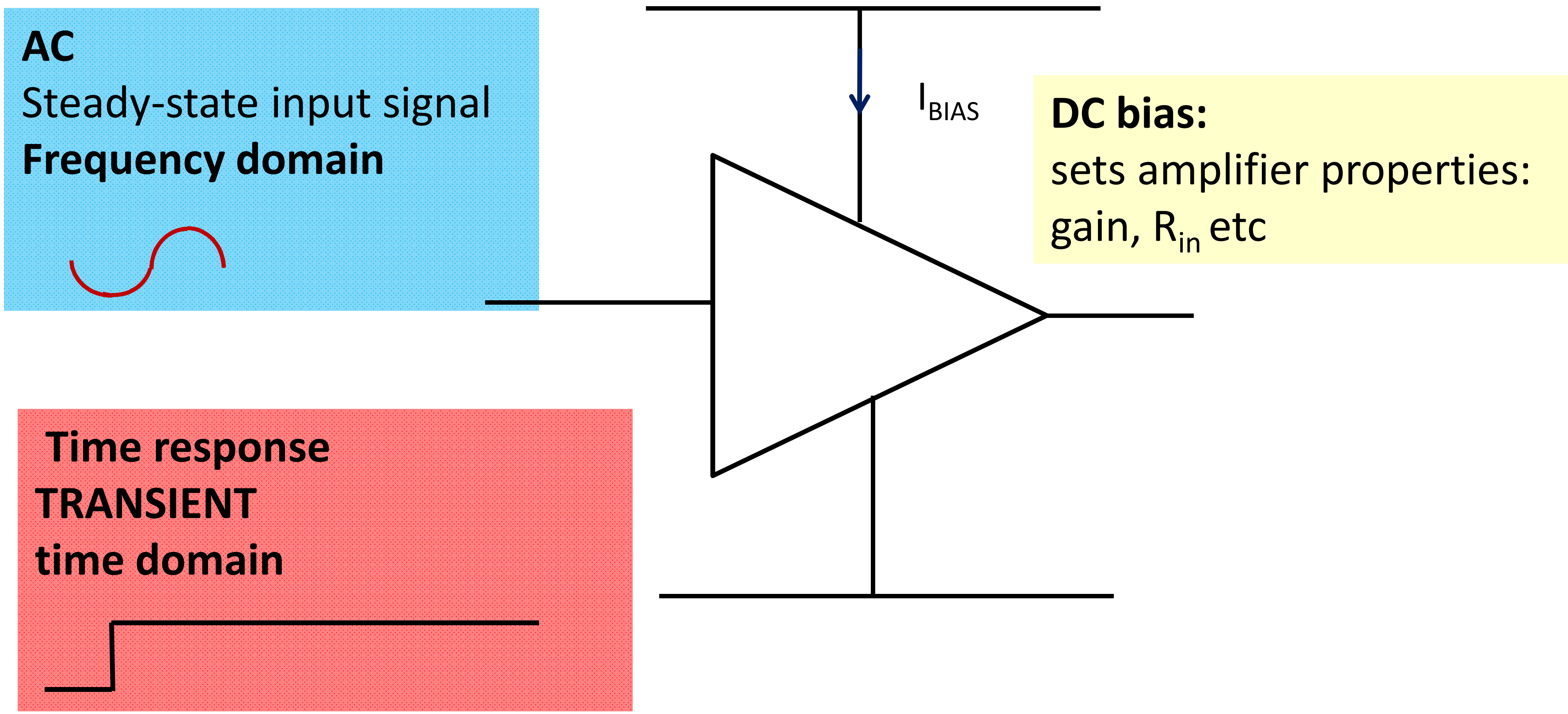
# Electronic circuits and systems
# ELEC271

## Part 9
## Step Response of amplifiers

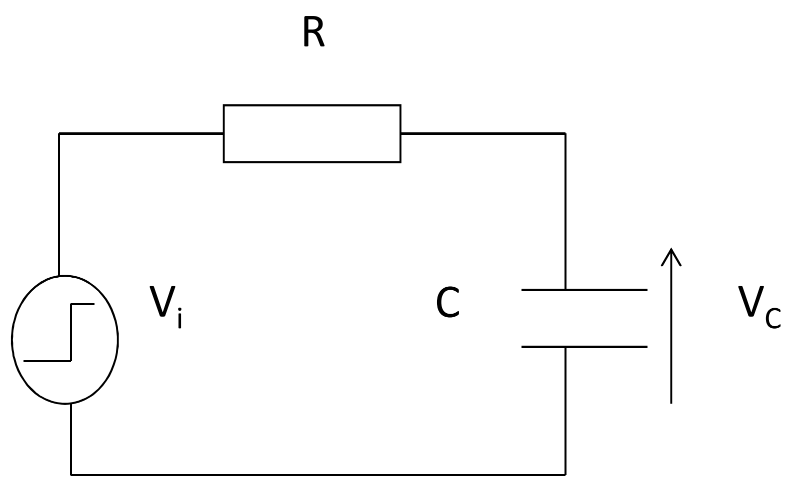
- Useful for characterising amplifiers:
  - close relationship between bandwidth ($f_H$) and rise time ($t_r$).
- Interesting to see how an electronic circuit responds to a different type of signal (transient)
  - Gives an indication of the time to 'power-up' an electronic system.

