ELEC 207 Part B

Control Theory Lecture 10: Control System Design (2)

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UNIVERSITY OF LIVERPOOL

Compensators

Cascade Compensators

Feedback Compensators

This lecture covers:

- Design of a Control System via root locus.

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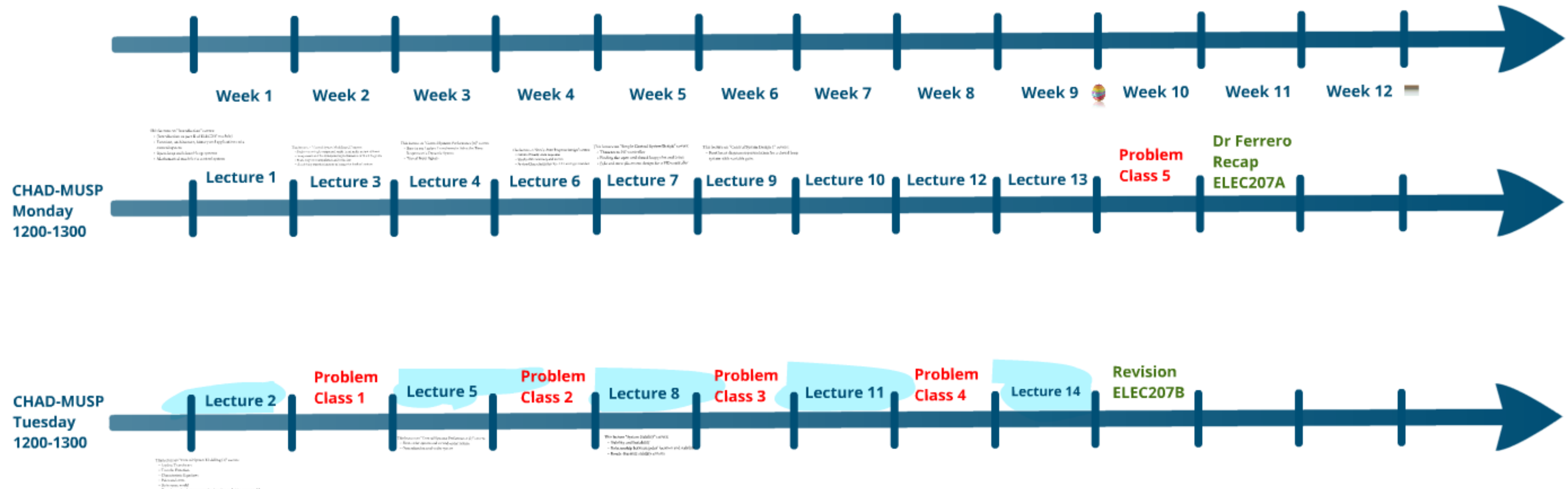
ELEC 207B: Timeline



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- Design of a Control System via root locus.

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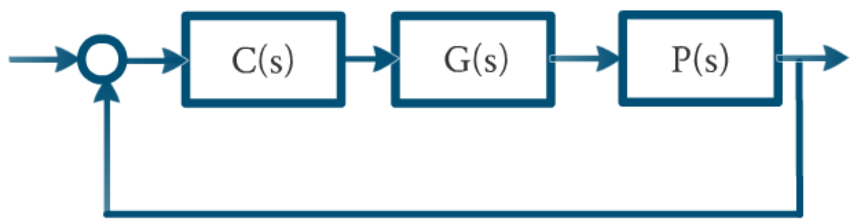
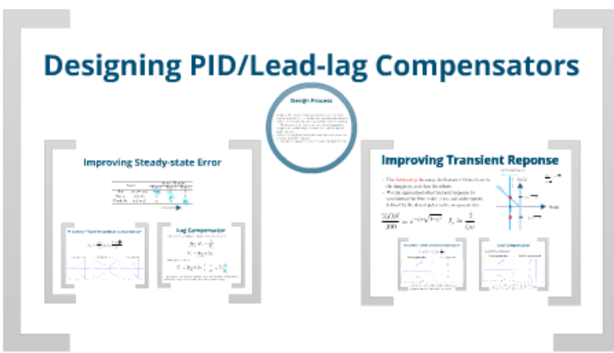
Cascade Compensators





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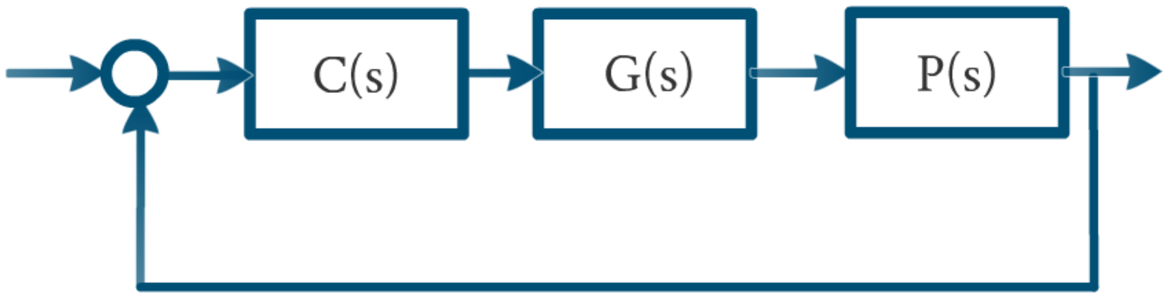

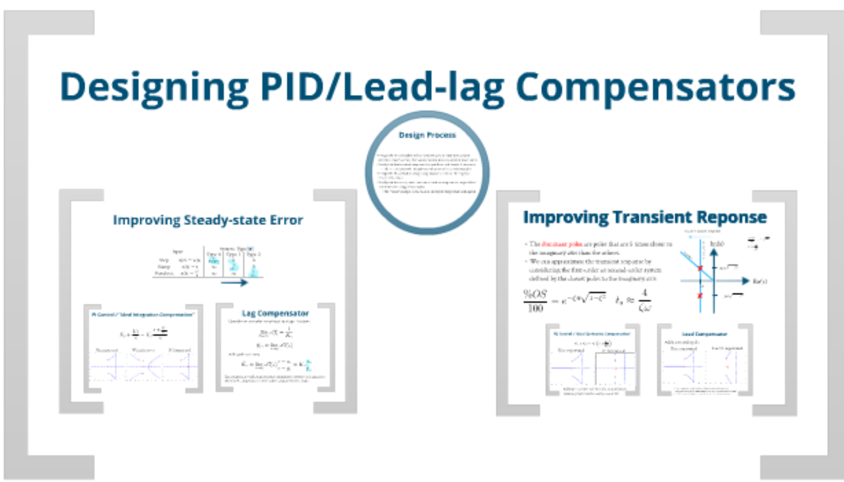

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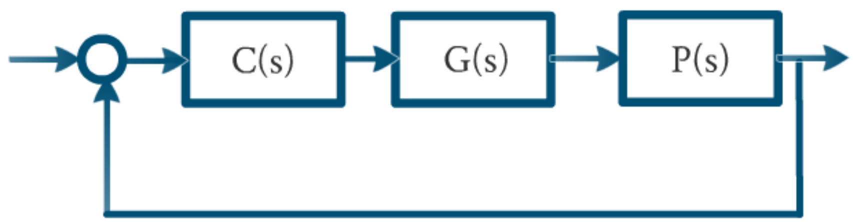

