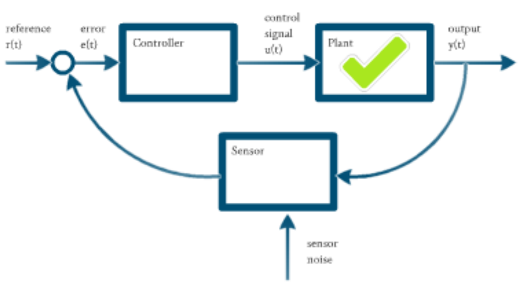
Closed Loop Control System (recap)

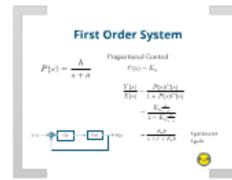


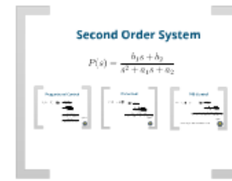
PID Controller Design

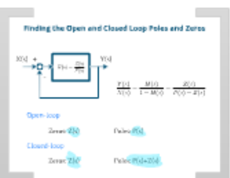
$$u(t) = \underbrace{K_p e(t)}_{\text{(P)roportional control}} + \underbrace{K_i \int_0^t e(\tau)d\tau}_{\text{(I)ntegral control}} + \underbrace{K_d \frac{d}{dt}e(t)}_{\text{(D)ifferential control}}$$

	Error Type			
Component	Stability	Fast transient response	Zero steady-state error	Small overshoot
Proportional	[illegible]	[illegible]	[illegible]	[illegible]
Integral	[illegible]	[illegible]	[illegible]	[illegible]
Differential	[illegible]	[illegible]	[illegible]	[illegible]

Application of PID Controllers







ELEC 207 Part B

Control Theory Lecture 7: Simple Control System Design

Prof Simon Maskell
CHAD-G68
s.maskell@liverpool.ac.uk
0151 794 4573

UNIVERSITY OF LIVERPOOL

This lecture covers:
- Three term PID controller
- Finding the open and closed loop poles and zeros
- Pole and zero placement design for a PID controller

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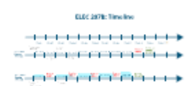
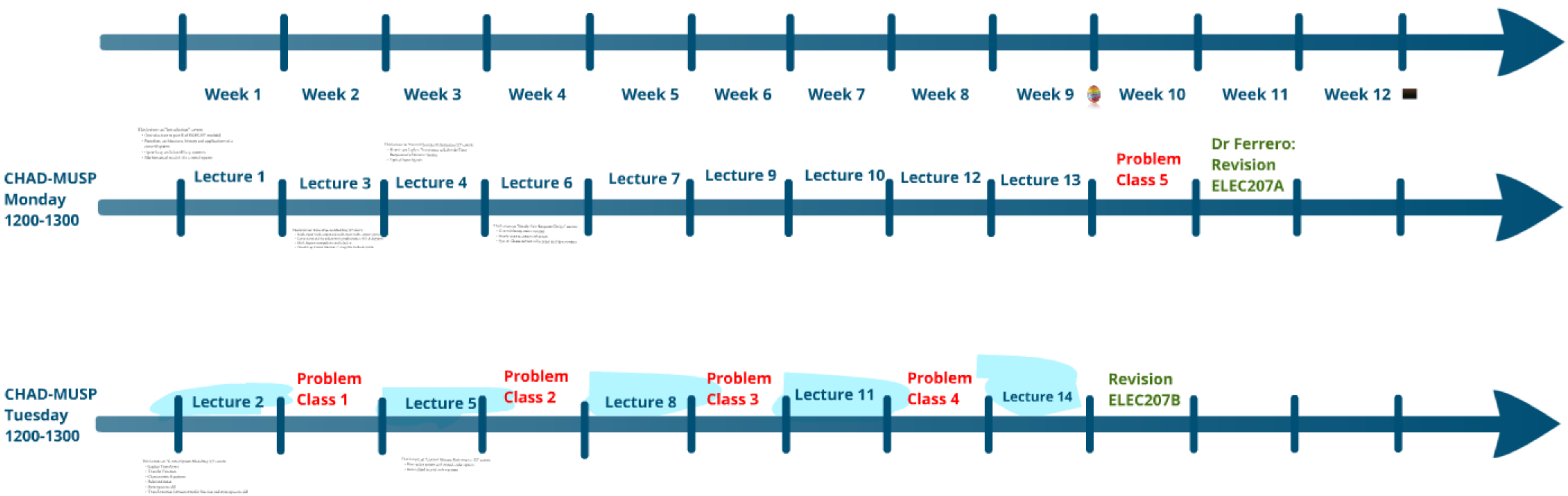


