Revision

D.C. considerations

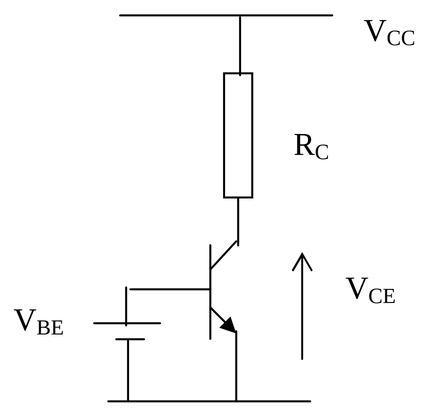
The collector current of an ideal bipolar transistor is given by

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \qquad \text{where } V_T = kT/q = 25 \text{ mV @ 300K}$$

[This is a very important equation and should be remembered. It states that the collector current is dependent *only on the emitter-base bias* and not on the collector bias – recall that transistor output characteristics are very flat. In reality, there is a slight dependence of I_C on V_{CE} and this arises from the 'Early Effect', that is, I_S has a slight collector bias dependence.]

Exercise: Consider the circuit opposite where the transistor has infinite output resistance. (flat output characteristic). Assume that the transistor is in the active regime of operation. What do you expect to happen to the collector current (I_C) if

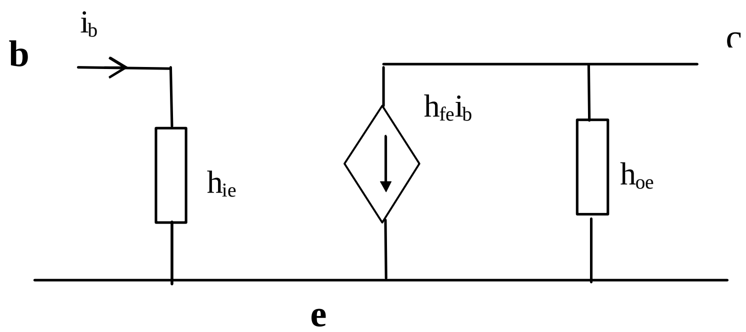
a) R_C is increased
b) V_{CC} is increased
c) V_{BE} is increased



What happens to V_{CE} in each case?

A.C. Model

The model used in year 1 was the 'h' parameter model which was derived by considering the transistor as a two port network to give:



Recall that the input resistance, h_{ie} and a.c. current gain, h_{fe} depend on the d.c. bias level, I_C. Note that this model works at low frequencies only. Using this model, amplifiers can be designed for given specifications. This year, we seek a more accurate model.

IMPORTANT: IN PREVIOUS YEARS, WE HAVE IDENTIFIED TWO MAIN CONCEPTS THAT STUDENTS FIND DIFFICULT TO UNDERSTAND.

If you can spend some time to really understand these two concepts by BACKGROUND READING and STUDY, you will find the module much easier to follow.

DIFFICULT CONCEPT 1: The small signal ac equivalent circuit

Q: What is the meaning of ‘small signal’ and ‘equivalent circuit?

A: Electronic circuits require dc bias to be applied so that the active devices (transistors) operate in thecorrect part of their characteristics. However, the circuits often operate with ac signal levels (amplitudes) that are small compared to the bias currents Transistors are very **non-linear** devices so full analysis requires the use of complex computer models

We need the ability to analyse the circuits for the ac properties that constitute the purpose of the circuit – eg an amplifier, a filter etc etc

In such circumstances we reduce the full schematic circuit to a *small signal equivalent* – a simple LINEAR circuit (a combination of resistors, capacitors, dependent current sources) that is ‘easy’ to analyse. Using this approach we can estimate the voltage and current gain, input and output resistance and other important properties of the circuit

By this means we can get a ‘first cut’ design – we may need to then use a full computer model to optimise our design to produce a working prototype.

DIFFICULT CONCEPT 2: Current sources

A) DC, Independent current sources

An ideal, independent current source has **infinite resistance.** We will use current sources to provide the dc bias for circuits. A.c signals would not pass through such an energy source – they would see an **open circuit**. Of course in reality, we cannot make ideal things and we represent the ‘losses’ by a parallel resistance R_o across our ideal dc current source. This allows ac to pass but if we can make good current sources, R_o is very big so the ac current ‘lost’ is very small. We can make good dc current sources with a transistor and some resistors (see later lectures).

