Electronic circuits and systems
ELEC271

Part 13: Feedback Theory

Harold S. Black

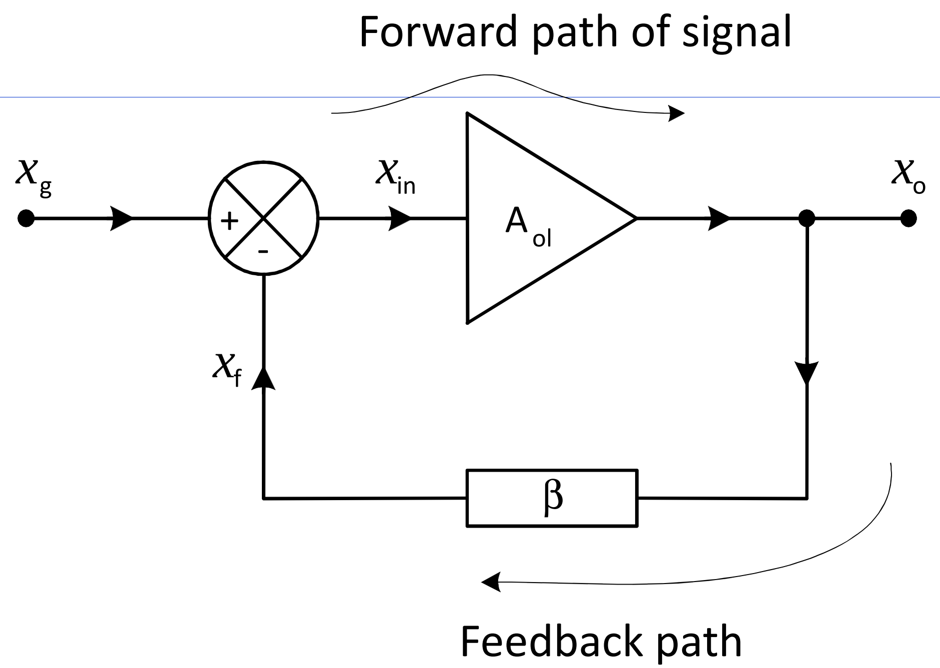
Harold Stephen Black (April 14, 1898 – December 11, 1983) was an American electrical engineer, who revolutionized the field of applied electronics by inventing the negative feedback amplifier in 1927.

To some, his invention is considered the most important breakthrough of the twentieth century in the field of electronics

According to Black he got his inspiration to invent the negative feedback amplifier when he was travelling from New Jersey to New York City by taking a ferry to cross the Hudson River in **August 1927.** Having nothing to write on he sketched his thoughts on a misprinted page of the New York Times and then signed and dated it. At that time, Bell Laboratories headquarters were located in 463 West Street, Manhattan, New York City instead of New Jersey and he lived in New Jersey such that he took the ferry every morning to go to work.

Theoretical matters

Consider the block diagram of a (negative) feedback system



NB arrows indicate the direction of ***signal*** flow - NOT current !

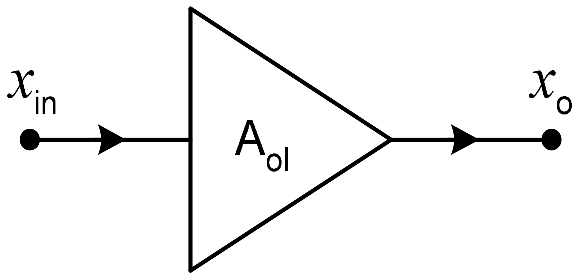
We use "x" because it represents a '**signal**' of some description, (but in our case it will be either a voltage or a current).

Note also that A_{ol} and β can **transform** a signal from one type into another so the signal can be changed as it passes through different parts of the system.

e.g. x_0 may be a voltage whilst x_g, x_f and x_{in} are currents, whilst in a **control system**, x_o might be displacement and x_g, x_f and x_{in} a voltage.

