ELEC 207 Part B

Control Theory Lecture 8: System Stability

Prof Simon Maskell
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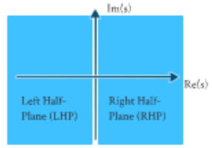
0151 794 4573

Stability and Instability

Bounded Input Bounded Output definition of stability:

- A system is stable if every bounded input yields a bounded output.
- Systems that aren't stable are unstable (and are pretty useless).

Relationship Between Poles' Location and Stability



Example 2: Numeric

$$s^4 + 6s^3 + 13s^2 + 12s + 4$$

s^4	1	13	4
s^3	6	12	0
s^2	$-\frac{1}{6}\begin{vmatrix}1 & 13\\ 6 & 12\end{vmatrix} = 11$	$-\frac{1}{6}\begin{vmatrix}1 & 4\\ 6 & 0\end{vmatrix} = 4$	0
s^1	$-\frac{1}{11}\begin{vmatrix}6 & 12\\ 11 & 4\end{vmatrix} = \frac{108}{11}$	0	0
s^0	$-\frac{11}{108}\begin{vmatrix}11 & 4\\ \frac{108}{11} & 0\end{vmatrix} = 4$	0	0

Routh-Hurwitz Stability Criteria

$$(s+r_1)(s+r_2)\ldots(s+r_n) = s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 = 0$$

$$r_i > 0 \quad \forall i \qquad a_i > 0 \quad \forall i$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Complete the remaining part of the Routh table until all terms generated are zero

Interpret the results

Example 5: Parameterised

Example 1

$$a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0 s^0$$

Example 4: Row of Zeros

Example 3: Zero First Column

This lecture covers:
- Stability and Instability
- Relationship between poles' location and stability
- Routh-Hurwitz stability criteria

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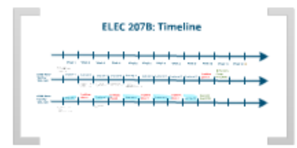
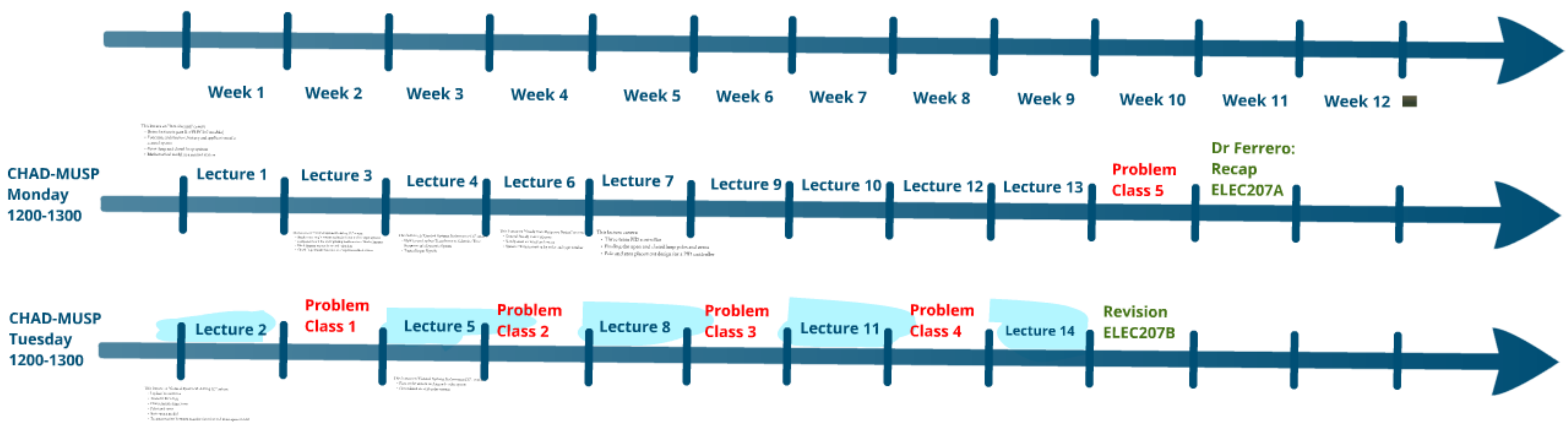


- Stability and Instability
- Relationship between poles' location and stability
- Routh-Hurwitz stability criteria

ELEC 207B: Timeline



This lecture on "Introduction" covers:

- (Introduction to part B of ELEC207 module)
- Function, architecture, history and applications of a control system
- Open-loop and closed-loop systems
- Mathematical model of a control system

Lecture 1

Lecture 2

This lecture on "Control System Modelling (1)" covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model

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Lecture 3

This lecture on "Control System Modelling (2)" covers:

- Single-input single-output and multi-input multi-output systems
- Components and the underpinning mathematics of block diagrams
- Block diagram manipulation and reduction
- Closed-loop transfer function of a negative feedback system

Lect

This lecture on "Control Sy

- How to use Laplace Tr
Response of a Dynamic
- Typical Input Signals

e 3

covers:

multi-output systems

atics of block diagrams

feedback system

Lecture 4

This lecture on "Control Systems Performance (1)" covers:

- How to use Laplace Transforms to Solve the Time Response of a Dynamic System
- Typical Input Signals

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Lecture 5

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This lecture on "Control Systems Performance (2)" covers:

- First-order system and second-order system
- Generalized second-order system

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Lecture 6

This lecture on "Steady-State Response Design" covers:

- General Steady-state response
- Steady-state accuracy and errors
- System Characterization by order and type number

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This lecture c

- Three ter
- Finding t
- Pole and

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gn" covers:

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Lecture 7

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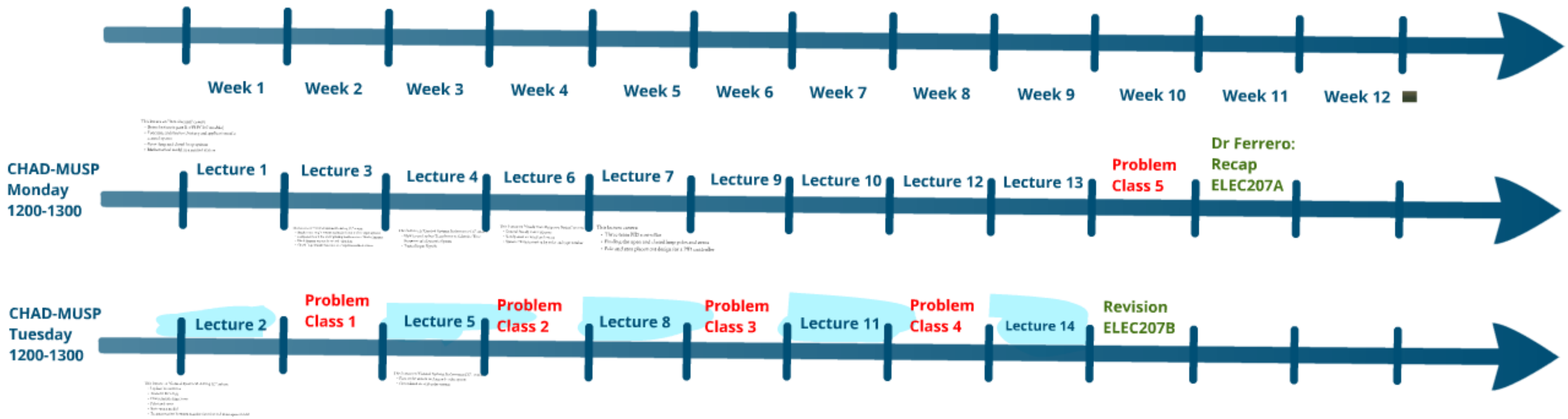
This lecture covers:

- Three term PID controller
- Finding the open and closed loop poles and zeros
- Pole and zero placement design for a PID controller

