





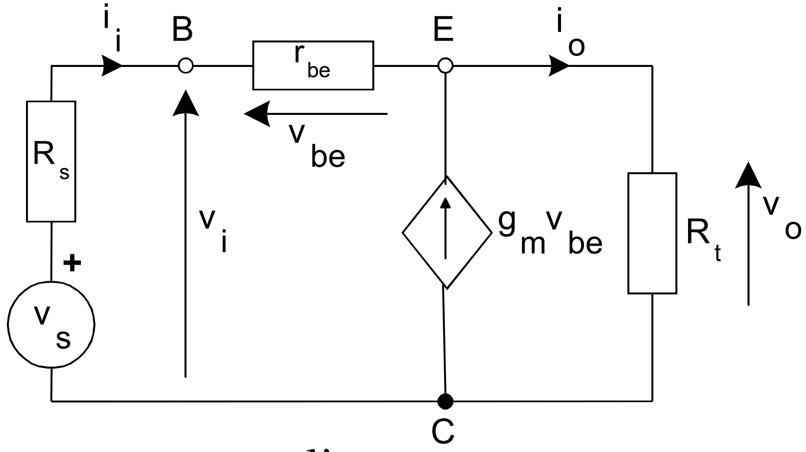
$R_t = R_E//R_L$

Assume $R_1//R_2$ very large

Q: what is meant by 'small signal'? (see earlier notes)
Does it mean 'small amplitude' or 'small frequency'?

Voltage Gain (v_o /v_i)

(we need to learn some techniques..)



$$v_o = i_o R_t \qquad (1)$$

KCL at node E: $i_i + g_m v_{be} = i_o = \dfrac{v_o}{R_t}$ (2)

also $v_{be} = v_i - v_o$ and $i_i = \dfrac{v_i - v_o}{r_{be}}$ (3)

Substitute for i_i and v_{be} in Eqn.(2): $\dfrac{(v_i - v_o)}{r_{be}} + g_m(v_i - v_o) = \dfrac{v_o}{R_t}$

Multiply across by r_{be}: $v_i - v_o + \beta_o v_i - \beta_o v_o = v_o \dfrac{r_{be}}{R_t}$

Re-arranging gives: $v_o + \beta_o v_o + v_o \dfrac{r_{be}}{R_t} = +v_i + \beta_o v_i$

(where we've used)

$(\beta_o = g_m r_{be})$

(note the technique; we have gathered all the terms involving v_o on the LHS and those involving v_i on the RHS)

$$v_o\left(1 + \beta_o + \frac{r_{be}}{R_t}\right) = v_i(1 + \beta_o) \Rightarrow A_V \equiv \frac{v_o}{v_i} = \frac{(1+\beta_o)}{1 + \beta_o + \dfrac{r_{be}}{R_t}}$$

Interpretation (CC)

$$A_V \equiv \frac{v_o}{v_i} = \frac{(1+\beta_o)}{(1+\beta_o) + \frac{r_{be}}{R_t}}$$

We notice straight away that the voltage gain must be less than 1!

Assuming now, that $\beta_o >> 1$,

$$A_V = \frac{v_o}{v_i} = \frac{\beta_o}{\beta_o + \frac{r_{be}}{R_t}}$$

an approximate but quite accurate estimate for the voltage gain

Which can also be written as $A_V \equiv \frac{v_o}{v_i} = \frac{g_m \times R_t}{1 + g_m \times R_t}$ for the common-collector gain.

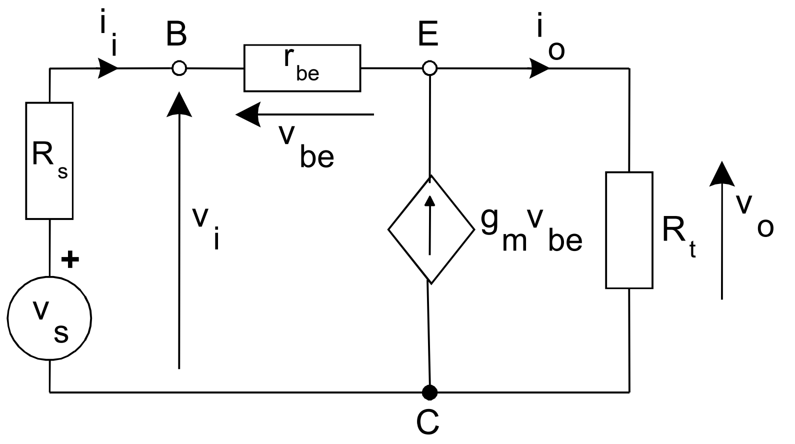
$$(\beta_o = g_m r_{be})$$

Q: What use is an amplifier with voltage gain less than 1!

Let's look at the input resistance.........

