PROBLEMS I

1. An amplifier has a feedback fraction β equal to 0.1. Calculate the overall gain with feedback, A_f, if the open loop gain A_{ol} has the following values (a) 10; (b) 50; © 100; (d) 500; (e) 1000. From these results what would you guess the limiting value of A_f would be as A_{ol} tends to infinity? Compare this guess with the value of 1/β.

β=0.1

A_{ol}	$A_f = \frac{A_{ol}}{1+A_{ol}\beta}$, β=0.1
10	5
50	8.33
100	9.09
500	9.8
1000	9.9
∞	10

$A_{ol}\beta = T$ is the LOOP GAIN, if T very large $A_f \cong \frac{1}{\beta}$.

2. An amplifier with an open loop gain A_{ol} of 2500 can have the following possible β values (a) 0.2; (b) 0.4; © 0.8; (d) 1.0. Calculate the values of the overall gain with feedback A_f in each case and compare with the limiting values of A_f when A_{ol} tends to infinity (i.e. 1/β).

A_{ol}=2500, A_f=?

β	$A_f = \frac{A_{ol}}{1+A_{ol}\beta}$, **for A_{ol}=2500**	$A_f = \frac{A_{ol}}{1+A_{ol}\beta}$, **for $A_{ol}=\infty$**
0.2	**4.99**	**5**
0.4	**2.498**	**2.5**
0.8	**1.25**	**1.25**
1	**1**	**1**

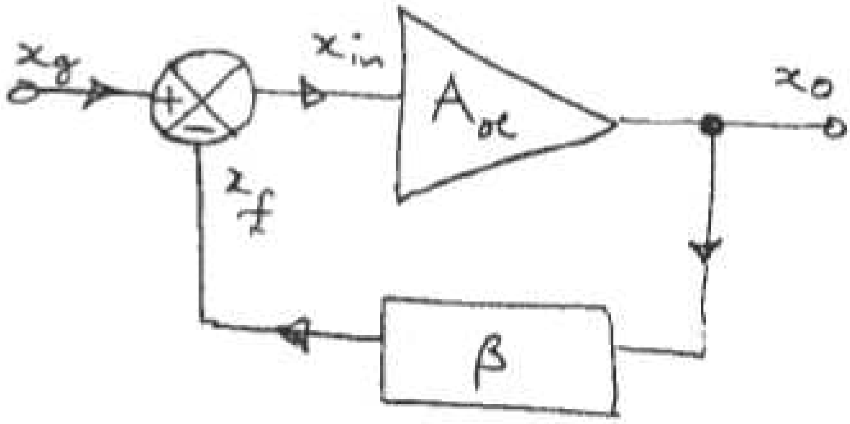
$A_{ol}\beta = T$ is the LOOP GAIN, if T is very large

$$A_f \cong \frac{1}{\beta}.$$

Closed loop gain is practically independent of amplifier open loop gain, A_{ol}, provided A_{ol}β is big.

3. Calculate the value of x_{in} for the circuit shown in fig.P3.1 when $x_g = 1$, if $A_{ol} = 1000$, $\beta = 0.1$. Has the feedback increased or decreased x_{in}? (Without feedback $x_{in} = x_g$). What would the limiting value of x_{in} be as A_{ol} tends to infinity?

Fig. P3.1



$$x_{in} = x_g - x_f = x_g - \beta x_o = x_g - \beta A_{ol} x_{in}$$

$$x_g = (1 + \beta A_{ol}) x_{in}$$

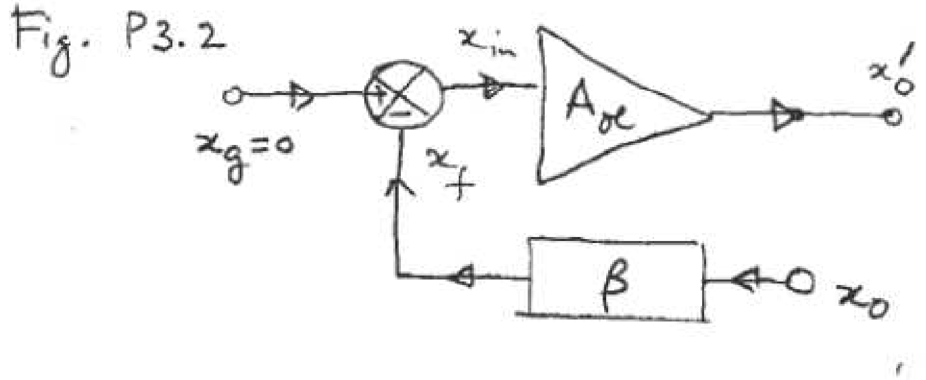
$$x_{in} = \frac{x_g}{1 + \beta A_{ol}} = 0.0099$$

The feedback reduces x_{in}!

