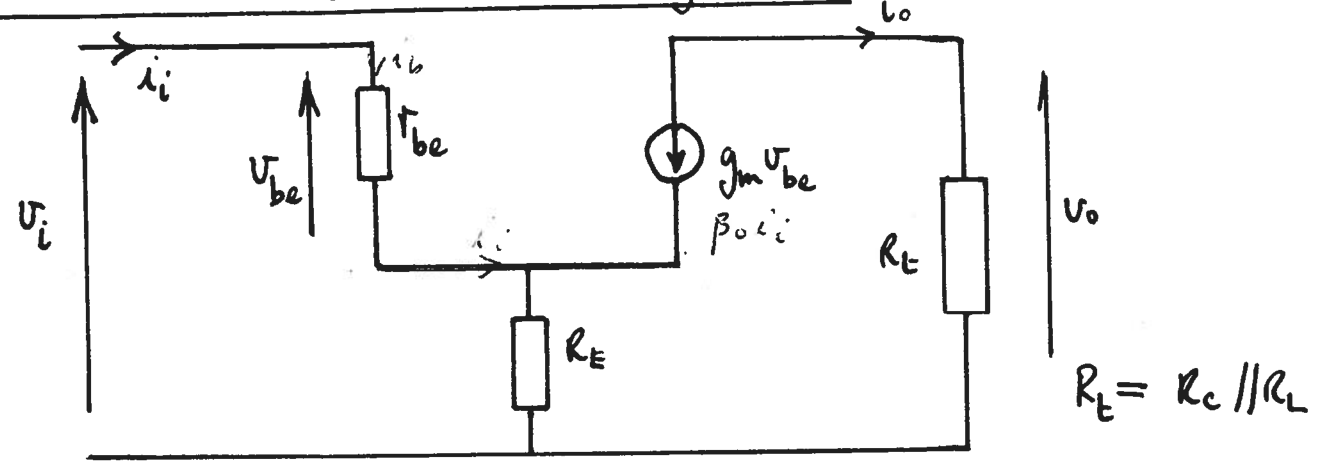
SOLUTIONS (COURSE DOC.)

1. Common Emitter with Emitter Degradation



$A_I /$ $\quad i_o = -g_m v_{be}, \; v_{be} = i_i r_{be} \quad i_o/i_i = -g_m r_{be} = -\beta_o$

$R_i /$ $\quad v_i = i_i r_{be} + (i_i + \overset{\beta_o i_i}{g_m v_{be}}) R_E$

$\quad = i_i r_{be} + (i_i + g_m i_i r_{be}) R_E \quad$ as $\quad v_{be} = i_i r_{be}$

$\therefore \quad \dfrac{v_i}{i_i} = r_{be} + (1+\beta_o) R_E \quad$ as $\quad \beta_o = g_m r_{be}$

same as common collector / emitter follower

$A_V /$ $\quad v_o = i_o R_t = -g_m v_{be} R_t$

$v_i = i_i \left[r_{be} + (1+\beta_o) R_E \right]$

$\quad = \dfrac{v_{be}}{r_{be}} \left[r_{be} + (1+\beta_o) R_E \right]$

$$A_V = \frac{v_o}{v_i} = \frac{-g_m v_{be} R_t}{\frac{v_{be}}{r_{be}} \left[r_{be} + (1+\beta_o) R_E \right]}$$

$$\therefore A_V = \frac{v_o}{v_i} = -\frac{\beta_o R_t}{r_{be} + (1+\beta_o) R_E}$$