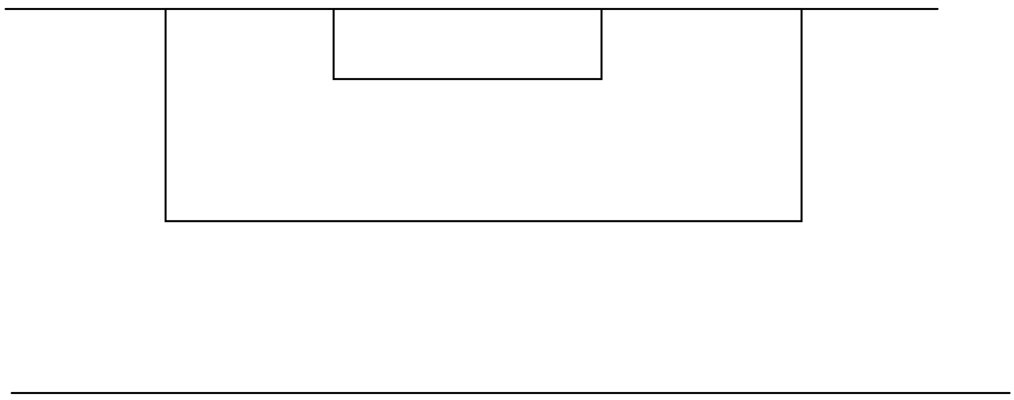
B) AC, dependent current sources

The other type of current source we have seen is in the ac equivalent circuit itself. This is a current source that depends on some other parameter – ie h_{fe} x i_{in} where h_{fe} is the transistor ac gain and i_{in} is the ac current into the base. Sometimes we label this current source as $\beta_o i_{in}$ where $\beta_o = h_{fe}$; or $g_m v_{be}$ (see later). We use whichever form is most convenient but note that all are dependent sources.

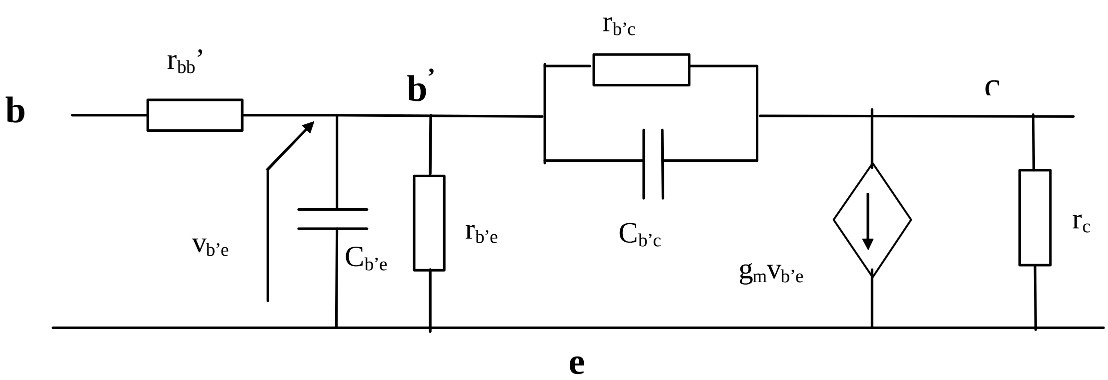
NOTE THAT THE DEPENDENT CURRENT SOURCE DOES NOT HAVE INFINITE **<u>RESISTANCE - IT IS DIFFERENT FROM THE INDEPENDENT CURRENT SOURCE</u>**

High frequency model for the bipolar transistor: hybrid-π model

The figure below shows a cross section of a bipolar junction transistor (BJT). In order to design amplifiers etc., we need an engineering model of this physical structure and the easiest way to proceed is to relate important regions of the device to appropriate circuit elements. Semiconductor regions could be represented by resistances, p-n junction depletion regions by capacitances etc.



In most cases, we can see a fairly clear relationship between the physical structure and the model shown below. Note that **e, b, c** refer to the device external terminals.



The need for a current source element between collector and emitter is evident by looking at the output characteristics which are seen to be flat, that is constant current with collector-emitter bias. The model parameters are summarised in the table overleaf.

