Digital Electronics and Microprocessor Systems (ELEC211)

Dave McIntosh and Valerio Selis

dmc@liv.ac.uk

v.selis@liv.ac.uk

Week 6 Q&A

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Week 6 – Lecture 14

Digital Electronics

Basic gates and function implementation

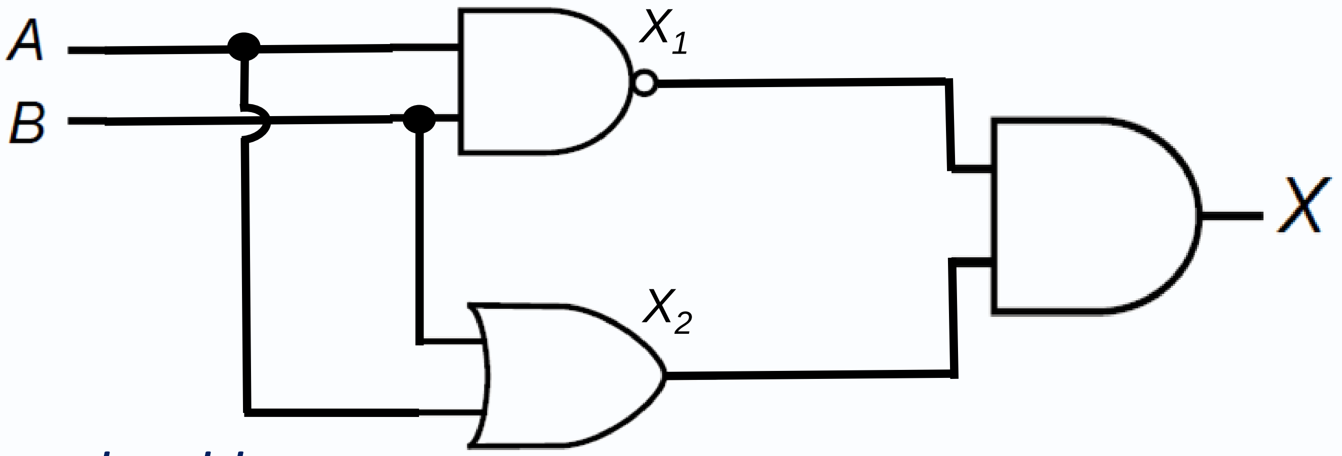
Question

Which logic gate is "(A NAND B) AND (A OR B)" equivalent to?

$$X = (\overline{A \cdot B}) \cdot (A + B)$$

Answer

Method part 1: Draw circuit diagram



Method part 2: Construct a combined truth table

A NAND B

A	B	X_1
0	0	**1**
0	1	**1**
1	0	**1**
1	1	**0**

A OR B

A	B	X_2
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**1**

(Combined)

A	B	X_1	X_2	X
0	0	**1**	**0**	**0**
0	1	**1**	**1**	**1**
1	0	**1**	**1**	**1**
1	1	**0**	**1**	**0**

(A NAND B) **AND** (A OR B)

A	B	X
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**

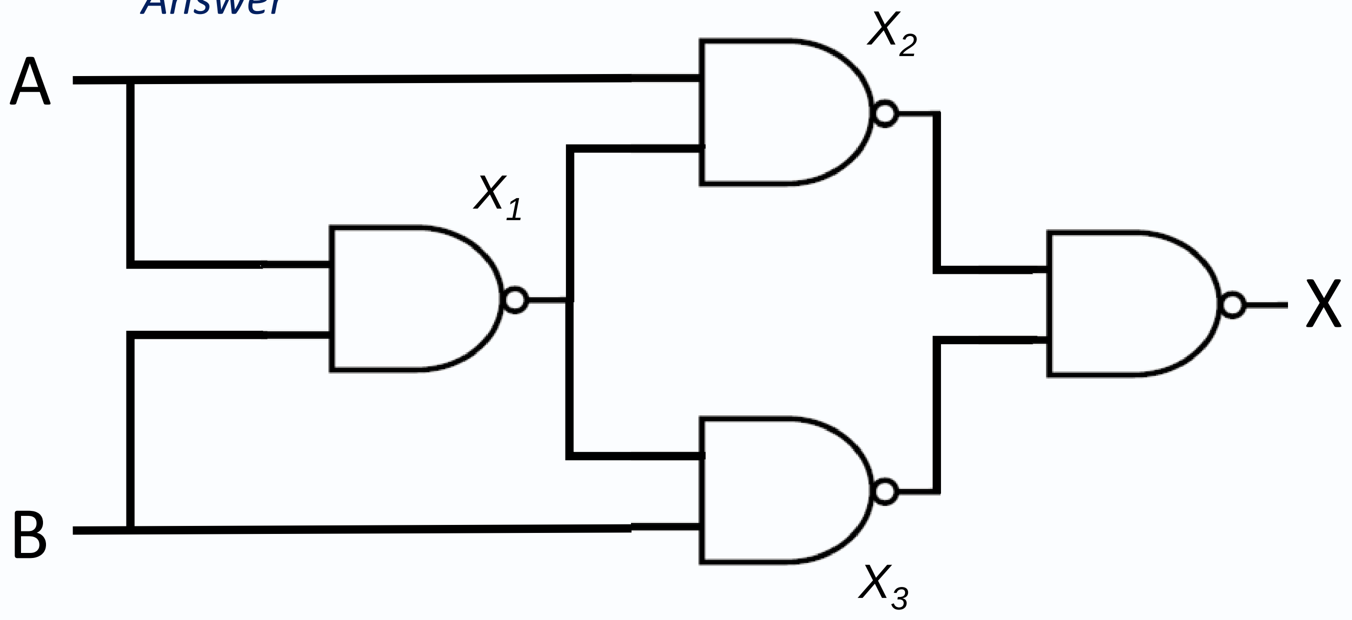
Answer: $\boldsymbol{X = A \oplus B}$ **(XOR)**

Question

Make the equivalent of a XOR gate using four NAND gates

Answer

Answer



Proof

A	B	X_1	$\boldsymbol{X_2}$	$\boldsymbol{X_3}$	$\boldsymbol{X}$
0	0	1	**1**	**1**	**0**
0	1	1	**1**	**0**	**1**
1	0	1	**0**	**1**	**1**
1	1	0	**1**	**1**	**0**

A	B	$\boldsymbol{X}$
0	0	**0**
0	1	**1**
1	0	**1**
1	1	**0**

$$\therefore X = A \oplus B \text{ (XOR)}$$

Question

- Substitute in the minterms to give the following 'canonical sum of products' as a sum of product terms, and simplify (if possible).

