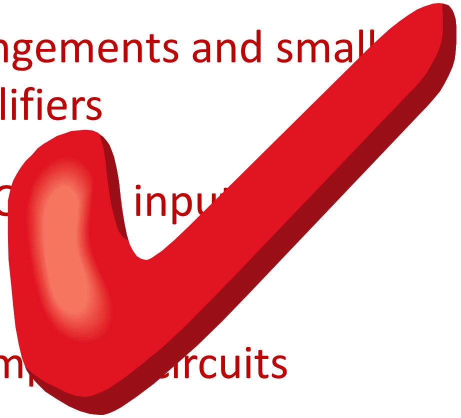
The story so far....

Module summary / syllabus

- Hybrid-pi model of the bipolar junction transistor, biasing arrangements and small signal analysis of basic single stage amplifiers, multi-stage amplifiers
- Differential amplifiers – differential and common mode gains, C inpu resistance
- Current mirrors and Active loads – application to differential am ircuits
- **High frequency amplifiers – effect of collector current, load and source resistances on high frequency response**
- Field Effect devices: amplifiers discrete and integrated, switched capacitor resistor
- Step response of amplifiers
- Negative feedback – circuit analysis, effect on bias condition, gain, bandwidth, input/output resistances, stability.
- Operational amplifiers – role of feedback, limitations, application to filter design



Electronic circuits and systems
ELEC271

Part 6
Frequency response of amplifiers – I
Gain-bandwidth product, f_T

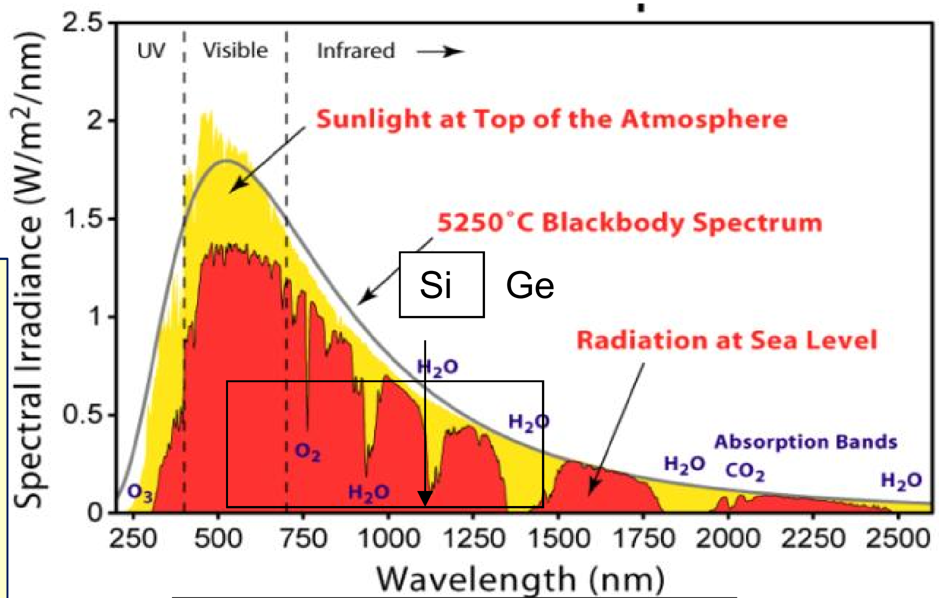
Called the 'transition frequency' on datasheets

What's the highest frequency that can be amplified with a transistor amplifier?

What do you know about frequency!

- Human hearing range? ~12 kHz
- Audio frequency range? 20 kHz
- FM radio? 90-100 MHz
- Mobile phones? 0.8-2.6 GHz
- Personal computer clock? 2 GHz
- Microwave oven? 2.45 GHz
- Satellite dishes? 12 GHz
- radar? Many different bands: 10’s MHz to 100’s GHz
- Sunlight?

•Very wide range of frequencies!
•What can be done with Si transistors?
•What can be done with regular electronics?

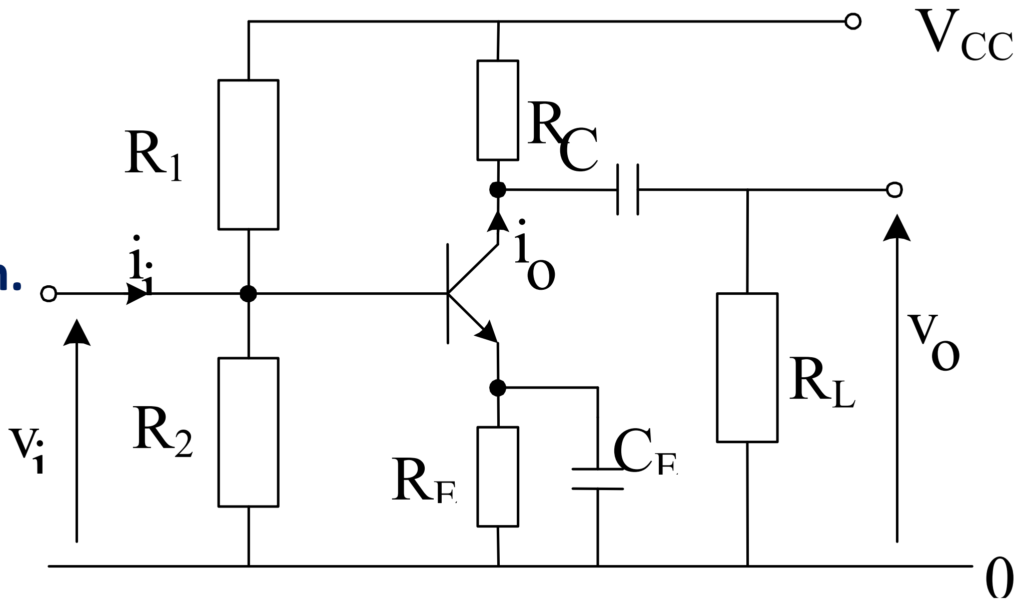


750 THz to 215 THz

Lower frequencies, and energy

Common Emitter short-circuit, high frequency current gain, f_T

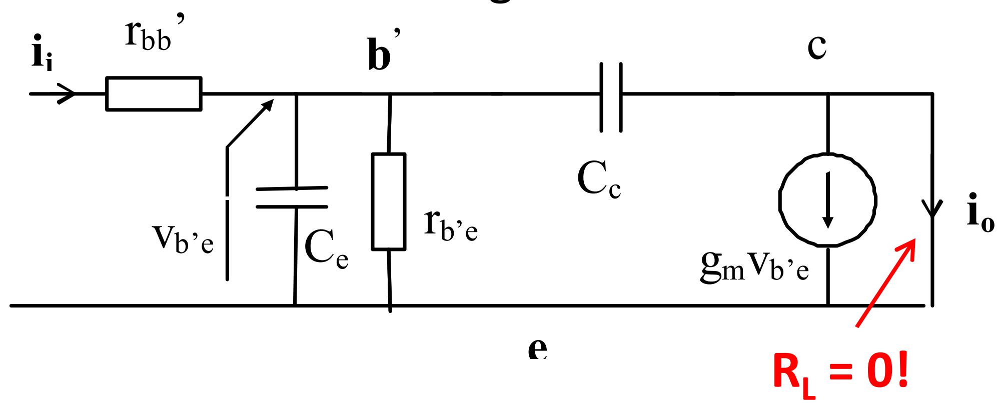
Consider a single stage CE amplifier with the load resistor, **$R_L = 0$**
use this condition to find the **highest frequencies that the amplifier can attain.**



find an expression for **the high frequency current gain, i_o/i_i**. Consider the hybrid-π equivalent circuit for the amplifier with $R_L = 0$ and bias resistors ignored:

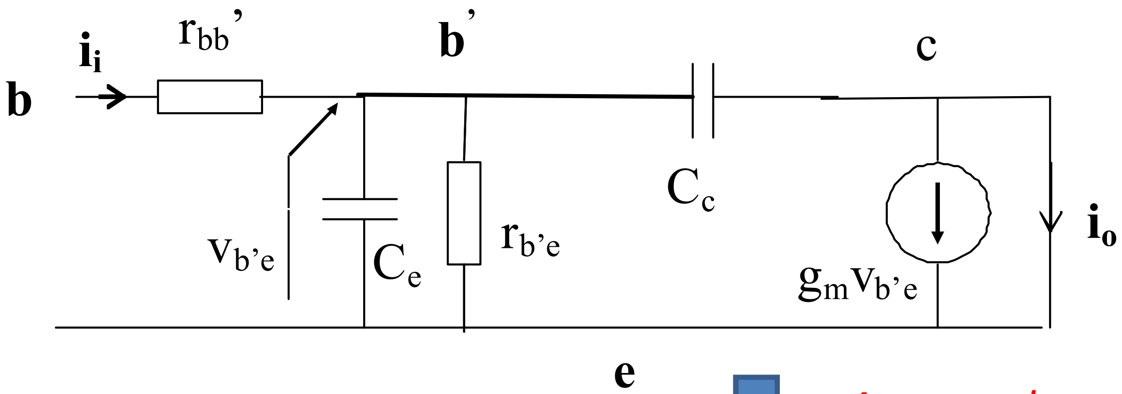
presence of C_c complicates the analysis!

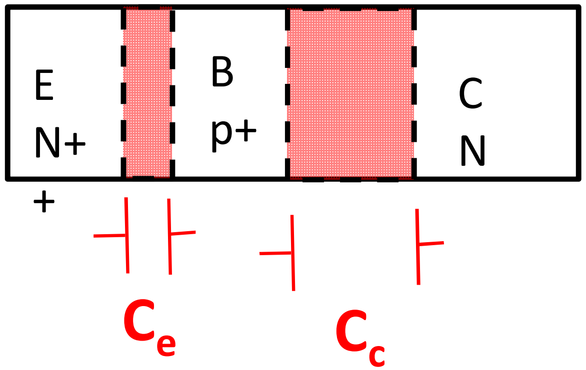
- *Assume that the a.c. current through to the output via C_c is very small*
- justify this approximation later on. The equivalent circuit then reduces to:



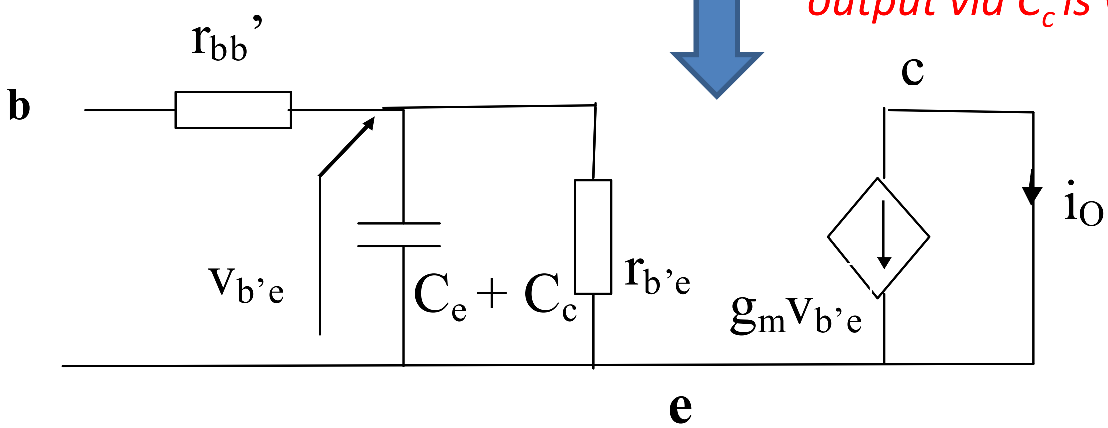
Simplify the equivalent circuit

Repeated from previous slide:





Assume that the a.c. current through to the output via C_c is very small

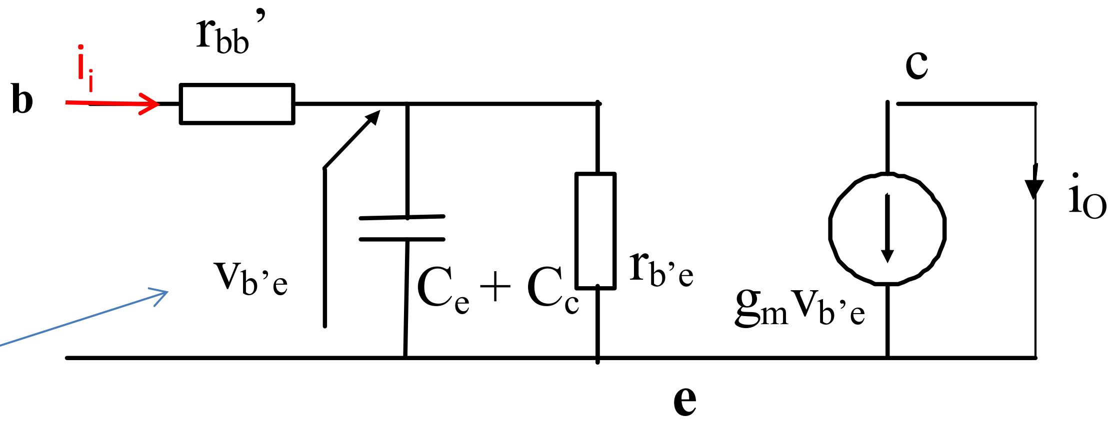


Note:
simple 1st order
RC circuit!

Notice that the capacitor C_c still draws current in the input circuit and is therefore included in parallel with C_e.

Now do the analysis..........

Analysis



$i_o = -g_m v_{b'e}$ **(1)**

$v_{b'e} = i_i (r_{b'e} // X_{e+c})$, that is $v_{b'e} = i_i (r_{b'e} // X_C) = i_i \dfrac{r_{b'e}/j\omega C}{r_{b'e} + 1/j\omega C}$

$X_C = \dfrac{1}{j\omega C}$

where $C = C_e + C_c$, $\omega = 2\pi f$ (f = frequency), X_C is the impedance of C and $j = \sqrt{-1}$

Multiplying top and bottom by $j\omega C$ gives $v_{b'e} = \dfrac{i_i r_{b'e}}{1 + j\omega r_{b'e}(C_e + C_c)}$ **(2)**

Substituting (1) into (2) gives the required expression $A_i \equiv \dfrac{i_o}{i_i} = -\dfrac{g_m r_{b'e}}{1 + j\omega r_{b'e}(C_e + C_c)}$

which can also be written: $A_i = -\dfrac{\beta_o}{1 + j(f/f_\beta)}$ **(3)**

Recall
$\beta_o = g_m r_{b'e}$

where $f_\beta = \dfrac{1}{2\pi r_{b'e}(C_e + C_c)}$

When f = f_T, A_i = 1 **(definition of f_T !)**

Take the modulus of A_i gives: $$|A_i| = \frac{\beta_o}{\left[1+(f/f_\beta)^2\right]^{1/2}} \qquad (4)$$

when f = f_β, the gain is reduced by a factor 1/√2 or –3 dB.

This is the *corner frequency* and f_β defines the **BANDWIDTH** of the amplifier.

A_i = 1 when f = f_T so **Eqn. (4) then becomes:** $$\left[1+(f_T/f_\beta)^2\right]^{1/2} = \beta_o$$

Squaring both sides and noting that $\beta_o^2 \gg 1$: $\boldsymbol{f_T = \beta_o f_\beta}$ (5)

So f_T is therefore defined as the **GAIN-BANDWIDTH PRODUCT** for the amp.