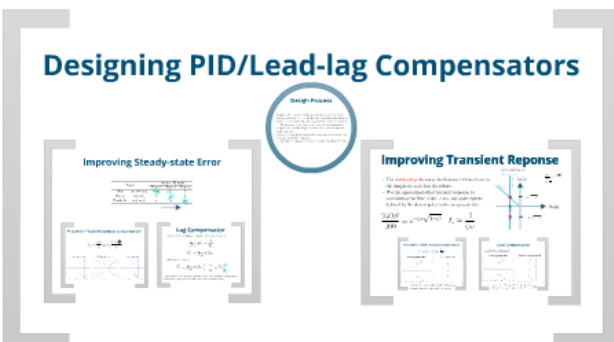

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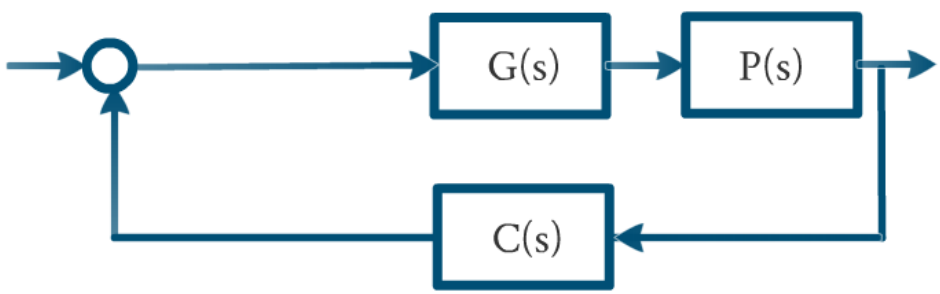

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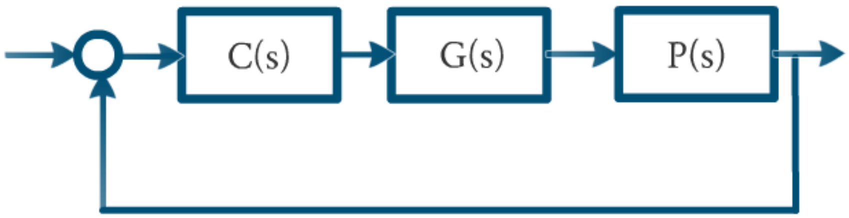

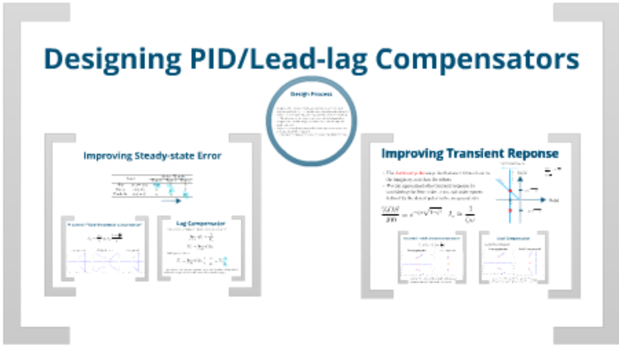

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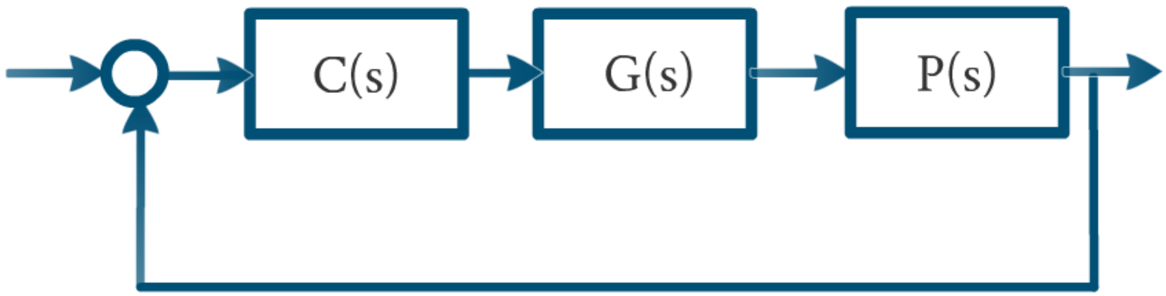
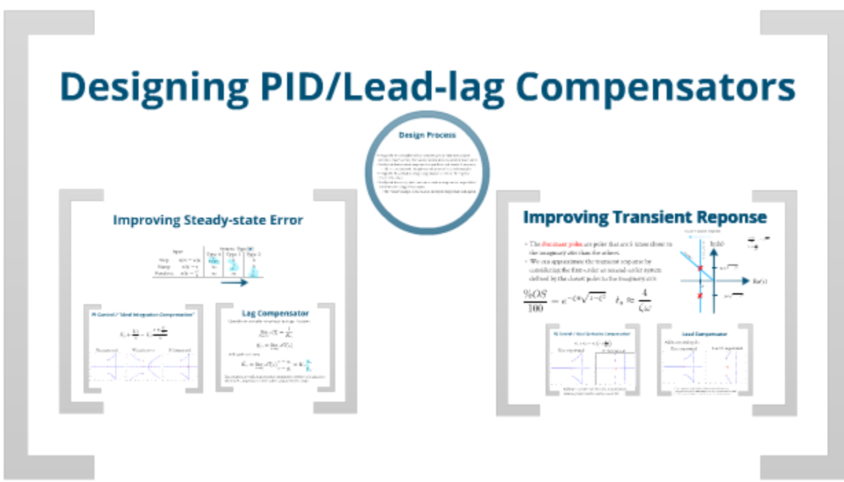


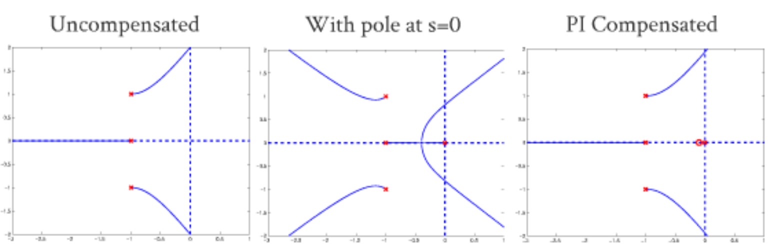


Improving Steady-state Error

Input		System Type		
		Type-0	Type-1	Type-2
Step	$x(t) = u(t)$	$\frac{1}{1+K_p}$	0	0
Ramp	$x(t) = t$	∞	$\frac{1}{K_v}$	0
Parabola	$x(t) = \frac{t^2}{2}$	∞	∞	$\frac{1}{K_a}$