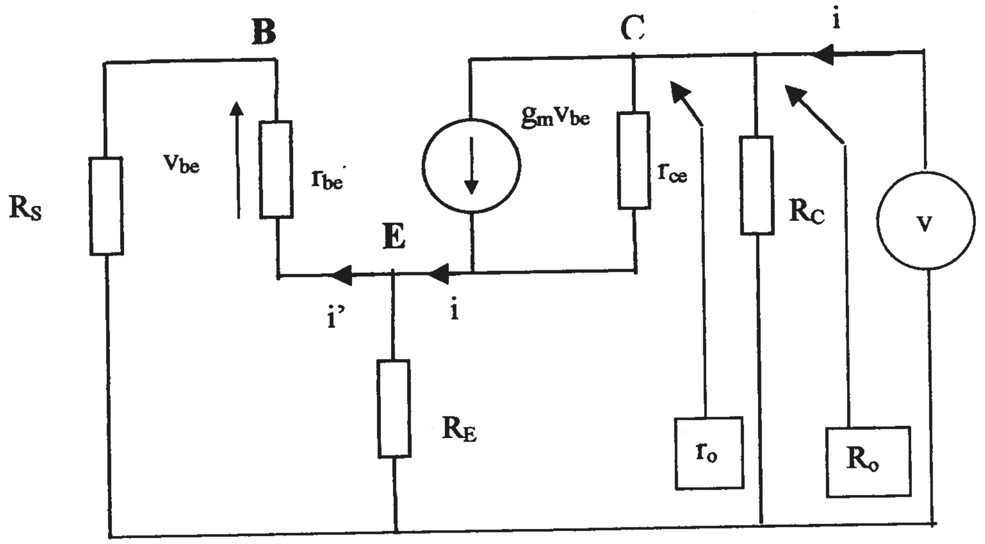
$$\simeq -\frac{g_m R_t}{1 + g_m R_E}$$

as $\beta_o \gg 1$ and $\beta_o / r_{be} = g_m$

If $g_m R_E \gg 1$, $\quad A_V \rightarrow -\dfrac{R_t}{R_E}$

independent of transistor parameters and I_c (g_m)

Output resistance, Ro



$R_o = r_o // R_C$ So remove R_C and calculate r_o (redefine i as the current into r_o)

With R_C removed: $$i = g_m v_{be} + \frac{v - v_E}{r_{ce}} \tag{1}$$

$$i' = i\frac{R_E}{R_E + R_S + r_{be}} = -\frac{v_{be}}{r_{be}} \tag{2}$$

so that $$v_{be} = -i\frac{r_{be} R_E}{R_E + R_S + r_{be}} \tag{3}$$

also need v_E: $$v_E = i'(R_S + r_{be}) = -v_{be}\left(1 + \frac{R_S}{r_{be}}\right) \quad \text{(using (2))} \tag{4}$$

Sub. (3) and (4) in (1): $$i = -g_m \frac{r_{be} R_E}{R_E + R_S + r_{be}} i + \frac{v}{r_{ce}} - \frac{r_{be} R_E}{R_E + R_S + r_{be}}\left(1 + \frac{R_S}{r_{be}}\right)\frac{1}{r_{ce}} i$$

that is, $$i\left[1 + \frac{r_{be} R_E}{R_E + R_S + r_{be}}\left(g_m + \left(1 + \frac{R_S}{r_{be}}\right)\frac{1}{r_{ce}}\right)\right] = \frac{v}{r_{ce}}$$

and $$r_o = \frac{v}{i} = r_{ce}\left[1 + \frac{r_{be} R_E}{R_E + R_S + r_{be}}\left(g_m + \frac{1}{r_{ce}}\left(1 + \frac{R_S}{r_{be}}\right)\right)\right]$$

generally $g_m >> \frac{1}{r_{ce}}\left(1 + \frac{R_S}{r_{be}}\right)$ (r_{ce} large, generally $R_S < r_{be}$)

$r_o \approx r_{ce}\left[1 + \frac{\beta_o R_E}{R_E + R_S + r_{be}}\right]$ and $R_o = R_C // r_o$. Often $R_C << r_o$ and $R_o \approx R_C$

G_M / Transconductance Gain $G_M = \frac{i_o}{v_i}$

$$v_i = i_i r_{be} + (i_i + g_m v_{be}) R_E$$
$$= i_i [r_{be} + (1+\beta_0) R_E] \quad \text{as} \quad v_{be} = i_i r_{be} \quad \text{and} \quad \beta_0 = g_m r_{be}$$

$$i_o = - g_m v_{be}$$
$$= - g_m i_i r_{be}$$
$$= - \beta_0 i_i$$

$$\therefore \quad G_M = \frac{i_o}{v_i} = \frac{-\beta_0}{r_{be} + (1+\beta_0) R_E}$$
$$\approx \frac{-g_m}{1 + g_m R_E} \quad \text{as } \beta_0 \gg 1$$

$G_m = G_M \big|_{R_L \to 0} = G_M$ as G_M is independant of R_L

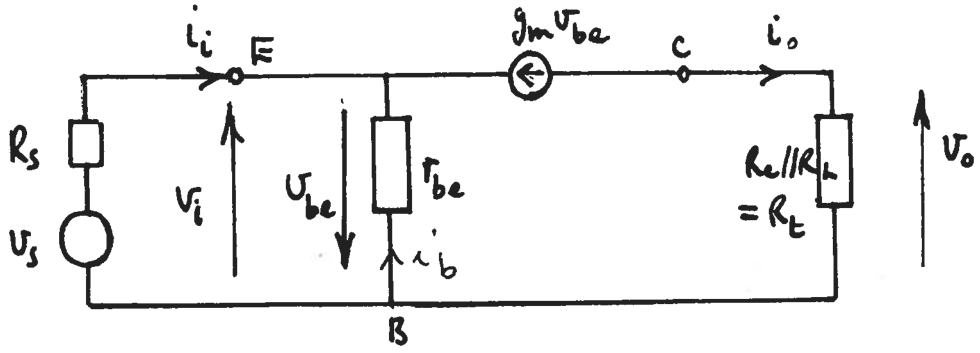
R_M / Transresistance Gain $R_M = \frac{v_o}{i_i}$

$$v_o = - g_m v_{be} R_t$$
$$= - g_m i_i r_{be} R_t$$

$$\therefore \quad R_M = \frac{v_o}{i_i} = -\beta_0 R_t \quad \text{as} \quad \beta_0 = g_m r_{be}$$

$$R_m = R_M \big|_{R_L \to \infty} = -\beta_0 R_c$$

L Common Base



Assume $R_E \gg R_i$ so treat R_E as open cct.

