ELEC 207
Instrumentation and Control

4 – Strain Gauge

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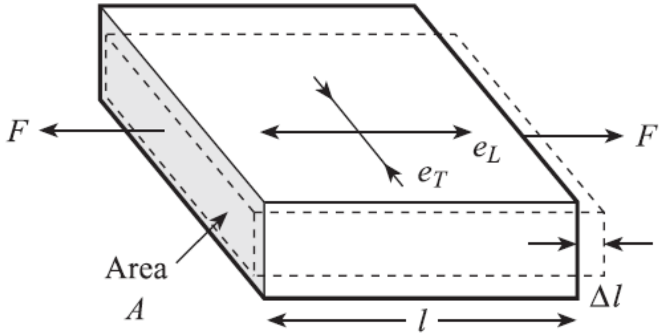
Stress and strain

Definitions

A strain gauge is a widely used transducer to measure strain:

- Strain is the deformation of a physical body under the action of an applied force (stress):
  - **Strain** = change in length per unit length: $e = \dfrac{\Delta l}{l}$
  - **Stress** = force per unit area: $\sigma = \dfrac{F}{A}$
  - **Elastic modulus** = ratio between stress and strain: $E = \dfrac{\sigma}{e}$

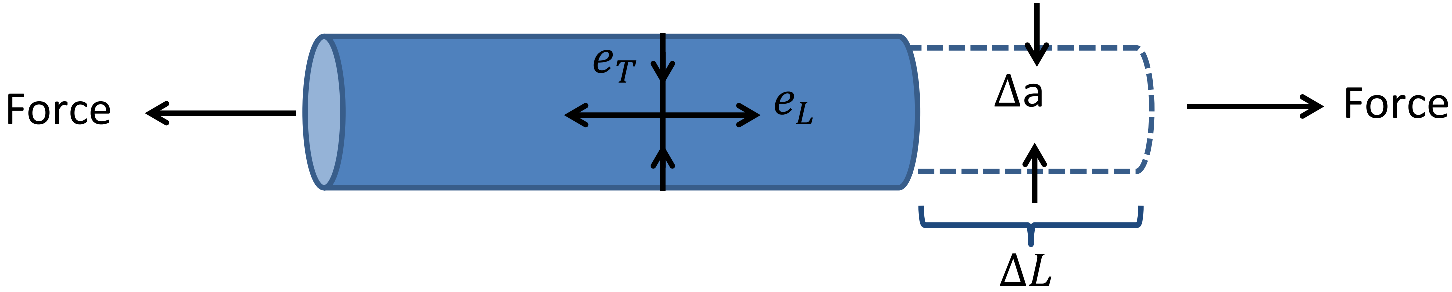


Stress and strain

Poisson's ratio

The longitudinal tensile strain is accompanied by a transverse compressive strain, and vice-versa:

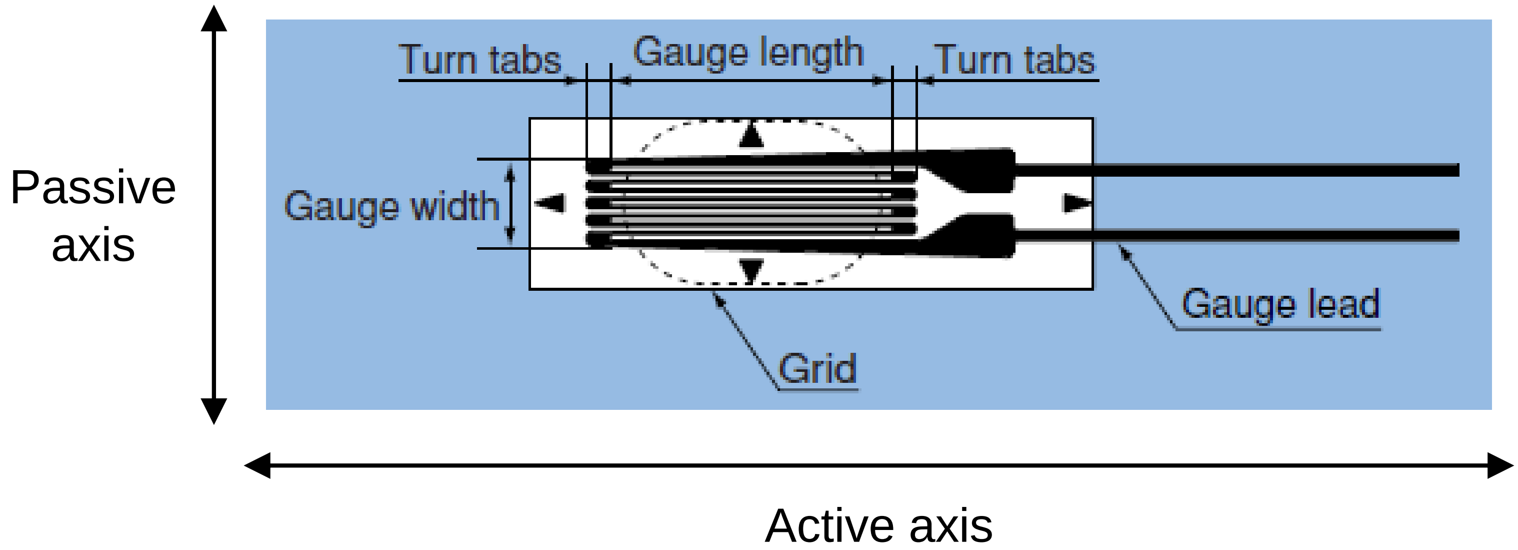
- The ratio between them is called **Poisson's ratio**: $v = -\frac{e_T}{e_L}$
  - It is usually between 0.25 and 0.4 for most materials;
  - The negative sign is used because e_T and e_L have opposite signs.



Strain gauge
Operating principle (1)

A **strain gauge** is a metal or semiconductor element that experiences a change in resistance when it is strained:

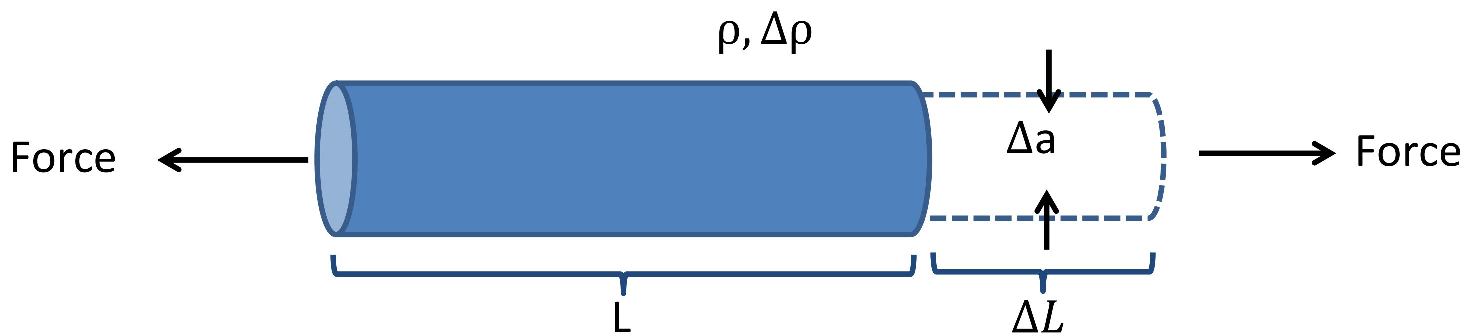
- It converts a strain into a change in **electrical resistance**, which can be measured.



Strain gauge
Operating principle (2)

The applied strain leads to changes in physical parameters (length *l*, cross-section area *A*, resistivity *ρ*):

- These parameters determine the **resistance** of the material: $R = \rho \dfrac{l}{A}$



- The **Gauge factor** is defined as: $G = \dfrac{\Delta R/R}{e}$ $\Rightarrow$ $\dfrac{\Delta R}{R} = Ge$

Strain gauge
Input-output relationship

The input-output relationship (**transfer function**) of a strain gauge can be written as:

$$\Delta R = R - R_o = R_o G e \quad \Longrightarrow \quad R = R_o(1 + Ge)$$

- Typical values of gauge factors (*G*) for **metal** strain gauges are around 2.0;
- Strain values (*e*) are usually very small, with maximum values around 5%:
  - Strain is often measured in **microstrain** ($\cdot 10^{-6}$);
- Nominal (unstrained) resistance values (R_o) are around 100 Ω.

