Three Different Ways to Think About Errors

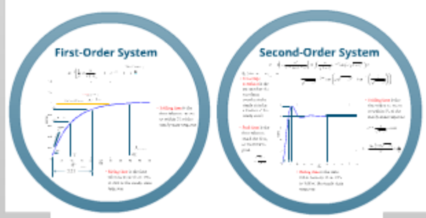
Time

s-Domain

General Steady-state Response

Time Domain (Representation) of the System's Transient Response

Approximate as first-order or second-order using the pole(s) nearest the imaginary axis.



Finding the Steady-state Error

Closed-loop Transfer Function (Representation) of the System

We can write the Laplace Transform of the error signal:

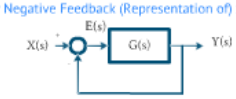
$$E(s) = X(s) - Y(s)$$
$$= X(s) - H(s)X(s)$$
$$= (1 - H(s))X(s)$$

The final value theorem can then be used:

$$e(\infty) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s(1 - H(s))X(s)$$

System Type and Error Constants

Unity Negative Feedback (Representation of) System



Number of "pure integrators" (ie poles that G(s) has at s=0) is the system type

Position constant $k_p = \lim_{s \to 0} G(s)$

Velocity constant $k_v = \lim_{s \to 0} sG(s)$

Acceleration constant $k_a = \lim_{s \to 0} s^2 G(s)$

Input		System Type Type-0	Type-1	Type-2
Step	$x(t) = u(t)$	$\frac{1}{1+K_p}$	0	0
Ramp	$x(t) = t$	∞	$\frac{1}{K_v}$	0
Parabola	$x(t) = \frac{t^2}{2}$	∞	∞	$\frac{1}{K_a}$

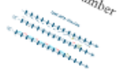
Steady-state Errors

ELEC 207 Part B

Control Theory Lecture 6:
Steady-State (and Transient) Response Design

Prof Simon Maskell
CHAD-G68
s.maskell@liverpool.ac.uk
0151 794 4573
UNIVERSITY OF LIVERPOOL

This lecture covers:
- General Steady-state response
- Steady-state accuracy and errors
- System Characterization by order and type number



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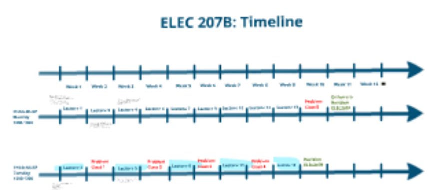
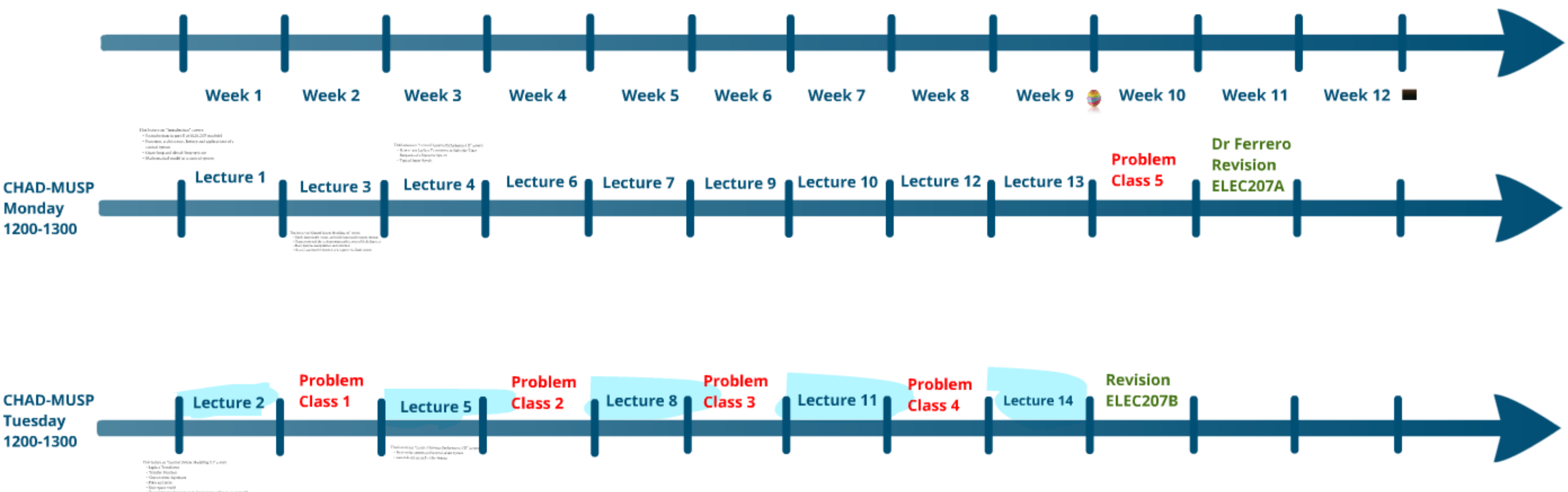


ELEC 207B: Timeline



This lecture on "Introduction" covers:

- (Introduction to part B of ELEC207 module)
- Function, architecture, history and applications of a control system
- Open-loop and closed-loop systems
- Mathematical model of a control system

Lecture 1

Lecture 2

This lecture on "Control System Modelling (1)" covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model

1

Lecture 3

This lecture on "Control System Modelling (2)" covers:

- Single-input single-output and multi-input multi-output systems
- Components and the underpinning mathematics of block diagrams
- Block diagram manipulation and reduction
- Closed-loop transfer function of a negative feedback system

Lect

This lecture on "Control

- How to use Laplace T
  Response of a Dynam
- Typical Input Signals

This lecture on "Control Systems Performance (1)" covers:

- How to use Laplace Transforms to Solve the Time Response of a Dynamic System
- Typical Input Signals

3

Lecture 4

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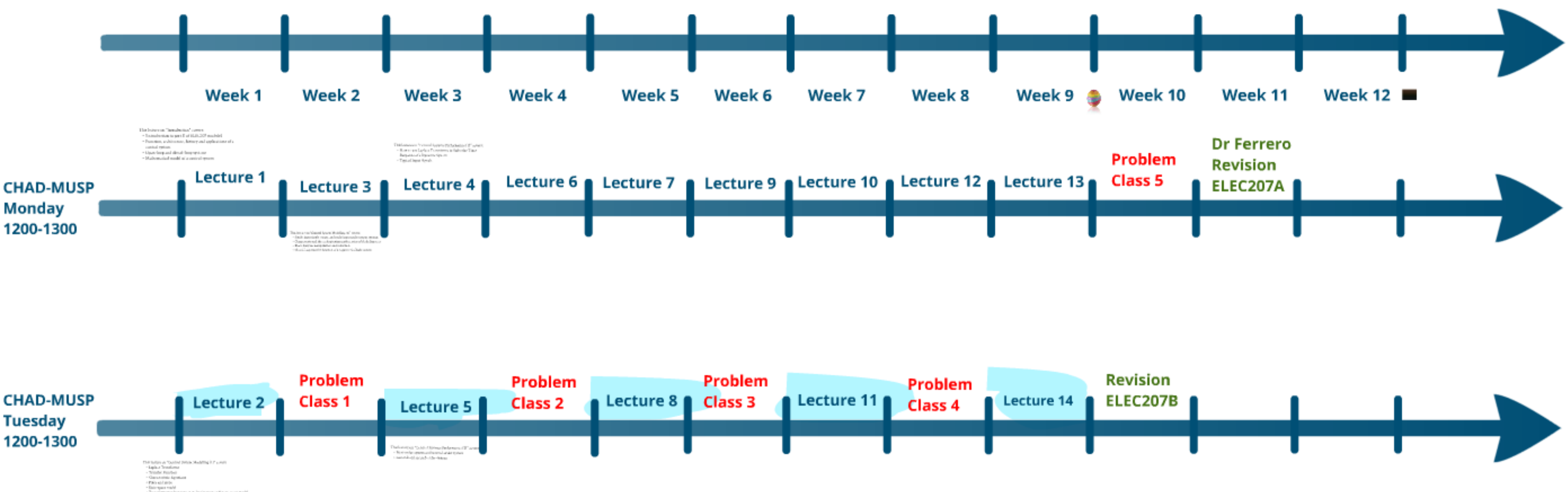
Lecture 5