PARAMETER	PHYSICAL DEFINITION
$\mathbf{r_{bb'}}$	*ohmic* resistance of the semiconductor bulk from external base contact to the intrinsic transistor region.
$\mathbf{r_{b'e}}$	*dynamic* resistance of the emitter-base junction ($\Delta V_{BE}/ \Delta I_B = v_{be}/i_b$)
$\mathbf{r_{ce}}$	*dynamic* resistance of the transistor output characteristic ($\Delta V_{CE}/ \Delta I_C = v_{ce}/i_c$)
$\mathbf{r_{b'c}}$	Takes care of feedback. Will always be assumed very large and hence ignored.
$\mathbf{C_{b'e}}$	differential capacitance of the forward biased emitter-base junction comprising the sum of the depletion and diffusion capacitances. Also written as $\mathbf{C_e}$
$\mathbf{C_{b'c}}$	differential capacitance of the reverse biased collector-base junction (capacitance of the depletion region). Also written as $\mathbf{C_C}$
$\mathbf{g_m}$	transistor transconductance ($\Delta I_C/\Delta V_{BE} = 40\ I_C$)

Derivation of parameters from transistor device physics

The collector current of a bipolar transistor is given by (ref: year 1 Integrated Electronics)

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right) \qquad \text{where } V_T = kT/q = 25 \text{ mV @ 300K}$$

We can derive other parameters from this equation.

Transconductance

$$g_m = \frac{\Delta I_C}{\Delta V_{BE}} = \frac{I_S}{V_T}\exp\left(\frac{V_{BE}}{V_T}\right) = \frac{I_C}{V_T}$$

hence **$g_m = 40\ I_C$ Amps/V** **(IMPORTANT – REMEMBER)**

(N.B $r_e \equiv g_m^{-1}$)

Input dynamic resistance

$$r_{b'e} = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{\Delta V_{BE}}{\Delta I_C}\frac{\Delta I_C}{\Delta I_B} = \frac{\beta_o}{g_m}$$

i.e., **$r_{b'e} = \beta_o /(40\ I_C) = \beta_o/g_m$** **(IMPORTANT – REMEMBER)**

(N.B $r_{be} = \beta_o\ r_e$)

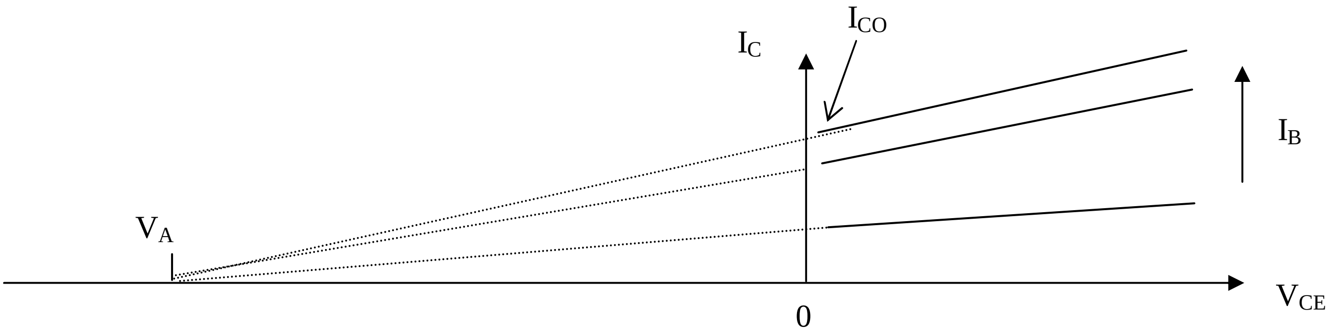
Note that this *dynamic* resistance depends on the d.c. or bias current level unlike the *ohmic* component $\mathbf{r_{bb'}}$ which is independent of current at least for moderately low current densities.

[$r_{bb'}$ becomes dependent on current level at high current densities due to the effect of *conductivity modulation* but this effect is beyond the scope of this course.]

Note also that β_o is the same as the h-parameter, h_{fe}.

Output dynamic resistance, r_{ce}

Consider a real transistor output characteristic (only active regime shown). Note that there is slope on the characteristic due to the Early Effect (see text books for an explanation).



Extrapolating the currents back to zero as shown, we find that the characteristics pass through a common point: the 'Early voltage', V_A. (named after J.M. Early who first explained the effect)

Can represent the slope on the output characteristic, as:

$$I_C = I_S \exp\left(\frac{V_{BE}}{V_T}\right)\left[1+\frac{V_{CE}}{V_A}\right] \qquad (*)$$

Typical values: V_A (npn) ≈ 150 V
V_A (pnp) ≈ 50 V (note that pnp's are inferior devices)