Substituting $\beta_o = g_m r_{b'e}$ and f_β from Eqn. (3): $$f_T = g_m \cancel{r_{b'e}} \frac{1}{2\pi \cancel{r_{b'e}}(C_e + C_c)}$$

hence $$f_T = \frac{g_m}{2\pi(C_e + C_c)}$$

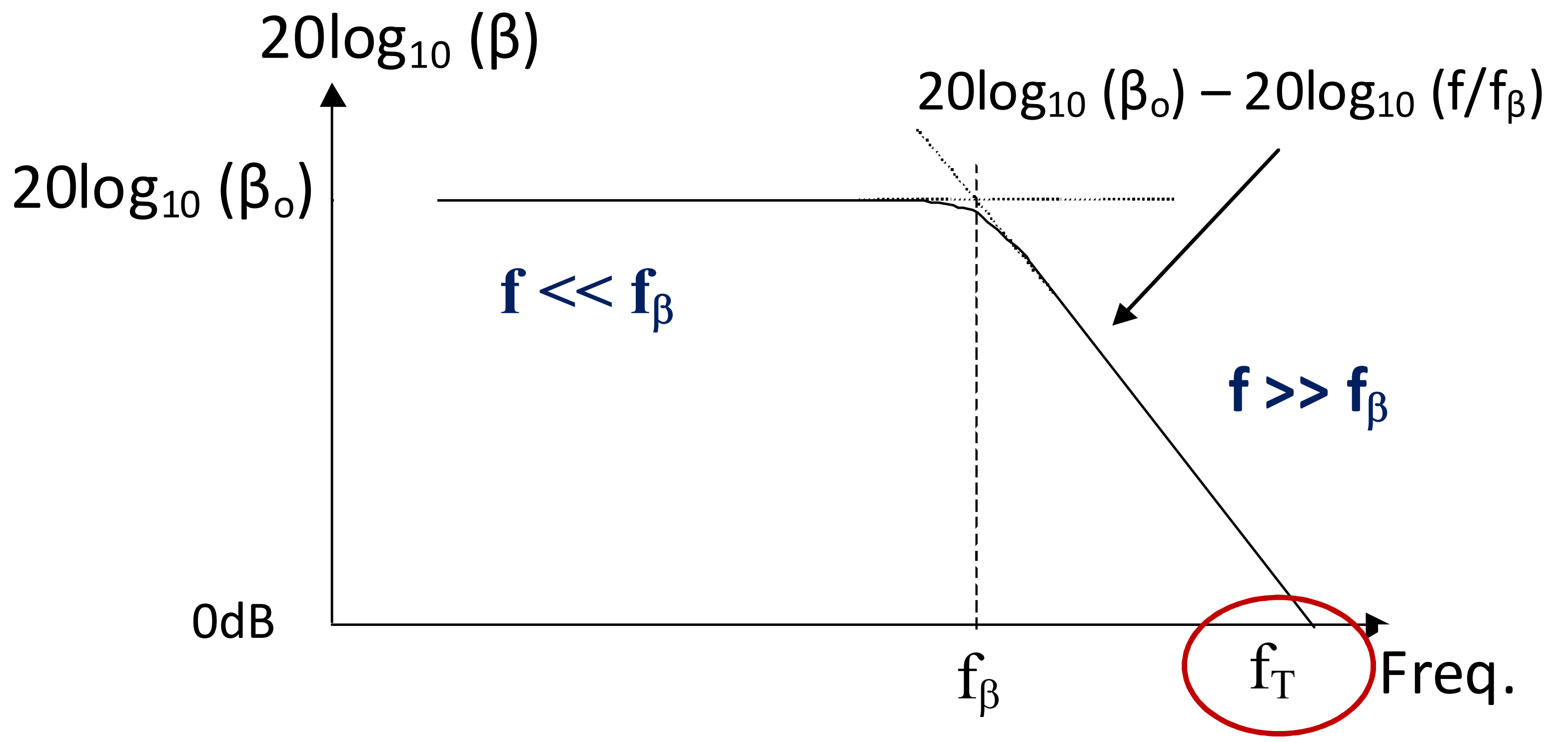
Small devices for high-frequency! – Moore's Law

Represent Equation (4) on a **Bode plot**

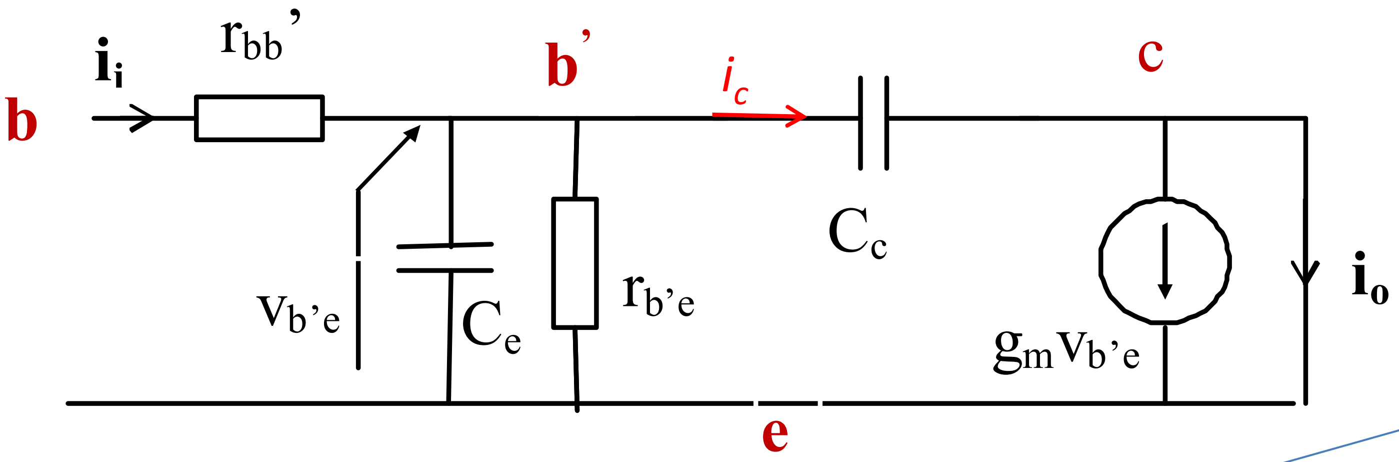
- Write A_i as β (not to be confused with d.c. current gain!), take logs to base 10 and write Eqn. (4) as

$$|A_i| = \frac{\beta_o}{\left[1+\left(f/f_\beta\right)^2\right]^{1/2}} \Rightarrow |\beta| = \frac{\beta_o}{\left[1+\left(f/f_\beta\right)^2\right]^{1/2}}$$

[Convention used in most textbooks]



Justification of previous approximation



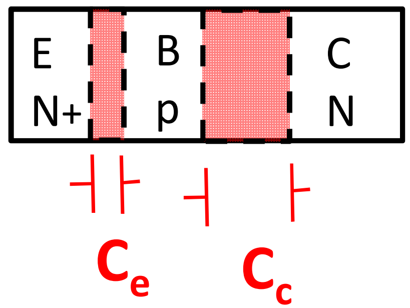
$$X_c = \frac{1}{2\pi f_T C_c}$$

$$v_{b'e} = X_c i_c$$

- The current through C_c at frequency f_T is: $i_C = v_{b'e}(2\pi f_T C_c)$. **(Ohm's Law)**
- The ratio of this current to that in the output circuit current source $g_m v_{b'e}$ is therefore

$$f_T = \frac{g_m}{2\pi(C_e + C_c)}$$

$$\frac{2\pi f_T C_c v_{b'e}}{g_m v_{b'e}} = \frac{\cancel{2\pi f_T} C_c}{\cancel{2\pi f_T}(C_e + C_c)} = \frac{C_c}{(C_e + C_c)}$$



- as $C_c << C_e$ for bipolar transistors (IMPORTANT – REMEMBER), $\frac{C_c}{(C_e + C_c)} << 1$

thus justifying the use of the approximation

Relation to models in Part 1

- Recall from the first lecture, that $C_e \approx I_C\tau/V_T$

τ is the **transit time** for the electrons to go through the base of the transistor

assume $C_c << C_e$, then

$$f_T \approx \frac{g_m}{2\pi C_e} = \frac{I_C}{V_T}\frac{1}{2\pi C_e}$$

$$f_T = \frac{g_m}{2\pi(C_e + \cancel{C_c})}$$

- Substituting in the expression for $C_e \approx I_C\tau/V_T$ to obtain

$$\boxed{f_T = \frac{1}{2\pi\tau}}$$

- that is to say, physically, the **gain-bandwidth** is a measure of the time that it takes electrons to move through the transistor, from emitter to collector. **This is obviously the ultimate limit of transistor speed!**

Data sheet for 2N2222 BJT

2N2222

Low Power Bipolar Transistors

multicomp

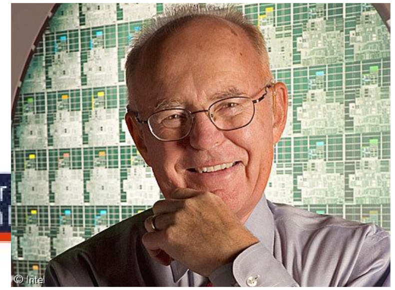
Electrical Characteristics (T_a = 25°C unless specified otherwise)

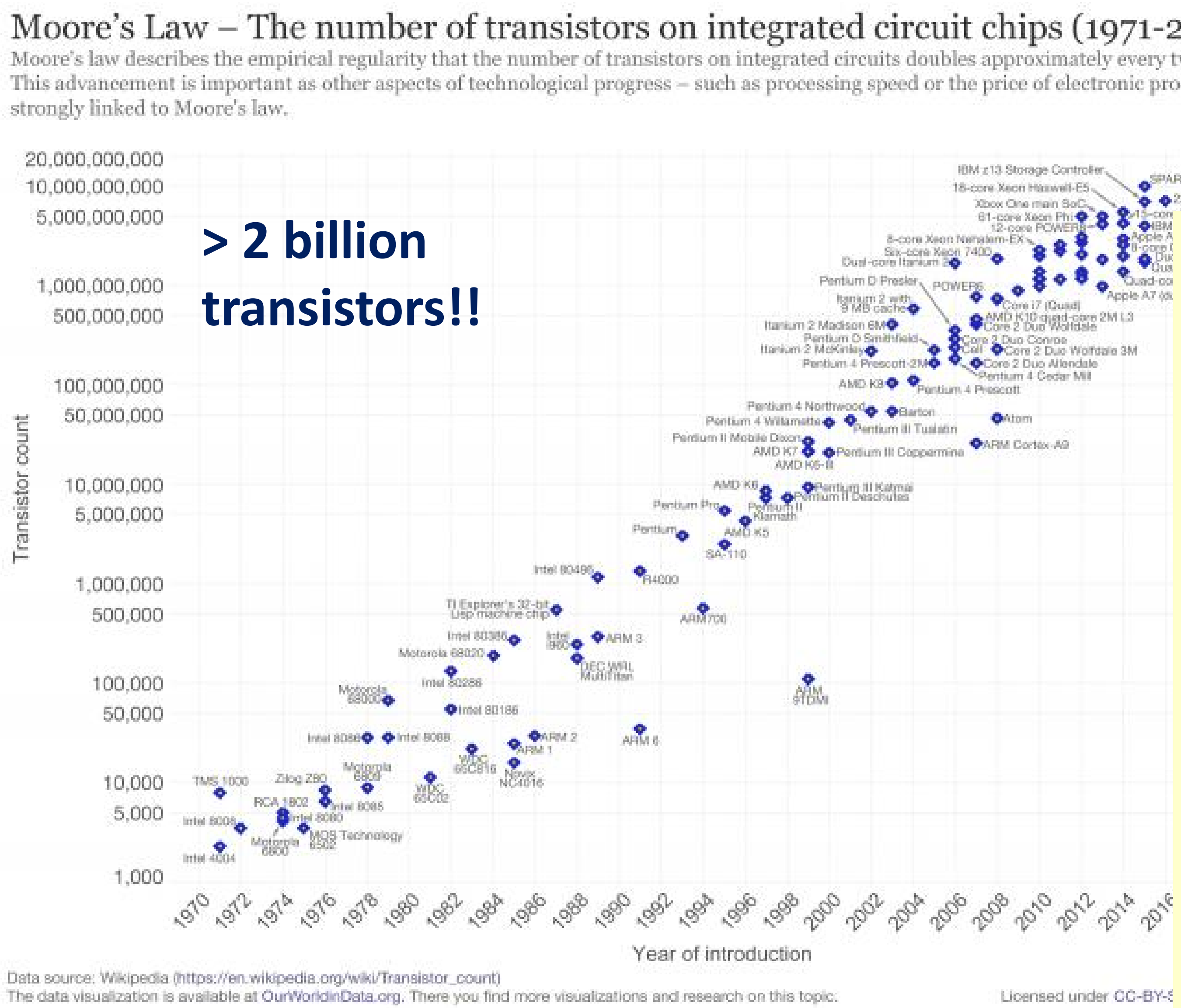
Parameter	Symbol	Test Condition	2N2222		Unit
			Minimum	Maximum	
DC Current Gain	h_{FE}	I_C = 0.1mA, V_{CE} = 10V* I_C = 1mA, V_{CE} = 10V I_C = 10mA, V_{CE} = 10V* I_C = 150mA, V_{CE} = 1V* I_C = 150mA, V_{CE} = 1V* I_C = 500mA, V_{CE} = 10V*	35 50 75 50 100 30	300	-
Dynamic Characteristics					
Transition Frequency	f_t	I_C = 20mA, V_{CE} = 20V f = 100MHz	250	-	MHz
		V_{CB} = 10V, I_E = 0		-	

Note that the f_T is quoted at a particular value of I_C – this is the maximum value

Moore’s Law

Intel





> 2 billion transistors!!

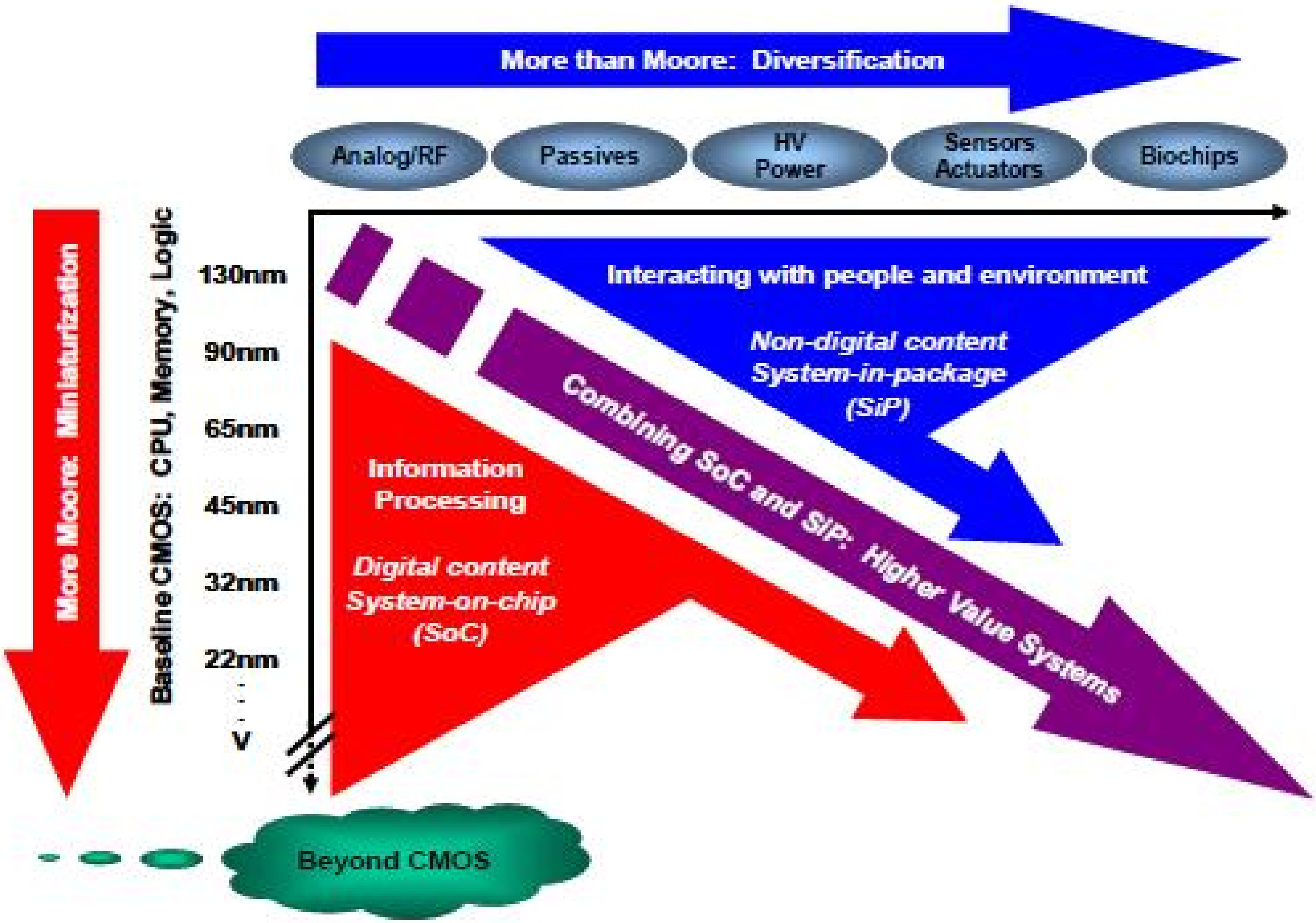
Mainly applies to microprocessors and memory

But analogue circuits tend to be produced alongside digital ones! (mixed signal)

Smaller devices, means smaller C_e, C_c

Higher frequencies!