ELEC 207B: Timeline



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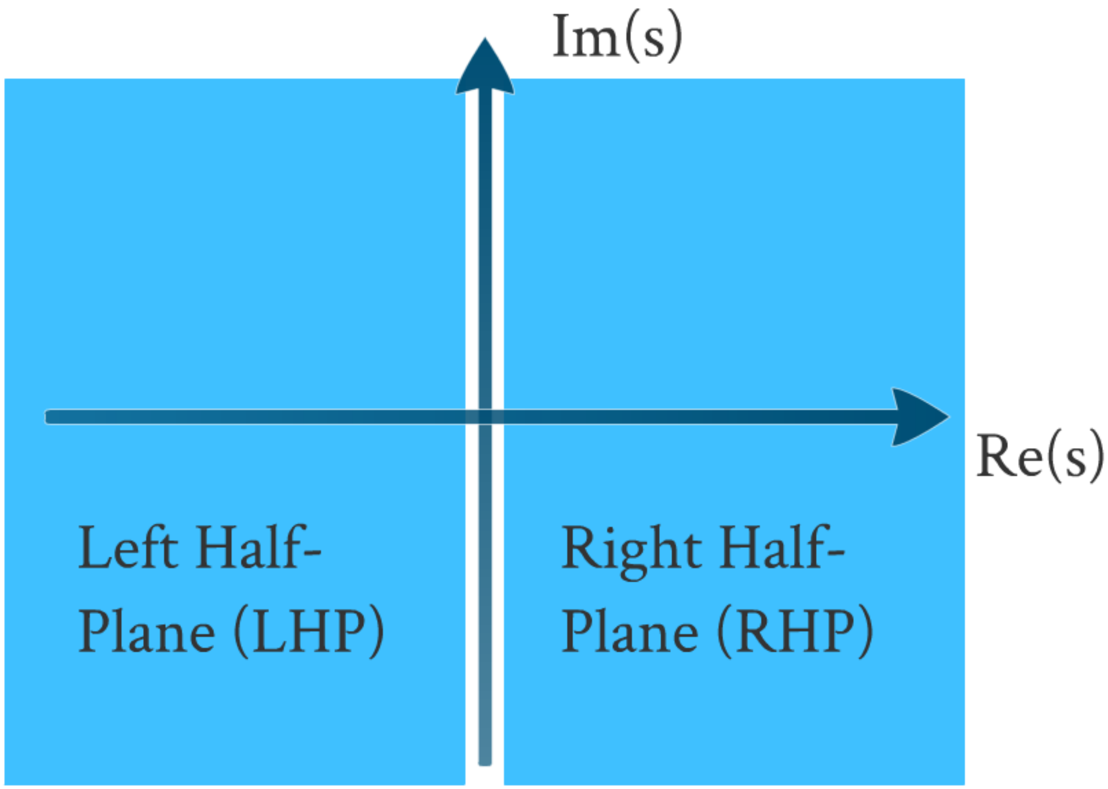
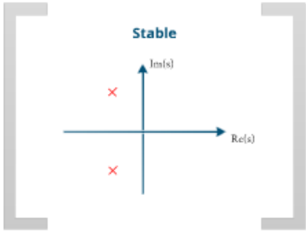
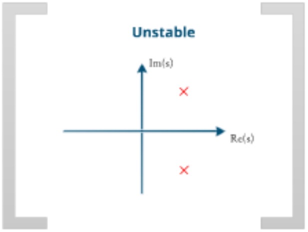
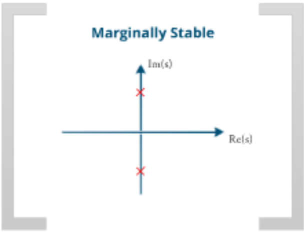
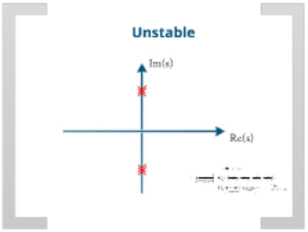