PI Control / "Ideal Integration Compensation"

$$K_p + \frac{K_I}{s} = K_p \frac{s + \frac{K_I}{K_p}}{s}$$



Lag Compensator

Consider an exemplar ramp input to a type-1 system:

$$\lim_{t \to \infty} e(t) = \frac{1}{K_v}$$

$$K_v = \lim_{s \to 0} sG(s)$$

Add a pole and zero:

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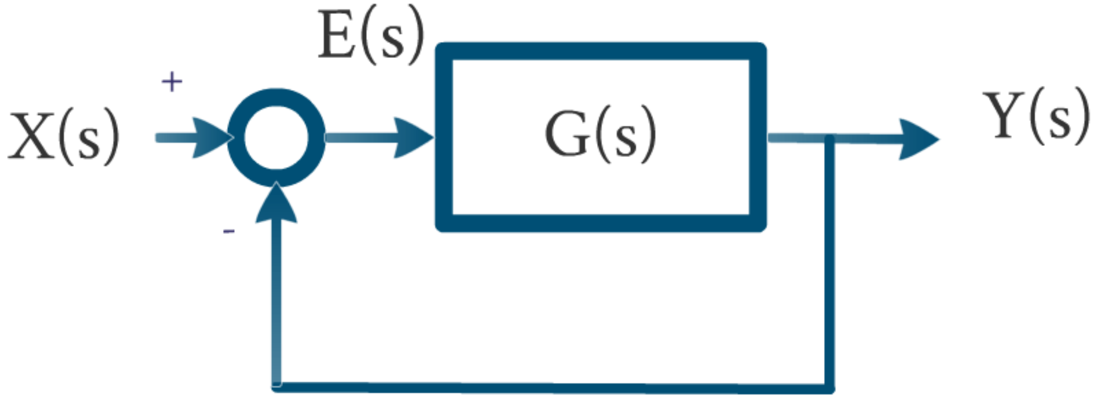
Lag compensators only require passive components (resistors and capacitors) whereas PI compensators require active components (Op Amps).

System Type

Unity Negative Feedback (Representation of) System



$$G(s) = \frac{1}{(s-3)(s-0)(s-4)(s-5)(s-0)}$$

$$= \frac{1}{s^2(s-3)(s-4)(s-5)}$$

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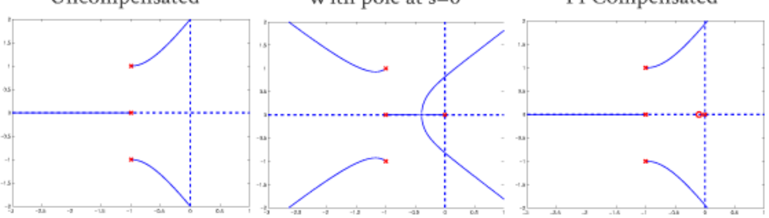
Uncompensated   With pole at s=0   PI Compensated



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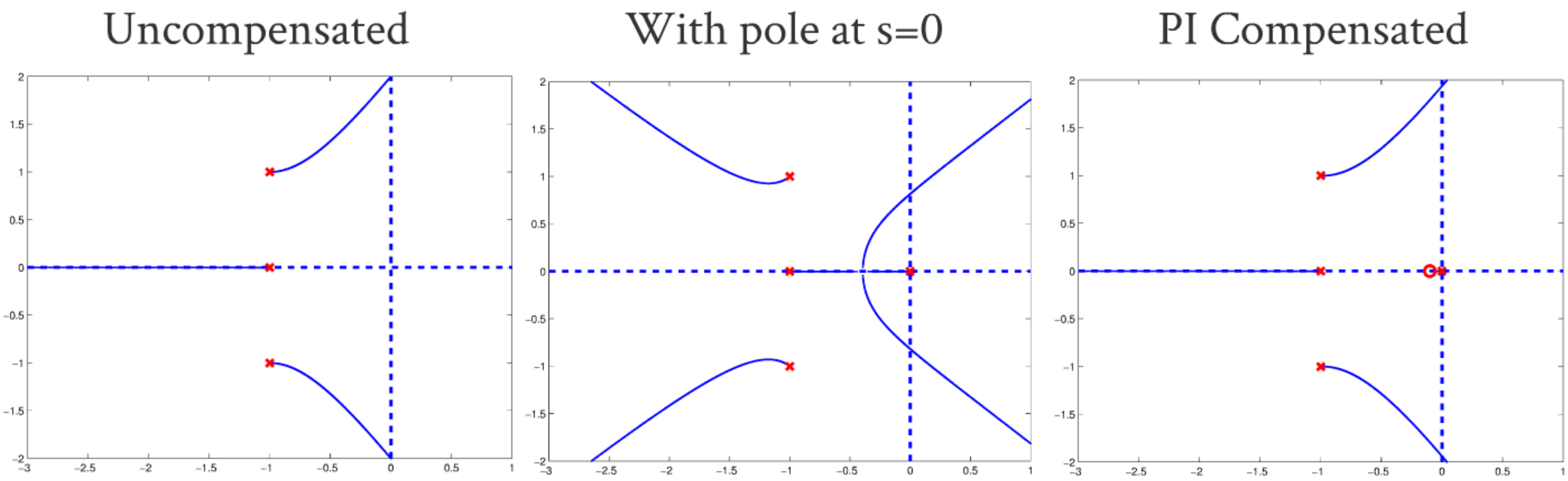

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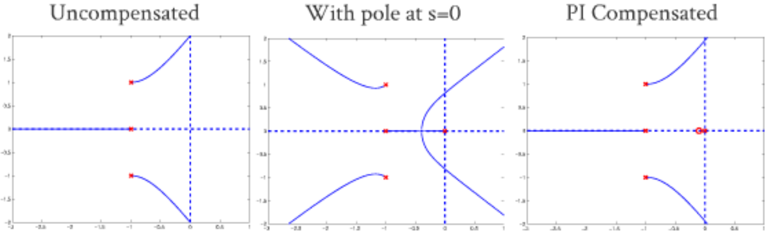

