ELEC 207
Instrumentation and Control

9 – Instrument Transient Response

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Dynamic characteristics of instruments

Transient response

All the static characteristics of an instrument (accuracy, non-linearity error, etc.) refer to measured quantities in **steady-state conditions**.

In **dynamic conditions**, also the transient response of the instrument must be considered:

- It describes the behaviour of the system after a change in the input quantity is applied, before a steady-state condition is reached;
- Two types of standard input functions are commonly used to describe the transient response:
  - **Step input**;
  - **Sinusoidal input**.

Step response

Mathematical modelling

The general response of a **linear and time-invariant system** to a step change in the input quantity can be written as:

$$a_n \frac{d^n q_o}{dt^n} + a_{n-1} \frac{d^{n-1} q_o}{dt^{n-1}} + \cdots + a_1 \frac{d q_o}{dt} + a_0 q_o = b_0 q_i$$

q_i: input quantity (measurand);

q_o: output quantity (instrument reading);

a_n, a_{n-1}, …, a_0, b_0: constant coefficients.

- This is a ***n***

Zero-order system

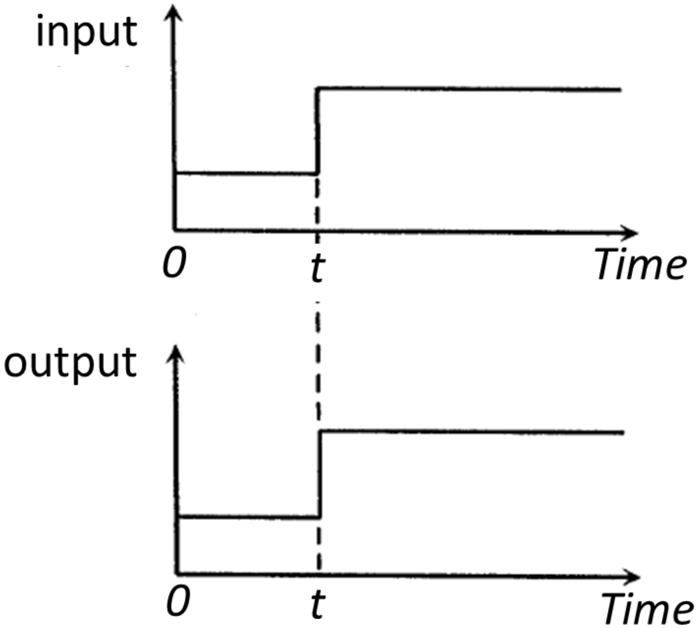
Mathematical modelling and physical interpretation

The response of a **zero-order system** is described by the following equation:

$$a_0 q_o = b_0 q_i \quad \Longrightarrow \quad q_o = \frac{b_0}{a_0} q_i = K q_i$$

$K = b_0/a_0$: static sensitivity

- This means that the output follows **immediately** the input change;
- For example, a resistive potentiometer is a zero-order system.



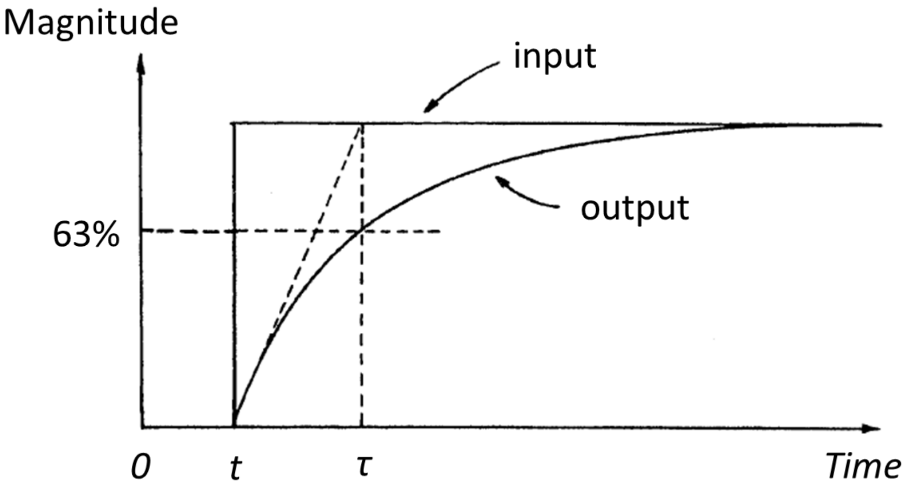
First-order system

Mathematical modelling and physical interpretation

The response of a **first-order system** is described by the following equation:

$$a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

- An **exponential transient** in the output follows a step change in the input;
- For example, a thermocouple is a first-order system;
- In general, a first-order response arises from a single energy storage component in the system (e.g., a capacitor in a RC circuit).