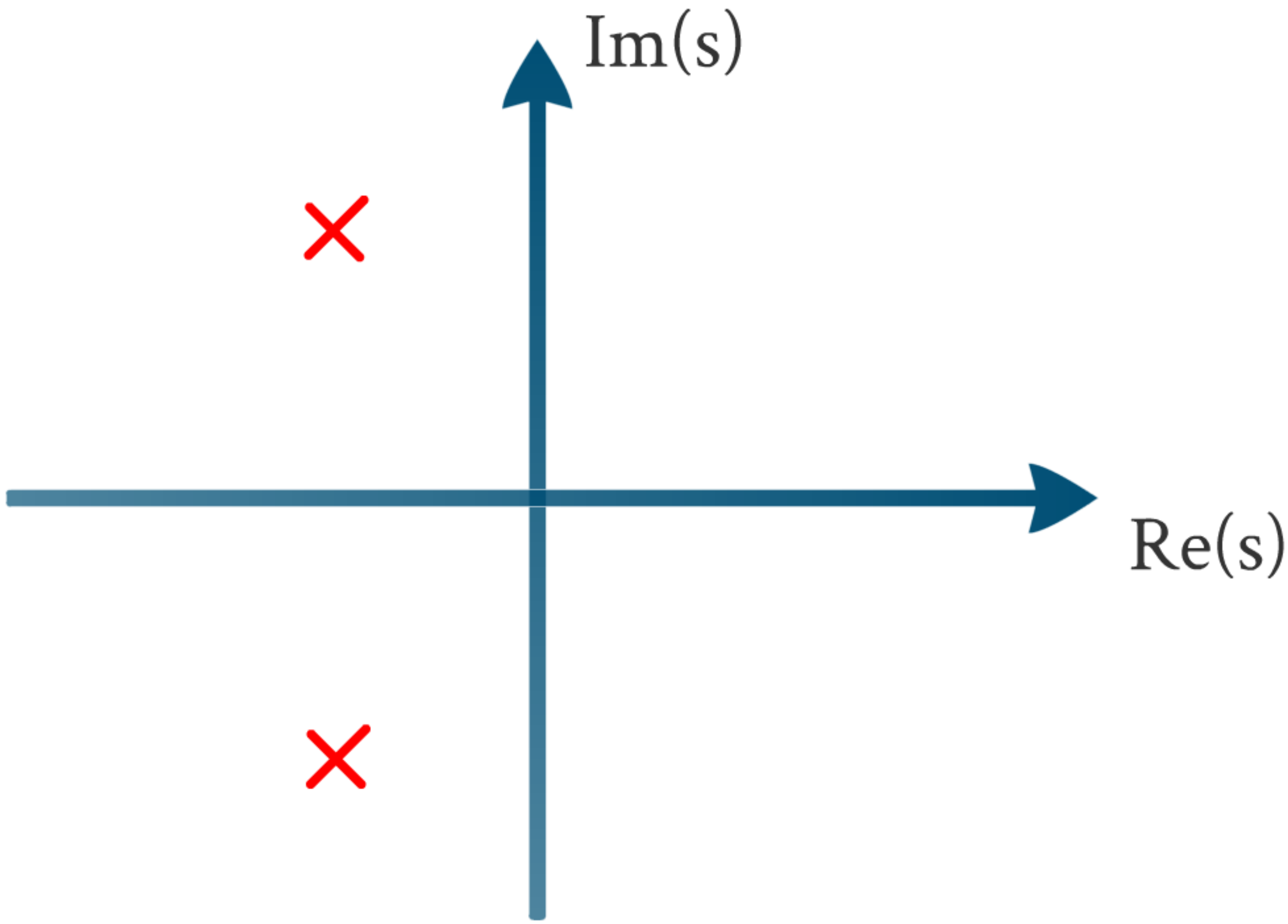




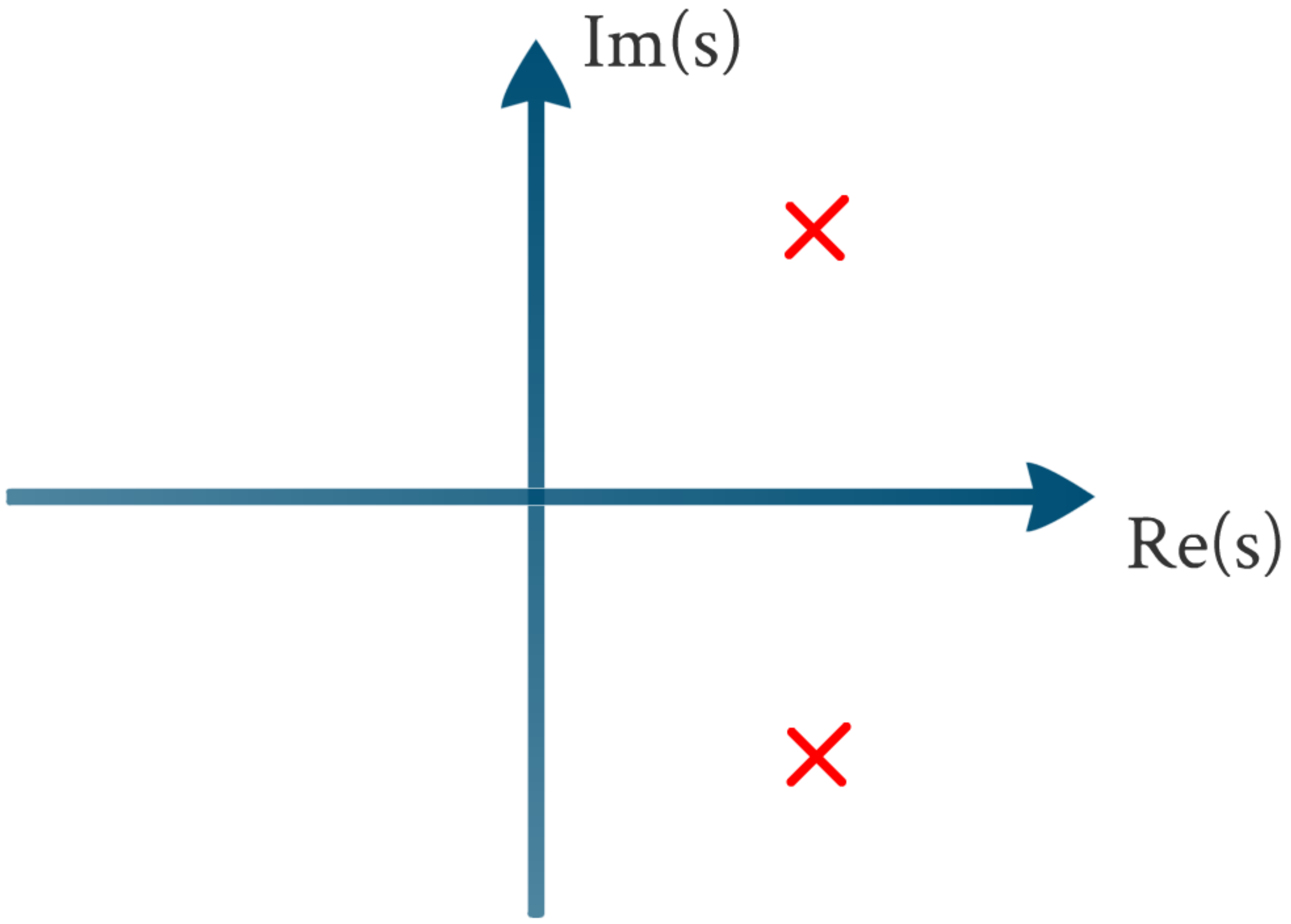




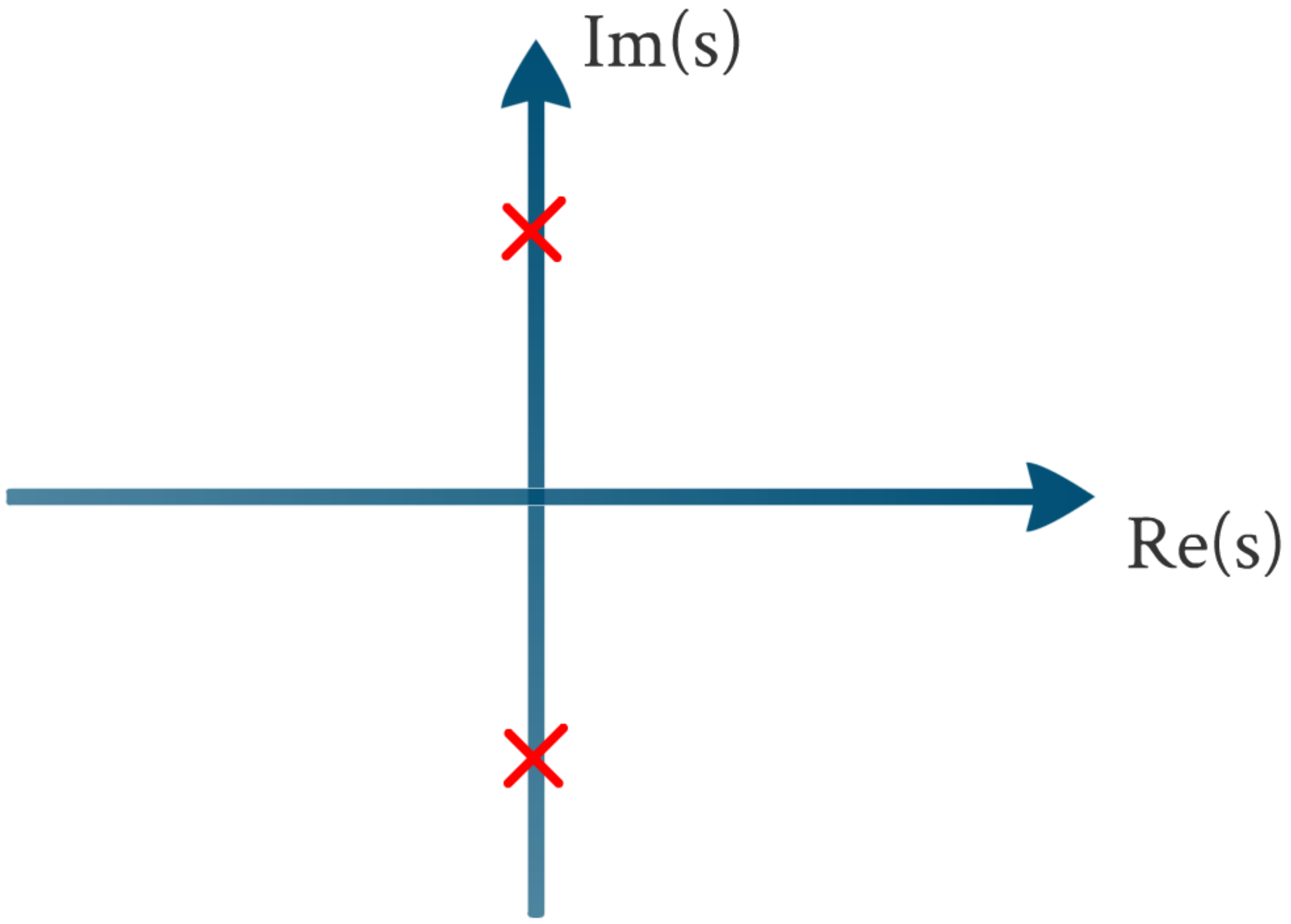
Stable



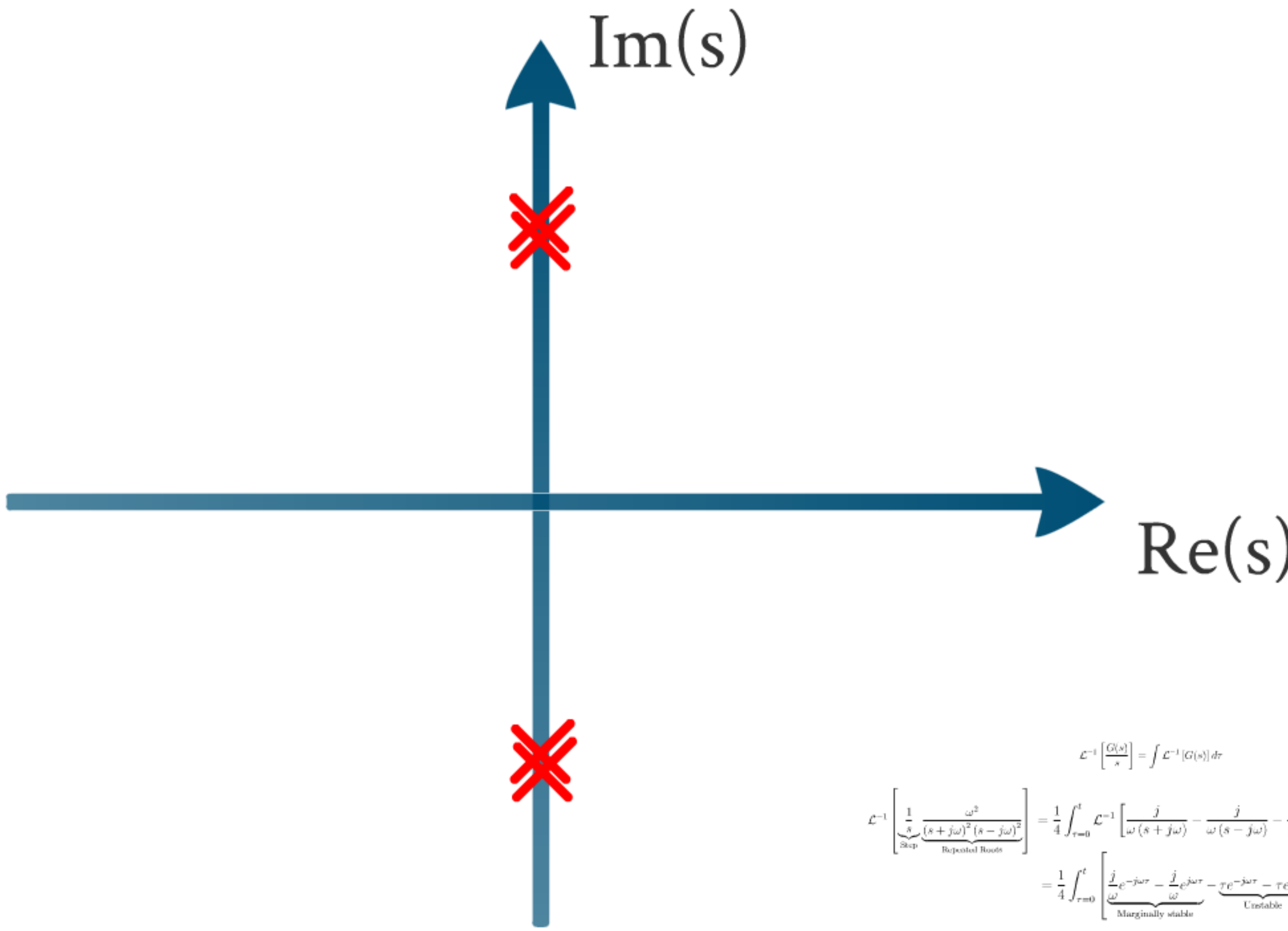
Unstable



Marginally Stable



Unstable



$$\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right] = \int \mathcal{L}^{-1}\left[G(s)\right] d\tau$$

