COMP323 - Introduction to Computational Game Theory

Tutorial 5 - Questions

Problem 1. Suppose we have 3 machines M_1, M_2, M_3 with speeds $s_1 = 3$, $s_2 = 10$ and $s_3 = 13$, respectively. We also have 6 tasks (players), with weights $w_1 = 30$, $w_2 = 27$, $w_3 = 21$, $w_4 = 18$, $w_5 = 14$, $w_6 = 12$. Find a pure Nash equilibrium using the LPT rule (see lecture slides). Is this also an optimum allocation?

Problem 2. Consider the following instance of the load balancing game where the number of tasks is equal to the number of machines, and in particular we have:

- m identical machines $M_1, M_2, \ldots, M_m$ (all of speed 1),
- m identical tasks $w_1 = w_2 = \cdots = w_m = 1$.

Consider also the mixed strategy profile A where each of the tasks is assigned to all machines equiprobably (i.e. with probability $1/m$). As shown in the set of problems of the previous week, A is a Nash equilibrium. Calculate the ratio $cost(A)/cost(OPT)$ in the following special cases:

(a) $m = 2$,

(b) $m = 3$.

Discuss what this ratio is for arbitrary m. What does this imply about the Price of Anarchy on identical machines for mixed Nash equilibria?