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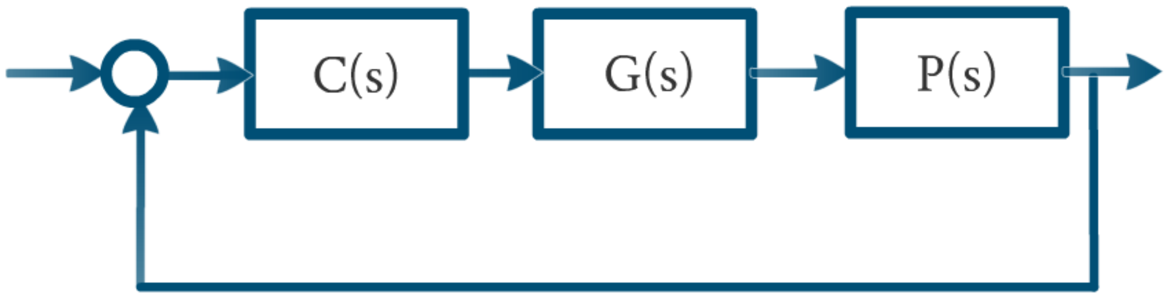
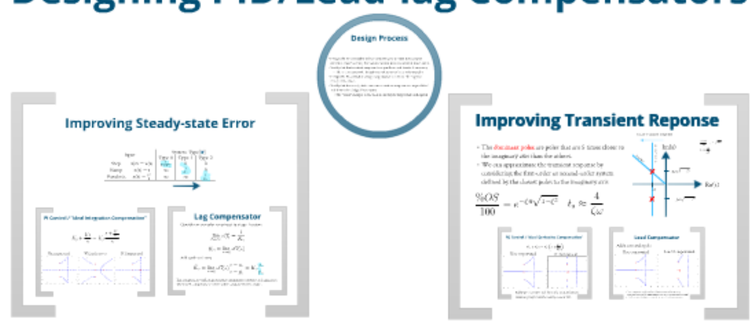
Cascade Compensators



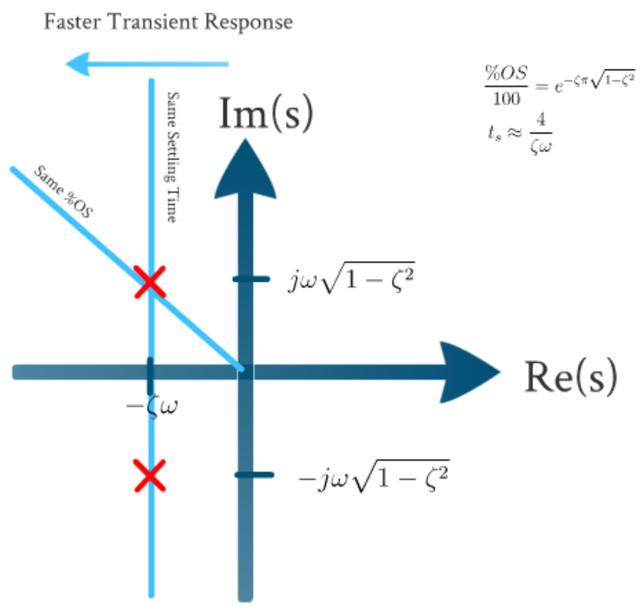
Designing PID/Lead-lag Compensators



Improving Transient Reponse

- The dominant poles are poles that are 5 times closer to the imaginary axis than the others.
- We can approximate the transient response by considering the first-order or second-order system defined by the closest poles to the imaginary axis

$$\frac{\%OS}{100} = e^{-\zeta\pi\sqrt{1-\zeta^2}} \quad t_s \approx \frac{4}{\zeta\omega}$$



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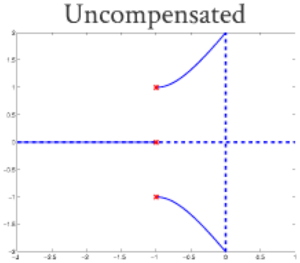
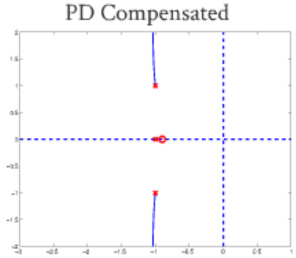
PD Control / "Ideal Derivative Compensation"

$$K_p + K_d s = K_d\left(s + \frac{K_p}{K_d}\right)$$

Uncompensated



PD Compensated

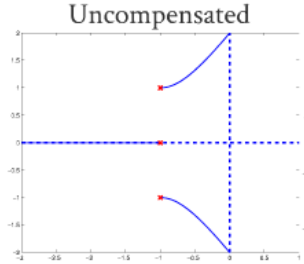


Adding PD control will be likely to alter (ie not necessary improve) the steady-state error.

Lead Compensator

Add a zero and a pole:

Uncompensated



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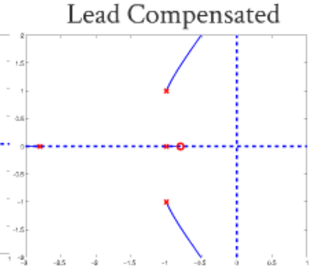

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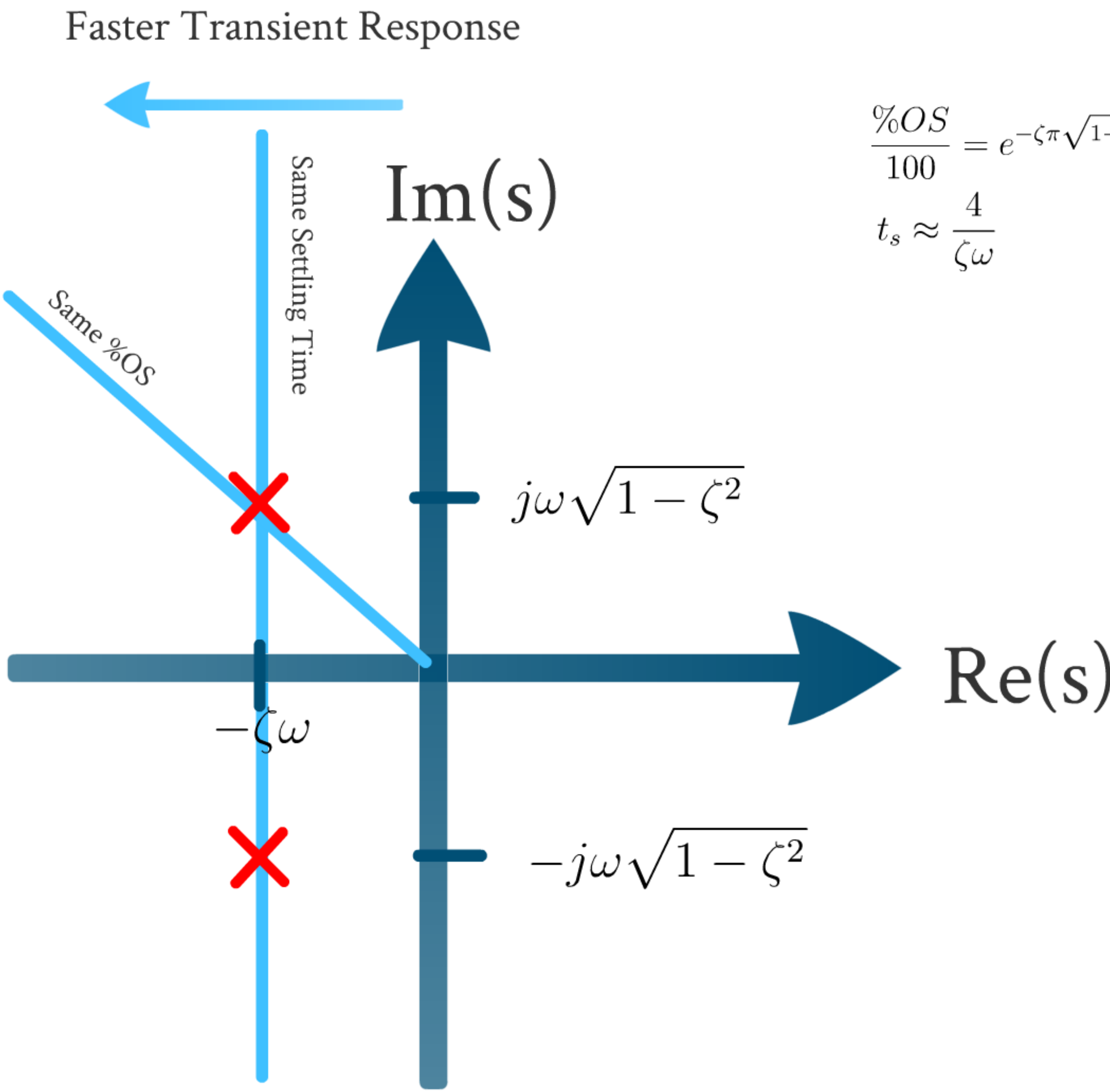