$$\mathcal{L}^{-1}\left[\underbrace{\frac{1}{s}}_{\text{Step}} \underbrace{\frac{\omega^2}{(s+j\omega)^2 (s-j\omega)^2}}_{\text{Repeated Roots}}\right] = \frac{1}{4}\int_{\tau=0}^{t} \mathcal{L}^{-1}\left[\frac{j}{\omega(s+j\omega)} - \frac{j}{\omega(s-j\omega)} - \frac{1}{(s+j\omega)^2} - \frac{1}{(s-j\omega)^2}\right] d\tau$$

$$= \frac{1}{4}\int_{\tau=0}^{t}\left[\underbrace{\frac{j}{\omega}e^{-j\omega\tau} - \frac{j}{\omega}e^{j\omega\tau}}_{\text{Marginally stable}} - \underbrace{\tau e^{-j\omega\tau} - \tau e^{j\omega\tau}}_{\text{Unstable}}\right] d\tau$$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = t$$

$$\mathcal{L}^{-1}\left[G(s-a)\right] = e^{at}\mathcal{L}^{-1}\left[G(s)\right]$$

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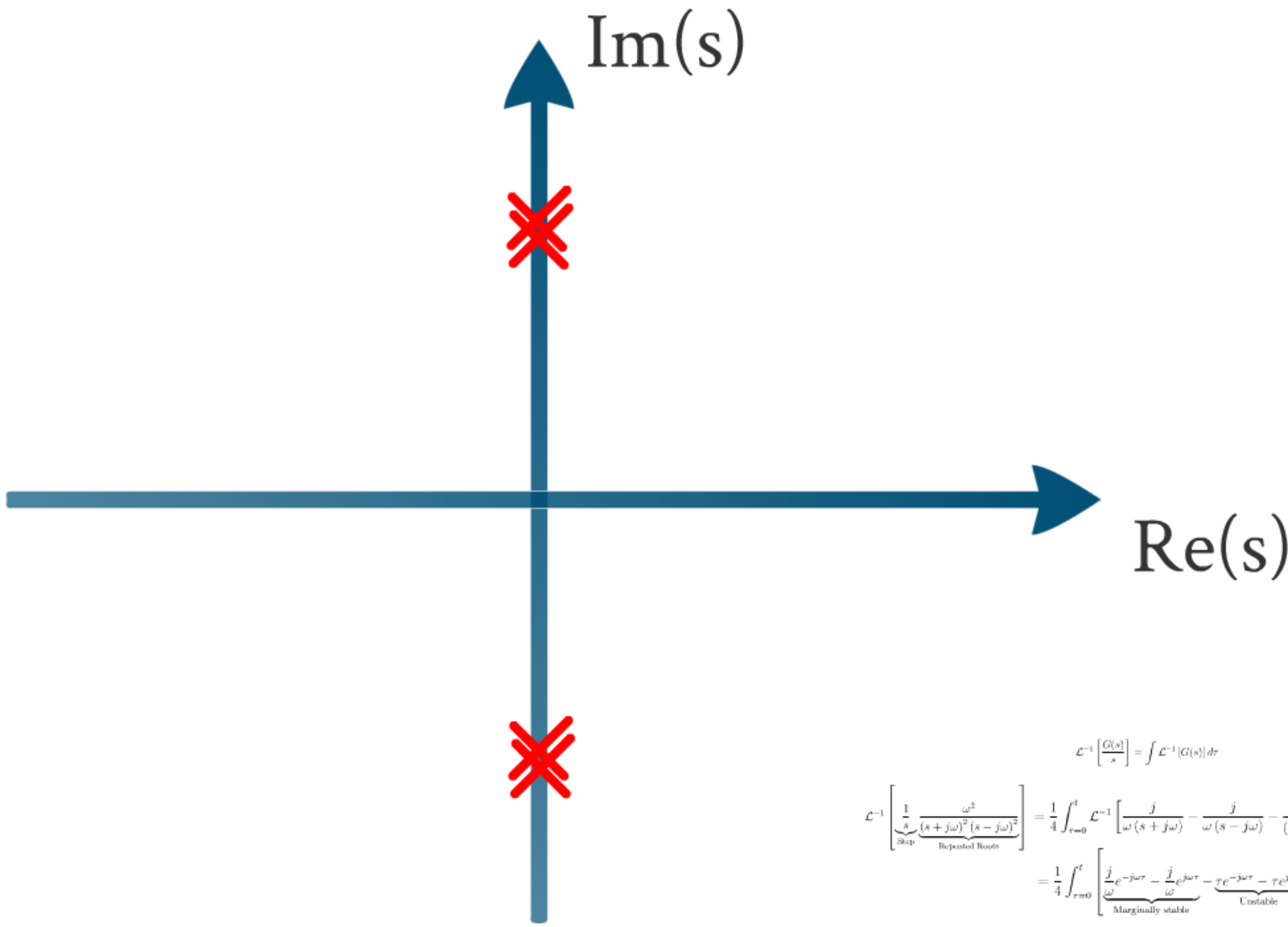

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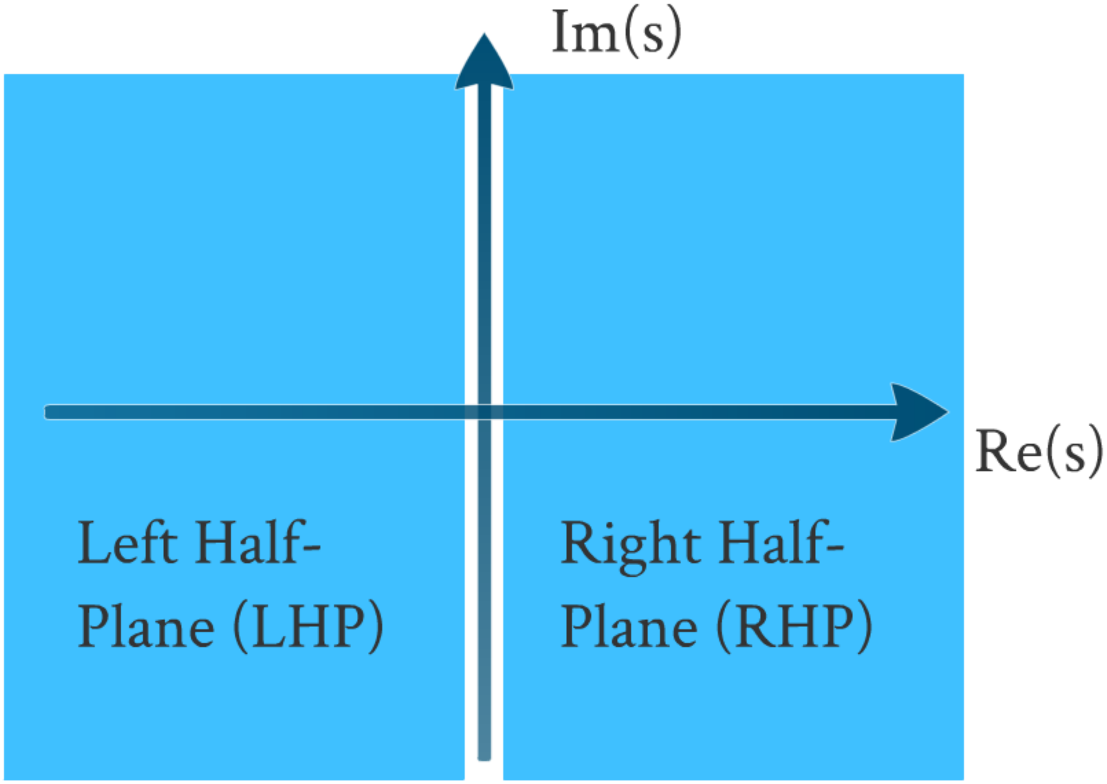
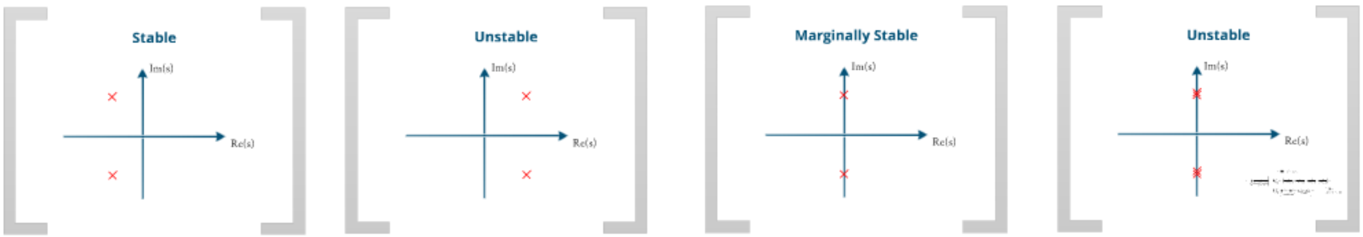