This lecture on "Control Systems Performance (2)" covers

- First-order system and second-order system
- Generalized second-order system

ELEC 207B: Timeline



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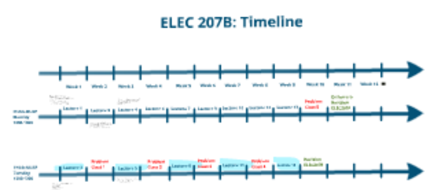

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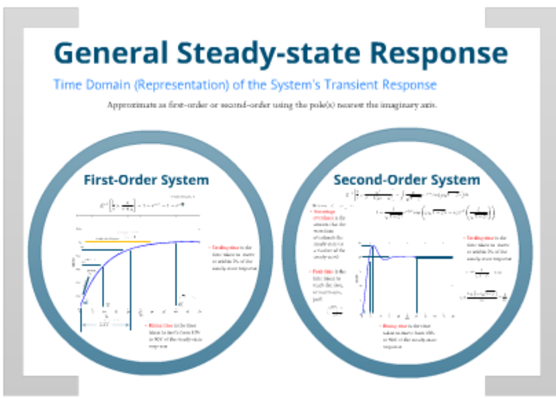
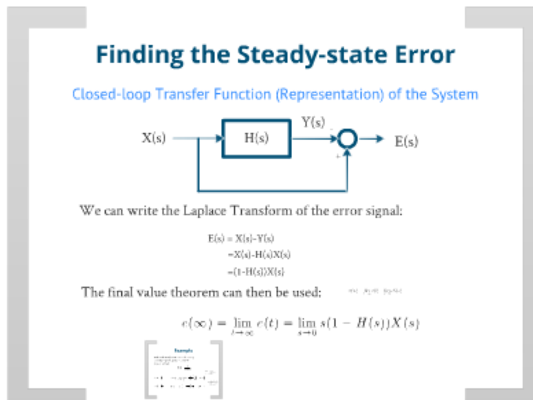
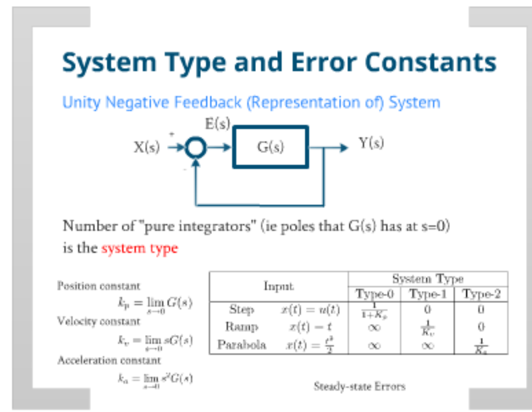






General Steady-state Response

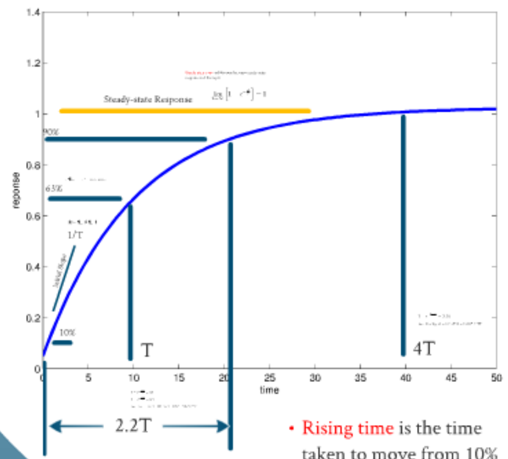
Time Domain (Representation) of the System's Transient Response

Approximate as first-order or second-order using the pole(s) nearest the imaginary axis.

First-Order System

Time Constant, T

$$\mathcal{L}^{-1}\left[\frac{1}{s} \times \frac{a}{s+a}\right] = 1 - e^{-at} = 1 - e^{-\frac{t}{T}}$$



- Settling time is the time taken to move to within 2% of the steady-state response
- Rising time is the time taken to move from 10% to 90% of the steady-state response

Second-Order System

$$\mathcal{L}^{-1}\left[\frac{1}{s} \times \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}\right] = \int \frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta\omega\tau} \sin\left(\omega\sqrt{1-\zeta^2}\tau\right) d\tau$$

$$= 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \cos\left(\omega\sqrt{1-\zeta^2}t - \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\right)$$

- Percentage overshoot is the amount that the waveform overshoots the steady-state (as a fraction of the steady-state)
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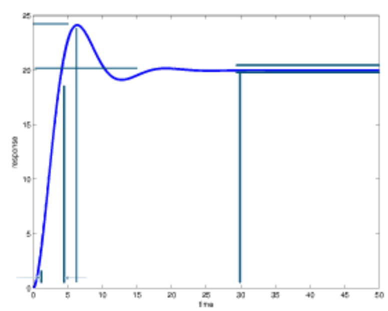

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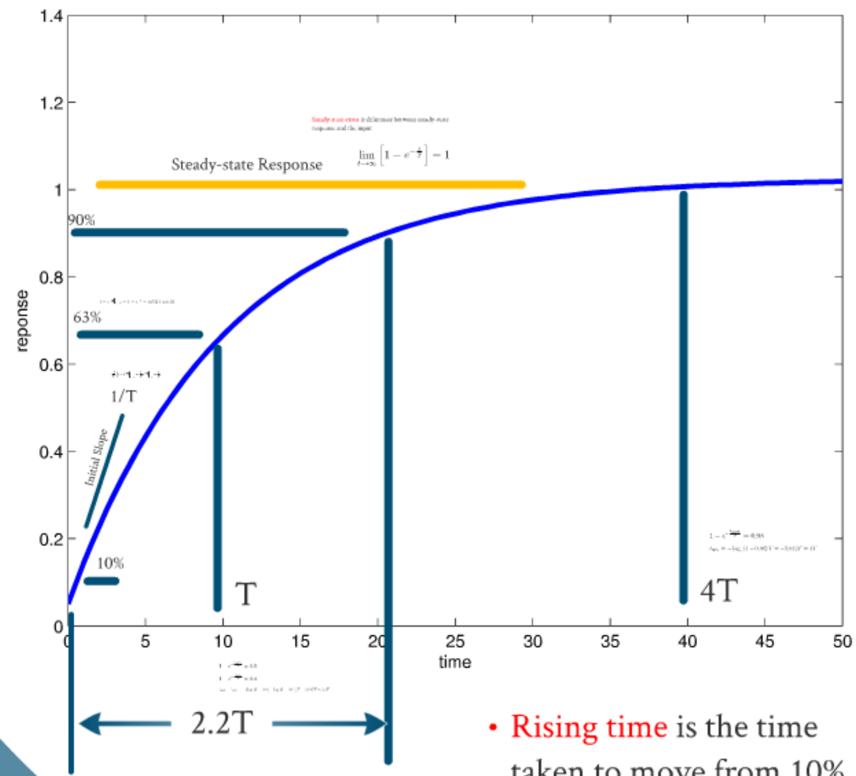

