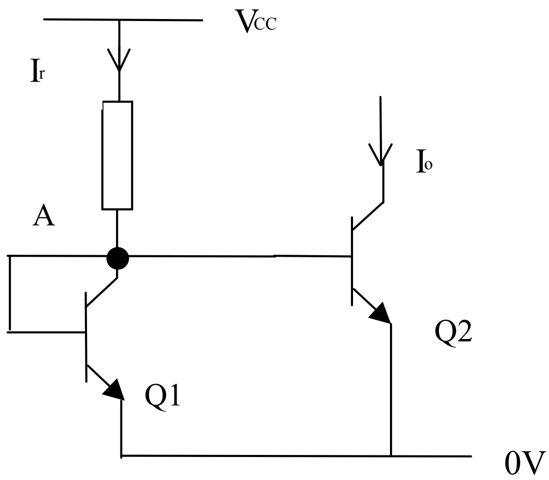
Current mirror problems

For the simple current mirror shown below, show that

$$\frac{I_o}{I_r} = \frac{1}{1+2/\beta} = 1 - \frac{2}{\beta+2}$$



where $\beta = I_C/I_B$.
Assume identical transistors.

<u>Solution</u>

Transistors are identical, so

$I_{C1} = I_{C2} = I_C$ (as $V_{BE1} = V_{BE2}$)

$I_{B1} = I_{B2} = I_B$

KCL at A

$$I_r = I_{B1} + I_{B2} + I_{C1}$$
$$I_r = 2I_B + I_o$$

Now $I_B = \dfrac{I_C}{\beta} = \dfrac{I_o}{\beta}$

Therefore: $I_r = \dfrac{2I_o}{\beta} + I_o = I_o\left(\dfrac{2}{\beta} + 1\right)$

$$\frac{I_o}{I_r} = \frac{1}{1+2/\beta}$$

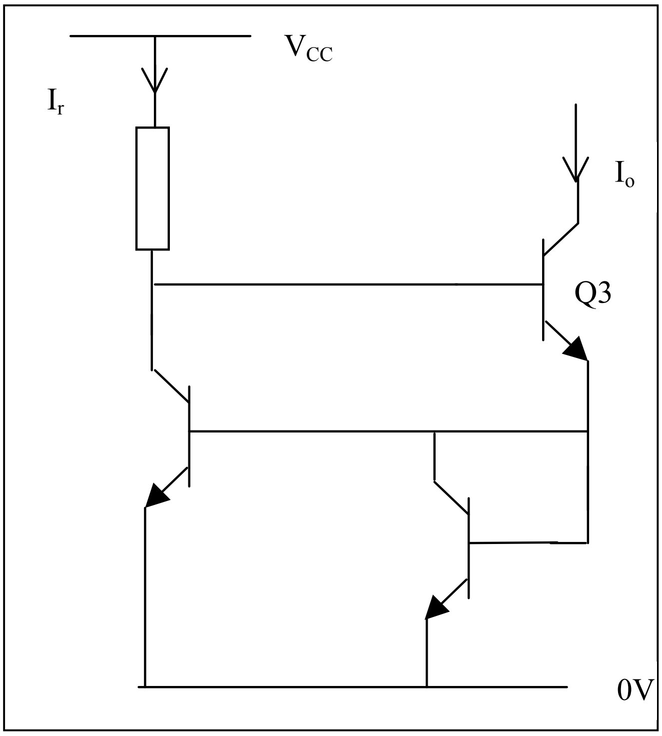
Now: $1 - \dfrac{2}{\beta+2} = \dfrac{\beta+2-2}{\beta+2} = \dfrac{1}{1+\beta/2}$

$$\underline{\frac{I_o}{I_r} = 1 - \frac{2}{\beta+2}}$$

Similarly show that for the current mirror below,

For the circuit shown, show that

$$\frac{I_o}{I_r} = 1 - \frac{2}{\beta(\beta+2)+2}$$



Solution

KCL at A

$$I_{E3} = I_o + I_{B3}$$

Now

$$I_{E3} = I_o + \frac{I_o}{\beta}$$

$$I_{E3} = \frac{\beta+1}{\beta} I_o \quad (1)$$

Also

$$I_{E3} = I_C + 2I_B$$

$$= \frac{\beta+2}{\beta} I_C \qquad (2)$$

Equate 1 and 2 to get

$$I_C = \frac{\beta+1}{\beta+2} I_o \quad (3)$$

$$I_r = I_C + \frac{I_o}{\beta}$$

Sub. (3) gives $I_r = \frac{\beta+1}{\beta+2} I_o + \frac{I_o}{\beta}$

$$I_r = \frac{\beta+1}{\beta+2} I_o + \frac{I_o}{\beta} = I_o\left[\frac{\beta+1}{\beta+2} + \frac{1}{\beta}\right]$$

$$I_r == I_o\left[\frac{\beta(\beta+1)+\beta+2}{\beta(\beta+2)}\right], \qquad \frac{I_o}{I_r} = \left[\frac{\beta(\beta+2)}{\beta^2+\beta+\beta+2}\right]$$

$$\frac{I_o}{I_r} = \left[\frac{\beta(\beta+2)}{\beta^2+2\beta+2}\right], \quad \frac{I_o}{I_r} = \left[\frac{\beta(\beta+2)}{\beta(\beta+2)+2}\right]$$

Hence $\frac{I_o}{I_r} = 1 - \frac{2}{\beta(\beta+2)+2}$

Calculate the percentage change in I_o/I_r for each circuit if β varies from 5 to 100. Hence show that the second circuit is approximately six times superior to the first in maintaining I_o constant.