PART 2: Basic bipolar transistor amplifier configurations

Having established our model for the bipolar transistor, we are in a position to analyse the basic amplifier configurations.

1. Common emitter (CE)
2. Common emitter with emitter degradation (CE-ED)
3. Common collector (CC) also known as emitter follower (EF)
4. Common base (CB)

in terms of voltage and current gains, input and output resistance etc. We will see that each configuration has particular properties that we can usefully employ when building up complex electronic circuits.

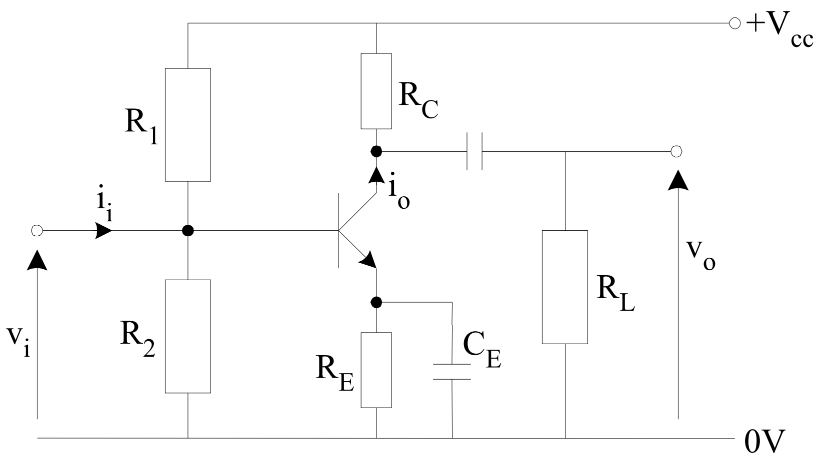
The following pages show the circuit schematic diagrams for each configuration and the associated a.c. equivalent circuit for each is shown underneath. We will go through the analyses for CE and CC in the lecture period, to illustrate the basic analytical techniques. The analyses for the other two configurations (CE-ED, CB) are left for you to try.

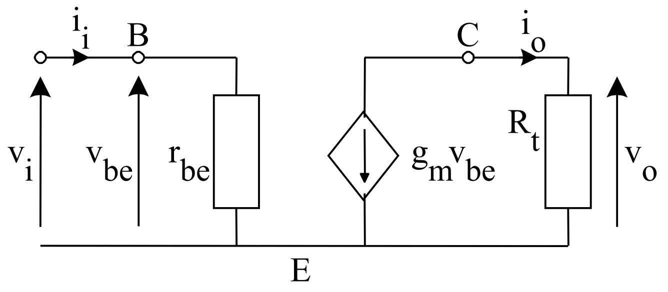
We will also go through examples of two and three stage amplifiers and the diagrams are given towards the end of this handout.

Golden Rules for drawing equivalent circuits

1. Draw the equivalent circuit of the transistor first
2. Identify the a.c. grounds (voltage sources look like short circuits for a.c. currents!)
3. Convert capacitors to short circuits (we are interested in ‘mid-frequency’ regimes where the impedance of coupling/de-coupling capacitors is zero
4. Add the other circuit components (resistors)to obtain the complete equivalent circuit.

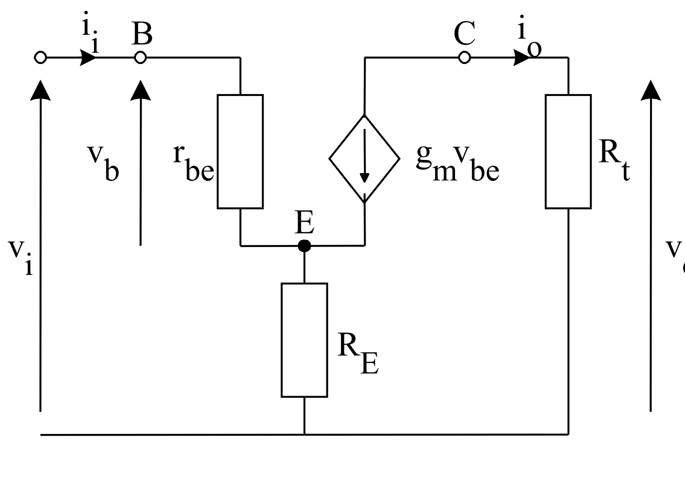
CE





(analysed overleaf)

CE - ED



Remove C_E in CE circuit above

(for analysis see exercise 2 – you do this one!)

COMMON EMITTER AMPLIFIER

Assume amp. biased to some current level I_C (see 1st year notes or equivalent). Knowing I_C, can calculate g_m and r_{be} ; $\beta_O (\equiv h_{fe})$ obtained from manufacturers specification sheet.

