

# Digital Electronics and Microprocessor Systems (ELEC211)

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Digital 4: Introduction to sequential logic

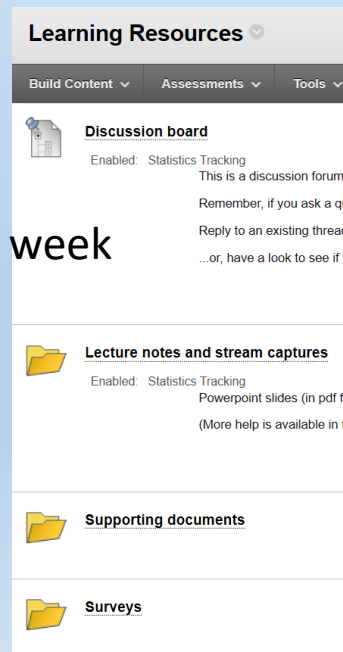
# Outline

- Sequential circuits
- Propagation / gate delay
- Timing diagrams
- Latches
- Flip flops

## Use VITAL!:

- Stream lectures
- Handouts
- Notes and Q&A each week
- Discussion Board
- Exam resources

[www.liv.ac.uk/vital](http://www.liv.ac.uk/vital)

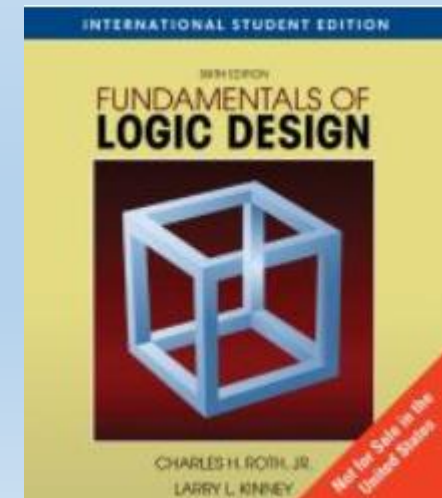


## Previous material

Basic gates ✓

Combinational and sequential logic ✓

Course textbook – please borrow and use it! →



Current channel depends on previous channel  
... which depends on previous channel  
... which depends on previous channel  
...

**Previous state is significant!**



# Sequential logic – revision

**Sequential** switching circuits have the property that the output depends not only on the present input but also on the **past sequence** of inputs.

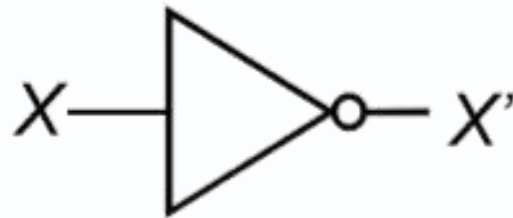
So the “state” becomes significant to determining the output.

**Latches** and **flip-flops** are commonly used as memory devices in sequential circuits.

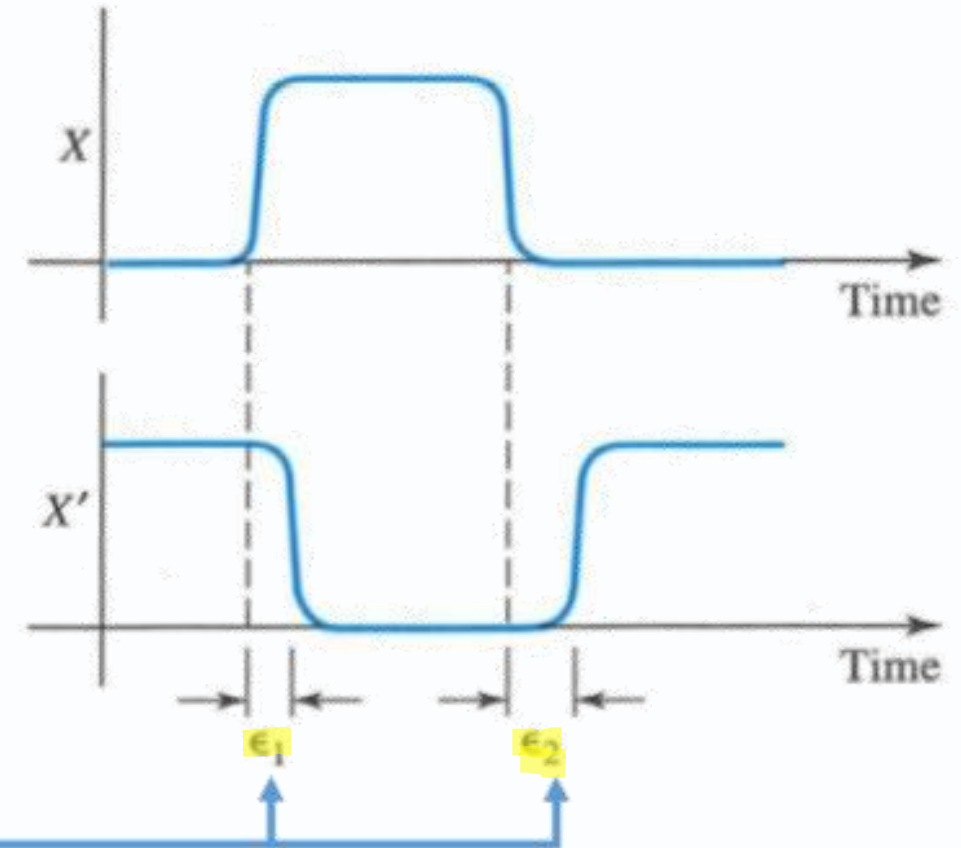
In order to construct a switching circuit that has memory, such as a latch or flip-flop, we must introduce **feedback** into the system.

# Gate delays and timing diagrams

- The output of a logic gate does not change instantaneously.
  - The output change is going to be delayed with respect to the input change.
- This delay is called propagation delay
  - Propagation delay from 0 to 1,  $\epsilon_1$ .
  - Propagation delay from 1 to 0,  $\epsilon_2$ .

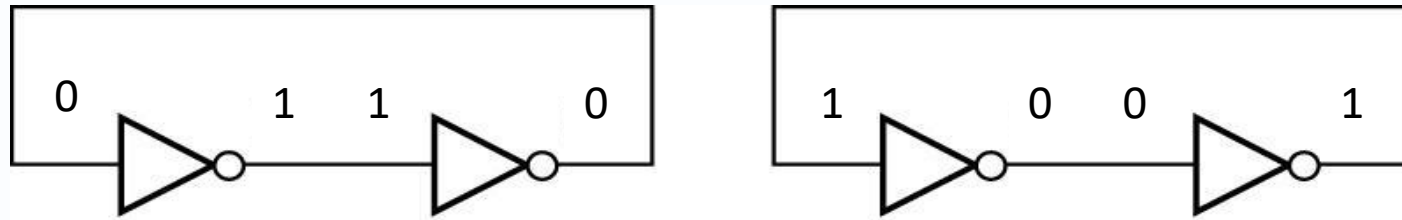


In practice  $\epsilon_1$  and  $\epsilon_2$  can be different

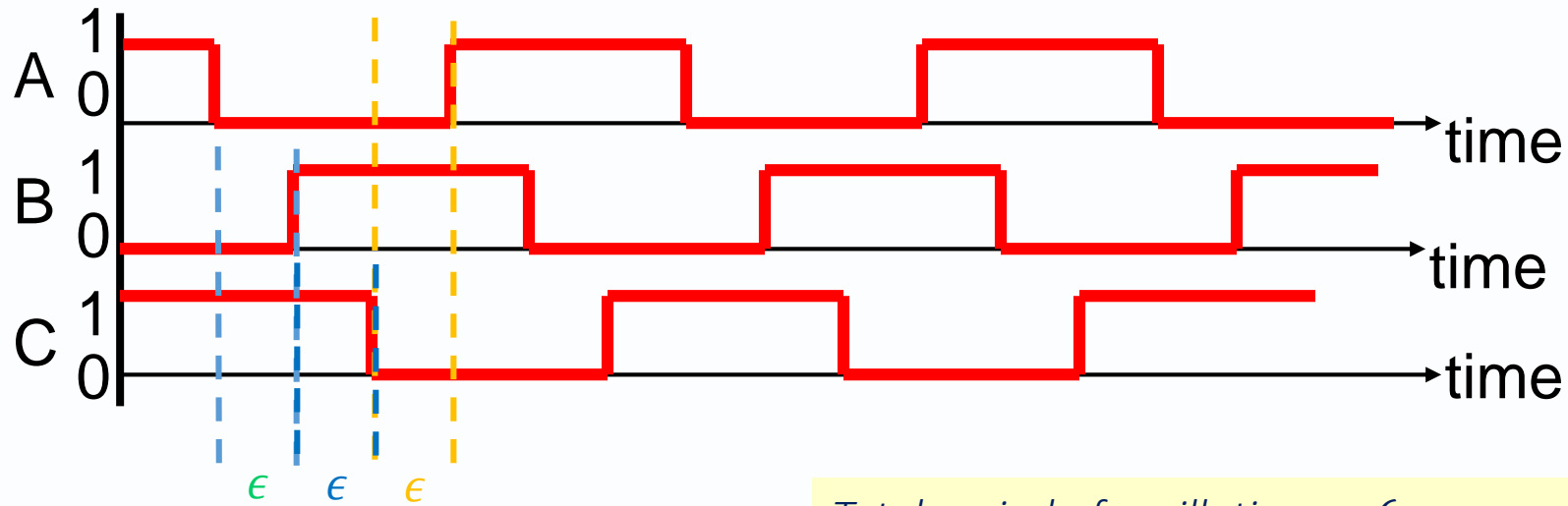
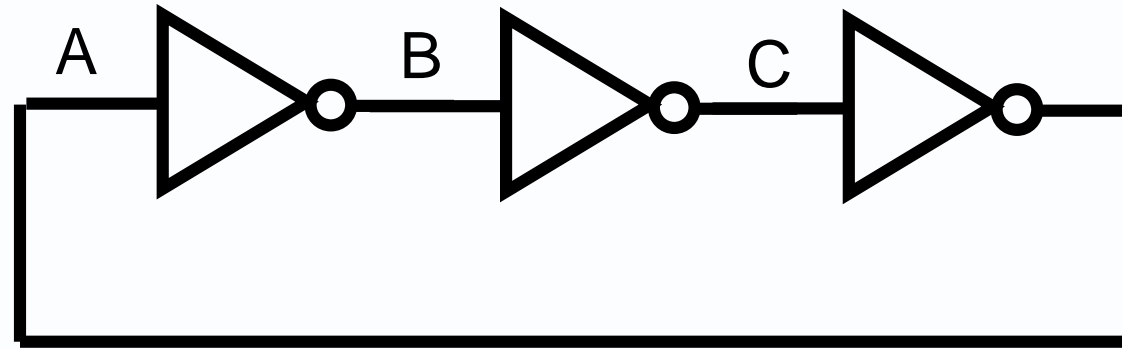