ELEC 207B: Timeline



This lecture on "Introduction" covers:

- (Introduction to part B of ELEC207 module)
- Function, architecture, history and applications of a control system
- Open-loop and closed-loop systems
- Mathematical model of a control system

Lecture 1

Lecture 2

This lecture on "Control System Modelling (1)" covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model

This lecture on "Control Systems F

- How to use Laplace Transform
  Response of a Dynamic System
- Typical Input Signals

e 1

Lecture 3

Lectu

This lecture on "Control System Modelling (2)" covers:

- Single-input single-output and multi-input multi-output systems
- Components and the underpinning mathematics of block diagrams
- Block diagram manipulation and reduction
- Closed-loop transfer function of a negative feedback system

This lecture on "Control Systems Performance (1)" covers:

- How to use Laplace Transforms to Solve the Time Response of a Dynamic System
- Typical Input Signals

Lecture 4

Lecture 5

This lecture on "Control Systems Performance (2)" covers:

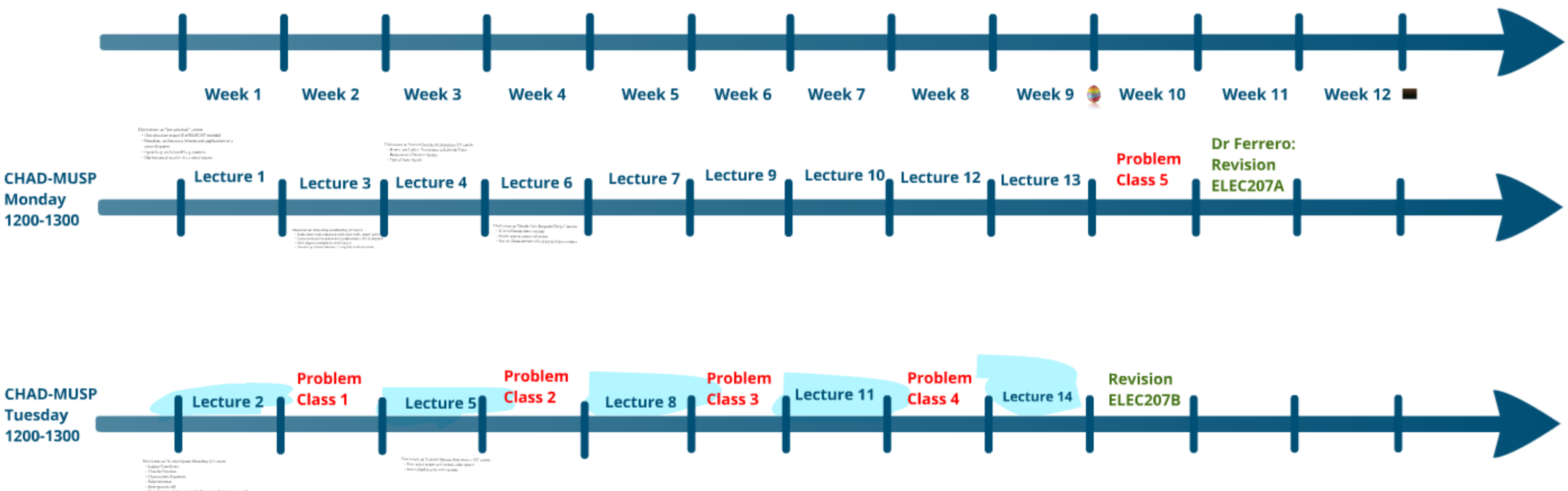
- First-order system and second-order system
- Generalized second-order system

