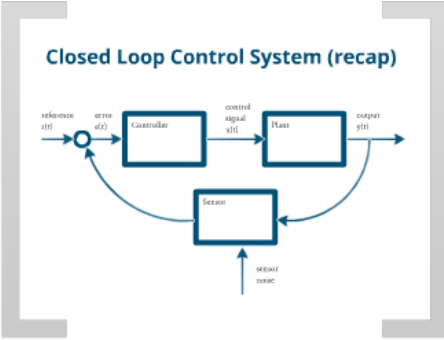

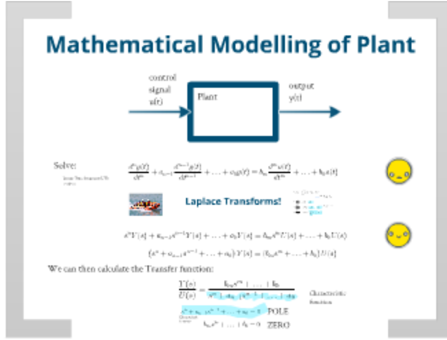

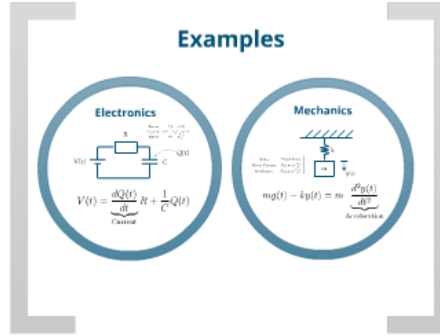

This lecture covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model

ELEC 207 Part B

Control Theory Lecture 2: Control System Modelling (1)

Prof Simon Maskell
CHAD-G68
s.maskell@liverpool.ac.uk
0151 794 4573

UNIVERSITY OF LIVERPOOL

This lecture covers:

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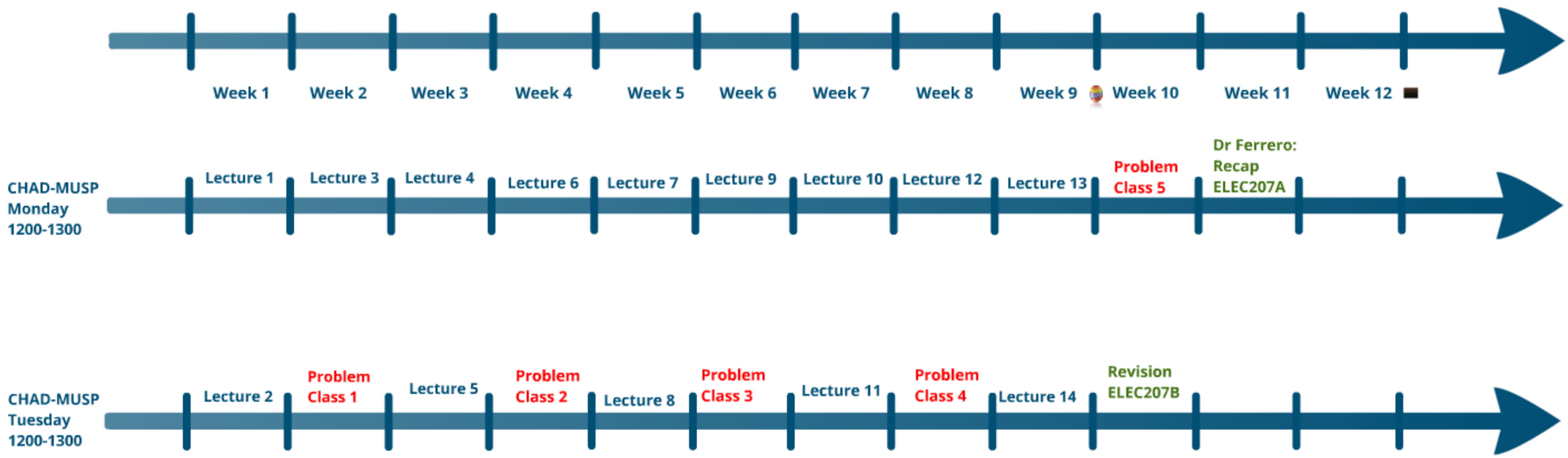
ELEC 207B: Timeline