# Feedback loop with two Inverters

Consider a feedback loop circuit with two inverters.  
This circuit has two stable states:



# Ring Oscillator – timing diagram



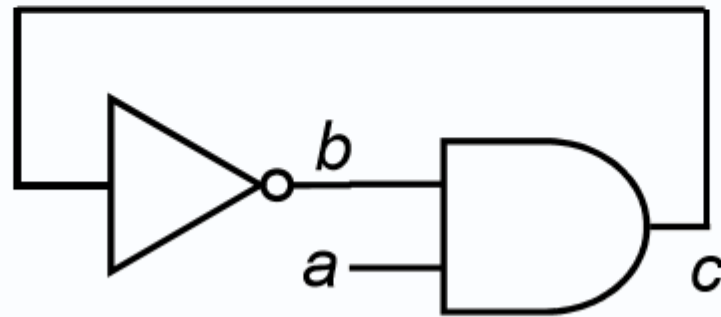
*Total period of oscillation =  $6\epsilon$   
(assumes propagation delay is  
identical for each of the three gates)*



# Question



The inverter in the figure has a propagation delay of 4 ns and the AND gate of 8 ns. Draw a timing diagram for the circuit showing a, b, c. a and c are initially equal to 0, b is initially one. After 20 ns a becomes 1 for 90 ns and then 0 again.





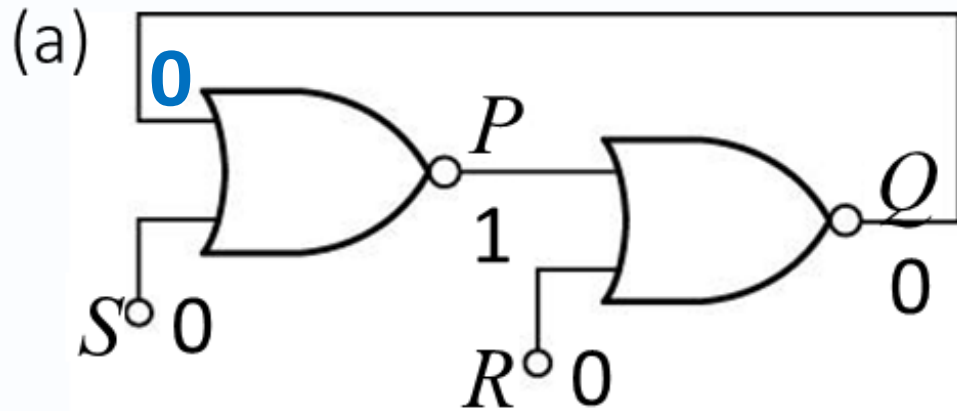
# Latches – two stable states



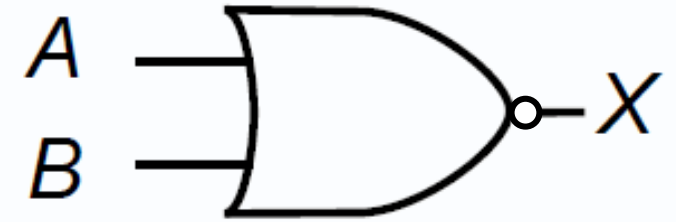
# Set-Reset Latch

The S-R latch can be put into one of two stable output states.

These are triggered by an input pulse at input S (to **set** to one stable state,  $P=0$ ,  $Q=1$ ) or at R (to **reset** to the other stable state,  $P=1$ ,  $Q=0$ )



NOR

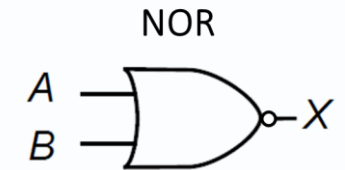


$A$	$B$	$X$
0	0	1
0	1	0
1	0	0
1	1	0

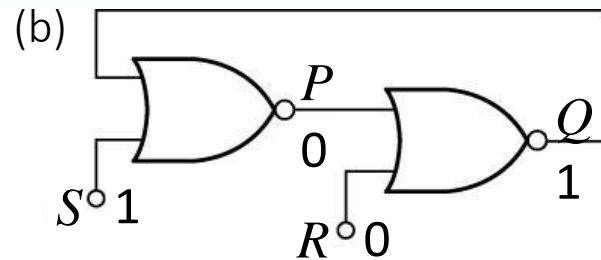
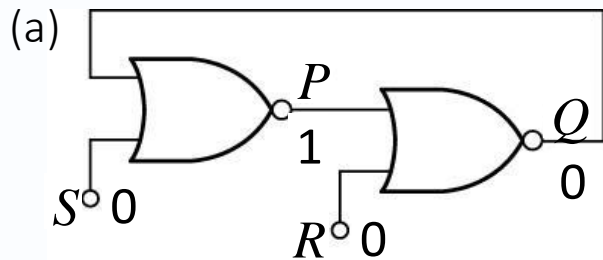
# Set-Reset Latch

The S-R latch can be put into one of two stable output states.

These are triggered by an input pulse at input S (to **set** to one stable state,  $P=0$ ,  $Q=1$ ) or at R (to **reset** to the other stable state,  $P=1$ ,  $Q=0$ )

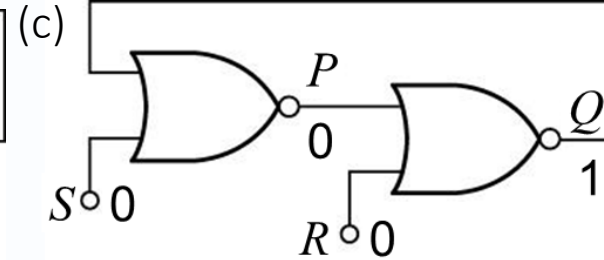


A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

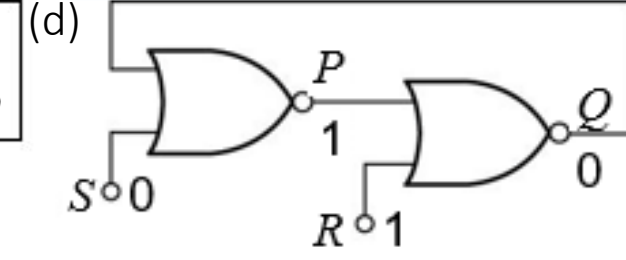


Initial state:  $S=R=0$ ,  $P=Q'=1$

Pulse at S:  $S=1$ ,  $R=0$ ,  $P=Q'=0$

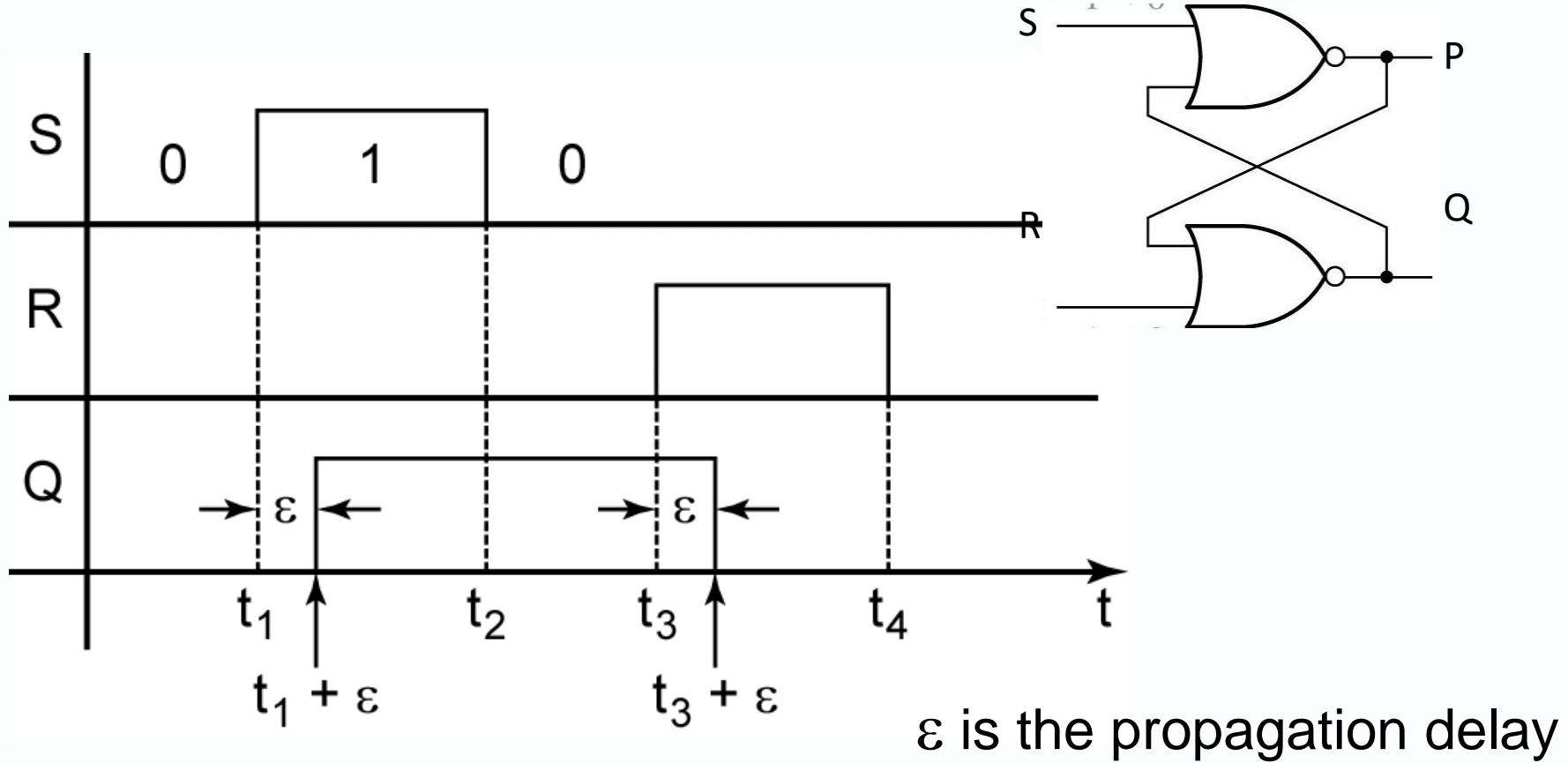


Pulse at S removed:  $S=0$ ,  $R=0$ ,  $P=Q'=0$



Pulse at R:  $S=0$ ,  $R=1$ ,  $P=Q'=1$

# S-R latch timing



# S-R Latch table

The circuit is said to have “memory” because output depends not just on present inputs, but also on the previous sequence of inputs

