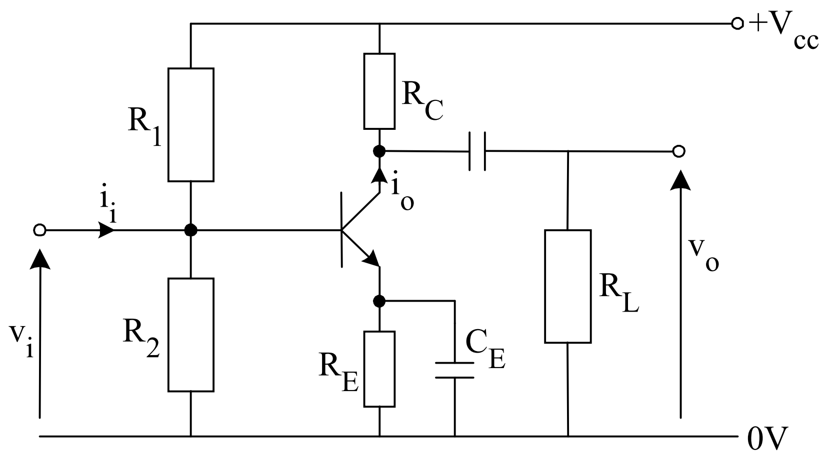
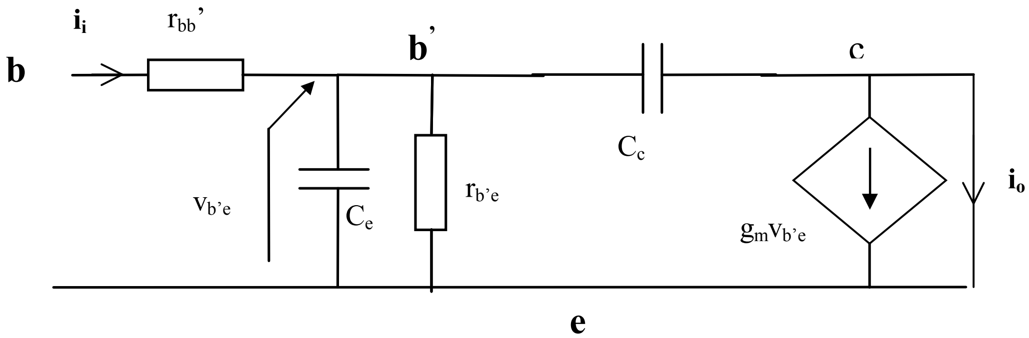
Common Emitter short-circuit, high frequency current gain

We consider a single stage CE amplifier with (conventionally) the load resistor, $R_L = 0$ and use this condition to find the **highest frequency that the amplifier can usefully attain.**

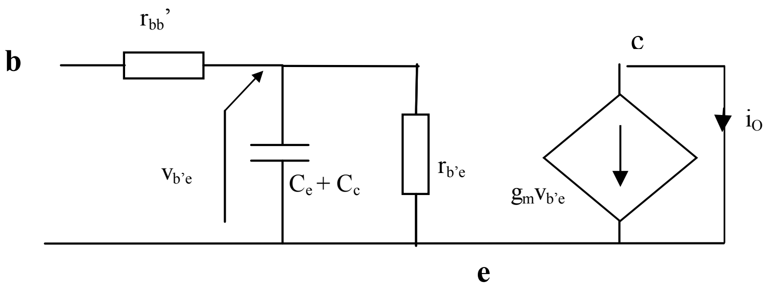


Now proceed to find an expression for the high frequency current gain, i_o/i_i. Consider the hybrid-π equivalent circuit for the amplifier with $R_L = 0$ and bias resistors ignored:

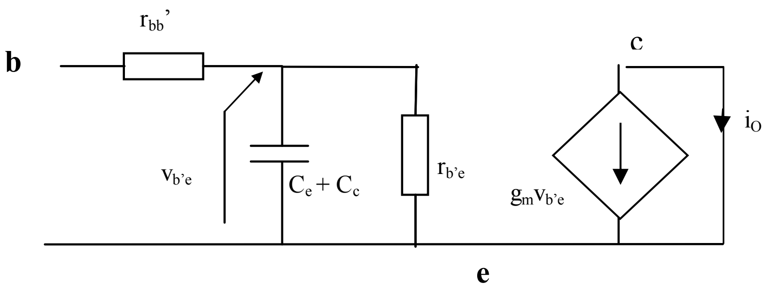


(Note that $r_{b'c}$ and r_{ce} have been assumed to be very large)

The presence of the capacitor C_c complicates the analysis. We will assume that the a.c. current through to the output via C_c is very small and will justify this approximation later on. The equivalent circuit then reduces to:



(repeated diagram):



Notice that the capacitor C_c still draws current in the input circuit and is therefore included in parallel with C_e. We proceed with the analysis:

$i_o = - g_m v_{b'e}$ (1)

$v_{b'e} = i_i (r_{b'e}//X_{e+c})$, that is $v_{b'e} = i_i\left(r_{b'e} // X_C\right) = i_i \dfrac{r_{b'e}/j\omega C}{r_{b'e} + 1/j\omega C}$

where $C = C_e + C_c$, $\omega = 2\pi f$ (f = frequency), X_C is the impedance of C and $j = \sqrt{-1}$

Multiplying top and bottom by $j\omega C$ gives

$$v_{b'e} = \frac{i_i r_{b'e}}{1 + j\omega r_{b'e}(C_e + C_c)} \qquad (2)$$

Substituting (1) into (2) gives the required expression

$$A_i = -\frac{g_m r_{b'e}}{1 + j\omega r_{b'e}(C_e + C_c)}$$

which can also be written:

$$A_i = -\frac{\beta_o}{1 + j\left(f / f_\beta\right)}$$

where $$f_\beta = \frac{1}{2\pi r_{b'e}(C_e + C_c)} \qquad (3)$$

Taking the modulus of this expression gives:

$$|A_i| = \frac{\beta_o}{\left[1 + \left(f / f_\beta\right)^2\right]^{1/2}} \qquad (4)$$

we see that when $f = f_\beta$, the gain is reduced by a factor $1/\sqrt{2}$ or –3 dB. This is therefore the *corner frequency* and f_β defines the BANDWIDTH of the amplifier.