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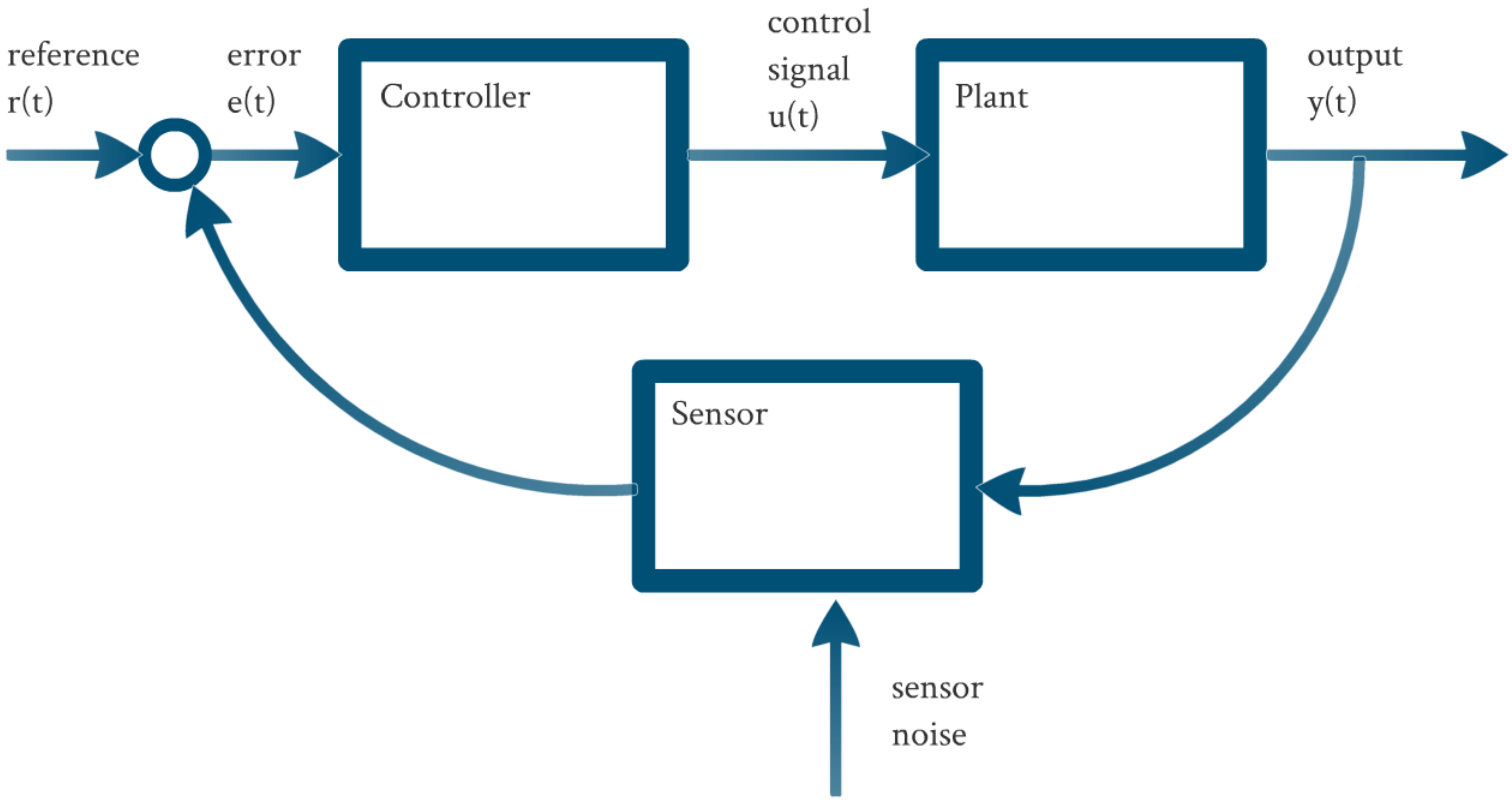
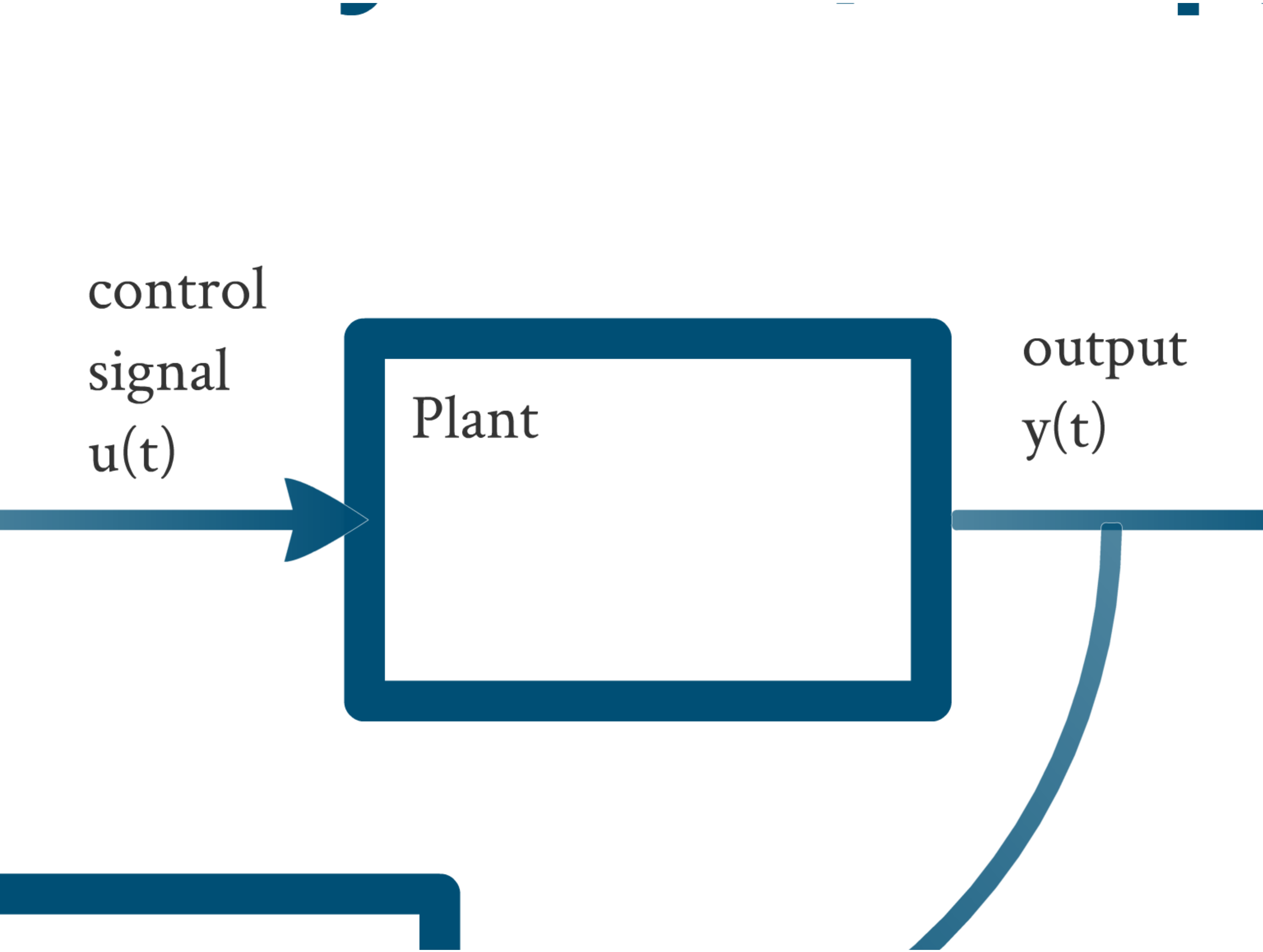
Closed Loop Control System (recap)