$$f(C,B,A) = \sum m(0,3,6)$$

Answer

$$f(C,B,A) = \sum m(0,3,6)$$

$$f(C,B,A) = m_0 + m_3 + m_6$$

Answer: $$f(C,B,A) = C'B'A' + C'BA + CBA'$$

Note that it can't be simplified:

f	$C'B'$	$C'B$	CB	CB'
A'	1		1	
A		1		

Question

- By substituting in the minterms and simplifying, rewrite the following 'canonical sum of products' expression as a minimum sum of products (minimum SOP).

$$f(D,C,B,A) = \sum m(1,4,8,9,14)$$

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Answer

$$f(D,C,B,A) = \sum m(1,4,8,9,14)$$

$$f(D,C,B,A) = m_1 + m_4 + m_8 + m_9 + m_{14}$$

$$= D'C'B'A + D'CB'A' + DC'B'A' + DC'B'A + DCBA'$$

	D'C'	D'C	DC	DC'
B'A'		1		1
B'A	1			1
BA				
BA'			1	

Answer: $f = C'B'A + D'CB'A' + DC'B' + DCBA'$

Answer – alternative method

A solution using only Boolean algebra.
Note: In this case the Boolean simplification is fairly straightforward. In general, for similar (and more complicated) examples, K-maps may be simpler, and allow confidence that you have fully rather than partially minimised an expression.

$$f = D'C'B'A + D'CB'A' + DC'B'A' + \underline{DC'B'A} + DCBA'$$

Using the **idempotent law**: $X = X + X$

So that: $DC'B'A = DC'B'A + DC'B'A$

We duplicated *DC'B'A* so that it could then be used twice for variable elimination

$$f = \boxed{D'C'B'A + \underline{DC'B'A}} + D'CB'A' + \boxed{DC'B'A' + \underline{DC'B'A}} + DCBA'$$

$$f = \boxed{(D' + D)C'B'A} + D'CB'A' + \boxed{DC'B'(A' + A)} + DCBA'$$

Answer: $f = C'B'A + D'CB'A' + DC'B' + DCBA'$

Week 6 – Lecture 15

Digital Electronics

Multiplexers, decoders and encoders

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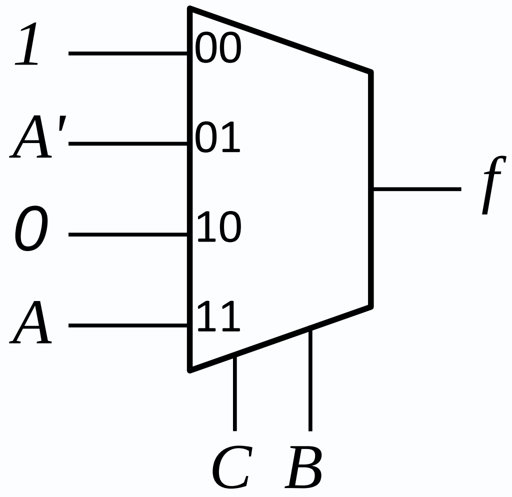
Question

- What function does this 4 to 1 MUX circuit implement?



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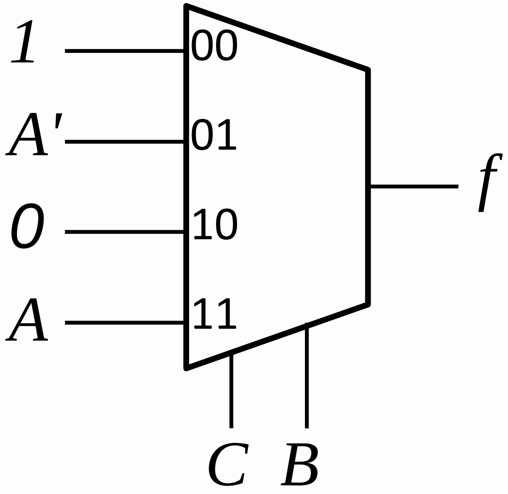
Answer



C	B	f
0	0	1
0	1	A'
1	0	0
1	1	A

	C	B	A	f
$C'.B'.\mathbf{1}$	0	0	0	1
	0	0	1	1
$C'.B.\mathbf{A'}$	0	1	0	1
	0	1	1	0
$C.B'.\mathbf{0}$	1	0	0	0
	1	0	1	0
$C.B.\mathbf{A}$	1	1	0	0
	1	1	1	1

$$f = C'B' + C'BA' + CBA$$

Question

- Design a circuit for the following expression using a 4 to 1 mux with B and C connected to the select inputs.

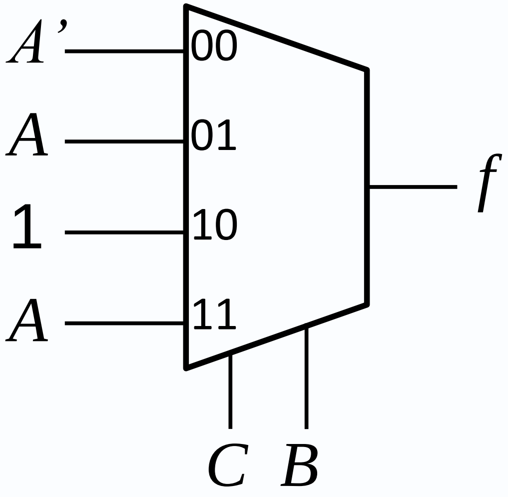
$$f = B'A' + BA + CB'$$

Answer

Method

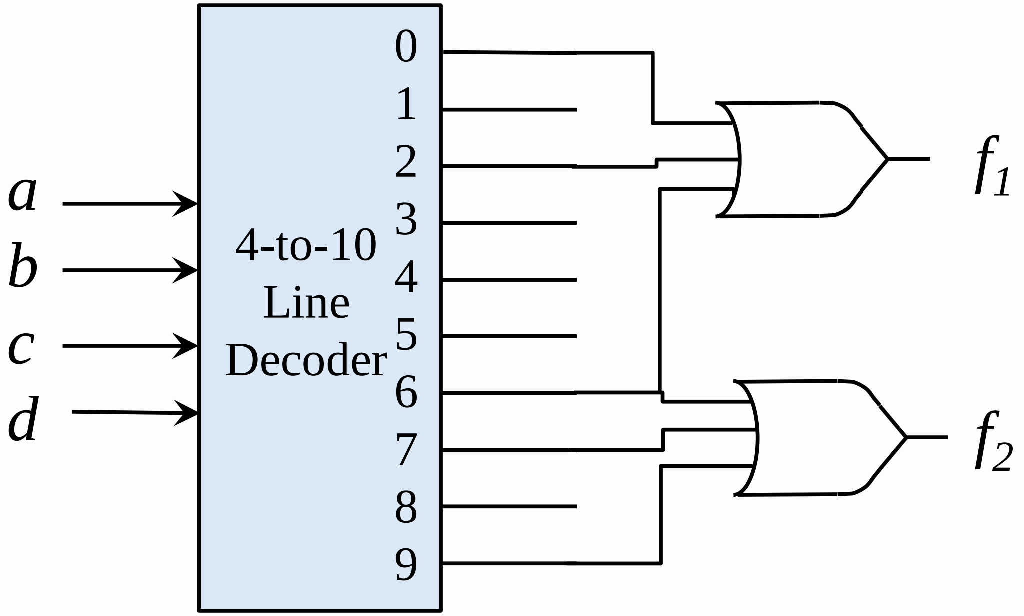
C	B	A	f	
0	0	0	1	A'
0	0	1	0	
0	1	0	0	A
0	1	1	1	
1	0	0	1	1
1	0	1	1	
1	1	0	0	A
1	1	1	1	