Input resistance of the CC amplifier

This is defined as: $R_i \equiv \dfrac{v_i}{i_i}$



Take a KVL: $v_i = i_i r_{be} + v_o$ (1)

Where $v_o = i_o R_t$ (2)

Now take KCL at node 'E': $i_o = i_i + g_m v_{be}$

Which can also be written as $i_o = i_i + \beta_o i_i \Rightarrow i_o = i_i(1+\beta_o)$ (3)

Sub. (3) into (2): $v_o = i_i R_t (1+\beta_o)$

Sub. into (1): $v_i = i_i r_{be} + i_i R_t (1+\beta_o) \Leftarrow$ (note again the technique; we have gathered all the terms involving v_i on the LHS and those involving i_i on the RHS)

$$R_i \equiv \frac{v_i}{i_i} = r_{be} + R_t(1+\beta_o)$$

Interpretation

$$R_i \equiv \frac{v_i}{i_i}$$
$$= r_{be} + R_t\left(1 + \beta_o\right)$$

- Recall the input resistance of the CE amp (r_{be}).
- So the input resistance of the CC amp is **boosted by the factor R_t (1 + β_o)!**
- The CC amp has high input resistance!

Q: Is this any use??
Let's also look at the output resistance – a rather long analysis .....

Common Collector, Output resistance

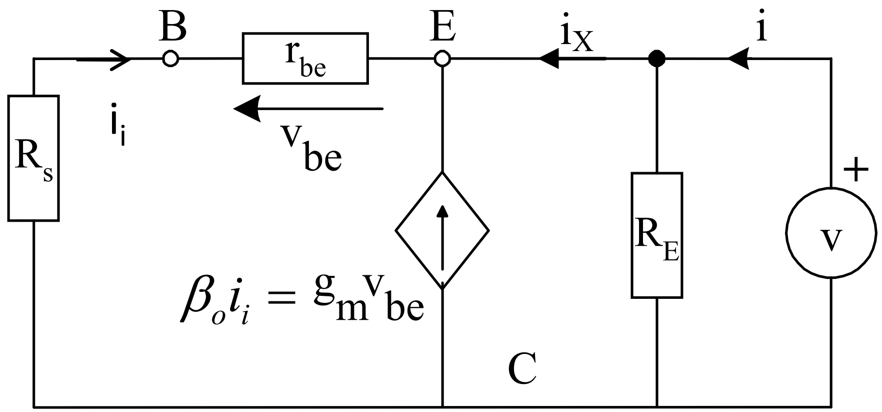
Set up the analysis

Suppress (ie short out), the signal source v_s.

Apply a voltage source to the output – to force a current into the amplifier.

The output resistance is then given by $\boxed{R_o = \dfrac{v}{i}}$

. **We must find this by appropriate circuit analysis.**



We can write $i = \dfrac{v}{R_E} + i_X$ (1)

Apply KCL at 'E': $i_X + \beta_o i_i + i_i = 0$

$$\therefore i_X = -(1+\beta_o) i_i \qquad (2)$$

Need to get rid of i_i! : write, $v = -i_i(R_s + r_{be})$ $\Rightarrow$ $i_i = -\dfrac{v}{R_S + r_{be}}$ (3)

Sub for i_X in (1) $i = \dfrac{v}{R_E} - (1+\beta_o) i_i$

Substitute (3) to get $i = \dfrac{v}{R_E} + \dfrac{(1+\beta_o)}{R_S + r_{be}} v$ (4)

Common Collector, Output resistance $R_o = \dfrac{v}{i}$

From previous page $i = \dfrac{v}{R_E} + \dfrac{(1+\beta_o)}{R_S + r_{be}} v$ (4)

Nearly there.... We have 'i' on RHS and terms in 'v' on RHS

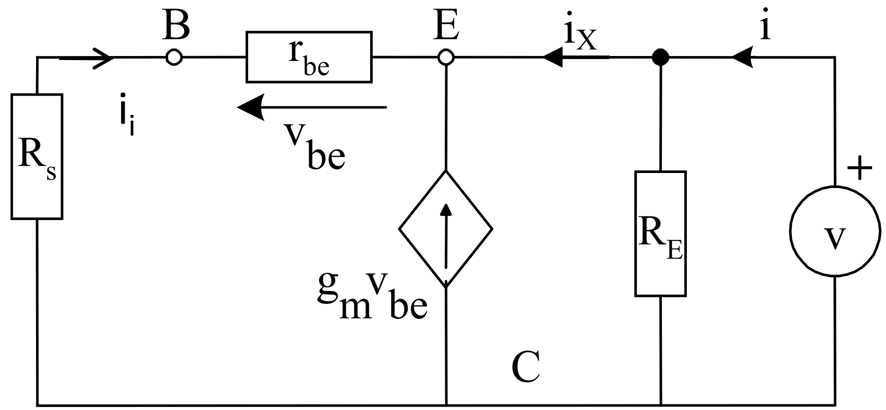
That is (4) can be written as $\dfrac{i}{v} = \dfrac{1}{R_E} + \dfrac{1+\beta_o}{R_S + r_{be}}$

That is, $R_o = \dfrac{v}{i} = R_E \,//\, \dfrac{R_S + r_{be}}{1+\beta_o}$ (Note that $\dfrac{R_S + r_{be}}{1+\beta_o}$ has units of ohms

β_o is large, so we can predict that R_o is small…

Conclusions:
The CC has large (-ish) input resistance
and small output resistance
– but no voltage gain!

Use it to 'match' high impedance source to low impedance load stages (see 'Design an Op-amp' Expt 5 later in the year.)