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Strain gauge

Gauge factor (1)

For small changes in the resistance value, the following approximation applies:

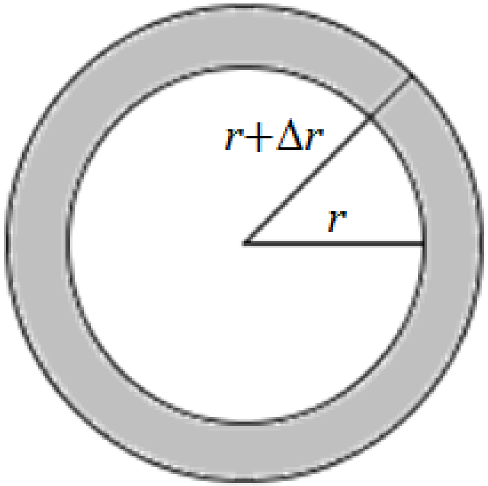
$$\Delta R \approx \left(\frac{\partial R}{\partial l}\right)\Delta l + \left(\frac{\partial R}{\partial A}\right)\Delta A + \left(\frac{\partial R}{\partial \rho}\right)\Delta \rho \quad \Longrightarrow \quad \frac{\Delta R}{R} \approx \frac{\Delta l}{l} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho}$$

- Consider a circular cross section as an example:

$$A = \pi r^2$$

$$\Delta A \approx (2\pi r) \cdot \Delta r$$

$$\frac{\Delta A}{A} \approx \frac{2\pi r \Delta r}{\pi r^2} \approx 2\frac{\Delta r}{r}$$



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Strain gauge

Gauge factor (2)

$$e_T = -ve_L \quad \Longrightarrow \quad \frac{\Delta r}{r} = -v\frac{\Delta l}{l} \quad \Longrightarrow \quad \frac{\Delta A}{A} \approx -2v\frac{\Delta l}{l}$$

$$\Longrightarrow \quad \frac{\Delta R}{R} \approx \frac{\Delta l}{l} - \left(-2v\frac{\Delta l}{l}\right) + \frac{\Delta \rho}{\rho} = (1+2v)\frac{\Delta l}{l} + \frac{\Delta \rho}{\rho}$$

- Therefore the **Gauge factor** can be written as:

$$G \approx \underbrace{1 + 2v}_{\textbf{deformation effect}} + \underbrace{\frac{1}{e}\frac{\Delta \rho}{\rho}}_{\textbf{piezoresistive effect}}$$