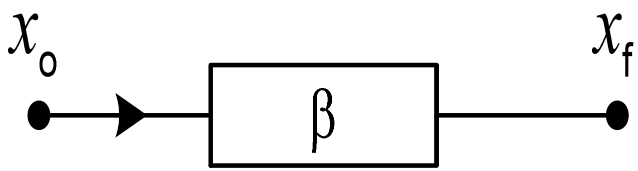
Symbols used

High gain amp
with open loop gain A_{ol}



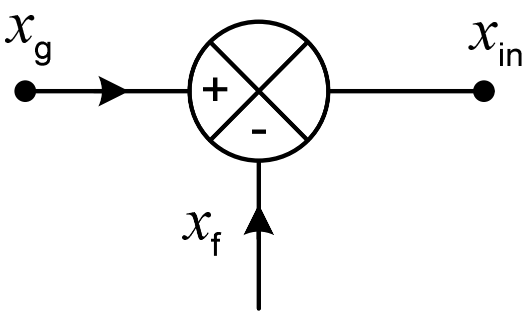
$$A_{ol} = \frac{x_o}{x_{in}}$$

Feedback network with a feedback fraction β



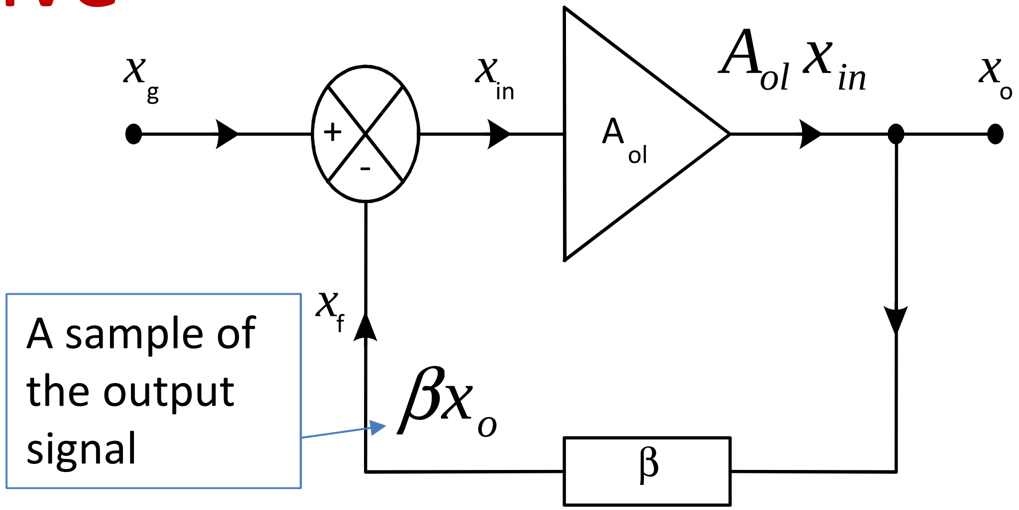
$$\beta = \frac{x_f}{x_o}$$

Input “summer”