Simplified hybrid - π equivalent circuit

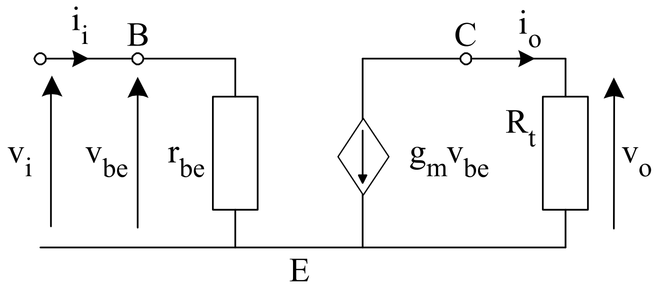


Fig.1

Voltage Gain, A_V:

$$A_V = \frac{v_o}{v_i} = \frac{v_o}{v_{be}}$$

$$v_o = i_o R_t = -g_m v_{be} R_t$$

so $\underline{A_V = -g_m R_t}$

Include effect of $r_{bb'}$ and source resistance, R_S :

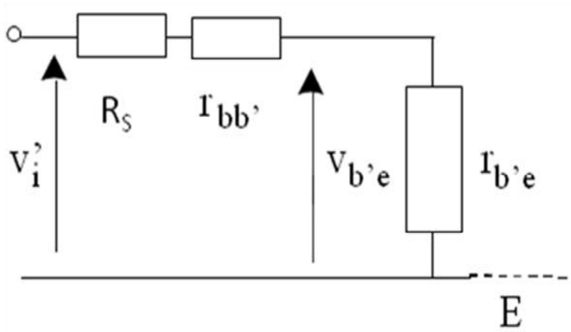


Fig.2

$$A_V = \frac{v_o}{v_i'} = \frac{v_o}{v_{b'e}} \frac{v_{b'e}}{v_i'}$$

$$v_{b'e} = v_i' \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s}$$

$$\underline{A_V' = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s}}$$

Note the degradation of voltage gain

Full treatment – include effect of bias resistors

R = R₁ // R₂

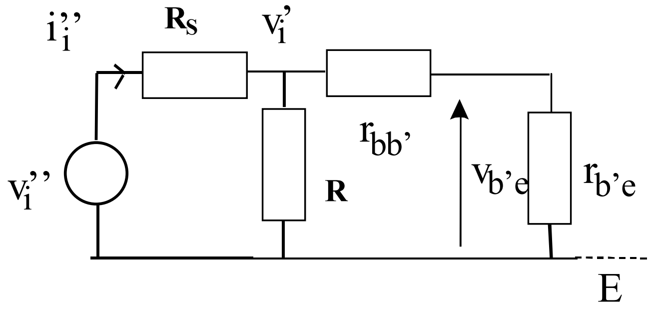


Fig.3

$$A_V'' = \frac{v_o}{v_i''} = \frac{v_o}{v_{b'e}} \frac{v_{b'e}}{v_i'} \frac{v_i'}{v_i''}$$

$$v_{b'e} = v_i' \frac{r_{b'e}}{r_{b'e} + r_{bb'}}$$

$$v_i' = v_i'' \frac{R // (r_{bb'} + r_{b'e})}{R_S + R // (r_{b'e} + r_{bb'})}$$

$$A_V'' = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'}} \frac{R // (r_{bb'} + r_{b'e})}{R_S + R // (r_{b'e} + r_{bb'})}$$

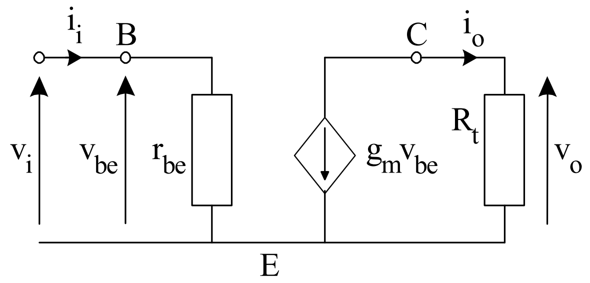
Input resistance (see Fig.1) $R_i = \frac{v_i}{i_i} = r_{b'e}$ **include r_{bb}':** $R_i = r_{b'e} + r_{bb}'$ (see Fig.2)

easy to include the effect of R (bias resistors) (see Fig.3): $R_i' = \frac{v_i'}{i_i'} = R_S + R \,//\, (r_{b'e} + r_{bb}')$

usually make R >> $(r_{b'e} + r_{bb}')$ - so signal goes into the transistor not through the bias resistors!!