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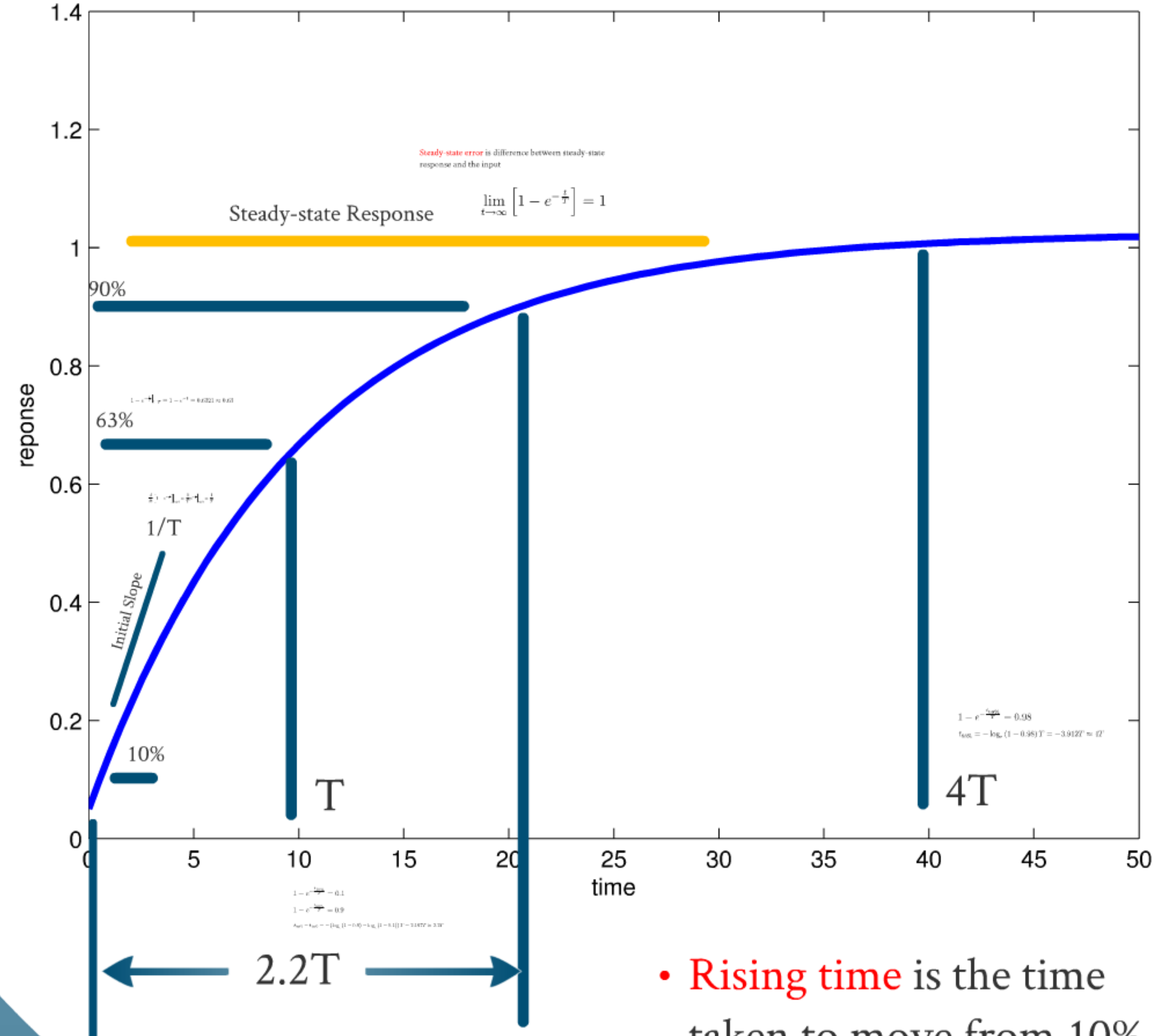
Steady-state error is difference between steady-state response and the input

Steady-state Response

$$\lim_{t \to \infty}\left[1 - e^{-\frac{t}{T}}\right] = 1$$

$$\mathcal{L}^{-1}\left[\frac{1}{s}\times\frac{a}{s+a}\right]=1-e^{-at}=1-e^{-\frac{t}{T}}$$



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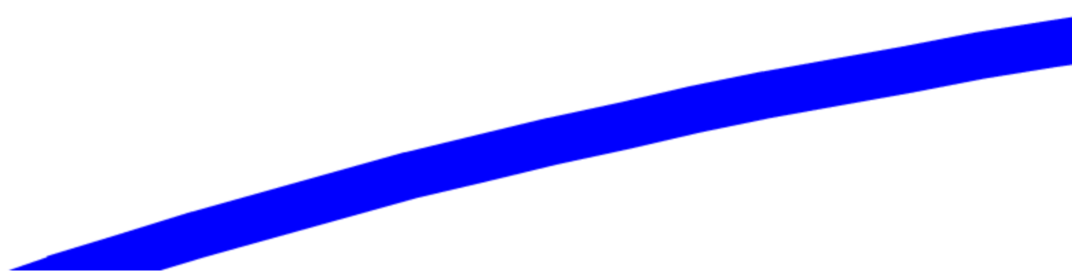

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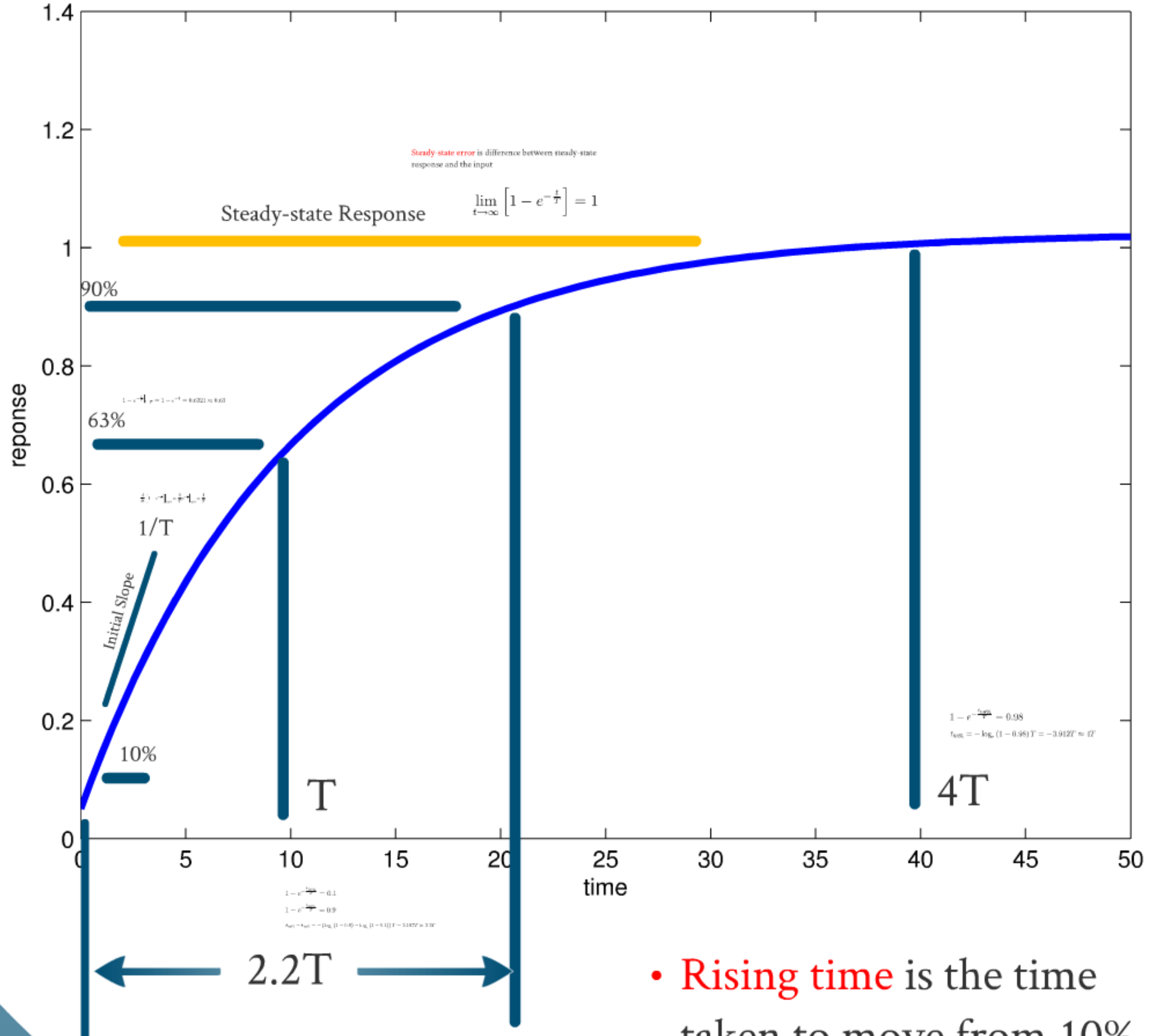