Lecture 6

This lecture on "Steady-State Response Design" covers:

- General Steady-state response
- Steady-state accuracy and errors
- System Characterization by order and type number

ELEC 207B: Timeline



This lecture covers:

- Three term PID controller
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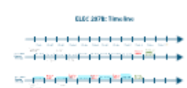
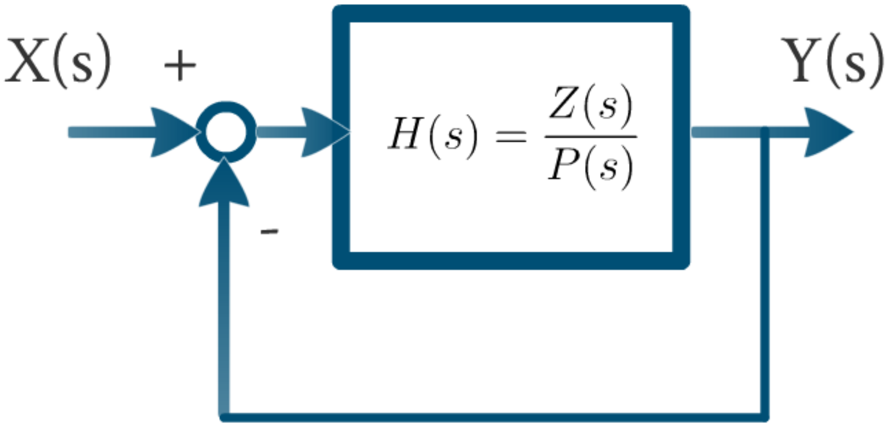


Finding the Open and Closed Loop Poles and Zeros



$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + H(s)} = \frac{Z(s)}{P(s) + Z(s)}$$