Routh-Hurwitz Stability Criteria

$$(s+r_1)(s+r_2)\dots(s+r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0 = 0$$

$$r_i > 0 \quad \forall i \qquad\qquad a_i > 0 \quad \forall i$$

$$\underbrace{(s-1+3j)(s-1-3j)}_{\text{Roots in RHP}}(s+3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the s° row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number, ϵ, continue and reinterpret the results.

Case 2: There is a zero across an entire row.

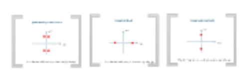
- Consider the Auxiliary Equation, A(s), formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from dA(s)/ds and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is stable.
- A sign change implies that the system is unstable.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system unstable.
- Multiple distinct roots on the imaginary axis make the system marginally stable.

$$\ldots + \ldots + a_1 s + a_0 = 0$$

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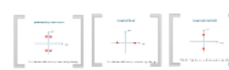

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Example 1

$$a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0 s^0$$

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4				
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$			
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$		
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3				
s^2				
s^1				
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s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1				
s^0				

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s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1	$e_1 = -\frac{1}{d_1}\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$	0	0	0
s^0				

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s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
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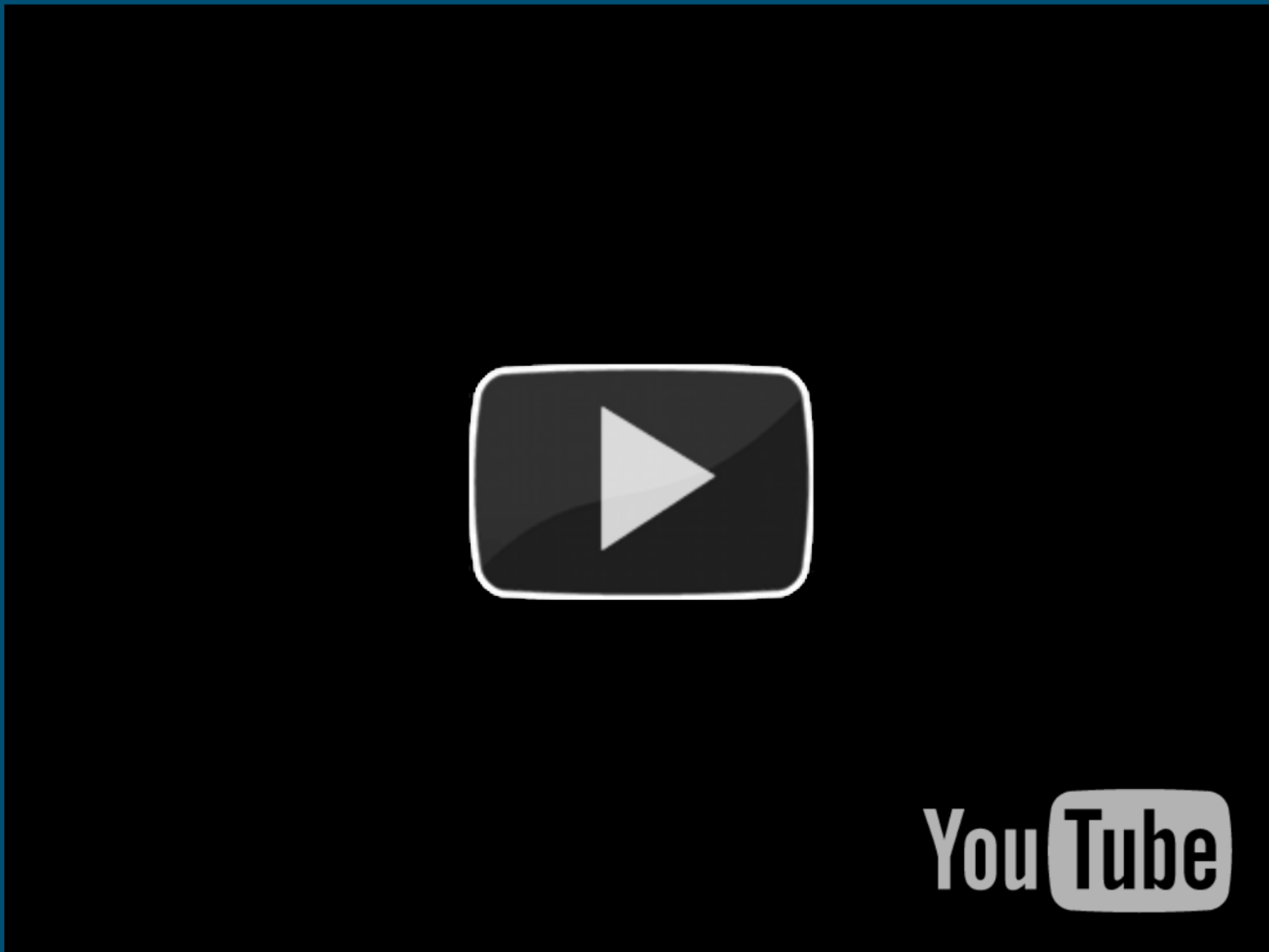
s^1	$e_1 = -\frac{1}{d_1}\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$	0	0	0
s^0	$f_1 = -\frac{1}{e_1}\begin{vmatrix} d_1 & d_2 \\ e_1 & 0 \end{vmatrix}$	0	0	0

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4				
s^3				
s^2				
s^1				
s^0				


YouTube

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4				
s^3				
s^2				
s^1				
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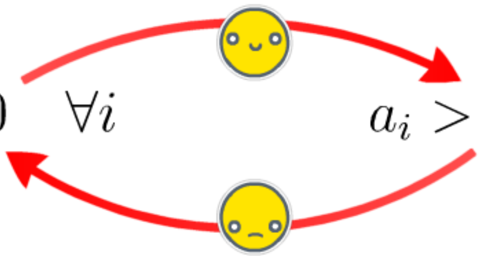

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Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number, ϵ, continue and reinterpret the results.

Case 2: There is a zero across an entire row.

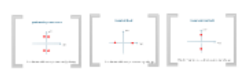
- Consider the Auxiliary Equation, A(s), formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from dA(s)/ds and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is stable.
- A sign change implies that the system is unstable.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system unstable.
- Multiple distinct roots on the imaginary axis make the system marginally stable.

Example 1

$$a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s^1 + a_0 s^0$$

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4				
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$			
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$		
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1	$e_1 = -\frac{1}{d_1}\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$	0	0	0
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1	$e_1 = -\frac{1}{d_1}\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$	0	0	0
s^0	$f_1 = -\frac{1}{e_1}\begin{vmatrix} d_1 & d_2 \\ e_1 & 0 \end{vmatrix}$	0	0	0

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$			
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$		
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0

s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{}$ $\begin{vmatrix} a_6 & a_4 \end{vmatrix}$	$b_2 = -\frac{1}{}$ $\begin{vmatrix} a_6 & a_2 \end{vmatrix}$	$b_3 = -\frac{1}{}$ $\begin{vmatrix} a_6 & a_0 \end{vmatrix}$	0

s				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5} \begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5} \begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5} \begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3				
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2				
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1				
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1	$e_1 = -\frac{1}{d_1}\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$	0	0	0
s^0				

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1	$e_1 = -\frac{1}{d_1}\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$	0	0	0
s^0	$f_1 = -\frac{1}{e_1}\begin{vmatrix} d_1 & d_2 \\ e_1 & 0 \end{vmatrix}$	0	0	0

Routh-Hurwitz Stability Criteria

$$(s+r_1)(s+r_2)\dots(s+r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0 = 0$$

$$r_i > 0 \quad \forall i \qquad\qquad a_i > 0 \quad \forall i$$

$$\underbrace{(s-1+3j)(s-1-3j)}_{\text{Roots in RHP}}(s+3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the s° row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

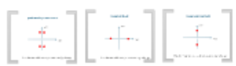
Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.
- We can replace the zero with a small number, ϵ, continue and reinterpret the results.