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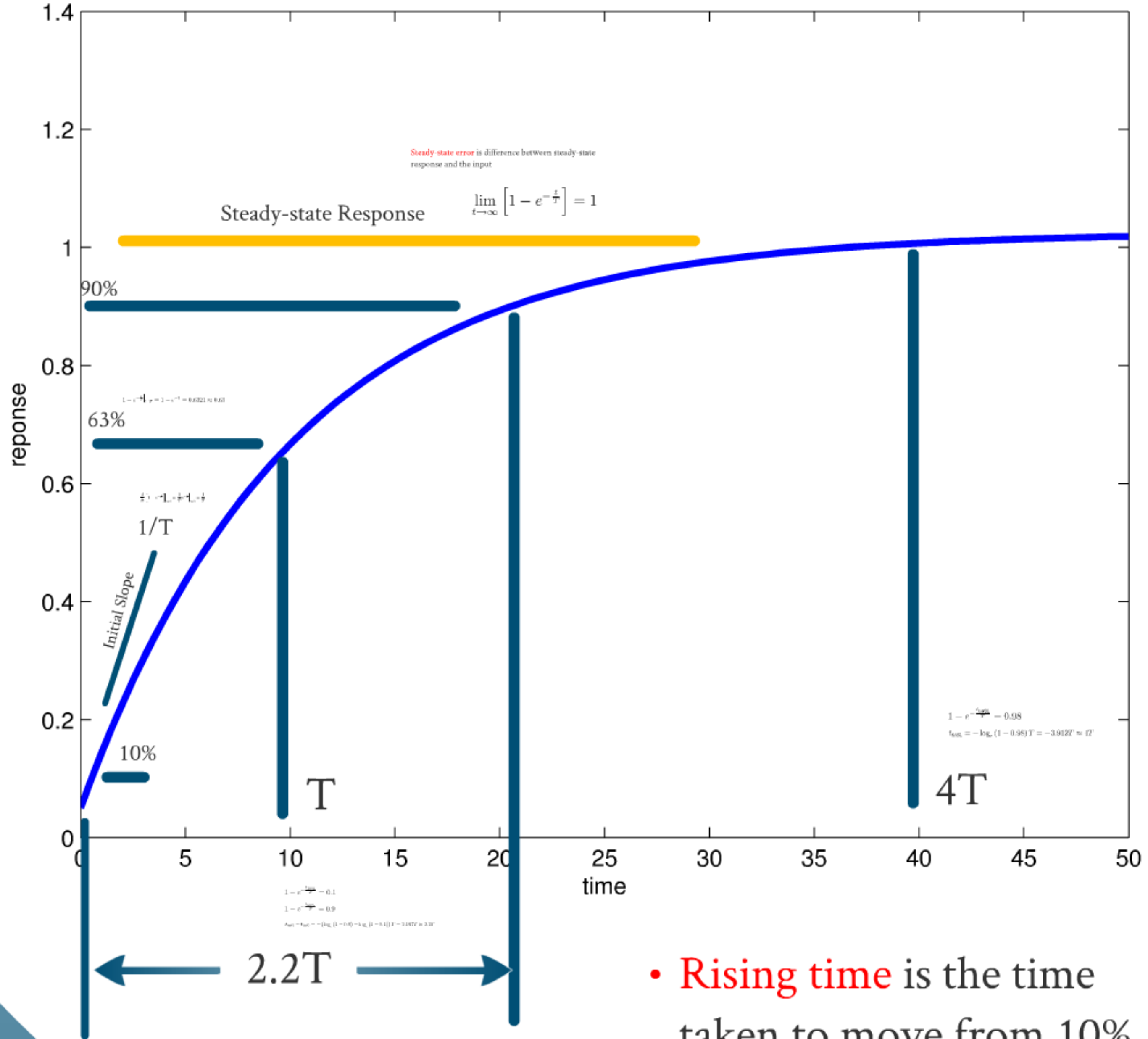
$$1 - e^{-\frac{t}{T}}\Big|_{t=T} = 1 - e^{-1} = 0.6321 \approx 0.63$$

%

$$\mathcal{L}^{-1}\left[\frac{1}{s}\times\frac{a}{s+a}\right] = 1 - e^{-at} = 1 - e^{-\frac{t}{T}}$$



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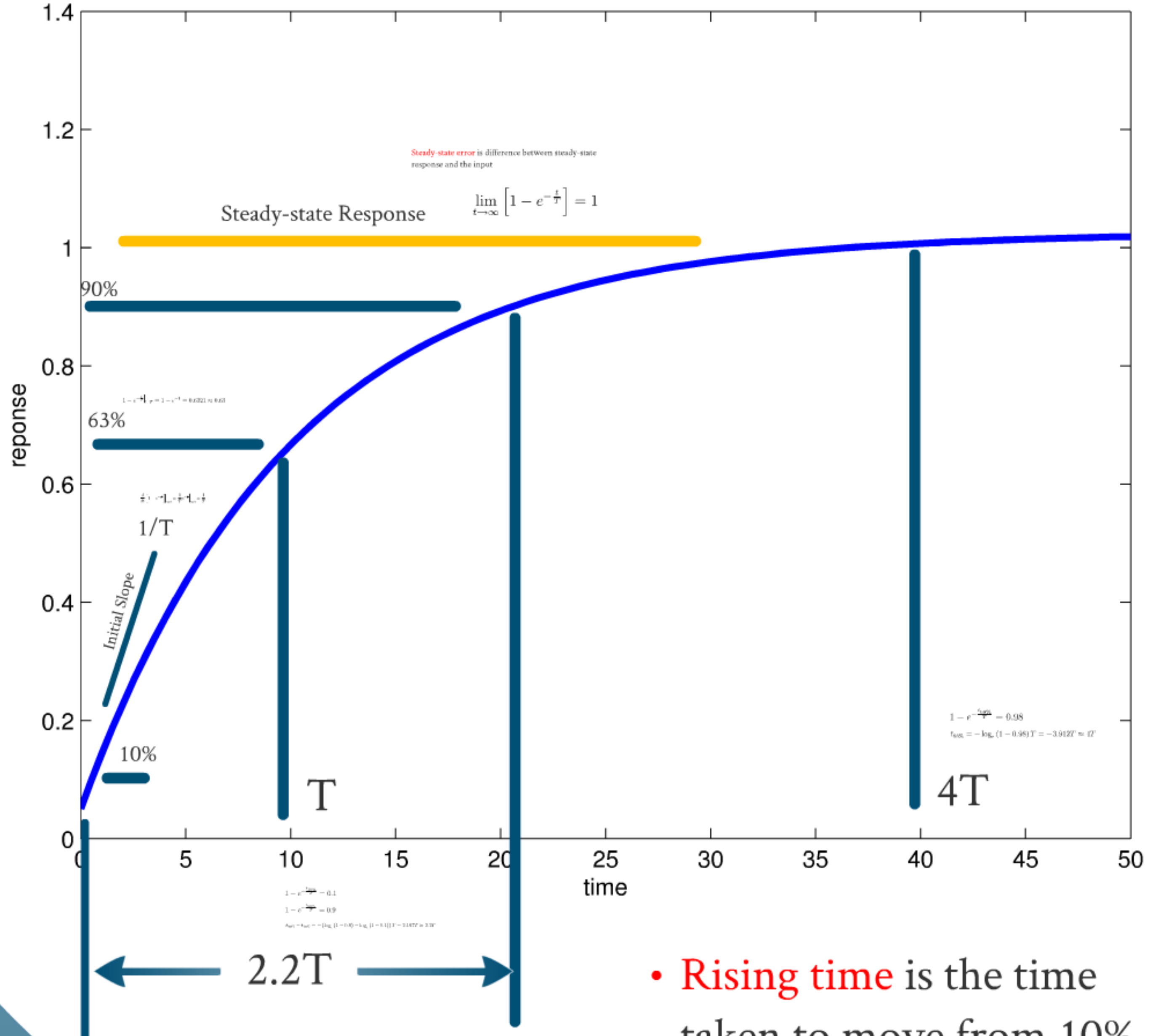
$$\frac{d}{dt}\left[1 - e^{-\frac{t}{T}}\right]\Bigg|_{t=0} = \frac{1}{T}e^{-\frac{t}{T}}\Bigg|_{t=0} = \frac{1}{T}$$

1/T

$$\mathcal{L}^{-1}\left[\frac{1}{s}\times\frac{a}{s+a}\right] = 1 - e^{-at} = 1 - e^{-\frac{t}{T}}$$