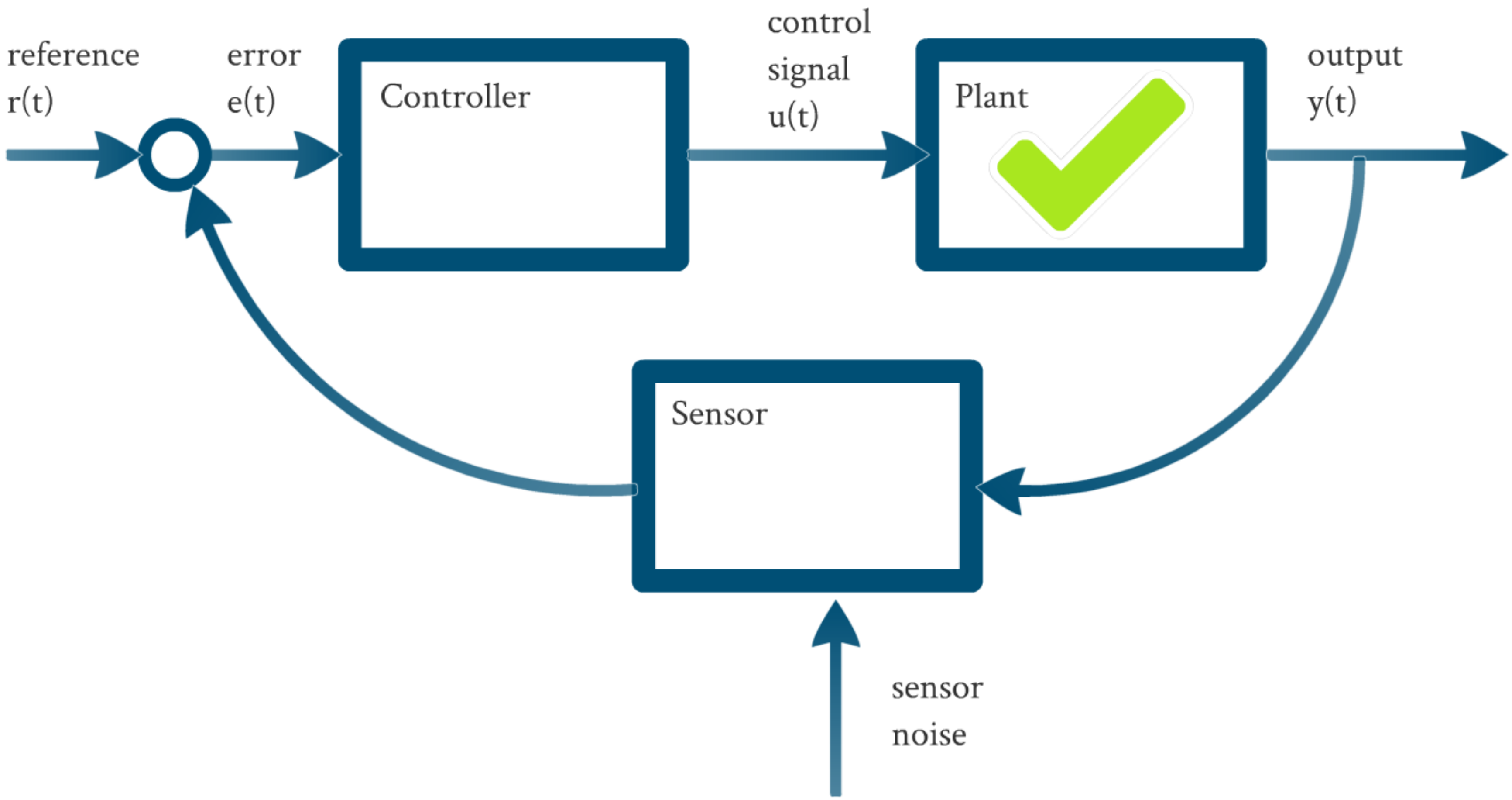
Open-loop

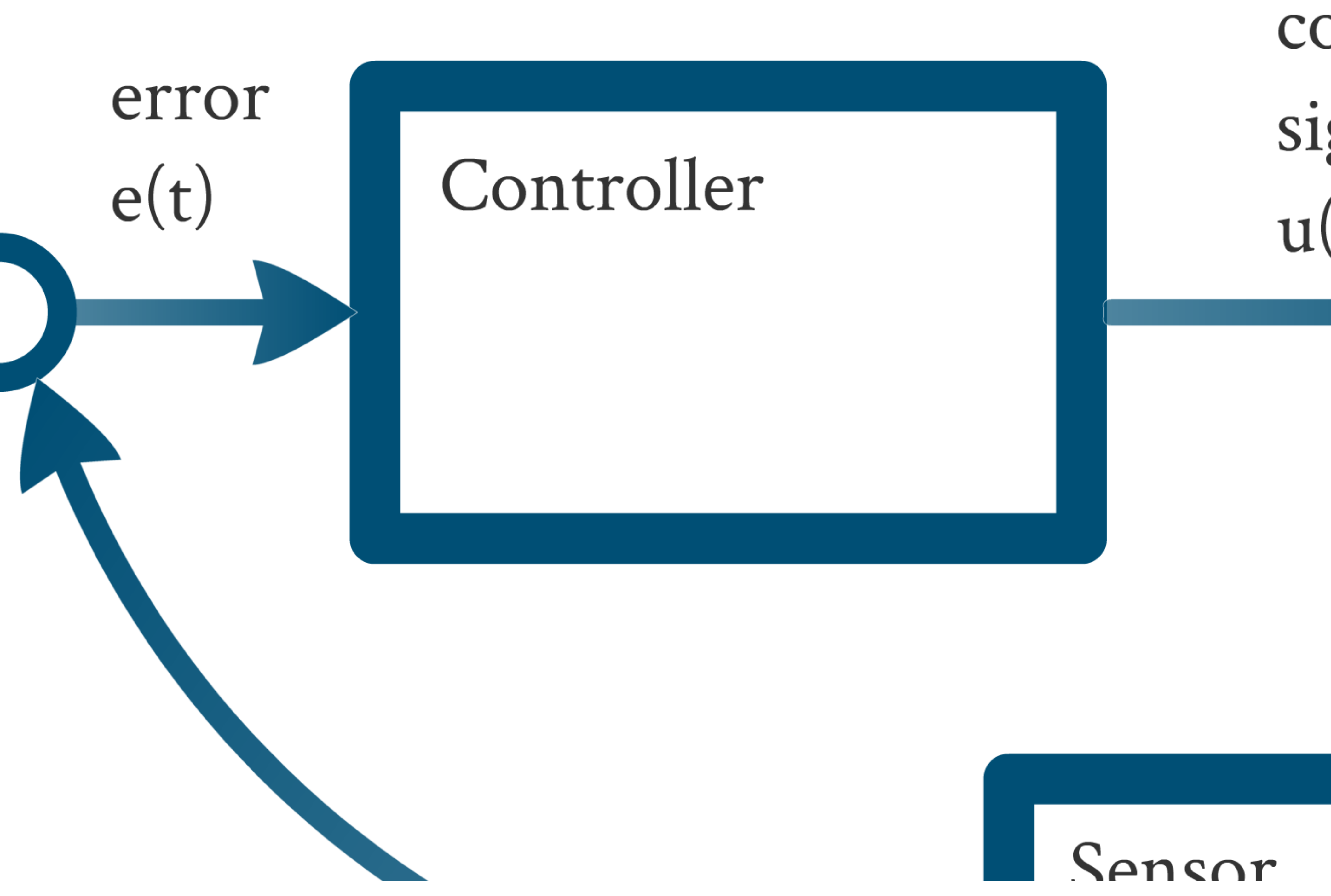
Zeros: Z(s) Poles: P(s)