Example of ‘Matching’

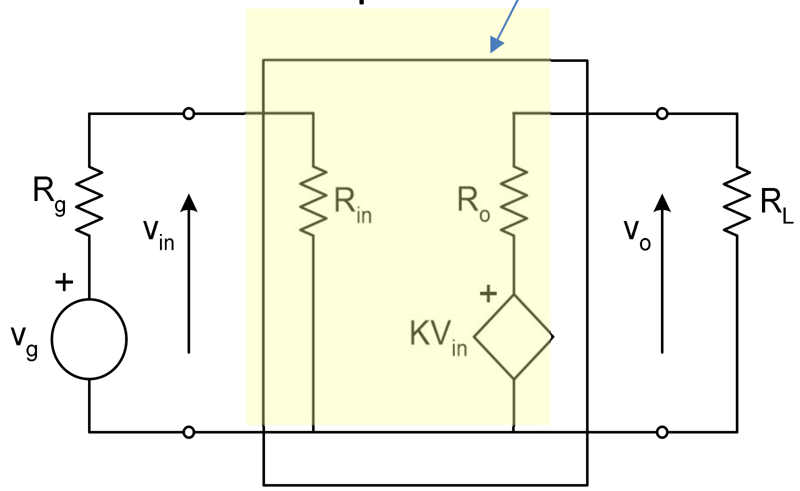
Consider a voltage amplifier and Thevenin source.

Should R_{in} be big or small?

$$v_{in} = \frac{R_{in}}{R_{in} + R_g} v_g$$

Ideally, want $V_{in} = V_g$

Equivalent circuit of a V-amp



K = constant

So want $R_{in} >> R_g$, ideally infinity!
To get good coupling between source and amplifier

All the signal, v_g is ‘coupled’ into the amplifier!

Bootstrapping

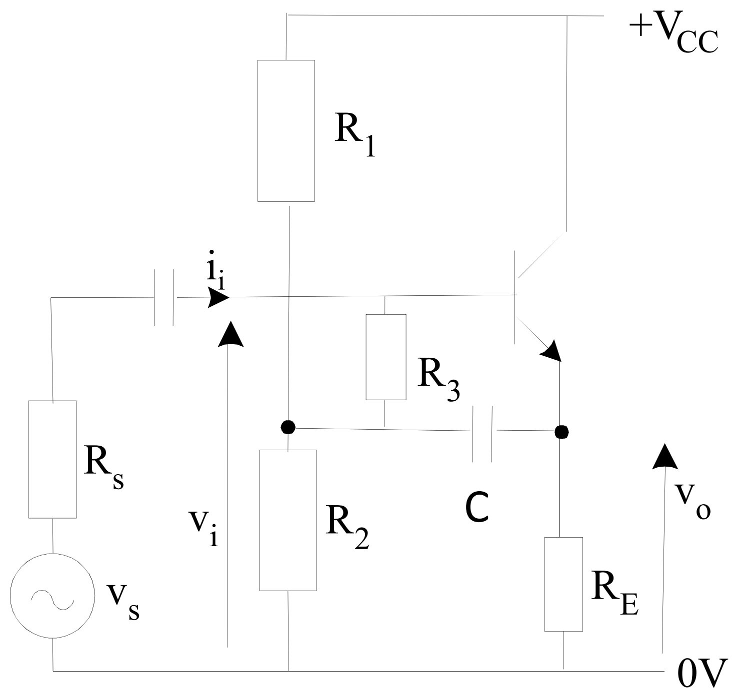
Although the CC input resistance (R_{in}) is large, the bias resistors (R_1, R_2) can effectively reduce it, as they appear in parallel with R_{in}. One way to get round this is to use 'bootstrapping'

- **C** is short circuit at freq of interest
- R_3 is small value (doesn't affect bias)
- change in ac current through R_3 is

$$i_{R3} = \frac{v_i - v_o}{R_3}$$

And as $v_i \sim v_o$, $i_{R3} \sim 0$!

That is, ideally, the path via R_3 has infinite resistance – **all signal goes into BJT!**