control
signal
u(t)
Plant
output
y(t)

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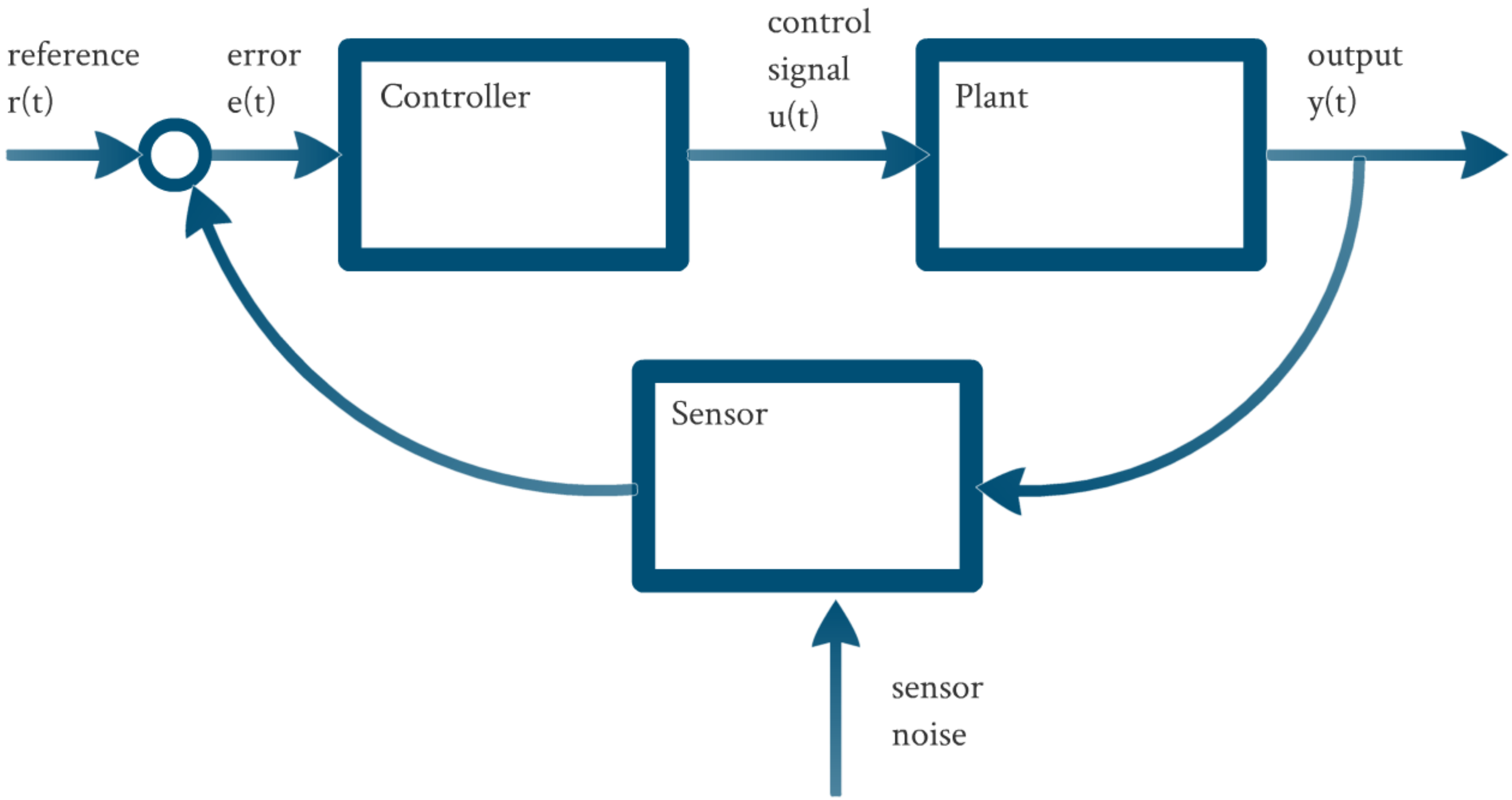
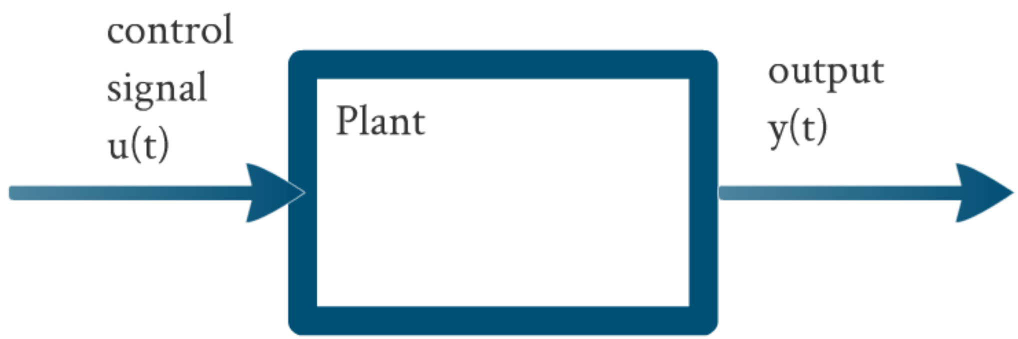


Mathematical Modelling of Plant



Solve:

Linear Time Invariant (LTI)
Order n

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \ldots + b_0 u(t)$$

