ELEC 207 Part B

Control Theory Lecture 14: Frequency Design (2)

Prof Simon Maskell
CHAD-G68
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0151 794 4573

Design Process

Transient Response using Gain Adjustment and Bode Plots

Second Order Systems

Open loop:

Closed loop:

Peak Time and Settling Time

Phase Margin

Design Process

Designing Phase-lag Compensators using Bode Plots

$$G(s) = \frac{s+z}{s+p}$$

Design Process

Designing Phase-lead Compensators using Bode Plots

$$G(s) = \frac{1}{\beta}\frac{s+z}{s+\frac{1}{\beta}}$$

Design Process

This lecture covers:
- Optimising controller parameters

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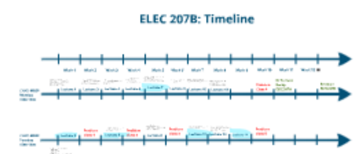
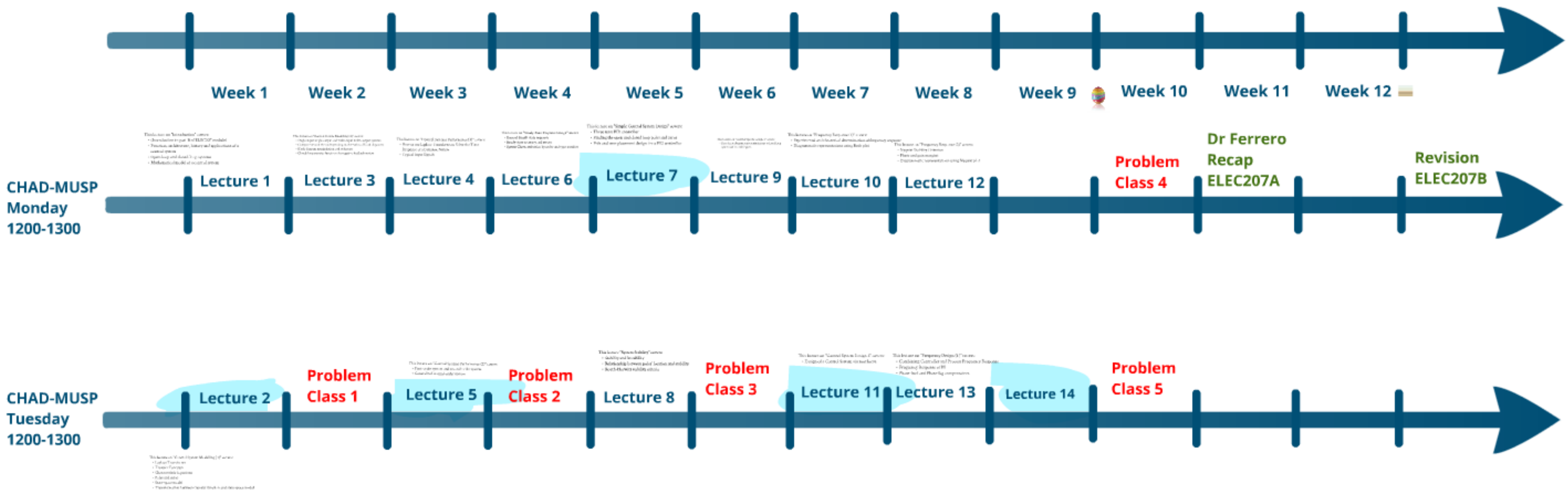
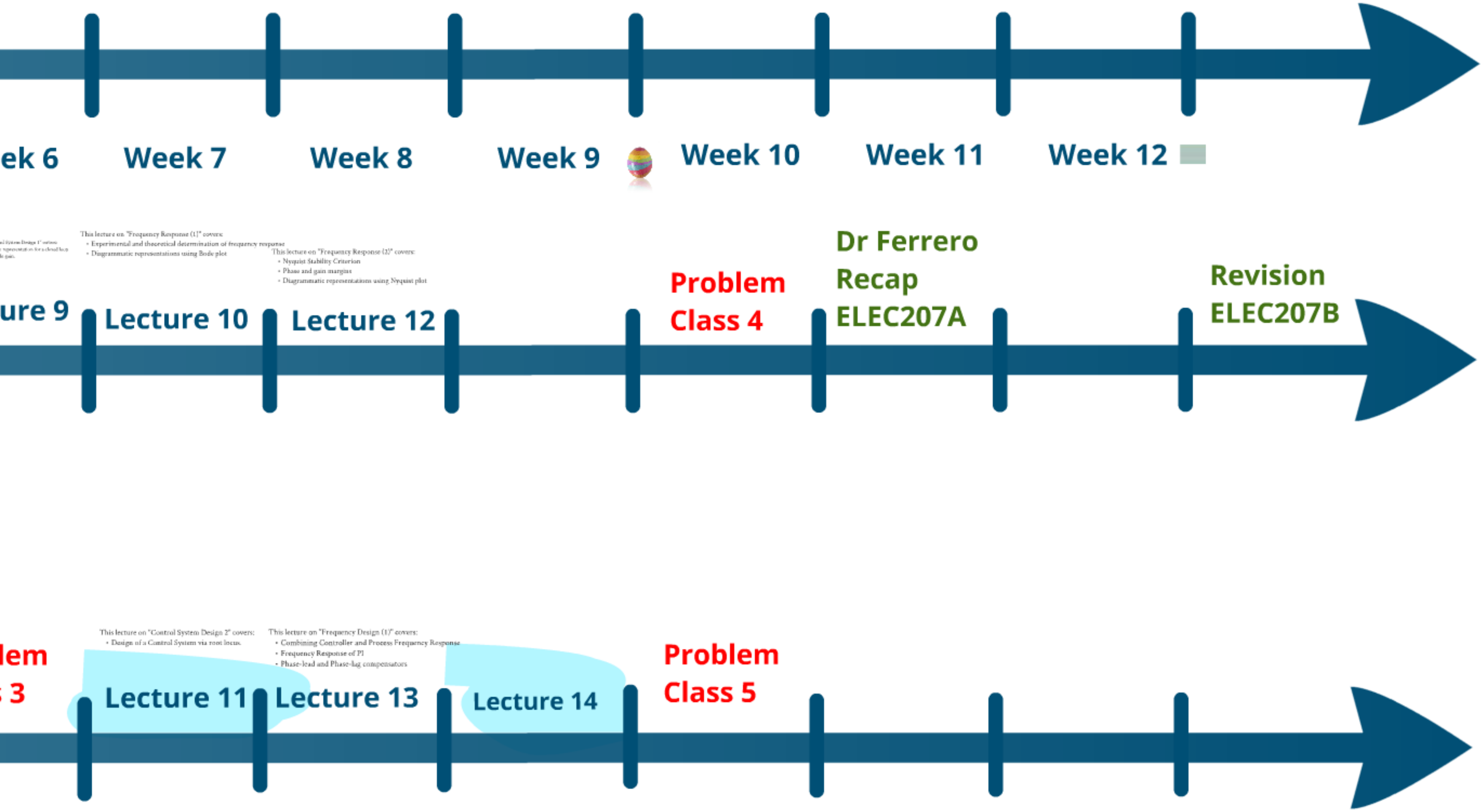


ELEC 207B: Timeline




ek 6
Week 7
Week 8
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Week 12
This lecture on "Frequency Response (1)" covers:
• Experimental and theoretical determination of frequency response
• Diagrammatic representations using Bode plot
This lecture on "Frequency Response (2)" covers:
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• Phase and gain margins
• Diagrammatic representations using Nyquist plot
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Lecture 10
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Problem Class 4
Dr Ferrero Recap ELEC207A
Revision ELEC207B
This lecture on "Control System Design 2" covers:
• Design of a Control System via root locus.
This lecture on "Frequency Design (1)" covers:
• Combining Controller and Process Frequency Response
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• Phase-lead and Phase-lag compensators
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Lecture 11
Lecture 13
Lecture 14
Problem Class 5