Answer



Question

- What functions f_1 and f_2 does this circuit implement? Find a sum of products expression for each one.



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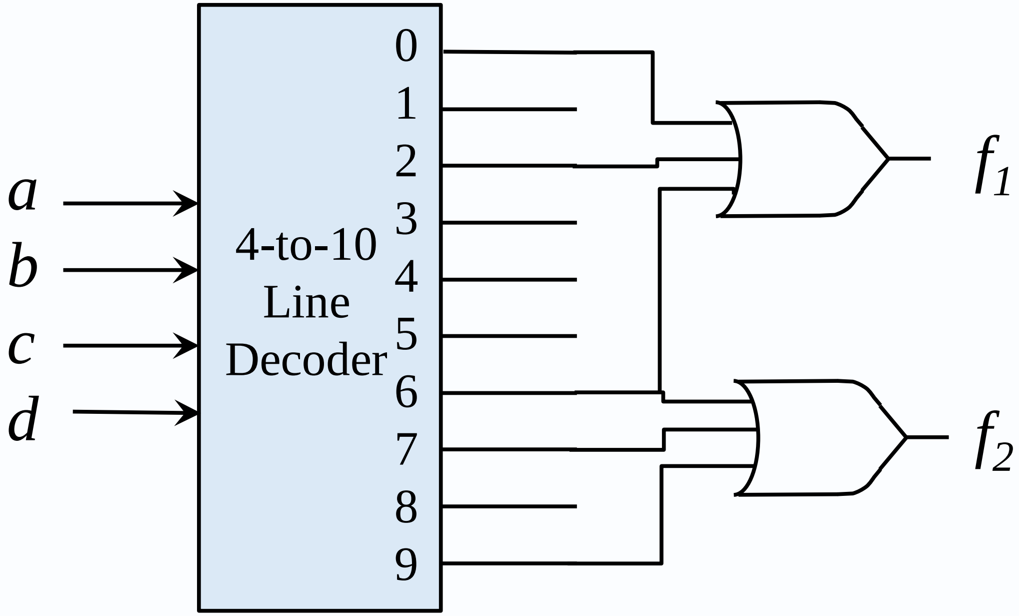
Answer



$$f_1 = m_0 + m_2 + m_6 \ldots = a'.b'.c'.d' + a'.b'.c.d' + a'.b.c.d' \ldots$$

$$\boldsymbol{f_1 = a'.b'.d' + a'.c.d'}$$ ***(answer part 1)***

$$f_2 = m_6 + m_7 + m_9 \ldots = a'.b.c.d' + a'.b.c.d + a.b'.c'.d \ldots$$

$$\boldsymbol{f_2 = a'.b.c + a.b'.c'.d}$$ ***(answer part 2)***

Question

- Design a circuit for the following expression using a BCD to decimal decoder and two OR gates.

$$f_1 = a'b'c' + a'c'd$$

$$f_2 = a'bd + ab'c'd'$$

Method

$$f_1 = a'b'c' + a'c'd$$

$$= \overset{m_0}{a'b'c'd'} + \overset{m_1}{a'b'c'd} + a'b'c'd + \overset{m_5}{a'bc'd}$$

$$f_2 = a'bd + ab'c'd'$$

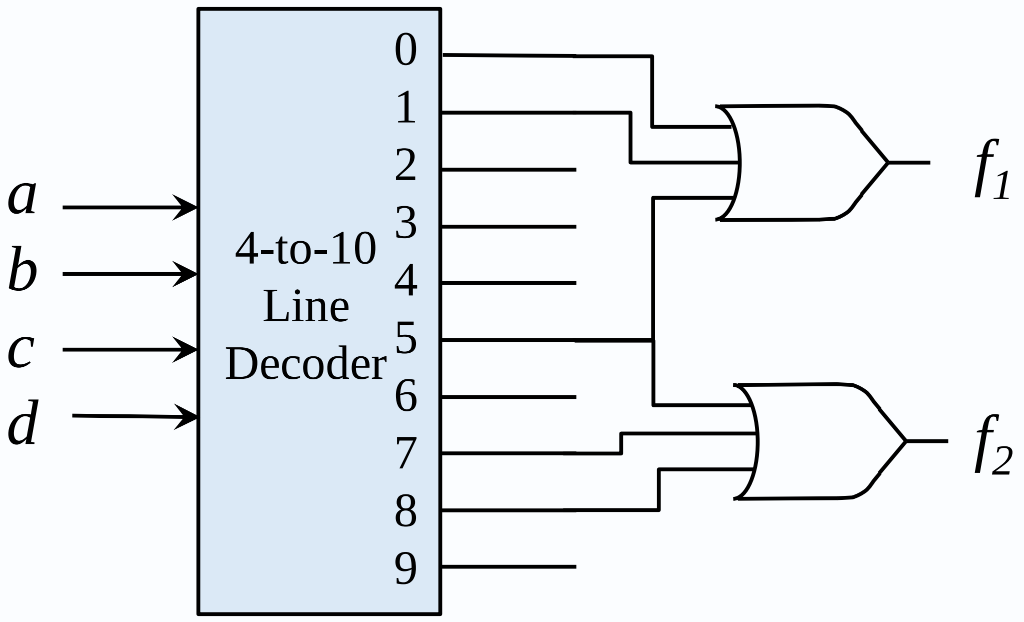
$$= \overset{m_5}{a'bc'd} + \overset{m_7}{a'bcd} + \overset{m_8}{ab'c'd'}$$

$$f_1 = m_0 + m_1 + m_5$$

$$f_2 = m_5 + m_7 + m_8$$

a	*b*	*c*	*d*		f_1	f_2	
0	0	0	0	m_0	1	0	
0	0	0	1	m_1	1	0	
0	0	1	0		0	0	
0	0	1	1		0	0	
0	1	0	0		0	0	
0	1	0	1	m_5	1	1	m_5
0	1	1	0		0	0	
0	1	1	1		0	1	m_7
1	0	0	0		0	1	m_8
1	0	0	1		0	0	
1	0	1	0		0	0	
1	0	1	1		0	0	
1	1	0	0		0	0	
1	1	0	1		0	0	
1	1	1	0		0	0	
1	1	1	1		0	0	

