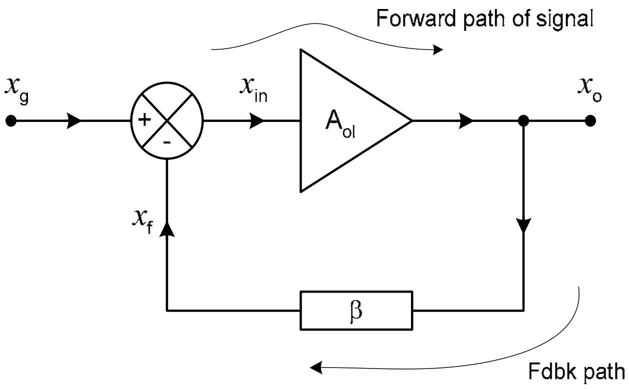
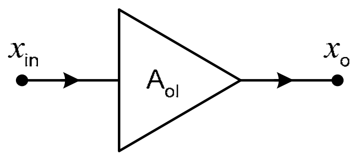
PART 13: Theoretical Matters

Consider block diagram of a negative feedback amplifier system

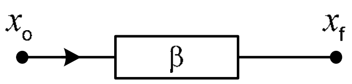


Arrows indicate signal flow direction NOT current !

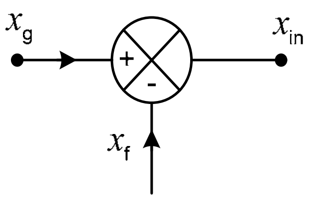
Note also that A_{OL} and β can <u>transform</u> a signal from one type into another so the signal can be changed as it passes through different parts of the system. e.g. x_0 may be a voltage whilst x_g, x_f and x_{in} are currents, whilst in a control system, x_o might be displacement and x_g, x_f and x_{in} a voltage.



≡ <u>High gain amp</u> with " open loop gain" $A_{0l} = \dfrac{x_0}{x_{in}}$



≡ <u>Feedback network</u> with a "feedback fraction" $\beta = \dfrac{x_f}{x_0}$



≡ <u>Input "summer"</u> $x_{in} = x_g - x_f$

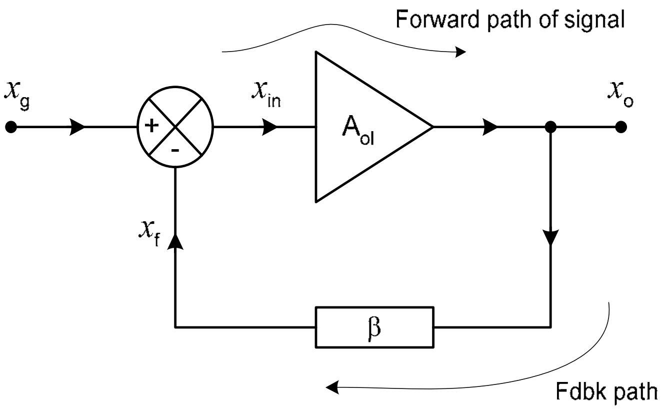
Note: we use "x" because the quantities can be EITHER a voltage or a current

e.g. x_o is either i_o or v_o

x_g is either i_g or v_g
x_{in} is either i_{in} or v_{in}
x_f is either i_f or v_f

Analysis – derivation of the feedback equation

$x_o = A_{ol}\, x_{in}$



$$x_{in} = x_g - x_f$$
$$= x_g - \beta x_o$$

$\therefore x_o = A_{ol}\,(x_g - \beta x_o) = A_{ol}\, x_g - A_{ol}\,\beta\, x_o$

$\therefore x_o + A_{ol}\,\beta x_o = A_{ol}\, x_g$

$x_o\,(1 + A_{ol}\,\beta) = A_{ol}\, x_g$

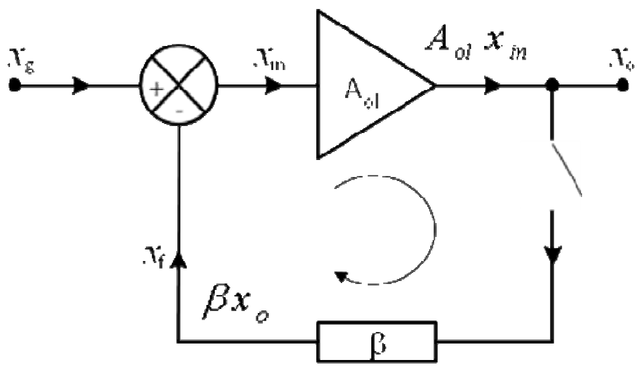
$\therefore \dfrac{x_o}{x_g} = \dfrac{A_{ol}}{1 + A_{ol}\beta} = A_f \leftarrow$ the "overall gain" also known as the 'closed loop gain'

$$\boxed{A_f = \frac{A_{ol}}{1 + A_{ol}\beta}}$$

This is the classic equation of negative feedback theory

Note that the loop being referred to is the feedback loop

If the loop is broken (anywhere) then the gain given by simply going round the loop once = 'loop gain' = $-\beta A_{ol}$ Note the negative sign is because it is negative feedback,