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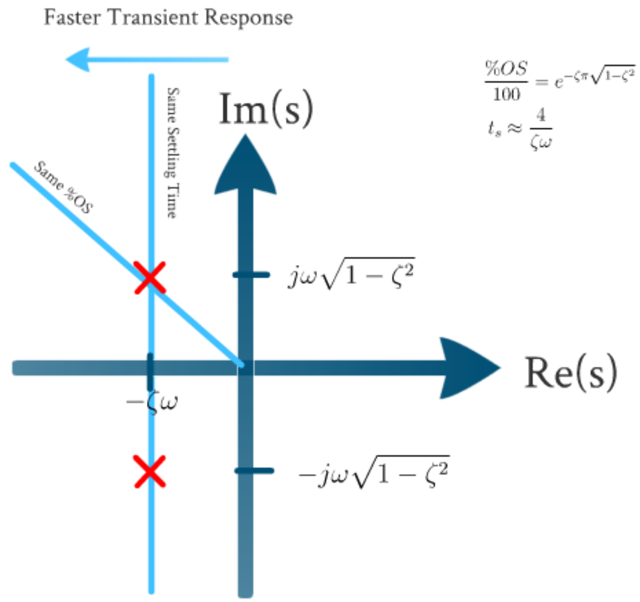

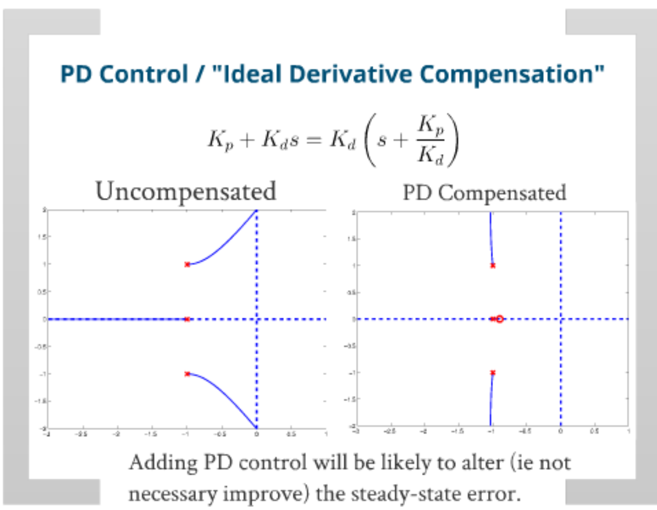

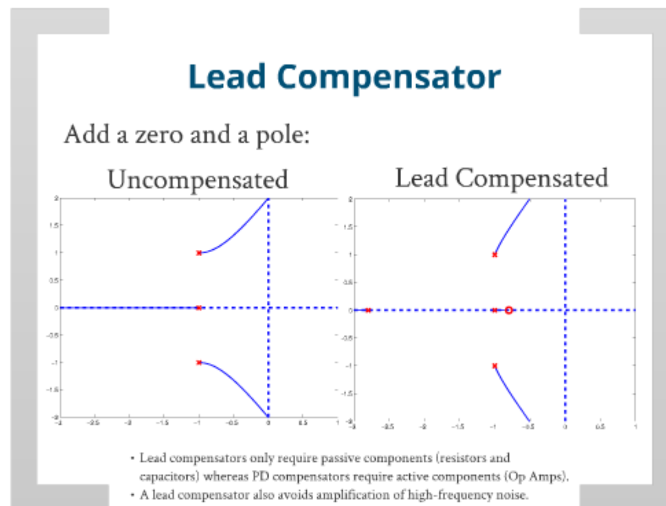

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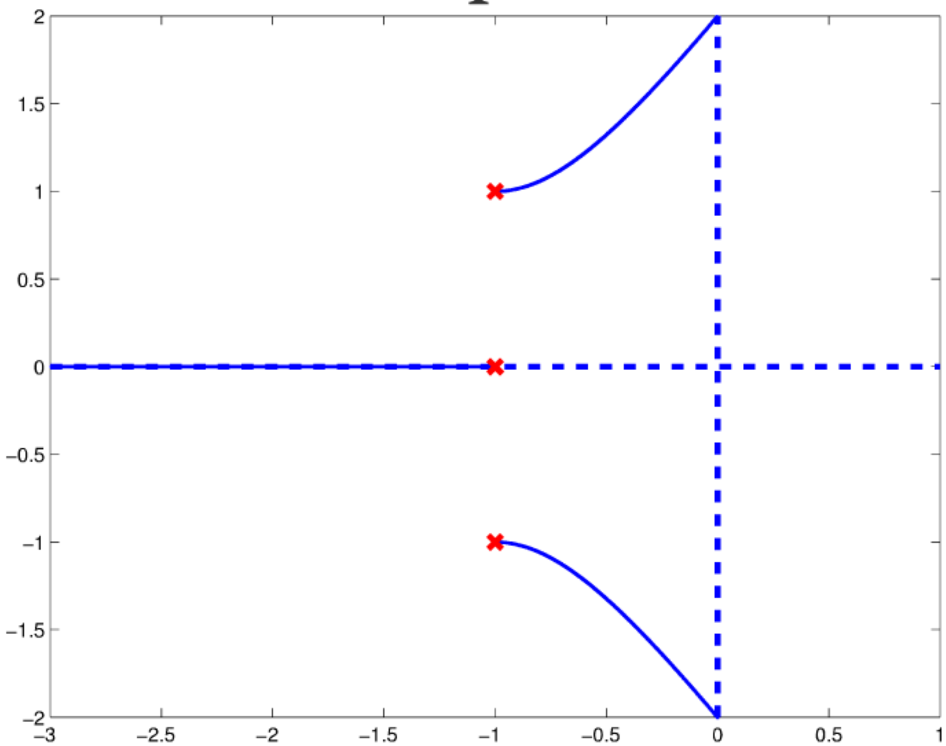