(without feedback, $x_{in}=x_g=1$).

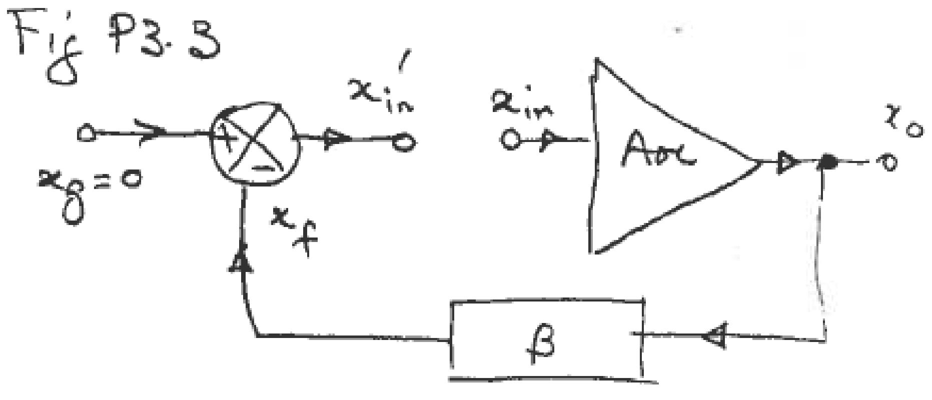
If $A_{ol} \to \infty$, $x_{in} \to 0$.

4. Consider the circuit shown in fig. P3.2, in which the input signal x_g has been reduced to zero and the feedback path is broken at the output. Calculate the ratio x'_o/x_o. Now consider the circuit shown in fig. P3.3, in which the connection between the summing junction and the amplifier input has been broken. Calculate x'_{in}/x_{in}. [In both cases, the answer is $-A_{ol}.\beta$. I hope you can now see why this quantity, (without the negative sign) is called the *loop gain*.].



$$x'_o = A_{ol}x_{in} = A_{ol}(0 - x_f) = -A_{ol}\beta x_o$$

$$\frac{x'_o}{x_o} = -\beta A_{ol}$$



$$x'_{in} = (0 - x_f) = 0 - \beta x_o = -\beta A_{ol}x_{in}$$

$$\frac{x'_{in}}{x_{in}} = -\beta A_{ol}$$

βA_{ol} = LOOP GAIN (same all the way round/ the loop).

5. An amplifier has an open loop gain of 2400 when the transistors used in its construction have a common emitter gain of 100. If the feedback fraction is 1/21, calculate the overall gain with feedback, A_{f1}. (You will need to express your answer to 3 decimal places). If now the transistors are replaced, so that the open loop gain rises to 2600, calculate the new gain with feedback, A_{f2}. Hence calculate the fractional change in overall gain given by

$$\frac{\Delta A_f}{A_f} = \frac{A_{f2} - A_{f1}}{A_{f1}}$$

Compare this result with that using the equation given in the lecture.

$$A_{f1} = \frac{A_{ol1}}{1 + \beta A_{ol1}} = \frac{2400}{1 + \left(\frac{1}{21}\right) 2400} = 20.818$$

$$A_{f2} = \frac{A_{ol2}}{1 + \beta A_{ol2}} = \frac{2600}{1 + \left(\frac{1}{21}\right) 2600} = 20.832$$

$$\frac{\Delta A_f}{A_f} = \frac{20.832 - 20.818}{20.818} = 6.7 \cdot 10^{-4}$$

Using the formula from the lecture:

$$\frac{\Delta \boldsymbol{A_f}}{\boldsymbol{A_{f1}}} = \frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{\beta A_{ol1}}} \frac{\Delta \boldsymbol{A_{ol}}}{\boldsymbol{A_{ol1}}} = \frac{\mathbf{1}}{\mathbf{1} + \mathbf{2400}(\frac{\mathbf{1}}{\mathbf{21}})} \frac{\mathbf{200}}{\mathbf{2400}}$$

$$= \mathbf{7.23 \cdot 10^{-4}}$$

$$\frac{\Delta \boldsymbol{A_f}}{\boldsymbol{A_{f2}}} = \frac{\mathbf{1}}{\mathbf{1} + \boldsymbol{\beta A_{ol2}}} \frac{\Delta \boldsymbol{A_{ol}}}{\boldsymbol{A_{ol2}}} = \frac{\mathbf{1}}{\mathbf{1} + \mathbf{2600}(\frac{\mathbf{1}}{\mathbf{21}})} \frac{\mathbf{200}}{\mathbf{2600}}$$

$$= \mathbf{6.16 \cdot 10^{-4}}$$

$$\frac{\Delta \boldsymbol{A_f}}{\boldsymbol{A_f}} = \frac{\mathbf{1}}{\mathbf{2}} \left(\frac{\Delta \boldsymbol{A_f}}{\boldsymbol{A_{f1}}} + \frac{\Delta \boldsymbol{A_f}}{\boldsymbol{A_{f2}}} \right) = \mathbf{6.7 \cdot 10^{-4}}$$

6. We found that A_f can be approximated by $1/\beta$. The error made in making this assumption can be defined as the *fractional gain error (FGE):*

$$FGE = \left(\frac{1}{\beta} - \frac{A_{ol}}{(1+A_{ol}.\beta)}\right) \Big/ \frac{A_{ol}}{(1+A_{ol}.\beta)}$$

Calculate the FGE if $\beta = 1/21$ and $A_{ol} = 2400$ as a percentage.

FGE=0.874%.

7. The feedback network is normally made up of resistors. These will have a finite tolerance, for example 1%. As a result, the value of β will be unknown to some tolerance as well. Suppose β lies in the range 0.100 ± 0.002. What is the range of A_f expected if A_{ol} is 1600. Can you use calculus to find the answer?

$$\boldsymbol{A_{f+} = \frac{1600}{1 + 1600 \cdot 0.102} = 9.744}$$

$$\boldsymbol{A_{f-} = \frac{1600}{1 + 1600 \cdot 0.098} = 10.139}$$

$$\boldsymbol{A_{f} = \frac{1600}{1 + 1600 \cdot 0.100} = 9.94}$$

Thus, $9.744 < A_f < 10.139$, i.e. $A_f = 9.94 \pm 0.1975$

$$\boldsymbol{\frac{\Delta A_f}{A_f} = \frac{\pm 0.1975}{9.9415} \cdot 100\% = \pm 1.99\%}$$

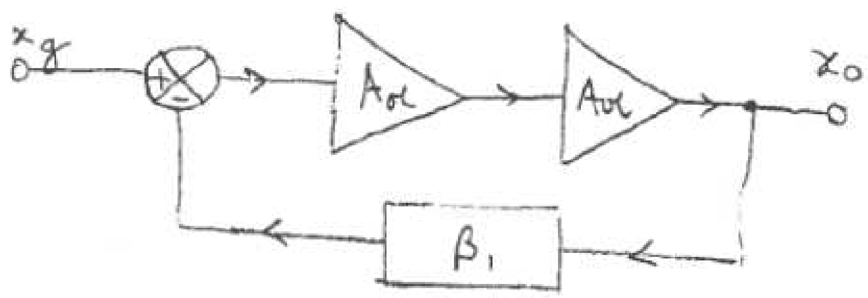
$$\boldsymbol{\frac{\Delta \beta}{\beta} = \frac{\pm 0.002}{0.1} \cdot 100\% = \pm 2\%}$$

8. Calculate the overall gains with feedback x_o/x_g for the circuits shown in fig. P3.4. Show that if the overall gains of these circuits are to be the same, then

$$\beta_1 = \frac{2.\beta_2}{A_{ol}} + \beta_2^2$$

Now calculate the overall gains on the assumption that A_{ol} is extremely large. Show that with this approximation the gains are equal when $\beta_1 \approx (\beta_2)^2$. If the overall gain of both circuits is to be 10 with Aol = 100, calculate suitable values for β_1 and β_2.