$$x_{in} = x_g - x_f$$

Derivation of negative feedback equation



$$x_{in} = x_g - x_f \;= x_g - \beta x_o$$

$$x_o = A_{ol}\;x_{in}$$

$$\therefore\; x_o = A_{ol}\,(x_g - \beta x_o)$$

$$= A_{ol}\;x_g - A_{ol}\;\beta\;x_o$$

$$x_o\,(1 + A_{ol}\;\beta\,) = A_{ol}\;x_g$$

‘open loop’ gain

$$\therefore\; \frac{x_o}{x_g} = \frac{A_{ol}}{1 + A_{ol}\,\beta} = A_f$$

‘closed loop’ gain

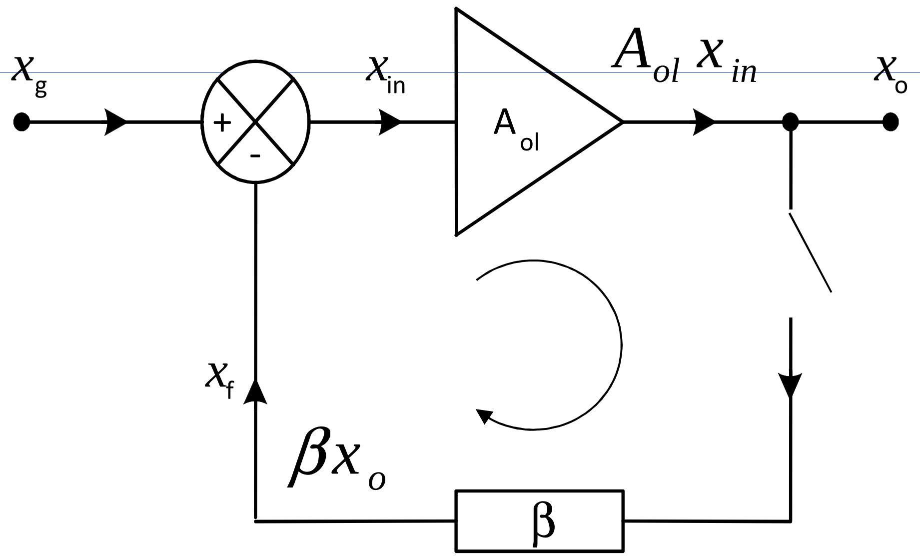
‘feedback *fraction*’ β

‘feedback *factor*’ = $1+\beta A_{ol}$ ‘loop gain’ = βA_{ol}

NB. This is the classic equation of feedback theory. ***Note carefully the terminology used to refer to the various parts of this equation.***

The feedback loop

Note that the loop being referred to is **the feedback loop**



If the loop is broken (anywhere) then the gain given by simply going round the loop once = 'loop gain' = $-\beta A_{ol}$ **(see problem sheet!)**

Note the negative sign is because it is negative feedback

The **closed loop gain** or **gain with feedback** is the gain of the whole system when the loop is closed = A_f

The **open loop gain** is the gain of the system when the loop is open (i.e. With no feedback applied) = A_{ol}

Application

$$A_f = \frac{A_{ol}}{1 + A_{ol}\beta}$$

Suppose we change transistors in an amplifier

Then A_{ol} will change to $A_{ol} + \Delta A_{ol}$

And A_f will change to $A_f + \Delta A_f$

How big is ΔA_f??

That is, how **sensitive** is the gain to changes in A_{ol}

Sensitivity of gain to changes

The sensitivity of A_f to changes in A_{ol} can be quantified by differentiating the feedback equation.

$$A_f = \frac{A_{ol}}{1+\beta A_{ol}}$$

First take logs of both sides:

$$ln(A_f) = ln(A_{ol}) - ln(1 + A_{ol}\beta)$$

Differentiate both sides

$$\frac{1}{A_f}\frac{dA_f}{d\,A_{ol}} = \frac{1}{A_{ol}} - \frac{\beta}{1+A_{ol}\beta} \equiv \frac{1}{(1+A_{ol}\beta)A_{ol}}$$

Letting ΔA_{ol} approximate to dA_{ol} and ΔA_f approximate to dA_f (OK if they are small!), gives

$$\frac{\Delta A_f}{A_f} = \left(\frac{1}{1+\beta A_{ol}}\right)\frac{\Delta A_{ol}}{A_{ol}}$$

So a given fractional change in A_{ol} is attenuated by the feedback factor to give the resultant fractional change in A_f.

Example

Suppose $A_{ol} = 1000$ and $\beta = 0.05$

Then $$A_f = \frac{A_{ol}}{1 + A_{ol}\beta} = \frac{10^3}{1 + 10^3 \times 0.05} = 19.608$$

If A_{ol} were now to increase by 10% to 1100 then

$$A_f' = \frac{1100}{1 + 1100 \times 0.05} = 19.643$$

i.e. an increase of only 0.18 %

Check

The percentage change in A_f should reduce by the feedback factor i.e $1 + A_{ol}\beta = 51$

$$\frac{10\%}{51} \approx 0.2\%$$

(The minor discrepancy is due to the fact that the 10% change is not a **small** incremental change, but quite a **large** one. Better agreement will be obtained if an ***average*** feedback factor of 53 were used.)

***The important point to remember* is that a change in A_{ol} has a significantly diminished effect on A_f when negative feedback is present**

– and by a factor given by the feedback factor, $1 + A_{ol}\beta$

Alternative Picture

$$A_f = \frac{A_{ol}}{1+\beta A_{ol}} \longrightarrow A_f = \frac{1}{\beta}\frac{A_{ol}\beta}{1+\beta A_{ol}}$$

Provided $A_{ol}\,\beta > 100$ (say), then $\frac{A_{ol}\beta}{1+A_{ol}\beta} \approx 1$ $\boxed{\rightarrow A_f \approx \frac{1}{\beta}}$

To < 1% **and the closed loop gain A_f becomes independent of open loop gain A_{ol} !!**

This is a very important result!