Closed-loop

Zeros: Z(s) Poles: P(s)+Z(s)

Closed Loop Control System (recap)




error
e(t)
Controller
co
si
u(
Sensor

PID Controller Design

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Component	Error Type			
	Stability	Fast transient response	Zero steady-state error	Small overshoot
Proportional	Excessively larg… …ause instability	Large values … the response	Non-zero stead… …rrors can result	Larg… …will cause … …shoot
Integral	Integral … …lps stability	The trans… …s to … …pensated for with t… …onsu… …vershoot	Integral … …ives the steady-s… …r to zero	Any… transie… …e… …ted for with overshoot
Differential	Exce… …y l… …alues amp… …s… …ause instability	Diffe… …o… …slows down t… …ns… …esponse	Non-ze… …dy-state error… …result	… …es decre… …rshoot

$$(t) = \underbrace{K_p e(t)}_{\text{(P)roportional control}} + \underbrace{K_i \int_0^t e(\tau)d\tau}_{\text{(I)ntegral control}} + \underbrace{K_d \frac{d}{dt} e(t)}_{\text{(D)ifferential contr}}$$

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Stability

Excessively larg  cause instability

Integral c lps stability

response

Large values  the response

The trans  s to  pensated for with th onsu overshoot

steady-state erro

Non-zero steady  errors can result

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Integral c  lps stability

Exces  ly la  values

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Application of PID Controllers

First Order System

$$P(s) = \frac{b}{s+a}$$

Proportional Control

$$C(s) = K_p$$

$$\frac{Y(s)}{X(s)} = \frac{P(s)C(s)}{1+P(s)C(s)}$$

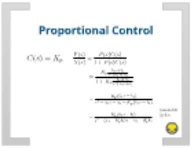
$$= \frac{K_p \frac{b}{s+a}}{1+K_p \frac{b}{s+a}}$$

$$= \frac{K_p b}{s+a+K_p b}$$

1 parameter
1 pole

Second Order System

$$P(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}$$



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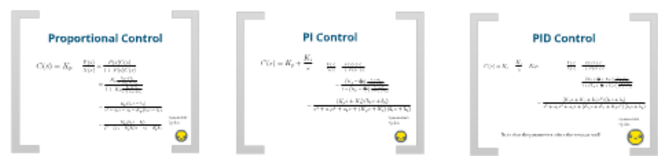