S-R latch table

S(t)	R(t)	Q(t)	Q(t+ $\epsilon$ )
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	-
1	1	1	-

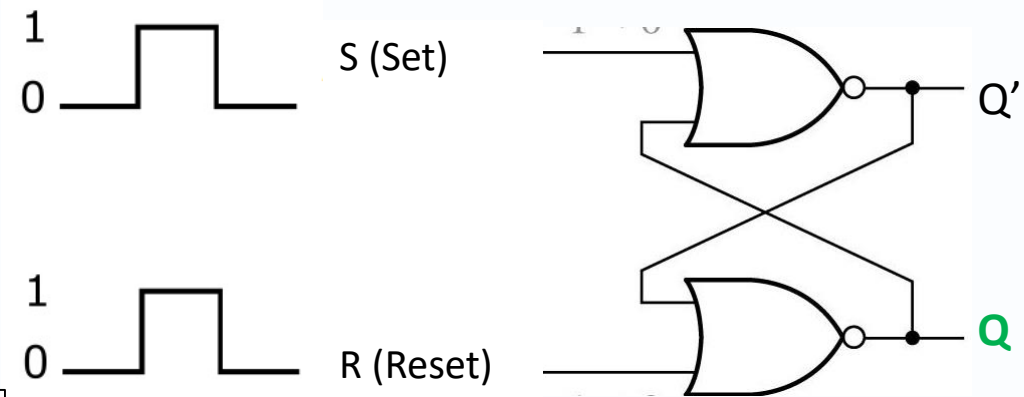
Q is usually referred to as the output of the S-R latch

Q = 1 is a ‘set’ state  
Q = 0 is a ‘reset’ state

Inputs not allowed

$\epsilon$  is the propagation delay

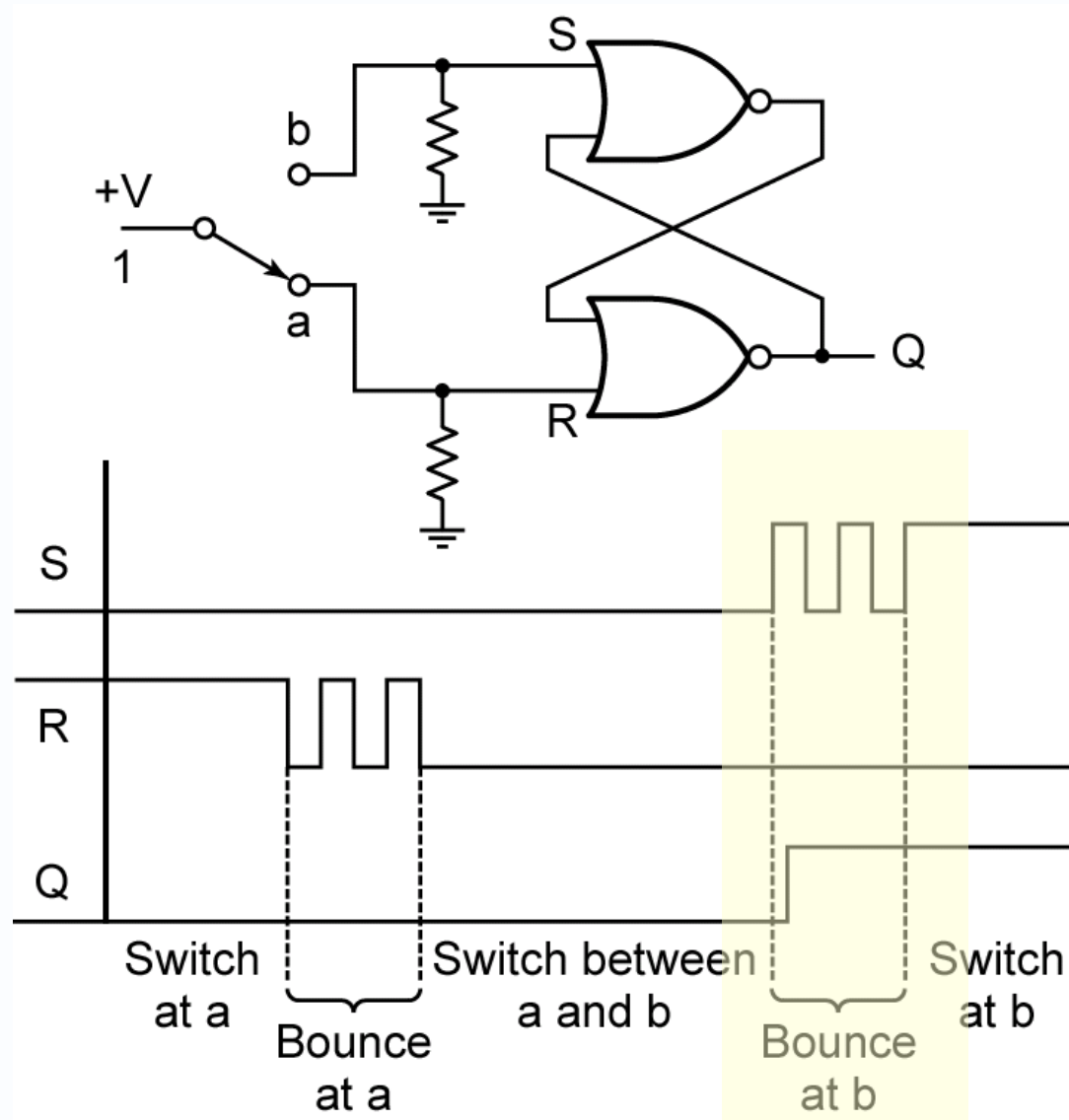
*Cross-coupled* form emphasises the gates’ symmetry:



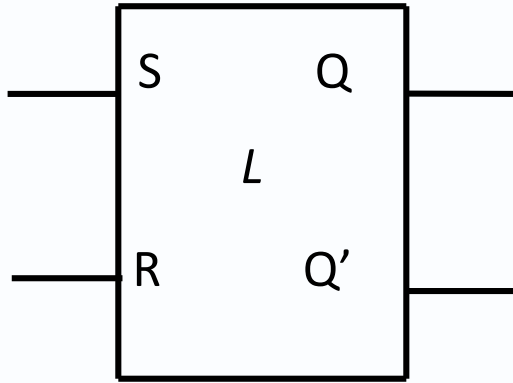
# Switch Debouncing with an S-R Latch

When a mechanical switch is opened or closed, the switch contacts vibrate or bounce open & closed several times.

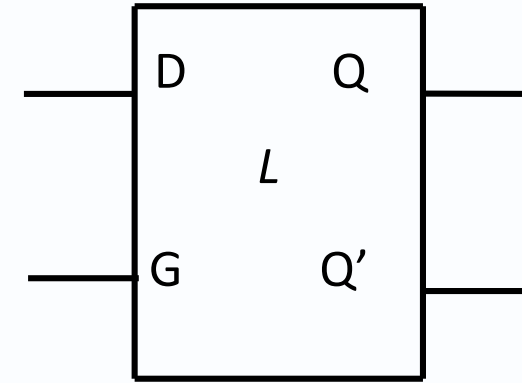
The pull-down resistors assure that when the switch is between *a* and *b*, the latch inputs S and R are at logic 0.



# Latch shorthand

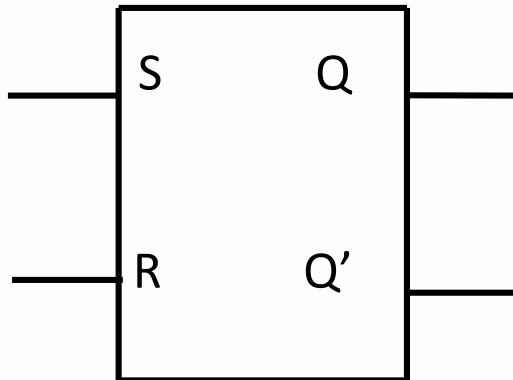


SR Latch

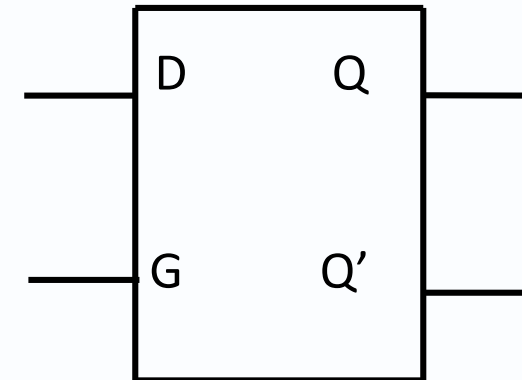


Gated D Latch

We won't often include the "L"



SR Latch



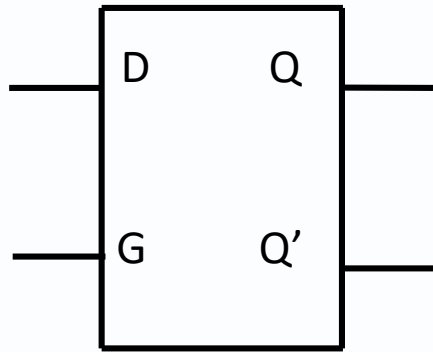
Gated D Latch

<b>G</b>	<b>Q<sup>+</sup></b>
<b>0</b>	mem
<b>1</b>	D

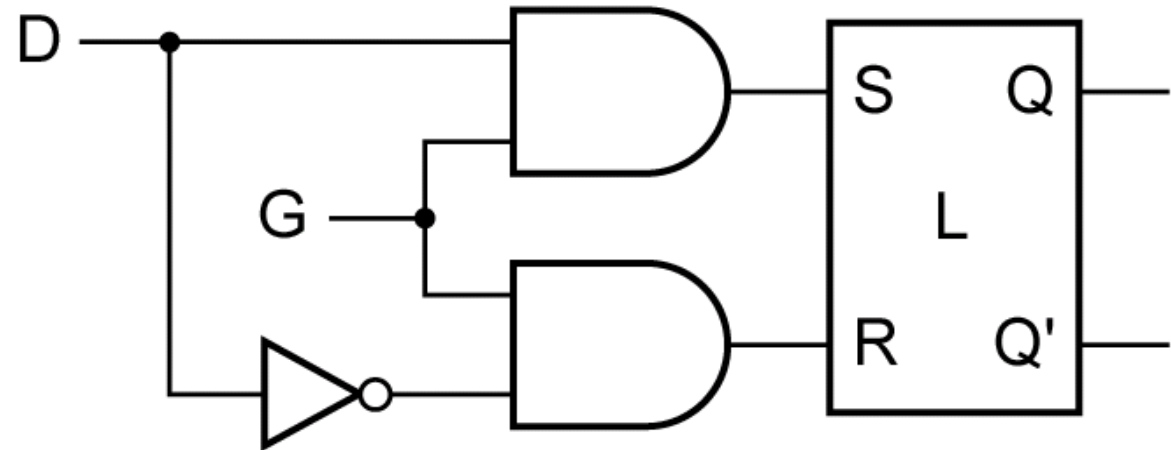
# Gated D-Latch or Transparent Latch

A gated D-latch has two inputs, a data input (D) and a gate input (G).