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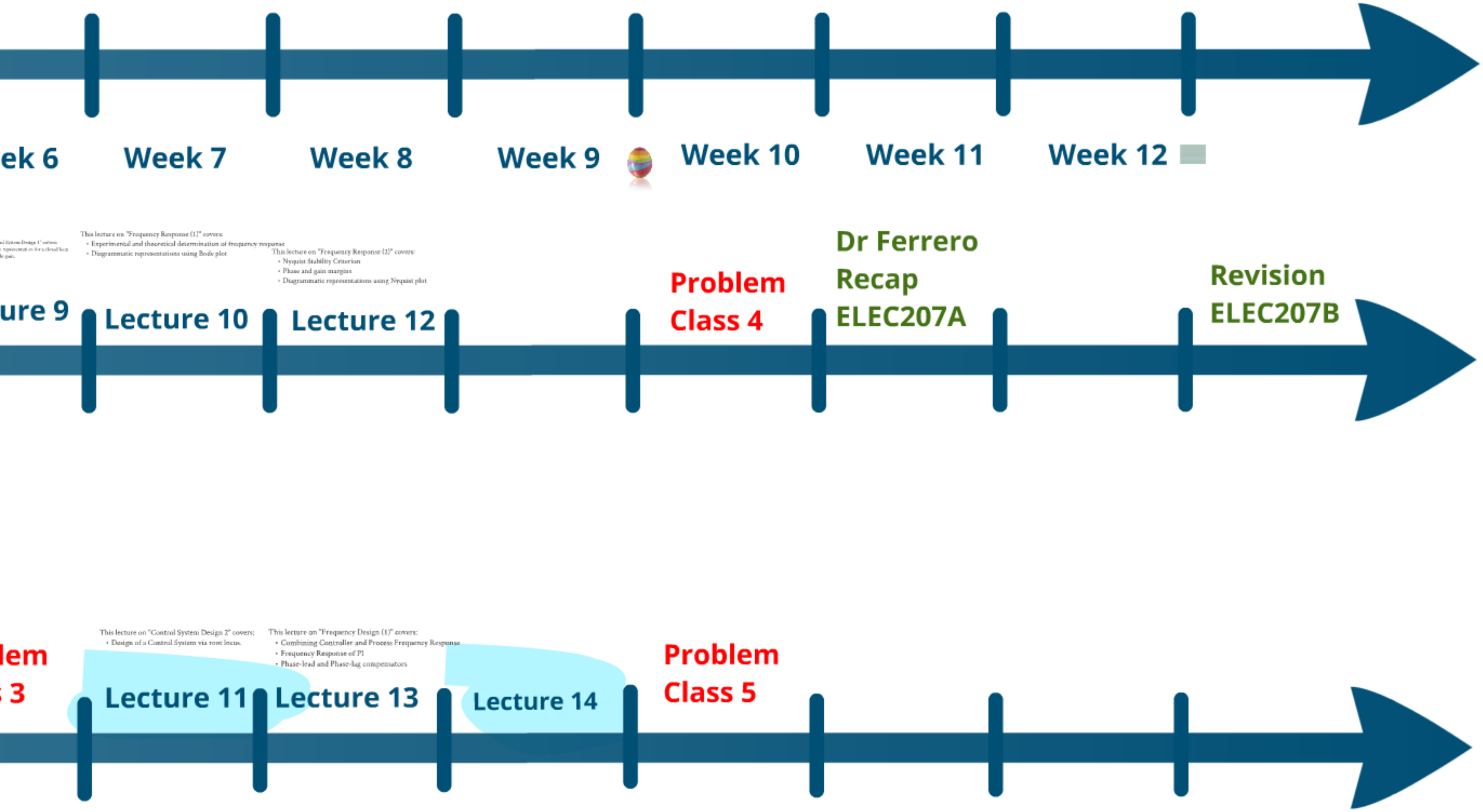
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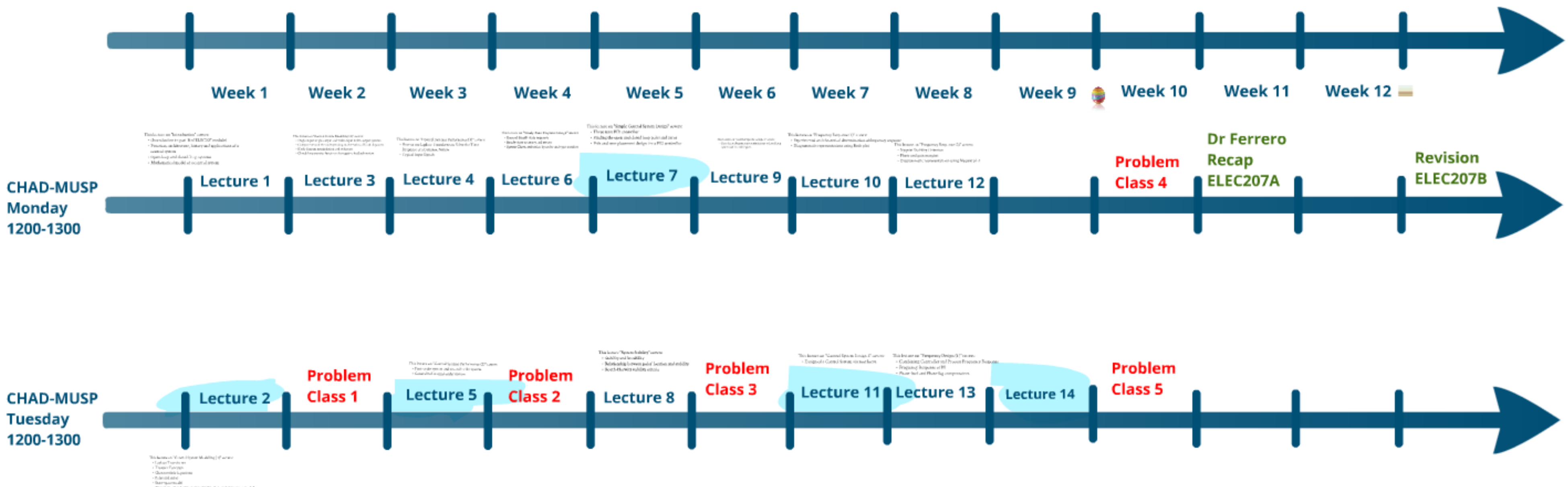

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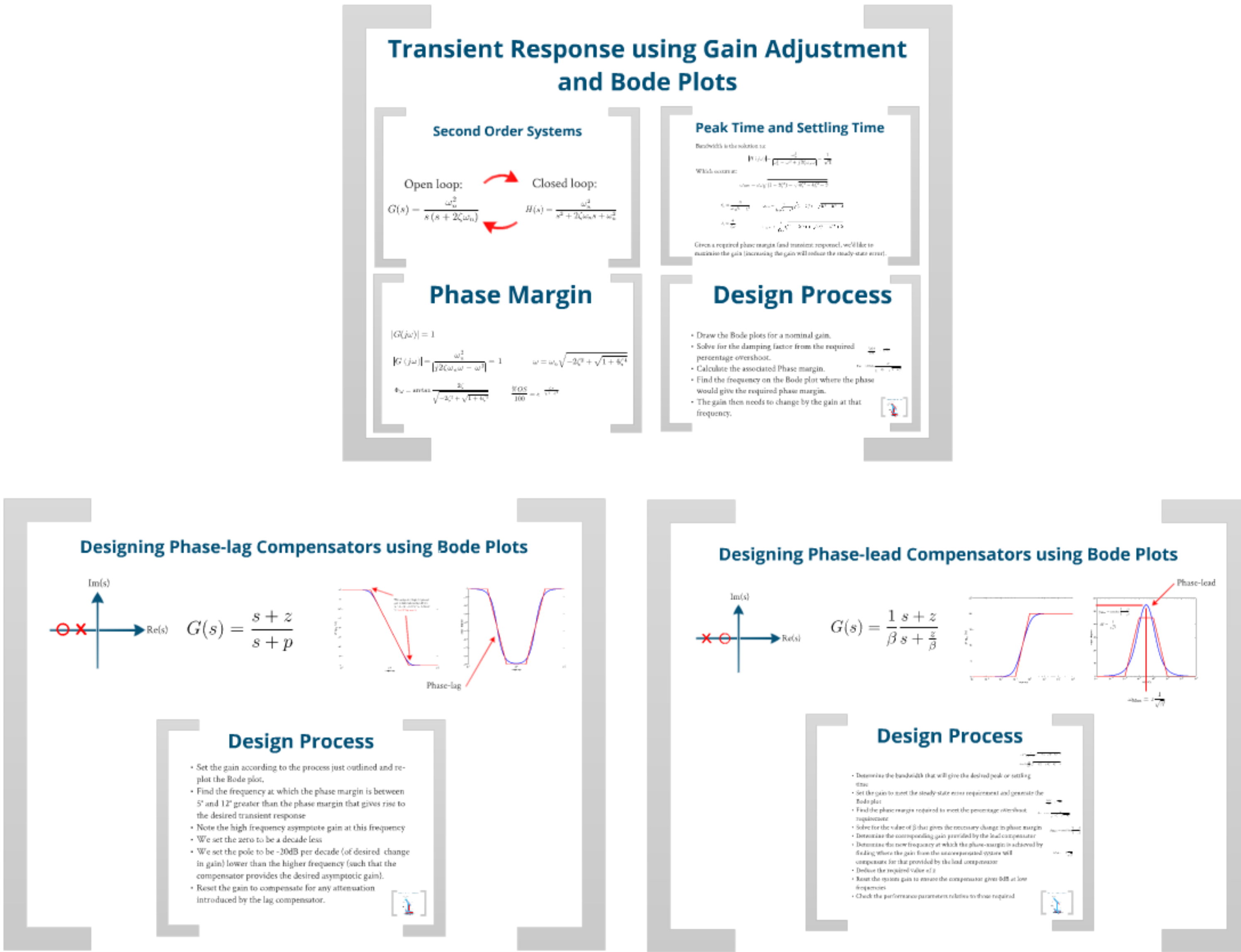