# Forms of excitation



# A simple series R-C (first order) circuit

Work out time for the voltage $V_C$, to change from the 10% to 90% level following an abrupt input voltage step, $V_i$




Proceed with the analysis using Laplace Transforms ($s = j\omega$):

**The transfer function is:** $$\frac{V_C(s)}{V_i(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{1+sRC}$$

Represent the input step of amplitude $V$, as $V_i(s) = V/s$, so $$V_C(s) = \frac{V}{s}\frac{1}{1+sRC}$$

It is necessary to split the expression into partial fractions: $$\frac{1}{s(1+sRC)} = \frac{A}{s} + \frac{B}{1+sRC}$$

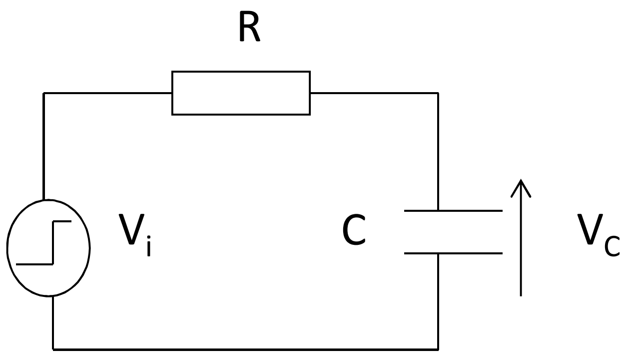
Clear the fractions: $$1 = A + AsRC + Bs$$

**Equate coefficients:**

Constants: $1 = A$

S: $0 = ARC + B \Rightarrow B = -1/RC$

**See also Control module!**




Therefore partial fractions are

$$\frac{1}{s(1+sRC)} = \frac{1}{s} - \frac{RC}{1+sRC}$$

Substitute back $$V_C(s) = V\left(\frac{1}{s} - \frac{RC}{1+sRC}\right) = V\left(\frac{1}{s} - \frac{1}{s+1/RC}\right)$$

The expression is now in **standard form** and the solution (time domain) is found from tables:

$$V_C(t) = V\left[1 - \exp\left(-\frac{t}{RC}\right)\right] \qquad (1)$$

**the 10% level** ($t = t_1$) is found as $V_C(t_1) = 0.1V = V\left[1 - \exp\left(-\frac{t_1}{RC}\right)\right]$

hence $\exp(-t_1/RC) = 0.9 \quad t_1 = -RC\ell n(0.9) \quad \Rightarrow$ **$t_1$ = 0.1*RC***

**The 90% level** ($t = t_2$) is found as $V_C(t_2) = 0.9V = V\left[1 - \exp\left(-\frac{t_2}{RC}\right)\right]$

hence $\exp(-t_2/RC) = 0.1 \quad t_2 = -RC\ell n(0.1) \quad \Rightarrow$ **$t_2$ = 2.3*RC***

**The rise time is therefore: $t_r = t_2 - t_1 = 2.2RC$** (2)

Typical job interview Q!

# Common source amplifier -1

Now apply the above theory to the case of a common source MOSFET amplifier (see last lecture for schematic diagram).

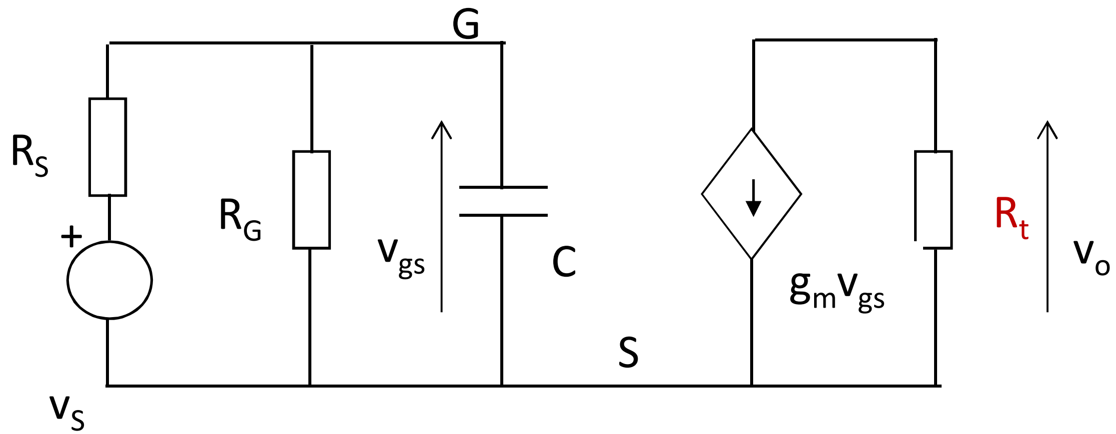
Apply Miller's theorem and the equivalent circuit reduces to:

**$R_G = R_1 // R_2$ (the bias resistors)**

**$C = C_{gs} + (1 – K)C_{gd}$**

**$K = v_o/v_{gs} = - g_m R_t$**

**with $R_t = R_D//R_L$**

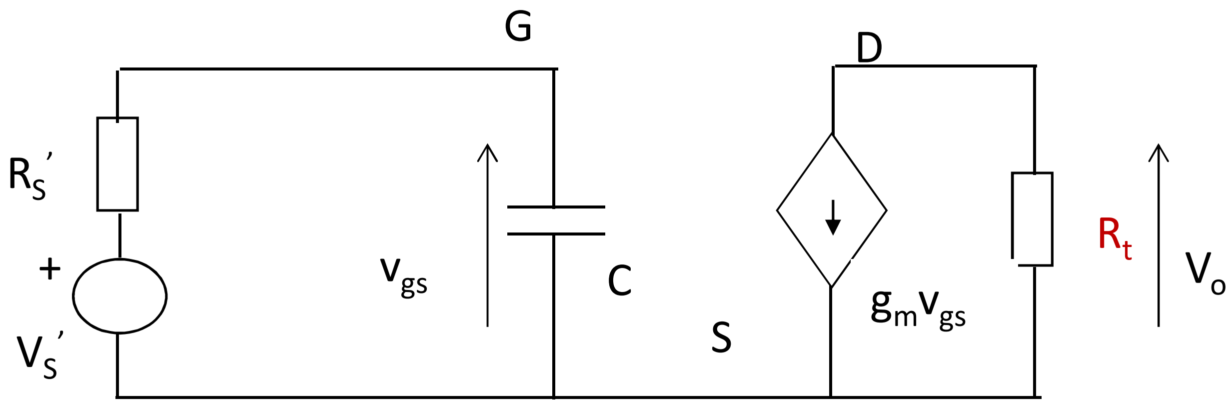



As before, transform the input circuit using **Thevenin** to get a simple series R-C network

**This side is just a simple RC circuit**

$$R_S' = R_G // R_S$$

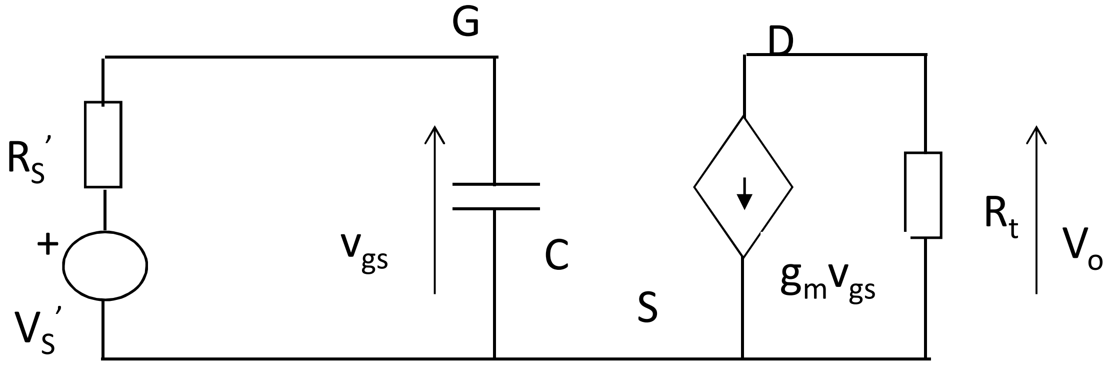
$$V_S' = \frac{R_G}{R_G + R_S} V_S$$



# Common source amplifier - 2

$$R_S' = R_G \,//\, R_S$$

$$V_S' = \frac{R_G}{R_G + R_S} V_S$$




Recognise the following equivalences with the analysis for the **R-C network:**

$$V_i = V_S' = \frac{R_G}{R_G + R_S} V_S$$ ($V_s$' is taken as applied voltage step)

$$V_C = v_{gs} = -V_o / g_m R_t$$ (voltage across the capacitor)

$$R = R_S' = R_G \,//\, R_S \qquad C = C_{gs} + (1 + g_m R_t) C_{gd}$$

**Substituting into Eqn. (1) gives**:

$$V_o(t) = -g_m R_t \frac{R_G}{R_G + R_S} V_S \left[ 1 - \exp\left( -\frac{t}{R_G \,//\, R_S \left( C_{gs} + (1 + g_m R_t) C_{gd} \right)} \right) \right] \qquad (3)$$

# Interpretation

$$V_o(t) = \underbrace{-g_m R_t \frac{R_G}{R_G + R_S}}_{\text{mid-freq. voltage gain!}} V_S \left[1 - \exp\left(-\frac{t}{\underbrace{R_G // R_S}_{\text{'R'}} \underbrace{\left(C_{gs} + (1 + g_m R_t) C_{gd}\right)}_{\text{'C'}}}\right)\right]$$

- **Looks complicated!!** But it just has the form: $V(t) = V_{MAX}\left[1 - \exp\left(-\frac{t}{RC}\right)\right]$
- Note that for $t = 0$, $V_O = 0$. For $t >> 0$, the exponential term goes to zero and the output is constant.