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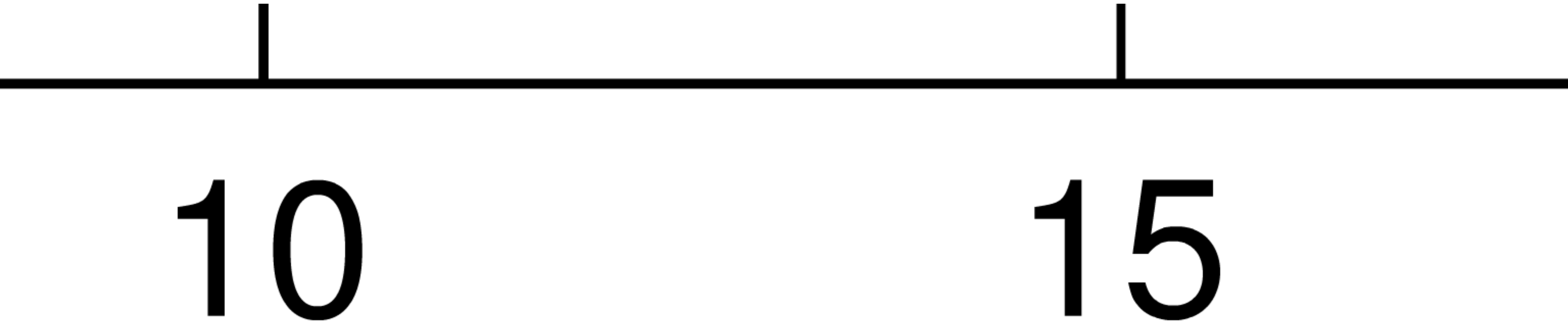


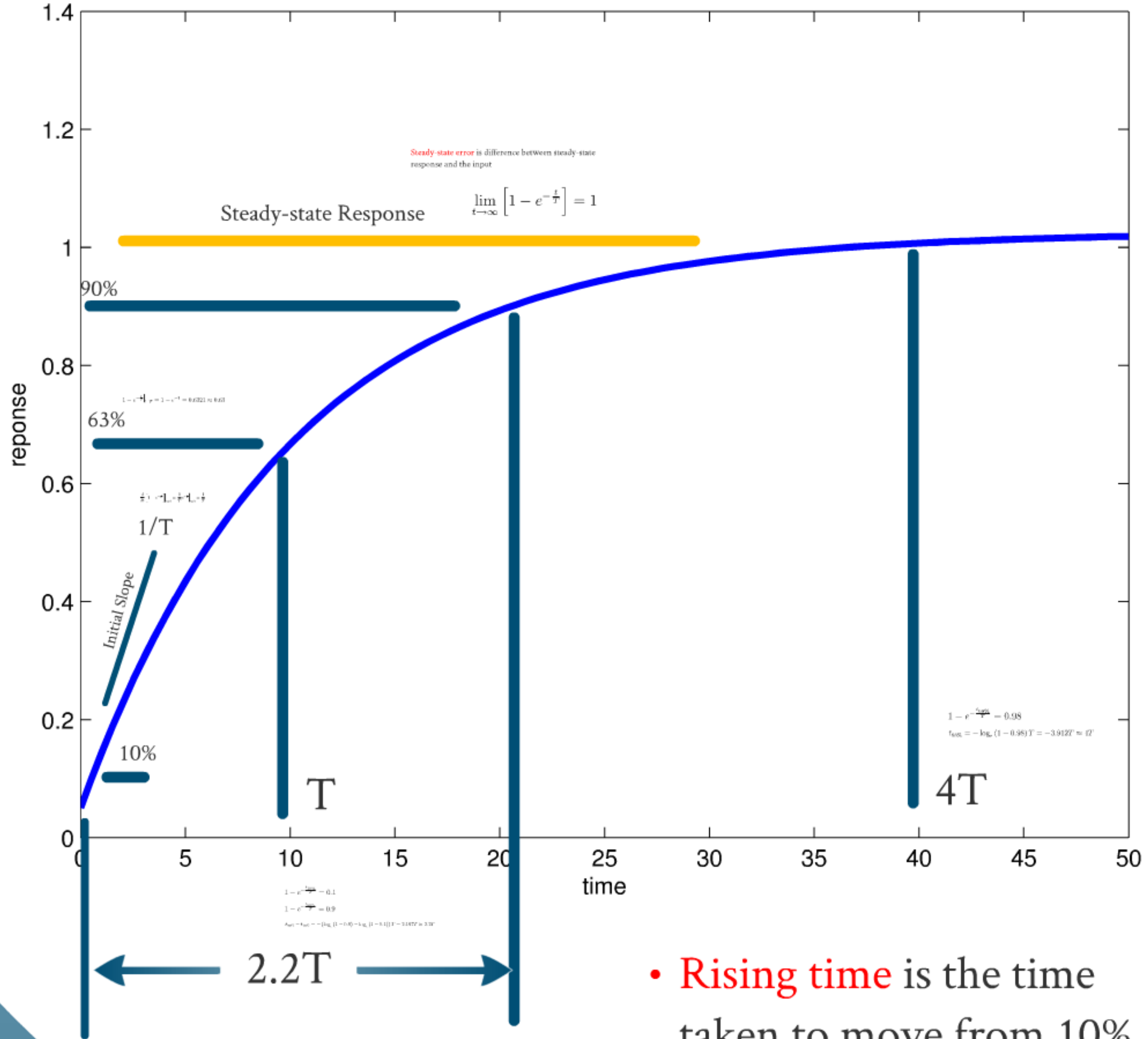
$$1 - e^{-\frac{t_{10\%}}{T}} = 0.1$$

$$1 - e^{-\frac{t_{90\%}}{T}} = 0.9$$

$$t_{90\%} - t_{10\%} = -\left(\log_e(1-0.9) - \log_e(1-0.1)\right)T = 2.197T \approx 2.2T$$

2.2T

$$\mathcal{L}^{-1}\left[\frac{1}{s}\times\frac{a}{s+a}\right] = 1 - e^{-at} = 1 - e^{-\frac{t}{T}}$$



- Settling time is the time taken to move to within 2% of the steady-state response

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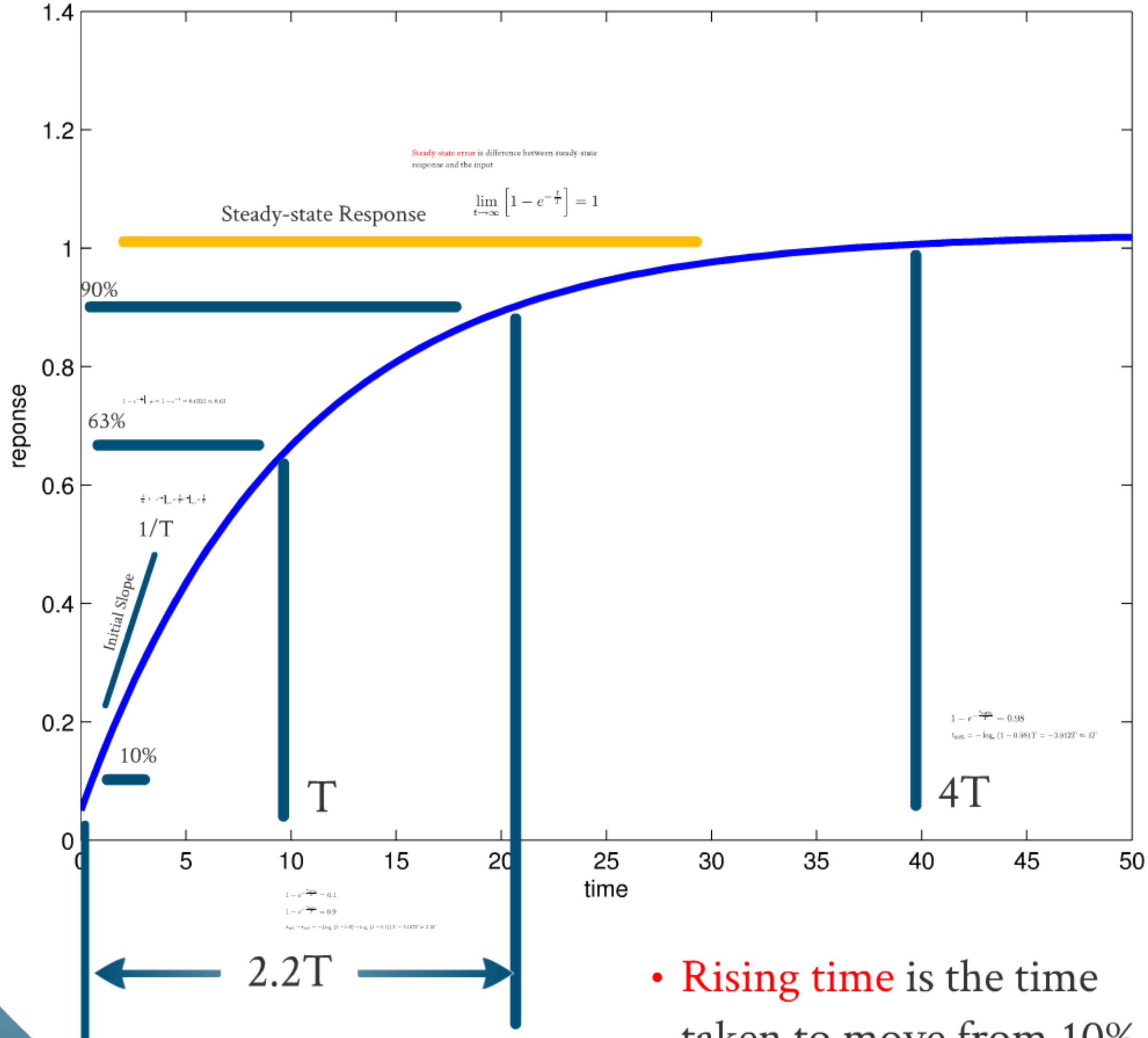
$$1 - e^{-\frac{t_{98\%}}{T}} = 0.98$$