To find the highest frequency at which the amplifer can produce current gain, we set $A_i = 1$ in Eqn. (4) which then becomes:

$$\left[1+\left(f / f_\beta\right)^2\right]^{1/2} = \beta_o$$

Squaring both sides and noting that $\beta_o^2 >> 1$, we find that

$$f_T = \beta_o f_\beta \qquad (5)$$

and f_T is therefore defined as the GAIN-BANDWIDTH PRODUCT for the amplifier. Substituting $\beta_o = g_m r_{b'e}$ and f_β from Eqn. (3):

$$f_T = g_m r_{b'e} \frac{1}{2\pi r_{b'e}\left(C_e + C_c\right)} \qquad \text{hence } f_T = \frac{g_m}{2\pi\left(C_e + C_c\right)}$$

Notice that f_T involves only parameters of the transistor and is therefore the GAIN-BANDWIDTH product of the transistor. It is an important figure of merit for the transistor and can be found on data sheets.

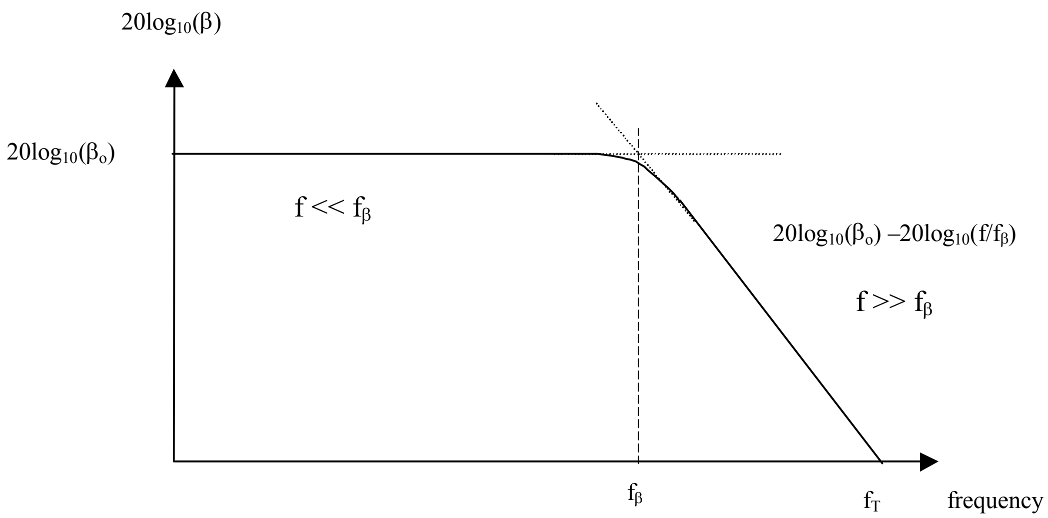
> The equation for f_T serves to indicate the importance of scaling transistors to their smallest dimensions so as to make the internal capacitamnces as small as possible and hence maximise f_T. This is the essence of the 'Moore's Law scaling which has driven the global electronics industry for decades. (see http://www.itrs.net/).

Note:

1. Equation (4) is usually represented on a Bode plot. We write A_i as β (not to be confused with d.c. current gain), take logs to base 10 and write Eqn. (4) as

$$20\log|\beta| = 20\log|\beta_o| - 20\log\left[1+\left(\frac{f}{f_\beta}\right)^2\right]^{1/2}$$

The graphical form of this equation is shown below as a Bode plot. .



2. The current through C_c at the frequency f_T is: $v_{b'e}(2\pi f_T C_c)$. The ratio of this current to that in the output circuit current source $g_m v_{b'e}$ is therefore

$$\frac{2\pi f_T C_c v_{b'e}}{g_m v_{b'e}} = \frac{2\pi f_T C_c}{2\pi f_T (C_e + C_c)} = \frac{C_c}{(C_e + C_c)}$$

as $C_c << C_e$ for bipolar transistors (IMPORTANT – REMEMBER), this ratio is << 1 and this justifies the use of the approximation introduced at the beginning of the analysis.

* It can be noted that mathematically, the argument here is somewhat circular in that the approximation was made and then used to prove that it is valid! It can be claimed however, that the 'analysis' really represents a rough estimate – often used in electronics.
Alternatively, one can estimate the ratio $\omega C/gm$ and put in typical numbers. This should produce an estimate of about 0.1 for this ratio – good enough for our purpose.

3. We recall from the first lecture, that

$C_e \approx I_C\tau/V_T$

where τ is the time for the electrons to move through the transistor (transit time). If we assume $C_c << C_e$, then

$f_T \approx g_m/(2\pi C_e) = f_T \approx \dfrac{g_m}{2\pi C_e} = \dfrac{I_C}{V_T}\dfrac{1}{2\pi C_e}$

Substituting in the expression for C_e, we obtain

$$f_T = \frac{1}{2\pi\tau}$$

that is to say, physically, the gain-bandwidth is a measure of the time that it takes electrons to move through the transistor, from emitter to collector. This is obviously the ultimate limit of transistor speed.