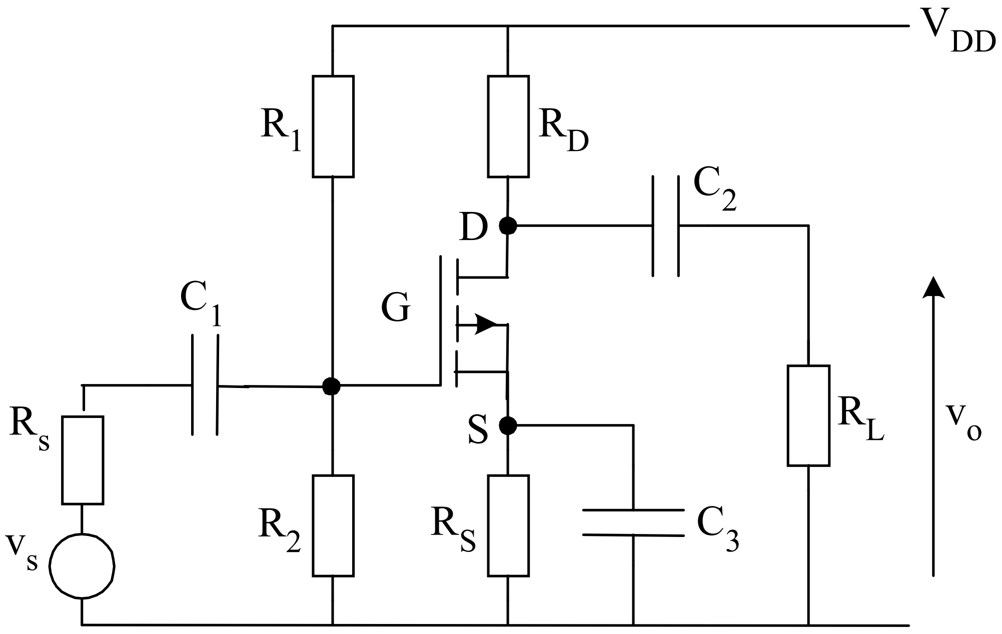
Recalling (from MOST lecture) that the bandwidth of the amplifier is $f_H = \frac{1}{2\pi R_S^{'} C}$

have (from Eqn. 2) ***($t_r$ = 2.2RC)***

$$t_r = 2.2 \frac{1}{2\pi f_H} = \frac{0.35}{f_H}$$

**The bandwidth of the amplifier, $f_H$ can be estimated from the rise time of the output voltage following an input step voltage!**

# Practical point




In reality, the voltage step would need to be **a.c. coupled** into the amplifier (that is, via an input capacitor).

Thus for very long times, $t \to \infty$, the output would eventually go to zero!

However the time constant for this would be very long, because circuit capacitors are very large compared to the device capacitances that we are concerned with here.

The equations (3,4) above do not predict this and are only true for times much less than the (long) time constants associated with the discharging of circuit capacitors.

**This point is demonstrated in the following PSPICE simulation.**

# Apply the method to the BJT

**A similar approach can be used for the bipolar transistor: common emitter amplifier. Recognise that:**

$$V_i = V_S' = V_S \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s} \qquad V_C = v_{b'e} = -V_o \big/ g_m R_t$$

$$C = C_e + C_c(1 - K) \qquad (K = -g_m R_L >> 1)$$

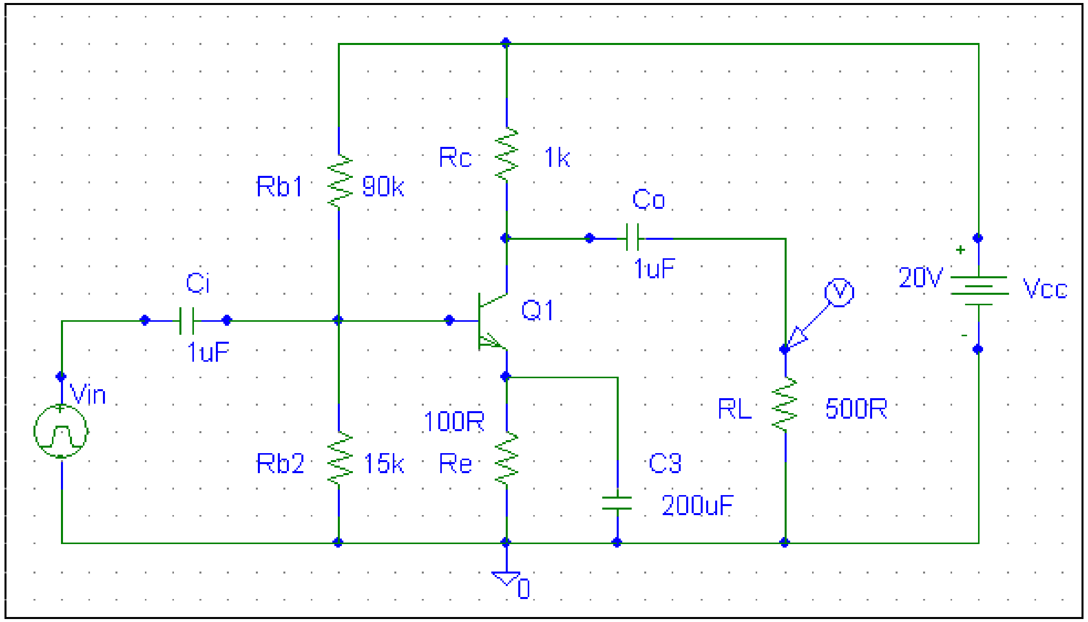
$$R = R_S' = (R_S + r_{bb'}) // r_{b'e}$$