The Roadmap: http://www.itrs.net/



End of Part 6...

- Next, we look at bandwidth of a voltage amplifier

Electronic circuits and systems
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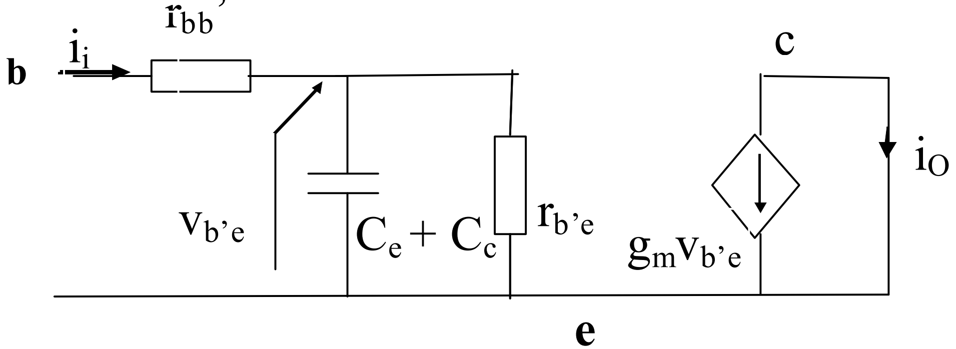
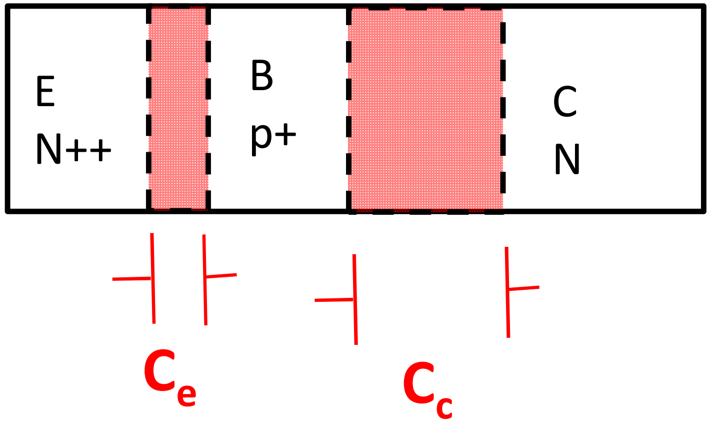
Part 7
Frequency response of amplifiers – II
Voltage gain

- What's the highest frequency for **voltage** amplification?
    - a model for amplifier bandwidth
- Design issues: what factors limit the bandwidth
- What are suitable circuit blocks for high-frequency, broadband amplifier operation?
    - And why...

Summary of last lecture

High frequency equivalent circuit

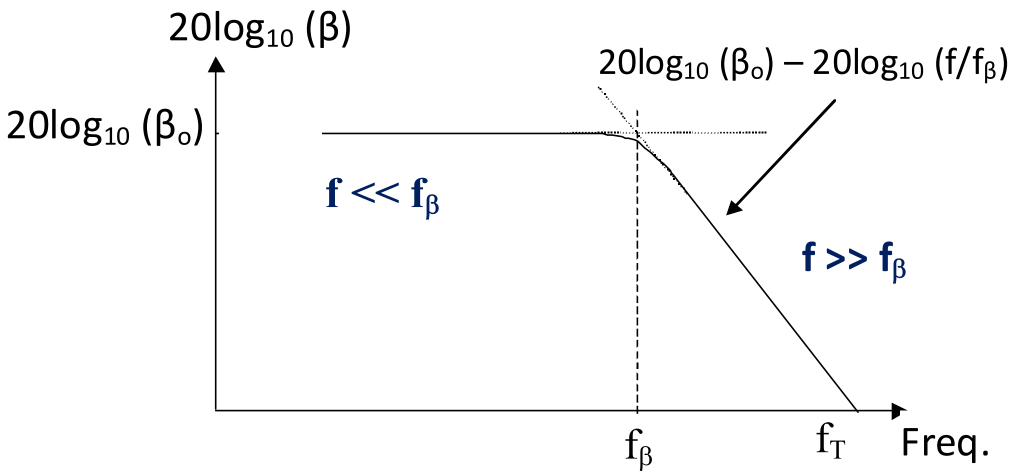




Bode plot

$$A_i = -\frac{\beta_o}{1 + j\left(f / f_\beta\right)}$$

bandwidth $$f_\beta = \frac{1}{2\pi r_{b'e}(C_e + C_c)}$$



Gain-bandwidth product (A_i = 1)

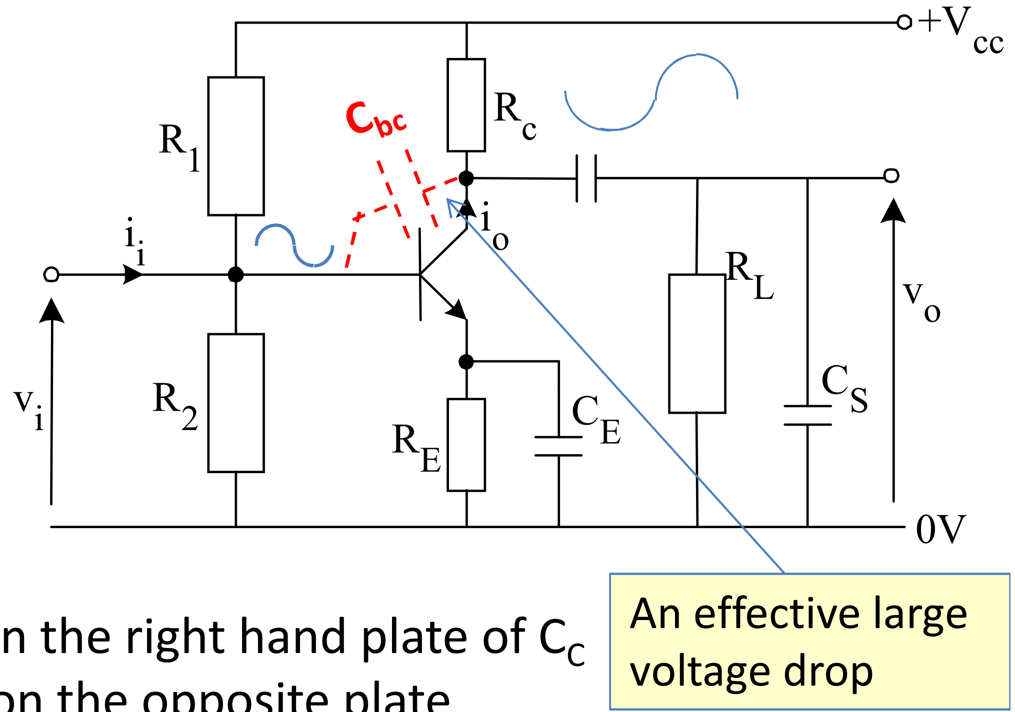
$$f_T = \frac{g_m}{2\pi\left(C_e + C_c\right)}$$

$$f_T = \frac{1}{2\pi\tau}$$

f_T is a figure of merit indicating the highest useful frequency for a transistor

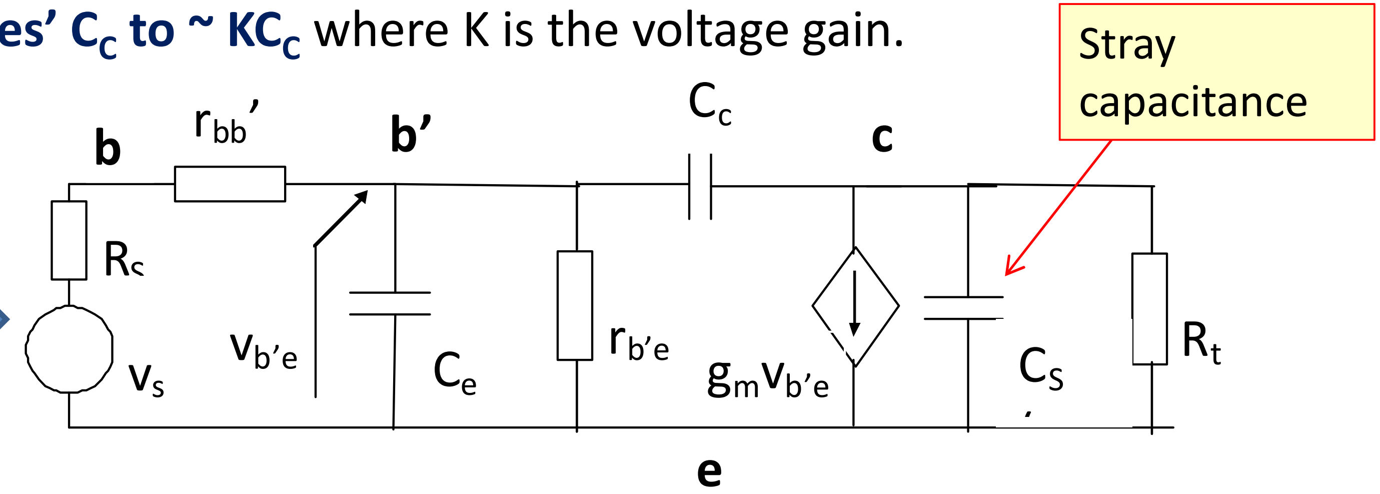
CE Voltage amplifier: A_V (f)

The presence of a collector load means that the effective collector capacitance C_c is greatly increased by the **Miller effect**



- The large (amplified) voltage signal on the right hand plate of C_C
- compared to the small input voltage on the opposite plate
- **effectively 'amplifies' C_C to ~ KC_C** where K is the voltage gain.

Equivalent circuit with transistor capacitances



Miller's Theorem

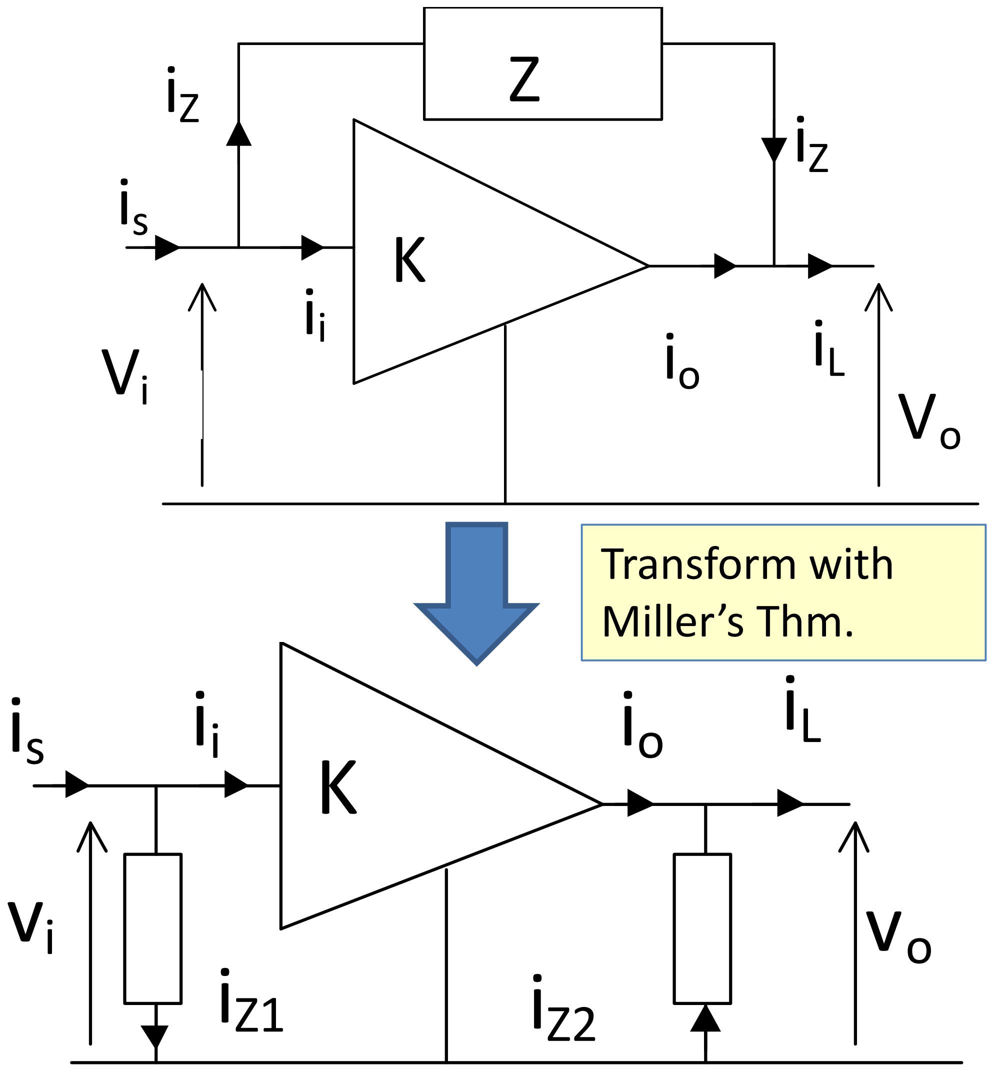
For the amplifier shown opposite

$$K = \frac{v_o}{v_i}$$

this configuration can be re-drawn as that shown underneath if the impedances are of value:

$$Z_1 = \frac{Z}{1-K}$$

$$Z_2 = Z\left[-\frac{K}{1-K}\right]$$



John Milton Miller (1882 - 1962)

- Noted American electrical engineer, best known for discovering the Miller effect and inventing fundamental circuits for quartz crystal oscillators (Miller oscillators).

- born in Hanover, Pennsylvania. In 1904 he graduated from Yale University, in 1907 he received an M.A. from Yale, and in 1915 he received his Ph.D. in physics from Yale.

- Miller was awarded the Distinguished Civilian Service Award in 1945 for:

  "initiation of the development of a new flexible radio-frequency cable urgently needed in radio and radar equipment which solved a desperate material shortage in the United States during World War II,"

- IRE Medal of Honor in 1953 for "his pioneering contributions to our basic knowledge of electron tube theory, of radio instruments and measurements, and of crystal controlled oscillators."

DEPENDENCE OF THE INPUT IMPEDANCE OF A THREE-ELECTRODE VACUUM TUBE UPON THE LOAD IN THE PLATE CIRCUIT

By John M. Miller

Published in IRE 1920

CONTENTS

. INTRODUCTION

In a ...er[1] was treated the theory of the use of a three-elect... ...um tube as an amplifier, showing the importance of the a... plification constant as determining the voltage amplification of the tube and the internal resistance of the tube in the plate or output circuit as determining the alternating current flowing in that circuit. A dynamic method was given for determining these important quantities directly.