Current gain $A_i = \frac{i_o}{i_i}$

We can write:



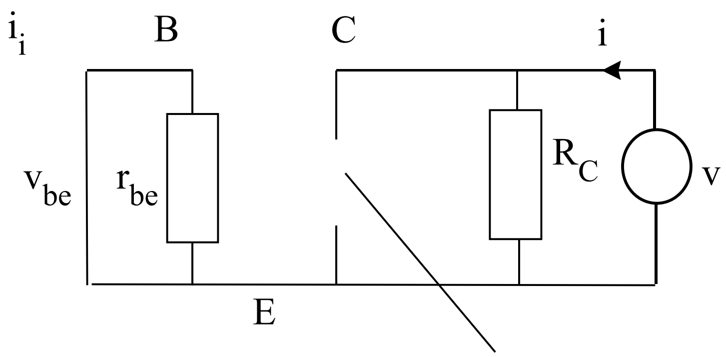
$i_o = -g_m v_{be}$

$v_{be} = i_i r_{be}$

so $A_i = \frac{i_o}{i_i} = -g_m r_{be} = -\beta_o$ (**Q:** what is the effect of including r_{bb}'??)

Output resistance:

Technique:
1. Set i/p voltage = 0
2. Let $R_L \rightarrow$ infinity
3. Drive o/p terminal by a volt source, v



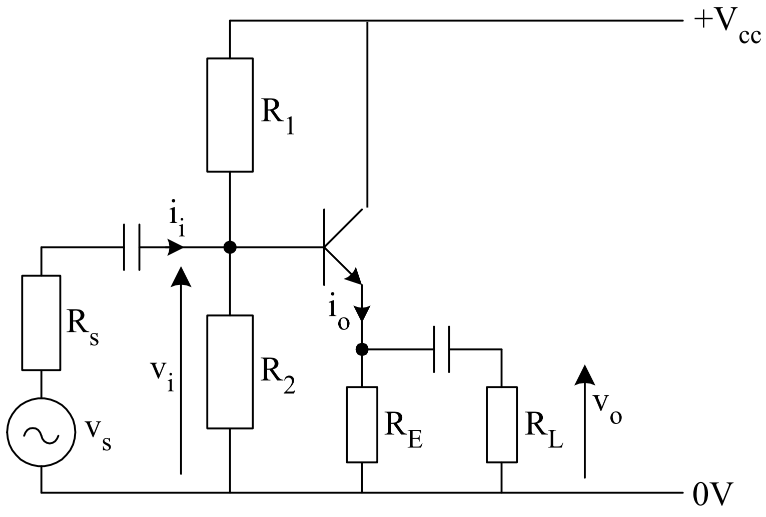
Then output resistance, $\boxed{R_o \equiv \frac{v}{i} = R_C}$ **V_{be} = 0, so no current**

So here, $R_o = R_C$

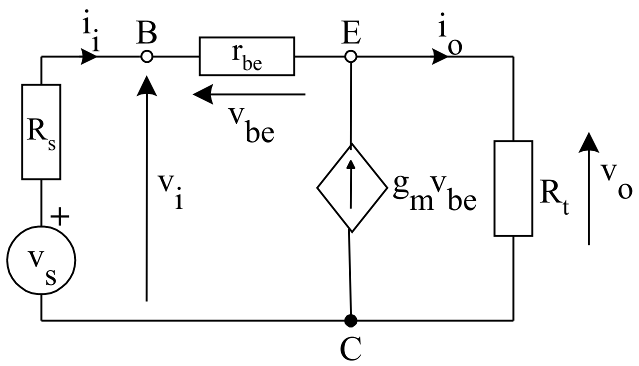
Including the effect of r_{ce} : $R_o = R_C \,//\, r_{ce}$

COMMON COLLECTOR (emitter-follower) AMPLIFIER

The schematic circuit.



The a.c., small signal equivalent circuit



$R_t = R_E // R_L$

Assume $R_1 // R_2$ very large

Q: what is meant by 'small signal'? (see earlier notes) Does it mean 'small amplitude' or 'small frequency'?

Voltage Gain

$$v_o = i_o R_t \qquad (1)$$

KCL at node E: $$i_i + g_m v_{be} = i_o = \frac{v_o}{R_t} \qquad (2)$$

also $v_{be} = v_i - v_o$ and $i_i = \dfrac{v_i - v_o}{r_{be}}$ (3)

Substitute for i_i and v_{be} in Eqn.(2): $$\frac{(v_i - v_o)}{r_{be}} + g_m (v_i - v_o) = \frac{v_o}{R_t}$$

Multiply across by r_{be}: $v_i - v_o + \beta_o v_i - \beta_o v_o = v_o \dfrac{r_{be}}{R_t}$ (where $\beta_o = g_m r_{be}$ is used - see part 1 notes)

Rearranging: $v_o + \beta_o v_o + v_o \frac{r_{be}}{R_t} = +v_i + \beta_o v_i$

(note the technique; all the terms involving v_o are gathered on the LHS and those involving v_i on the RHS)

$$v_o\left(1+\beta_o+\frac{r_{be}}{R_t}\right)=v_i(1+\beta_o) \text{ and finally, } A_V \equiv \frac{v_o}{v_i}=\frac{(1+\beta_o)}{1+\beta_o+\frac{r_{be}}{R_t}}$$

Notice straight away that the voltage gain must be less than 1!