Fig P3.4



$$x_o = A_{ol}x'_{in} = A_{ol} \cdot A_{ol}x_{in} = A_{ol}^2 x_{in} \qquad (1)$$

$$x_{in} = x_g - x_f = x_g - \beta_1 x_o \qquad (2)$$

Substituting (2) in (1),

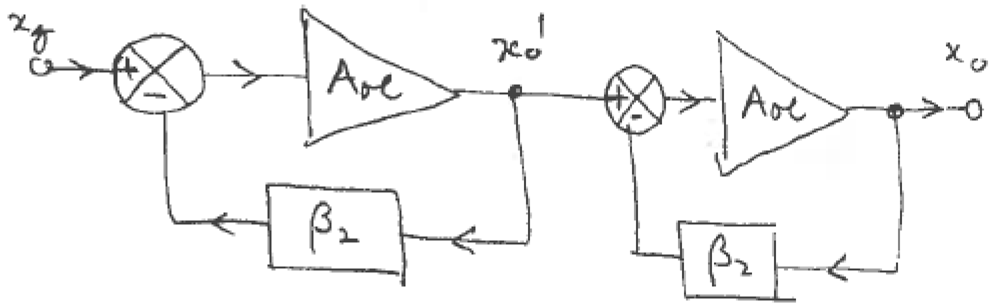
$$x_o = A_{ol}^2 \cdot [x_g - \beta_1 x_o]$$

and rearranging:

$$\frac{x_o}{x_g} = \frac{A_{ol}^2}{1+\beta_1 A_{ol}^2} \qquad (3)$$

If $A_{ol} \to \infty$, then $\frac{x_o}{x_g} = \frac{1}{\beta_1}$.

Now, let's look at the 2nd circuit:



$$x_o = A_{ol}x_{in2} = A_{ol}[x_o' - \beta_2 x_o] \qquad (4)$$

$$x_{in2} = x_o' - \beta_2 x_o \qquad (5)$$

$$x_o' = A_{ol}x_{in1} \qquad (6)$$

$$x_{in1} = x_g - \beta_2 x_o' \qquad (7)$$

Substituting (7) in (6) gives:

$$x_o' = \frac{A_{ol}}{1+\beta_2 A_{ol}} x_g \qquad (8)$$

Substituting (8) in (4) gives:

$$\frac{x_o}{x_g} = \frac{A_{ol}^2}{(1+\beta_2 A_{ol})^2} \qquad (9)$$

If $A_{ol} \to \infty$, then $\frac{x_o}{x_g} = \frac{1}{\beta_2^2}$

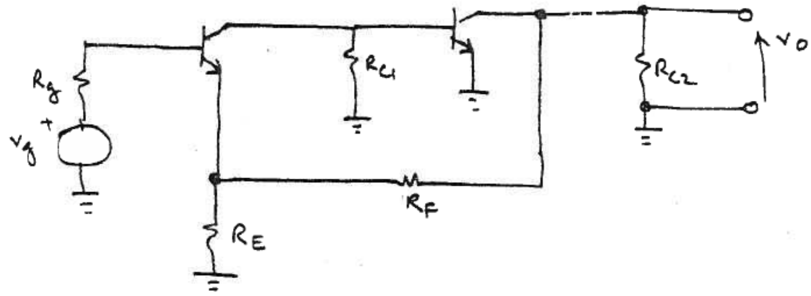
By equating (3) and (9), the following relation is obtained:

$$\beta_1 = \frac{2\beta_2}{A_{ol}} + \beta_2^2$$

For given values from Eq. (3): β_1=0.1, and from Eq. (9) β_2=0.306.

PROBLEMS II

3. The partial a.c equivalent circuit of an amplifier is shown below:



Calculate the overall gain (closed loop gain) $\frac{v_o}{v_g}$ assuming loop gain is very large.