Transient Response using Gain Adjustment and Bode Plots

Second Order Systems

Open loop:

$$G(s) = \frac{\omega_n^2}{s\left(s + 2\zeta\omega_n\right)}$$

Closed loop:

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Peak Time and Settling Time

Bandwidth is the solution to:

$$|H(j\omega)| = \frac{\omega_n^2}{|\omega_n^2 - \omega^2 + j2\zeta\omega_n\omega|} = \frac{1}{\sqrt{2}}$$

Which occurs at:

$$\omega_{\mathrm{BW}} = \omega_n\sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$t_p = \frac{\pi}{\omega\sqrt{1-\zeta^2}} \qquad \omega_{\mathrm{BW}} = \frac{\pi}{t_p\sqrt{1-\zeta^2}}\sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

$$t_s \approx \frac{4}{\zeta\omega} \qquad \omega_{\mathrm{BW}} \approx \frac{4}{\zeta t_s}\sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}}$$

Given a required phase margin (and transient response), we'd like to maximise the gain (increasing the gain will reduce the steady-state error).

Phase Margin

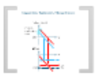
$$|G(j\omega)| = 1$$

$$|G(j\omega)| = \frac{\omega_n^2}{|j2\zeta\omega_n\omega - \omega^2|} = 1 \qquad \omega = \omega_n\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}$$

$$\Phi_M = \arctan\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} \qquad \frac{\%OS}{100} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

Design Process

- Draw the Bode plots for a nominal gain.
- Solve for the damping factor from the required percentage overshoot. $\frac{\%OS}{100} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$
- Calculate the associated Phase margin. $\Phi_M = \arctan\frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}}$
- Find the frequency on the Bode plot where the phase would give the required phase margin.
- The gain then needs to change by the gain at that frequency.



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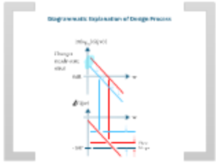
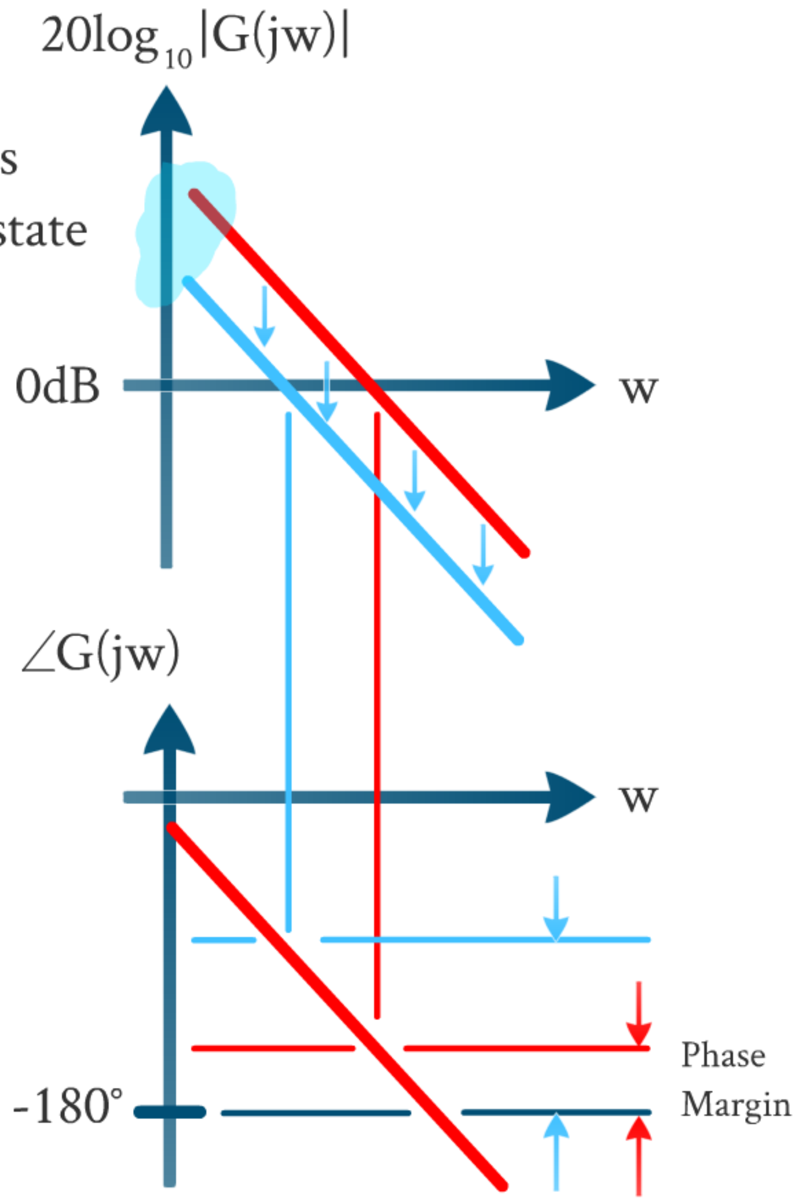


Diagrammatic Explanation of Design Process



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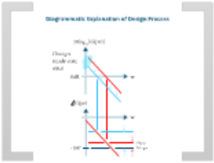

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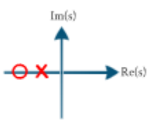
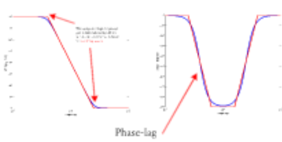
Designing Phase-lag Compensators using Bode Plots



$$G(s) = \frac{s + z}{s + p}$$



Design Process

- Set the gain according to the process just outlined and re-plot the Bode plot.
- Find the frequency at which the phase margin is between 5° and 12° greater than the phase margin that gives rise to the desired transient response
- Note the high frequency asymptote gain at this frequency
- We set the zero to be a decade less
- We set the pole to be -20dB per decade (of desired change in gain) lower than the higher frequency (such that the compensator provides the desired asymptotic gain).
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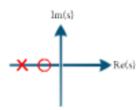
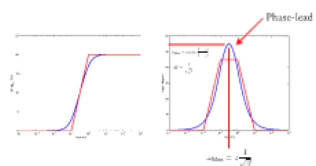
Designing Phase-lead Compensators using Bode Plots



$$G(s) = \frac{1}{\beta}\frac{s + z}{s + \frac{z}{\beta}}$$



Design Process

- Determine the bandwidth that will give the desired peak or settling time
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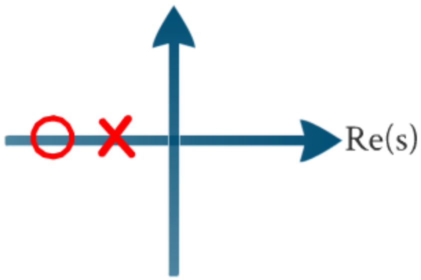
Designing Phase-lag Compensators using Bode Plots