$$t_{98\%} = -\log_e (1 - 0.98)\, T = -3.912T \approx 4T$$

4T

$$\mathcal{L}^{-1}\left[\frac{1}{s}\times\frac{a}{s+a}\right] = 1 - e^{-at} = 1 - e^{-\frac{t}{T}}$$



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Second-Order System

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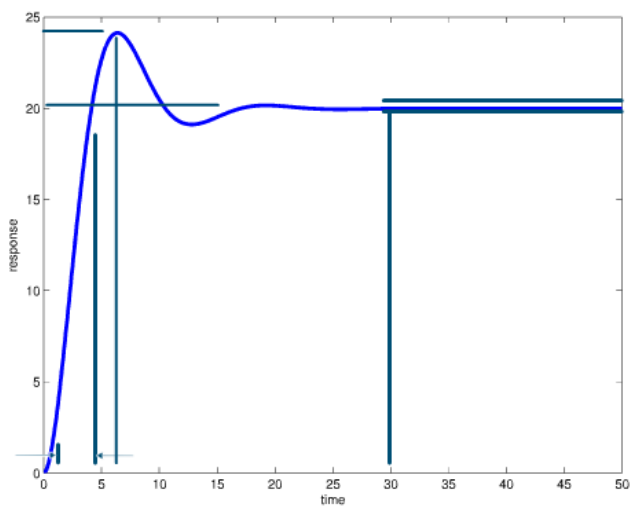

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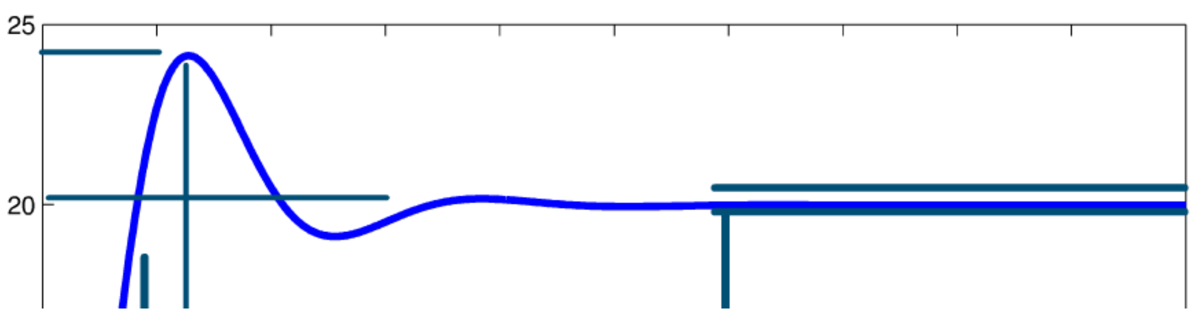

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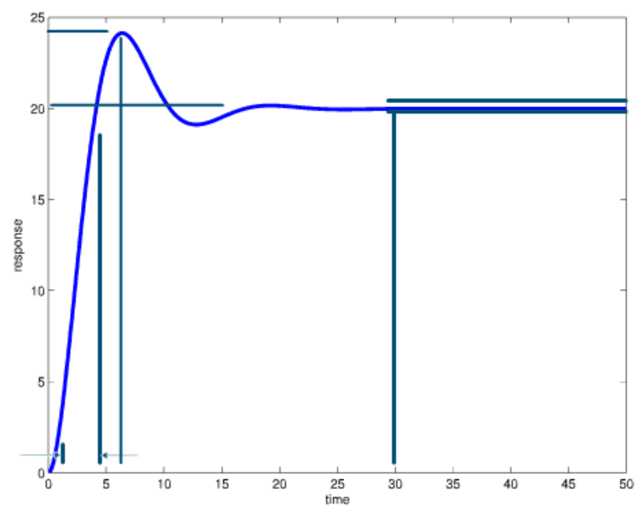

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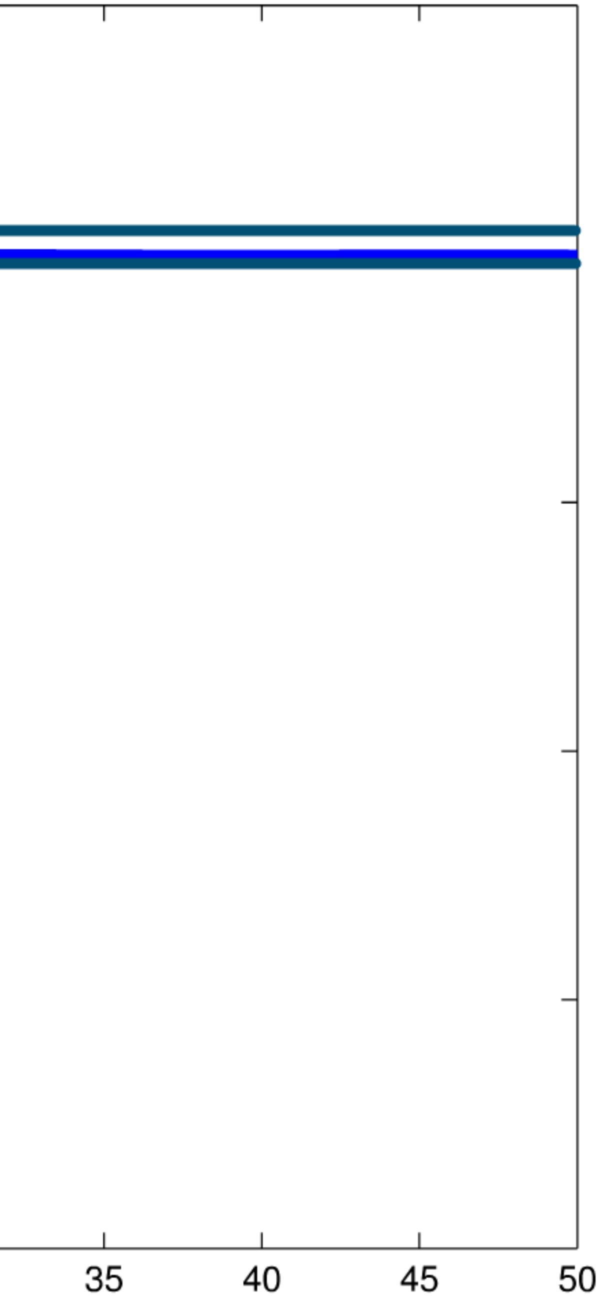

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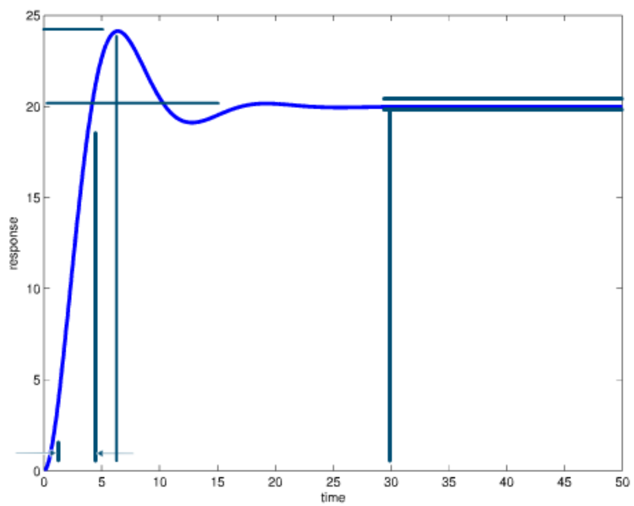