Assuming now, that $\beta_o >> 1$,

get an approximate but quite accurate estimate for the voltage gain: $A_V = \frac{v_o}{v_i} = \frac{\beta_o}{\beta_o + \frac{r_{be}}{R_t}}$

Which can also be written as $\boxed{A_V \equiv \frac{v_o}{v_i} = \frac{g_m R_t}{1+g_m R_t}}$ for the common-collector gain.

Q: What use is an amplifier with gain less than 1!

Let's look at the **input resistance**

This is defined as: $R_i \equiv \frac{v_i}{i_i}$

Take a KVL: $v_i = i_i r_{be} + v_o$ (1)

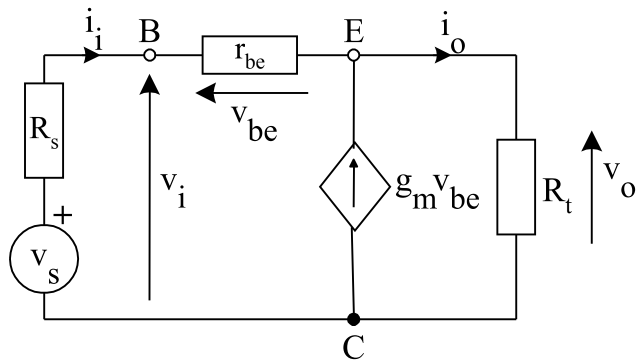
Where $v_o = i_o R_t$ (2)

Now take KCL at node 'E'

$i_o = i_i + g_m v_{be}$

Which can also be written as

$i_o = i_i + \beta_o i_i$ (3)



(Diagram repeated from previous page for clarity)

Sub. (3) into (2): $v_o = i_i R_t(1+\beta_o)$

Sub. into (1): $v_i = i_i r_{be} + i_i R_t(1+\beta_o)$

(note again the technique; we have gathered all the terms involving v_i on the LHS and those involving i_i on the RHS)

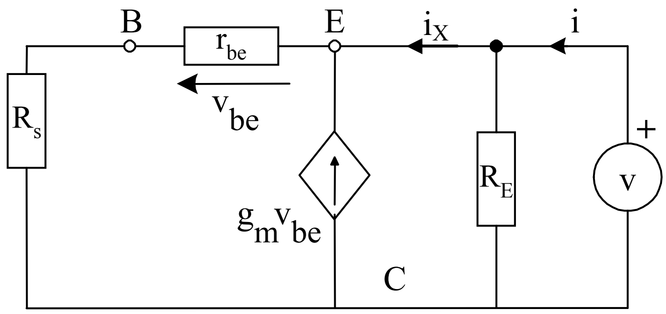
and so: $\boxed{R_i \equiv \frac{v_i}{i_i} = r_{be} + R_t(1+\beta_o)}$

Recall the input resistance of the CE amp (r_{be}). So the input resistance of the CC amp is boosted by the factor $R_t(1 + \beta_o)$! The CC amp has high input resistance!

Q: Is this any use?? Let's also look at the output resistance – a rather long analysis .....

Common Collector, Output resistance

Set up the analysis
Suppress (ie short out), the signal source v_s.
Apply a voltage source to the output – to force a current into the amplifier. The output resistance is then given by $\boxed{R_o = \frac{v}{i}}$. We must find this by appropriate circuit analysis.



We can write $i = \frac{v}{R_E} + i_X$ (1)

Apply KCL: $i_X + \beta_o i_i + i_i = 0$, $\therefore i_X = -(1+\beta_o)i_i$ (2)

Need to get rid of i_i!: write, $v = -i_i(R_s + r_{be})$ (3)

(1) implies, $i = \frac{v}{R_E} - (1+\beta_o)i_i$ which is $i = \frac{v}{R_E} + i_X$ from (2)

Substitute (3) to get, $i = \frac{v}{R_E} + \frac{(1+\beta_o)}{R_S + r_{be}}v$ (4)

(* Nearly there! - we have 'i' on LHS and terms in 'v' on RHS – which is what we wanted!

We can write (4) in the form: $\frac{i}{v} = \frac{1}{R_E} + \frac{1+\beta_0}{R_S + r_{be}}$