- It means that the overall system gain is **no longer dependent** on the precision with which the gain of the amplifier can be made.
- **If we just make A_{ol} large** (so that $\beta A_{OL} >> 1$, remembering that β is a *fraction* that usually lies between 10^{-3} and 1),
- then the overall gain will **instead be set by the feedback *fraction* β.**
- **This often just involves choosing some resistors that can be tightly controlled quite easily – and adjusted if desired.**

Conclusions

$$A_f \cong \frac{1}{\beta}$$ If $A_{ol}\beta$ is large

closed loop gain is practically **independent** of amplifier **open loop gain**

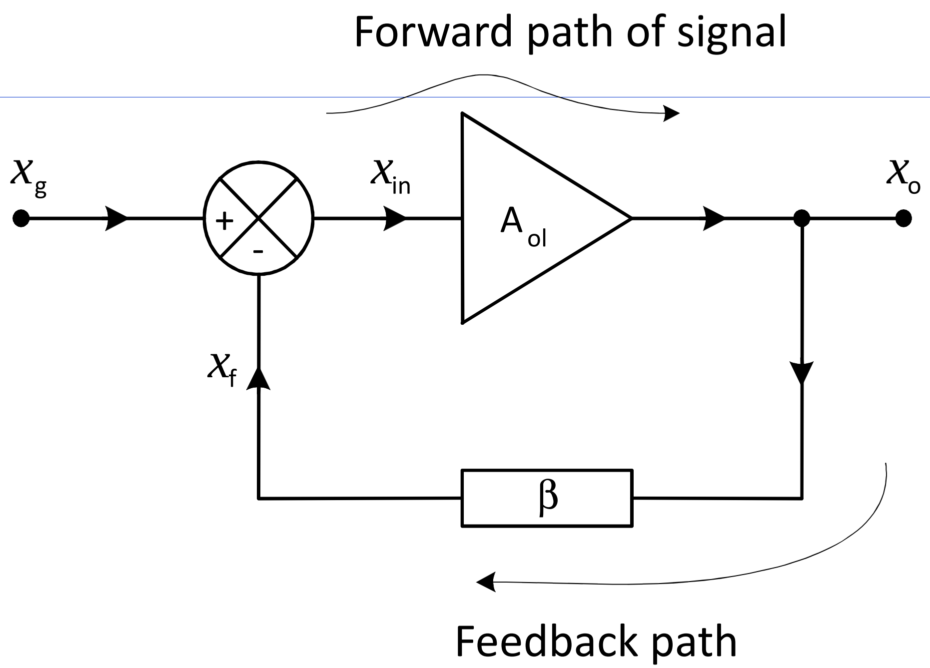
- Important quantity the **loop gain** and we often write it as 'T'

$$T \equiv A_{ol}\beta$$

- It's worth thinking about the relation. In **op-amps** (like the 741) we have VERY high open-loop gain (A_{ol}) so we can 'afford' to throw some of it away by feedback (β) because we can still usually maintain a large enough 'T' to make the simple approximation valid.
- Finally, note we will sometimes write $A_f = A_\infty \frac{T}{1+T}$ with $A_\infty = \frac{1}{\beta}$

Last time: Feedback Theory

block diagram of a (negative) feedback system



NB arrows indicate the direction of ***signal*** flow - NOT current !

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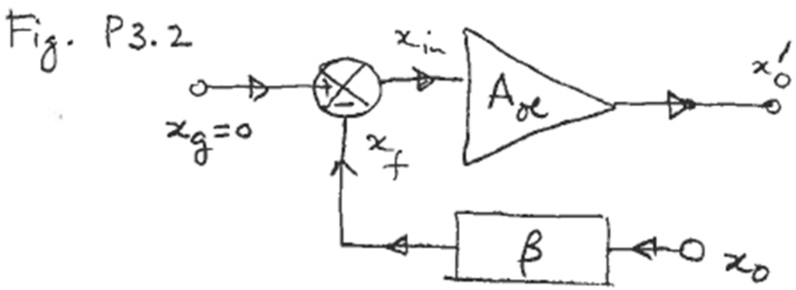
Gain with feedback