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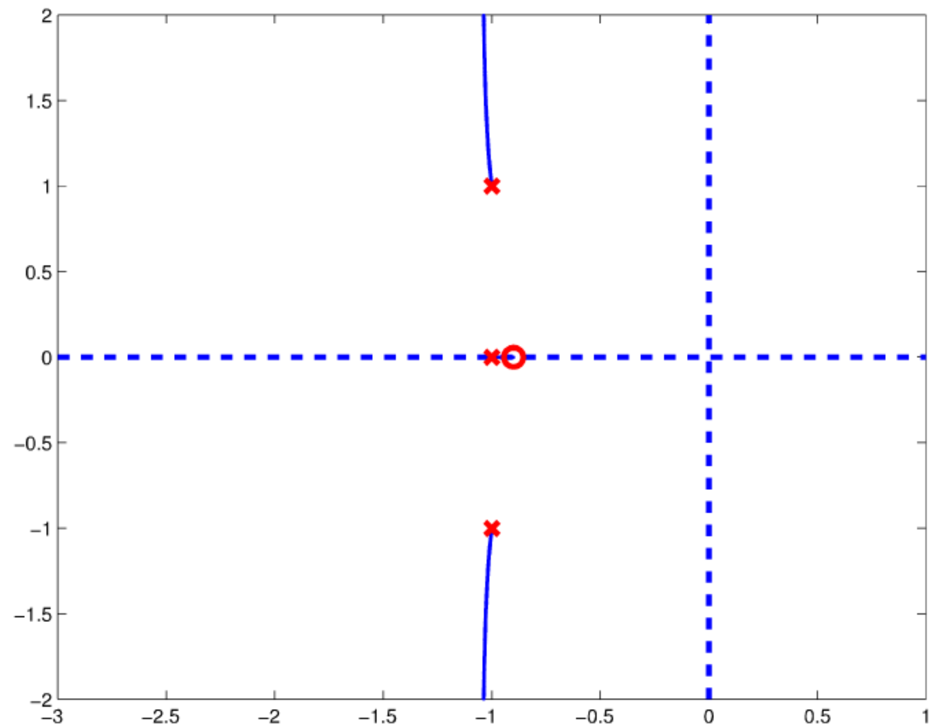

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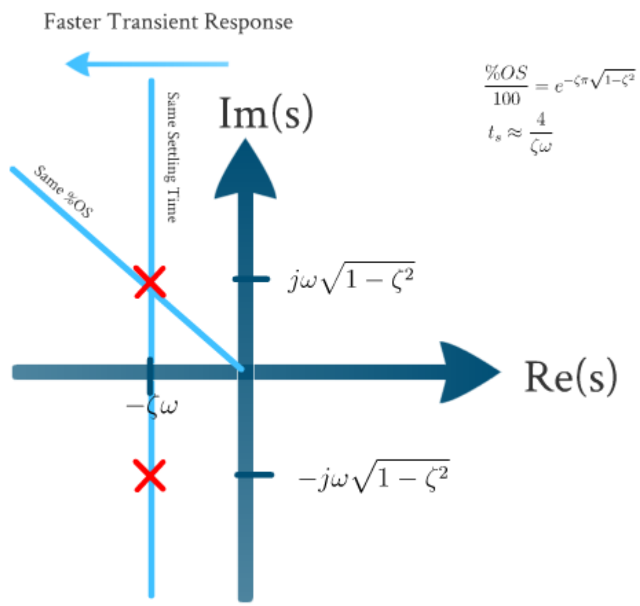
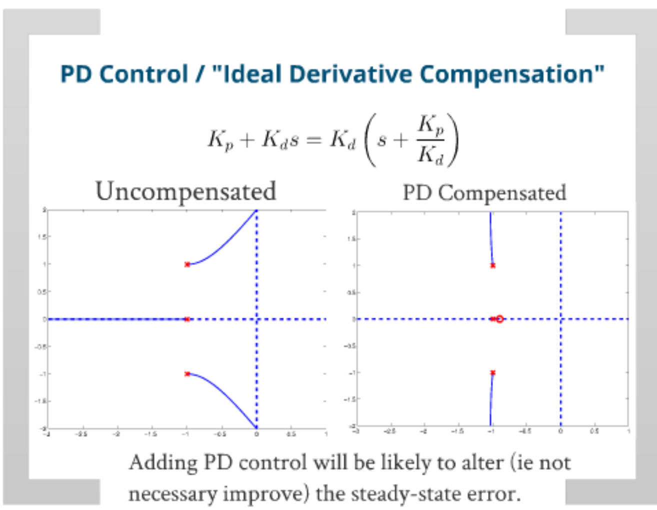
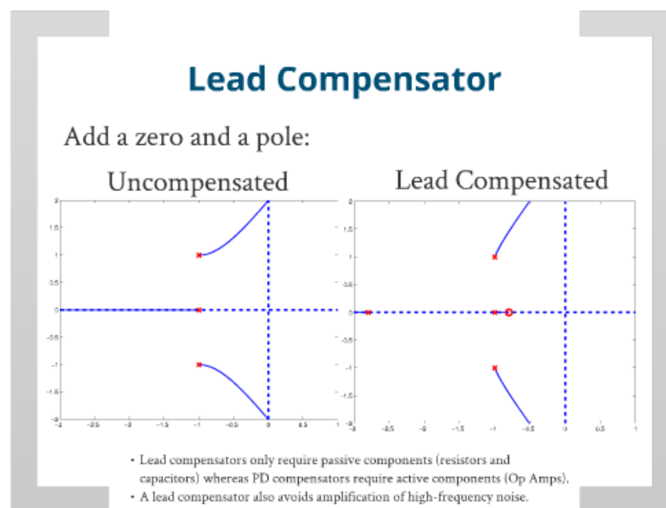
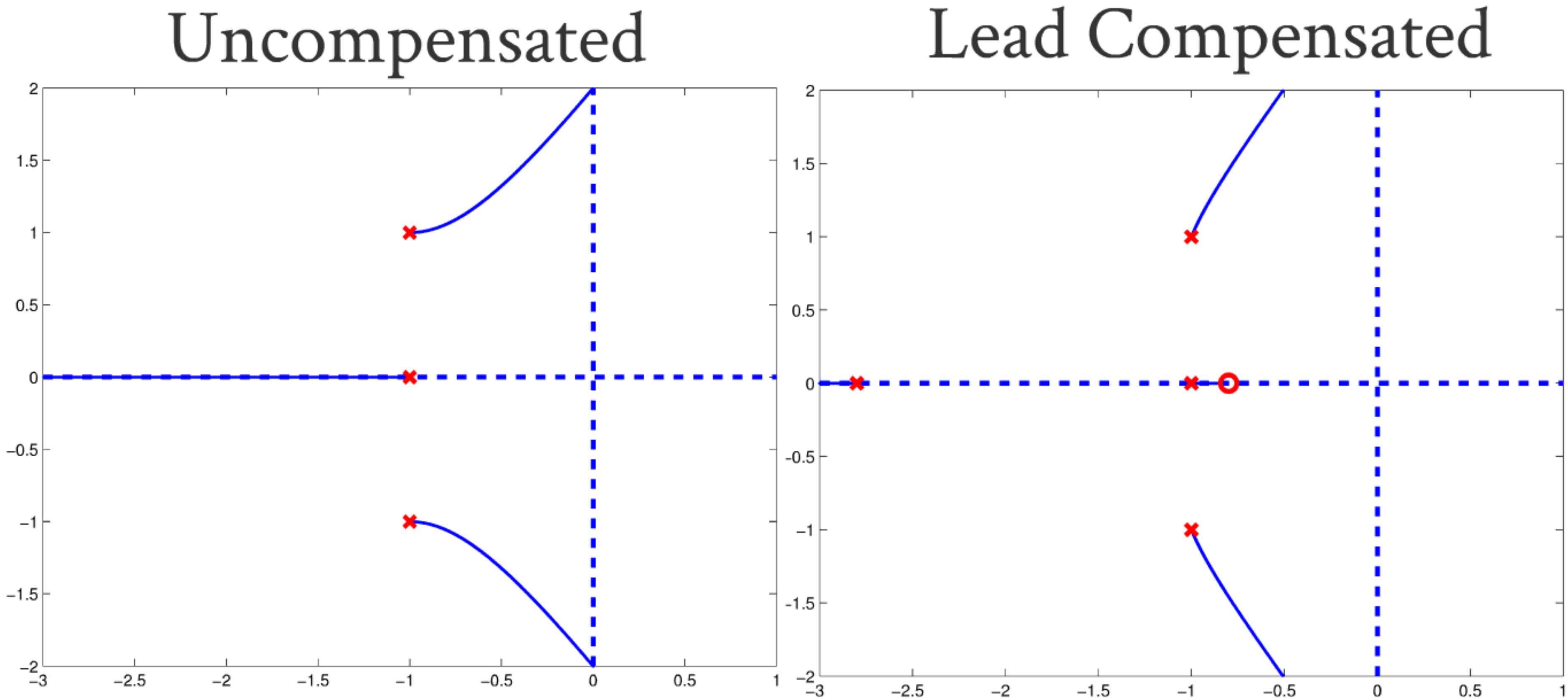