The present paper is an extension of the theory and is concerned with the characteristics of the grid or input circuit. The input impedance of the tube is of importance in determining the input power and the voltage supplied to the input terminals of the tube by the apparatus in the input circuit.

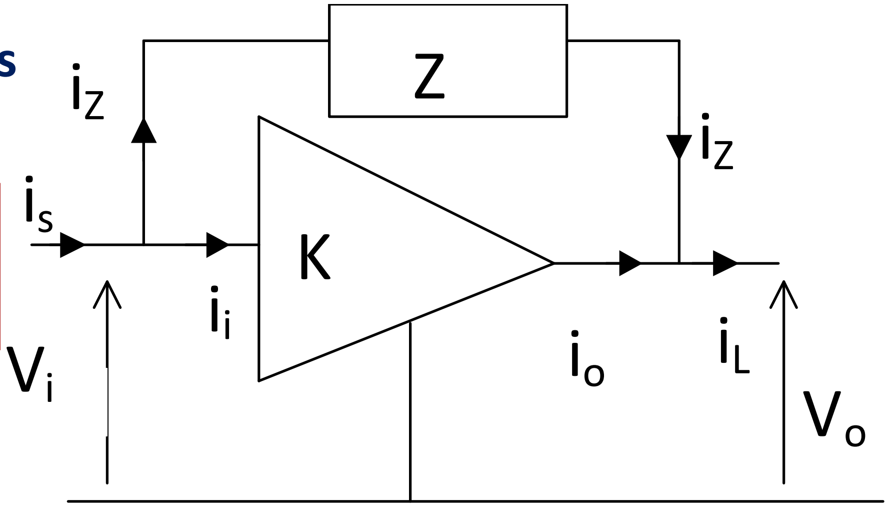
[1] Miller, Proc. I. R. E., 6, 141; 1918.

Proof of The Miller Theorem - 1

Consider the input side of the two amplifiers

$$i_S = i_Z + i_i \qquad i_S = i_{Z1} + i_i$$

$$\boxed{K = \frac{v_o}{v_i}}$$



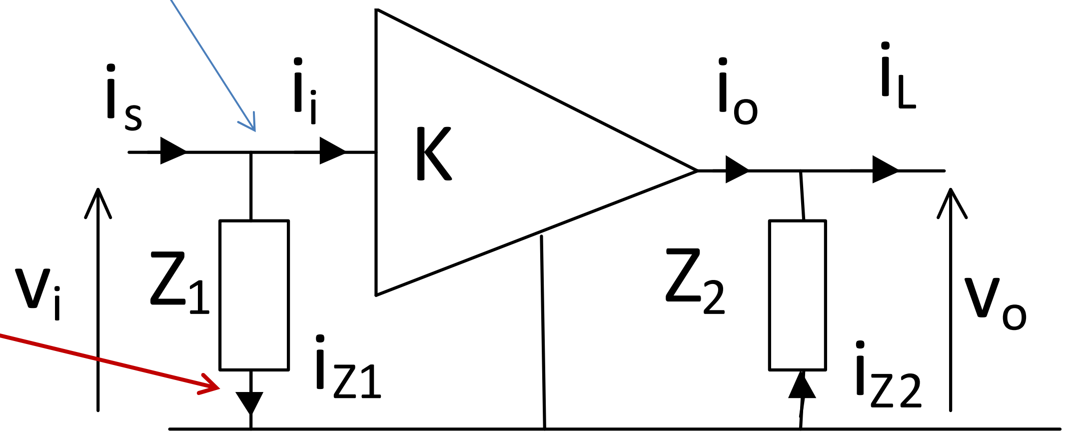
Top circuit

$$i_Z = \frac{v_i - v_o}{Z} = \frac{v_i}{Z}\left(1 - \frac{v_o}{v_i}\right)$$

$$i_Z = \frac{v_i}{Z}(1 - K)$$

if we make $Z_1 = \dfrac{Z}{1-K}$

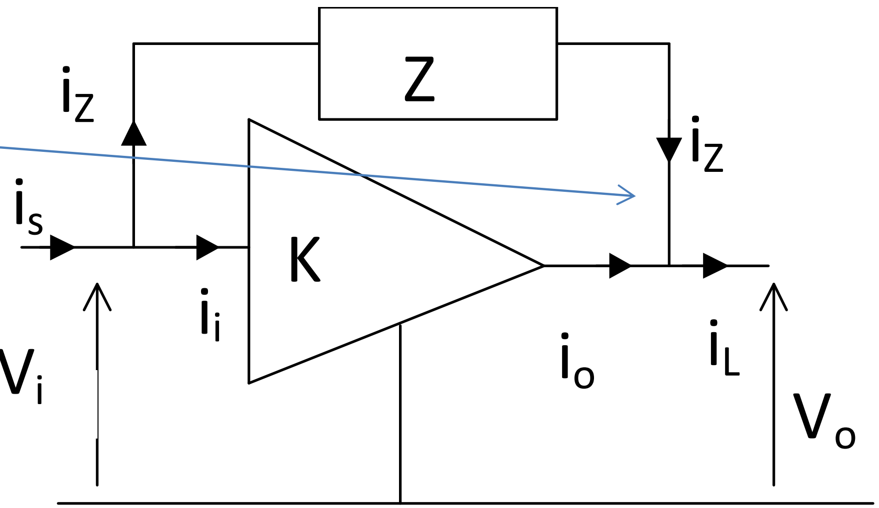
then we have $i_Z = \dfrac{v_i}{Z_1} = i_{Z1}$



and the currents on the input side of the 2 configurations are the same

Proof of The Miller Theorem - 2

Consider the output side of the two amplifiers



$$i_L = i_Z + i_o$$

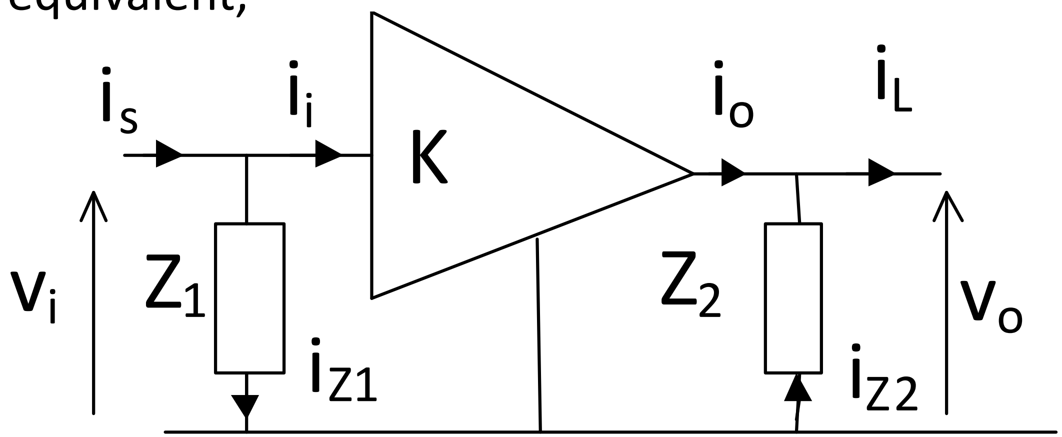
$$i_Z = \frac{v_i - v_o}{Z} = \frac{v_o}{Z}\left(\frac{v_i}{v_o} - 1\right)$$

$$i_Z = \frac{v_o}{Z}\left(\frac{1}{K} - 1\right)$$

$$\boxed{K = \frac{v_o}{v_i}}$$

require $i_{Z2} = i_Z$, to make the circuits equivalent,
That is,

$$-\frac{v_o}{Z_2} = \frac{v_o}{Z}\left(\frac{1}{K} - 1\right)$$

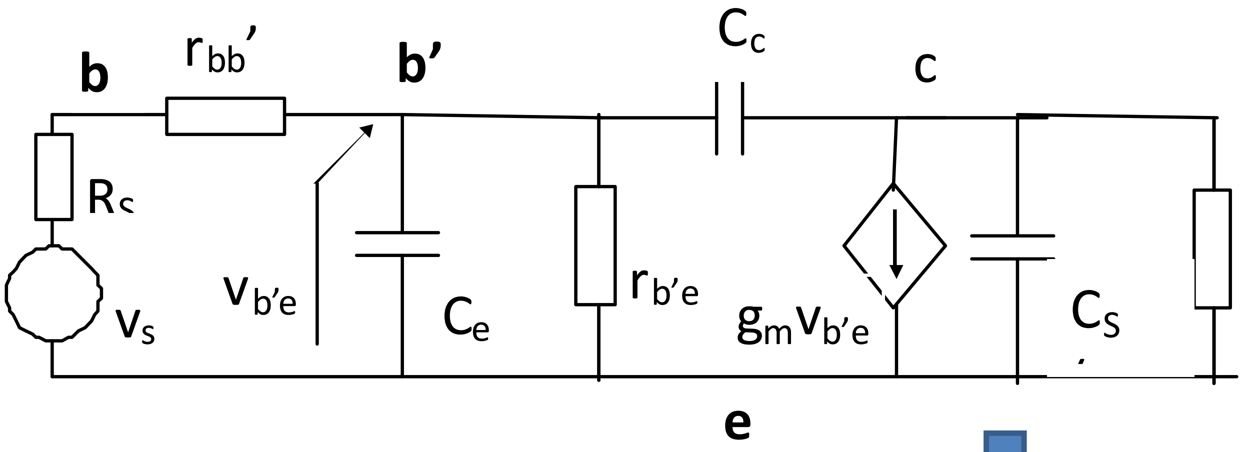


Re-arranging: $Z_2 = Z\left(-\frac{K}{1-K}\right)$

the currents on the output side of the 2 configurations are the same!

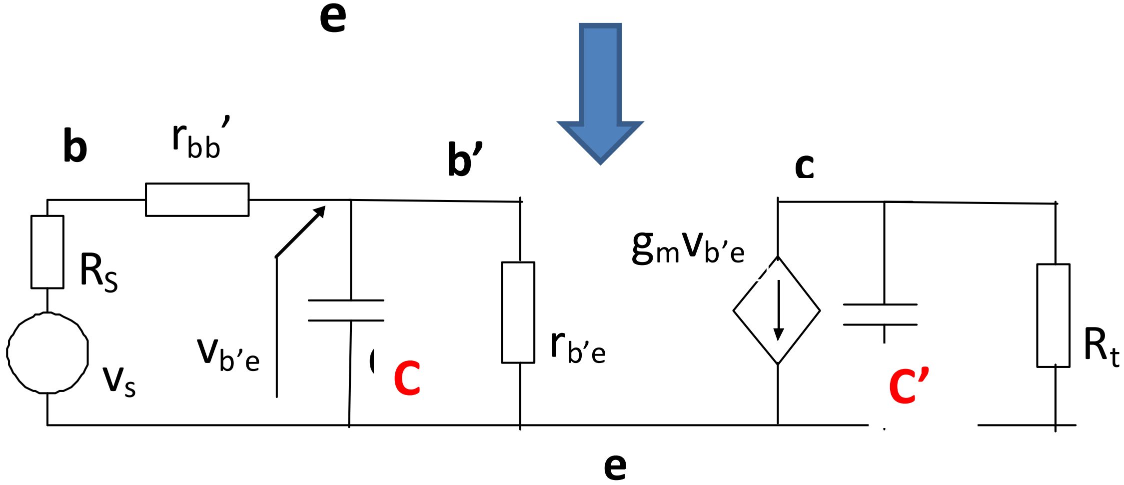
Apply Miller's Theorem

We use the Miller theorem to transform the equivalent circuit:



$$Z_C = \frac{1}{\omega \times C}$$

where



$\mathbf{C} = C_e + C_c(1 - K)$

$\approx C_e + C_c\,|K|$

$\mathbf{C'} = C_S + C_c(K - 1)/K$

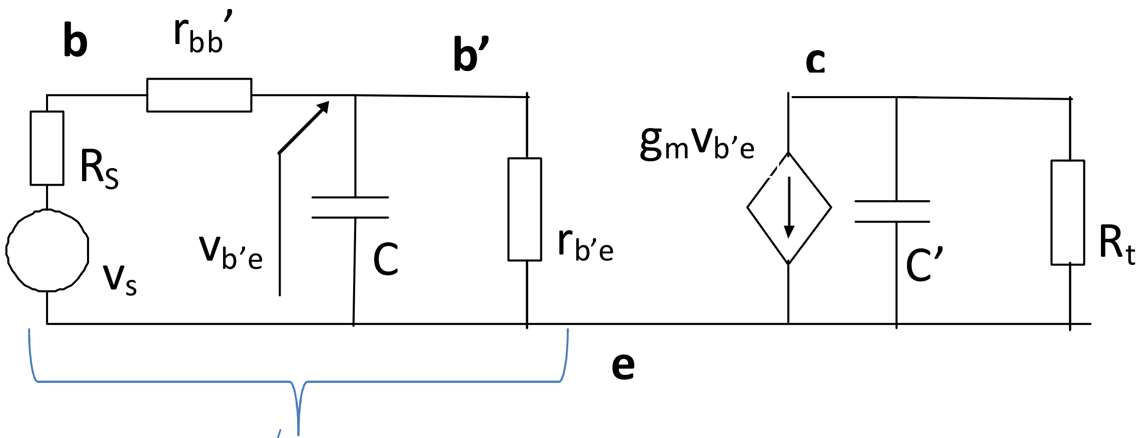
$\approx C_S + C_c$

because $K = g_m R_t >> 1$

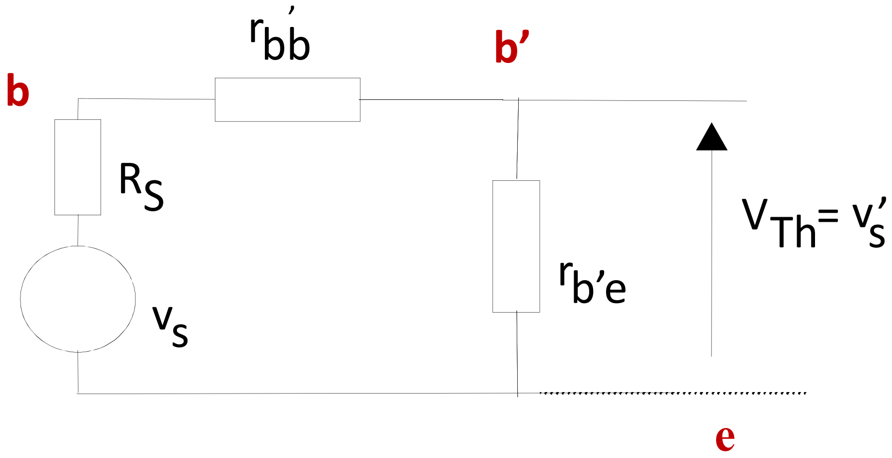
Much simpler to analyse!.....