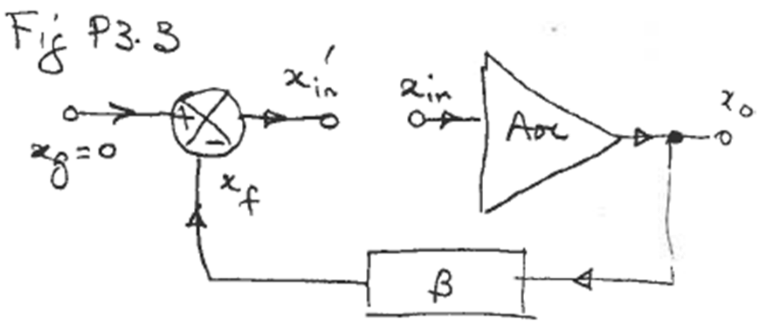
$$A_f = \frac{1}{\beta}\frac{A_{ol}\beta}{1+\beta A_{ol}} \quad \boxed{\rightarrow A_f \approx \frac{1}{\beta}}$$

If $A_{ol}\,\beta >> 1$

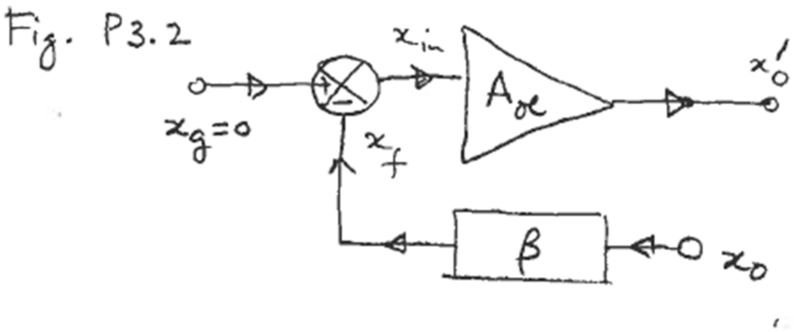
Exercise

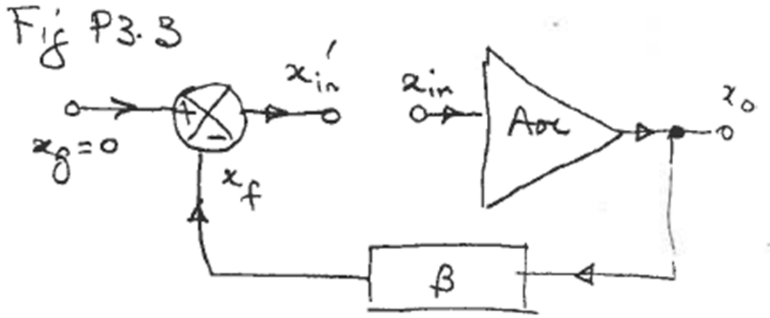
4. Consider the circuit shown in fig. P3.2, in which the input signal x_g has been reduced to zero and the feedback path is broken at the output. Calculate the ratio x'_o/x_o. Now consider the circuit shown in fig. P3.3, in which the connection between the summing junction and the amplifier input has been broken. Calculate x'_{in}/x_{in}. [In both cases, the answer is $-A_{ol}.\beta$. I hope you can now see why this quantity, (without the negative sign) is called the *loop gain*.].





Solution





$$x_f = \beta x_o$$

$$x_{in} = -\beta x_o$$

$$x_o' = -\beta x_o A_{ol}$$

$$\boxed{\frac{x_o'}{x_o} = -\beta A_{ol}}$$

$$x_o = A_{ol} x_{in}$$

$$x_f = \beta A_{ol} x_{in}$$

$$x_{in}' = -\beta A_{ol} x_{in}$$

$$\boxed{\frac{x_{in}'}{x_{in}} = -\beta A_{ol}}$$

Loop Gain!

Worked Example

An amplifier with an open loop gain of 500 operates in an environment where temperature rise causes the open loop gain to increase to 550.

i) Treating the amplifier as a negative feedback system, find the necessary feedback fraction, β to ensure that that the closed loop gain does not change by more than 0.2%. Assume that β does not change with temperature.