Answer



Week 6 – Lecture 16

Digital Electronics

Tristate gates, ROM, PLA and PAL

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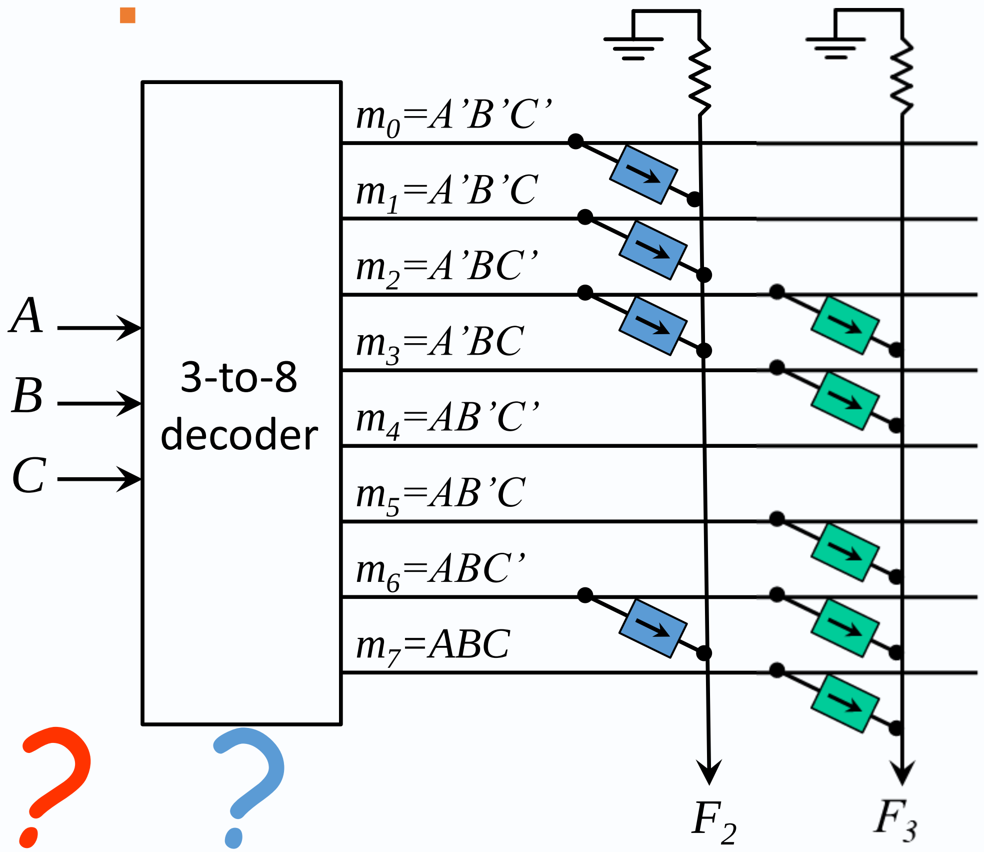
Question



- Find the 'sum of products' expressions for outputs F_2 and F_3 and simplify if possible.

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Answer (F_2)

Karnaugh map

F_2	$\boldsymbol{A'B'}$	$\boldsymbol{A'B}$	$\boldsymbol{AB}$	$\boldsymbol{AB'}$
$\boldsymbol{C'}$	1	1	1	0
$\boldsymbol{C}$	1	0	0	0

$$F_2 = \Sigma m(0,1,2,6)$$

$$= A'B'C' + A'B'C + A'BC' + ABC'$$

$$= A'B'(C + C') + (A + A')BC'$$

$$= \boldsymbol{A'B' + BC'}$$

Answer (F_3)

K-map

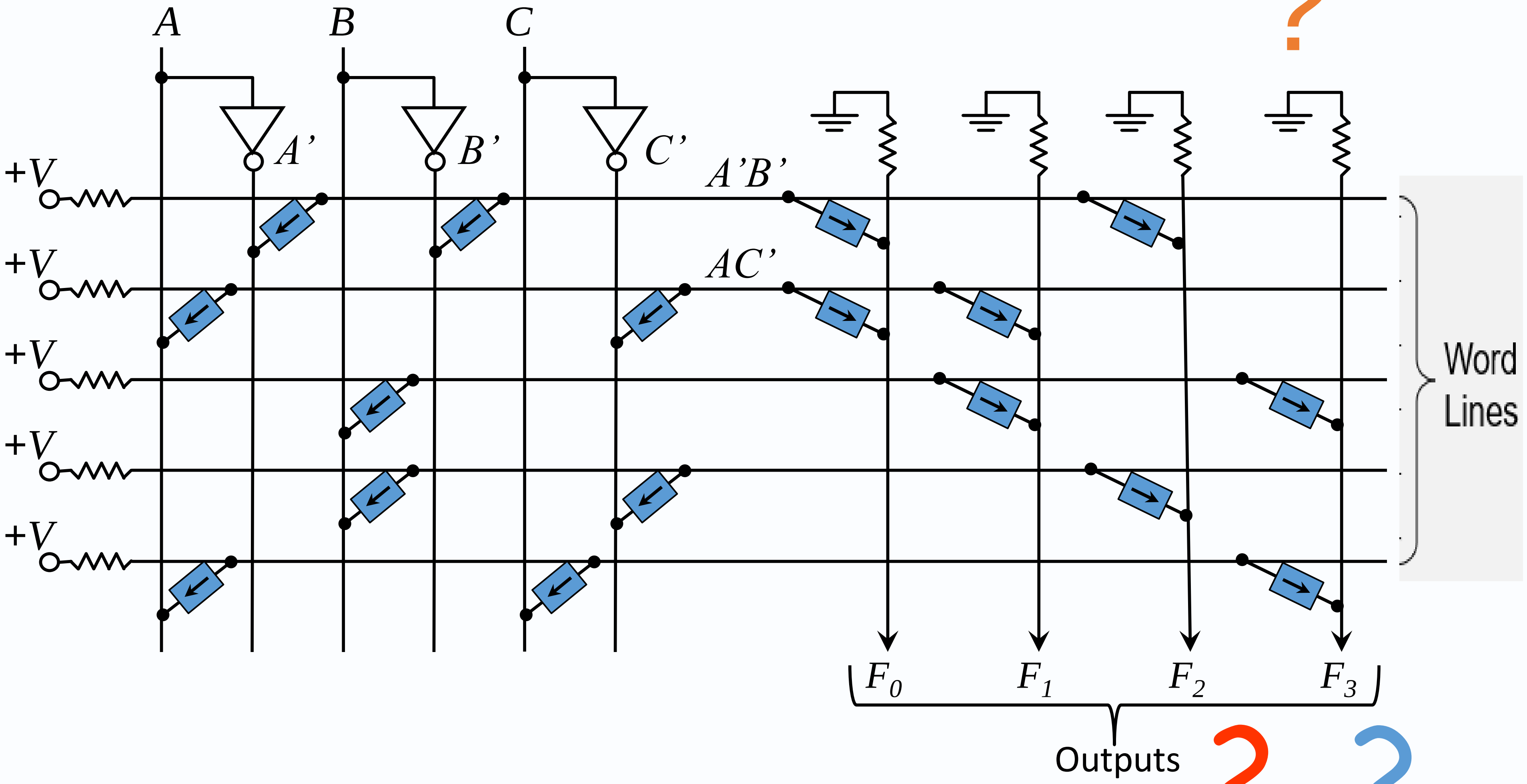
F_3	$A'B'$	$A'B$	AB	AB'
C'	0	1	1	0
C	0	1	1	1