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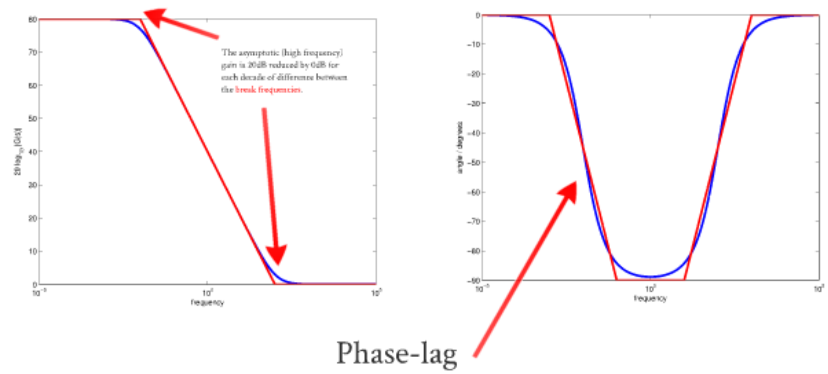

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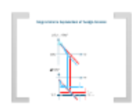
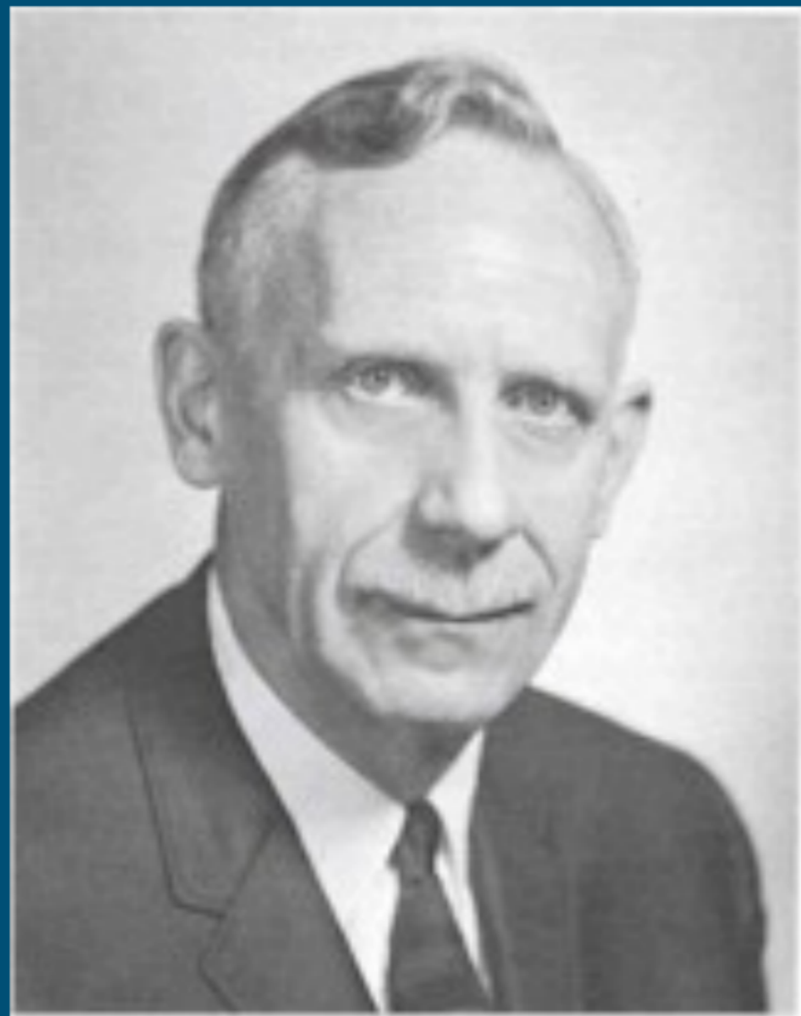
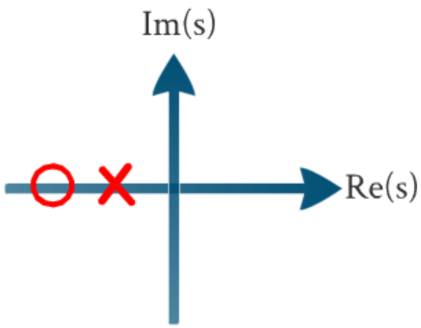


Bode



Designing Phase-lag Compensators using Bode Plots



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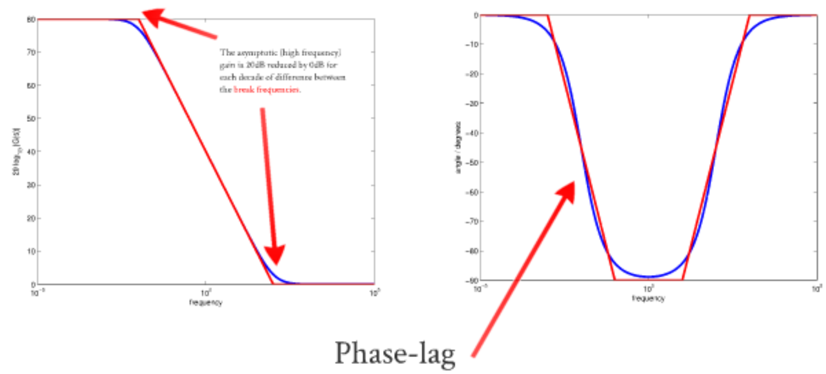