Analysis of circuit - 1

the equivalent circuit comprises of two 1st-order networks: input and output circuits.



now convert the input circuit to a **Thevenin equivalent**. Removing C, have:



$$V_{TH} = v_s' = v_s \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_S}$$

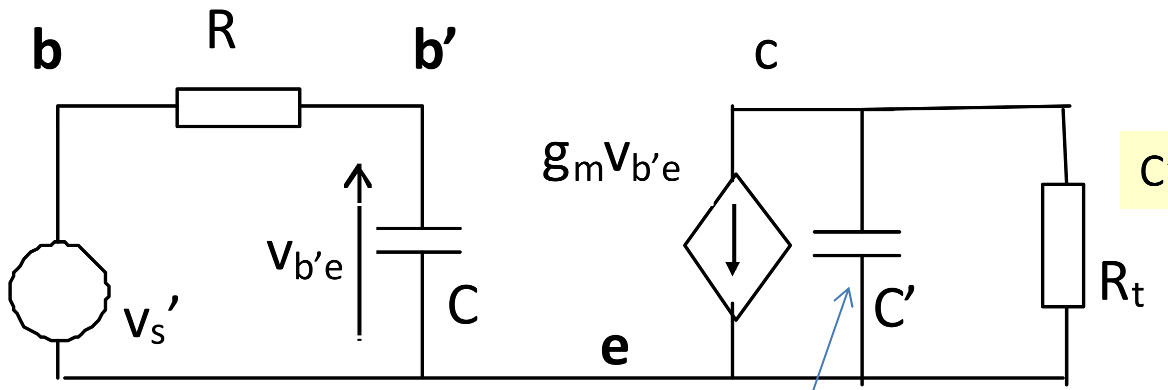
$$R_{TH} = r_{b'e} // \left(R_S + r_{bb'}\right)$$

Analysis of circuit - 2

$$v_s' = v_s \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_S}$$

$$R = r_{b'e} // (R_S + r_{bb'})$$

$C \approx C_e + C_c |K|$



$C' \approx C_S + C_c$

Both input and output sections of the amplifier are simple, first order circuits!
Consider the time constants ('RC') of the two first order circuits:

$$\tau_i = RC = [r_{b'e} // (R_S + r_{bb'})]C \qquad \tau_o = R_t C'$$

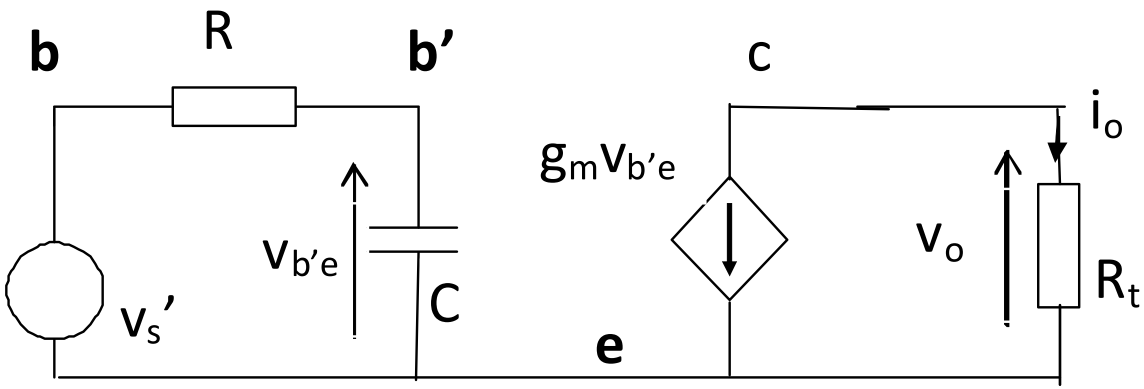
Generally the **input time constant is much greater than the output time constant** i.e., $\tau_i >> \tau_o$ so we can disregard the capacitor C' in the circuit.

Note than in any problem, it is important to justify the use of this approximation and this is easily done for given values of R_C, R_L, C_c etc.

Ohm's Law gives

$$i_o = -g_m v_{b'e} = v_o/R_t$$

$$v_o = -v_{b'e} g_m R_t \qquad (*)$$



Now **potential division** across R and C gives
$$v_{b'e} = v_s' \frac{1/j\omega C}{R + 1/j\omega C} = v_s' \frac{1}{1 + j\omega RC}$$

where we have multiplied top and bottom by $j\omega C$

Now substitute into (*).
$$v_o = -v_s' \frac{1}{1 + j\omega RC} g_m R_t$$

$$v_s' = v_s \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_S}$$
$$R = r_{b'e} // (R_S + r_{bb'})$$

Substituting for v_s' and R (see last slide) and re-arranging:
we get the desired expression for voltage gain in the standard form:

$$\boxed{A_{Vs} = \frac{v_o}{v_s} = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s} \frac{1}{1 + j\left(\frac{f}{f_H}\right)}}$$

$$f_H = \frac{1}{2\pi[r_{b'e} // (r_{bb'} + R_S)]C}$$

voltage gain bandwidth.

Interpretation

$$A_{Vs} = \frac{v_o}{v_s} = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s} \frac{1}{1 + j\left(\frac{f}{f_H}\right)}$$

The low frequency gain ($f \ll f_H$) is $A_{Vs} = \frac{v_o}{v_s} = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s}$

- Notice that, if $r_{bb}' = 0$ and $R_S = 0$, then $\mathbf{A_{Vs} = -g_m R_t}$:
  an approximation we have usually used!

- The more exact expression shows the need for a **low source resistance (R_S)** to achieve high voltage gain, that is a coupling factor close to unity.

  A low source resistance however can have implications for other properties of the amplifier such as distortion but that is beyond the scope of this course.

Approximations used

$$f_H = \frac{1}{2\pi\left[r_{b'e} // \left(r_{bb}' + R_S\right)\right]\left(C_e + C_c\left|K\right|\right)}$$

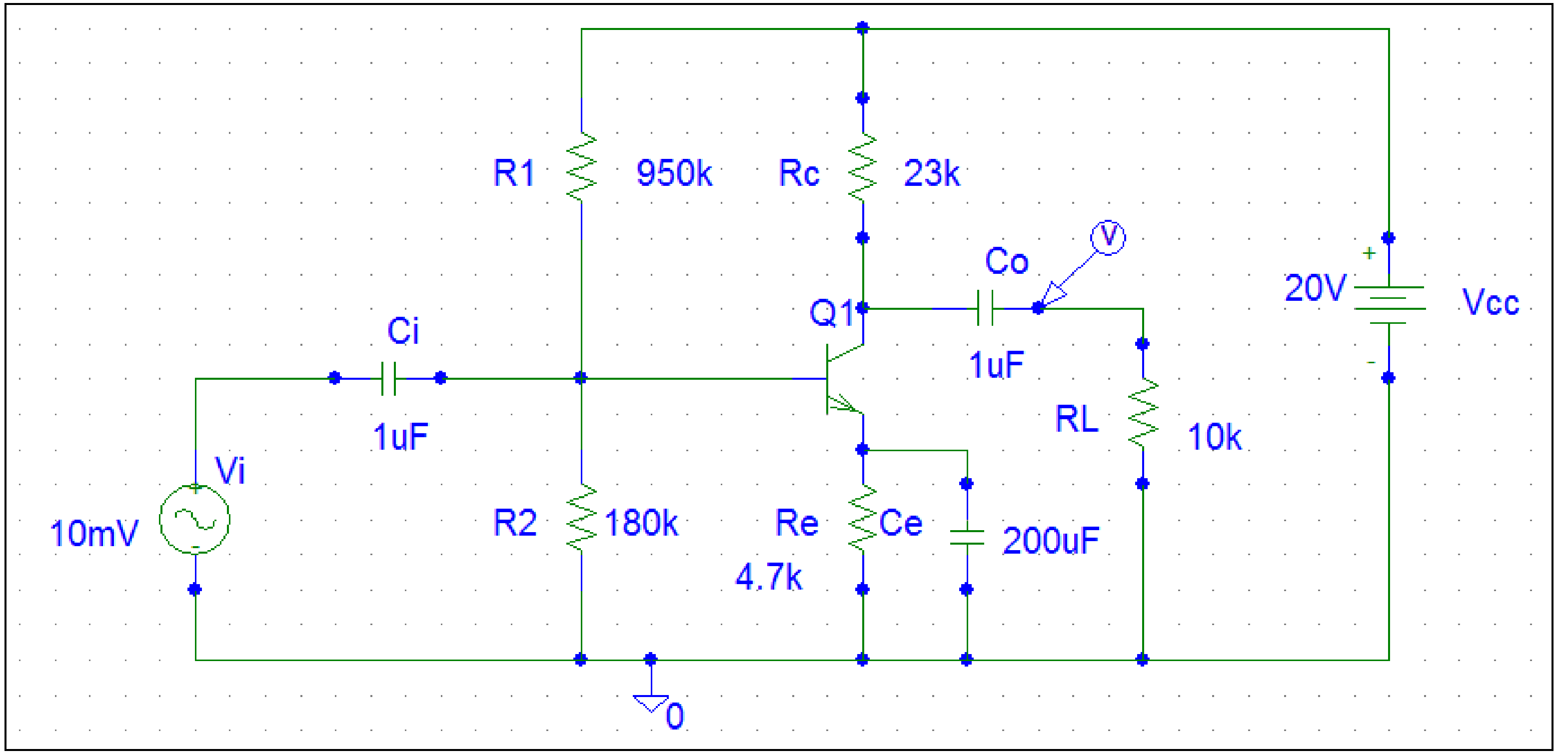
- An approximation inherent in our use of the Miller theorem is that the input capacitance is overestimated because we have used the ***low frequency*** expression for '**K**' although we are analysing up to **high frequencies.**
  - **This approximation leads to an *under-estimate* of $f_{H.}$**
- However, we also neglected the output time constant and this results in an ***over-estimate* of $f_{H.}$**

- The net effect of these approximations is that we ***over-estimate* the bandwidth f_H by 5-10%** (i.e. get a bigger value). This is often accurate enough for our purposes. Circuit simulators such as SPICE can provide a more accurate analysis.

PSPICE simulation

$$A_{Vs}=\frac{v_o}{v_s}=-g_m R_t \frac{r_{b'e}}{r_{b'e}+r_{bb'}+R_s}\frac{1}{1+j\left(\frac{f}{f_H}\right)}$$

$$f_H=\frac{1}{2\pi[r_{b'e}//(r_{bb}'+R_S)](C_e+C_c|K|)}$$

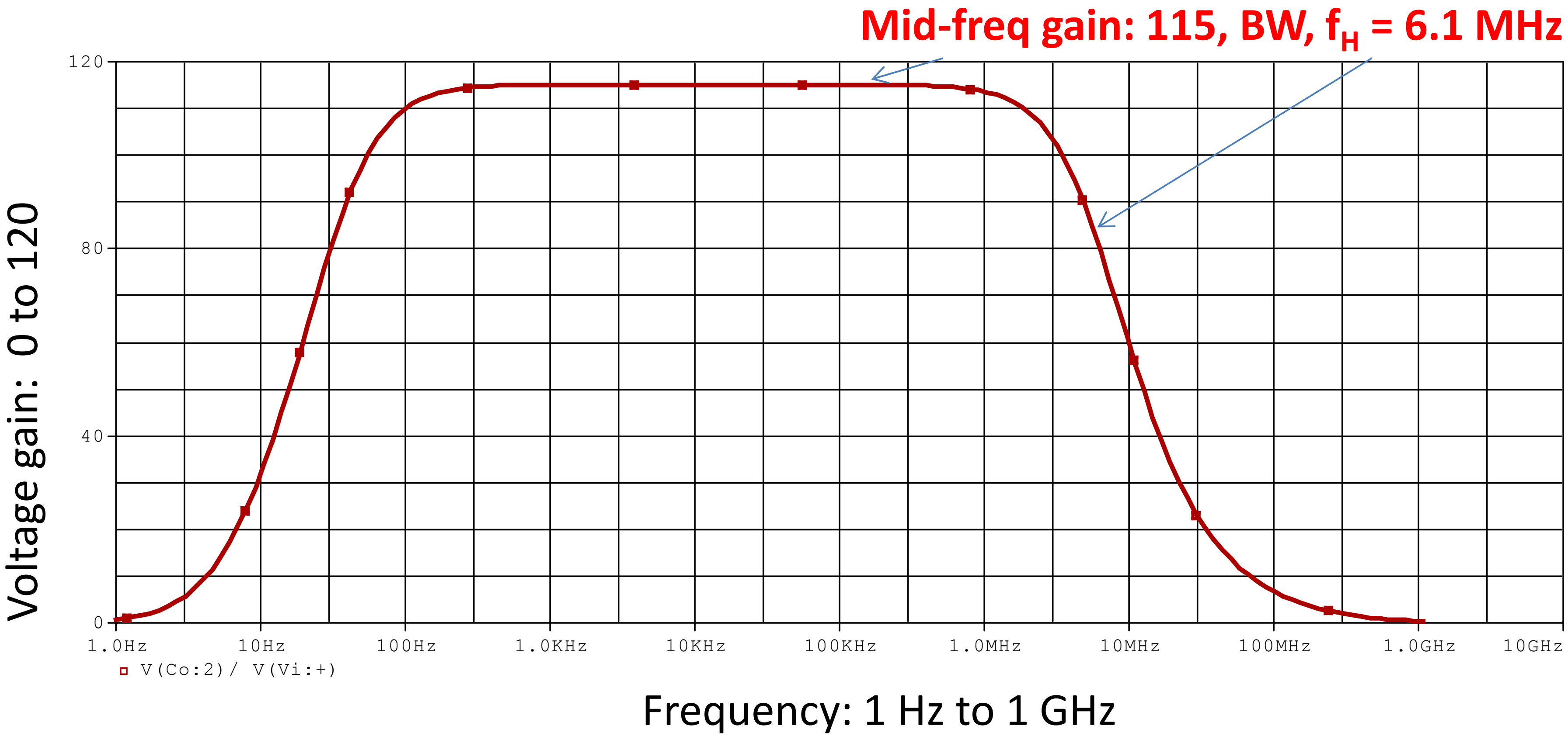


From transistor SPICE netlist: C_{be} = 52 pF, C_{bc} = 7 pF, $r_{bb'}$ = 23 Ω, β_o = 95 (at I_C= 0.5 mA)

I_C = 0.45 mA, so g_m = 18 mA/V, $r_{be'}$ = 2.5 kΩ, R_t = R_C //R_L = 7 kΩ, R_S = 0

K=125, f_H = 7.5 MHz

Results



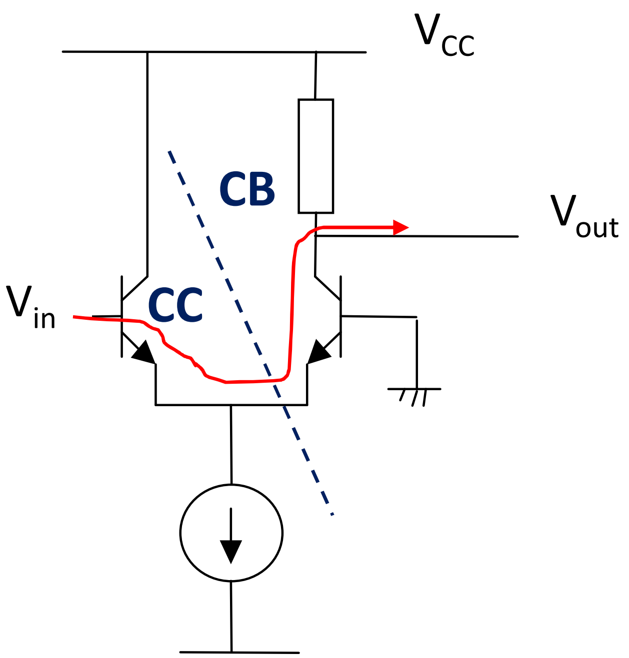
Theory: K=125, f_H = 7.4 MHz

Simul: K=115, f_H = 6.1 MHz

Two examples of **high bandwidth** circuits

$$f_H = \frac{1}{2\pi\left[r_{b'e} // (r_{bb'} + R_S)\right]\left(C_e + C_c|K|\right)}$$

- **Miller effect** causes a considerable reduction in bandwidth as it 'amplifies' the capacitor C_c by the voltage gain of the amplifier.
- R_S also reduces bandwidth.
- Below are two circuits which **reduce Miller effect** and so have greatly increased bandwidth:



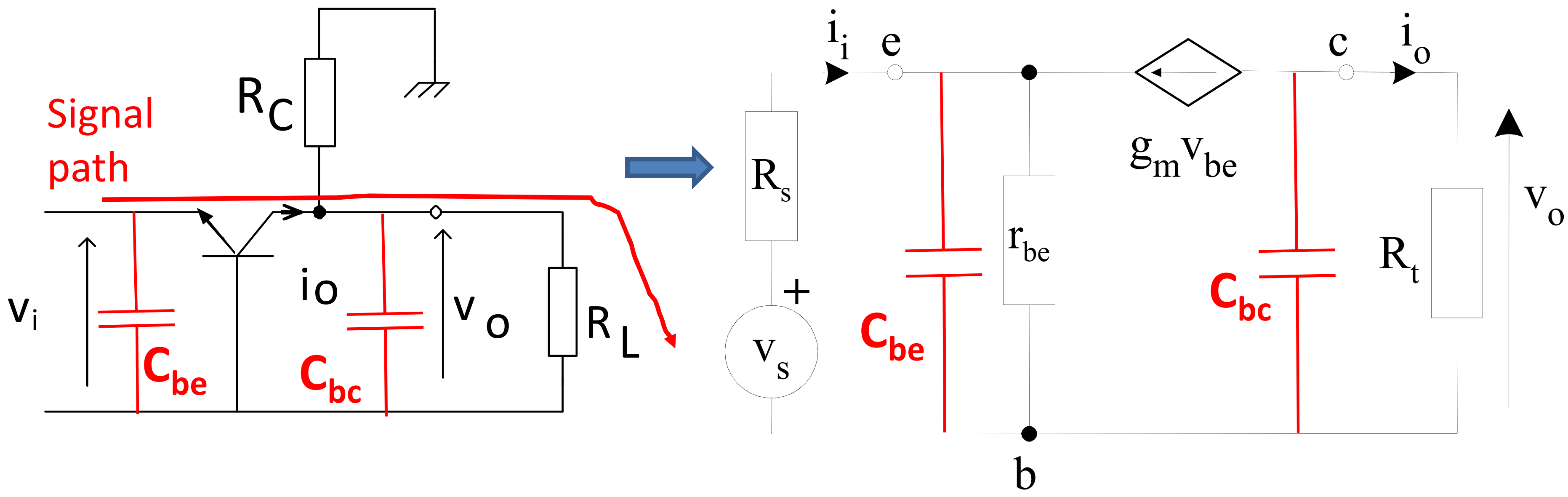
We have seen (Part 3, problem 5) that this differential amplifier configuration can be considered a two-stage, d.c. coupled amplifier: CC followed by CB.

