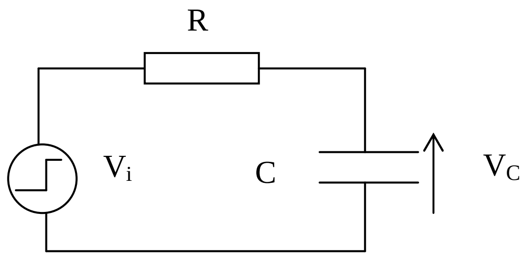
Step response of amplifiers

- Useful for characterising amplifiers: close relationship between bandwidth (f_H) and rise time (t_r).
- Interesting to see how an electronic circuit responds to a different type of signal (transient). Gives an indication of the time to 'power-up' an electronic system.

Consider a simple series R-C (first order) circuit:



Work out the time for the voltage across the capacitor, V_C, to change from the 10% to 90% level following an abrupt input voltage step, V_i.

Proceed with the analysis using Laplace Transforms (s = jω):

The transfer function is:

$$\frac{V_C(s)}{V_i(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{1+sRC}$$

Represent the input step of amplitude V, as $V_i(s) = V/s$, SO $V_C(s) = \frac{V}{s}\frac{1}{1+sRC}$

It is necessary to split the expression into partial fractions:

$$\frac{1}{s(1+sRC)} = \frac{A}{s} + \frac{B}{1+sRC} \qquad \Rightarrow \qquad 1 = A + AsRC + Bs$$

Equating coefficients: Constants: 1 = A

S: 0 = ARC +B ⇒B = -1/RC

Therefore partial fractions are $\frac{1}{s(1+sRC)} = \frac{1}{s} - \frac{RC}{1+sRC}$

And $$V_C(s) = V\left(\frac{1}{s} - \frac{RC}{1+sRC}\right) = V\left(\frac{1}{s} - \frac{1}{s+1/RC}\right)$$

The expression is now in standard form and the solution (time domain) is found from tables: $$V_C(t) = V\left[1-\exp\left(-\frac{t}{RC}\right)\right] \qquad (1)$$

the 10% level ($t = t_1$) is found as $V_C(t) = 0.1V = V\left[1-\exp\left(-\frac{t_1}{RC}\right)\right]$

hence $\exp(-t_1/RC) = 0.9$ $\Rightarrow t_1 = -RC\ell n(0.9)$ ⇒ $\boldsymbol{t_1 = 0.1RC}$

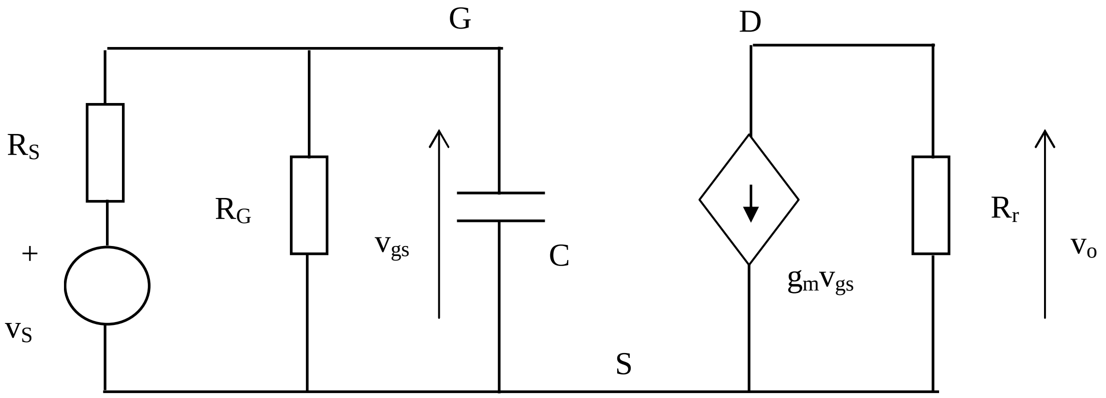
The 90% level ($t = t_2$) is found as $V_C(t_2) = 0.9V = V\left[1-\exp\left(-\frac{t_2}{RC}\right)\right]$

hence $\exp(-t_2/RC) = 0.1$ $\Rightarrow t_2 = -RC\ell n(0.1)$ ⇒ $\boldsymbol{t_2 = 2.3RC}$