Now to find r_{ce}, write (*) as

$$I_C = I_{CO}\left[1+\frac{V_{CE}}{V_A}\right]$$

where I_{CO} is the d.c. bias current level of the transistor. Hence

$$r_{ce} = \frac{\Delta V_{CE}}{\Delta I_C} = \frac{V_A}{I_{CO}}$$

or simply write as $r_{ce} = (V_A + V_{CE})/I_C$ with I_C the bias current level.
(note that often, $V_{CE} << V_A$, so we can estimate r_{ce} as ~ V_A / I_C)

This is an important equation which states that the output resistance of the transistor *increases* (generally a good thing) as the bias current is reduced.

e.g. I_C = 1 mA, r_{ce}(npn) = 150 kΩ
r_{ce}(pnp) = 50 kΩ
I_C = 10 μA, r_{ce}(npn) = 15 MΩ
r_{ce}(pnp) = 5 MΩ

Input capacitance

$C_{b'e}$ (also written as C_e) comprises the capacitance of the depletion region at the emitter-base pn junction plus the capacitance associated with the free charge which is carrying the current (electrons for an n-p-n transistor). The free charge or *diffusion* component can be found as

$$C_{diffusion} = \frac{dQ_B}{dV_{BE}}$$

which simply defines the change of base charge associated with the free carriers, with emitter-base voltage. Now charge is related to current *I*, and time *t*, thus

$$Q_B = I_C \tau_F$$

where the time is interpreted as an average time for the carriers to pass through the base, that is $t = \tau_F$: therefore we can write,

$$C_{diffusion} = \tau_F \frac{dI_C}{dV_{BE}} = \frac{I_C \tau_F}{V_T} \approx C_{b'e}$$

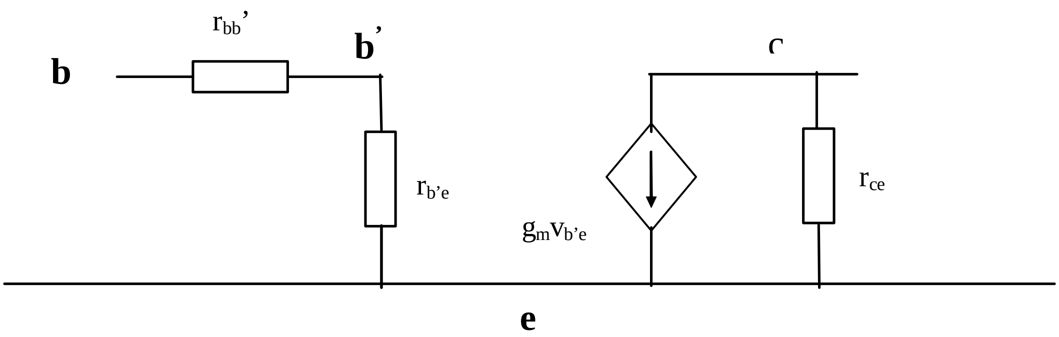
where we have differentiated Eqn. 1. Thus the emitter-base capacitance is not fixed but is dependent on the bias (collector) curent level. We will show later that :

$$C_{b'c} + C_{b'e} \approx C_{b'e} \approx \frac{g_m}{2\pi f_T}$$

where f_T is the frequency at which the common emitter current gain is unity; f_T is an important parameter for a transistor as it defines the ***bandwidth*** of the transistor and will be investigated later in the course.

Final notes

1. At low frequencies, the hyprid-pi model reduces to:



Note the similarity to the h-parameter model (ignoring $r_{b'c}$) as

$g_m v_{b'e} = h_{fe} i_b \Rightarrow h_{fe} = g_m r_{b'e}$
and $r_{bb'} + r_{b'e} = h_{ie}$.

2. Often in the course, we will assume $r_{bb}' = 0$ and r_{ce} = infinity; so both can be removed.

Exercises

1. Calculate g_m, r_e and r_{be} when $I_C = 10\ \mu A$ and when $I_C = 0.25mA$. Comment on the values ($\beta_o = 200$)
2. Why is the small-signal equivalent circuit of a battery (d.c. voltage source) taken to be a short circuit?
3. What would be the small signal equivalent circuit of an ideal current source?
4. What are the equivalent circuit parameters for a transistor whose $\beta_o = 200$ and whose Early voltage is 150V when it is operating at $I_C = 0.1mA$ and $V_{CE} = 5V$?
5. Calculate the values of I_C, I_E and I_B of a transistor in the common emitter configuration, when $V_{CE} = 8V$ and $V_{BE} = 0.63V$; given that $I_S = 10^{-13}$ A at $V_{CE} = 5V$, $\beta = 200$ and VA = 150V.
6. What is the output resistance of the transistor at the operating point in 5 above?