Case 2: There is a zero across an entire row.
- Consider the Auxiliary Equation, A(s), formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from dA(s)/ds and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.
- So, no sign change in the first column of Routh table implies the system is stable.
- A sign change implies that the system is unstable.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.
- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
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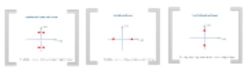

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Example 4: Row of Zeros

s^5	1	2	1
s^4	1	2	1
s^3			
s^2			
s^1			
s^0			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \to 4$	0
s^2	$-\frac{1}{4}\begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = 1$	$-\frac{1}{4}\begin{vmatrix} 1 & 1 \\ 4 & 0 \end{vmatrix} = 1$	0
s^1			
s^0			

s^6	a_6	a_4	a_2	a_0
s^5	a_5	a_3	a_1	0
s^4	$b_1 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$	$b_3 = -\frac{1}{a_5}\begin{vmatrix} a_6 & a_0 \\ a_5 & 0 \end{vmatrix}$	0
s^3	$c_1 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_3 \\ b_1 & b_2 \end{vmatrix}$	$c_2 = -\frac{1}{b_1}\begin{vmatrix} a_5 & a_1 \\ b_1 & b_3 \end{vmatrix}$	0	0
s^2	$d_1 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$	$d_2 = -\frac{1}{c_1}\begin{vmatrix} b_1 & b_3 \\ c_1 & 0 \end{vmatrix}$	0	0
s^1	$e_1 = -\frac{1}{d_1}\begin{vmatrix} c_1 & c_2 \\ d_1 & d_2 \end{vmatrix}$	0	0	0
s^0	$f_1 = -\frac{1}{e_1}\begin{vmatrix} d_1 & d_2 \\ e_1 & 0 \end{vmatrix}$	0	0	0

Routh-Hurwitz Stability Criteria

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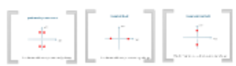

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Example 2: Numeric

$$s^4 + 6s^3 + 13s^2 + 12s + 4$$

s^4	1	13	4
s^3	6	12	0
s^2	$-\frac{1}{6}\begin{matrix} 1 & 13 \\ 6 & 12 \end{matrix} =11$	$-\frac{1}{6}\begin{matrix} 1 & 4 \\ 6 & 0 \end{matrix} = 4$	0
s^1	$-\frac{1}{11}\begin{matrix} 6 & 12 \\ 11 & 4 \end{matrix} = \frac{108}{11}$	0	0
s^0	$-\frac{11}{108}\begin{matrix} 11 & 4 \\ \frac{108}{11} & 0 \end{matrix} = 4$	0	0

Routh-Hurwitz Stability Criteria

$$(s+r_1)(s+r_2)\dots(s+r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0 = 0$$

$r_i > 0 \quad \forall i$ $\qquad$ $a_i > 0 \quad \forall i$

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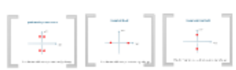

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e completed the s° row, working out

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number, ϵ, continue and reinterpret the results.

ber of roots in the RHP.

n of Routh table implies the system

Example 3: Zero First Column

s^5	1	3	5
s^4	2	6	3
s^3	$-\frac{1}{2}\begin{vmatrix}1 & 3\\ 2 & 6\end{vmatrix} = \epsilon$	$-\frac{1}{2}\begin{vmatrix}1 & 5\\ 2 & 3\end{vmatrix} = \frac{7}{2}$	0
s^2			
s^1			
s^0			

s^5	1	3	5
s^4	2	6	3
s^3	$-\frac{1}{2}\begin{vmatrix}1 & 3\\ 2 & 6\end{vmatrix} = \epsilon$	$-\frac{1}{2}\begin{vmatrix}1 & 5\\ 2 & 3\end{vmatrix} = \frac{7}{2}$	0
s^2	$\lim\limits_{\epsilon\to 0} -\frac{1}{\epsilon}\begin{vmatrix}2 & 6\\ \epsilon & \frac{7}{2}\end{vmatrix} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}\begin{vmatrix}2 & 3\\ \epsilon & 0\end{vmatrix} = 3$	0
s^1	$\lim\limits_{\epsilon\to 0} \frac{\epsilon}{7}\begin{vmatrix}\epsilon & \frac{7}{2}\\ -\frac{7}{\epsilon} & 3\end{vmatrix} = \frac{7}{2}$	0	0
s^0			

s^5	1	3	5
s^4	2	6	3
s^3	$-\frac{1}{2}\begin{vmatrix}1 & 3\\ 2 & 6\end{vmatrix} = \epsilon$	$-\frac{1}{2}\begin{vmatrix}1 & 5\\ 2 & 3\end{vmatrix} = \frac{7}{2}$	0
s^2	$\lim\limits_{\epsilon\to 0} -\frac{1}{\epsilon}\begin{vmatrix}2 & 6\\ \epsilon & \frac{7}{2}\end{vmatrix} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}\begin{vmatrix}2 & 3\\ \epsilon & 0\end{vmatrix} = 3$	0
s^1			
s^0			

s^5	1	3	5
s^4	2	6	3
s^3	$-\frac{1}{2}\begin{vmatrix}1 & 3\\ 2 & 6\end{vmatrix} = \epsilon$	$-\frac{1}{2}\begin{vmatrix}1 & 5\\ 2 & 3\end{vmatrix} = \frac{7}{2}$	0
s^2	$\lim\limits_{\epsilon\to 0} -\frac{1}{\epsilon}\begin{vmatrix}2 & 6\\ \epsilon & \frac{7}{2}\end{vmatrix} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}\begin{vmatrix}2 & 3\\ \epsilon & 0\end{vmatrix} = 3$	0
s^1	$\lim\limits_{\epsilon\to 0} \frac{\epsilon}{7}\begin{vmatrix}\epsilon & \frac{7}{2}\\ -\frac{7}{\epsilon} & 3\end{vmatrix} = \frac{7}{2}$	0	0
s^0	$\lim\limits_{\epsilon\to 0} -\frac{2}{7}\begin{vmatrix}-\frac{7}{\epsilon} & 3\\ \frac{7}{2} & 0\end{vmatrix} = 3$	0	0

s^5	1	3	5
s^4	2	6	3
s^3	$-\frac{1}{2}\begin{vmatrix}1 & 3\\ 2 & 6\end{vmatrix} = \epsilon$	$-\frac{1}{2}\begin{vmatrix}1 & 5\\ 2 & 3\end{vmatrix} = \frac{7}{2}$	0
s^2			
s^1			
s^0			

s^5	1	3	5
s^4	2	6	3
s^3	$-\frac{1}{2}\begin{vmatrix}1 & 3\\ 2 & 6\end{vmatrix} = \epsilon$	$-\frac{1}{2}\begin{vmatrix}1 & 5\\ 2 & 3\end{vmatrix} = \frac{7}{2}$	0
s^2	$\lim\limits_{\epsilon\to 0} -\frac{1}{\epsilon}\begin{vmatrix}2 & 6\\ \epsilon & \frac{7}{2}\end{vmatrix} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}\begin{vmatrix}2 & 3\\ \epsilon & 0\end{vmatrix} = 3$	0
s^1			
s^0			

s^5	1	3	5
s^4	2	6	3
s^3	$-\frac{1}{2}\begin{matrix}1 & 3\\ 2 & 6\end{matrix} = \epsilon$	$-\frac{1}{2}\begin{matrix}1 & 5\\ 2 & 3\end{matrix} = \frac{7}{2}$	0
s^2	$\lim_{\epsilon\to 0} -\frac{1}{\epsilon}\begin{matrix}2 & 6\\ \epsilon & \frac{7}{2}\end{matrix} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}\begin{matrix}2 & 3\\ \epsilon & 0\end{matrix} = 3$	0
s^1	$\lim_{\epsilon\to 0} \frac{\epsilon}{7}\begin{matrix}\epsilon & \frac{7}{2}\\ -\frac{7}{\epsilon} & 3\end{matrix} = \frac{7}{2}$	0	0
s^0			

s^5	1			3			5
s^4	2			6			3
s^3	$-\frac{1}{2}$	$\begin{matrix}1 & 3\\ 2 & 6\end{matrix}$	$=\epsilon$	$-\frac{1}{2}$	$\begin{matrix}1 & 5\\ 2 & 3\end{matrix}$	$=\frac{7}{2}$	0
s^2	$\lim\limits_{\epsilon\to 0}-\frac{1}{\epsilon}$	$\begin{matrix}2 & 6\\ \epsilon & \frac{7}{2}\end{matrix}$	$=-\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}$	$\begin{matrix}2 & 3\\ \epsilon & 0\end{matrix}$	$=3$	0
s^1	$\lim\limits_{\epsilon\to 0}\frac{\epsilon}{7}$	$\begin{matrix}\epsilon & \frac{7}{2}\\ -\frac{7}{\epsilon} & 3\end{matrix}$	$=\frac{7}{2}$	0			0
s^0	$\lim\limits_{\epsilon\to 0}-\frac{2}{7}$	$\begin{matrix}-\frac{7}{\epsilon} & 3\\ \frac{7}{2} & 0\end{matrix}$	$=3$	0			0