Laplace Transforms!

$$s^n Y(s) + a_{n-1}s^{n-1}Y(s) + \ldots + a_0 Y(s) = b_m s^m U(s) + \ldots + b_0 U(s)$$

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Characteristic function

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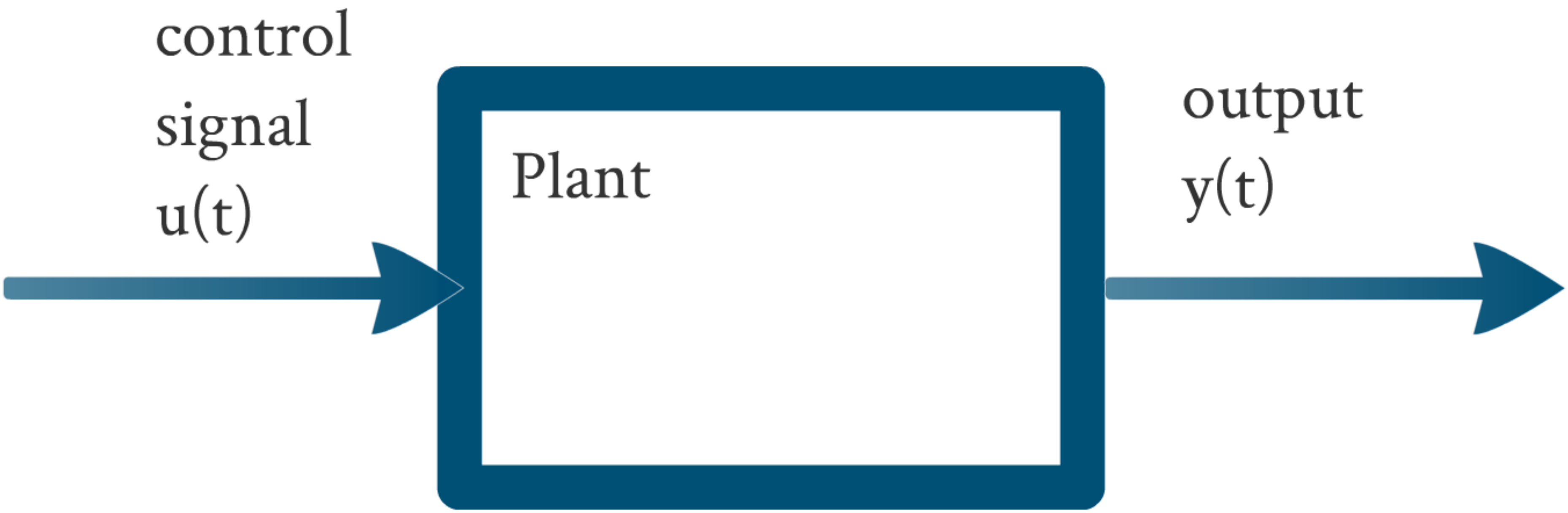
ematical Modelling of