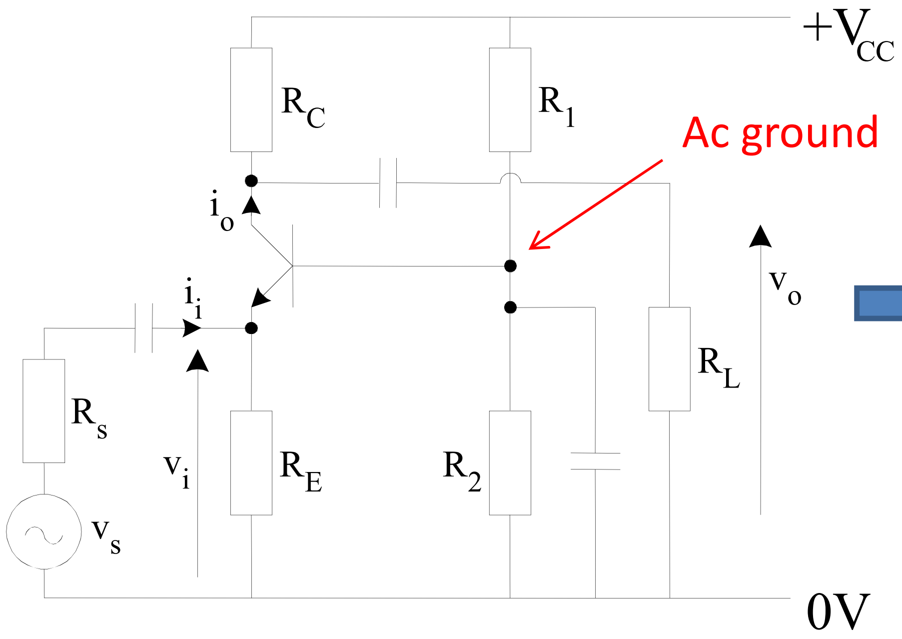


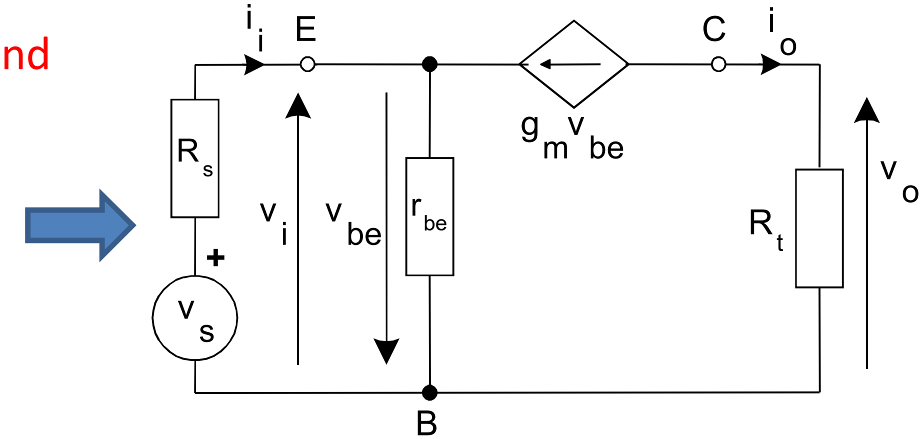
In fact, v_i is not equal to v_o as the gain of the CC is not quite unity. So write

$$R_3(effective) = \frac{v_i}{i_{R3}} = \frac{R_3}{1 - A_V}$$

Still a big value as ($1-A_V$) is very small

Common-base



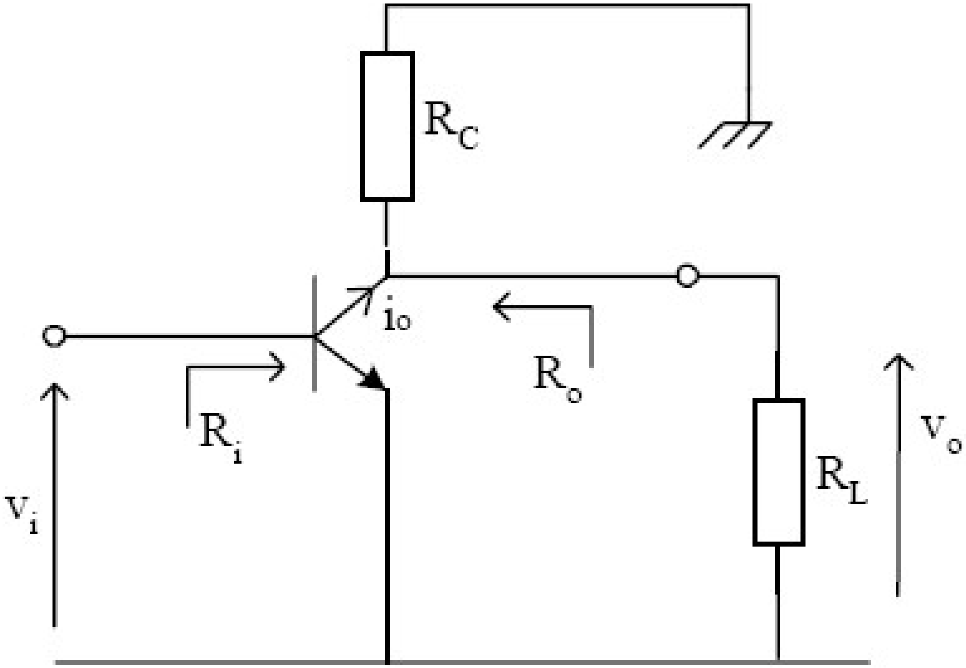


Note
R_E assumed to be much greater than R_{in} (= V_i /i_i)

(for analysis see exercise 3 – you do this one!)

Glossary of amplifier properties
(provided in examinations)