First-order system

Exponential transient

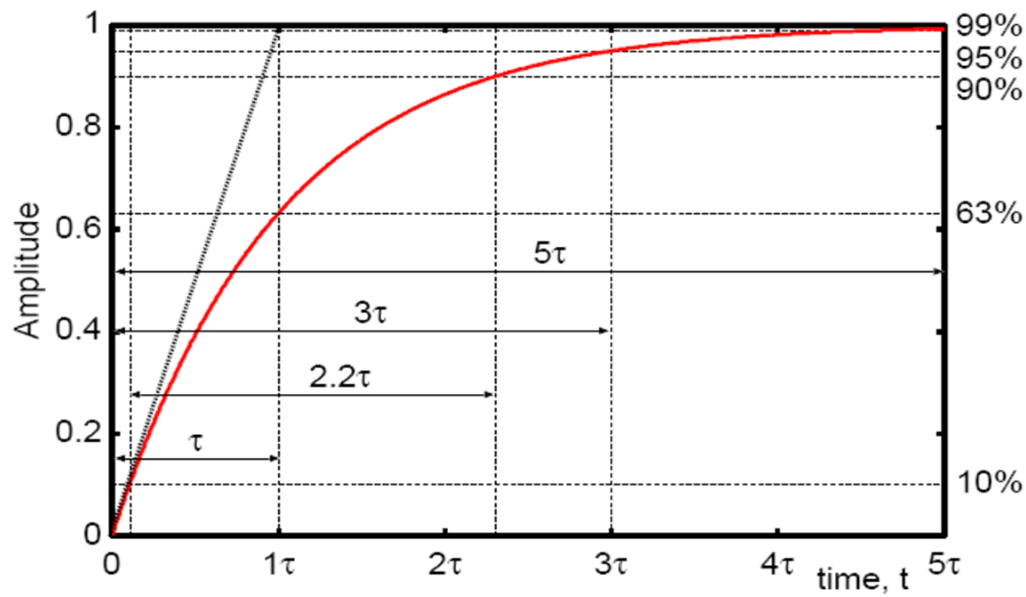
The output dynamics is entirely identified by the **time constant** τ of the exponential curve:

$$q_o(t) = q_{o,\infty} + \left(q_{o,0} - q_{o,\infty}\right) \cdot e^{-t/\tau}$$

$q_{o,0}$: initial value
$q_{o,\infty}$: final value

- τ corresponds to the time required to reach 63% of the final output value;

- The transient can usually be considered ended after 4τ or 5τ.



Second-order system

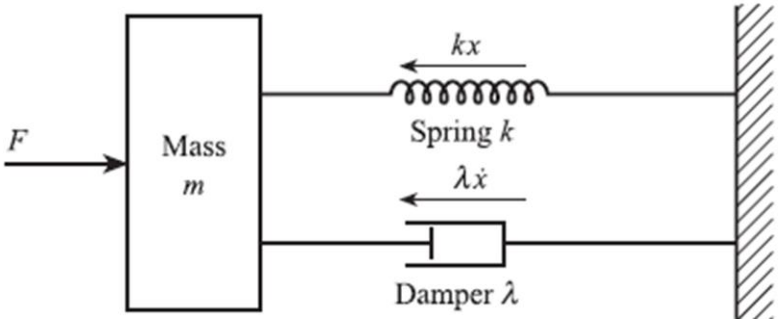
Mathematical modelling and physical interpretation

The response of a **second-order system** is described by the following equation:

$$a_2 \frac{d^2 q_o}{dt^2} + a_1 \frac{dq_o}{dt} + a_0 q_o = b_0 q_i$$

- There are now two energy storage components in the system (e.g., a capacitor and an inductor in a RLC circuit), and energy is exchanged between them:
  - ➢ Therefore, the output transient may be characterised by **oscillations**;

- An example of a second-order system is a mass connected to a spring and a damper:



$$F = m\ddot{x} + \lambda\dot{x} + kx$$

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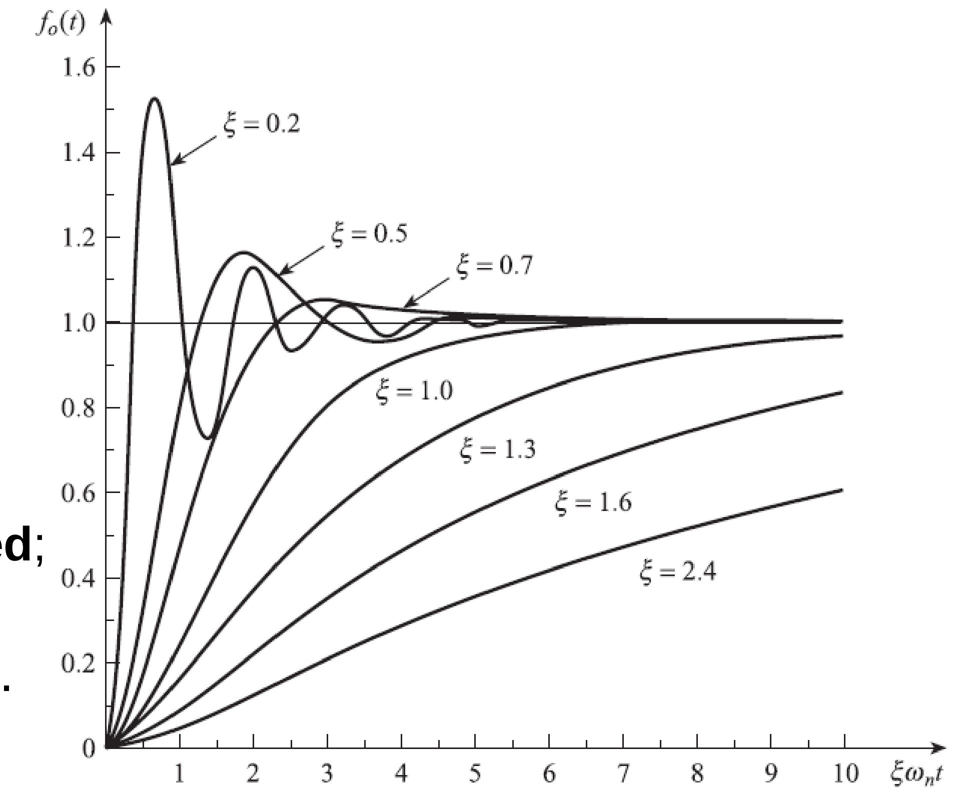
Second-order system
Natural frequency and damping ratio

The second-order differential equation can be rewritten as:

$$\frac{1}{\omega_n^2}\frac{d^2 q_o}{dt^2}+\frac{2\xi}{\omega_n}\frac{dq_o}{dt}+q_o=Kq_i$$

$\omega_n=\sqrt{a_0/a_2}$: **natural frequency**;
$\xi=\omega_n a_1/(2a_0)$: **damping ratio**;
$K=b_0/a_0$: static sensitivity.

- If ξ<1, the response is **underdamped**;
- If ξ>1, the response is **overdamped**.



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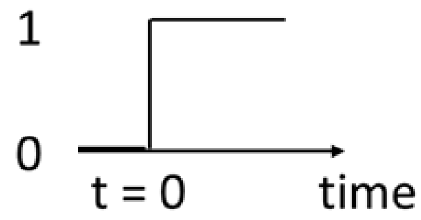
Second-order system

Underdamped vs overdamped response (1)

The damping ratio ξ determines the type of response, and in particular the presence and amplitude of **oscillations**:



- If ξ<1 (**underdamping**) the response to the unit step is:

$$q_0(t) = 1 - e^{-\xi\omega_n t}\left(\cos\omega_n\sqrt{(1-\xi^2)}t + \frac{\xi}{\sqrt{1-\xi^2}}\sin\omega_n\sqrt{(1-\xi^2)}t\right)$$

- If ξ=1 (**critical damping**) the response to the unit step is:

$$q_0(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$$

- If ξ>1 (**overdamping**) the response to the unit step is:

$$q_0(t) = 1 - e^{-\xi\omega_n t}\left(\cosh\omega_n\sqrt{(\xi^2-1)}t + \frac{\xi}{\sqrt{\xi^2-1}}\sinh\omega_n\sqrt{(\xi^2-1)}t\right)$$

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Second-order system

Underdamped vs overdamped response (2)

- A very underdamped response rises fast, but it has high oscillations:
  - In particular there is an **overshoot** (difference above the steady-state value), whose maximum value depends only on ξ and is:

$$overshoot_{max} = e^{-\pi\frac{\xi}{\sqrt{1-\xi^2}}}$$

- An overdamped response does not have oscillations (nor overshoots), but it is slow in reaching the steady-state value.

The best trade-off depends on the particular application, but it usually involves a damping ratio ξ between 0.6 and 0.8.

References

Textbook: Principles of Measurement Systems, 4th ed.

For further explanation about the points covered in this lecture, please refer to the following chapters and sections in the **Bentley** textbook:

- Chapter 4, Sec. 4.1.1: **First-order elements**;
- Chapter 4, Sec. 4.1.2: **Second-order elements**;
- Chapter 4, Sec. 4.2.1: **Step response of first- and second-order elements**.

NOTE: Topics not covered in the lecture are not required for the exam.

References

Textbook: Measurement and Instrumentation, 2nd ed.

For further explanation about the points covered in this lecture, please refer to the following chapters and sections in the **Morris-Langari** textbook:

- Chapter 2, Sec. 2.4: **Dynamic characteristics of instruments**.

NOTE: Topics not covered in the lecture are not required for the exam.