Strain gauge

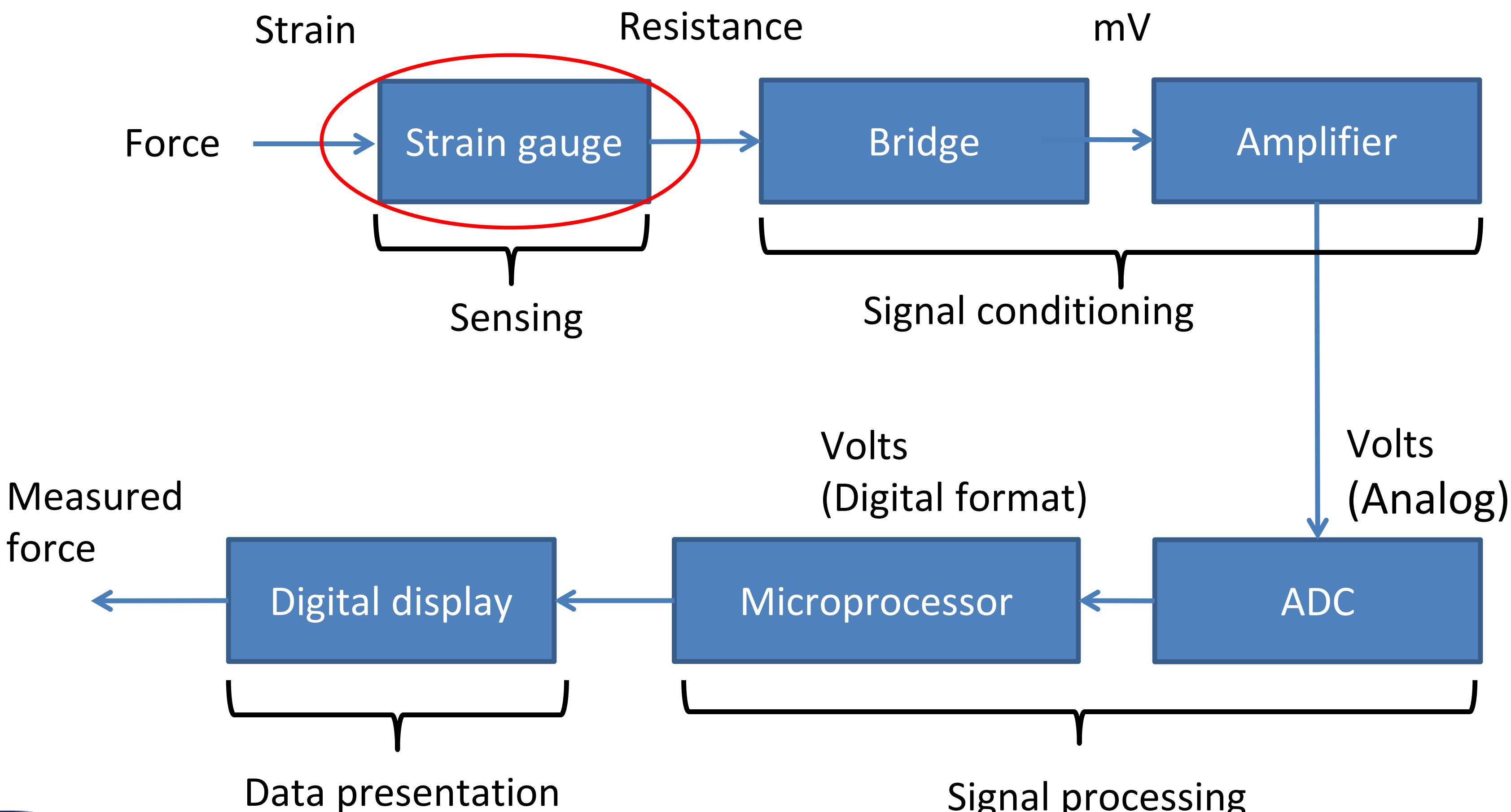
Metal vs semiconductor materials

- In **metal strain gauges** the deformation effect is dominant:
  - They have a low cost and therefore are very common;
  - They are less sensitive to temperature changes;
  - A typical material is the ‘advance’ alloy: 54% copper, 44% nickel, 1% manganese.

- In **semiconductor strain gauges** the piezoresistive effect is dominant:
  - They have a higher sensitivity, with Gauge factors higher than 100;
  - They are more sensitive to temperature changes;
  - The most common material is silicon doped with p-type or n-type material.

Strain measurement

Strain gauge in a measurement system



References

Textbook: Principles of Measurement Systems, 4th ed.

For further explanation about the points covered in this lecture, please refer to the following chapters and sections in the **Bentley** textbook:

- Chapter 8, Sec. 8.1.3: **Metal and semiconductor resistive strain gauges**.

NOTE: Topics not covered in the lecture are not required for the exam.

References

Textbook: Measurement and Instrumentation, 2nd ed.

For further explanation about the points covered in this lecture, please refer to the following chapters and sections in the **Morris-Langari** textbook:

- Chapter 13, Sec. 13.7: **Strain Gauges**;
- Chapter 13, Sec. 13.8: **Piezoresistive Sensors**.

NOTE: Topics not covered in the lecture are not required for the exam.