Common Emitter



$$R_i = r_{be}$$

$$R_o = R_C$$

$$A_V = \frac{v_o}{v_i} = -g_m R_C \,//\, R_L$$

$$G_M = \frac{i_o}{v_i} = -g_m$$

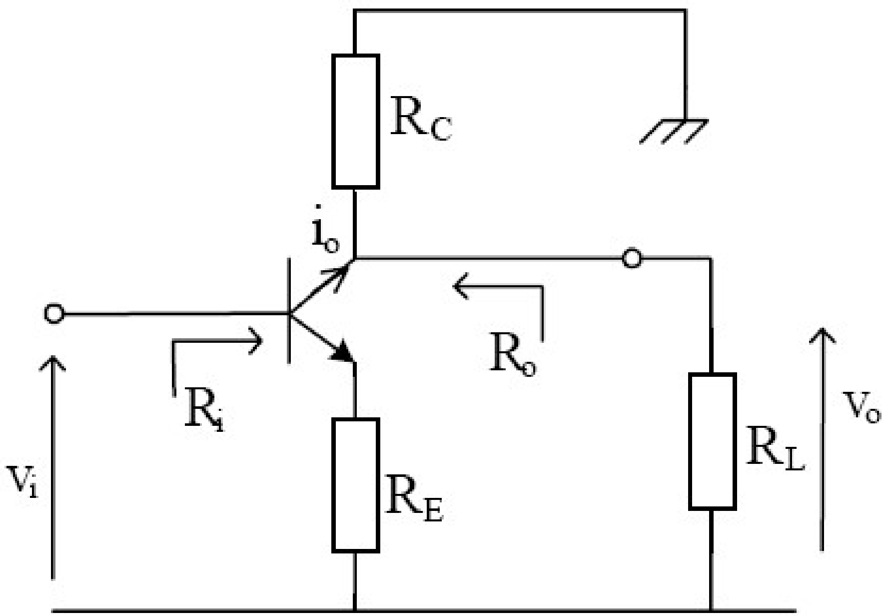
$$R_M = -\beta_o R_C \,//\, R_L$$

Note: $r_{be} \equiv r_\pi$ $\quad v_{be} \equiv v_\pi$

$r_{ce} \equiv r_o$

CE-ED

Common emitter with emitter degradation



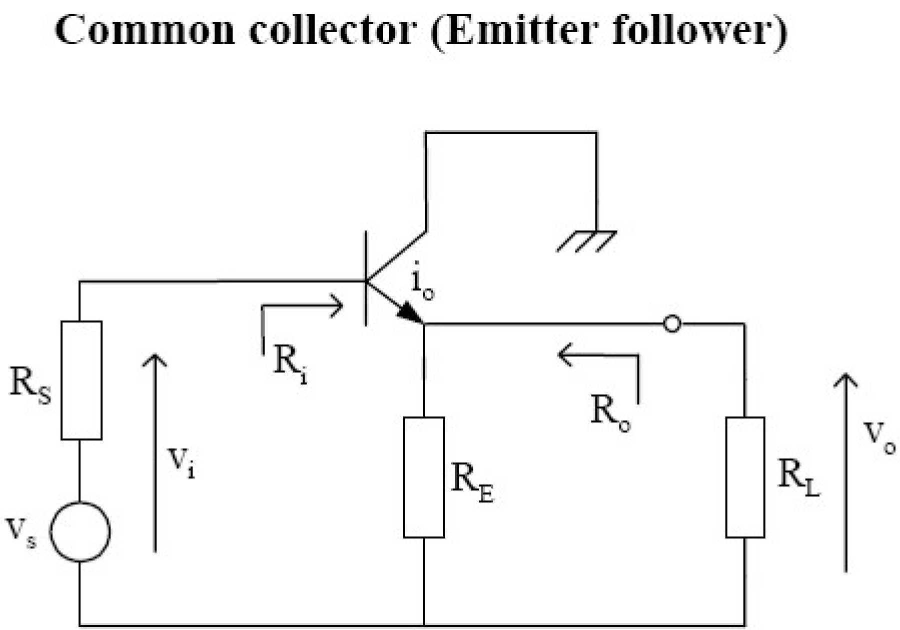
$$R_i = r_{be} + (1+\beta_o)R_E \qquad R_o = R_C$$

$$A_V = -\frac{g_m R_C // R_L}{1+g_m R_E}$$

$$G_M = -\frac{g_m}{1+g_m R_E}$$

$$R_M = -\beta_o R_C // R_L$$

CC (EF)



$$R_i = r_{be} + (1+\beta_o) R_E // R_L$$

$$R_o = \frac{r_{be} + R_S}{1+\beta_o} // R_E$$

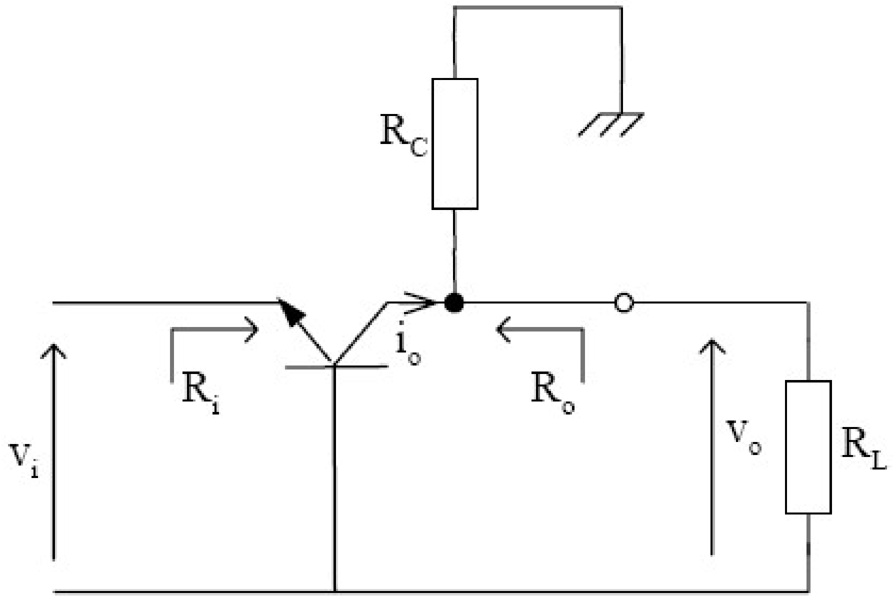
$$A_V = \frac{g_m R_E // R_L}{1 + g_m R_E // R_L}$$