The closed loop gain is the gain of the whole system when the loop is closed = A_{ol}

The open loop gain is the gain of the system when the loop is open (i.e. no with feedback applied) = A_{ol}

In practice, it is important that the bias conditions on the amplifier etc are not disturbed when the loop is broken. Some corrective action may be needed to compensate this.

Application

Suppose we change transistors in an amplifier

Then A_{ol} will change to $A_{ol} + \Delta A_{ol}$

And A_f will change to $A_f + \Delta A_f$ **How big is ΔA_f??**

Proof: Change in A_f due to a change in A_{ol}

We know that $A_f = \dfrac{A_{ol}}{1+A_{ol}\beta}$

Suppose change in A_{ol} is ΔA_{ol}, what is change in A_f ?

Problem can be solved using a bit of calculus provided ΔA_{ol} is small. (an alternative method is shown in problem 5).

First take logs of both sides:-

$$\ln(A_f) = \ln(A_{ol}) - \ln(1 + A_{ol}\beta)$$

Then differentiate both sides with respect to A_{ol} -since we are interested in the effects of changing A_{ol}, and remembering β is a constant gives:

$$\frac{1}{A_f}\frac{dA_f}{d\,A_{ol}} = \frac{1}{A_{ol}} - \frac{\beta}{1+A_{ol}\beta} \equiv \frac{1}{(1+A_{ol}\beta)A_{ol}}$$

Letting ΔA_{ol} approximate to dA_{ol} and ΔA_f approximate to dA_f (OK if they are small!), gives

$$\boxed{\frac{\Delta A_f}{A_f} = \frac{1}{1+A_{ol}\beta}\frac{\Delta A_{ol}}{A_{ol}}}$$

ie % change in A_f is smaller than % change in A_{ol} ! by a factor $1 + A_{ol}\beta$

<u>Example</u> Let $A_{ol} = 10^3$ *and* $\beta = 0.05$

Then $A_f = \dfrac{A_{ol}}{1+A_{ol}\beta} = \dfrac{10^3}{1+10^3 \times 0.05} = 19.61$

Assume A_{ol} changes to 1100, so $\Delta A_{ol} = 100$

Then formula gives $\dfrac{\Delta A_f}{19.61} = \dfrac{1}{1+10^3 \times 0.05} \times \dfrac{100}{10^3}$ $\quad \therefore \Delta A_f = 0.04$

So when A_{ol} changes from $1000 \to 1100$, A_f changes from $19.61 \to 19.65$

ie only **0.2%** increase in A_f, while A_{ol} change was 10%!!

- Negative feedback has stabilised the overall gain of the amplifier against changes in component values of amplifier itself.

Alternative Picture

We can write $A_f = \dfrac{A_{ol}}{1+A_{ol}\beta}$

As $A_f = \dfrac{1}{\beta}\dfrac{A_{ol}\beta}{1+A_{ol}\beta}$

Provided $A_{ol}\,\beta > 100$, then $\dfrac{A_{ol}\beta}{1+A_{ol}} \approx 1$ to $<1\%$

$$\boxed{A_f = \frac{1}{\beta} \text{ to } <1\%}$$

Important Conclusion

Equation $A_f \cong \dfrac{1}{\beta}$

means that:

1. <u>closed loop gain</u> is practically <u>independent</u> of amplifier open loop gain A_{ol} provided $A_{ol}\,\beta$ is big
2. A_f is determined solely by feedback network only (since this controls β).

Important quantity $A_{ol}\,\beta$ is called the **loop gain** and we often write it as 'T'

It's worth thinking about the relation, $T = A_{ol}\beta$. In op-amps we have VERY high open-loop gain (A_{ol}) so we can 'afford' to throw some of it away by feedback (β) because we can still usually maintain a large enough 'T' to make our simple approximation valid.

Exercises

You will find a set of exercises entitled 'S2 Problems 1' on VITAL, together with worked solutions.
You are encouraged to attempt these questions which start with simply substituting numbers into equations to build familiarity with concepts. Later questions require a little more thought but should not take you much time.

It is very important you work through this problems sheet, which forms an integral part of the module.