$$F_3 = \Sigma m(2,3,5,6,7)$$
$$= A'BC' + A'BC + AB'C + ABC' + ABC$$
$$= A(B' + B)C + B(A'C' + A'C + AC' + AC)$$
$$= \boldsymbol{AC + B}$$

Question

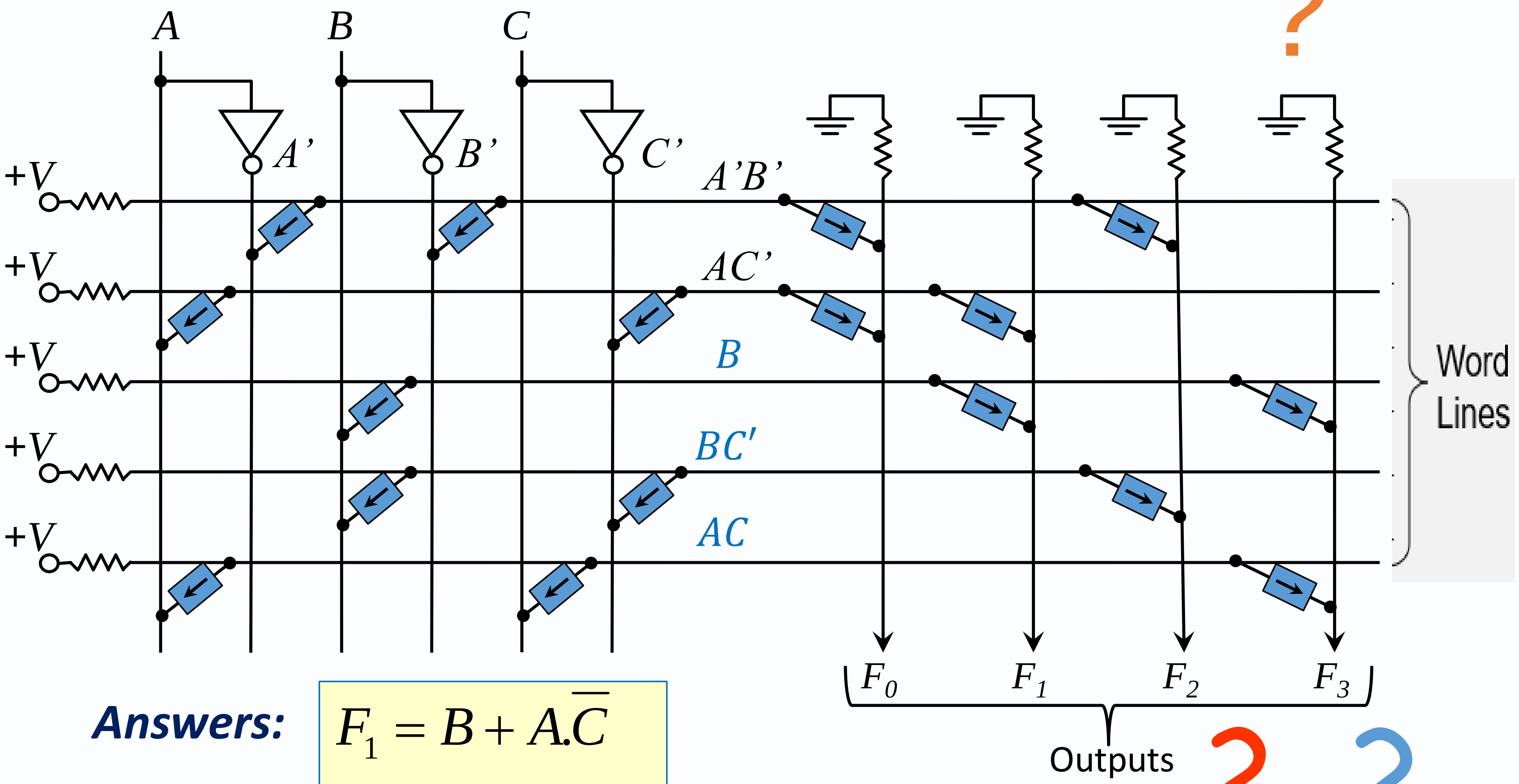
Find the 'sum of products' expressions for the outputs

F_1, F_2, F_3.



