Tutorial 1

Example 1 (Prisoner's Dilemma)

$N = \{1, 2\}$ Set of players

$S_1 = S_2 = \{\text{Quiet}, \text{Fink}\}$ Set of pure strategies for each player (actions)

① \ ②	Quiet	Fink
Quiet	2, 2	0, 3
Fink	3, 0	1, 1

Payoff function for each player (Preference relation)

Question: Which are the P.N.E.'s (if any)?

→ 1 P.N.E.: (Fink, Fink)

Example 2 (Matching Pennies)

① \ ②	Head	Tails
Head	1, -1	-1, 1
Tails	-1, 1	1, -1

⇝ All information of bimatrix game captured.

→ no P.N.E.

Example 3
(Battle of the Sexes)

Boy \ Girl	Theatre!	OK, football...
OK, theatre...	1 , 5	0, 0
Football!	0, 0	5, 1

↳ 2 P.N.E : (OK, theatre..., Theatre!) , (Football! , OK, football...)

... a Modification

Boy \ Girl	Theatre!	OK, football...	Football great, I will invite my dad
OK, theatre...	1, 5	0, 0	0 , 0
Football!	0, 0	5, 1	-1, 2

↳ 1 P.N.E. : (OK, theatre..., Theatre!)

So, there can be 0, 1, or multiple PNEs in a finite normal form game.

But, there is at least 1 (mixed) N.E. in every finite normal form game.

Exercise

① \ ②	L	R
T	3, 2	1, 6
M	5, 6	0, 5
B	0, 7	2, 2

a) $p = (\frac{1}{4}, \frac{1}{4}, \frac{1}{2})$, $q = (\frac{2}{3}, \frac{1}{3})$

a1) What is each player's expected payoff?

a2) Is (p, q) a N.E.?

b) $p = (\frac{1}{5}, \frac{4}{5}, 0)$, $q = (\frac{1}{3}, \frac{2}{3})$

b1) What is each player's expected payoff?

b2) Is (p, q) a N.E.?

a1) $$u_1(p,q) = \frac{1}{4}\cdot\frac{2}{3}\cdot 3 + \frac{1}{4}\cdot\frac{1}{3}\cdot 1 + \frac{1}{4}\cdot\frac{2}{3}\cdot 5 + \frac{1}{4}\cdot\frac{1}{3}\cdot 0 + \frac{1}{2}\cdot\frac{2}{3}\cdot 0 + \frac{1}{2}\cdot\frac{1}{3}\cdot 2 = \boxed{\frac{21}{12}}$$

$$u_2(p,q) = \frac{1}{4}\cdot\frac{2}{3}\cdot 2 + \frac{1}{4}\cdot\frac{1}{3}\cdot 6 + \frac{1}{4}\cdot\frac{2}{3}\cdot 6 + \frac{1}{4}\cdot\frac{1}{3}\cdot 5 + \frac{1}{2}\cdot\frac{2}{3}\cdot 7 + \frac{1}{2}\cdot\frac{1}{3}\cdot 2 = \boxed{\frac{59}{12}}$$

a2) N.E.: For each player, the actions (pure strategies) in her support when played with probability 1 against the other player's mixed strategy, should yield the same expected payoff.
The actions not in her support when played with probability 1 against the other player's mixed strategy, should yield at most the same expected payoff as that of an action in the support.

$$\left.\begin{aligned} u_1(T,q) &= \frac{2}{3}\cdot 3 + \frac{1}{3}\cdot 1 = \frac{7}{3} \\ u_1(M,q) &= \frac{2}{3}\cdot 5 + \frac{1}{3}\cdot 0 = \frac{10}{3} \end{aligned}\right\} \longrightarrow u_1(T,q) \neq u_1(M,q)$$

So $\boxed{(p,q) \text{ not a N.E}}$

b1) $u_1(p,q) = \frac{1}{5}\cdot\frac{1}{3}\cdot 3 + \frac{1}{5}\cdot\frac{2}{3}\cdot 1 + \frac{4}{5}\cdot\frac{1}{3}\cdot 5 + \frac{4}{5}\cdot\frac{2}{3}\cdot 0 + 0\cdot\frac{1}{3}\cdot 0 + 0\cdot\frac{2}{3}\cdot 2 = \boxed{\frac{25}{15}}$

$u_2(p,q) = \frac{1}{5}\cdot\frac{1}{3}\cdot 2 + \frac{1}{5}\cdot\frac{2}{3}\cdot 6 + \frac{4}{5}\cdot\frac{1}{3}\cdot 6 + \frac{4}{5}\cdot\frac{2}{3}\cdot 5 + 0\cdot\frac{1}{3}\cdot 7 + 0\cdot\frac{2}{3}\cdot 2 = \boxed{\frac{78}{15}}$

b2)
$$\left.\begin{aligned} u_1(T,q) &= \frac{1}{3}\cdot 3 + \frac{2}{3}\cdot 1 = \frac{5}{3} \\ u_1(M,q) &= \frac{1}{3}\cdot 5 + \frac{2}{3}\cdot 0 = \frac{5}{3} \\ u_1(B,q) &= \frac{1}{3}\cdot 0 + \frac{2}{3}\cdot 2 = \frac{4}{3} \end{aligned}\right\} \longrightarrow u_1(T,q) = u_1(M,q) \geq u_1(B,q)$$

support of player 1: $\{T, M\}$ ✓

$$\left.\begin{aligned} u_2(p,L) &= \frac{1}{5}\cdot 2 + \frac{4}{5}\cdot 6 + 0\cdot 7 = \frac{26}{5} \\ u_2(p,R) &= \frac{1}{5}\cdot 6 + \frac{4}{5}\cdot 5 + 0\cdot 2 = \frac{26}{5} \end{aligned}\right\} \longrightarrow u_2(p,L) = u_2(p,R)$$

support of player 2: $\{L, R\}$ ✓

Therefore, $\boxed{(p,q) \text{ is a N.E.}}$