



ELEC 207 Part B

Control Theory Lecture 2: Control System Modelling (1)

Prof Simon Maskell CHAD-G68 s.maskell@liverpool.ac.uk 0151 794 4573





ELEC 207 Part B

Control Theory Lecture 2: Control System Modelling (1)

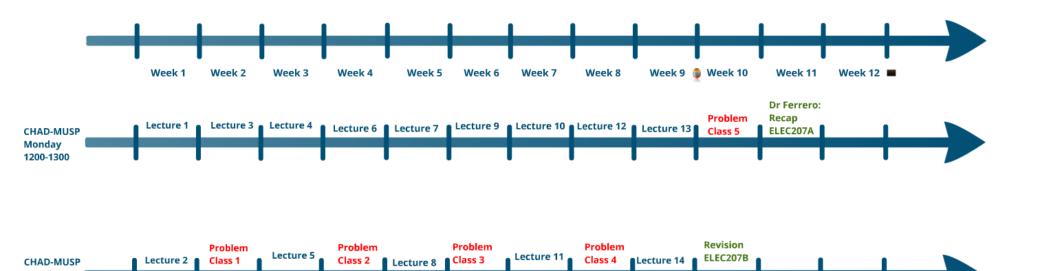
Prof Simon Maskell
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This lecture covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model

ELEC 207B: Timeline

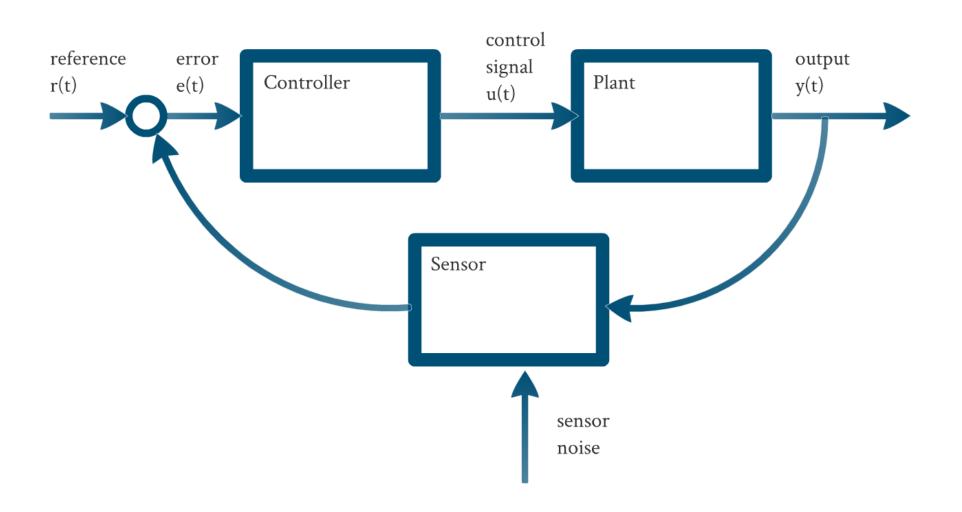


Tuesday 1200-1300

This lecture covers:

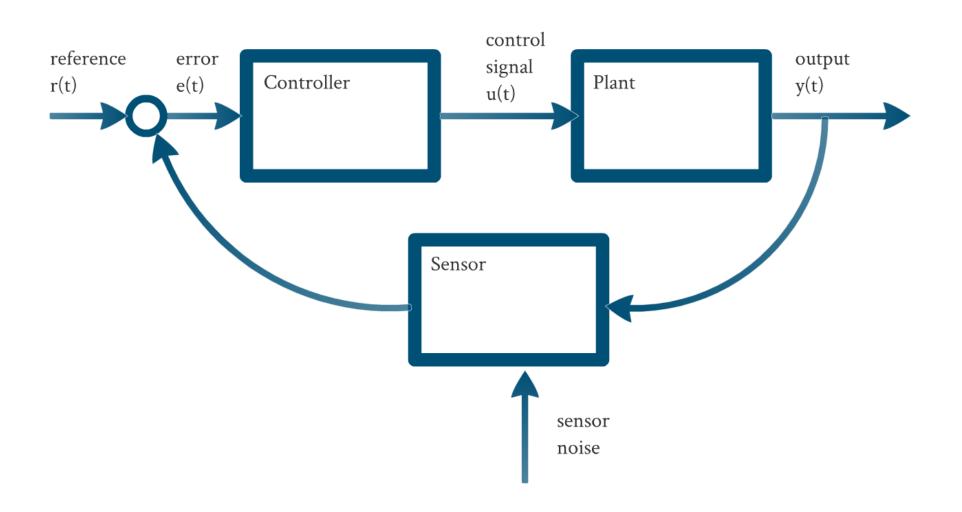
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Closed Loop Control System (recap)