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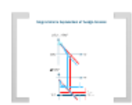
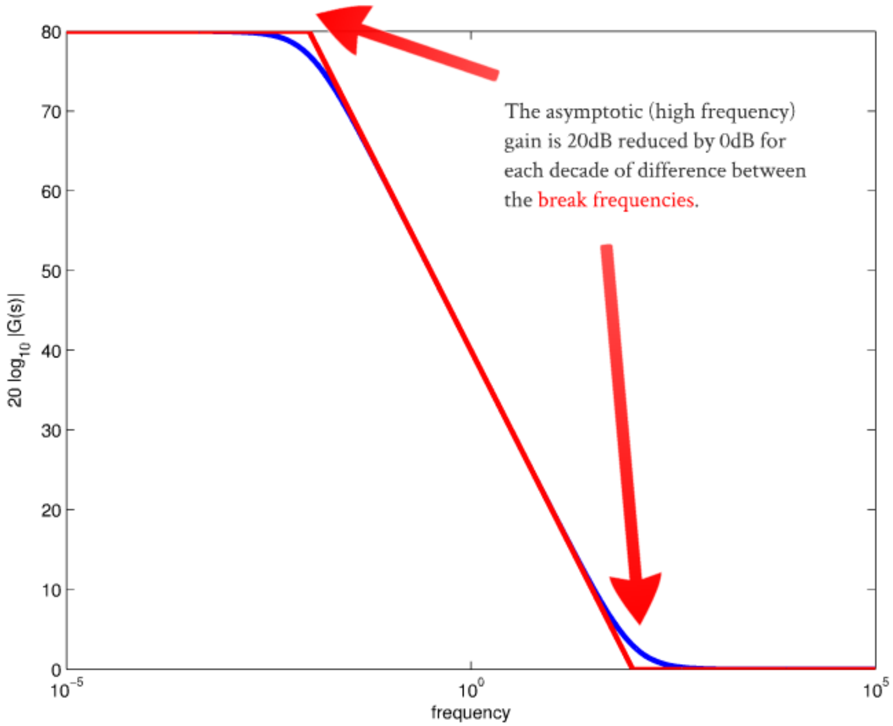
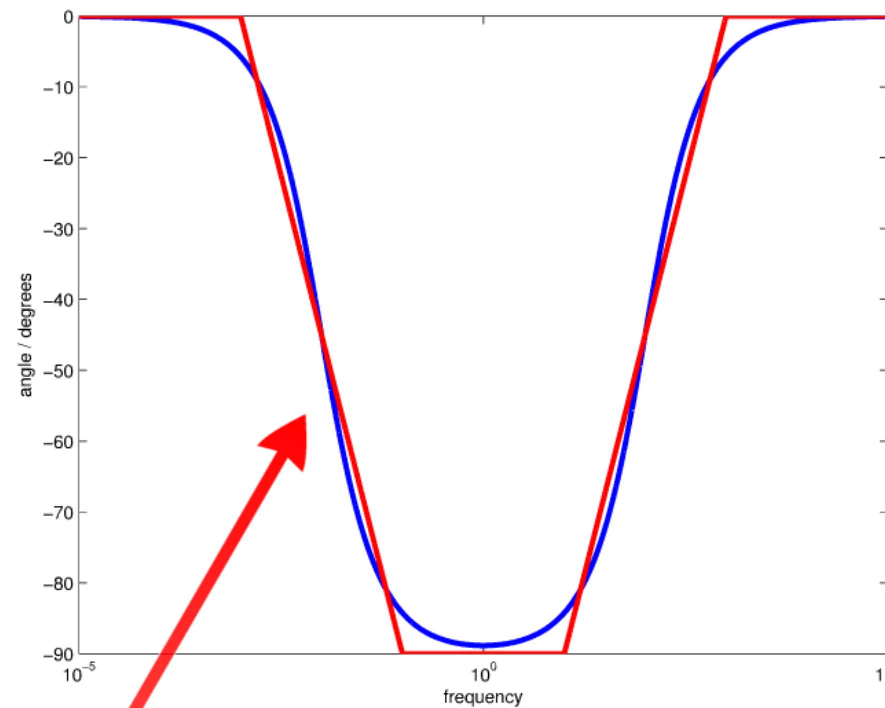
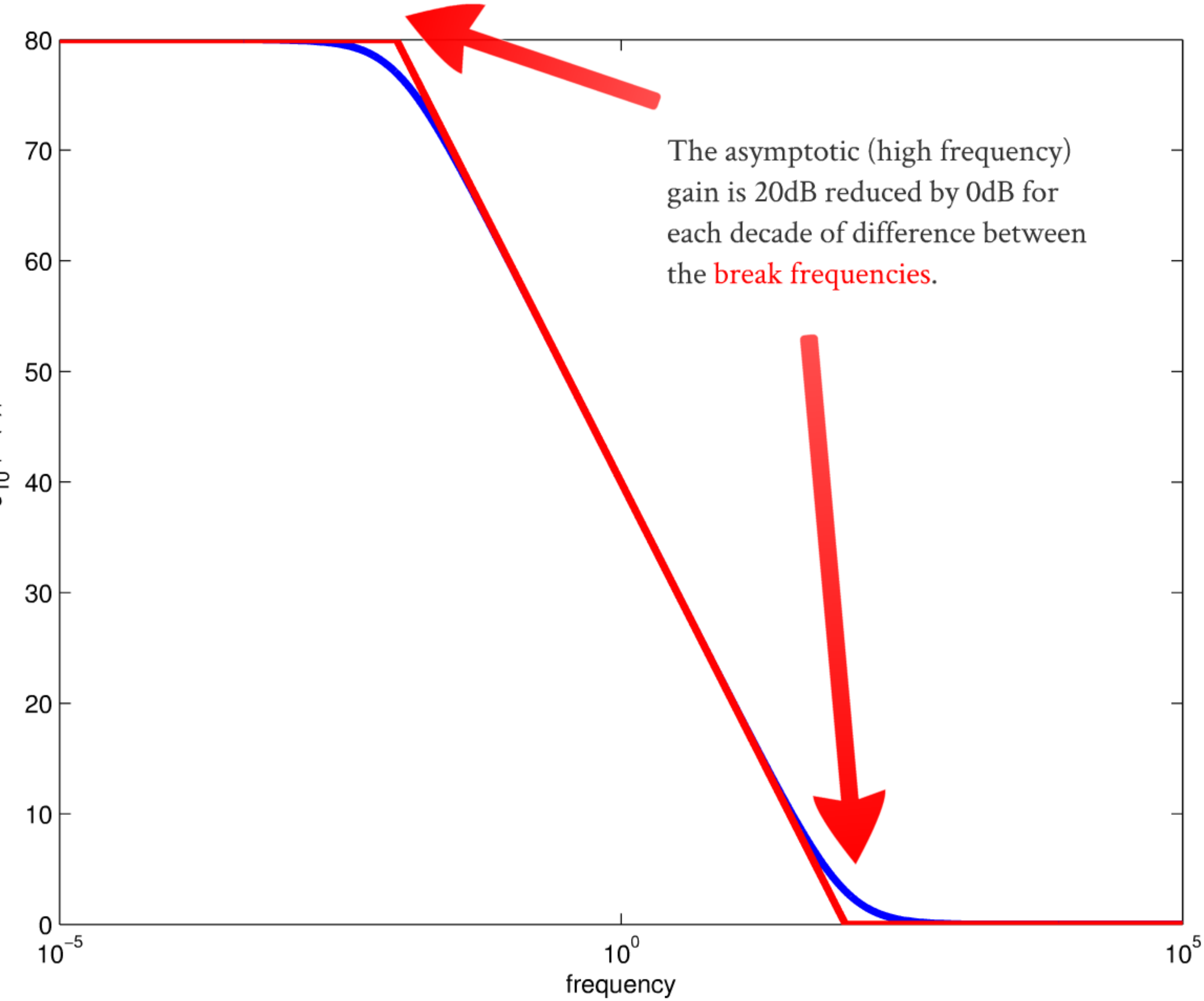



80
70
60
50
40
30
20
10
0
$20 \log_{10} |G(s)|$
10^{-5}
10^{0}
10^{5}
frequency
The asymptotic (high frequency) gain is 20dB reduced by 0dB for each decade of difference between the break frequencies.


0
−10
−20
−30
−40
−50
−60
−70
−80
−90
angle / degrees
10^{-5}
10^{0}
frequency

Phase-lag


80
70
60
50
40
30
20
10
0
10^{-5}
10^{0}
10^{5}
frequency
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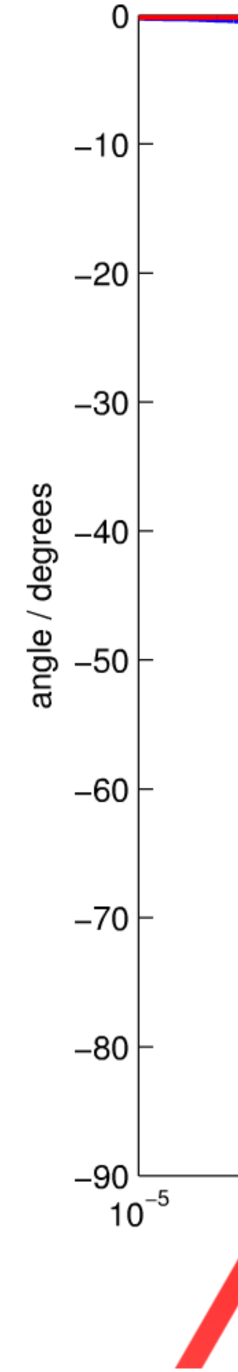
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angle / degrees

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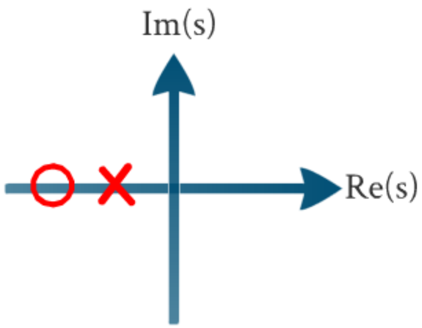

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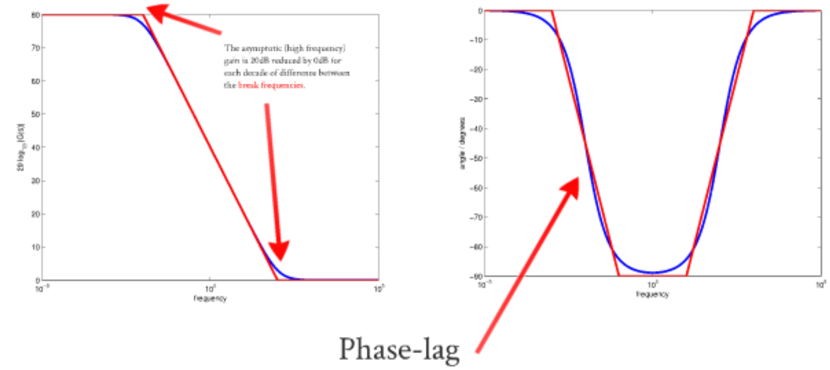

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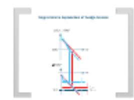