This is a level-sensitive latch with an enabling input (G)



≡



Implementation using S-R latch

'Transparent' because if **G=1**:  
output (Q) = D (data) input

$G=0 \rightarrow S=R=0 \rightarrow$  no change (memory)

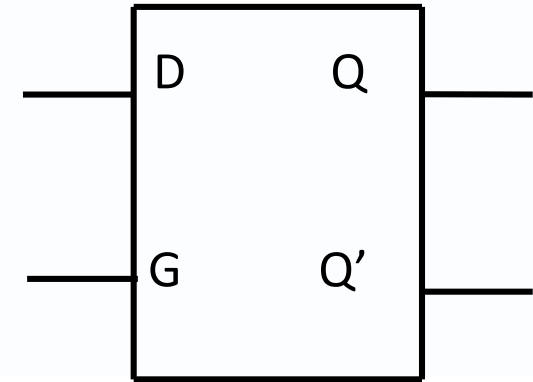
$G=1, D=0 \rightarrow S=0, R=1 \rightarrow$  RESET ( $Q^+$  becomes or stays 0)

$G=1, D=1 \rightarrow S=1, R=0 \rightarrow$  SET ( $Q^+$  becomes or stays 1)



# Gated D-Latch or Transparent Latch

$G=0 \rightarrow S=R=0 \rightarrow$  no change  
 $G=1, D=0 \rightarrow S=0, R=1 \rightarrow$  RESET ( $Q^+$  becomes or stays 0)  
 $G=1, D=1 \rightarrow S=1, R=0 \rightarrow$  SET ( $Q^+$  becomes or stays 1)



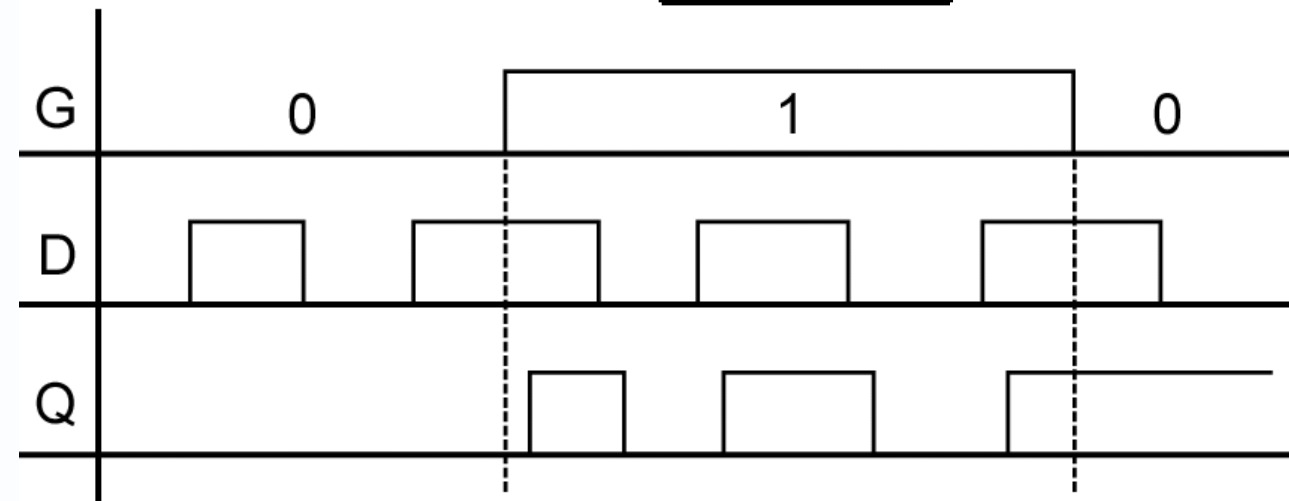
G	D	Q	Q <sup>+</sup>
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Q is existing state, Q<sup>+</sup> is the next state

"R" = GD'

"S" = GD

Output data **changes** only for these two rows.



G	Q <sup>+</sup>
0	mem
1	D

# Flip-flops

## Flip-flops:

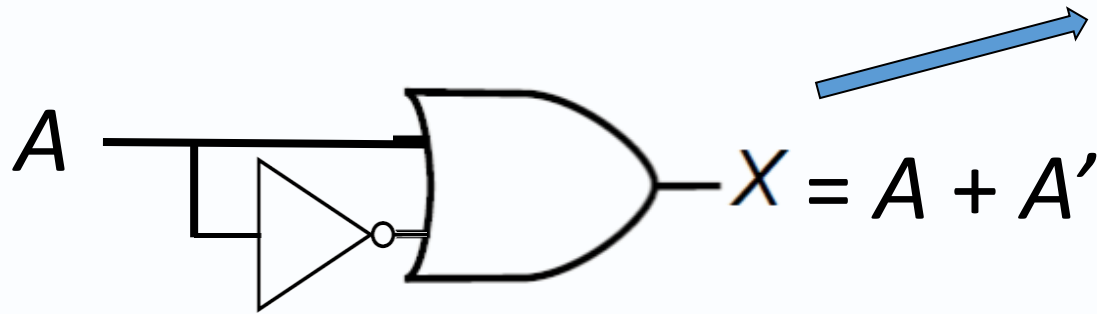
- We define the flip-flop as being a memory device (e.g. latch) that changes its output state **in response to a clock input**. Clocked circuits prevent response to ‘glitches’.

## Don't be confused....:

- SR latches are sometimes referred to by other sources as “SR flip-flops”
- In that case, our ‘SR flip-flop’ is referred to as a ‘gated’ or ‘clocked’ SR flip-flop

# Why use flip-flops not latches?

- Timing avoids the effect of “glitches”
- Example of a glitch:

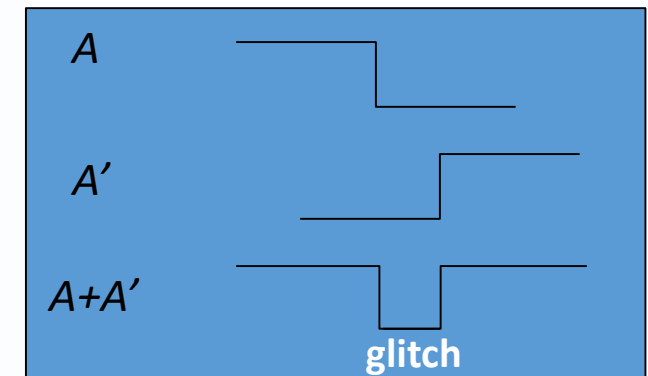


Logical outcome:  $X = \text{TRUE}$  for any  $A$

Example of actual outcome:

$A \rightarrow 0$ , during inverter propagation delay,  $A'$  still 0 so  $X = \text{FALSE}$  temporarily. This is a **glitch**.

A glitch at D might trigger an unwanted change (latch), but this can be avoided if the clock is in control (flip-flop)

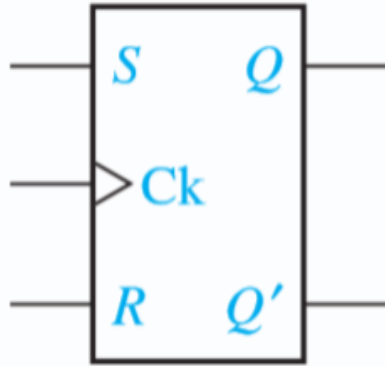


D	Action at D change
0	“RESET”, $Q^+ = 0$
1	“SET”, $Q^+ = 1$

vs

D	Action at next clock pulse
0	“RESET”, $Q^+ = 0$
1	“SET”, $Q^+ = 1$

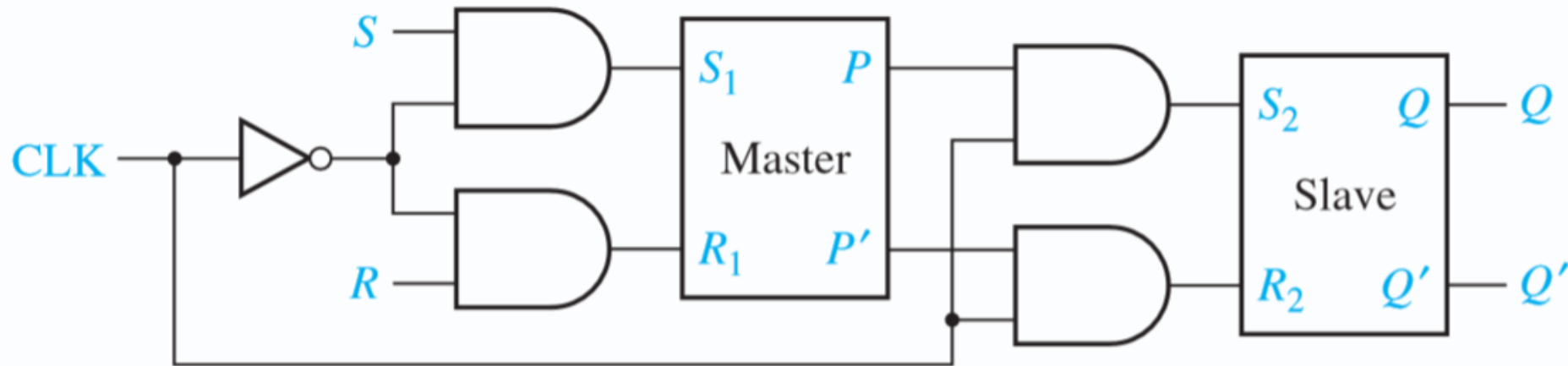
# S-R flip-flop



Operation summary:

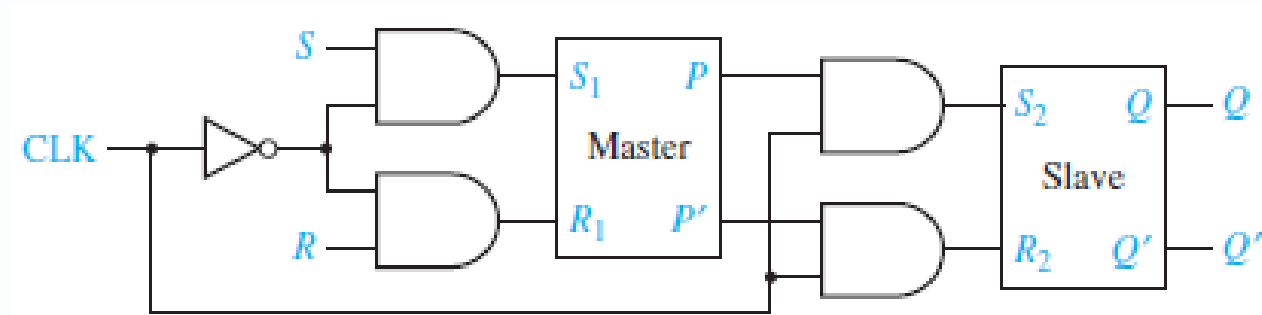
$S = R = 0$	No state change
$S = 1, R = 0$	Set $Q$ to 1 (after active Ck edge)
$S = 0, R = 1$	Reset $Q$ to 0 (after active Ck edge)
$S = R = 1$	Not allowed

This flip-flop changes state after the rising edge of the clock.

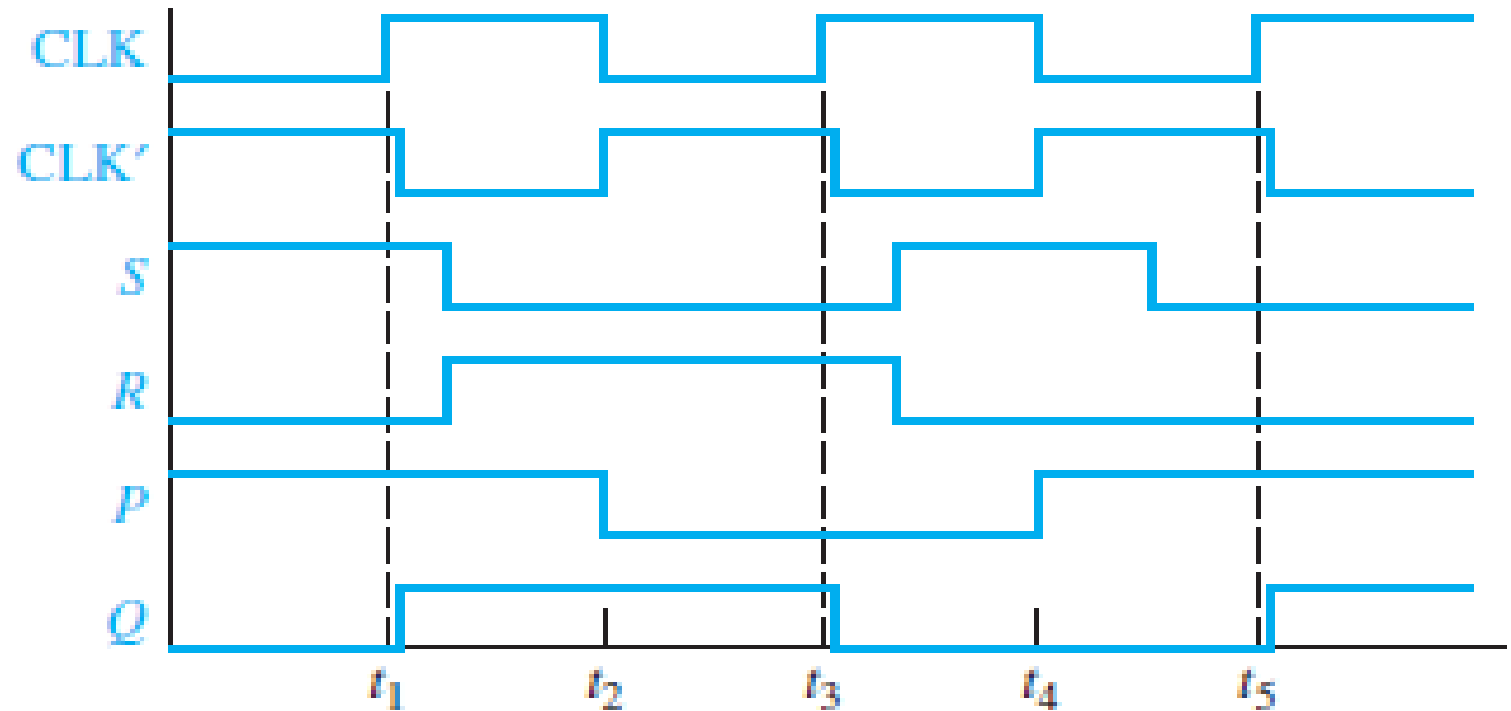


Implementation with two latches

# S-R flip-flop timing analysis



S-R flip-flop implemented with two latches



Timing analysis

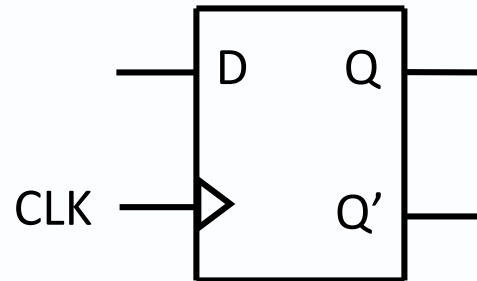
# Edge-Triggered D-type Flip-Flop

The **flip-flop** output changes only in response to the clock (CLK), not to a change in D.

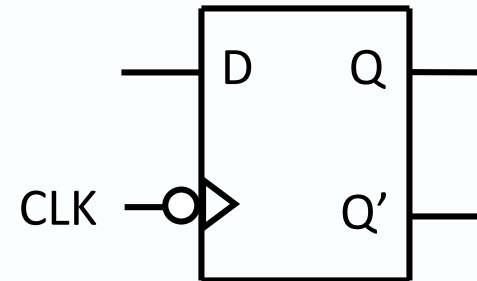
(It is still affected by D, but does not respond directly to the change in D.)

Arrowhead > identifies the clock input

Rising-edge:  
0 to 1 clock  
transition



*rising-edge-trigger*



*falling-edge trigger*

Falling-edge:  
1 to 0 clock  
transition

An inversion bubble on the clock input indicates a falling-edge trigger.

The “active edge” refers to the clock edge that triggers the change (i.e. rising or falling)

# Edge-Triggered D-type Flip-Flop

A *falling-edge triggered* D flip-flop can be constructed from two gated D latches and an inverter – Master/Slave.

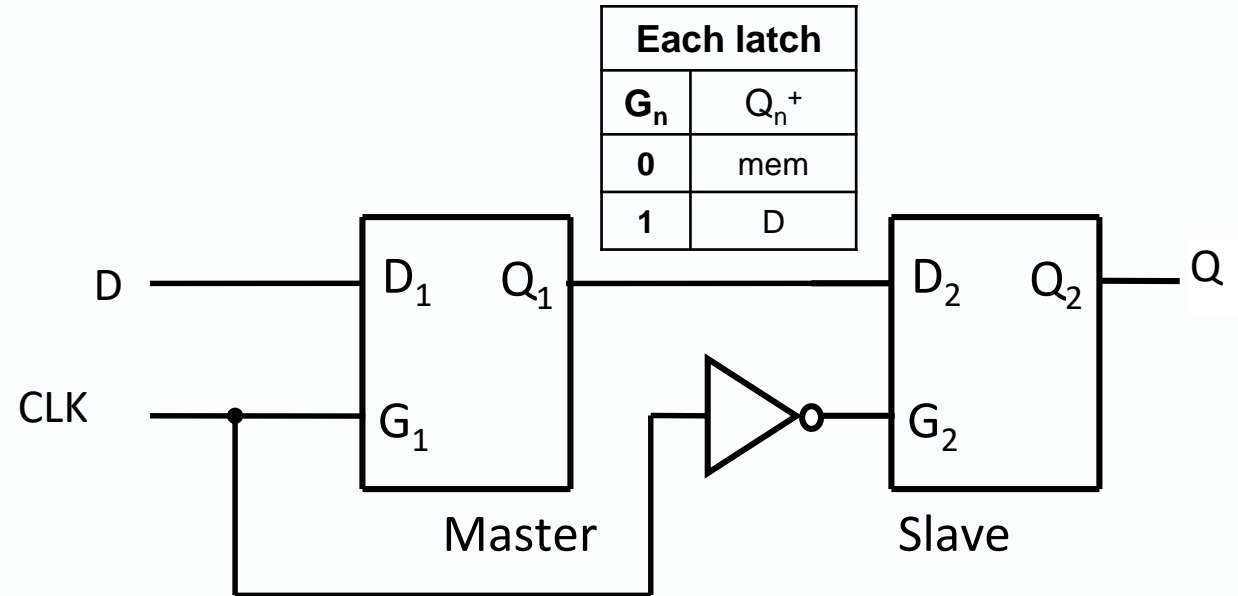
CLK 1→0:

$G_1 \rightarrow 0$  so  $D_2 = Q_1 = \text{mem}$

Before the edge,  $G_1 = 1$  so  $Q_1$  was D.  
So now  $Q_1$  and  $D_2$  store this.