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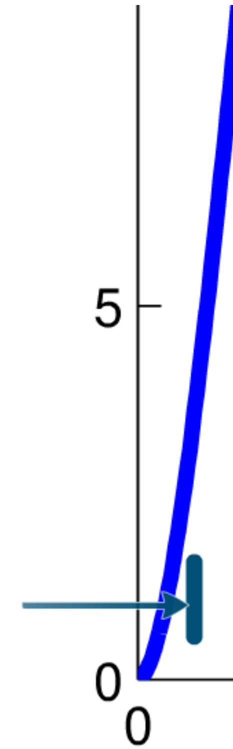

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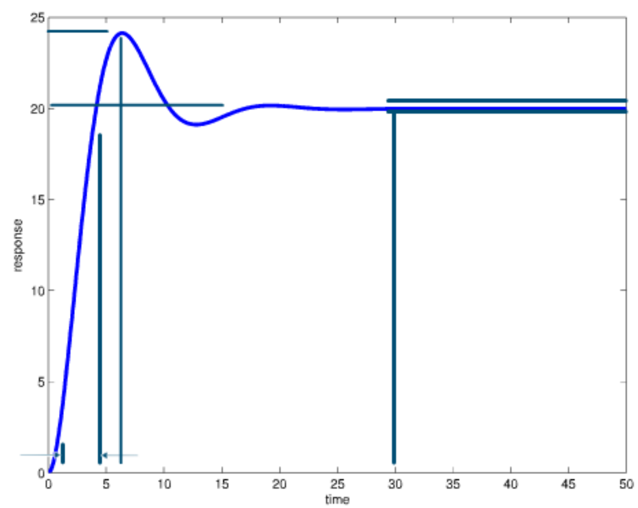

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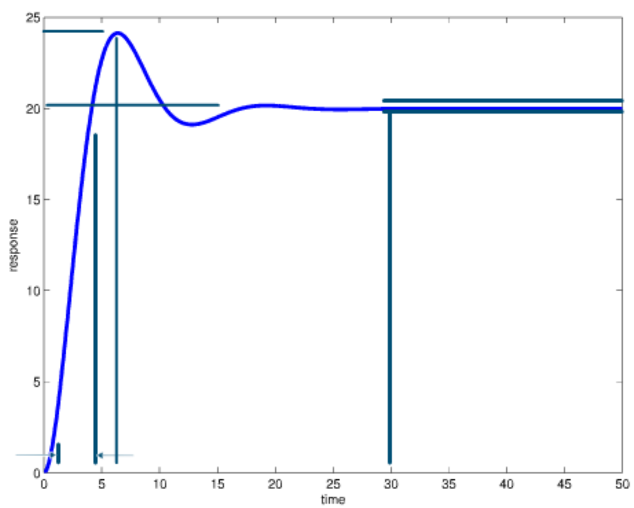
$$\frac{\%OS}{100}=\int\frac{\omega}{\sqrt{1-\zeta^2}}e^{-\zeta\omega\tau}\sin\left(\omega\sqrt{1-\zeta^2}\tau\right)d\tau\Big|_{t=\frac{\pi}{\omega\sqrt{1-\zeta^2}}}=e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

- Percentage overshoot is the amount that the waveform overshoots the steady-state (as a fraction of the steady-state)

- Peak time is the time taken to reach the first, or maximum, peak

$$\omega\sqrt{1-\zeta^2}t=n\pi$$

$$t_p=\frac{\pi}{\omega\sqrt{1-\zeta^2}}$$



- Rising time is the time taken to move from 10% to 90% of the steady-state response

- Settling time is the time taken to move to within 2% of the steady-state response

$$e^{-\zeta\omega t_s}\frac{1}{\sqrt{1-\zeta^2}}=0.02$$

$$t_s=\frac{\log_e\left(0.02\sqrt{1-\zeta^2}\right)}{\zeta\omega}\approx\frac{4}{\zeta\omega}$$

General Steady-state Response

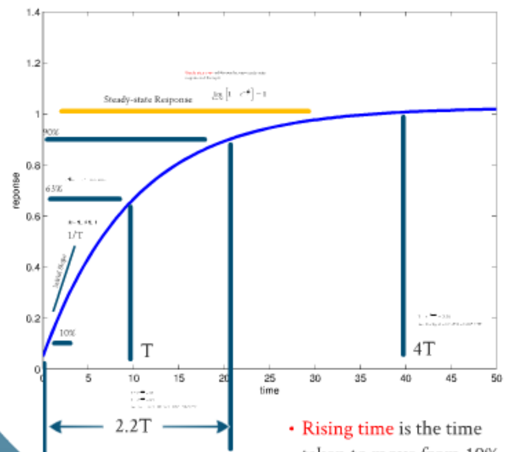
Time Domain (Representation) of the System's Transient Response

Approximate as first-order or second-order using the pole(s) nearest the imaginary axis.

First-Order System

Time Constant, T

$$\mathcal{L}^{-1}\left[\frac{1}{s}\times\frac{a}{s+a}\right]=1-e^{-at}=1-e^{-\frac{t}{T}}$$



- Settling time is the time taken to move to within 2% of the steady-state response

- Rising time is the time taken to move from 10% to 90% of the steady-state response

Second-Order System

$$\mathcal{L}^{-1}\left[\frac{1}{s}\times\frac{\omega^2}{s^2+2\zeta\omega s+\omega^2}\right]=\int\frac{\omega}{\sqrt{1-\zeta^2}}e^{-\zeta\omega\tau}\sin\left(\omega\sqrt{1-\zeta^2}\tau\right)d\tau$$

$$=1-\frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega t}\cos\left(\omega\sqrt{1-\zeta^2}t-\tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)\right)$$

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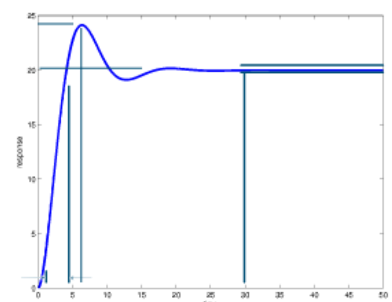

- Settling time is the time taken to move to within 2% of the steady-state response

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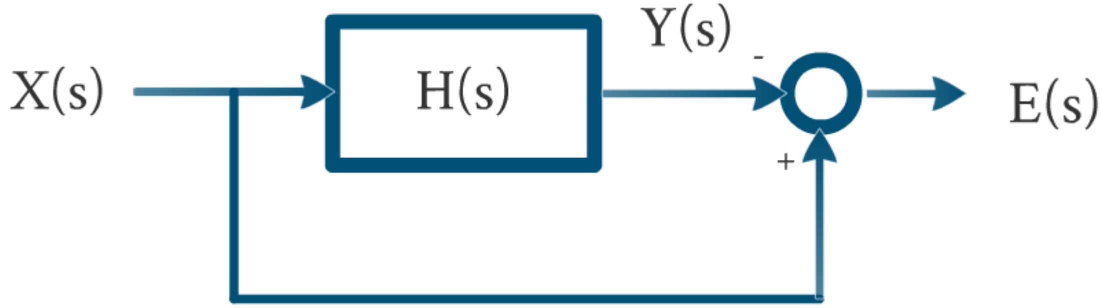
Finding the Steady-state Error

Closed-loop Transfer Function (Representation) of the System

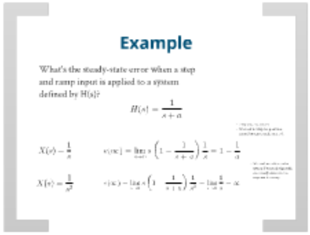


We can write the Laplace Transform of the error signal:

$$E(s) = X(s) - Y(s)$$
$$= X(s) - H(s)X(s)$$
$$= (1 - H(s))X(s)$$

The final value theorem can then be used: $x(\infty) = \lim_{t \to \infty} x(t) = \lim_{s \to 0} sX(s)$

$$e(\infty) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s(1 - H(s))X(s)$$



Example

What's the steady-state error when a step and ramp input is applied to a system defined by H(s)?

