ELEC 207
Instrumentation and Control

Example – Transient and Frequency Response

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Example 1 – Current transducer
Dynamic specifications (1)

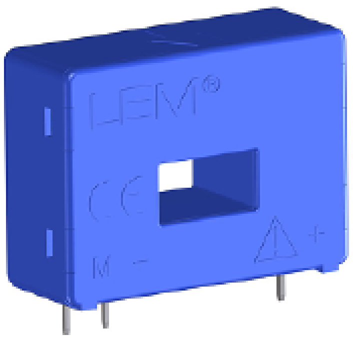
Let's consider again the LEM current transducer analysed in a past example:

LEM

Current Transducer LA 25-P

I_{PN} = 25 A

For the electronic measurement of currents: DC, AC, pulsed..., with galvanic separation between the primary circuit and the secondary circuit.



5 Year LEM TRANSDUCERS WARRANTY   CE   RoHS

Electrical data

I_{PN}	Primary nominal rms current	25	A
I_{PM}	Primary current, measuring range	0 .. ± 55	A

Example 1 – Current transducer
Dynamic specifications (2)

In addition to the static characteristics, the data sheet provides also **dynamic characteristics**.

In particular:

- **Step response time** to 90% of the nominal current;
- **Frequency bandwidth** (-1 dB).

t_{ra}	Reaction time	< 500	ns
t_r	Step response time to 90 % of I_{PN}	< 1	µs
di/dt	di/dt accurately followed	> 200	A/µs
BW	Frequency bandwidth (- 1 dB)	DC .. 200	kHz

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Example 1 – Current transducer

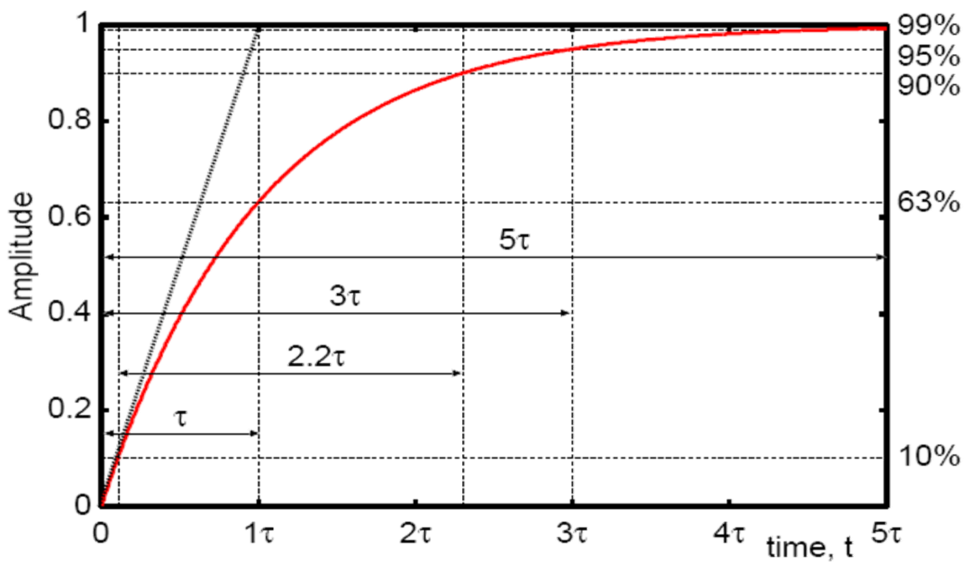
Step response time

The transient response of the transducer is not described in terms of a differential equation (the order of the equation is not known either), but in terms of a single parameter, the **step response time**.