Hence

$$V_o(t) = \underbrace{-g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_S}}_{\text{Note That this is the mid-freq. Voltage gain!}} V_S \left[ 1 - \exp\left( - \frac{t}{\underbrace{(R_S + r_{bb'}) // r_{b'e}}_{\text{'R'}} \underbrace{(C_e + (1 + g_m R_t) C_c)}_{\text{'C'}}} \right) \right] \qquad (4)$$

# SPICE simulation of response of a CE amplifier to a small voltage step

**$V_{in}$ = 10 mV step**

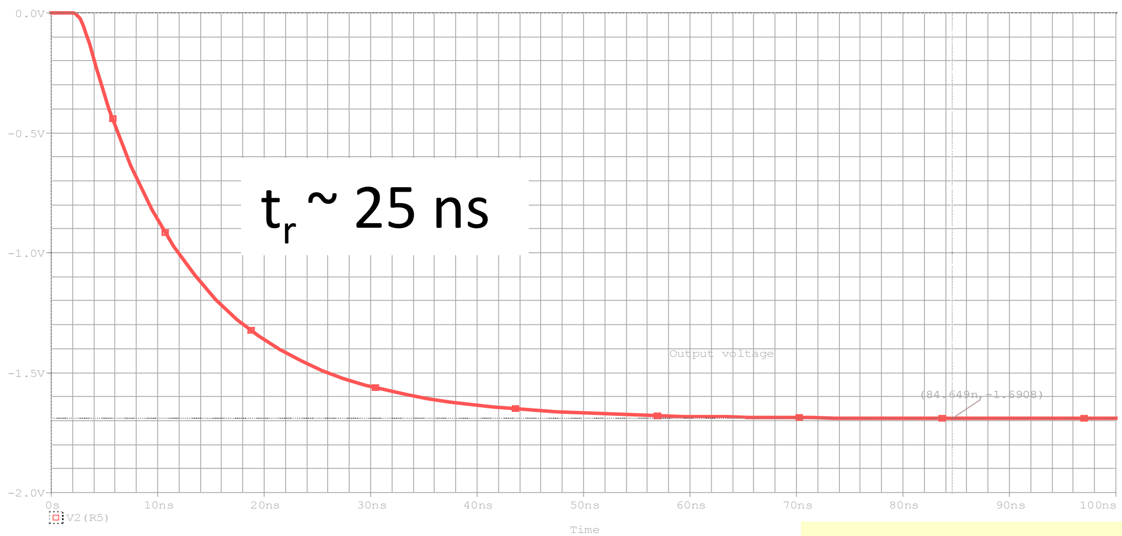



Prediction of gain

1.69/10 mV = **169**

**Output voltage**

**0 V to -2 V**




**Prediction of BW**

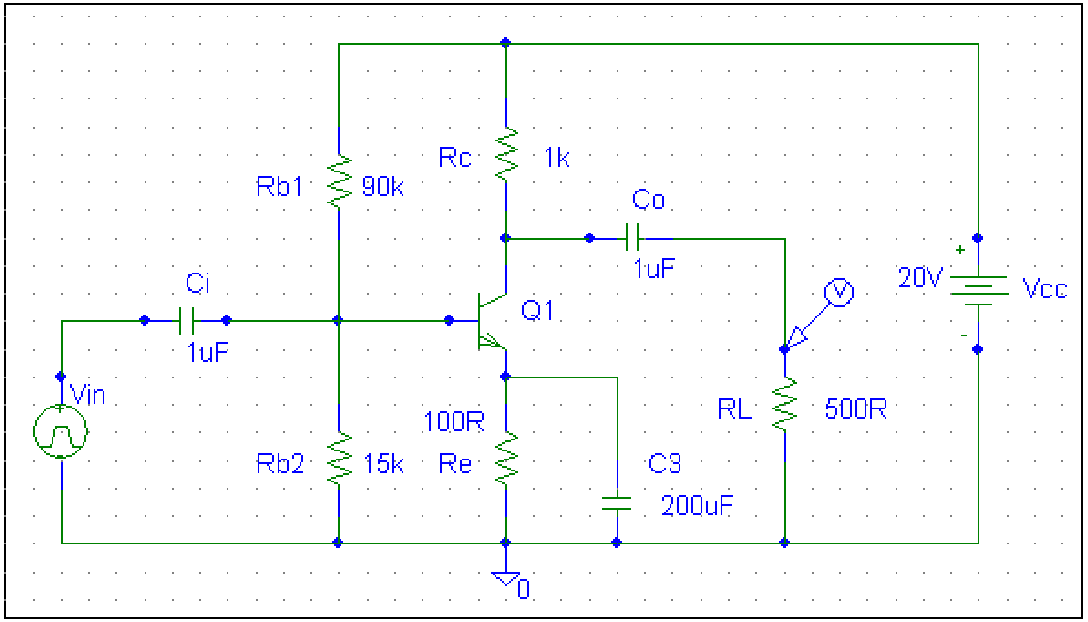
$$t_r \sim \frac{0.35}{f_H}$$

**$f_H$ ~ 14 MHz**

$V_o$ = 1.69 V

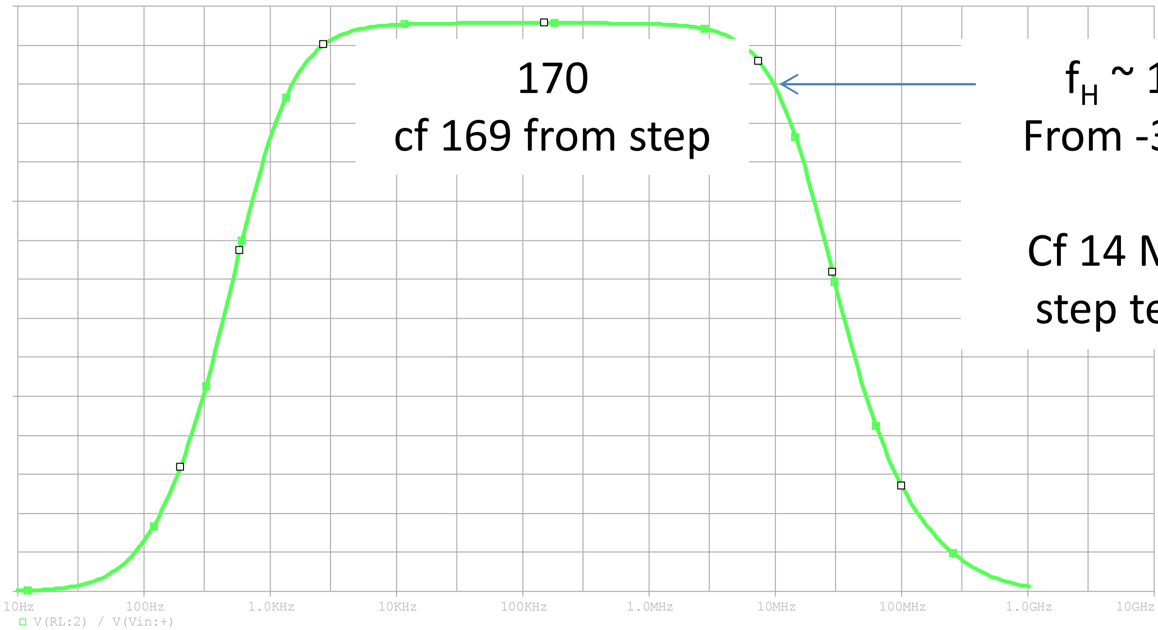
**Time:** 0 – **100 ns** ← **Note the short time scale!!**