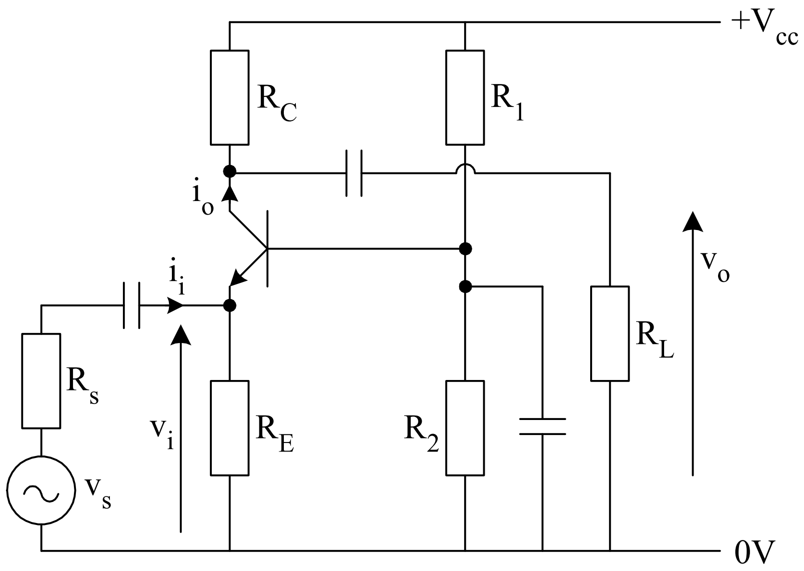
That is, $R_o = \frac{v}{i} = R_E \,//\, \frac{R_S + r_{be}}{1+\beta_o}$ (Note that $\frac{R_S + r_{be}}{1+\beta_o}$ has units of ohms)

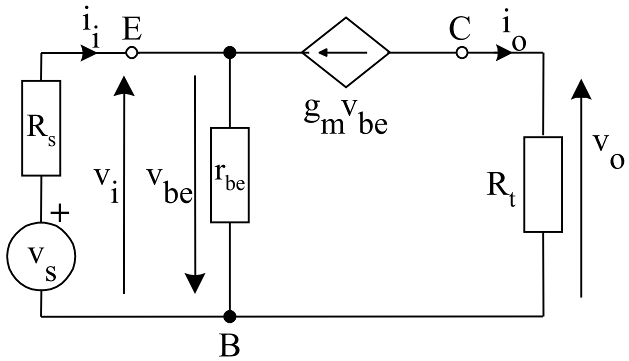
β_o is large, so we can predict that R_o is small…

Conclusions: The CC has large (ish) input resistance and small output resistance – but no voltage gain!

Use it to 'match' high impedance source to low impedance load stages (see 'Design an Op-amp' assignment later in the year.)

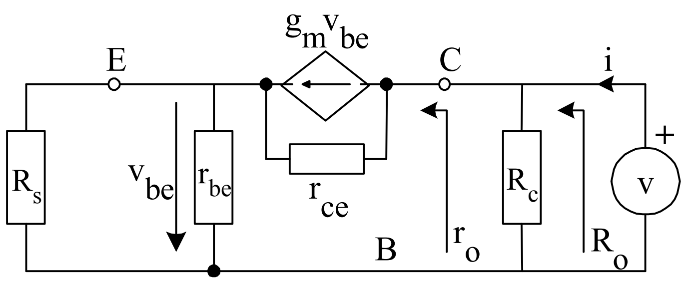
CB





Note

R_E assumed to be much greater than R_{in}

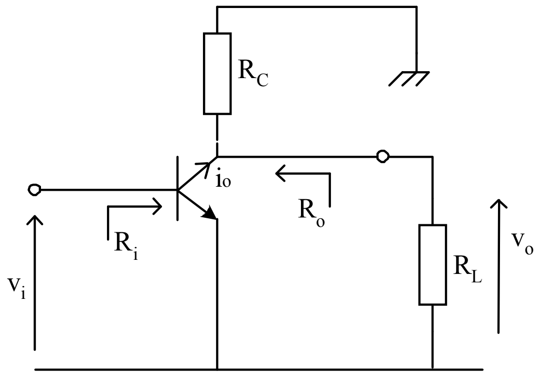


Equivalent circuit for finding output resistance, R_o

(for analysis see exercise 3 – you do this one!)