$G_2 \rightarrow 1$  so  $Q = Q_2 = D_2 = \text{mem}$  so:

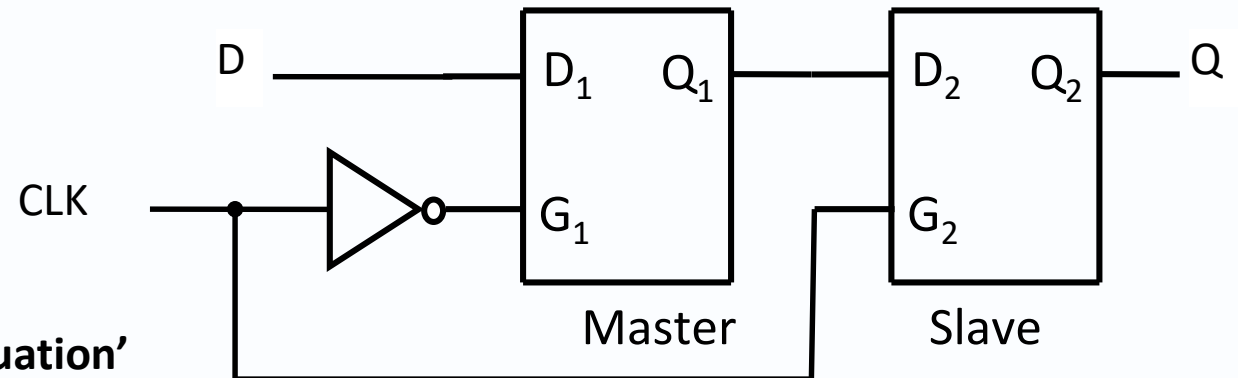
$Q^+ = D$ , meaning  $Q^+ = \text{whatever D was AT THE TIME OF the clock edge.}$



A *rising-edge-triggered* D flip-flop

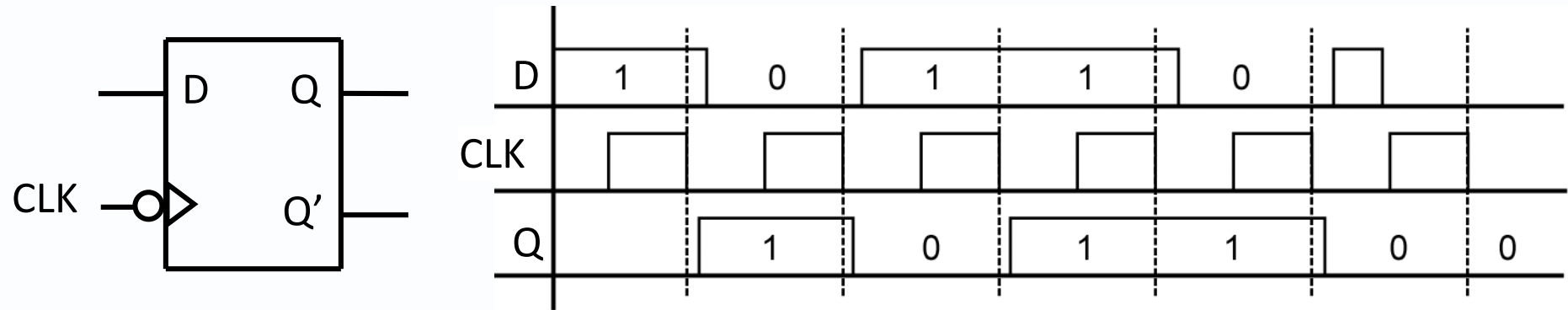
CLK 0→1:

Same applies:  $Q^+ = D$



$Q^+ = D$  is the 'characteristic equation'

# Edge-Triggered D-type Flip-Flop



The operation can be summarised in an action table.

D	Action at next clock pulse
0	“RESET”, $Q^+ = 0$
1	“SET”, $Q^+ = 1$

**The output after the clock pulse,  $Q^+$ , is equal to the D input at the time of the clock pulse.**



# Setup and Hold Times for an Edge-Triggered D Flip-Flop

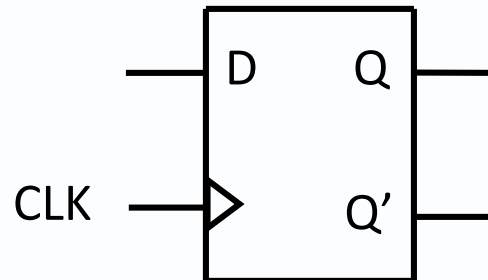
The **propagation delay** of a flip-flop ( $t_p$ ) is the time between the active edge of the clock and the resulting change in the output. 'Flip-flop delay'.

The D input also creates timing issues:

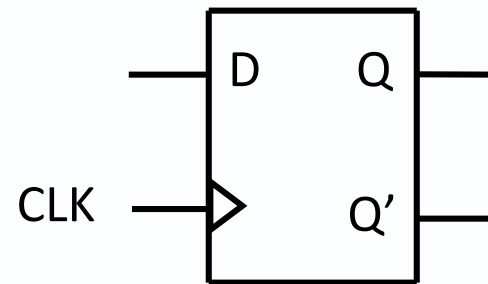
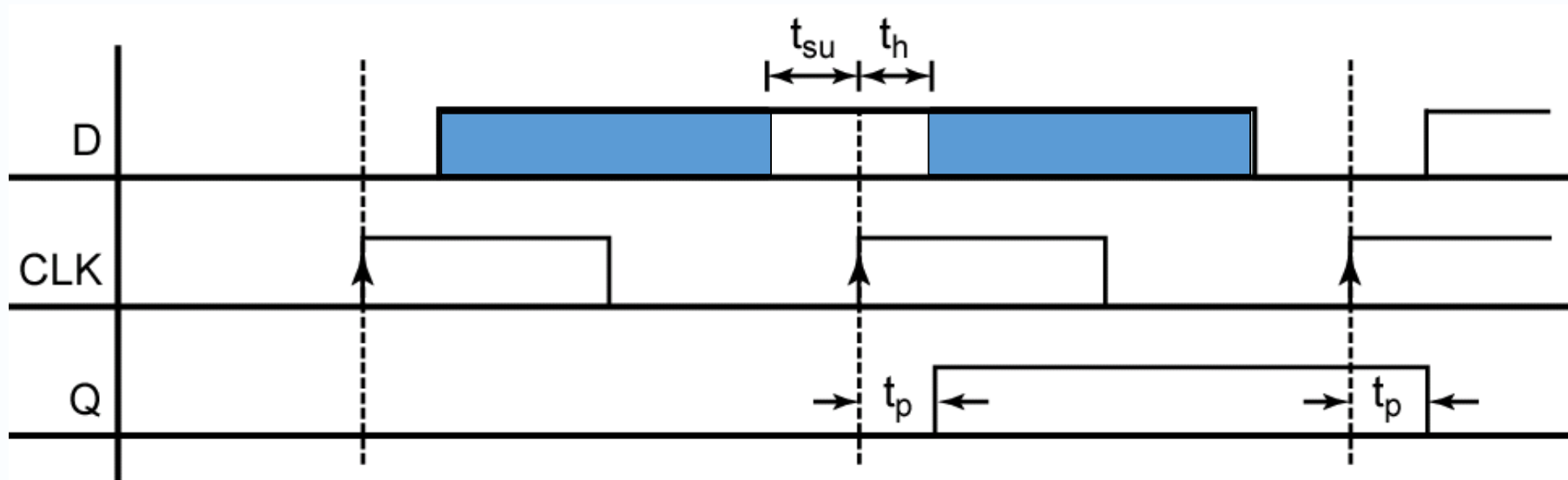
Amount of time that D must be stable **before** the active edge, is called the **setup time** ( $t_{su}$ ).

The amount of time that D must hold the same value **after** the active edge, is called the **hold time** ( $t_h$ ).

If D changes in the prohibited time interval, i.e. 'at or too close to the time of the active edge', the output is unpredictable.



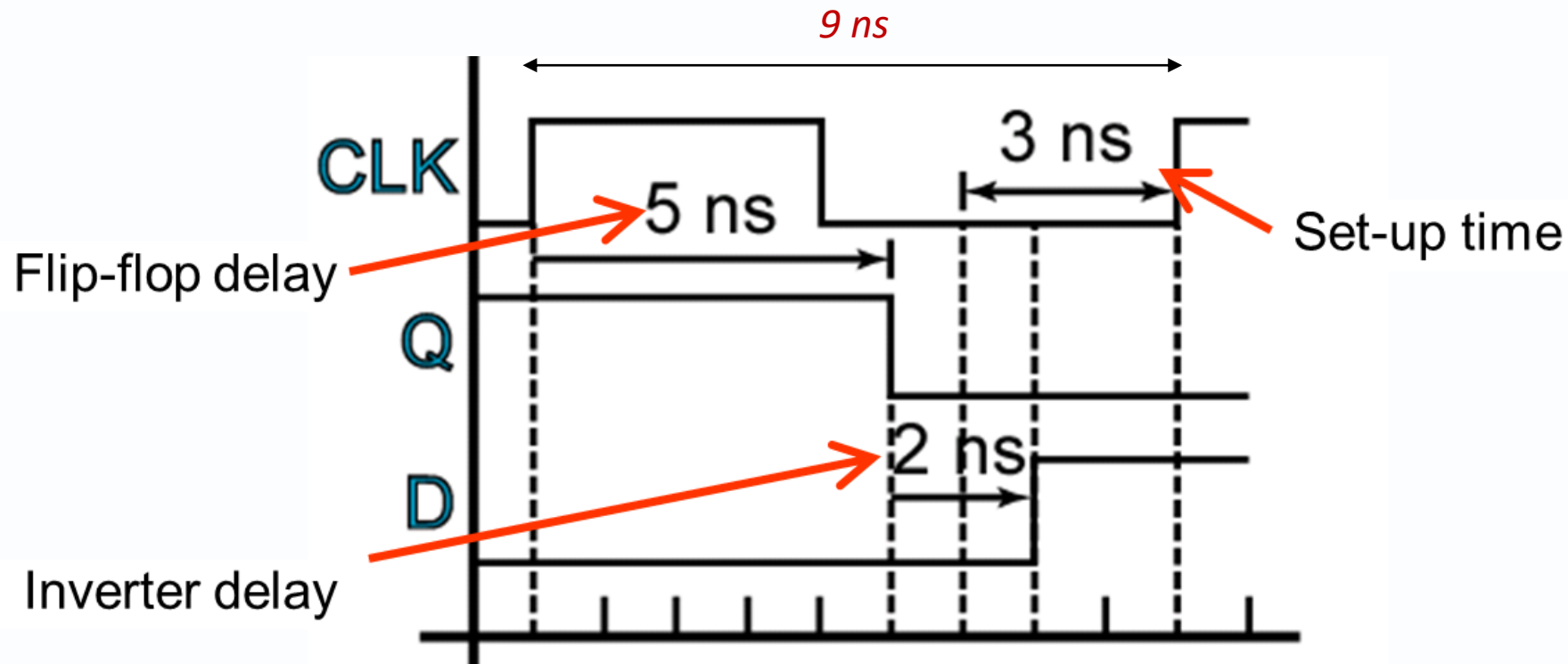
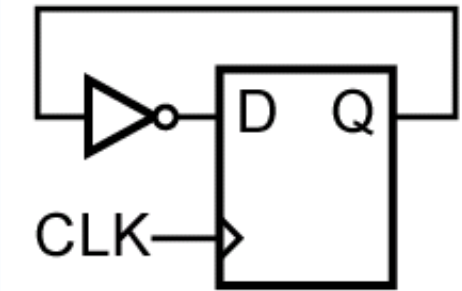
# Setup and Hold Times for a Rising-Edge-Triggered D Flip-Flop