Simple interpretation and modelling of the transient response can be obtained by assuming a **first-order response**:

- The time required to reach 90% of the steady-state value is 2.2τ;
- The maximum response time indicated in the data sheet is 1 μs, therefore it corresponds to an **equivalent time constant**:

$$\tau = \frac{1\ \mu\text{s}}{2.2} = 0.455\ \mu\text{s}$$



Example 1 – Current transducer

Frequency bandwidth (1)

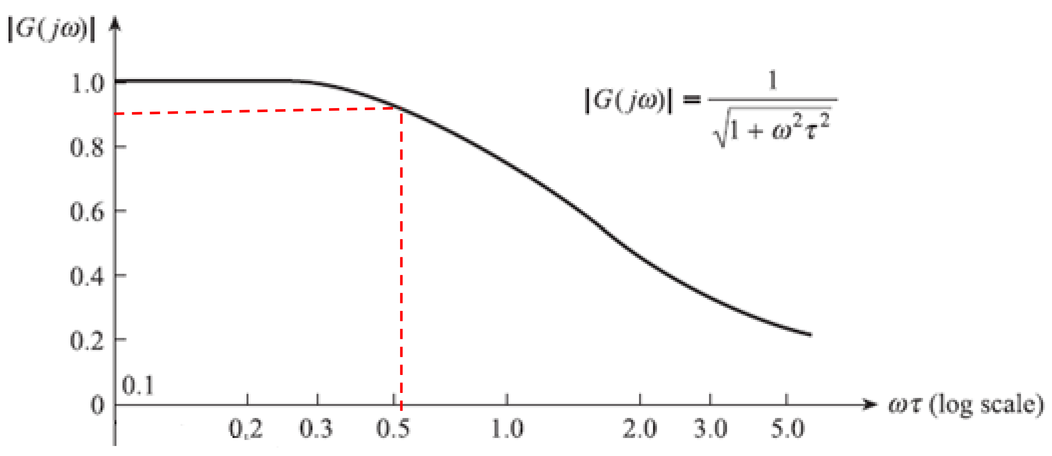
The **frequency bandwidth** provides similar information in the frequency domain (more useful in case of AC current measurements):

- It represents the frequency range in which the instrument can be used without significant errors (in this case, with an error below 1 dB in magnitude).

Again, a **first-order response** can be assumed as a first approximation:

- -1 dB = 0.89 in linear scale:
  - ➢ It corresponds to:

$$\omega\tau = 0.51$$



Example 1 – Current transducer

Frequency bandwidth (2)

- The maximum frequency indicated in the data sheet (corresponding to a maximum error of -1 dB) is 200 kHz:

$$\omega = 2\pi f = 1.26 \cdot 10^6 \text{ rad/s}$$

- This corresponds to an **equivalent time constant**:

$$\tau = \frac{0.51}{\omega} = 0.405\ \mu s$$

This result is similar to the equivalent time constant calculated from the step response time. The small difference is due to the approximation with a first-order response, while the actual response is likely to be more complex.

Example 2 – Accelerometer

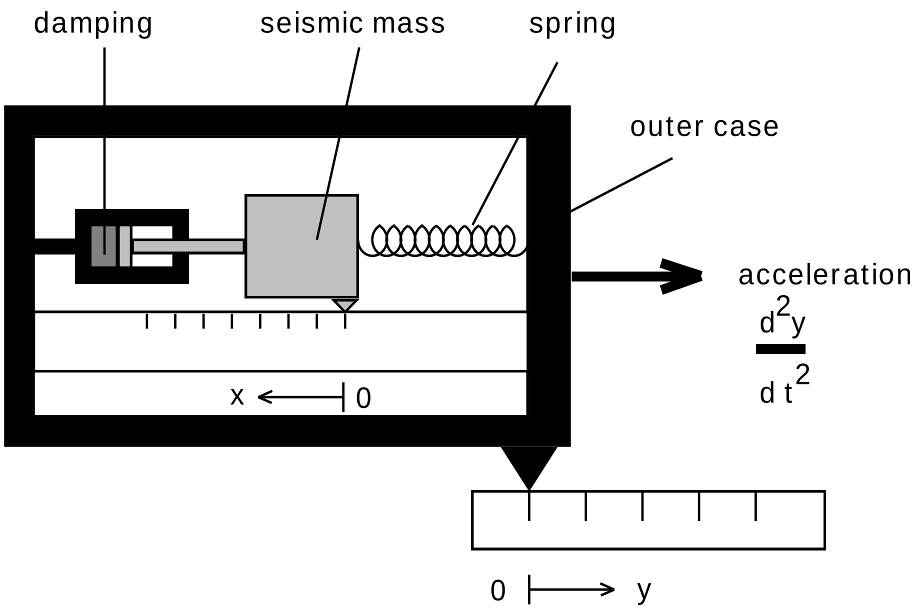
Principle of operation

An accelerometer is a good example of a **second-order** instrument:

- It is composed of a seismic mass restrained by spring and damping forces within a housing;

- The output of the accelerometer is proportional to the displacement of the mass within the housing (*x*);

- The direction of the mass displacement is opposite to the housing displacement (*y*).



Example 2 – Accelerometer

Mathematical modelling

The force balance on the seismic mass is described by a second-order differential equation:

$$F = m\frac{d^2x}{dt^2} + \lambda\frac{dx}{dt} + kx \qquad \Longrightarrow \qquad \frac{m}{k}\frac{d^2x}{dt^2} + \frac{\lambda}{k}\frac{dx}{dt} + x = \frac{F}{k}$$

- The equation can be rewritten in the usual form:

$$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2} + \frac{2\xi}{\omega_n}\frac{dx}{dt} + x = KF$$

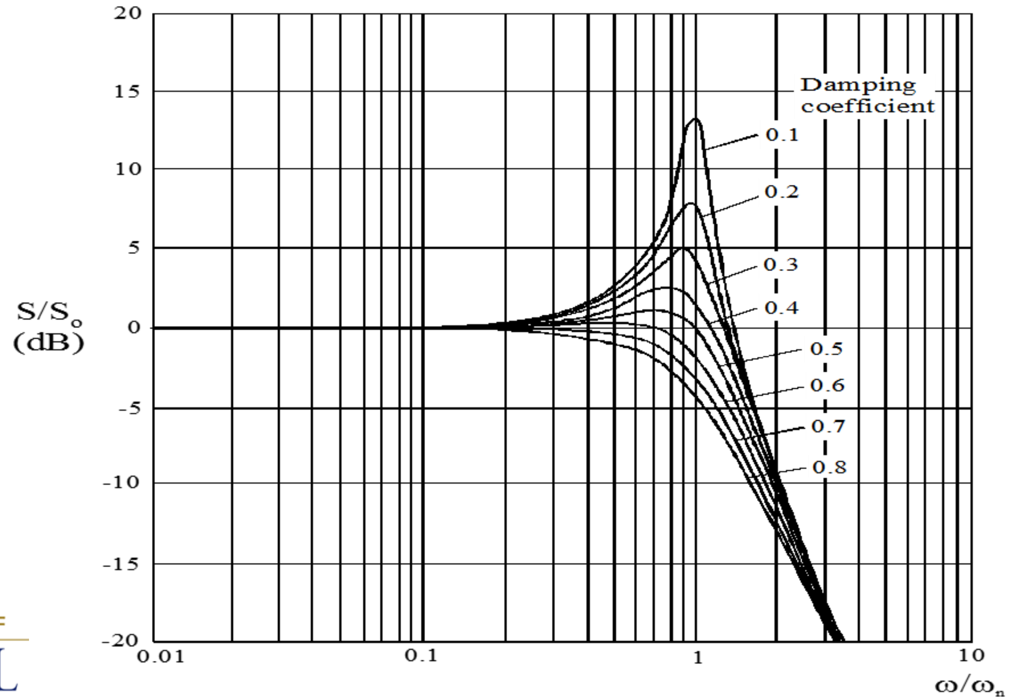
$$\omega_n = \sqrt{\frac{k}{m}} \quad , \qquad \xi = \frac{\lambda}{2\sqrt{km}} \quad , \qquad K = \frac{1}{k}$$

Example 2 – Accelerometer

Dynamic characteristics

The actual sensitivity of the accelerometer depends on the choice of the natural frequency and the damping ratio:

- The actual sensitivity can be normalised with respect to the low-frequency sensitivity and plotted vs frequency:



Example 2 – Accelerometer
Complete measurement system