Hint: $F_0 = \overline{A}\,\overline{B} + A\overline{C}$

Answer



Answers:

$$F_1 = B + A.\overline{C}$$

$$F_2 = \overline{A}.\overline{B} + B.\overline{C}$$

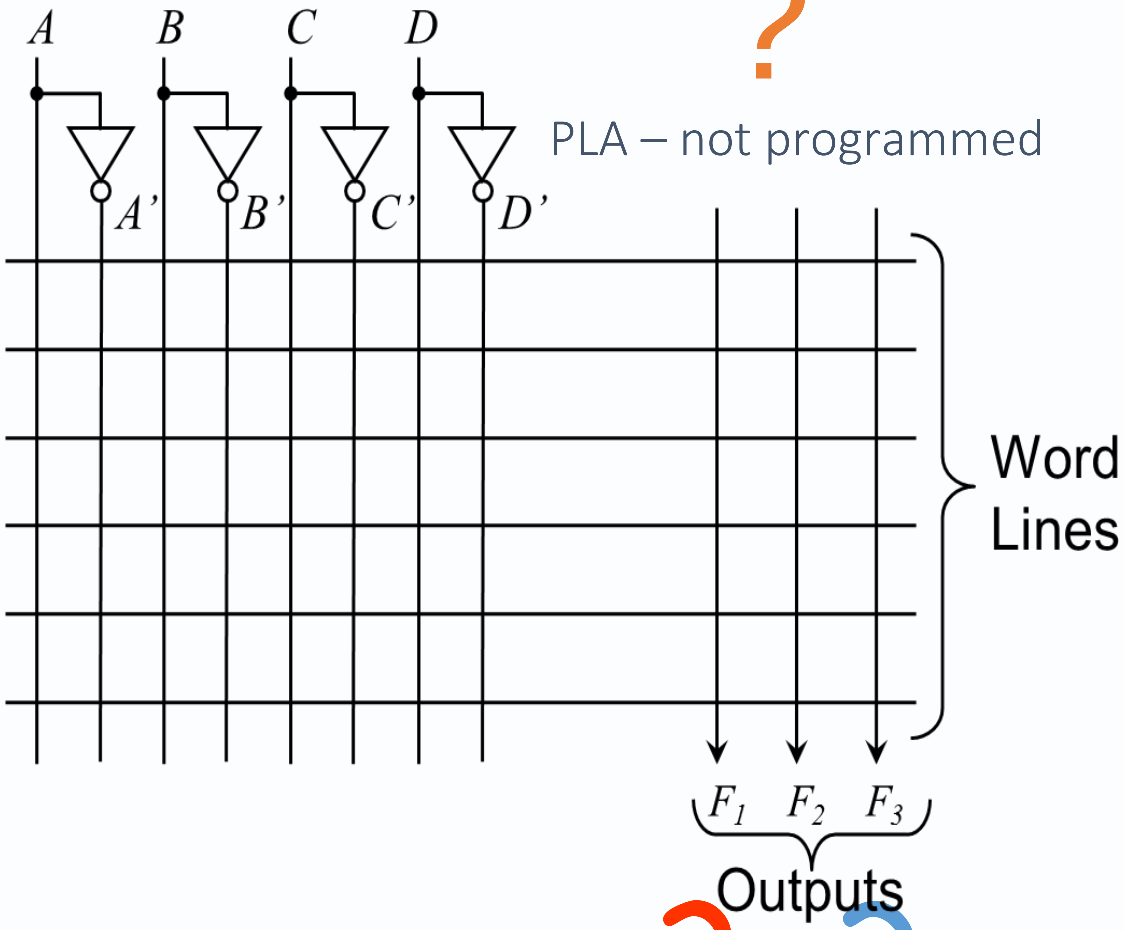
$$F_3 = B + A.C$$

Question

- Design a circuit with outputs F_1, F_2 and F_3 using a PLA

$$F_1 = BC' + A'D + B'D$$
$$F_2 = A'C' + B'D + CD$$
$$F_3 = BC' + AC'D$$

PLA – not programmed

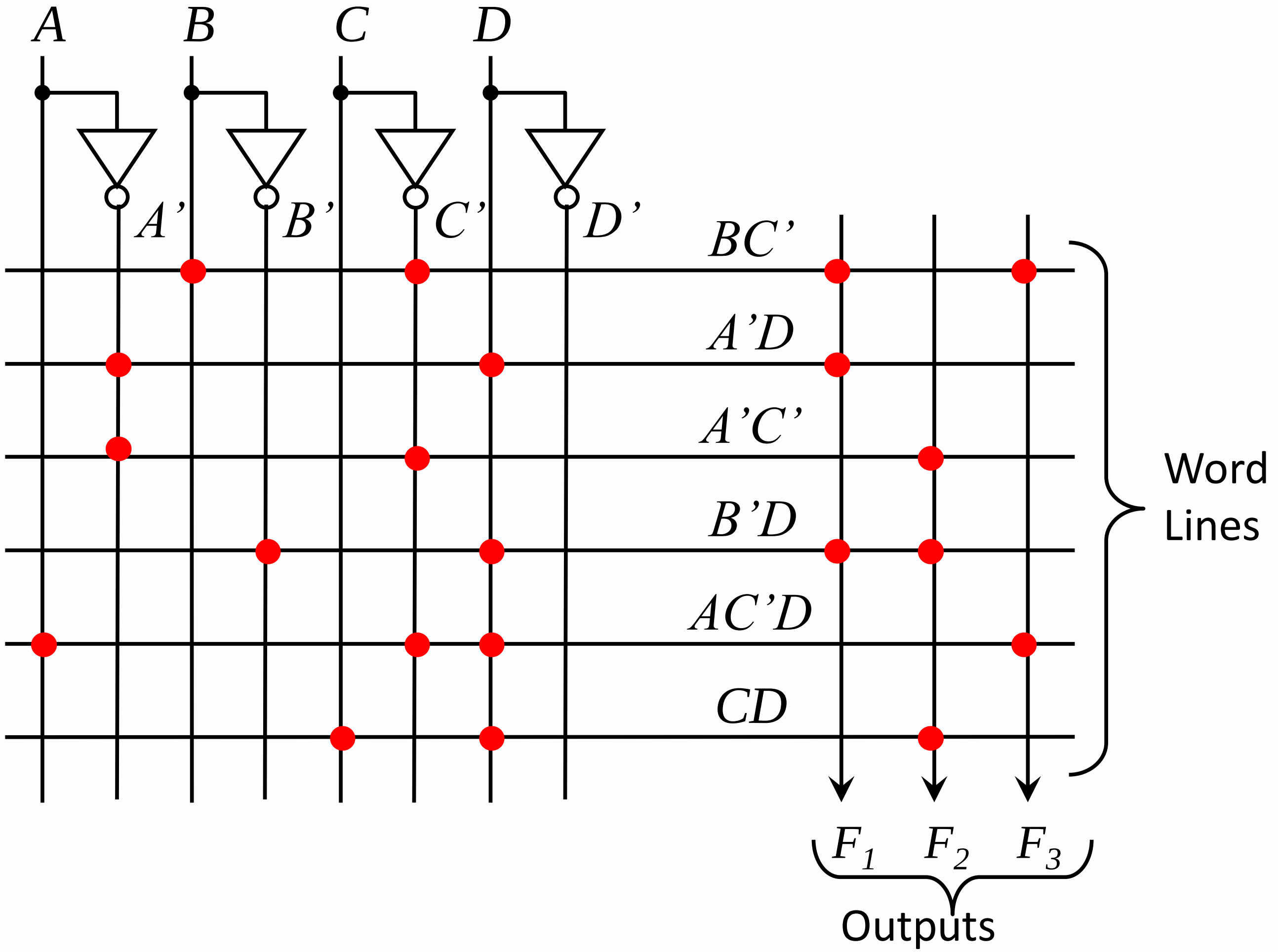


Answer

Method: PLA table

INPUTS				OUTPUTS		
A	B	C	D	F_1	F_2	F_3
-	1	0	-	1	0	1
0	-	-	1	1	0	0
0	-	0	-	0	1	0
-	0	-	1	1	1	0
1	-	0	1	0	0	1
-	-	1	1	0	1	0