Routh-Hurwitz Stability Criteria

$$(s+r_1)(s+r_2)\dots(s+r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1 s + a_0 = 0$$

$$r_i > 0 \quad \forall i \qquad\qquad a_i > 0 \quad \forall i$$

$$\underbrace{(s-1+3j)(s-1-3j)}_{\text{Roots in RHP}}(s+3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the s° row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.
- We can replace the zero with a small number, ϵ, continue and reinterpret the results.

Case 2: There is a zero across an entire row.
- Consider the Auxiliary Equation, A(s), formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from dA(s)/ds and continue Routh's tabulation.

Interpret the results

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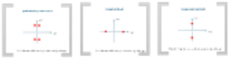
If there were a row of zeros, then there are symmetric roots about the origin



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ut negative of scaled determinant of m

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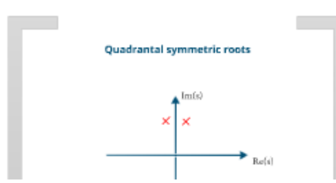

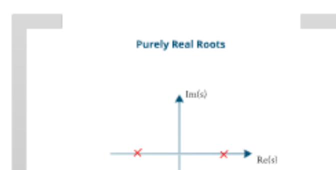

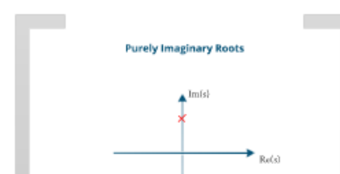

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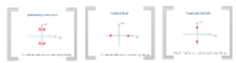

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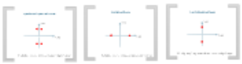

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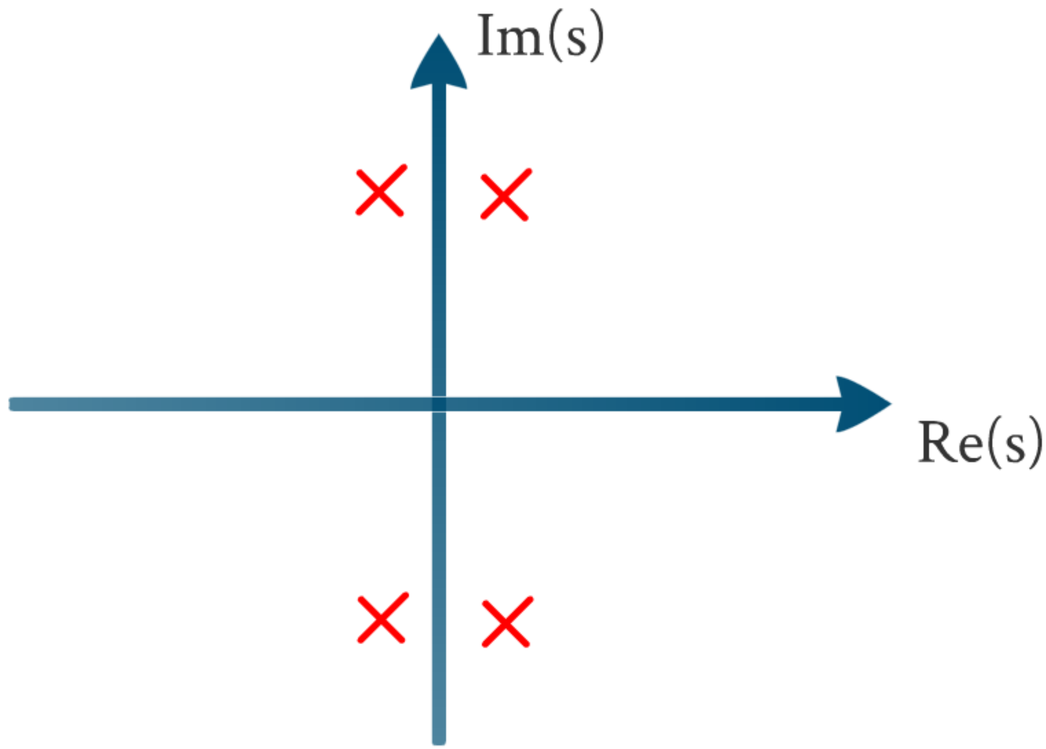
Example 4: Row of Zeros

s^5	1	2	1
s^4	1	2	1
s^3			
s^2			
s^1			
s^0			

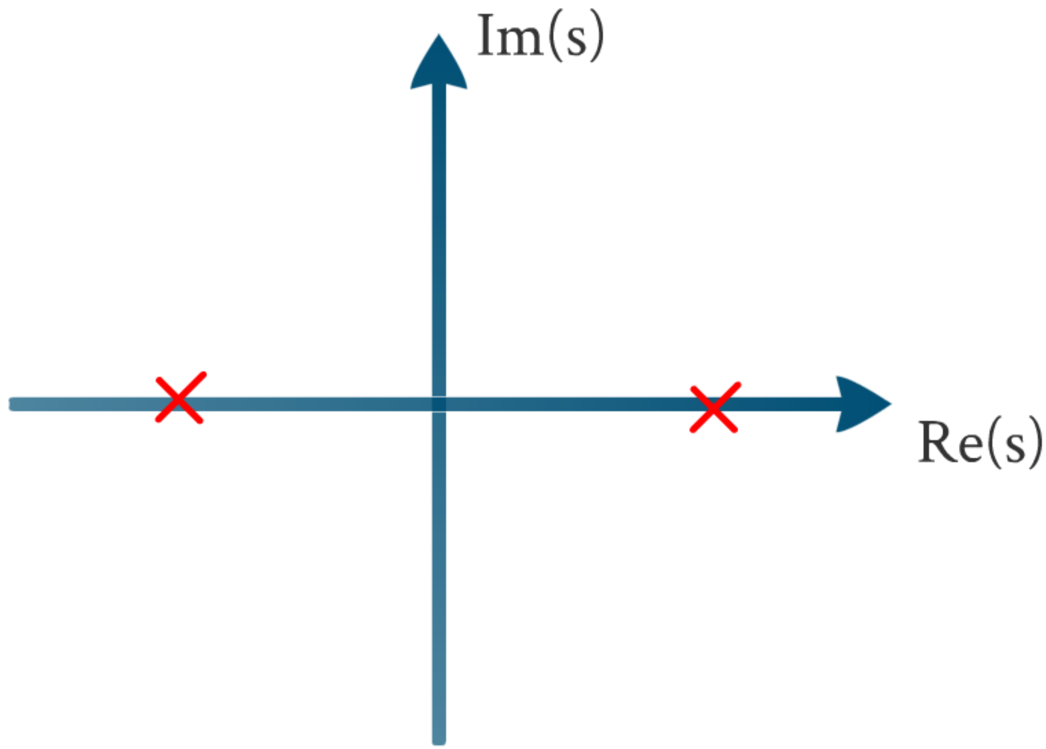
s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0 \rightarrow 4$	$-\frac{1}{1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \rightarrow 4$	0
s^2	$-\frac{1}{4}\begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = 1$	$-\frac{1}{4}\begin{vmatrix} 1 & 1 \\ 4 & 0 \end{vmatrix} = 1$	0
s^1			
s^0			