Glossary of amplifier properties

Common Emitter



$R_i = r_{be}$

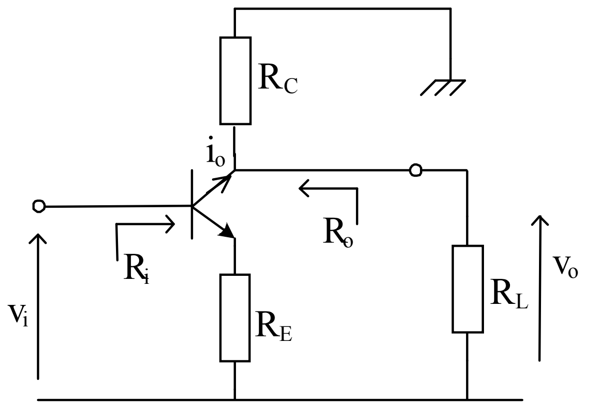
$R_o = R_C$

$$A_V = \frac{v_o}{v_i} = -g_m R_C \,//\, R_L$$

$$G_M = \frac{i_o}{v_i} = -g_m$$

$$R_M = -\beta_o R_C \,//\, R_L$$

Common emitter with emitter degradation



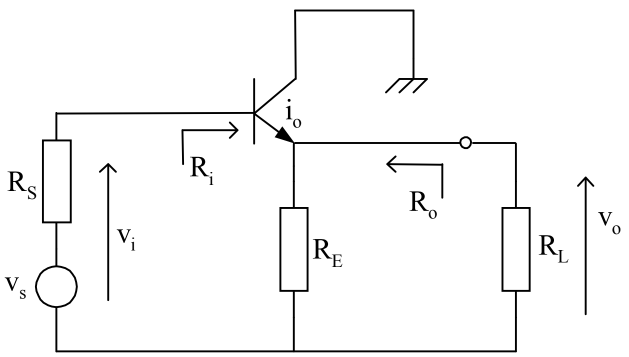
$R_i = r_{be} + (1+\beta_o) R_E$ $\qquad$ $R_o = R_C$

$$A_V = -\frac{g_m R_C \,//\, R_L}{1 + g_m R_E}$$

$$G_M = -\frac{g_m}{1 + g_m R_E}$$

$$R_M = -\beta_o R_C \,//\, R_L$$

Common collector (Emitter follower)



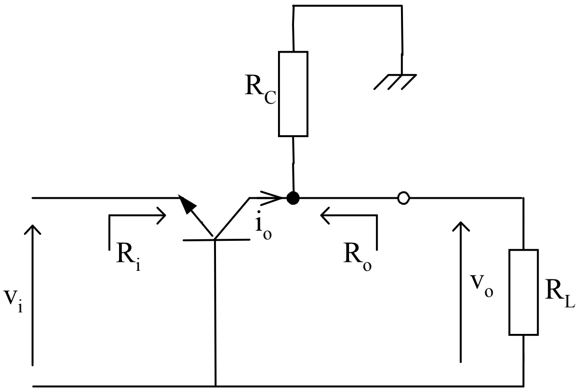
$R_i = r_{be} + (1+\beta_o) R_E \,//\, R_L$

$$R_o = \frac{r_{be} + R_S}{1+\beta_o} \,//\, R_E$$

$$A_V = \frac{g_m R_E \,//\, R_L}{1 + g_m R_E \,//\, R_L}$$

$$G_M = \frac{g_m}{1 + g_m R_E \,//\, R_L}$$

$$R_M = (1+\beta_o) R_E \,//\, R_L$$



$$R_i = \frac{r_{be}}{1+\beta_o} \approx 1/g_m = r_e \qquad R_o \sim R_C$$

$$A_V = g_m R_C // R_L$$

$$G_M = g_m \qquad R_M = \frac{\beta_o R_C // R_L}{1+\beta_o}$$

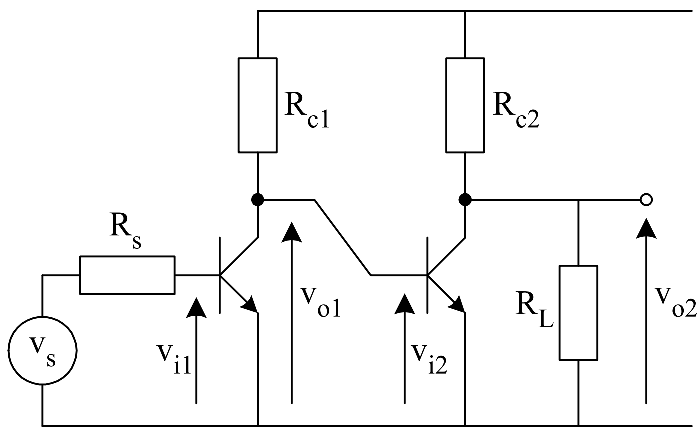
Common base

Summary of properties of amplifier configurations

Property	A_V	A_I	R_{in}	R_{out}	Usage
CE	high	high	Medium	high	Most useful, general purpose
CE-ED	Low	high	high	high	R_E constitutes feedback – sacrifice A_V for increased stability: less dependent on β_o (also temperature), increase R_{in}
CC	Low (< 1)	high	high	low	Impedance matching – high R-source to low-R load (also very linear – used in power amp. Output stages)
CB	high	low (<1)	low	high	Impedance matching – low R-source to high-R load (also features in Diff.Amp. – see later notes)

Multi-stage amplifiers can thus be configured to provide, for example, very high input impedance, high voltage and current gain and low output impedance.

