PART 17: Input and output resistances of feedback amplifiers

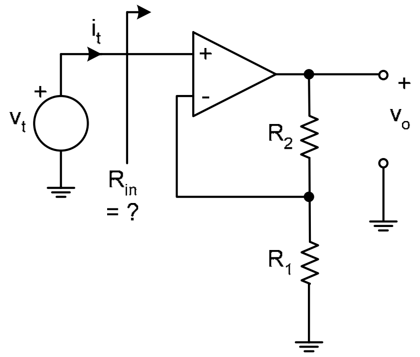
We have frequently stated that
"Negative Feedback derives R_{in} *and* R_0 towards ∞ *or* 0"

Justification – using op-amp as an example

1. Input Resistance of Non Inverting Amp

If we can calculate i_t due to the applied test voltage, v_t then:

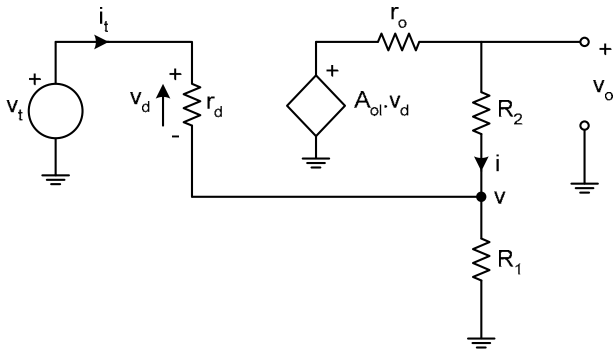
$$R_{in} \equiv \frac{v_t}{i_t}$$



Need to use equivalent circuit of an op-amp

We can write:

$$v_t = i_t\, r_d + v\,, \qquad v_d = i_t\, r_d$$
$$i_t + i = v/R_1\,, \qquad i = \left[A_{ol} v_d - v\right]/\left(r_0 + R_2\right)$$



Some long-winded algebra should give us

$$R_{in} = \frac{v_t}{i_t} = r_d\left[1 + \frac{A_{ol}}{1 + \frac{(R_2 + r_o)}{R_1}}\right] + R_1 \| (R_2 + r_o)$$

Too complicated !

Make sensible **approximations:**

1) $A_{ol} = BIG$
2) 2^{nd} term << 1^{st} term
3) $r_o << R_2$ normally

Then $R_{in} \approx r_d\left[1 + \frac{A_{ol}}{1 + \frac{R_2}{R_1}}\right]$

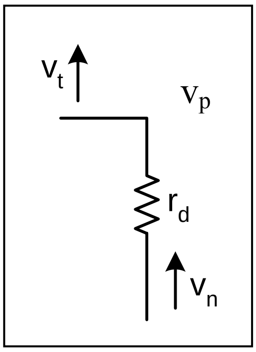
But $\dfrac{A_{ol}}{1 + \frac{R_2}{R_1}} = \dfrac{R_1}{R_1 + R_2} A_{ol} = \beta\, A_{ol} \equiv T(\textit{loop gain})$

$$\boxed{\therefore R_{in} = r_d\,[1 + T]}$$

Conclusion
<u>Without feedback</u>, input resistance of op-amp was r_d.
Application of negative feedback has increased R_{in} by a factor of (1 + T) - this can be very big!

Another picture of effect of feedback on R_{in}

v_n <u>tracks</u> v_p so $v_p - v_n$ is very small.



$\therefore\ i_t = \dfrac{v_p - v_n}{r_d}$ is also very small

But $v_n = \beta . v_o = \beta \dfrac{A_{ol}}{1+A_{ol}\beta} v_t = \dfrac{T}{1+T} v_t$

$\therefore i_t = \dfrac{v_t - \dfrac{T}{1+T} v_t}{r_d}$ since $v_p = v_t$

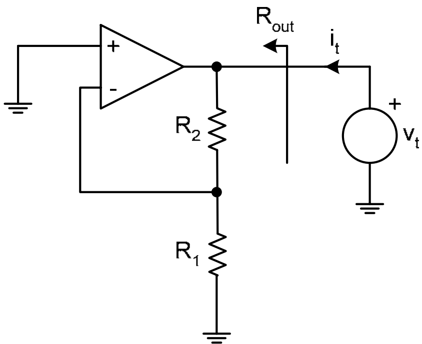
$\rightarrow \dfrac{v_t}{i_t} = R_{in} = r_d\left[1+T\right]$ as before r_d has been **"bootstrapped up"** by feedback