Quadrantal symmetric roots



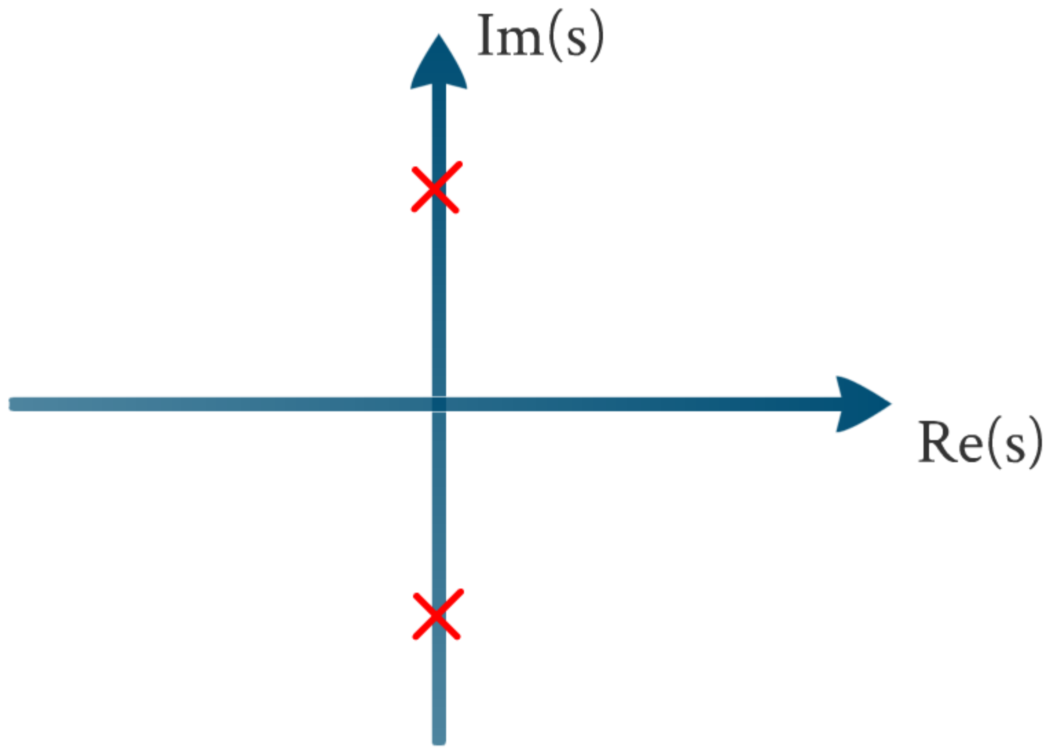
Includes unstable roots, so causes a sign change

Purely Real Roots



Includes unstable root, so causes a sign change

Purely Imaginary Roots



Purely imaginary roots do not cause a sign change

Example 4: Row of Zeros

s^5	1	2	1
s^4	1	2	1
s^3			
s^2			
s^1			
s^0			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix}1 & 2\\ 1 & 2\end{vmatrix} = 0$	$-\frac{1}{1}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} = 0$	0
s^2			
s^1			
s^0			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix}1 & 2\\ 1 & 2\end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} = 0 \to 4$	0
s^2			
s^1			
s^0			

$$\frac{d}{ds}\left(s^4 + 2s^2 + 1\right) = 4s^3 + 4s$$

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix}1 & 2\\ 1 & 2\end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} = 0 \to 4$	0
s^2	$-\frac{1}{4}\begin{vmatrix}1 & 2\\ 4 & 4\end{vmatrix} = 1$	$-\frac{1}{4}\begin{vmatrix}1 & 1\\ 4 & 0\end{vmatrix} = 1$	0
s^1			
s^0			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix}1 & 2\\ 1 & 2\end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} = 0 \to 4$	0
s^2	$-\frac{1}{4}\begin{vmatrix}1 & 2\\ 4 & 4\end{vmatrix} = 1$	$-\frac{1}{4}\begin{vmatrix}1 & 1\\ 4 & 0\end{vmatrix} = 1$	0
s^1	$-\frac{1}{1}\begin{vmatrix}4 & 4\\ 1 & 1\end{vmatrix} = \epsilon$	0	0
s^0			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix}1 & 2\\ 1 & 2\end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} = 0 \to 4$	0
s^2	$-\frac{1}{4}\begin{vmatrix}1 & 2\\ 4 & 4\end{vmatrix} = 1$	$-\frac{1}{4}\begin{vmatrix}1 & 1\\ 4 & 0\end{vmatrix} = 1$	0
s^1	$-\frac{1}{1}\begin{vmatrix}4 & 4\\ 1 & 1\end{vmatrix} = \epsilon$	0	0
s^0	$-\frac{1}{\epsilon}\begin{vmatrix}1 & 1\\ \epsilon & 0\end{vmatrix} = 1$	0	0

s^5	1	2	1
s^4	1	2	1
s^3			
s^2			
s^1			
s^0			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1} \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0$	$-\frac{1}{1} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$	0
s^2			
s^1			
s^0			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix}1 & 2\\1 & 2\end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix}1 & 1\\1 & 1\end{vmatrix} = 0 \to 4$	0
s^2			
s^1			
s^0			

$$\frac{d}{ds}\left(s^4 + 2s^2 + 1\right) = 4s^3 + 4s$$

ow of Zeros

s^5	1	2	1
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s^3	$-\frac{1}{1}\begin{vmatrix}1 & 2\\ 1 & 2\end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix}1 & 1\\ 1 & 1\end{vmatrix} = 0 \to 4$	0
s^2	$-\frac{1}{4}\begin{vmatrix}1 & 2\\ 4 & 4\end{vmatrix} = 1$	$-\frac{1}{4}\begin{vmatrix}1 & 1\\ 4 & 0\end{vmatrix} = 1$	0
s^1			
s^0			

s^5	1	2	1

s			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \to 4$	0
s^2	$-\frac{1}{4}\begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = 1$	$-\frac{1}{4}\begin{vmatrix} 1 & 1 \\ 4 & 0 \end{vmatrix} = 1$	0
s^1	$-\frac{1}{1}\begin{vmatrix} 4 & 4 \\ 1 & 1 \end{vmatrix} = \epsilon$	0	0
s^0			

s^5	1	2	1
s^4	1	2	1
s^3	$-\frac{1}{1}\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = 0 \to 4$	$-\frac{1}{1}\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0 \to 4$	0
s^2	$-\frac{1}{4}\begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = 1$	$-\frac{1}{4}\begin{vmatrix} 1 & 1 \\ 4 & 0 \end{vmatrix} = 1$	0
s^1	$-\frac{1}{1}\begin{vmatrix} 4 & 4 \\ 1 & 1 \end{vmatrix} = \epsilon$	0	0
s^0	$-\frac{1}{\epsilon}\begin{vmatrix} 1 & 1 \\ \epsilon & 0 \end{vmatrix} = 1$	0	0

Routh-Hurwitz Stability Criteria

$$(s+r_1)(s+r_2)\ldots(s+r_n) = s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 = 0$$

$$r_i > 0 \quad \forall i \qquad\qquad a_i > 0 \quad \forall i$$

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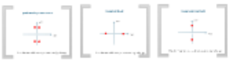

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Example 5: Parameterised

s^2	1	K
s^1	$0 \to 2$	$0 \to 0$
s^0		

s^2	1	K
s^1	$0 \to 2$	$0 \to 0$
s^0	$-\frac{1}{2} \begin{array}{cc} 1 & K \\ 2 & 0 \end{array} = K$	0

s^2	1	K
s^1	$0 \rightarrow 2$	$0 \rightarrow 0$
s^0		

<table>
<tr><td>s^2</td><td colspan="3">1</td><td>K</td></tr>
<tr><td>s^1</td><td colspan="3">$0 \to 2$</td><td>$0 \to 0$</td></tr>
<tr><td>s^0</td><td>$-\frac{1}{2}$</td><td>$\begin{matrix} 1 & K \\ 2 & 0 \end{matrix}$</td><td>$= K$</td><td>$0$</td></tr>
</table>

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s^2	1	K
s^1	$0 \to 2$	$0 \to 0$
s^0		

<table>
<tr><td>s^2</td><td colspan="3">1</td><td>K</td></tr>
<tr><td>s^1</td><td colspan="3">$0 \to 2$</td><td>$0 \to 0$</td></tr>
<tr><td>s^0</td><td>$-\frac{1}{2}$</td><td>$\begin{matrix} 1 & K \\ 2 & 0 \end{matrix}$</td><td>$= K$</td><td>$0$</td></tr>
</table>

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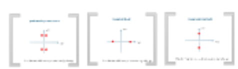
- So, no sign change in the first column of Routh table implies the system is stable.
- A sign change implies that the system is unstable.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.
- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system unstable.
- Multiple distinct roots on the imaginary axis make the system marginally stable.

This lecture covers:

- Stability and Instability
- Relationship between poles' location and stability
- Routh-Hurwitz stability criteria