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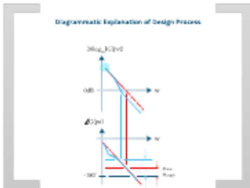
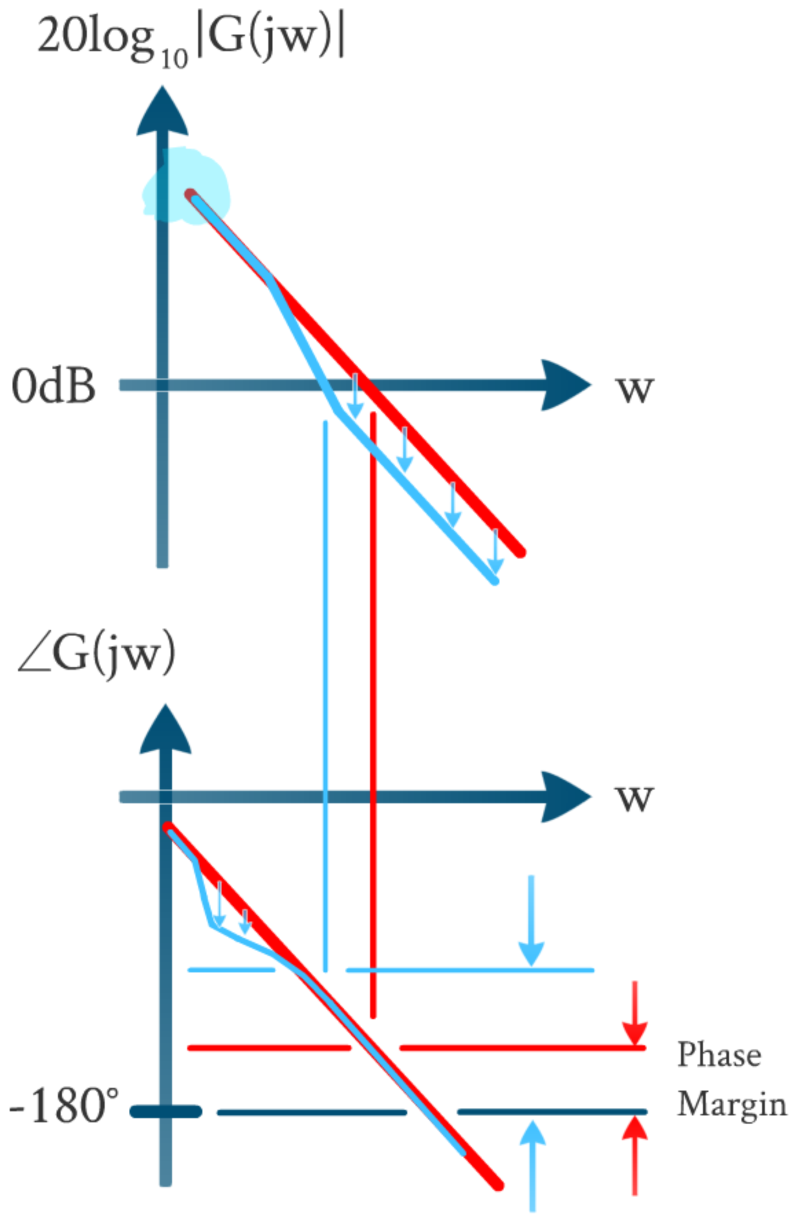


Diagrammatic Explanation of Design Process



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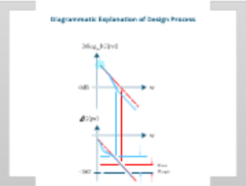
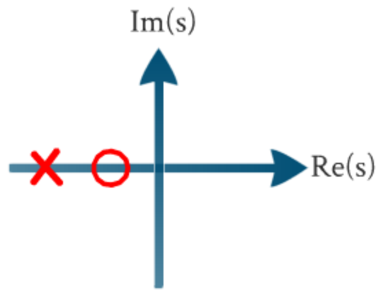


Designing Phase-lead Compensators using Bode Plots



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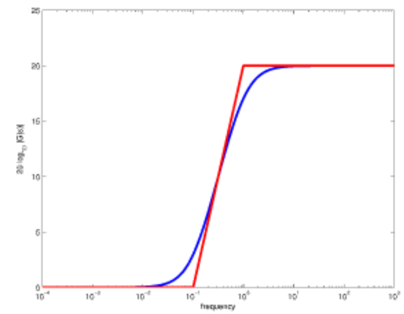

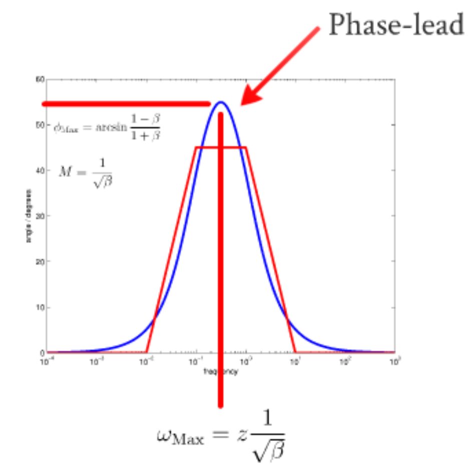

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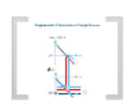
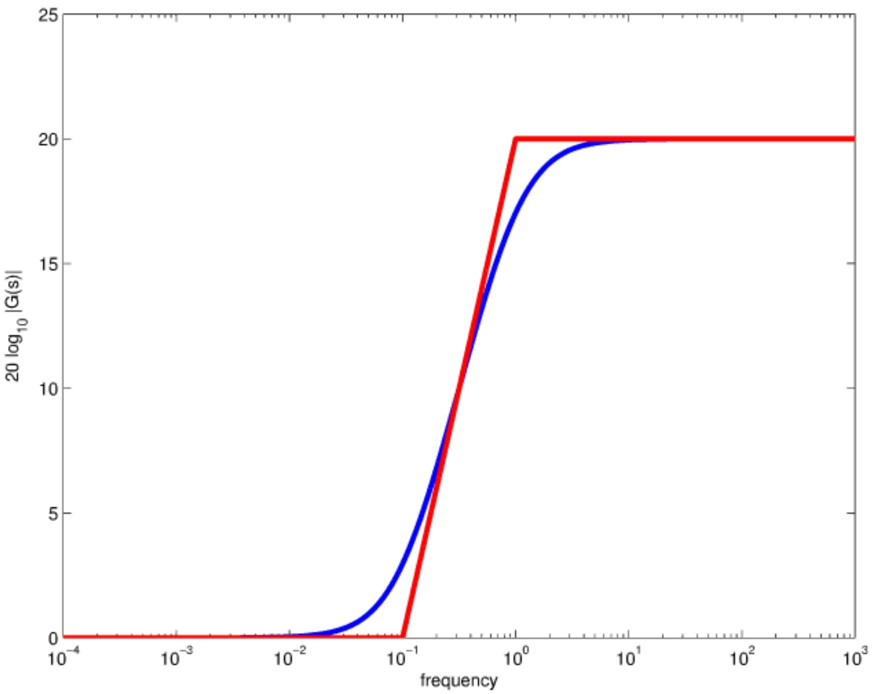
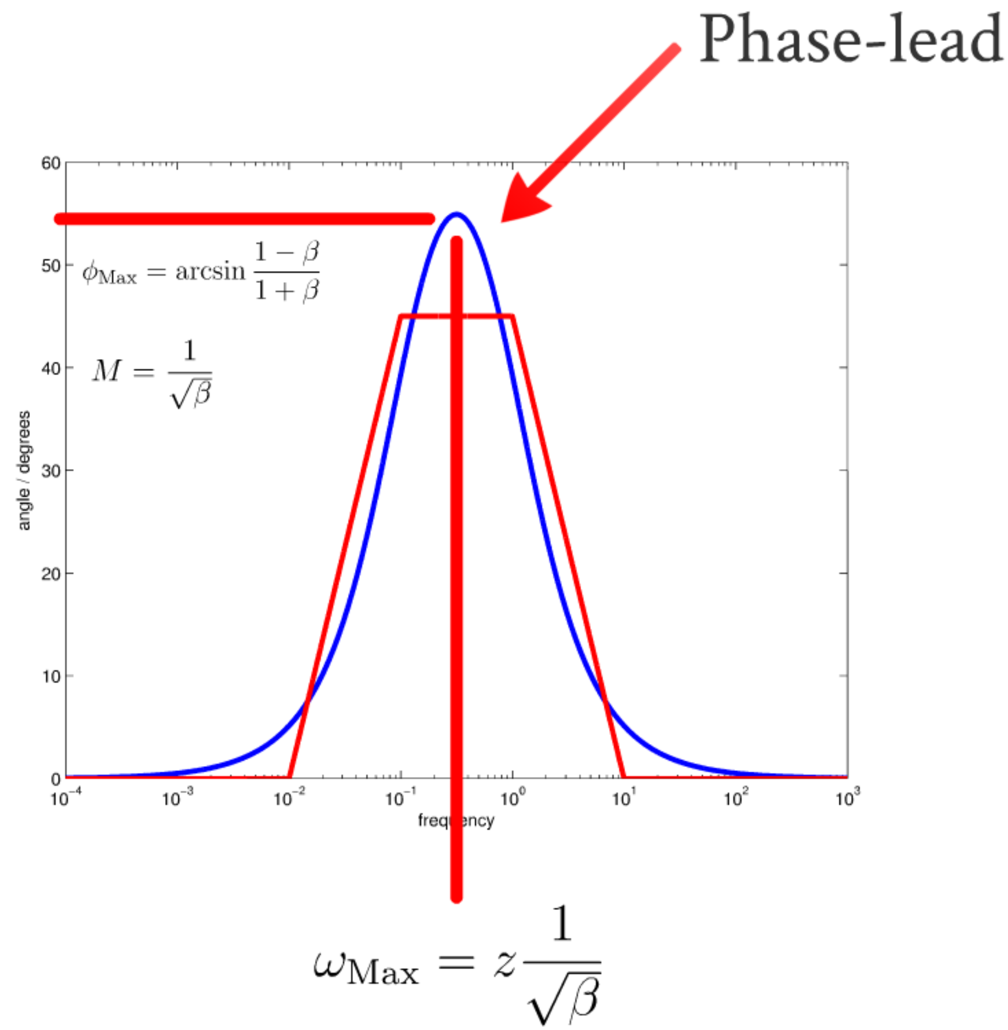