Input voltage applied to top of r_d

Causes **output** of op-amp to increase,
Causes v_n to increase – circuit stops voltage drops across r_d increasing

Output Resistance

Suppress input v_g ; apply a test voltage at output



Then $R_o \equiv \dfrac{v_t}{i_t}$

Use equivalent cct $\rightarrow$

$$R_0 = \frac{r_o}{1+\dfrac{A_{ol}+r_0/R_1+r_o/r_d}{1+\dfrac{R_2}{R_1}+\dfrac{R_2}{r_d}}}$$

With same approximations as before, $\boxed{R_0 \approx \dfrac{r_o}{1+T}}$

Summary
Output impedance of op-amp **without** feedback is r_o. Feedback **reduces** the output resistance by a factor of 1 + T

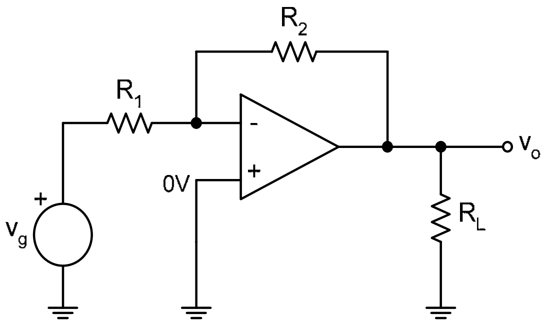
Conclusion
We said before that neg. feedback made R_{in} *and* R_o tend towards ∞ or 0 (open circuit or short circuit) depending on way feedback is connected.

Now we can be more quantitative and give a better estimate for R_{in} and R_0

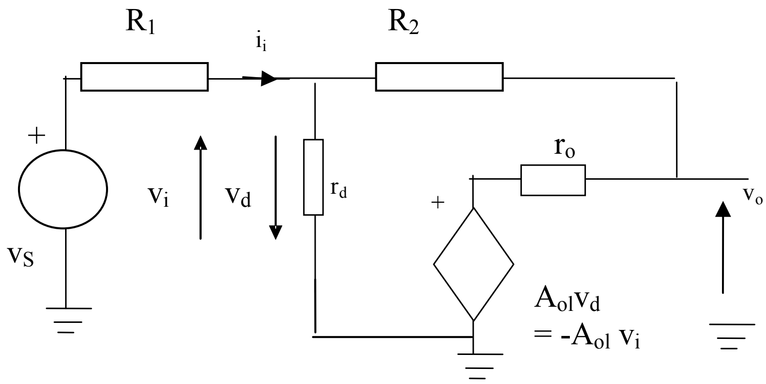
Generally, the **original** values of the input and output resistances of the **amplifier without feedback** are scaled up or down by the factor (1 + T) when feedback is applied. Whether up or down depends on the topology.

Example: What is the input resistance of the inverting amplifier

1) looking into the negative terminal (R_{in}')
2) looking from the source, v_g



Solution: The equivalent circuit for the amplifier is shown below.



1) Now $R_i' \equiv \frac{v_i}{i_i} = -\frac{v_d}{i_i}$ also, $i_i = i_{rd} + i_{R2}$

Applying Ohm's Law,

$$i_i = \frac{v_i}{r_d} + \frac{v_i - (-A_{ol} v_i)}{R_2 + r_o}$$

That is,

$$i_i = \frac{v_i}{r_d} + \frac{v_i(1 + A_{ol})}{R_2 + r_o}$$

Which can be written as:

$$\frac{i_i}{v_i} = \frac{1}{r_d} + \frac{1 + A_{ol}}{R_2 + r_o}$$

which can be expressed as: $R_i' \equiv \frac{v_i}{i_i} = r_d \,//\, \frac{R_2 + r_o}{1 + A_{ol}}$

Take representative values: R_2 = 10k, R_1 = 1k (gain of 10); r_o ~ 50R, $A_{ol} = 10^5$, r_d = 1MΩ
Then $R_i' \sim \frac{R_2}{A_{ol}} = \frac{10k}{10^5}$ **R_i = 0.1Ω which is small - as expected!**

2) looking from the source, we see $R_1 + R_i'$ ~ **R_1 = 1k**
that is, the input resistance of this **trans-resistance** amplifier is set by R_1!