Answer:



Question

- Design a circuit with outputs B, C and D using a PAL14H4.

$$B = U \cdot V' + V \cdot W' + U' \cdot W \cdot X + X' \cdot Y \cdot Z$$

$$C = S' \cdot Y' + T \cdot W' + S \cdot V \cdot W$$

$$D = P \cdot Z + R' \cdot S \cdot W + P' \cdot R \cdot V + W \cdot X$$

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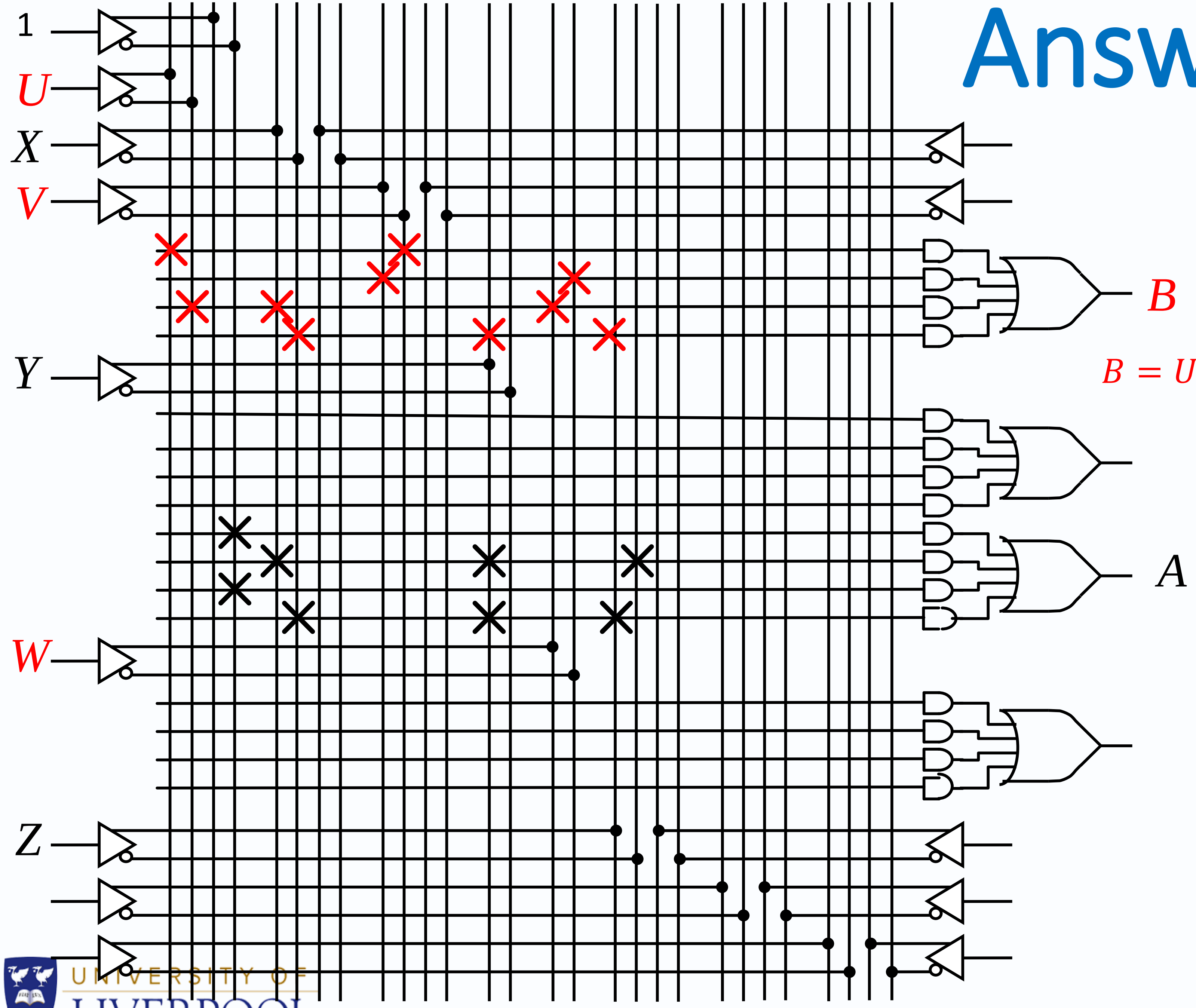
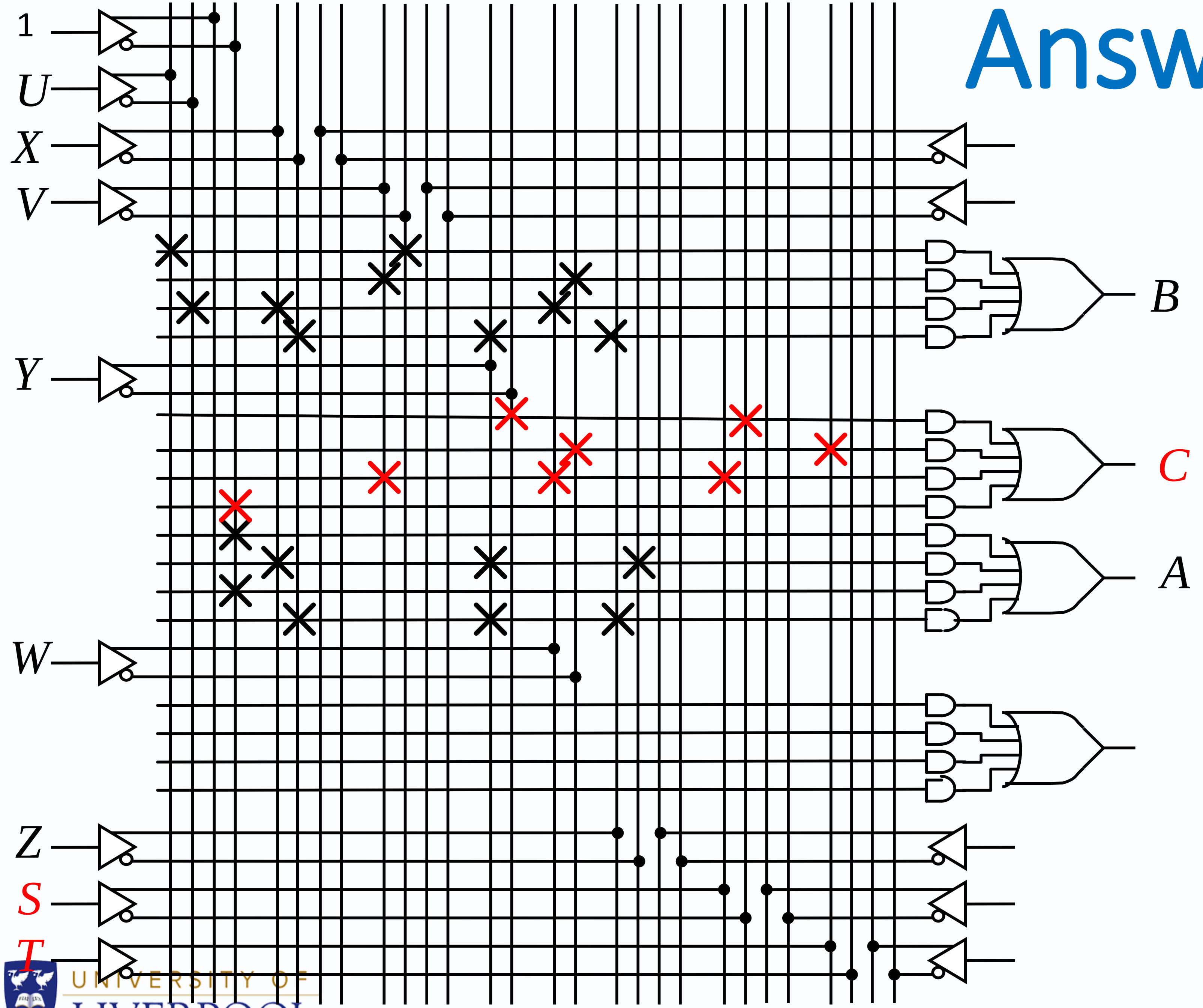
Answer (B)