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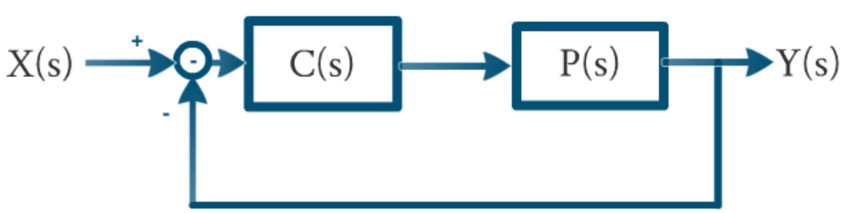

Second Order System

$$P(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}$$

Proportional Control

$$C(s) = K_p$$

$$\begin{aligned}\frac{Y(s)}{X(s)} &= \frac{P(s)C(s)}{1+P(s)C(s)} \\ &= \frac{K_p \frac{b_1 s + b_2}{s^2+a_1 s+a_2}}{1 + K_p \frac{b_1 s + b_2}{s^2+a_1 s+a_2}} \\ &= \frac{K_p (b_1 s + b_2)}{s^2 + a_1 s + a_2 + K_p (b_1 s + b_2)} \\ &= \frac{K_p (b_1 s + b_2)}{s^2 + (a_1 + K_p b_1) s + a_2 + K_p b_2}\end{aligned}$$

1 parameter
2 poles

PI Control

$$C(s) = K_p + \frac{K_i}{s}$$

$$\begin{aligned}\frac{Y(s)}{X(s)} &= \frac{P(s)C(s)}{1+P(s)C(s)} \\ &= \frac{\left(K_p + \frac{K_i}{s}\right) \frac{b_1 s + b_2}{s^2+a_1 s+a_2}}{1 + \left(K_p + \frac{K_i}{s}\right) \frac{b_1 s + b_2}{s^2+a_1 s+a_2}} \\ &= \frac{(K_p s + K_i)(b_1 s + b_2)}{s^3 + a_1 s^2 + a_2 s + (K_p s + K_i)(b_1 s + b_2)}\end{aligned}$$

2 parameters
3 poles

PID Control

$$C(s) = K_p + \frac{K_i}{s} + K_d s$$

$$\begin{aligned}\frac{Y(s)}{X(s)} &= \frac{P(s)C(s)}{1+P(s)C(s)} \\ &= \frac{\left(K_p + \frac{K_i}{s} + K_d s\right) \frac{b_1 s + b_2}{s^2+a_1 s+a_2}}{1 + \left(K_p + \frac{K_i}{s} + K_d s\right) \frac{b_1 s + b_2}{s^2+a_1 s+a_2}} \\ &= \frac{(K_p s + K_i + K_d s^2)(b_1 s + b_2)}{s^3 + a_1 s^2 + a_2 s + (K_p s + K_i + K_d s^2)(b_1 s + b_2)}\end{aligned}$$

3 parameters
3 poles

Note that the parameters affect the zeros as well

Proportional Control

$$C(s) = K_p$$

$$\frac{Y(s)}{X(s)} = \frac{P(s)C(s)}{1 + P(s)C(s)}$$

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$$= \frac{K_p \left(b_1 s + b_2\right)}{s^2 + a_1 s + a_2 + K_p \left(b_1 s + b_2\right)}$$

$$= \frac{K_p \left(b_1 s + b_2\right)}{s^2 + \left(a_1 + K_p b_1\right) s + a_2 + K_p b_2}$$

1 parameter

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2 parameters
3 poles

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3 parameters

3 poles

Note that the parameters affect the zeros as well

Second Order System

$$P(s) = \frac{b_1 s + b_2}{s^2 + a_1 s + a_2}$$

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2 parameters
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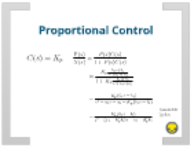

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