The change in gain is quite large so write:

$$\frac{\Delta A_f}{A_f} = \frac{A_{f2} - A_{f1}}{A_{f1}}$$

$$\frac{\Delta A_f}{A_f} = \left[\frac{550}{1+550\times\beta} - \frac{500}{1+500\times\beta}\right]\times\frac{1+500\times\beta}{500} = \frac{0.2}{100}$$

$$\beta = \frac{49}{550}$$

β = 0.089

Problems

- See VITAL for a problems sheet on Feedback Theory
- Solutions are also there, but try the problems yourself first

End of Part 13

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Part 14
Feedback theory applied to generic amplifier types

Look at feedback applied to the 4 amplifier types

- Voltage
- Current
- Transresistance
- transconductance

Application of feedback theory to amplifiers

Summary so far:

- Negative feedback converts a **high gain but poorly stabilised amp**. Into one which is nearly **ideal**
  - having practically constant gain independent of source and load impedances

- An ideal amplifier has R_{in}, R_{out} values which are either
  - ∞ (open circuit)
  - 0 (short circuit)

THE FEEDBACK TOPOLOGY DETERMINES WHICH!

If the loop gain, $A_{ol}\,\beta$ is sufficiently large, then the closed loop gain, A_f is equal to $1/\beta$: TO A GOOD APPROXIMATION

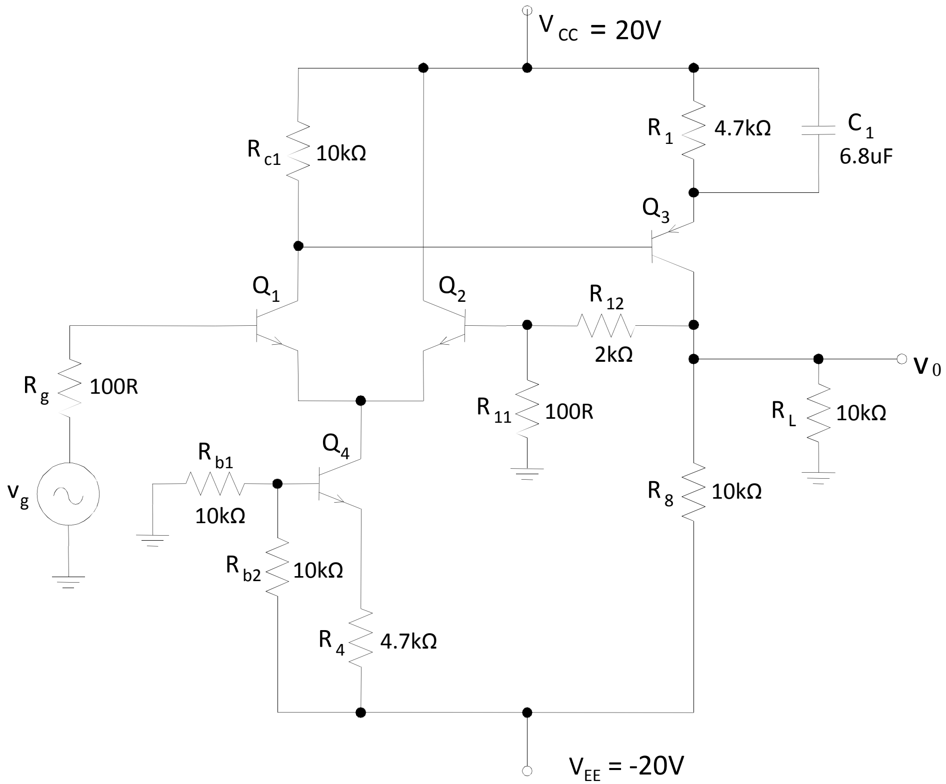
- **We will now apply these ideas to some real amplifier circuits and establish a simple method for finding the gain.**

1st Circuit type

Output sampling?

Input summing?