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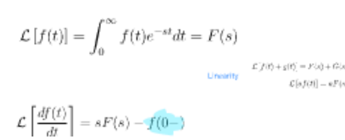
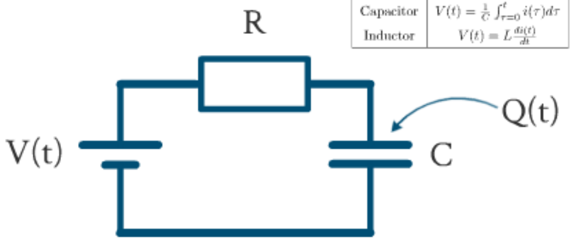


Examples

Electronics

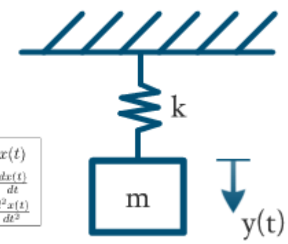
Resistor	$V(t) = i(t)R$
Capacitor	$V(t) = \frac{1}{C}\int_{\tau=0}^{t} i(\tau)d\tau$
Inductor	$V(t) = L\frac{di(t)}{dt}$



$$V(t) = \underbrace{\frac{dQ(t)}{dt}}_{\text{Current}} R + \frac{1}{C}Q(t)$$

Mechanics

Spring	$F(t) = Kx(t)$
Viscous Damper	$F(t) = f_v\frac{dx(t)}{dt}$
Acceleration	$F(t) = m\frac{d^2x(t)}{dt^2}$



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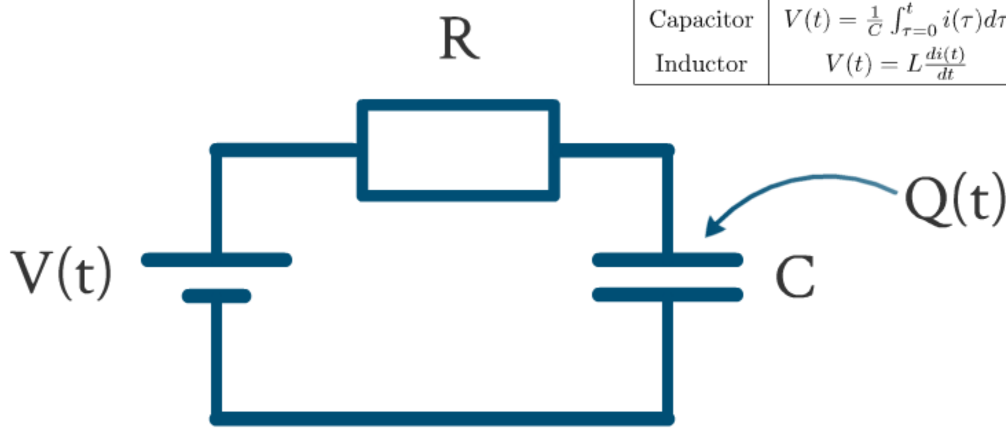

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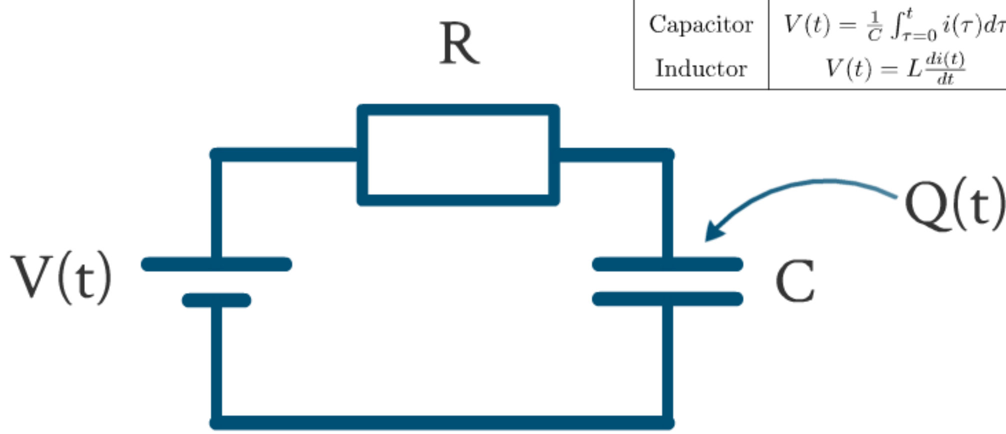