The rise time is therefore: $\boldsymbol{t_r = t_2 - t_1 = 2.2RC}$ **(2)**

Common source amplifier

Now apply the above theory to the case of a common source MOSFET amplifier (see last lecture for schematic diagram). Applying Miller's theorem, the equivalent circuit reduces to:

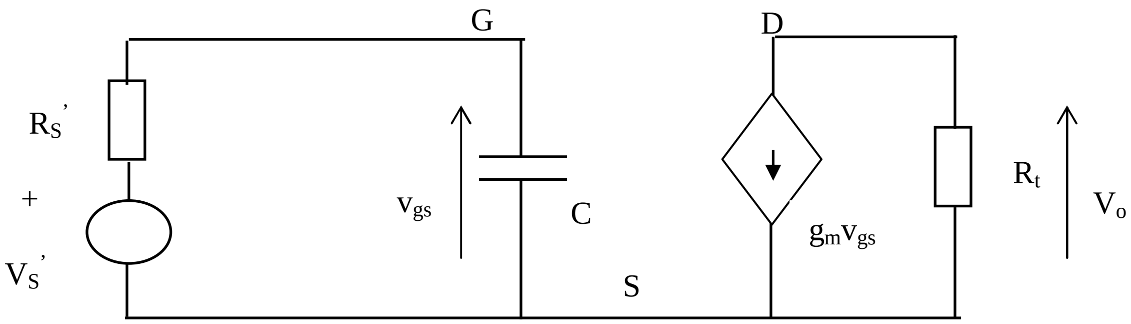


Where $R_G = R_1 // R_2$ (the bias resistors)

$C = C_{gs} + (1 - K)C_{gd}$ $K = v_o/v_{gs} = - g_m R_t$ with $R_t = R_D//R_L$

Make the approximation that the input time constant is much greater than the output time constant. (see notes on CS MOST and CE BJT).

As before, transform the input circuit using a Thevenin equivalent circuit to get a simple series R-C network



Where $R_S^{'} = R_G // R_S$ $V_S' = \frac{R_G}{R_G + R_S} V_S$

Recognise the following equivalences with the analysis for the R-C network:

$V_i = V_S^{'} = \frac{R_G}{R_G + R_S} V_S$ (V_S is taken as applied voltage step)

$V_C = v_{gs} = -{V_o}/{g_m R_t}$ (voltage across the capacitor)

$R = R_S^{'} = R_G // R_S$ $C = C_{gs} + (1 + g_m R_t)C_{gd}$

Substituting into Eqn. gives (1):

$$V_o(t) = -g_m R_t \frac{R_G}{R_G + R_S} V_S \left[1 - \exp\left(-\frac{t}{R_G // R_S \left(C_{gs} + \left(1 + g_m R_t\right) C_{gd} \right)} \right) \right] \qquad (3)$$

Note that for $t = 0$, $V_S = 0$ and hence $V_O = 0$. For $t >> 0$, the exponential term goes to zero and the output is constant.

Recalling (from MOST lecture) that the bandwidth of the amplifier is $f_H = \dfrac{1}{2\pi R_S^{'} C}$

have (from Eqn. 2) $t_r = 2.2 \dfrac{1}{2\pi f_H} = \dfrac{0.35}{f_H}$

That is, the bandwidth of the amplifier, f_H can be estimated from the rise time of the output voltage following an input step voltage.

Step response of bipolar transistor, CE amplifier

A similar approach can be used for the bipolar transistor: common emitter amplifier. Recognise that:

$$V_i = V_S^{'} = V_S \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s} \qquad V_C = v_{b'e} = -V_o / g_m R_t$$

$C = C_e + C_c(1 - K)$ $\quad$ ($K = -g_m R_L >> 1$) $\quad$ $R = R_S^{'} = \left(R_S + r_{bb'}\right) // r_{b'e}$