Lead Compensator

Add a zero and a pole:



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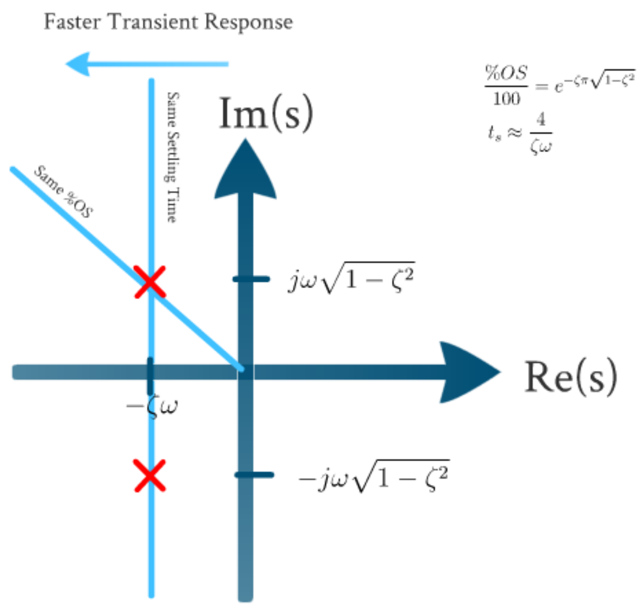

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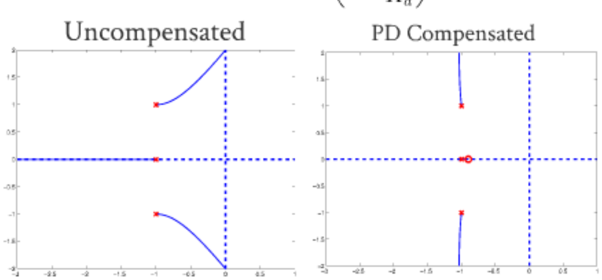

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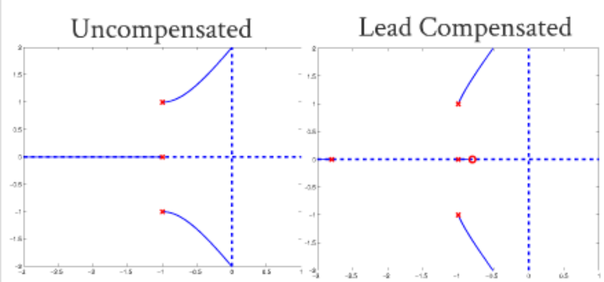

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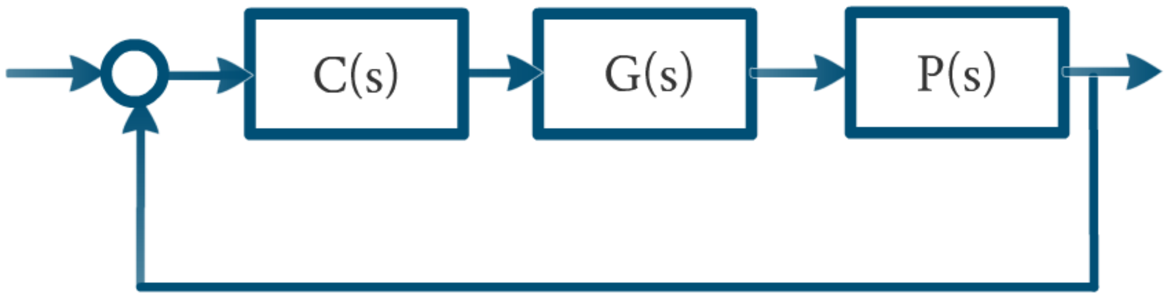