$$G_M = \frac{g_m}{1 + g_m R_E // R_L}$$

$$R_M = (1+\beta_o) R_E // R_L$$

CB

Common base



$$R_i = \frac{r_{be}}{1+\beta_o} \approx 1/g_m = r_e \qquad R_o \sim R_C$$

$$A_V = g_m R_C // R_L$$

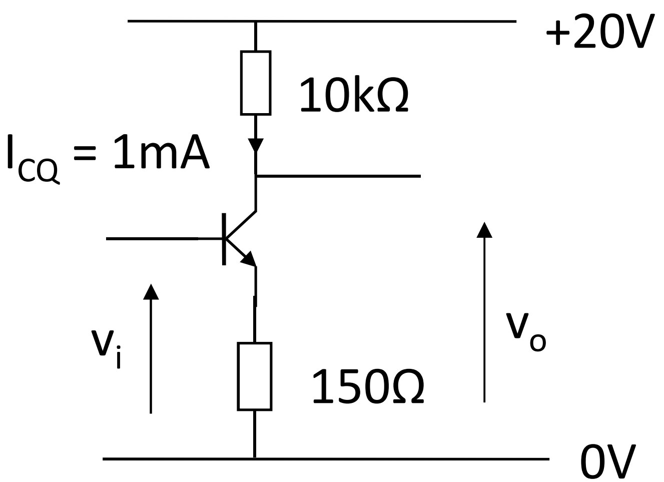
$$G_M = g_m \qquad R_M = \frac{\beta_o R_C // R_L}{1+\beta_o}$$

Summary of properties of amplifier configurations

Property	A_V	A_I	R_{in}	R_{out}	Usage
CE	high	high	Medium	high	Most useful, general purpose
CE-ED	Low	high	high	low	R_E constitutes feedback – sacrifice A_V for increased stability: less dependent on β_o (also temperature), increase R_{in}
CC	Low (< 1)	high	high	low	Impedance matching – high R-source to low-R load (also very linear – used in power amp. Output stages)
CB	high	low (<1)	low	high	Impedance matching – low R-source to high-R load (also features in Diff.Amp. – see later notes)

Multi-stage amplifiers can thus be configured to provide, for example, very high input impedance, high voltage and current gain and low output impedance.

Example



Calculate the voltage gain

Note that $R_t = R_C$

The amplifier is CE-ED.
From the amplifier properties sheet, $$\frac{v_o}{v_i} = -\frac{g_m R_t}{1 + g_m R_E}$$

For I_C = 1mA, $g_m = 40 I_{CQ} = 40 \text{ mA/V}$

$$\therefore \frac{v_o}{v_i} = -\frac{40\times10^{-3}\times10^{4}}{1+40\times10^{-3}\times150} = -57$$

Electronic circuits and systems
ELEC271

Design example 1

•The following is a ‘rough’ design with a number of approximations.

• It can be **validated** by PSPICE simulation and fine-tuning of the parameter values undertaken

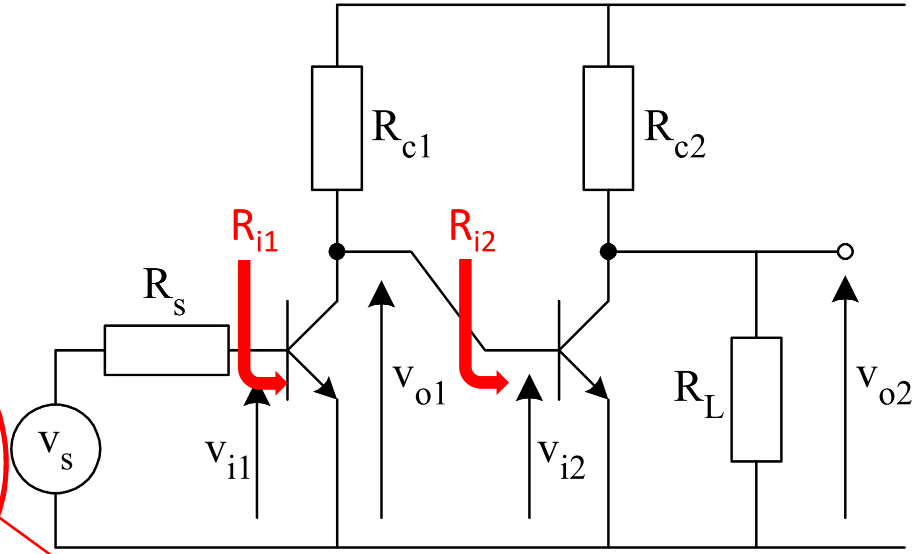
Specification

- Design a common-emitter amplifier to the following specification
  - ○ V_{CC} = 20V, A_V > 100, $V_C \sim V_{CC}/2$, a load of 10k
- Given
  - ○ β_o =250, make $R_1 // R_2 \sim 10 \times R_{in}$, V_{RE} =10%V_{CC}
- Hints
  - First draw the schematic circuit
  - Estimate values for R_C, I_C, R_E, V_B, R_2, R_1

SEE VITAL for the solution – try it before the next lecture!