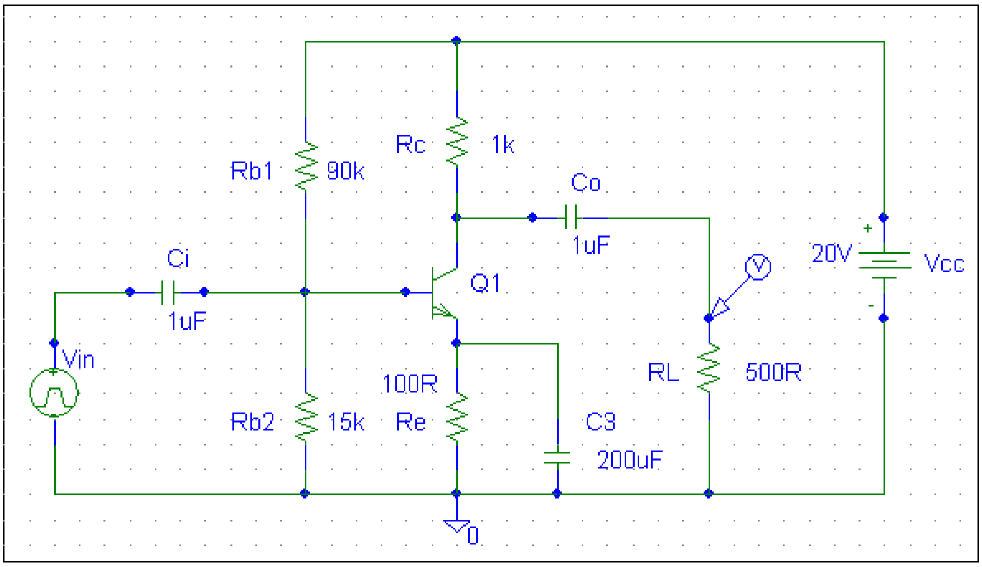
Hence

$$V_o(t) = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_S} V_S \left[1 - \exp\left(-\frac{t}{\left(R_S + r_{bb'}\right) // r_{b'e} \left(C_e + \left(1 + g_m R_t\right) C_c \right)} \right) \right] \qquad (4)$$

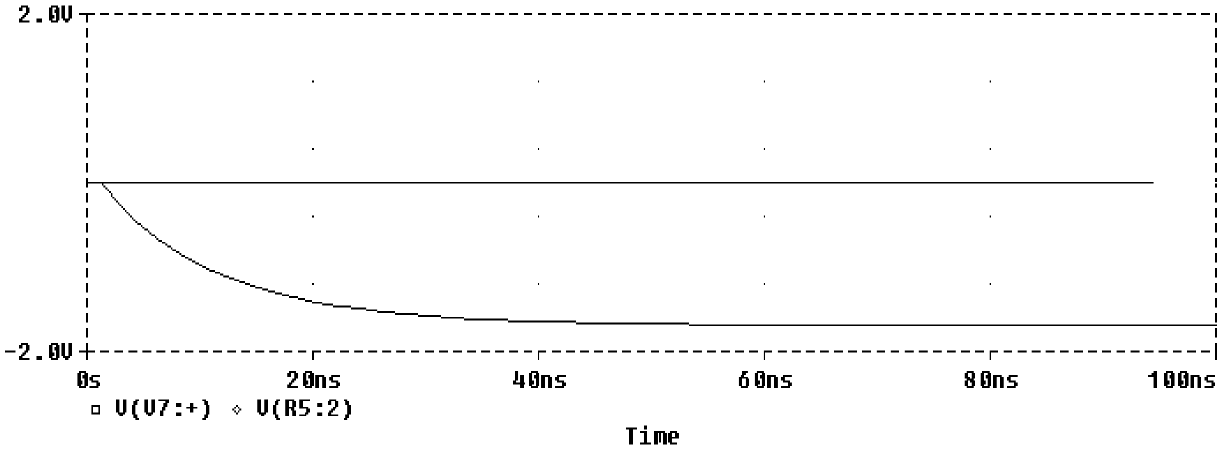
Practical point: In reality, the voltage step would need to be a.c. coupled into the amplifier (that is, via an input capacitor). Thus for very long times, $t \to \infty$, the output would eventually go to zero. However the time constant for this would be very long, because any circuit capacitors would be very large compared to the device capacitances that we are concerned with here. The equations (3,4) above do not predict this and are only true for times much less than the (long) time constants associated with the discharging of circuit capacitors.

This point is demonstrated in the following PSPICE simulation.

