ELEC 207
Instrumentation and Control

3 – Measurement Uncertainty

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Measurement uncertainty

Instrument contribution

Many sources of measurement uncertainties are associated with the **measurement instrument** (e.g. sensor, transducer, etc.):

- The extent of this uncertainty contribution can (and must) be provided by the manufacturer in terms of **instrument accuracy**, which is typically expressed as an uncertainty or maximum error. Its main contributions (often detailed by the manufacturer in the **instrument specifications**) can be:
  - Noise;
  - Resolution;
  - Gain error and/or offset;
  - Non-linearity;
  - Hysteresis.

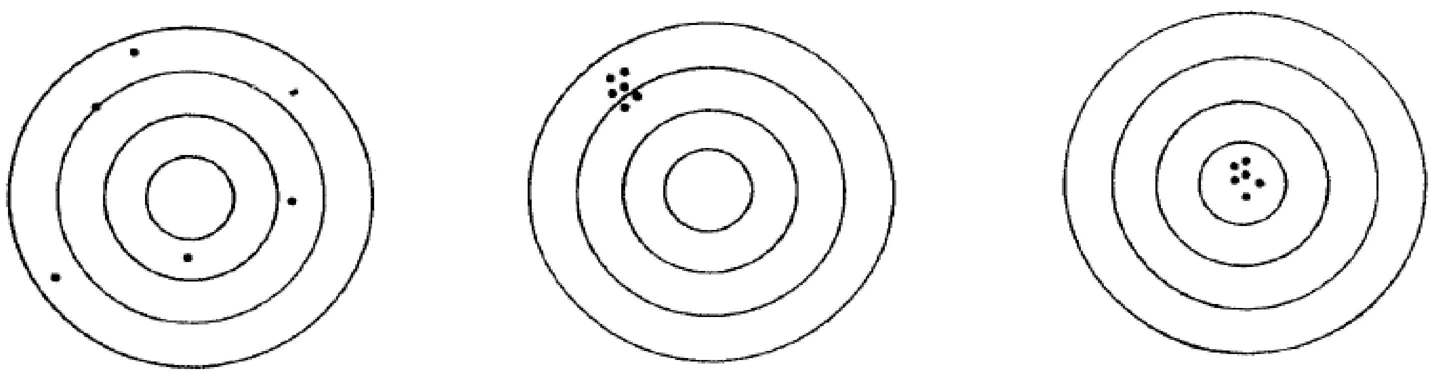
- This uncertainty contribution may or may not be the dominant one.

Instrument specifications

Accuracy vs precision

Accuracy and precision must not be confused:

- The **accuracy** of an instrument is a measure of how close the output reading of the instrument is to the correct value;
- The **precision** describes an instrument’s degree of freedom from random errors; it is similar to **repeatability**.



Low precision
Low accuracy

High precision
Low accuracy

High precision
High accuracy

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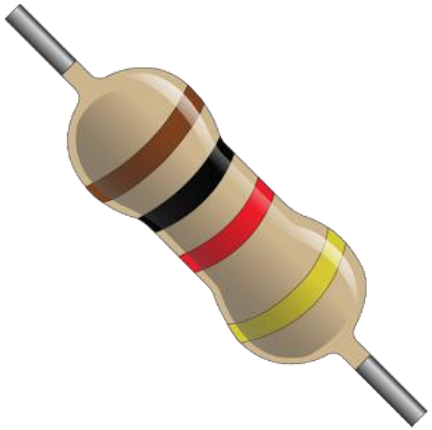
Instrument specifications

Accuracy vs tolerance

Accuracy must not be confused with tolerance either:

- Although the **tolerance** is also an estimate of the uncertainty of a quantity, it is not a characteristic of a measurement instrument, but of a **manufactured component**:
  - It describes the maximum deviation of a manufactured component from some specified value (nominal value).
- For example, a 1 kΩ resistor with 5% tolerance means that the actual resistance value can be between 0.95 and 1.05 kΩ.