A two-stage voltage amplifier

Bias resistors & coupling capacitors omitted



Analysis:

Use the chain rule to split the overall gain into three components

$$A_{Vs} = \frac{v_{o2}}{v_s} = \frac{v_{o2}}{v_{i2}} \times \frac{v_{i2}}{v_{i1}} \times \frac{v_{i1}}{v_s}$$

But $\mathbf{v_{i2} = v_{o1}}$ so:-

$$A_{Vs} = \frac{v_{o2}}{v_{i2}} \times \frac{v_{o1}}{v_{i1}} \times \frac{v_{i1}}{v_s}$$

Which is equal to

$$A_{Vs} = A_{V2} \times A_{V1} \times \frac{R_{i1}}{R_{i1} + R_s}$$

'Coupling' term: R_S 'loads' the input

$$A_{V2} = -g_{m2} R_L // R_{C2}$$

that is, 'g_m x the ac load'

$$A_{V1} = -g_{m1} R_{C1} // R_{i2}$$

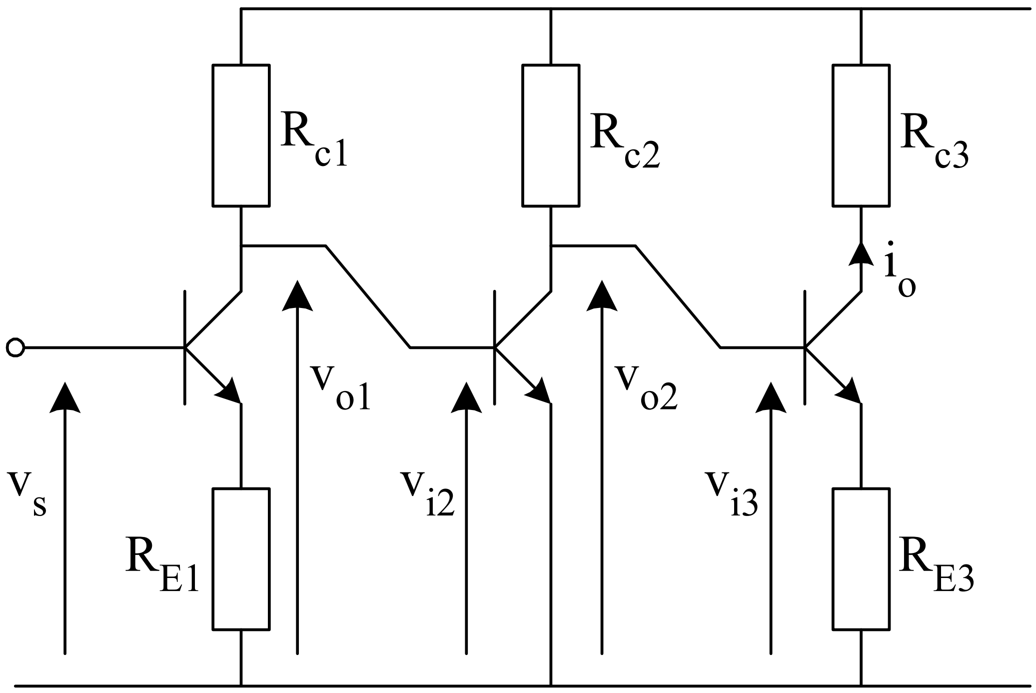
must take account of the loading effect of the second stage on the first (R_{i2})

Note that $R_{i2} = r_{be2}$, $g_{m2} = 40 \times I_{C2}$, $R_{i1} = r_{be1}$ **($r_{be} g_m = \beta_o$)**

A 3-stage transconductance amplifier

Function: *transconductance* amplifier – turns an input **voltage** signal v_s into an output **current** signal i_o with appropriate enlargement (amplification) of the signal.
In this case the current is into the collector load R_{C3}. A practical example of the use of such an amplifier could be in driving a light emitting diode (l.e.d) which would replace the resistor.

- First stage provides for high input resistance and some voltage gain,
- second stage provides high voltage gain, third stage converts voltage to current.



$$G_M = \frac{i_o}{v_s} = \frac{i_o}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_s}$$

$$v_{o2} = v_{i3}$$

$$v_{o1} = v_{i2}$$

$$\therefore G_M = \frac{i_o}{v_{i3}} \times \frac{v_{o2}}{v_{i2}} \times \frac{v_{o1}}{v_s}$$