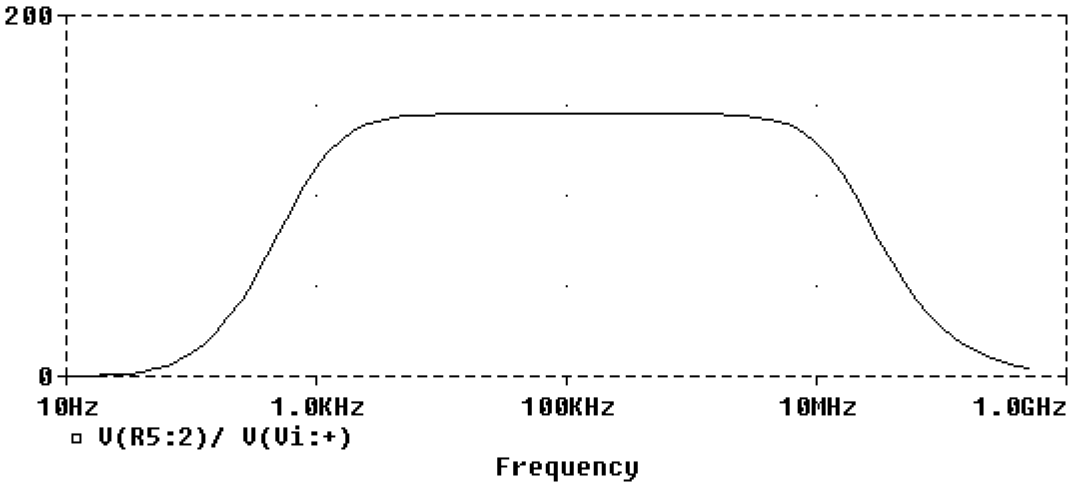
SPICE simulation of response of a CE amplifier to a small voltage step