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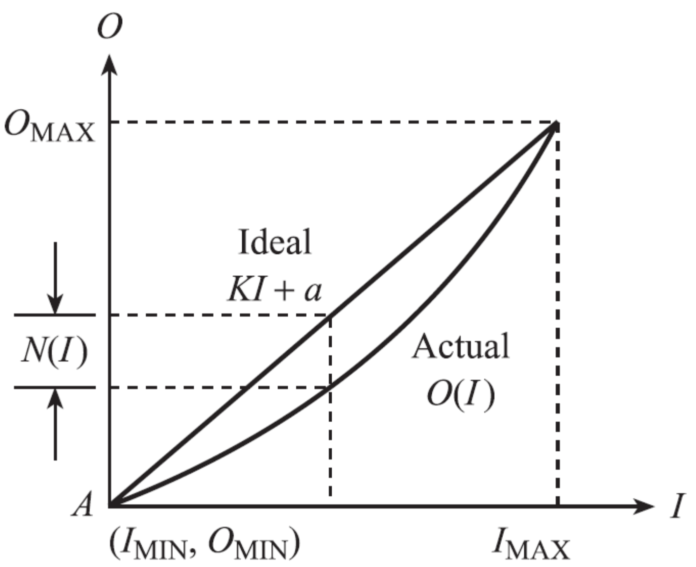
Instrument specifications
Measurement range

The accuracy and other specifications are often provided as a percentage of the measurement range:

- The **range** of an instrument defines the minimum and maximum values of a quantity that the instrument is designed to measure;

The **transfer function** of the instrument (input-output relationship) is often designed to be linear, but in practice there might be a non-linearity:

- The **non-linearity** of an instrument is defined as the maximum deviation of any of the output readings from the theoretical linear function.

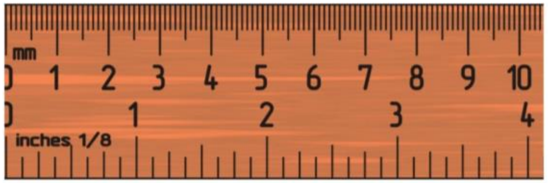


Instrument specifications

Resolution and sensitivity

Other two important instrument specifications are resolution and sensitivity:

- The **resolution** of an instrument describes how finely its output scale is divided into subdivisions:



- The **sensitivity** of an instrument is a measure of the change in its output that occurs when the quantity being measured (input) changes by a given amount:
  - ➢ It corresponds to the slope of the transfer function in linear instruments.

Instruments with higher resolution and/or higher sensitivity are not necessarily more accurate.

Instrument specifications

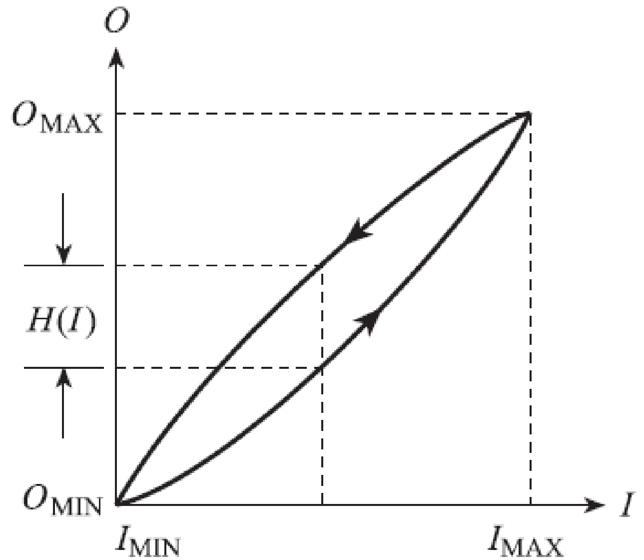
Hysteresis

Some instruments may also be affected by errors arising from hysteresis effects:

- The **hysteresis** is the deviation of the sensor output when the input signal is approached from opposite directions:

  - Typical causes of hysteresis are **friction** and **structural changes** in the material;

  - The maximum hysteresis error is usually expressed as a percentage of the instrument range.



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Uncertainty evaluation
Calculation from instrument specifications

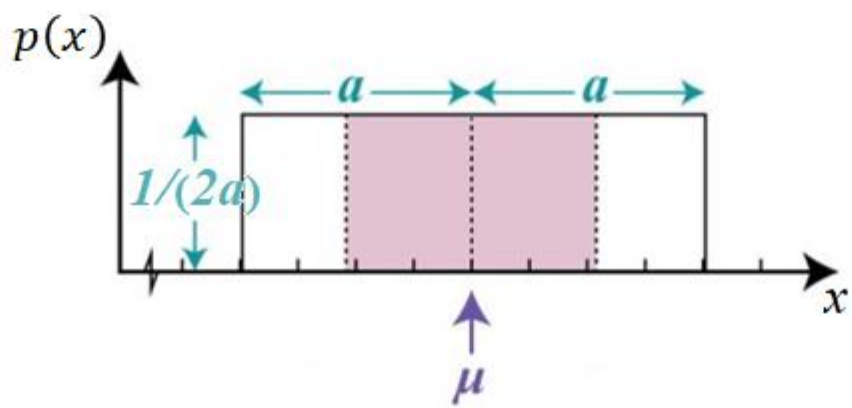
The information provided by the instrument manufacturer can be used to estimate the **measurement uncertainty due to the instrument alone**:

- It is usually defined as a maximum error (e.g., accuracy, resolution, non-linearity, hysteresis);

- The uncertainty is however the **standard deviation** of a probability distribution, therefore the maximum error must be converted into a standard deviation:

  - A **uniform distribution** is usually assumed for the calculation:

$$u = \sigma = \sqrt{\int_{\mu-a}^{\mu+a} (x-\mu)^2 p(x) \mathrm{d}x} = \frac{a}{\sqrt{3}}$$



Uncertainty evaluation

Experimental evaluation from repeated measurements (1)

Other random uncertainty contributions (e.g. due to noise or other environmental factors) can be estimated by repeated measurements:

- The best estimate of the measurement result is the **mean value** of all measured values:

$$\bar{x} = \mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- The best estimate of the measurement **uncertainty due to random contributions alone** is the **standard deviation** of all measured values:

$$u(x) = \sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2}$$

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Uncertainty evaluation

Experimental evaluation from repeated measurements (2)

- Note that the standard deviation $u(x)$ defined in the previous slide is the **uncertainty of a single measurement** x due to random errors:

$$u(x) = \sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2}$$

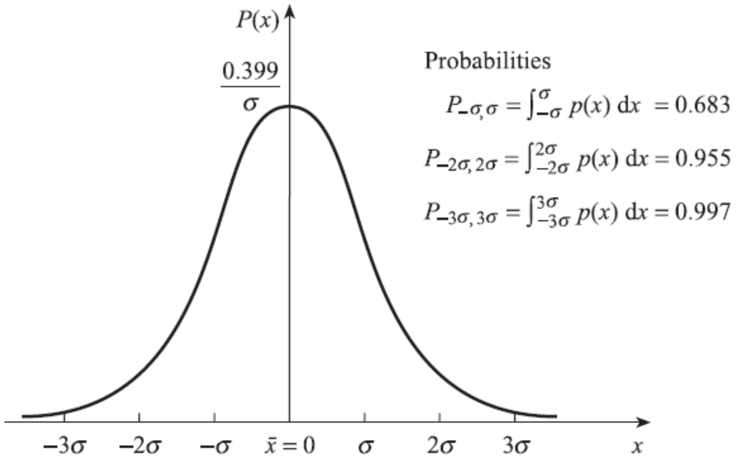
- The **uncertainty of the mean value** $\bar{x}$ is lower because the averaging process decreases the errors caused by random noise:

$$u(\bar{x}) = \frac{u(x)}{\sqrt{N}} = \sqrt{\frac{1}{N(N-1)}\sum_{i=1}^{N}(x_i - \mu)^2}$$

Uncertainty evaluation
Combined uncertainty and confidence level

Different uncertainty contributions can be **combined** in the following way:

$$u(x) = \sqrt{\sum_{i=1}^{N} u_i^2(x)}$$



Assuming a Gaussian probability distribution, this **standard uncertainty** means that the 'true' value has a 68.3% probability to be in the interval $x \pm u(x)$;

- If a higher probability (**confidence level**) is required, an **expanded uncertainty** can be used:

$$U(x) = ku(x)$$

k = 1 → P = 68.3%
k = 2 → P = 95.5%
k = 3 → P = 99.7%

References

Textbook: Principles of Measurement Systems, 4^{th} ed.

For further explanation about the points covered in this lecture, please refer to the following chapters and sections in the **Bentley** textbook:

- Chapter 2, Sec. 2.1: **Systematic characteristics [of measurement systems]**;

- Chapter 2, Sec. 2.3: **Statistical characteristics [of measurement systems]**;

<u>NOTE</u>: Topics not covered in the lecture are not required for the exam.

References

Textbook: Measurement and Instrumentation, 2nd ed.

For further explanation about the points covered in this lecture, please refer to the following chapters and sections in the **Morris-Langari** textbook:

- Chapter 2, Sec. 2.3: **Static characteristics of instruments**;
- Chapter 4, Sec. 4.2: **Mean and median values**;
- Chapter 4, Sec. 4.3: **Standard deviation and variance**;
- Chapter 4, Sec. 4.5: **Gaussian (normal) distribution**.

NOTE: Topics not covered in the lecture are not required for the exam.