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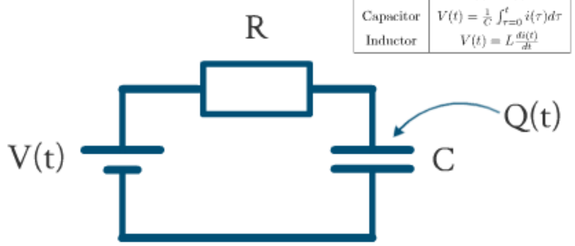

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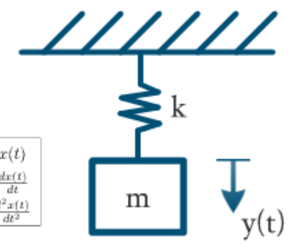

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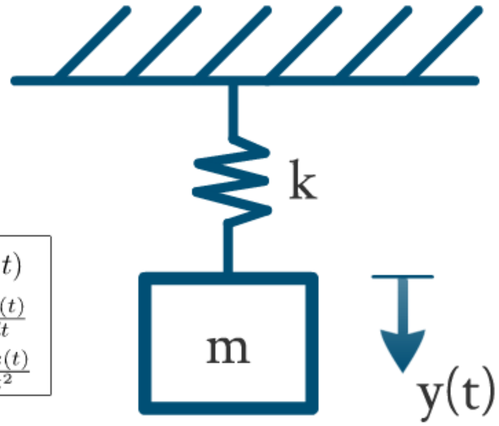

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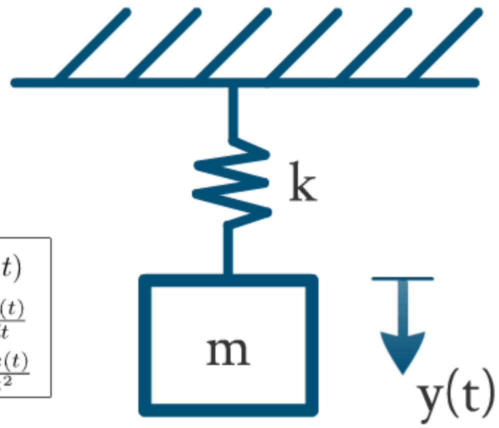

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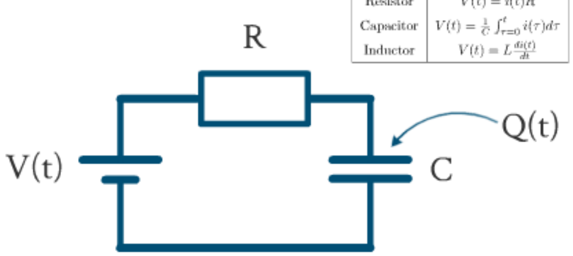

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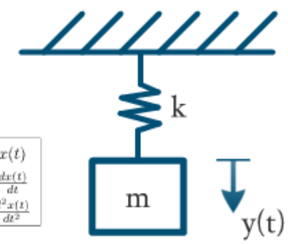

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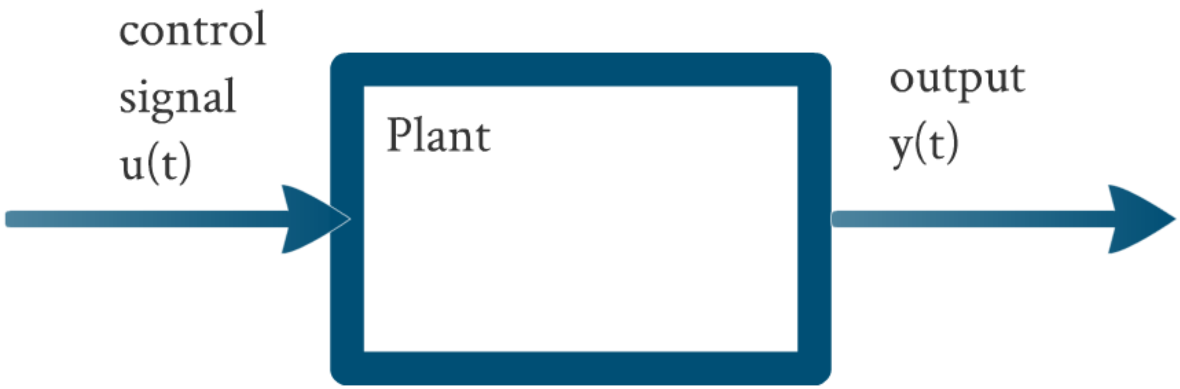