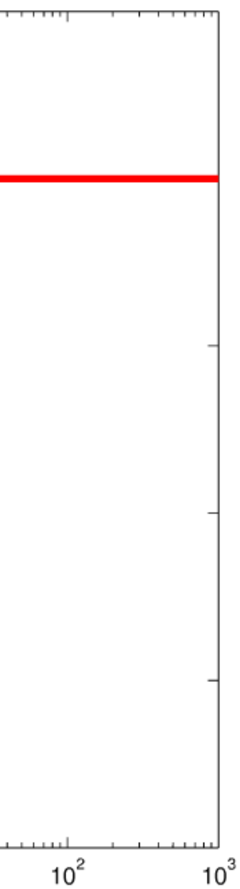
ensators using Bode Plots

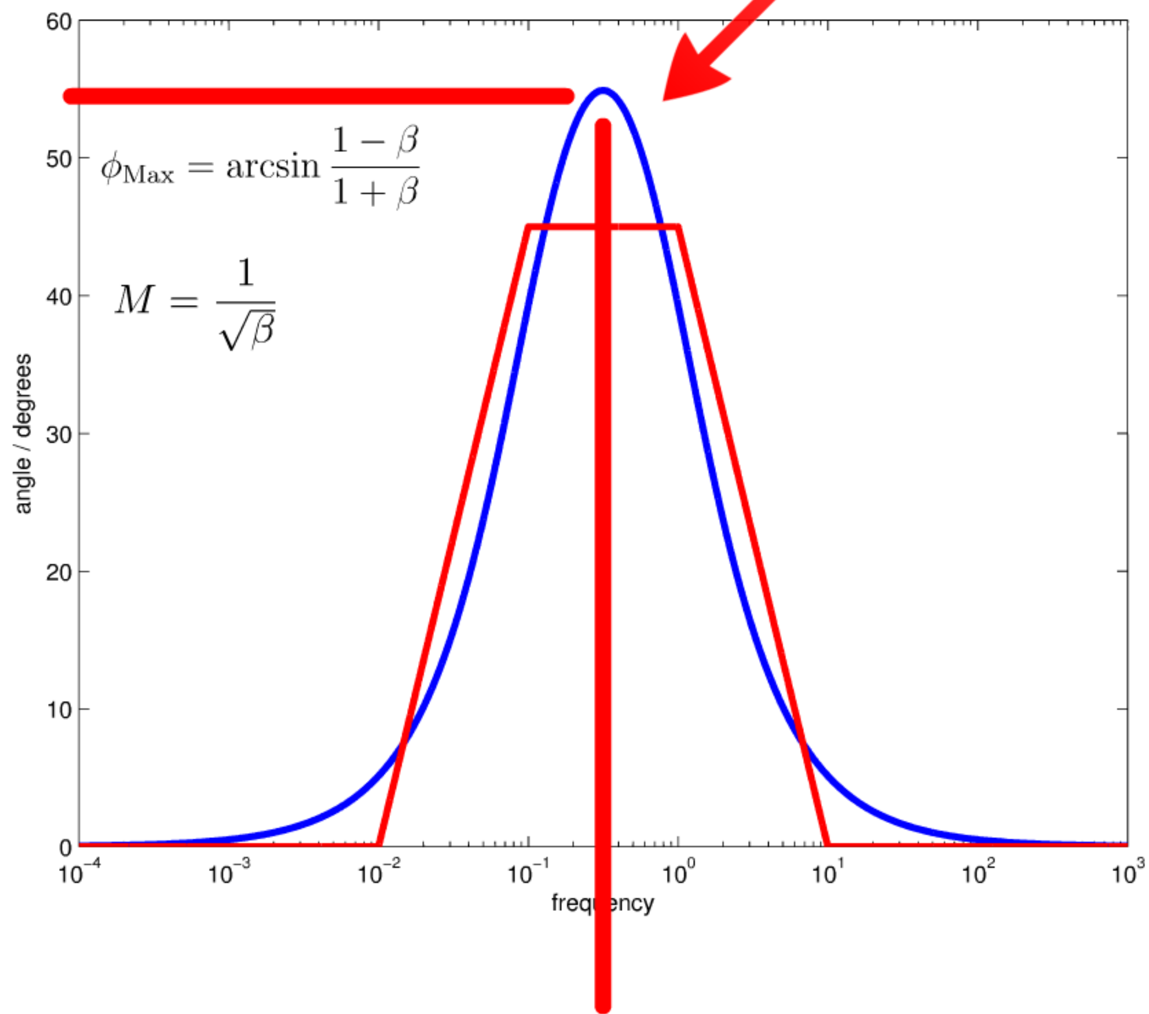




Process

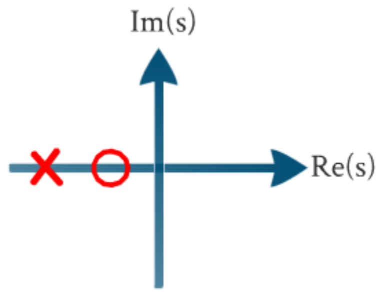
Phase-lead



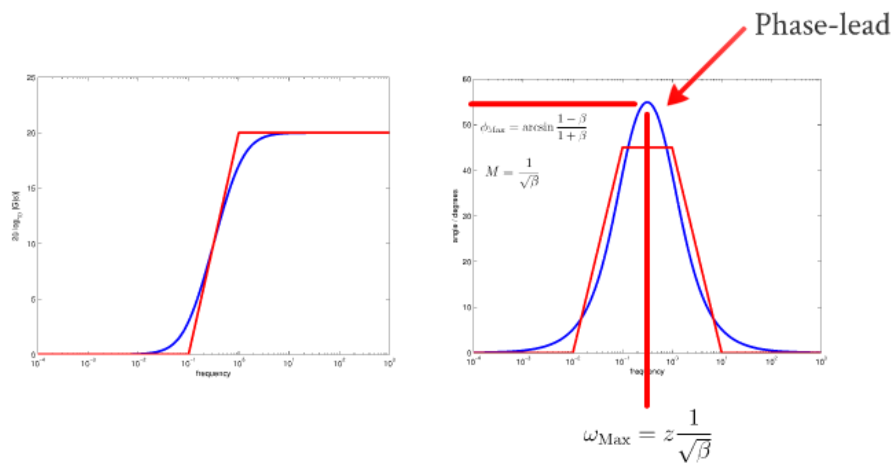


$$\omega_{\mathrm{Max}} = z\frac{1}{\sqrt{\beta}}$$

Designing Phase-lead Compensators using Bode Plots

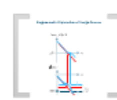


$$G(s) = \frac{1}{\beta}\frac{s+z}{s+\frac{z}{\beta}}$$



Design Process

- Determine the bandwidth that will give the desired peak or settling time
- Set the gain to meet the steady-state error requirement and generate the Bode plot
- Find the phase margin required to meet the percentage overshoot requirement
- Solve for the value of β that gives the necessary change in phase margin $\phi_{\mathrm{Max}} = \arcsin\frac{1-\beta}{1+\beta}$
- Determine the corresponding gain provided by the lead compensator
- Determine the new frequency at which the phase-margin is achieved by finding where the gain from the uncompensated system will compensate for that provided by the lead compensator $\omega_{\mathrm{Max}} = z\frac{1}{\sqrt{\beta}}$
- Deduce the required value of z
- Reset the system gain to ensure the compensator gives 0dB at low frequencies
- Check the performance parameters relative to those required