1. Output voltage versus time following a 10mV step voltage

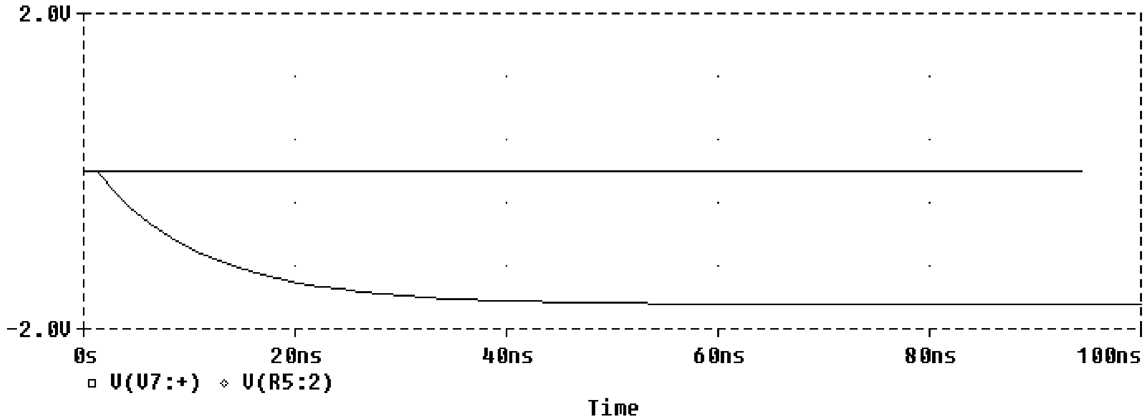


2. Small signal gain versus frequency

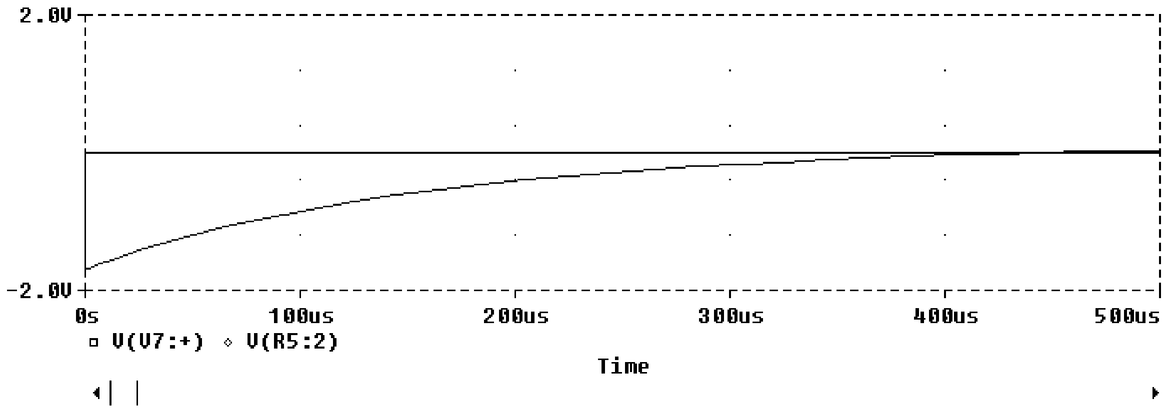


Compare plot from 1, above (first 100ns) with much longer time (500us)

<u>100ns time window:</u> output settles at ~ 1.7V [NON-STEADY STATE SITUATION!]



<u>500 µs time window:</u> output 'relaxes back to zero' WITH MUCH LONGER TIME CONSTANT (set by circuit capacitances)

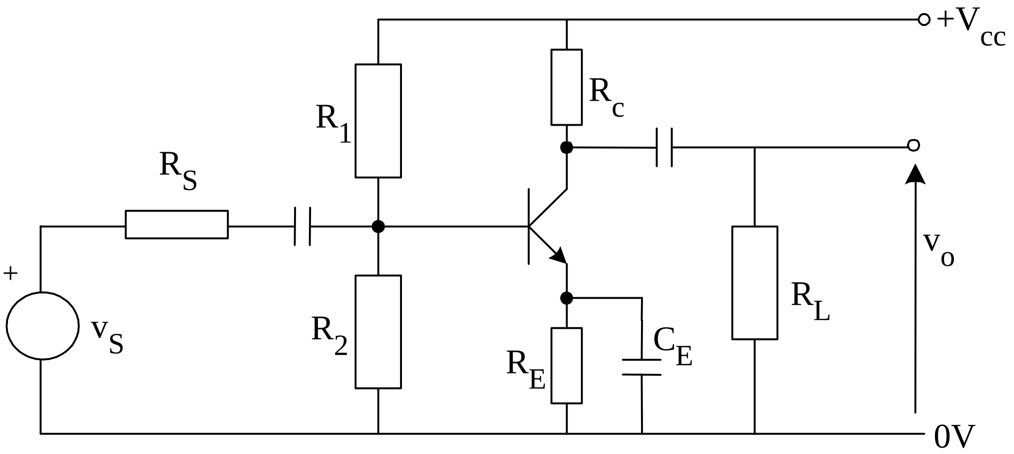


Worked example

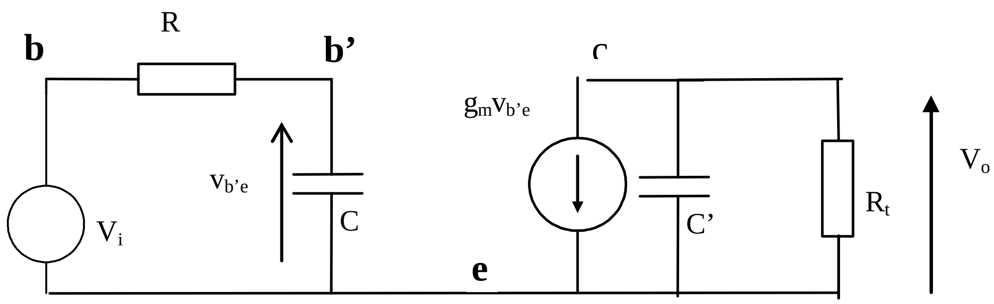
Work out values for the mid-frequency voltage gain, bandwidth and rise time of the transient response of the amplifier shown below.

R_S = 100Ω, R_C = 2 kΩ and the bias resistors can be assumed large.

The hybrid-π parameters of the transistor are:-
g_m = 40 mAV $^{-1}$, $r_{bb'}$ = 100 Ω, $r_{b'e}$ = 2.5 kΩ, $C_{b'e}$ = 200 pF and $C_{b'c}$ = 2 pF. $r_{b'c}$ and r_{ce} may be considered infinite.