CC is inherently **immune to Miller Effect** because it has less than unity voltage gain (K < 1)!

The CC also presents a low impedance into the CB stage. CB does not experience Miller Effect **(why?).**

Common-base equivalent circuit

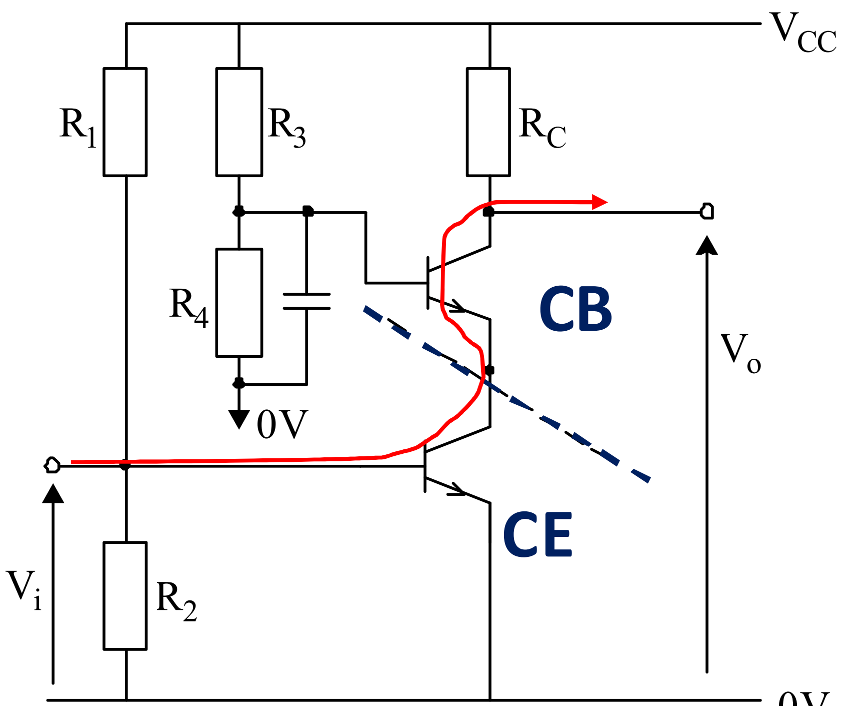
CB does not experience Miller Effect **(why?).**



C_{be}, C_{bc} are not directly in the signal path! (unlike CE amp)

So don't get 'Miller multiplied'!

‘Cascode’configuration



CE stage with a CB stage in the collector load. The CE stage ‘sees’ the small input resistance of the CB stage as its load (R_i (CB) ~ $1/g_m$). There is therefore very little change in collector voltage for the CE and hence no Miller Effect!

$$A_{VCE} \sim -g_m R_t = -g_m \times r_e = -1 \quad (r_e \equiv 1/g_m)$$

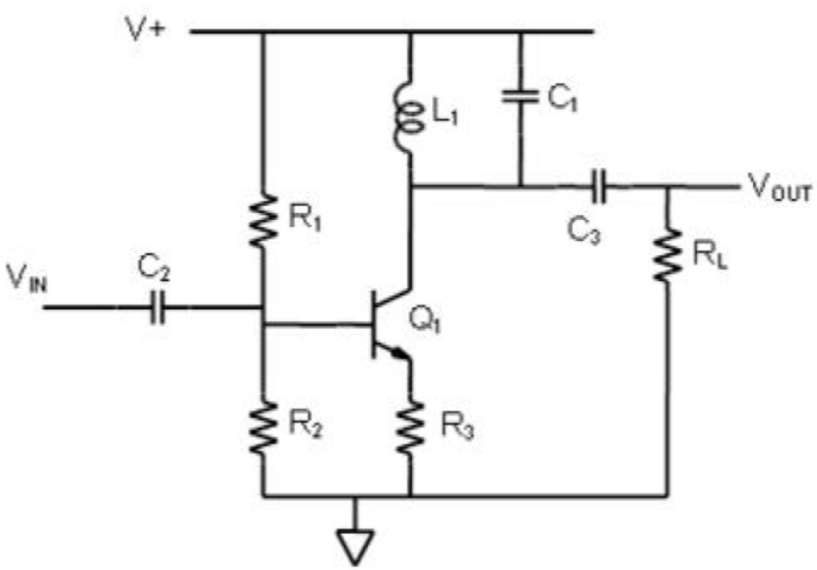
The collector current however, passes through to the top transistor and hence the output voltage is developed across R_C. As before, the CB does not suffer from Miller Effect.

Again, this can be analysed as a multi-stage amplifier; CE followed by CB. Note that the resistors $R_1 - R_4$ provide the biasing and the capacitor provides an a.c. ground on the base of CB.

Voltage gain of the CB stage is $A_{VCB} \sim g_m R_t = g_m \times R_C$

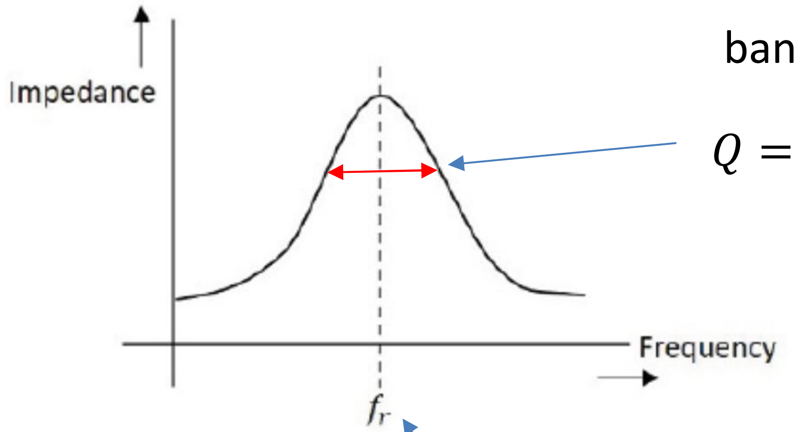
So overall gain is $A_V = A_{VCE} \times A_{VCB} = -1 \times g_m R_C = -g_m R_C$

Tuned amplifier



$A_V = g_m \times$ a.c.load

$$Z_R = \frac{L}{CR}$$



bandwidth

$$Q = \frac{2\pi f_r L}{R}$$

Resonant frequency $f_r = \dfrac{1}{2\pi\sqrt{L \times C}}$

Parallel LC circuit exhibits high impedance at resonant frequency – high gain

Narrow bandwidth (Q) makes amplifier highly selective to a band of frequencies

Just amplify the frequencies you want – power efficient

Electronic circuits and systems
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Part 7 (contd.)
Frequency response of amplifiers – II
Voltage gain

- What’s the highest frequency for **voltage** amplification?
    - a model for amplifier bandwidth
- Design issues: what factors limit the bandwidth
- What are suitable circuit blocks for high-frequency, broadband amplifier operation?
    - And why...

Phase relationships - 1

Write **Voltage gain** as

$$A_{Vs} = \frac{v_o}{v_s} = -\frac{A_{Vso}}{1 + j\dfrac{f}{f_H}} \qquad A_{Vso} = g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s}$$

to find phase information between v_o and v_s, convert to polar co-ordinates:

$$A_{Vs} = -\frac{A_{Vso}}{1 + j\dfrac{f}{f_H}} \frac{1 - j\dfrac{f}{f_H}}{1 - j\dfrac{f}{f_H}}$$

Multiply top and bottom by same factor

$$A_{Vs} = -\frac{A_{Vso}}{1 + \left(\dfrac{f}{f_H}\right)^2}\left(1 - j\frac{f}{f_H}\right) = -\frac{A_{Vso}}{\sqrt{1 + \left(\dfrac{f}{f_H}\right)^2}}\left(\frac{1}{\sqrt{1 + \left(\dfrac{f}{f_H}\right)^2}} - \frac{j\dfrac{f}{f_H}}{\sqrt{1 + \left(\dfrac{f}{f_H}\right)^2}}\right) \quad *$$

$$x = \sqrt{x}.\sqrt{x}$$

Phase relationships - 2

$$A_{Vs} = -\frac{A_{Vso}}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}}\left(\frac{1}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}} - \frac{j\frac{f}{f_H}}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}}\right) \quad *$$

$$\cos\phi - j\sin\phi = e^{-j\phi}$$

$$\cos\phi = \frac{1}{\sqrt{1+\left(\frac{f}{f_h}\right)^2}}$$

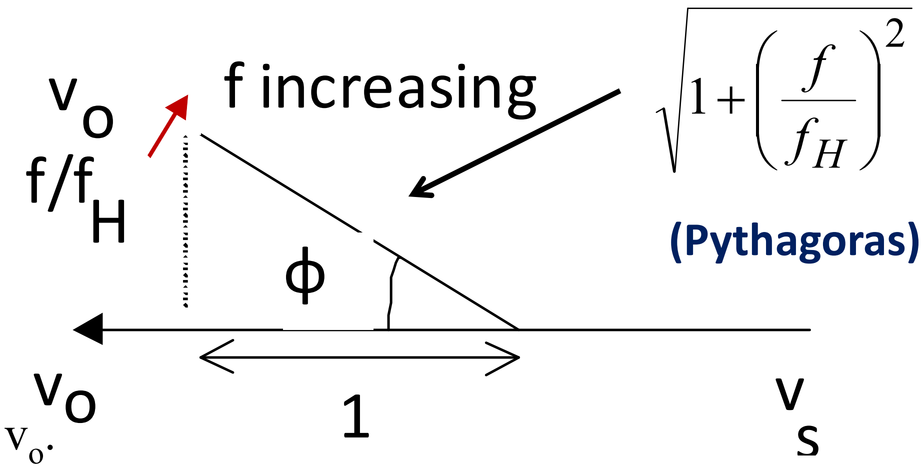
$$\sin\phi = \frac{f/f_H}{\sqrt{1+\left(\frac{f}{f_h}\right)^2}}$$

This can be represented in the standard form:

$$A_{Vs} = \frac{A_{Vso}}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}}\underbrace{e^{j(\pi-\phi)}}_{\text{This term = 1}}$$

where $\tan(\phi) = \frac{f}{f_H}$

and ϕ gives the phase angle between v_s and v_o.



$$e^{j\pi}.e^{-j\phi} = e^{j(\pi-\phi)}$$ *$e^{j\pi}$ = -1 and this accounts for the -ve sign in front of the gain* (Eqn *)

magnitude of $e^{j(\pi-\phi)}$ = 1, *so the exp. term does not affect the overall magnitude of the gain.*

Interpretation

$$A_{Vs} = \frac{A_{Vso}}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}} e^{j(\pi-\phi)}$$

where $\tan(\phi) = \frac{f}{f_H}$

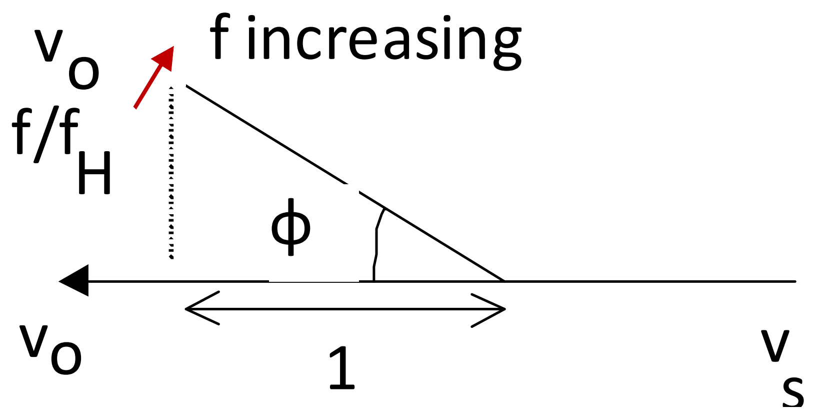
Gain term: $A_{Vs}(f)$

Phase term

ϕ is phase angle between v_s and v_o.

magnitude of $e^{j(\pi - \phi)}$ = 1!

Use of complex numbers has allowed us to split A_{Vs} into a magnitude term and a phase term!