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Characteristic Equation

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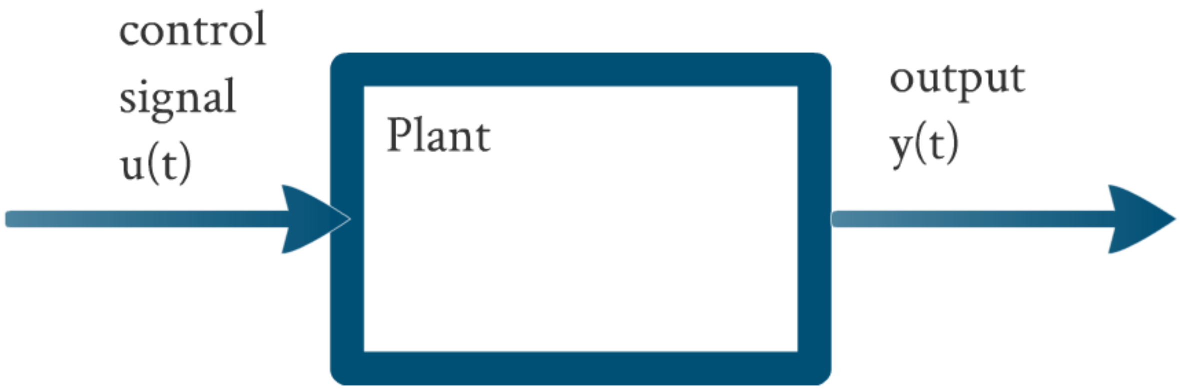

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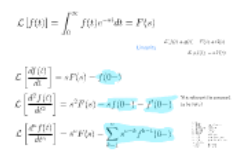

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Linearity

$$\mathcal{L}\left[f(t) + g(t)\right] = F(s) + G(s)$$

$$\mathcal{L}\left[af(t)\right] = aF(s)$$

$$\mathcal{L}\left[\frac{df(t)}{dt}\right] = sF(s) - f(0-)$$

$$\mathcal{L}\left[\frac{d^2f(t)}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$$

Not relevant (ie assumed to be zero)

$$\mathcal{L}\left[\frac{d^nf(t)}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$$

No	Theorem	
1	$\mathcal{L}\left[f(t)\right]$	$= F(s) = \int_{0-}^{\infty} f(t)e^{-st}dt$
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11	$f(\infty)$	$= \lim_{s\to 0} sF(s)$
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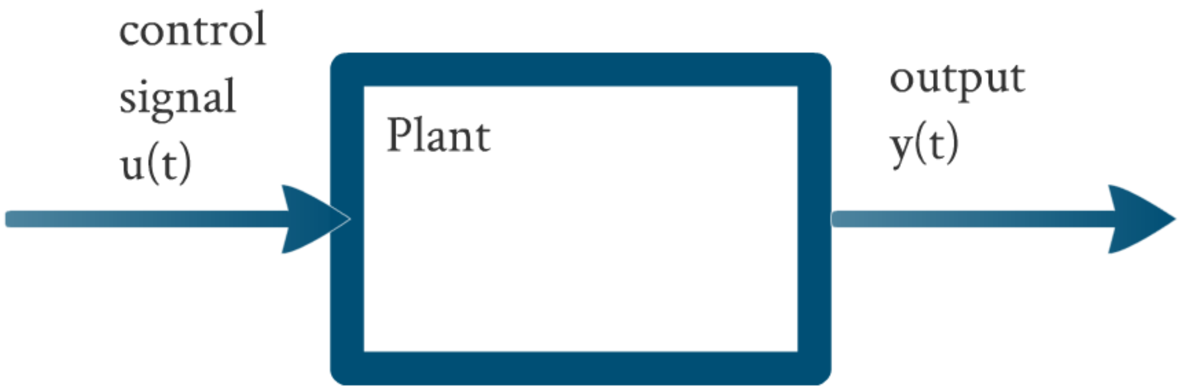

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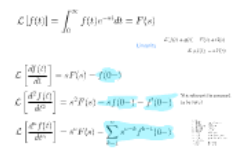