A_{ol} is very large, thus $T = \beta A_{ol}$ is very large and the closed loop gain is

$$A_v \sim \frac{1}{\beta_v}$$

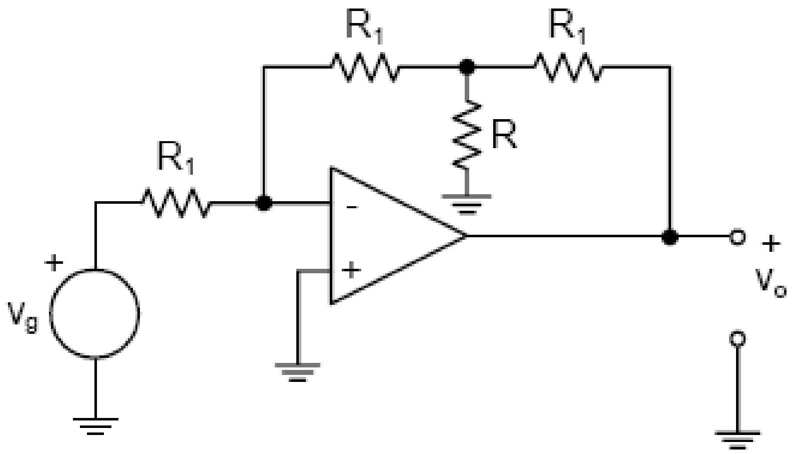
So, we need to find the feedback fraction $\beta_v = v_f/v_o$.

$$v_f = v_o \frac{R_E}{R_E + R_F}$$

$$A_v \cong \frac{R_E + R_F}{R_E}$$

PROBLEMS III

6. Find R such that $\dfrac{v_o}{v_g}$ = -100 when R_1 = 10k



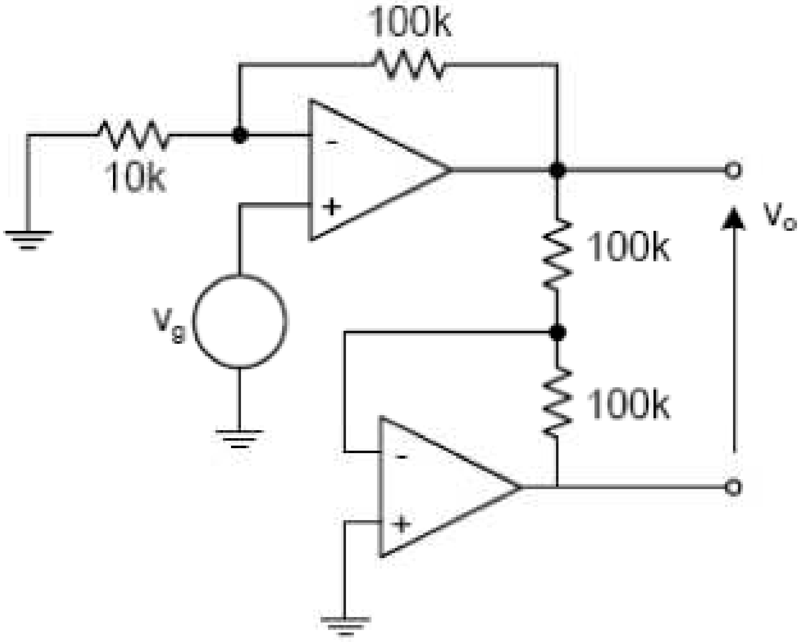
$$\frac{V_g - 0}{R_1} = \frac{0 - V}{R_1} \quad \text{so } V = -V_g \quad (1)$$

$$\frac{0 - V}{R_1} = \frac{V}{R} + \frac{V - V_o}{R_1}$$

$$\frac{V_o}{V_g} = -(2 + \frac{R_1}{R})$$

Hence R=102 Ω.

7. Find $\frac{v_o}{v_g}$ for



Label o/p of top op-amp as v and bottom as v'

Then $\mathbf{v_o = v - v'}$. Also, for top op-amp, $\mathbf{v_n = v_p = v_g}$

Then, for top op-amp: $\frac{0 - v_g}{10k} = \frac{v_g - v}{100k}$ giving $\frac{v}{v_g} = 11$

For bottom op-amp, $v_n = v_p = 0$

so $\frac{v - 0}{100k} = \frac{0 - v'}{100k}$ giving $v = -v'$

so, $v_o = v + v = 2v = 22v_g$ giving $\frac{v_o}{v_g} = 22$

*Note that $\frac{v'}{v_g} = -11$