# AC sweep




$V_{in}$ = 10 mV
Ac sweep

Gain
0 to 180



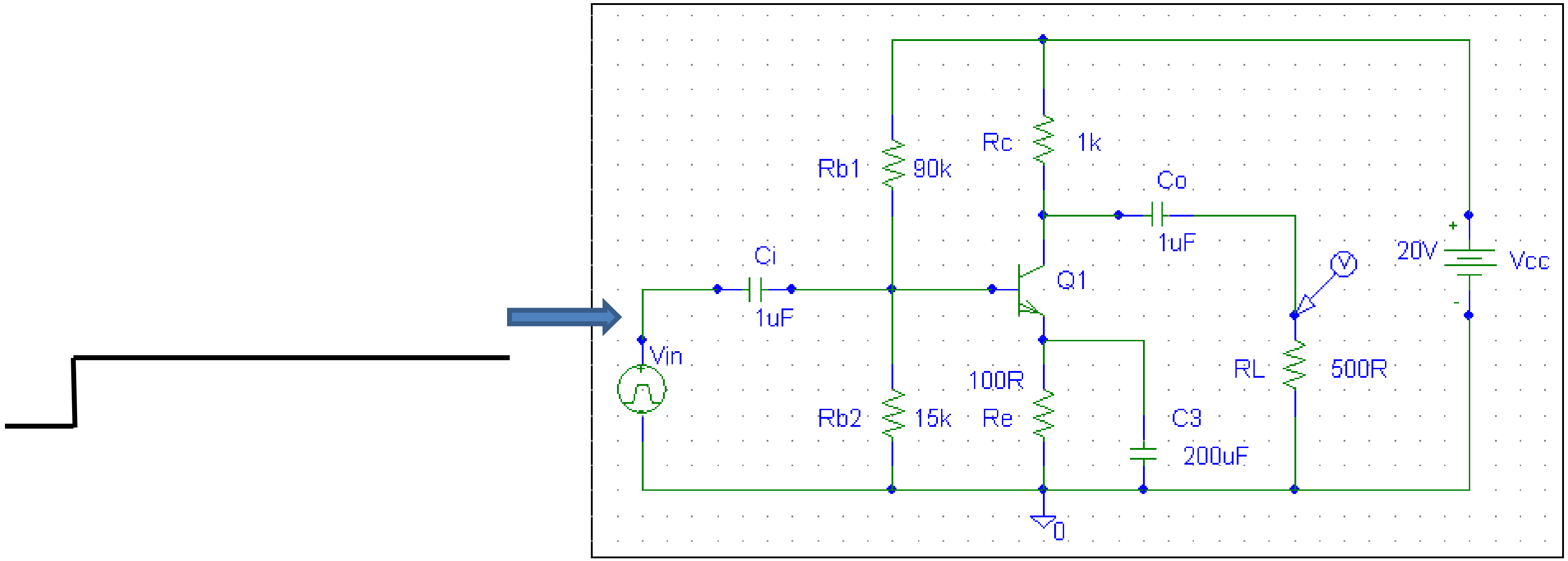

170
cf 169 from step

$f_H$ ~ 16 MHz
From -3dB point

Cf 14 MHz from
step technique

Frequency: 10 Hz- – **1 GHz**

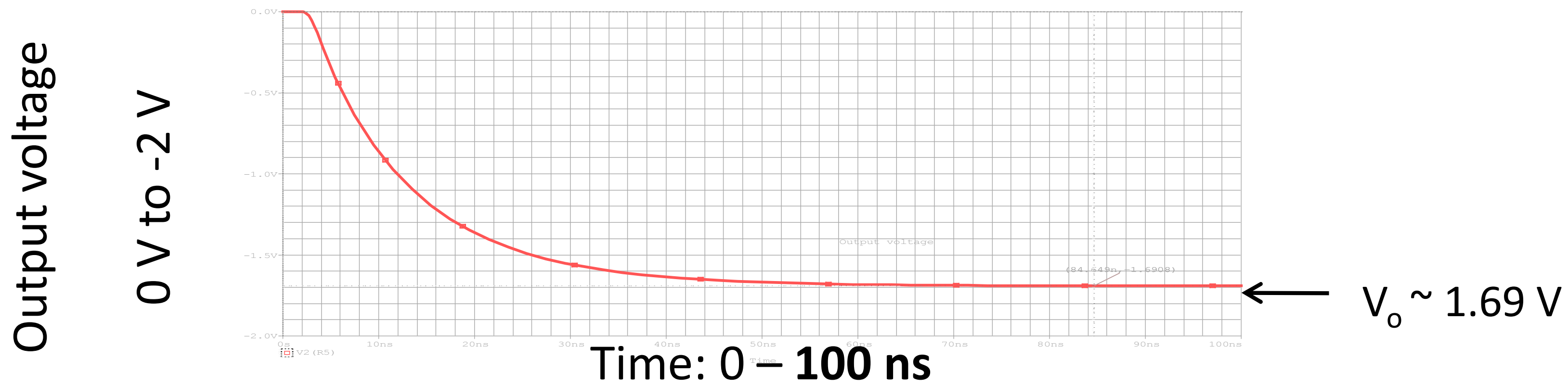
# After the transient.....