A_I/ $\quad i_o = -g_m v_{be} = -g_m (i_o - i_i) r_{be}$

as $\quad v_{be} = +(i_o - i_i) r_{be}$

$\therefore \quad i_o = -\beta_o (i_o - i_i)$

$A_I = \dfrac{i_o}{i_i} = \dfrac{\beta_o}{1+\beta_o}$ $\qquad$ As $\beta_o \gg 1$, $A \to 1$

R_i/ $\quad i_i = -\dfrac{v_{be}}{r_{be}} - g_m v_{be}$

$= \left(\dfrac{1}{r_{be}} + g_m\right) v_i$ $\qquad$ as $\quad v_i = -v_{be}$

$\therefore \quad R_i = \dfrac{v_i}{i_i} = \dfrac{1}{1/r_{be} + g_m}$

$= \dfrac{r_{be}}{1+\beta_o}$

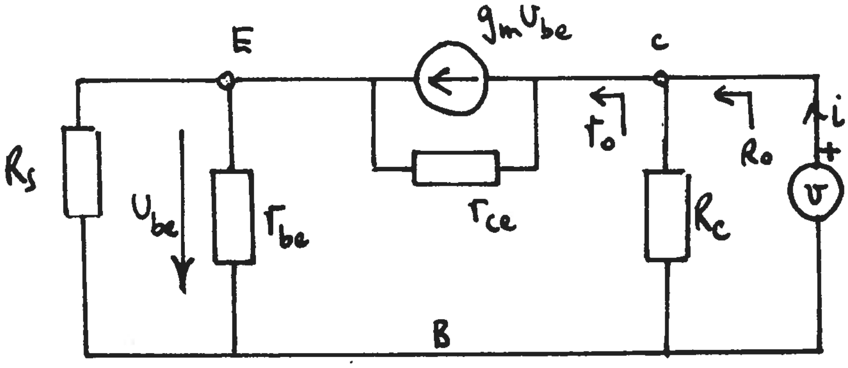
$= r_e$ $\qquad$ $r_e \ll R_E$ justifies original assumption

A_V/ $\quad v_o = -g_m v_{be} R_t$

$v_i = -v_{be}$

$\therefore \quad A_V = \dfrac{v_o}{v_i} = g_m R_t$ $\qquad$ Same magnitude as CE but +ve

R_o /



$$R_o = r_o // R_c$$

So remove R_c and calculate r_o

With R_c removed $\quad i = g_m v_{be} + \dfrac{v + v_{be}}{r_{ce}}$

$$i = \frac{-v_{be}}{R_s // r_{be}}$$

$$\therefore \quad i = -g_m (R_s // r_{be})\, i + \frac{v}{r_{ce}} - \frac{i\,(R_s // r_{be})}{r_{ce}}$$

$$i \left[1 + g_m R_s // r_{be} + \frac{R_s // r_{be}}{r_{ce}} \right] = \frac{v}{r_{ce}}$$

$$\therefore \quad \frac{v}{i} = r_{ce} \left[1 + \left(g_m + \frac{1}{r_{ce}} \right) R_s // r_{be} \right]$$

$$g_m \gg 1/r_{ce}$$

$$\therefore \quad r_o \approx r_{ce} \left[1 + g_m (r_{be} // R_s) \right]$$

$$\therefore \quad R_o = R_c // r_o$$

Often $R_c \ll r_o$ and $R_o \approx R_c$

G_M /

$$i_o = -g_m v_{be}$$
$$= +g_m v_i \quad \text{as} \quad v_i = -v_{be}$$
$$\therefore \quad G_M = \frac{i_o}{v_i} = g_m$$

R_M /

$$v_o = -g_m v_{be} R_t$$
$$v_{be} = -(i_i + g_m v_{be})\, r_{be}$$
$$v_{be}(1 + g_m r_{be}) = -i_i r_{be}$$

$$\therefore \quad v_o = +g_m R_t \frac{i_i r_{be}}{1 + g_m r_{be}}$$

$$\therefore \quad R_M = \frac{v_o}{i_i} = \frac{g_m r_{be}}{1 + g_m r_{be}} R_t$$
$$= \frac{\beta_o R_t}{1 + \beta_o}$$