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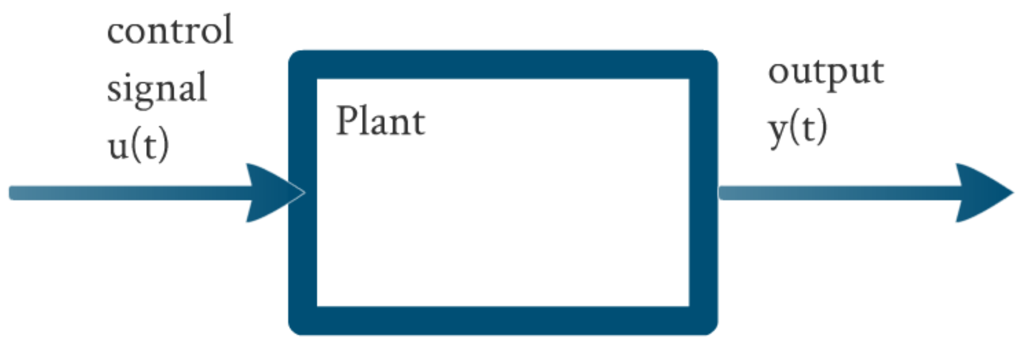

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Laplace Transforms!

$$s^n Y(s) + a_{n-1}s^{n-1}Y(s) + \ldots + a_0 Y(s) = b_m s^m U(s) + \ldots + b_0 U(s)$$

$$\left(s^n + a_{n-1}s^{n-1} + \ldots + a_0\right) Y(s) = \left(b_m s^m + \ldots + b_0\right) U(s)$$

We can then calculate the Transfer function:

$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + \ldots + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_0}$$

Characteristic function

$s^n + a_{n-1}s^{n-1} + \ldots + a_0 = 0$ POLE

Characteristic Equation

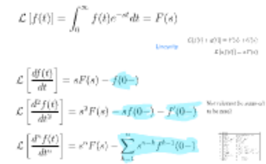
$b_m s^m + \ldots + b_0 = 0$ ZERO

Solve:

Linear Time Invariant (LTI)
Order n

$$\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \ldots + b_0 u(t)$$

Laplace Transforms!



$$s^n Y(s) + a_{n-1}s^{n-1}Y(s) + \ldots + a_0 Y(s) = b_m s^m U(s) + \ldots + b_0 U(s)$$

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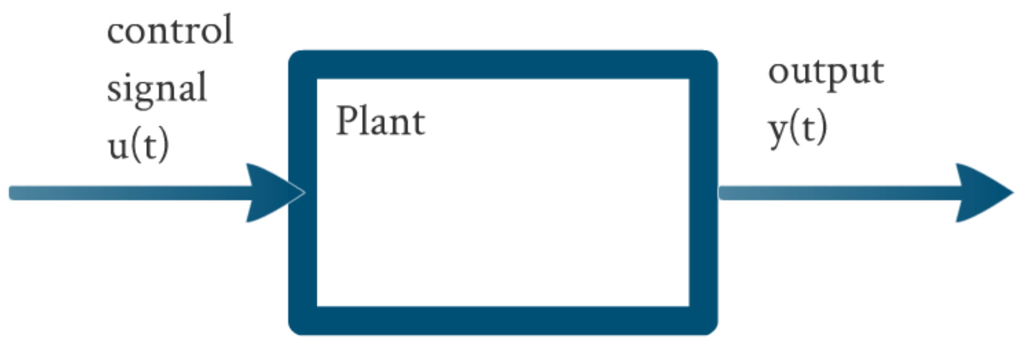
Mathematical Modelling of Plant



Solve:

Linear Time Invariant (LTI)
Order n

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Characteristic function

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Characteristic Equation

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function:

$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + \ldots + b_0}{s^n + a_{n-1}s^{n-1} + \ldots + a_0}$$

$$s^n + a_{n-1}s^{n-1} + \ldots + a_0 = 0$$ POLE

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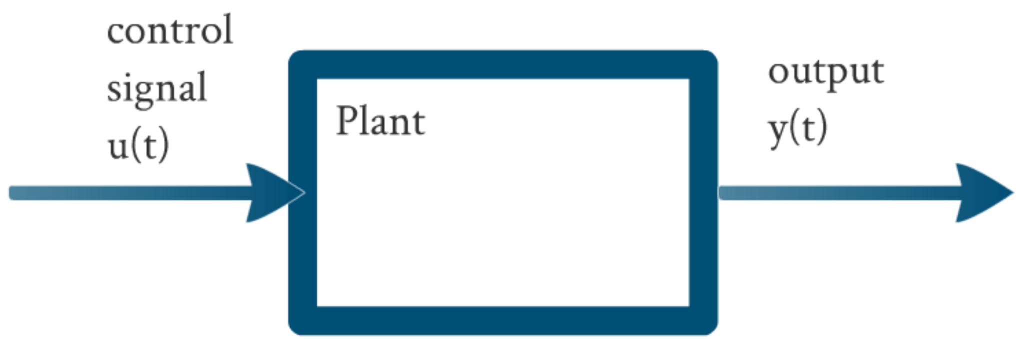
Mathematical Modelling of Plant



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This lecture covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model