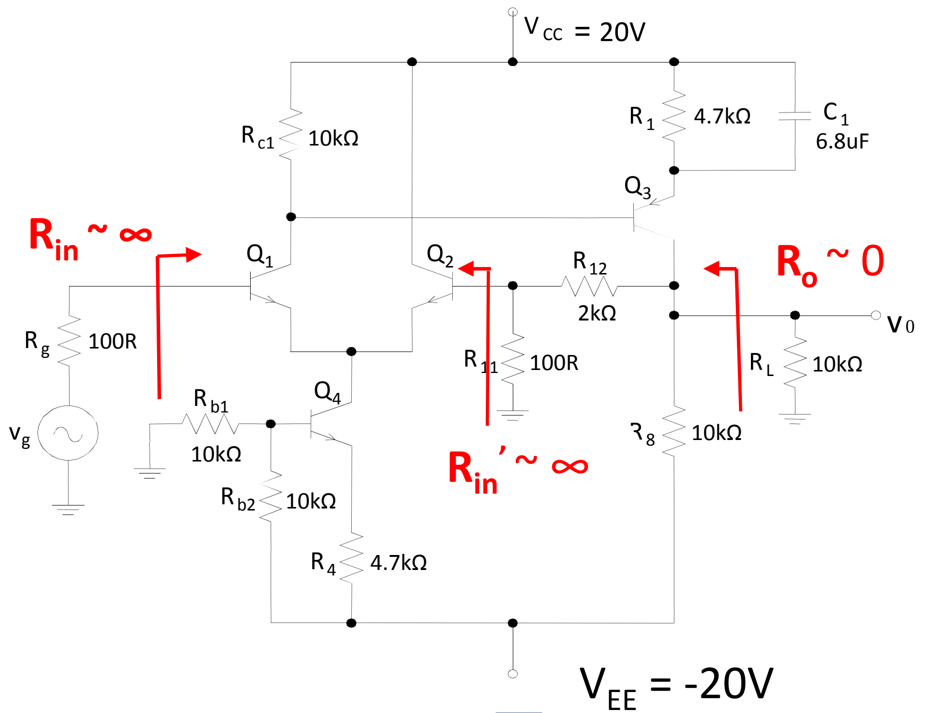
So amplifier type is?



1^{st} Circuit type

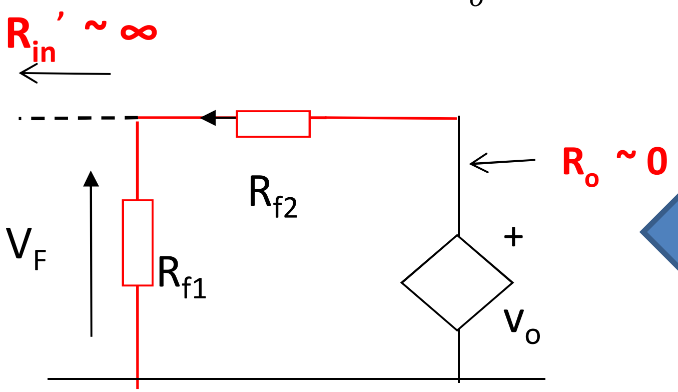
Voltage sensing at output

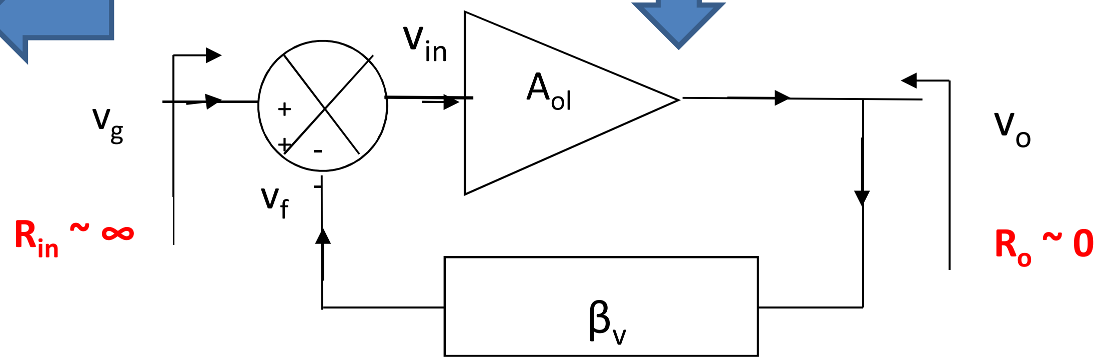
Voltage summing at input



So voltage amplifier

Want to find $\beta_v = \dfrac{v_f}{v_o}$





$$v_f = \frac{R_{f1}}{R_{f1}+R_{f2}} v_o$$

$$\beta_v = \frac{v_f}{v_o} = \frac{100}{100+2k} = \frac{1}{21}$$

$$\boxed{A_{fv} = \frac{v_o}{v_g} = \frac{1}{\beta_v} = 21}$$

Agrees very well with previous result using SPICE (see part 10)

Next lecture

- Apply the technique to estimate the gain of
  - A current amplifier
  - A transresistance amplifier
  - A transconductance amplifier