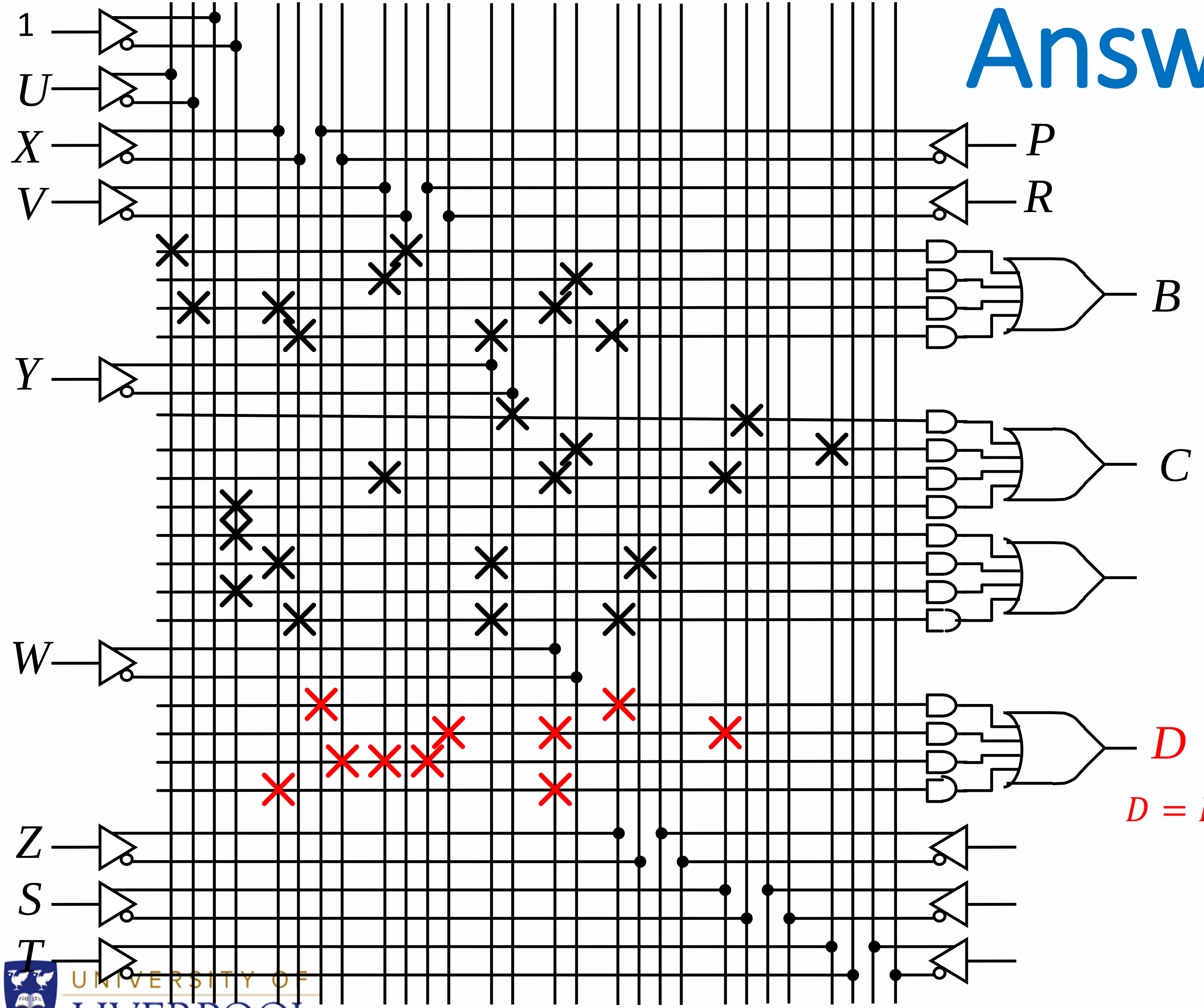
$B = U \cdot V' + V \cdot W' + U' \cdot W \cdot X + X' \cdot Y \cdot Z$

Answer (C)



$$C = S' \cdot Y' + T \cdot W' + S \cdot V \cdot W$$

Answer (*D*)



$D = P \cdot Z + R' \cdot S \cdot W + P' \cdot R \cdot V + W \cdot X$