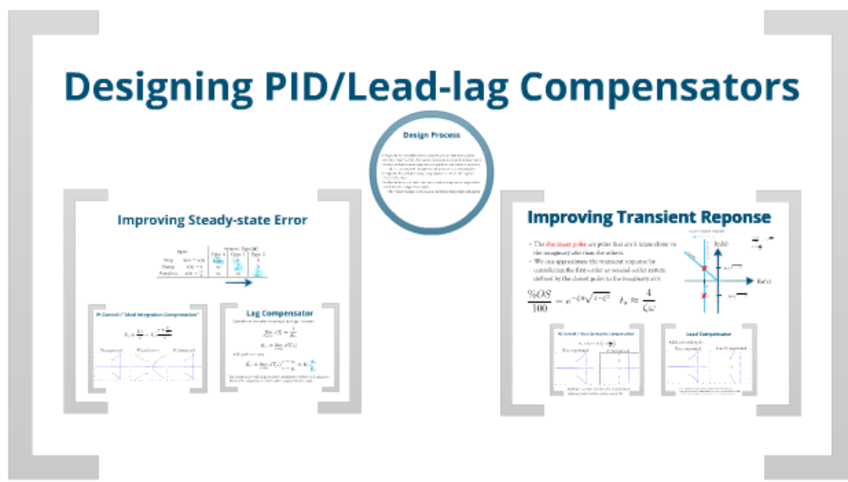

Designing PID/Lead-lag Compensators

Design Process

- Design the PD controller or lead-compensator to meet the transient response specifications; this moves the root locus to where it needs to be.
- Verify that the transient response is as predicted and iterate if necessary
  - The poles nearest the imaginary axis may not be dominant poles
- Design the PI controller or lag-compensator to achieve the required steady-state error.
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  - The "small" changes to the root locus may be larger than anticipated.

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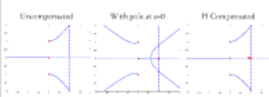

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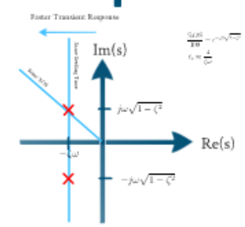

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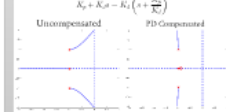

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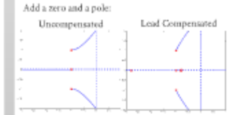

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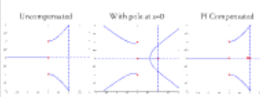
PI Control / "Ideal Integration Compensation"

$$K_p + \frac{K_I}{s} = K_p \frac{s + \frac{K_I}{K_p}}{s}$$



Lag Compensator

Consider an exemplar ramp input to a type-1 system:

$$\lim_{t \to \infty} e(t) = \frac{1}{K_v}$$

$$K_v = \lim_{s \to 0} sG(s)$$

Add a pole and zero:

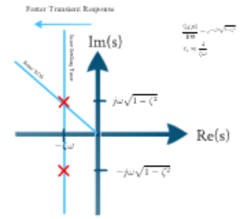
$$\bar{K}_v = \lim_{s \to 0} sG(s) \frac{s - z_c}{s - p_c} = K_v \frac{z_c}{p_c}$$

Lag compensators only require passive components (resistors and capacitors) whereas PI compensators require active components (Op Amps).

Improving Transient Reponse

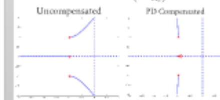
- The dominant poles are poles that are 5 times closer to the imaginary axis than the others.
- We can approximate the transient response by considering the first-order or second-order system defined by the closest poles to the imaginary axis

$$\frac{\%OS}{100} = e^{-\zeta\pi\sqrt{1-\zeta^2}} \quad t_s \approx \frac{4}{\zeta\omega}$$



PD Control / "Ideal Derivative Compensation"

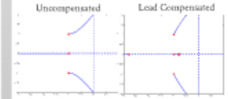
$$K_p + K_d s = K_d\left(s + \frac{K_p}{K_d}\right)$$



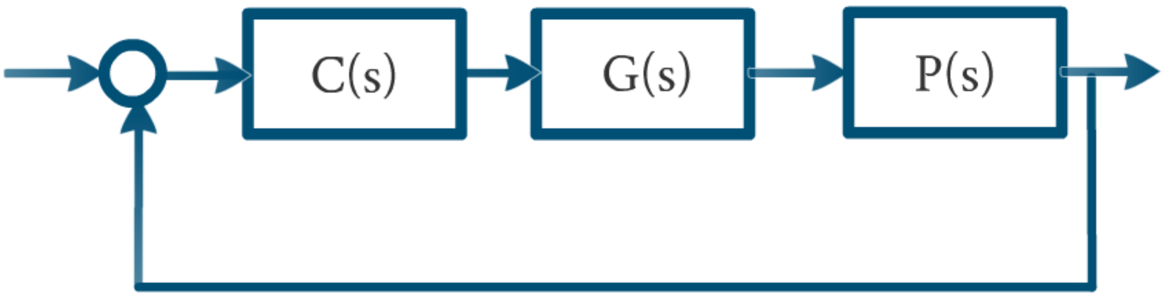
Adding PD control will be likely to alter (ie not necessary improve) the steady-state error.

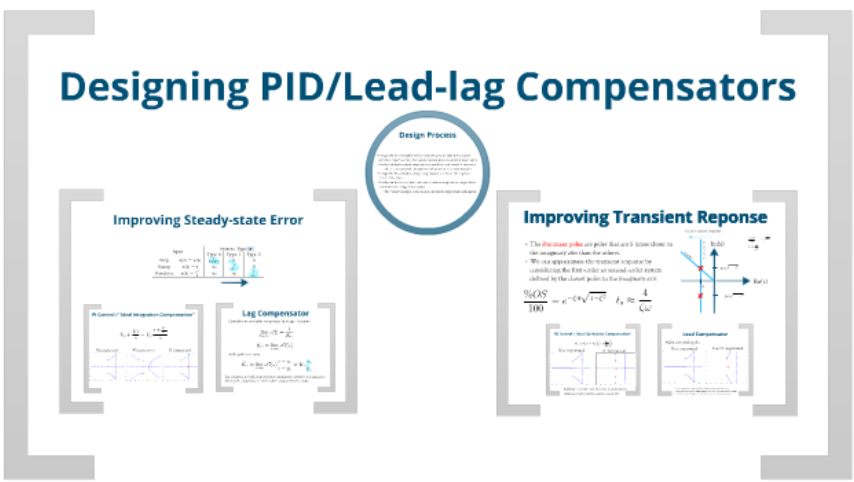
Lead Compensator

Add a zero and a pole:



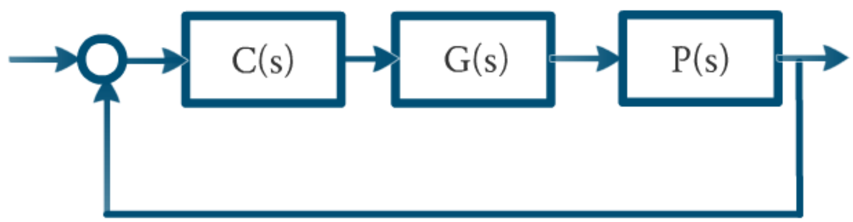
Cascade Compensators





Compensators

Cascade Compensators



Designing PID/Lead-lag Compensators

Design Process

Improving Steady-state Error

Lag Compensator

Improving Transient Reponse

Lead Compensator

Feedback Compensators

This lecture covers:

- Design of a Control System via root locus.