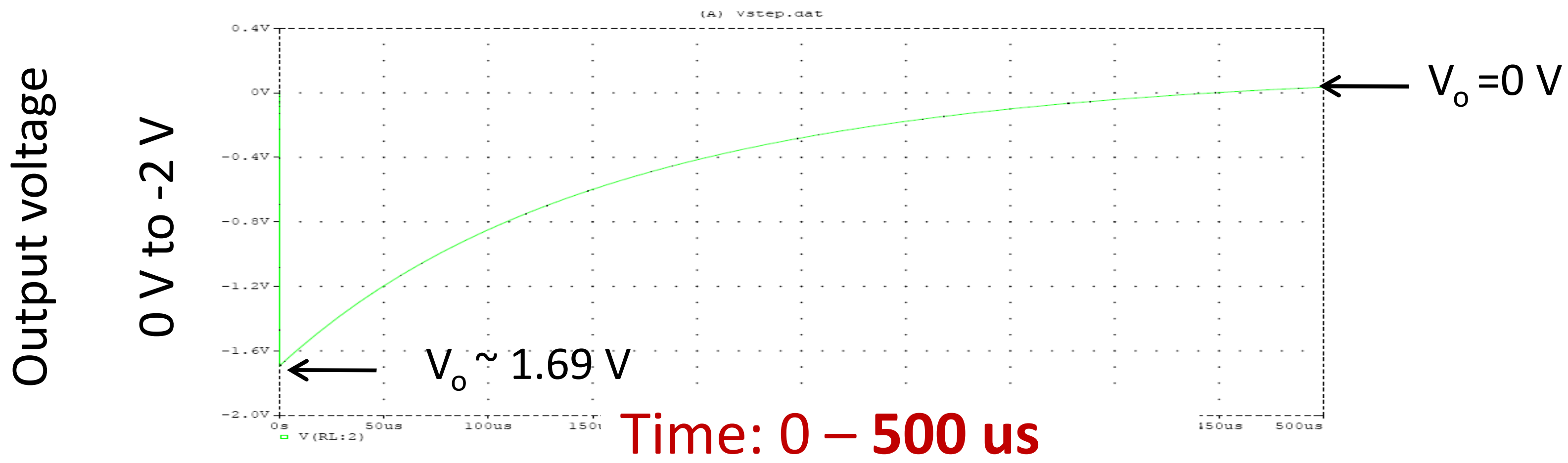

If we apply a voltage step to the input, what do we expect the output to be after a 'long time'....

# Compare plot from 1, above (first 100 ns) with much longer time (500 us)

**100ns time window:** output settles at ~ 1.7 V [NON-STEADY STATE SITUATION!]

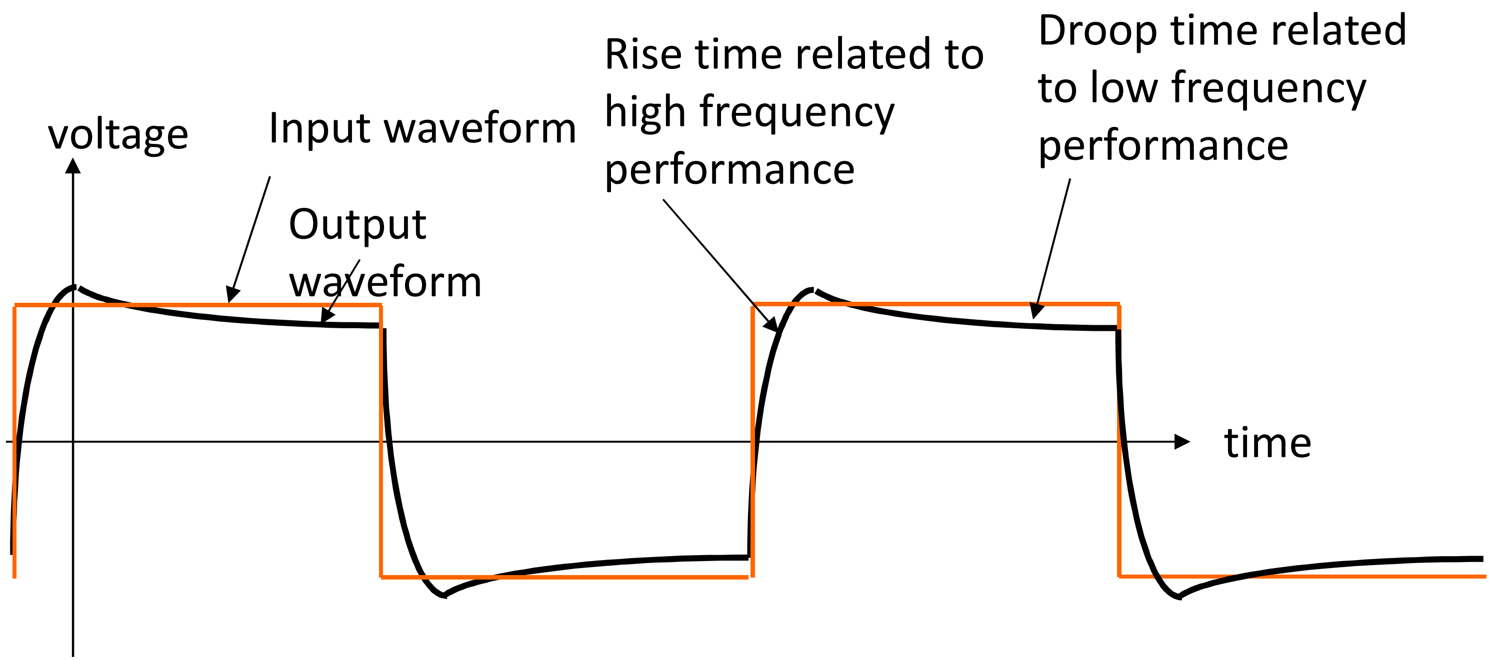



**500 μs time window:** output 'relaxes back to zero' WITH MUCH LONGER TIME CONSTANT (set by circuit capacitances)



# Can also estimate **low frequency** response

- The response of an amplifier to an input pulse is closely linked to the frequency response to sinusoidal input signals
- a relationship between the rise time and **droop time** of the pulse with the upper and lower cut off frequencies of the amplifier.



# End of 1st half: Electronic Circuits

- From now we deal with 'Electronic Systems'
- Remainder of module concerns 'negative feedback;
- A very clever way of building near to ideal, stable amplifier systems.....