D only allowed to change in the shaded regions  
(not during  $t_{su}$  or  $t_h$ )

# Determination of Minimum Clock Period

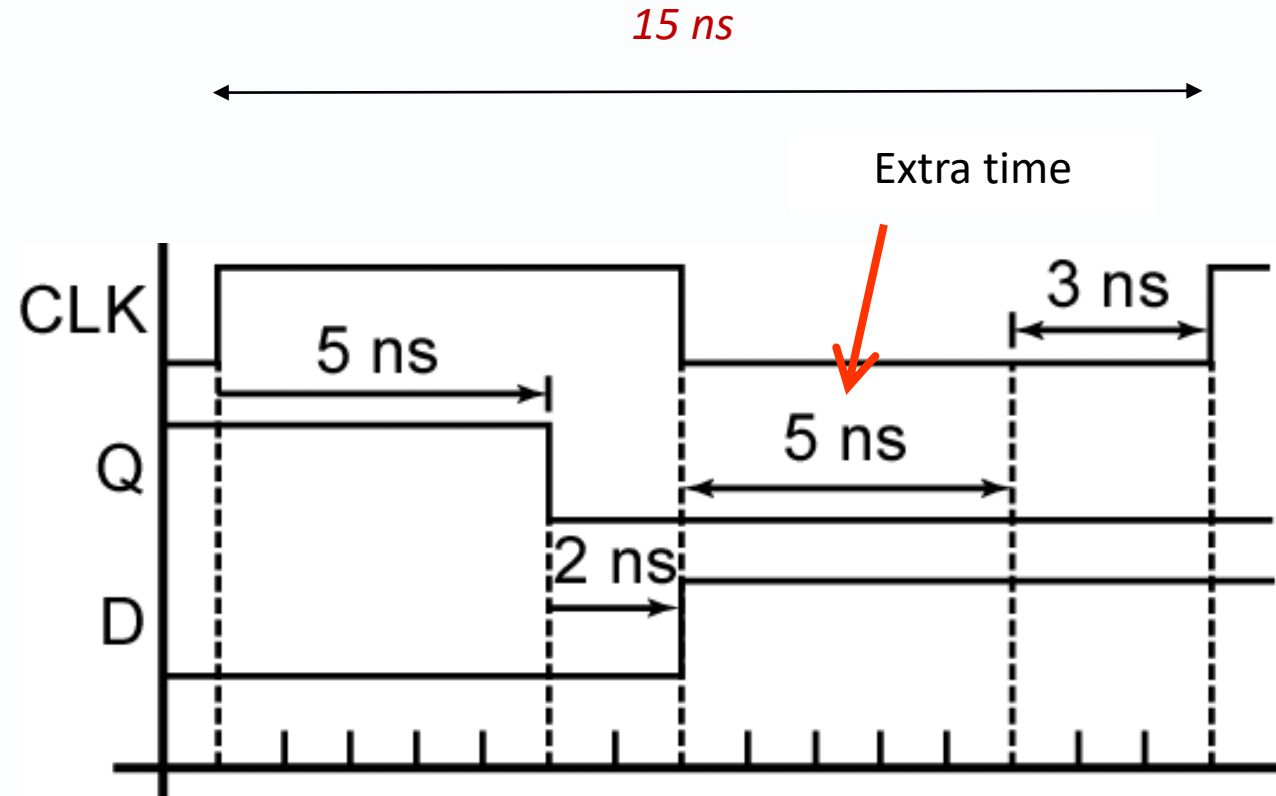
Suppose the inverter propagation delay is  $2\text{ ns}$   
the FF propagation delay is  $5\text{ ns}$ ,  
and its setup time is  $3\text{ ns}$ .  
(The hold time does not affect this calculation)



D is not in its new state long enough before the next rising edge (3ns set-up time required)

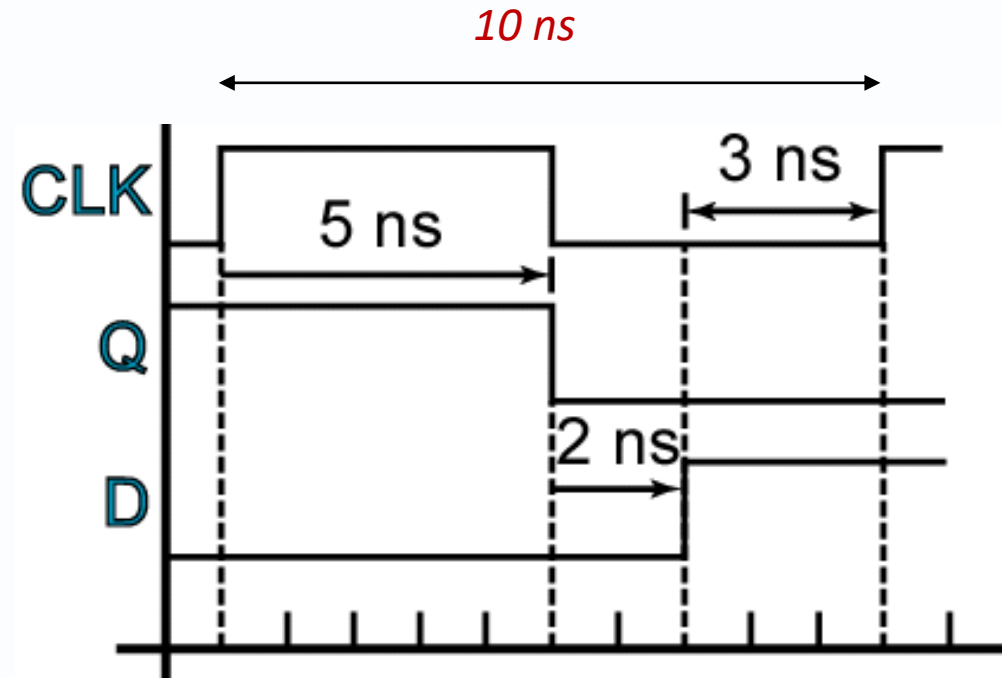
**$9\text{ ns}$**  is not enough for the clock period

# Determination of Minimum Clock Period



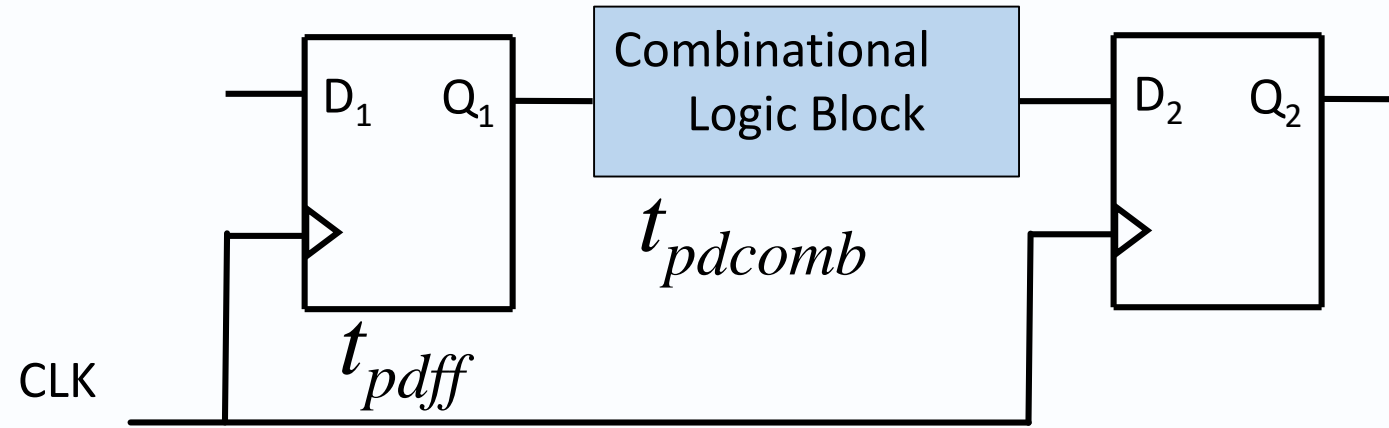
$15\text{ ns}$  is more than enough for the clock period.

# Determination of Minimum Clock Period



*10 ns* is just enough for the clock period.