Design Process

$$\omega_{BW} = \frac{\pi}{t_p\sqrt{1-\zeta^2}}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^4+2}}$$

$$\omega_{BW} \approx \frac{4}{\zeta t_s}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^4+2}}$$

- Determine the bandwidth that will give the desired peak or settling time
- Set the gain to meet the steady-state error requirement and generate the Bode plot

$$\frac{\%OS}{100} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$$\Phi_M = \arctan\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}}$$

- Find the phase margin required to meet the percentage overshoot requirement
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$$\phi_{Max} = \arcsin\frac{1-\beta}{1+\beta}$$

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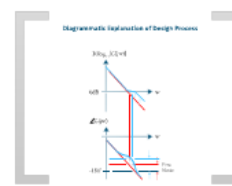
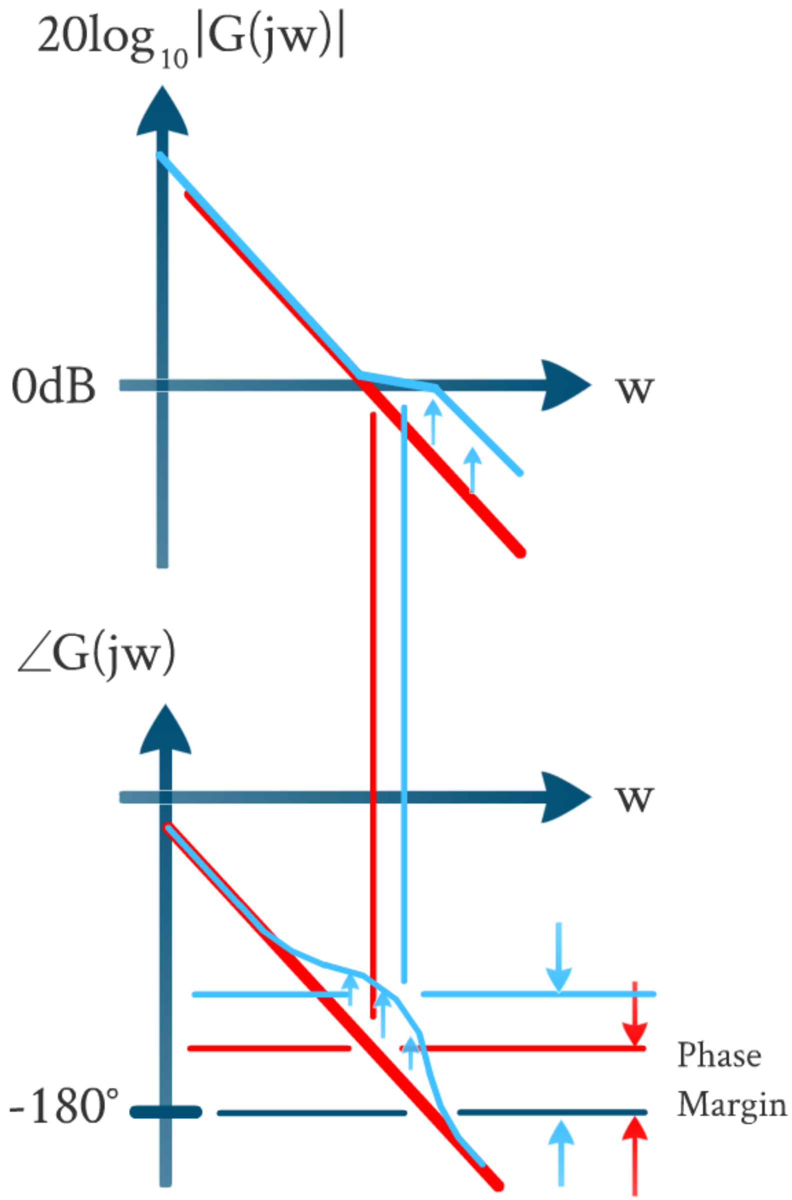


Diagrammatic Explanation of Design Process



Design Process

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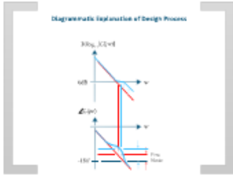
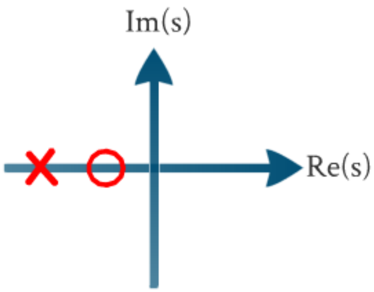


Designing Phase-lead Compensators using Bode Plots



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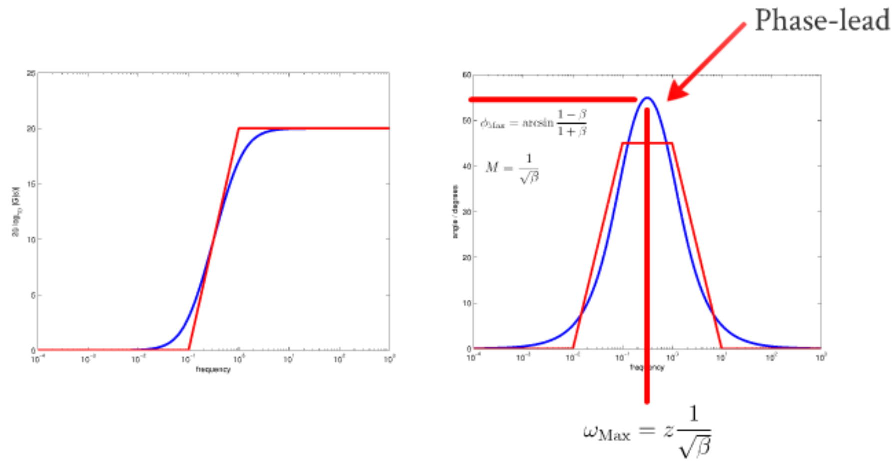

Design Process

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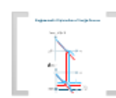


Design Process

Transient Response using Gain Adjustment and Bode Plots

Second Order Systems

Open loop:

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

Closed loop:

$$H(s) = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$$

Peak Time and Settling Time

Phase Margin

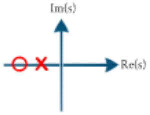
$$|G(j\omega)| = 1$$

$$\omega = \omega_n\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}$$

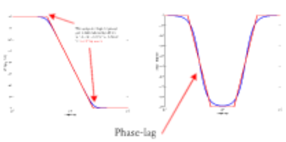
Design Process

- Draw the Bode plots for a nominal gain.
- Solve for the damping factor from the required percentage overshoot.
- Calculate the associated Phase margin.
- Find the frequency on the Bode plot where the phase would give the required phase margin.
- The gain then needs to change by the gain at that frequency.

Designing Phase-lag Compensators using Bode Plots



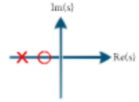
$$G(s) = \frac{s+z}{s+p}$$



Design Process

- Set the gain according to the process just outlined and re-plot the Bode plot.
- Find the frequency at which the phase margin is between 5° and 12° greater than the phase margin that gives rise to the desired transient response
- Note the high frequency asymptote gain at this frequency
- We set the zero to be a decade less
- We set the pole to be -20dB per decade (of desired change in gain) lower than the higher frequency (such that the compensator provides the desired asymptotic gain).
- Reset the gain to compensate for any attenuation introduced by the lag compensator.

Designing Phase-lead Compensators using Bode Plots



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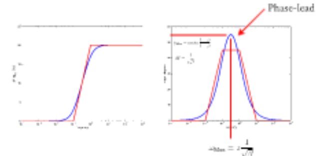

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This lecture covers:

- Optimising controller parameters

ELEC 207B: Timeline