Example of a two stage amplifier



Function: Voltage amplification - both stages have large voltage gain

Overall voltage gain is $A_{Vs} = \dfrac{v_{o2}}{v_s} = \dfrac{v_{o2}}{v_{i2}} \times \dfrac{v_{i2}}{v_{i1}} \times \dfrac{v_{i1}}{v_s}$

But $v_{i2} = v_{o1}$ so:- $A_{Vs} = \dfrac{v_{o2}}{v_{i2}} \times \dfrac{v_{o1}}{v_{i1}} \times \dfrac{v_{i1}}{v_s}$

that is, $A_{Vs} = A_{V2} \times A_{V1} \times \dfrac{R_{i1}}{R_{i1} + R_s}$

'Coupling' term: R_S 'loads' the input

$A_{V2} = -g_{m2} R_L // R_{C2}$

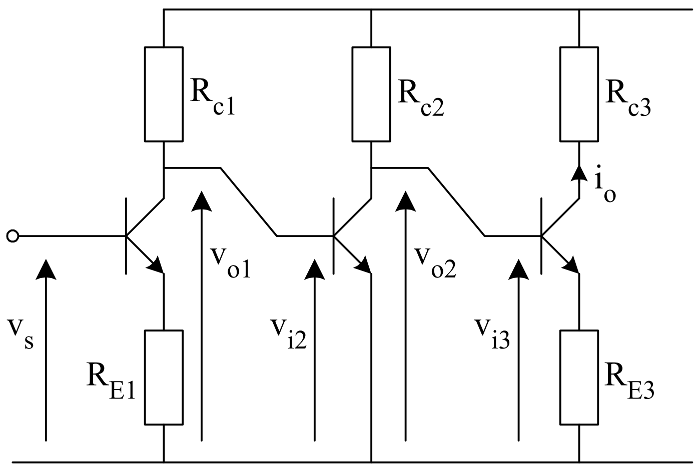
that is, 'g_m x the ac load'

$A_{V1} = -g_{m1} R_{C1} // R_{i2}$

must take account of the loading effect of the second stage on the first (R_{i2})

Note that $R_{i2} = r_{be2}$, $g_{m2} = 40\ I_{C2}$, $R_{i1} = r_{be1}$ ($r_{be} g_m = \beta_o$)

Example of a three stage amplifier



Function: *transconductance* amplifier – turns an input voltage signal v_s into an output current signal i_o with appropriate enlargement (amplification) of the signal. In this case the current is into the collector load R_{C3}. A practical example of the use of such an amplifier could be in driving a light emitting diode (l.e.d) which would replace the resistor. First stage provides for high input resistance and some voltage gain, second stage provides high voltage gain, third stage converts voltage to current.

Overall transconductance gain is $$G_M = \frac{i_o}{v_s} = \frac{i_o}{v_{i3}} \times \frac{v_{i3}}{v_{i2}} \times \frac{v_{i2}}{v_s}$$

That is, $$G_M = \frac{i_o}{v_{i3}} \times \frac{v_{o2}}{v_{i2}} \times \frac{v_{o1}}{v_{i1}}$$

or $$G_M = G_{M3} \times A_{V2} \times A_{V1}$$

$$A_{V1} = -\frac{g_{m1}R_{t1}}{1+g_{m1}R_{E1}}$$

First stage is CE-ED. Must take account of the loading effect of the second stage on the first

$$R_{t1} = R_{C1} // R_{i2}$$

R_{i2} is the input resistance of the 2nd stage; $R_{i2} = r_{be2}$

3rd stage is CE-ED
want the transconductance gain

$$G_{M3} = -\frac{g_{m3}}{1+g_{m3}R_{E3}}$$

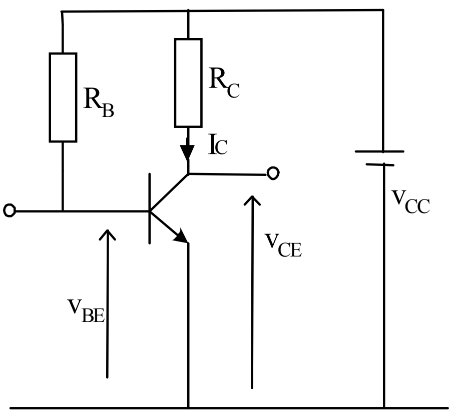
$$A_{V2} = -g_{m2}R_{t2}$$

2nd stage is CE, loaded by stage 3

$$R_{t2} = R_{C2} // R_{i3}$$

Exercises

1. Consider the circuit shown opposite.



   a) What are the disadvantages of the biasing arrangement?
   b) The circuit is to be designed such that V_{CE} is fixed at $\sim V_{CC}/2$. Why would this be desirable?
   c) What is the voltage gain of the circuit?
   d) What is the effect on the voltage gain of increasing R_C subject to the constraint of b)?