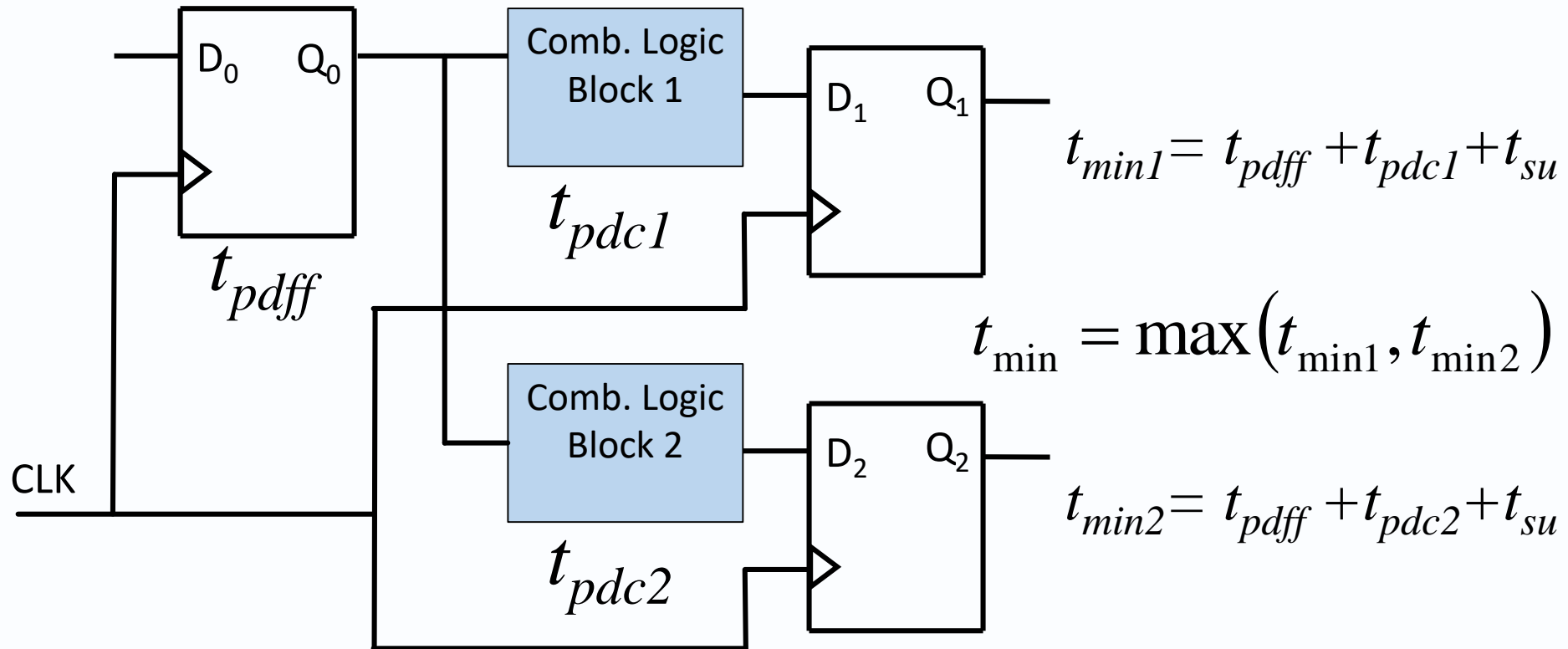
# Min. Clock Period in 2 Flip-flop System



$$t_{min} = t_{pdff} + t_{pdcomb} + t_{su}$$

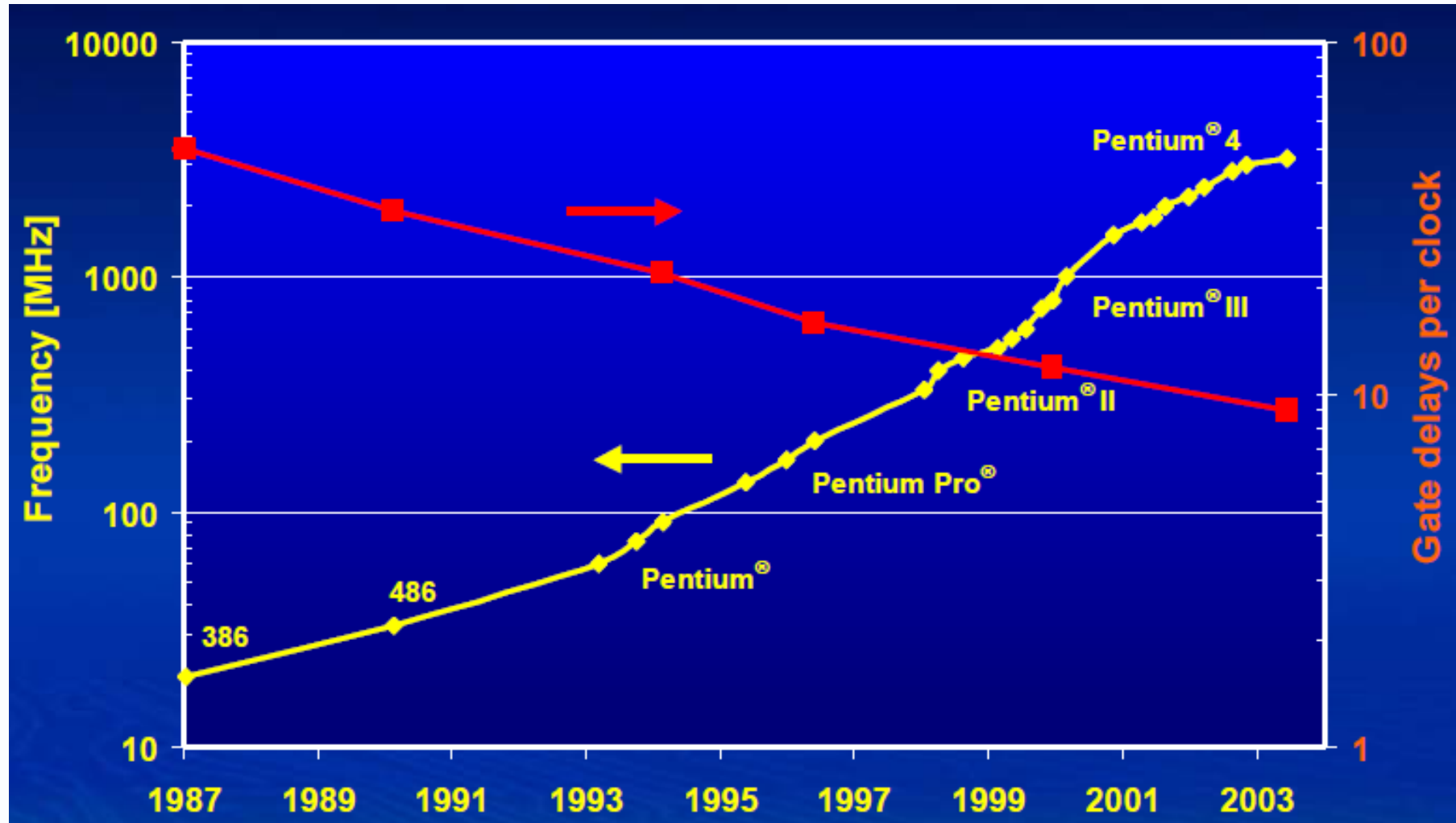
Set-up time and hold time are **not delays**. They are just timing constraints for the correct operation of flip-flops. The minimum clock period in a system does not depend on the hold time of flip flops.

# Min. Clock Period in a complex circuit



In a circuit with one clock & many flip-flops & combinational circuits, the longest propagation delay of the combinational circuits determines the minimum required clock period.

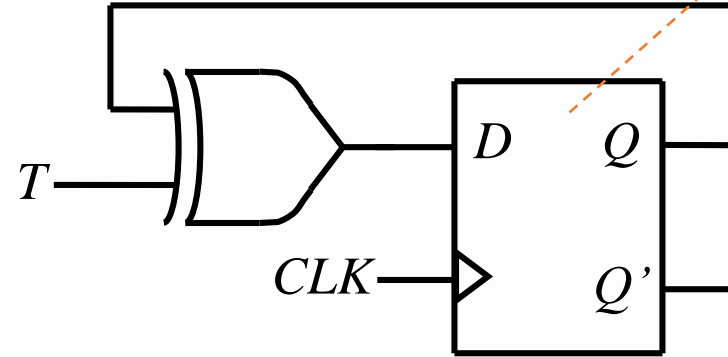
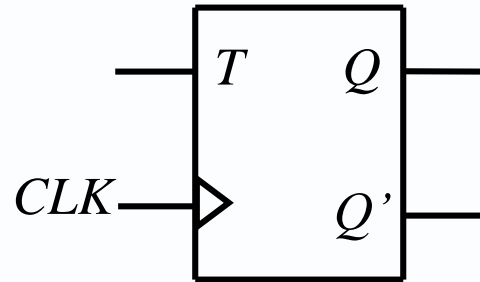
# Clock frequency and gate delay



From Intel presentation, 2004.



# T or Toggle Flip-Flop



(Edge-triggered D flip-flop:)

D	Action at next clock pulse
0	$Q^+ = 0$
1	$Q^+ = 1$

When  $T=1$  the flip-flop changes state after the active edge of the clock.

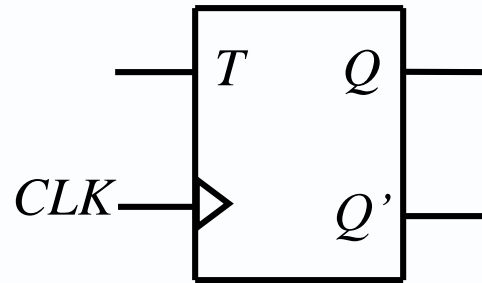
When  $T=0$  no state change occurs.

From the truth table:

$$Q^+ = D = \bar{T}Q + T\bar{Q} = T \oplus Q$$

$T$	$Q$	$Q^+$
0	0	0
0	1	1
1	0	1
1	1	0

# T or Toggle Flip-Flop



$T$	$Q$	$Q^+$
0	0	0
0	1	1
1	0	1
1	1	0

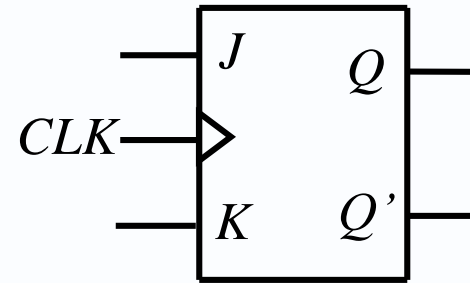
$T$	Action at next clock pulse
0	No change, $Q^+ = Q$
1	Toggle (change), $Q^+ = Q'$

Toggle flip-flop often used in building counters.

Most CPLDs and FPGAs can be programmed to implement toggle flip-flops.

# J-K Flip-Flop

$J$	$K$	$Q$	$Q^+$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0



$J$	$K$	Action at next clock pulse
0	0	No change, $Q^+ = Q$
0	1	Reset, $Q^+ = 0$
1	0	Set, $Q^+ = 1$
1	1	Toggle, $Q^+ = Q'$

Extended version of the S-R flip-flop ... simultaneous "1" allowed  
J is like S, K like R.

# Summary and suggested reading

- Gate delays, timing diagrams (Section 8.3)
- Set-Reset latch (Section 11.2)
- Gated D-latch (Section 11.3)
- Edge-triggered D flip-flop (Section 11.4)
- S-R, JK and T flip-flops (Sections 11.5-7)

**Roth and Kinney** *Fundamentals of Logic Design*