$$H(s) = \frac{1}{s+a}$$

$$X(s) = \frac{1}{s} \qquad e(\infty) = \lim_{s \to 0} s\left(1 - \frac{1}{s+a}\right)\frac{1}{s} = 1 - \frac{1}{a}$$

$$X(s) = \frac{1}{s^2} \qquad e(\infty) = \lim_{s \to 0} s\left(1 - \frac{1}{s+a}\right)\frac{1}{s^2} = \lim_{s \to 0} \frac{1}{s} = \infty$$

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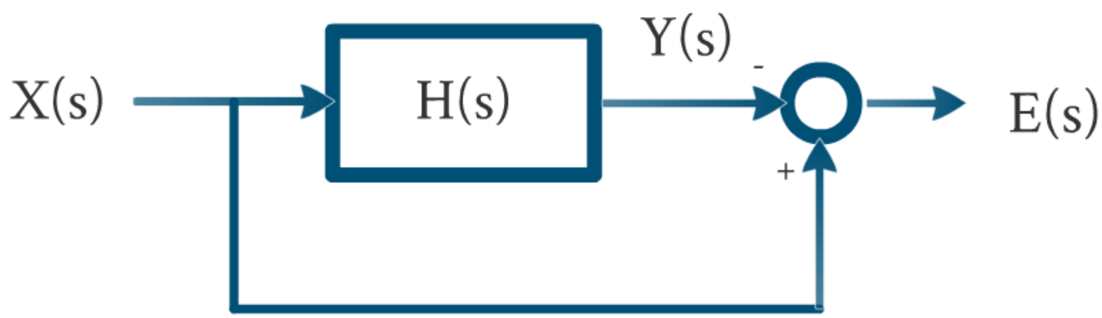

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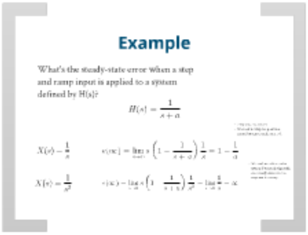

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What's the steady-state error when a step and ramp input is applied to a system defined by H(s)?

$$H(s) = \frac{1}{s+a}$$

- Only zero error if a=1
- We need to wrap the plant in a control system to make sure a=1

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- We can't use a first-order system if want a design with zero steady-state error in response to a ramp.

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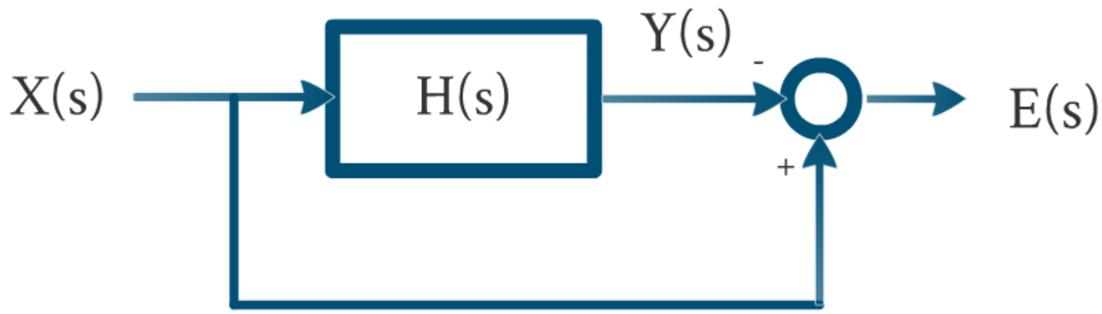

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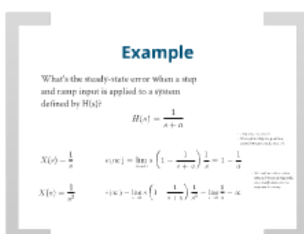

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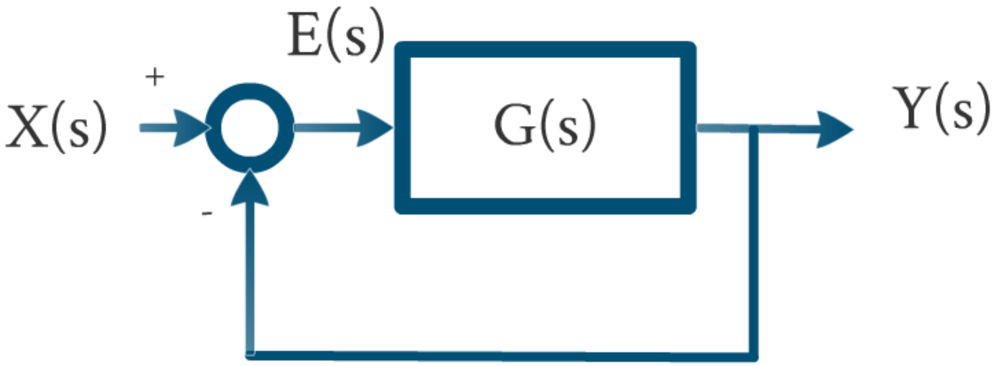
System Type and Error Constants

Unity Negative Feedback (Representation of) System



Number of "pure integrators" (ie poles that G(s) has at s=0) is the system type

Position constant

$$k_p = \lim_{s \to 0} G(s)$$

Velocity constant

$$k_v = \lim_{s \to 0} sG(s)$$

Acceleration constant

$$k_a = \lim_{s \to 0} s^2 G(s)$$

Input		System Type		
		Type-0	Type-1	Type-2
Step	$x(t) = u(t)$	$\frac{1}{1+K_p}$	0	0
Ramp	$x(t) = t$	∞	$\frac{1}{K_v}$	0
Parabola	$x(t) = \frac{t^2}{2}$	∞	∞	$\frac{1}{K_a}$

Steady-state Errors

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System Type and Error Constants

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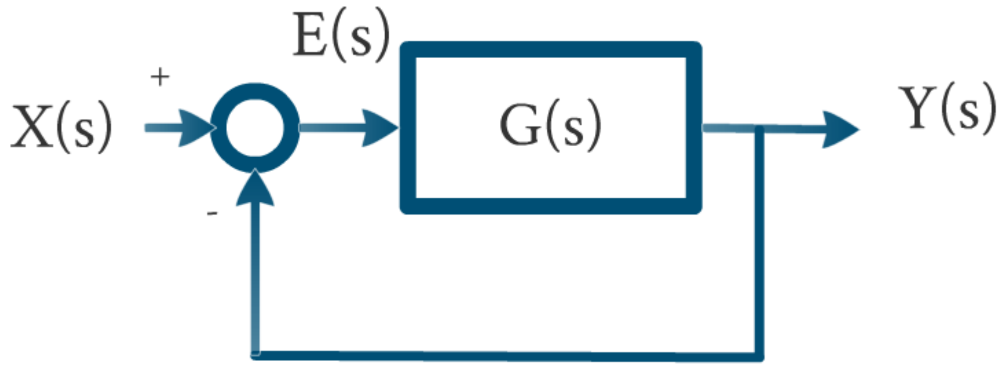

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Steady-state Errors

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Steady-state Errors

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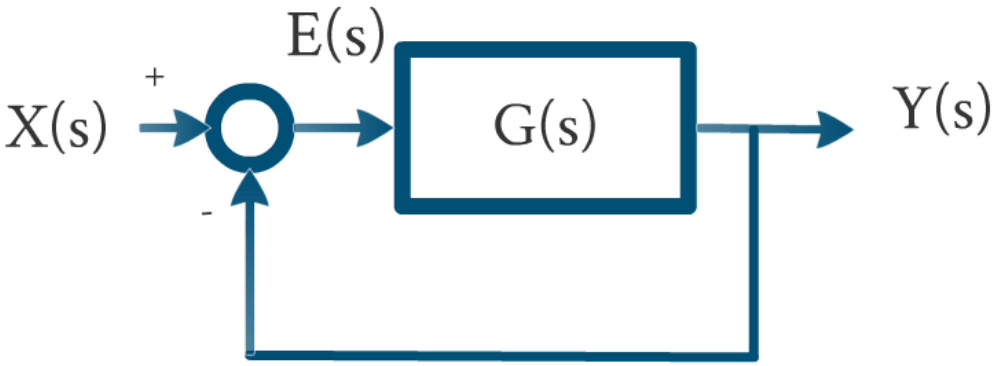

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Steady-state Errors

This lecture covers:

- General Steady-state response
- Steady-state accuracy and errors
- System Characterization by order and type number