Bode plots

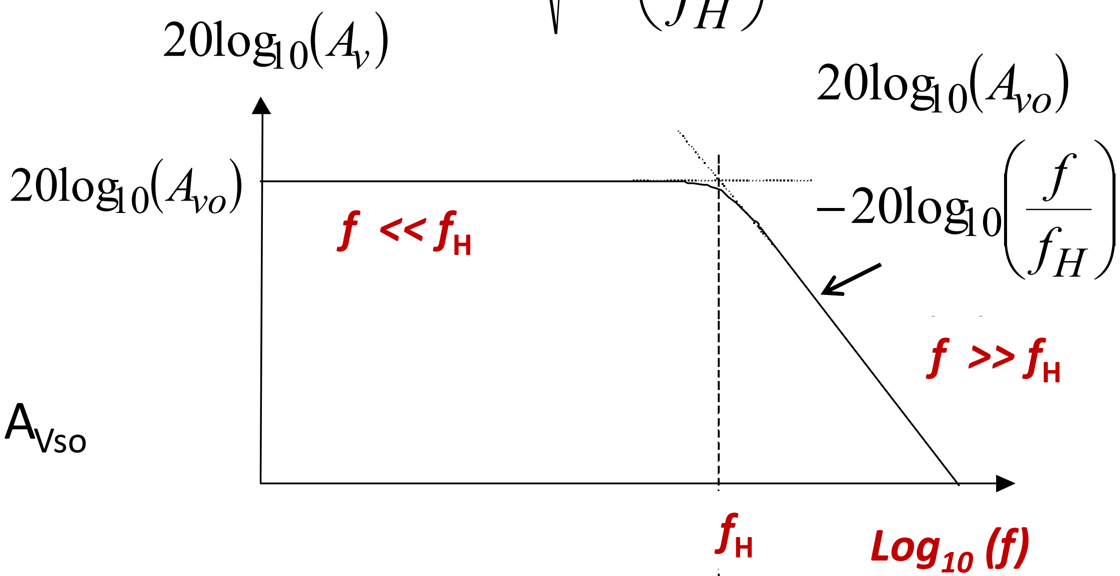
$$A_{Vs} = \frac{A_{Vso}}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}} e^{j(\pi-\phi)}$$

$$\boxed{\tan(\phi) = \frac{f}{f_H}}$$

***f* = 0 (or *f* << f_H) ,**

tan (ϕ) = 0 hence ϕ = 0°

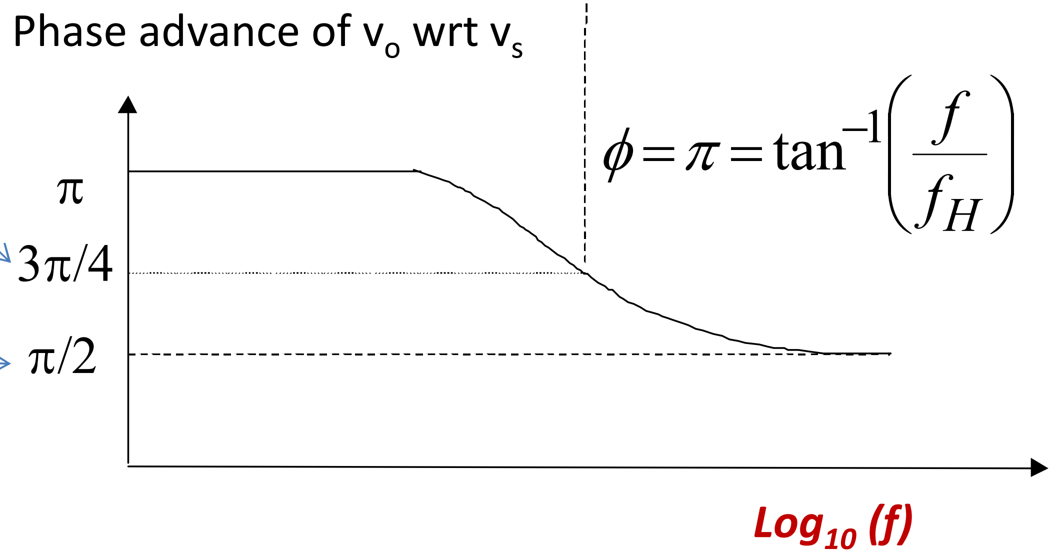
$e^{j(\pi-\phi)} = e^{j(\pi)} = -1$, hence $A_{Vs} = -A_{Vso}$



***f* = f_H,**

tan (ϕ) = 1 hence ϕ = 45° (π/4)

$A_{Vs} = -A_{Vso}/\sqrt{2}$



***f* = ∞,**

ϕ = 90° (= π/2) and A_{Vs} = 0 as required.

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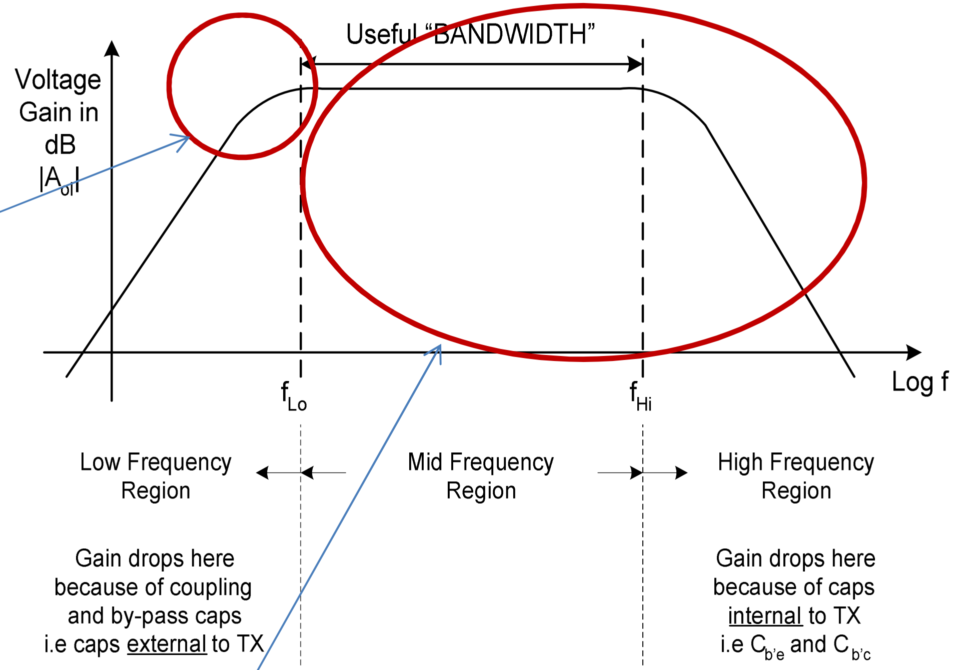
Part 7: Frequency response of Voltage gain (contd.)

Now this part Low freq roll-off



$$A_{Vs} = \frac{A_{Vso}}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}} e^{j(\pi-\phi)} \quad \text{where} \quad \tan(\phi) = \frac{f}{f_H}$$

Phase term

ϕ is phase angle between v_s and v_o.

Gain term: $A_{Vs}(f)$

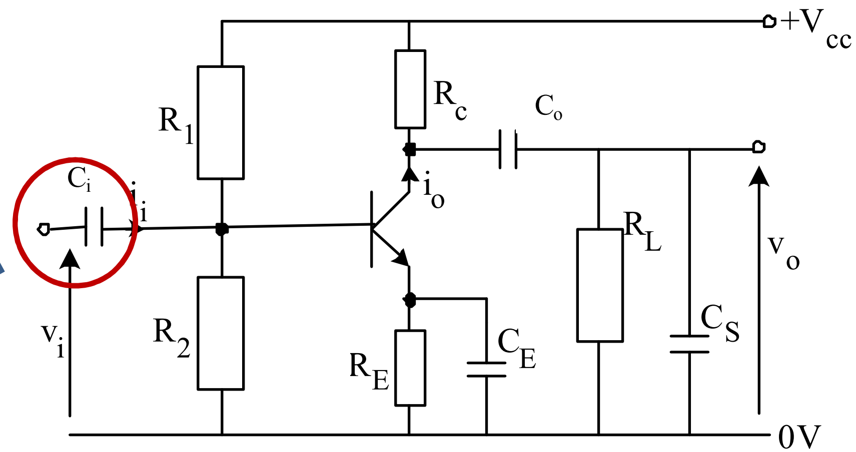
magnitude of $e^{j(\pi - \phi)} = 1$!

Design: calculate capacitor values to give a 100 Hz roll-off

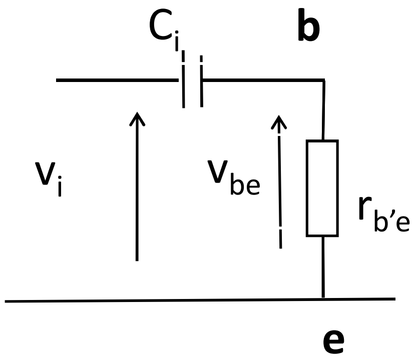
Q: 3 capacitors; which one should we use to set the roll-off?

A: C_E - the others (C_o and C_i)are for ac coupling (block the DC current)

Method: consider each capacitor in turn (assume the other 2 are s/c



Assume $R_{1,2} \gg r_{be}$



$$\frac{v_{be}}{v_i} = \frac{r_{be}}{r_{be} + \frac{1}{j\omega \times C_i}} = \frac{1}{1 + \frac{1}{j\omega \times r_{be} \times C_i}}$$

$$\left|\frac{v_{be}}{v_i}\right| = \frac{1}{\left[1 + \left(\frac{1}{\omega \times r_{be} \times C_i}\right)^2\right]^{1/2}}$$

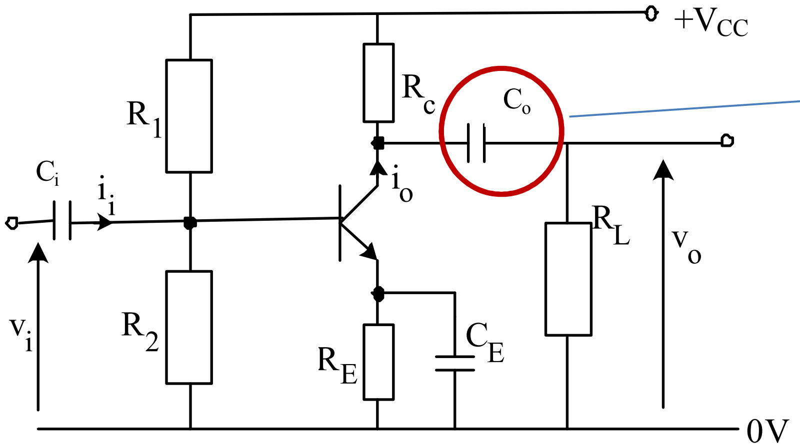
This term falls to 1/√2, when

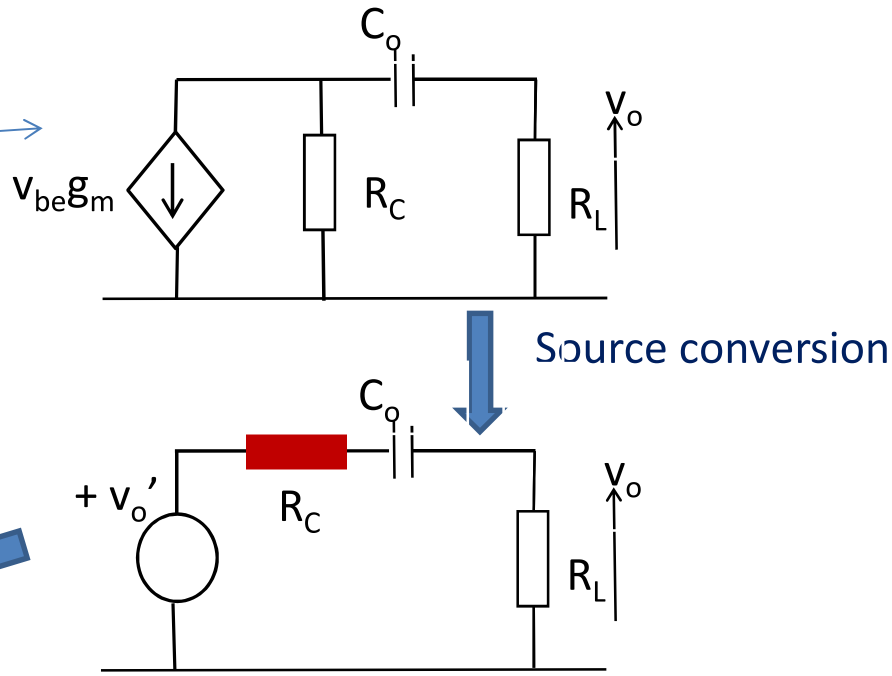
$$\omega \times C_i \times r_{be} = 1$$

$$\boxed{C_i = \frac{1}{2 \times \pi \times 100 \times r_{be}}}$$

(for 100 Hz cut-off)

Design: calculate capacitor values to give a 100 Hz roll-off





$$\frac{v_o}{v_o'} = \frac{R_L}{R_L + R_C + \dfrac{1}{j\omega \times C_o}}$$

$$= \frac{R_L}{R_L + R_C} \frac{1}{1 + \dfrac{1}{j\omega \times (R_L + R_C) \times C_o}}$$

$$\left|\frac{v_o}{v_o'}\right| = \frac{R_L}{R_L + R_C} \frac{1}{\left[1 + \left(\dfrac{1}{\omega \times (R_L + R_C) \times C_o}\right)^2\right]^{1/2}}$$

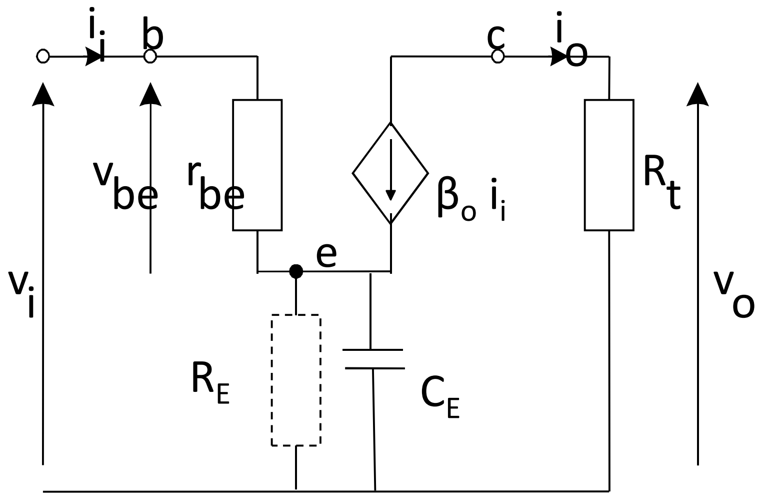
This term falls to $1/\sqrt{2}$, when

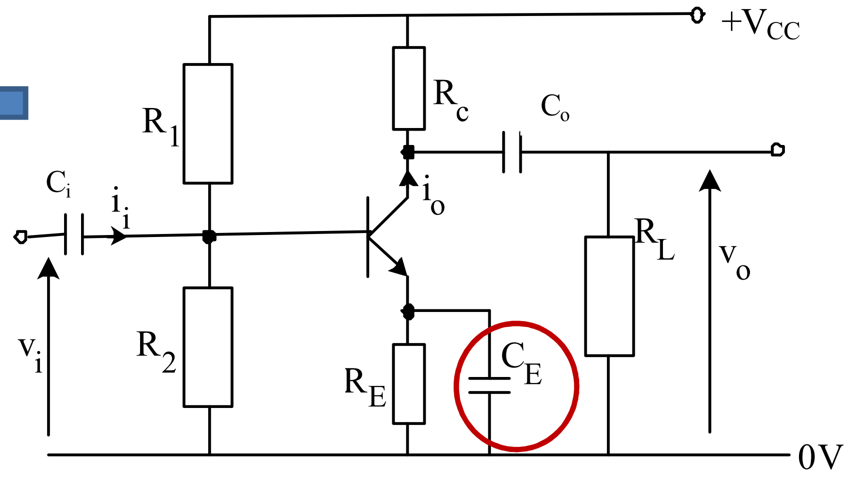
$$\omega \times (R_L + R_C) \times C_o = 1$$

$$\boxed{C_o = \frac{1}{2 \times \pi \times 100 \times (R_L + R_C)}}$$

(for 100 Hz cut-off)

C_E: To keep it simple, assume that the impedance of C_E , Z_{CE} << R_E at the cut-off frequency.





$$v_i = i_i r_{be} + (1+\beta_o)\times i_i \times \frac{1}{j\omega C_E}$$

$$\Rightarrow \quad i_i = \frac{v_i}{r_{be} + \dfrac{1+\beta_o}{j\omega \times C_E}}$$

Now $\quad v_o = -\beta_o i_i R_t$

Therefore the voltage gain is

$$\frac{v_o}{v_i} = -\frac{\beta_o R_t}{r_{be} + \dfrac{1+\beta_o}{j\omega \times C_E}} \quad \Rightarrow \quad \frac{v_o}{v_i} = -\frac{\beta_o R_t / r_{be}}{1 + \dfrac{1+\beta_o}{j\omega \times r_{be} \times C_E}}$$