2. For the common emitter amplifier with emitter degradation show that:-

a) $A_I = i_o/i_i = -\beta_o$

b) $R_i = v_i/i_i = r_{be} + (1+\beta_o)R_E$

c)

$$A_V = \frac{v_o}{v_i} = \frac{-\beta_o R_t}{r_{be} + (1+\beta_o)R_E}$$
$$\approx \frac{-g_m R_t}{1+g_m R_E}$$
$$\approx -\frac{R_t}{R_E} \text{ if } g_m R_E >> 1$$

d) $R_o \approx R_c$

e)

$$G_M = \frac{i_o}{v_i} = \frac{-\beta_o}{r_{be} + (1+\beta_o)R_E}$$
$$\approx \frac{-g_m}{1+g_m R_E}$$

(NOTE: this is the so-called ‘transconductance gain’)

f) $$R_M = \frac{v_o}{i_i} = -\beta_o R_t$$

(NOTE: this is the so-called ‘transresistance gain’)

For circuit see handout.

3. For the common base amplifier show that:-

a) $A_1 = i_o/i_i = \beta_o/(\beta_o + 1)$

b) $R_i = v_i/i_i = r_e = r_{be}/(1 + \beta_o)$

c) $A_V = \dfrac{v_o}{v_i} = g_m R_t;\ R_t = R_c // R_L$

d) $R_o = R_c//r_o$ where $r_o = r_{ce}\,[1 + g_m(r_{be}//R_s)]$

e) $G_M = i_o/v_i = g_m$

e) $R_M = v_o/i_i = \beta_o R_t/(1 + \beta_o)$

For circuit consult handout.

4. In figure 1 below, the biasing components have been omitted. The transistors are identical with $\beta_o = 200$. $I_{c1} = 1$ mA and $I_{c2} = 5$ mA. $R_{c1} = 5$ kΩ, $R_{c2} = 1$ kΩ, $R_{E2} = 150$ Ω, $R_s = 1$ kΩ.
   Calculate g_{m1}, g_{m2}, r_{be1} and r_{be2}.
   Calculate R_i, R_o, $A_v = v_o/v_i$ and $A_{vs} = v_o/v_s$.
   (Ans $R_i = 5$ kΩ, $R_o = 1$ kΩ, $A_v = + 1111$, $A_{vs} = +926$).

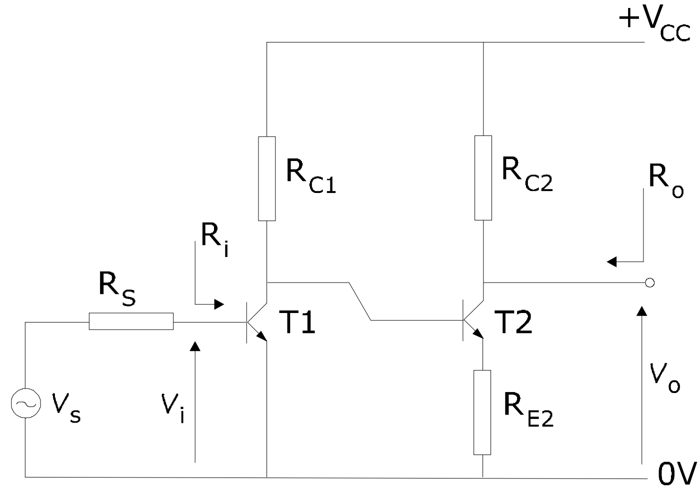


Figure 1

5. In figure 2 the biasing components have been omitted. The transistors are identical with $\beta_o = 99$ and $r_{be} = 2$ k. $R_c = 3$ k, $R_E = 50$ Ω and $R_s = 600$ Ω.
   Calculate the transresistance gain v_o/i_s.
   (Ans - 34.2×10^3 Ω).

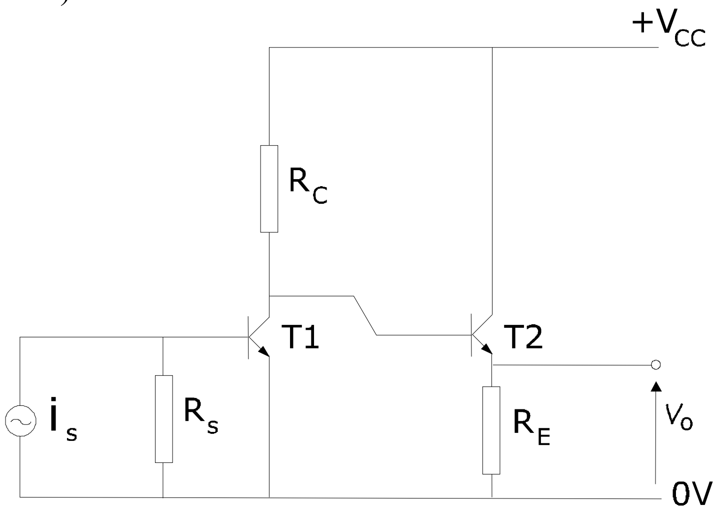


Figure 2

Hint: Apply the chain rule to find the components of the overall gain; you should find

$$R_M = A_{V2} \times A_{V1} \frac{R_S \times r_{be1}}{R_S + r_{be1}}$$