Solution:
Derive the ac equivalent circuit (see notes on high-frequency voltage gain of the CE configuration). Apply Miller's Theorem and use Thevenin's theorem to simplify the input circuit, the circuit. The circuit can be reduced to a simple first order RC network:



where $C = C_{b'e} + C_{b'c}(1 - K) \approx C_{b'e} + C_{b'c}\,|K|$ $\qquad$ $C' = C_S + C_c(K - 1)/K \approx C_S + C_c$

$$R = (R_S + r_{bb'}) // r_{b'e} \qquad V_i = V_S \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_s}$$

Both input and output sections of the amplifier are simple, first order circuits with time constants

$$\tau_i = RC = [r_{b'e} //(R_S + r_{bb'})]C \qquad \tau_o = R_t C'$$

Assume that the input time constant is much greater than the output time constant i.e., $\tau_i >> \tau_o$ and therefore disregard the capacitor C' in the circuit.

Can then write:

$v_o = -g_m v_{b'e} R_t$, ie voltage across the capacitor is $V_C = v_{b'e} = -{V_o}/{g_m R_t}$

The response to a voltage step is that of a first order network, ie $V_C(t) = V\left[1 - \exp\left(-\frac{t}{RC}\right)\right]$ which can be written in this context as

$$V_o(t) = -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_S} V_S \left[1 - \exp\left(-\frac{t}{(R_S + r_{bb'})//r_{b'e}\left(C_e + (1 + g_m R_t)C_c\right)}\right)\right]$$

The mid-frequency voltage gain can be found from the above equation for large t:

$$K \sim -g_m R_t \frac{r_{b'e}}{r_{b'e} + r_{bb'} + R_S} \quad = -40mx2000 \frac{2.5k}{2.5k + 100 + 100} \quad \mathbf{= -74}$$

Now $R = (R_S + r_{bb'})// r_{b'e}$ = 185Ω

$C \sim C_{b'e} + KC_{b'c}$ **=348pF**

******** input time constant = RC = 64ns whereas output time constant = $R_C C_{bc}$ ~4ns so approx above is justified

The rise time (10% - 90% levels) for a first order system is 2.2RC, so have $t_r = 2.2 \frac{1}{2\pi f_H} = \frac{0.35}{f_H}$

The bandwidth of the amplifier is $f_H = \frac{1}{2\pi RC} = \frac{1}{2\pi x 185 x 348 p}$ = 2.5MHz

So the rise time is **142ns**

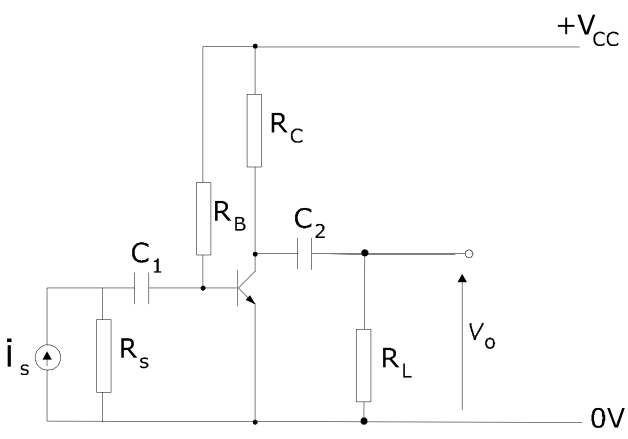
Exercise

In the amplifier shown, R_S = 1 kΩ, R_C = 1 kΩ and R_L = 1 kΩ.

The hybrid-π parameters of the transistor are:- g_m = 40 mAV^{-1}, $r_{bb'}$ = 100 Ω, $r_{b'e}$ = 2.5 kΩ, $C_{b'e}$ = 200 pF and $C_{b'c}$ = 2 pF. $r_{b'c}$ and r_{ce} may be considered infinite.

The source current is an abrupt positive change of magnitude 0.2 mA.

Derive an expression for the output voltage, v_o, as a function of time.



If there is a stray capacitance of 0.01 μF in parallel with R_L calculate the new output voltage neglecting the input time constant.