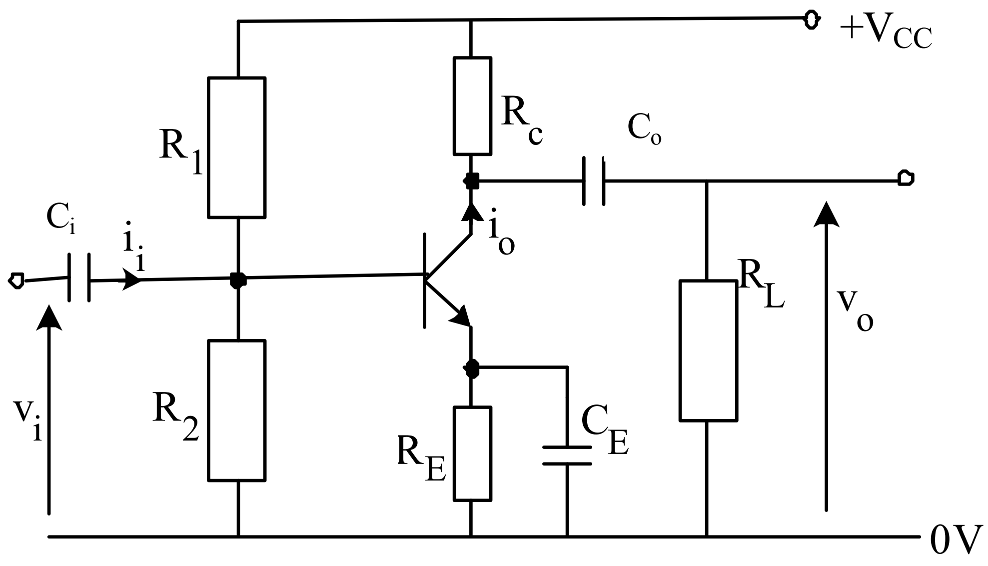
The magnitude of the gain will drop by 1/√2, when $\quad \dfrac{1+\beta_o}{\omega \times r_{be} \times C_E} = 1$

$$C_E = \frac{1+\beta_o}{2\times \pi \times f_o \times r_{be}}$$

f_o =100 Hz

Finally

- Multiply calculated values of C_i and C_o by a factor of 10 (say) so that they DON'T influence the roll-off,
- The roll-off is therefore controlled solely by C_E



End of lecture

- Next – field effect devices

Worked example

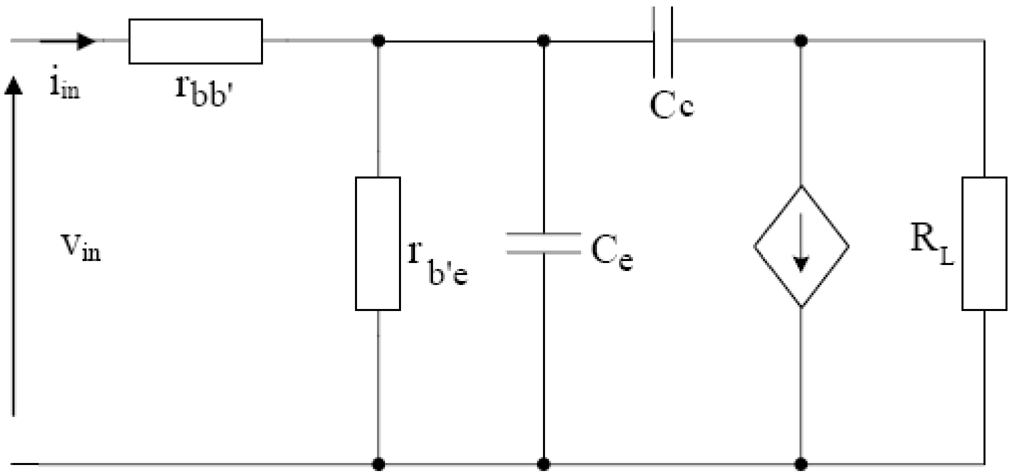
A. For the common emitter amplifier equivalent circuit shown, apply Miller's Theorem and hence show that the input impedance is given by:

$$Z_{in} = \frac{v_{in}}{i_{in}} = (r_{b'e} + r_{bb'})\frac{1 + j\dfrac{f}{f_1}}{1 + j\dfrac{f}{f_2}}$$

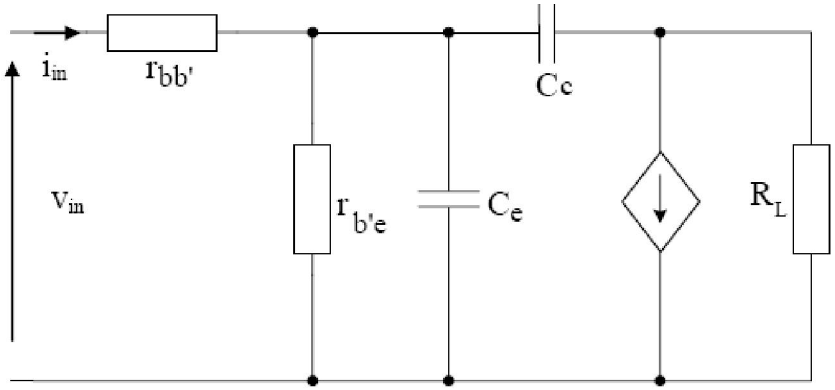
where $f_1 = \dfrac{1}{2\pi RC}$

$$f_2 = \frac{1}{2\pi r_{b'e} C}$$

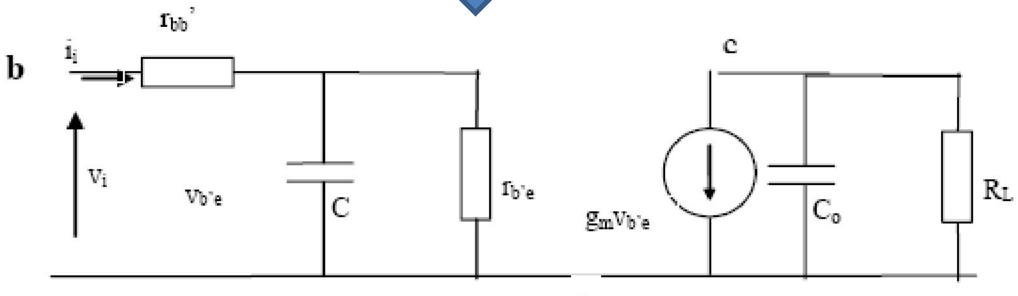
with $R = r_{b'e} // r_{bb'}$, $C = C_e + A_{Vo} C_c$, and A_{Vo} is the mid-frequency voltage gain.



Solution



Apply Miller's Th.



where $C \approx C_e + |g_m R_L|\ C_c$

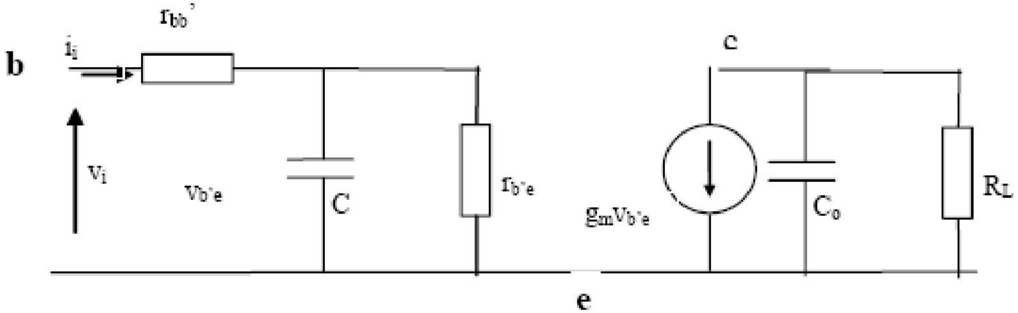
$$v_i = i_i\left(r_{bb'} + \frac{r_{b'e}/j\omega C}{r_{b'e} + 1/j\omega C}\right) \quad \text{so} \quad \frac{v_i}{i_i} = r_{bb'} + \frac{r_{b'e}}{1 + j\omega r_{b'e}C}$$

$$\frac{v_i}{i_i} = \frac{r_{bb'} + r_{b'e} + j\omega r_{b'e} r_{bb'} C}{1 + j\omega r_{b'e}C}$$

hence $\dfrac{v_i}{i_i} = (r_{bb'} + r_{b'e})\dfrac{1 + j\omega RC}{1 + j\omega r_{b'e}C}$

write in standard form: $$Z_{in} = \frac{v_{in}}{i_{in}} = (r_{b'e} + r_{bb'})\frac{1 + j\dfrac{f}{f_1}}{1 + j\dfrac{f}{f_2}}$$

as required.

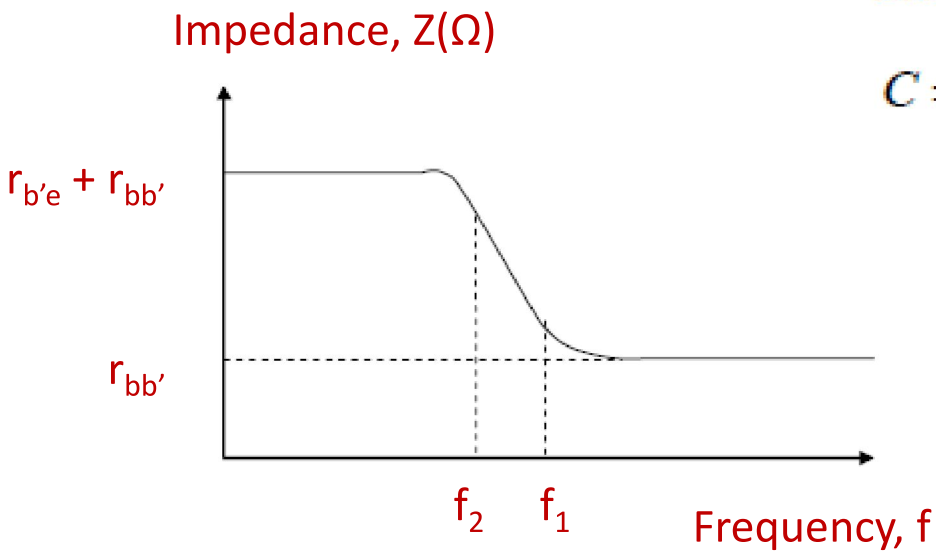


If f << f_1, f_2, then $Z_{in} \sim (r_{bb'} + r_{b'e})$

If f >> f_1, f_2 $Z_{in} \sim (r_{bb'} + r_{b'e})\dfrac{R}{r_{b'e}} = Z_{in} \sim (r_{bb'} + r_{b'e})\dfrac{r_{b'e} r_{bb'}}{r_{b'e}(r_{bb'} + r_{b'e})}$

$Z_{in} = r_{bb'}$

Sketch Z_{in} (f). labelling your diagram appropriately, for the case of $\beta_o = 200$, $r_{bb'} = 100\Omega$, a bias current of 1mA, load resistor of 2kΩ, $C_e = 10$pF and $C_c = 2$pF.



Magnitude of $A_{vo} = g_m R_L = 40m \times 2k = 80$

$C = C_e + A_{Vo} C_c = 10p + 80x2p = 170\text{pF},$

$$r_{b'e} = \frac{200}{40 * 1m} = 5000R$$

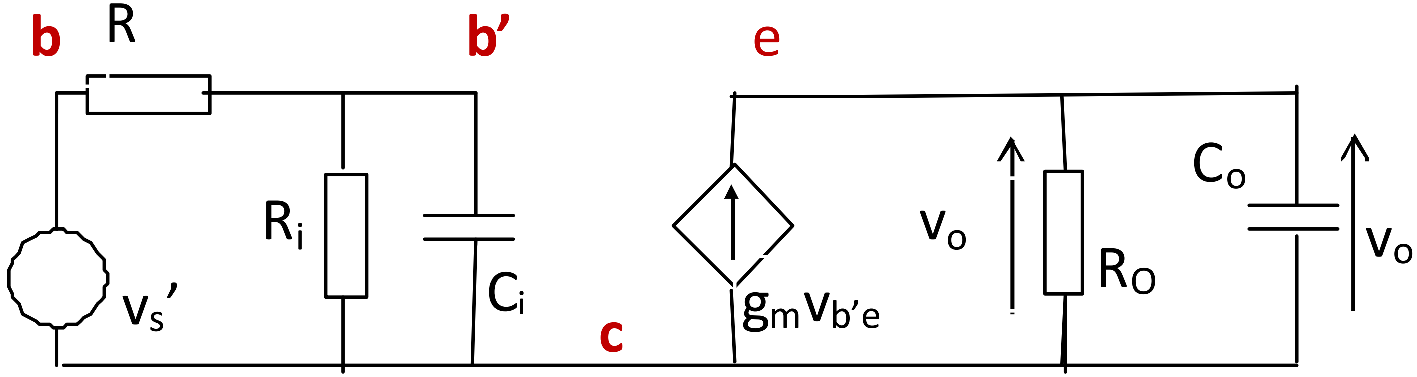
$$R = \frac{5000 * 100}{5000 + 100} = 98R \quad (R = r_{b'e} // r_{bb'})$$

$$f_1 = \frac{1}{2\pi RC} = \frac{1}{2\pi * 98 * 170p} = 9.6\text{MHz}$$

$$f_2 = \frac{1}{2\pi r_{b'e} C} = \frac{1}{2 \times \pi \times 5k \times 170p} = 1.8\text{MHz}$$

Bandwidth of CC (EF) - see VITAL

Draw an equivalent circuit, apply Miller's theorem to obtain:



$$C_i = C_c + (1-K)C_e$$

$$R_i = r_{b'e}(1-K)$$

$$R = R_s + r_{b'b}$$

$$K \equiv A_{Vo} \approx 1$$

$$C_o = C_{stray} + \left(\frac{K-1}{K}\right)C_e$$

$$R_o = R_E + r_{b'e}\left(\frac{K-1}{K}\right)$$

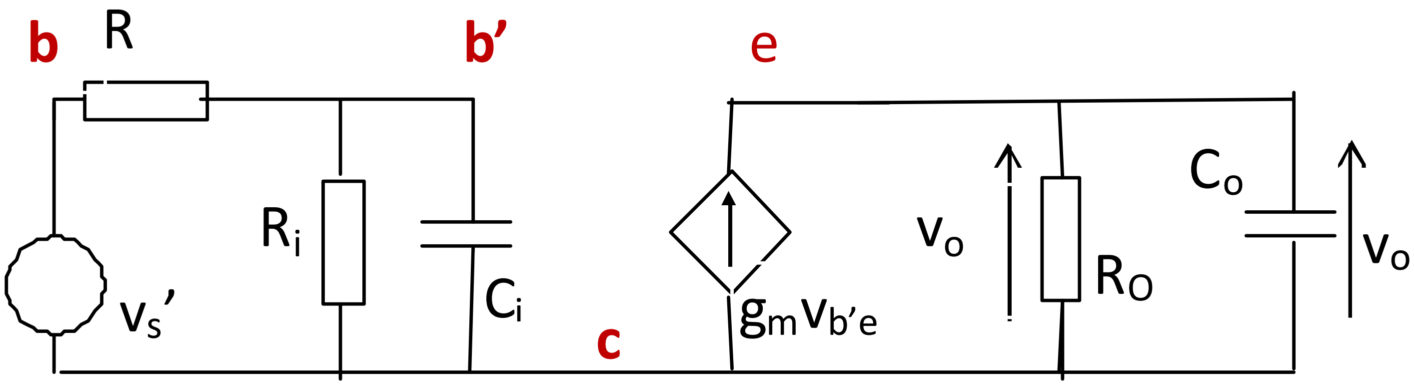
$$\boxed{A_V = \frac{v_o}{v_i'} = \frac{A_{Vo}}{1+j\left(\frac{f}{f_H}\right)}}$$

$$A_{Vo} = \frac{g_m R_E}{1+g_m R_E} \sim 1$$

$$C_i \approx C_c$$

$$C_o \approx C_{stray}$$

Bandwidth of CC (EF)



$$A_V = \frac{v_o}{v_i'} = \frac{1}{1 + j\left(\frac{f}{f_H}\right)}$$

Input and output time constants can be estimated

$$\tau_i \approx (R_s + r_{b'b})C_c$$

$$\tau_o \approx R_E C_{stray}$$

It is often the case that the output time constant is the largest (unlike the CE case), so the bandwidth can be estimated as:

$$f_H = \frac{1 + g_m R_E}{2\pi C_o R_E} \approx \frac{g_m}{2\pi C_o} = f_T \frac{C_e}{C_o}$$

$$f_T \approx \frac{g_m}{2\pi C_e}$$