Analysis

Overall transconductance gain is

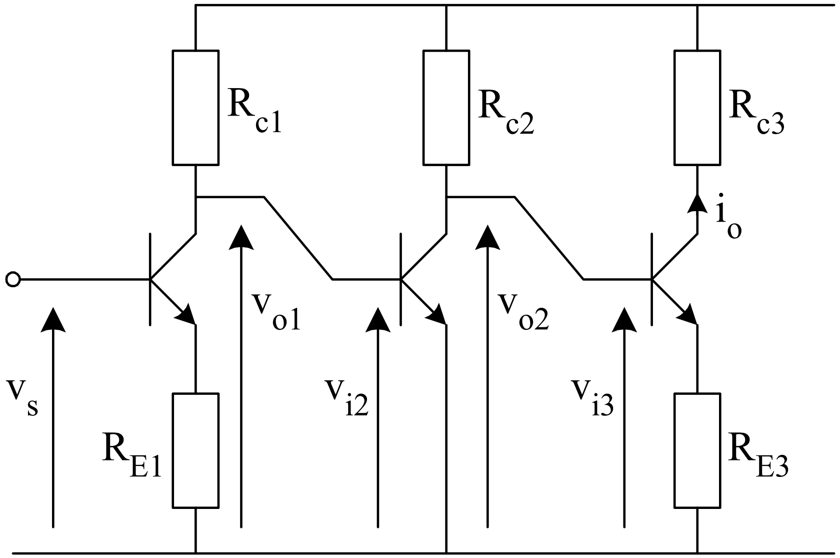
$$G_M = \frac{i_o}{v_s} = \frac{i_o}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_s}$$

That is,

$$G_M = \frac{i_o}{v_{i3}} \times \frac{v_{o2}}{v_{i2}} \times \frac{v_{o1}}{v_{i1}}$$

or

$$G_M = G_{M3} \times A_{V2} \times A_{V1}$$



$$A_{V1} = -\frac{g_{m1}R_{t1}}{1+g_{m1}R_{E1}}$$

First stage is CE-ED. Must take account of the loading effect of the second stage on the first

$$R_{t1} = R_{C1} // R_{i2}$$

R_{i2} is the input resistance of the 2nd stage; $R_{i2} = r_{be2}$

3rd stage is CE-ED want the transconductance gain

$$G_{M3} = -\frac{g_{m3}}{1+g_{m3}R_{E3}}$$

$$A_{V2} = -g_{m2}R_{t2}$$

2nd stage is CE, loaded by stage 3

$$R_{t2} = R_{C2} // R_{i3}$$

Worked example (vital)

Part 2 notes section

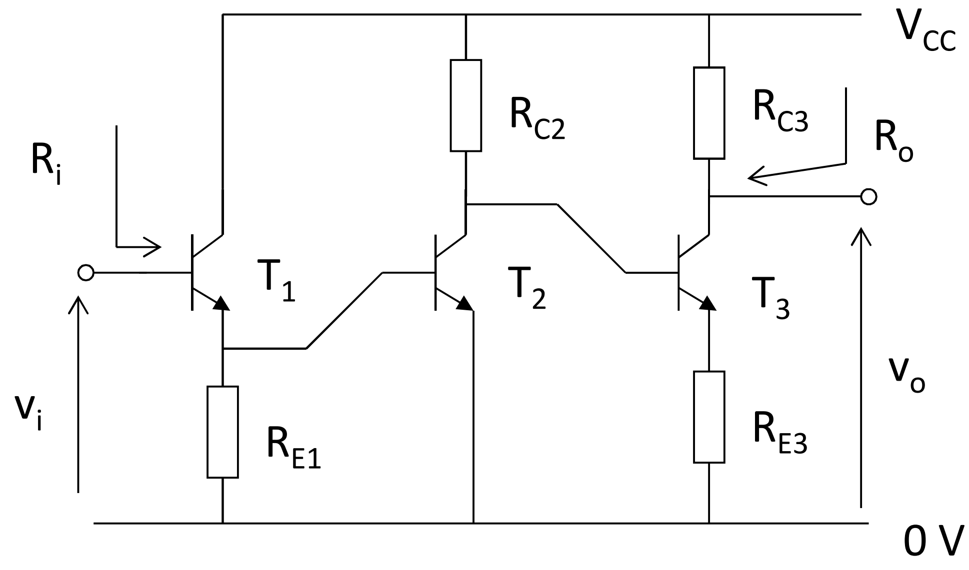
biasing components have been omitted.

I_{C1} = 0.1 mA

I_{C2} = 0.5 mA

I_{C3} = 2 mA

1. Find g_m, r_{be} of each transistor. (β_o = 100)
2. Find voltage gain v_o/v_i
3. Find R_i
4. Find R_o



R_{E1} = 1 kΩ, R_{C2} = 10 kΩ, R_{C3} = 2 kΩ and R_{E3} = 200 Ω; assume r_{ce} & $r_{b'c}$ are infinite and $r_{bb'}$= 0.

Solution Method – key points

1. Use the simple formulas for g_m, r_{be} - what are they? $g_m = 40 \times I_C$ $r_{be} = \beta_o / g_m$
2. Identify the circuit blocks

Start from the output side

Work out gain of stage 3; what is the amplifier type and ac load? CE-ED, R_{C3}

Work out gain of stage 2; what is the amplifier type and ac load? CE, $R_{C2}//R_{i3}$

Work out gain of stage 1; what is the amplifier type and ac load? CC, $R_{E1}//R_{i2}$

How should they be combined? $A_V = A_{V1} \times A_{V2} \times A_{V3}$

End of part 2

- Next lecture: the differential amplifier