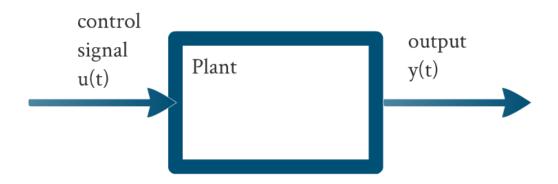


control output signal Plant u(t)

Closed Loop Control System (recap)



Mathematical Modelling of Plant



Solve:

Linear Time Invariant (LTI) Order n

$$\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \ldots + b_0 u(t)$$





Laplace Transforms!

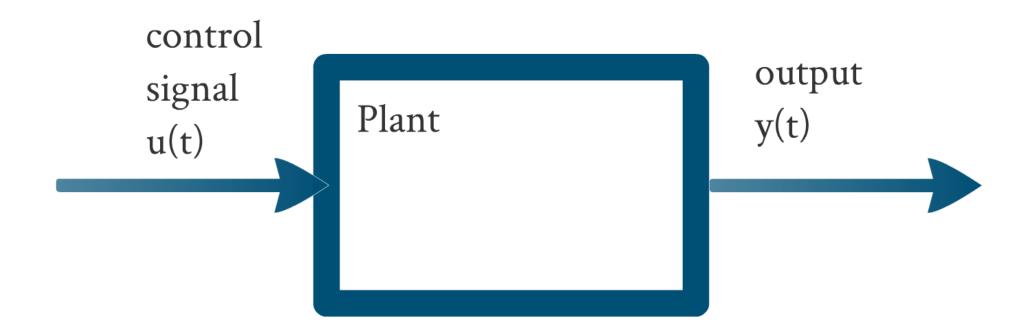
$$L\left[\frac{\partial G}{\partial t}\right] = A^{2}(t) - \frac{1}{2} \left[\frac{\partial G}{\partial t}\right] + \frac{1}{2} \left[\frac{\partial G}{\partial t}\right] + A^{2}(t) - A^{2}(t)$$

$$s^{n}Y(s) + a_{n-1}s^{n-1}Y(s) + \dots + a_{0}Y(s) = b_{m}s^{m}U(s) + \dots + b_{0}U(s)$$
$$(s^{n} + a_{n-1}s^{n-1} + \dots + a_{0})Y(s) = (b_{m}s^{m} + \dots + b_{0})U(s)$$



$$rac{Y(s)}{U(s)} = rac{b_m s^m + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$
 Characteristic function $s^n + a_{n-1} s^{n-1} + \ldots + a_0 = 0$ POLE $b_m s^m + \ldots + b_0 = 0$ ZERO

ematical Modelling of



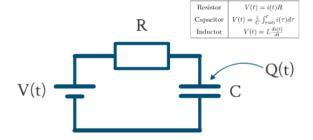
$$\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \dots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \dots + b_0 u(t)$$



 $\mathcal{L}\left[f(t)\right] = \int_{0}^{\infty} f(t)e^{-st}dt = F(s)$ $\underset{\mathcal{L}\left[s[t]\right] = sF(s)}{\text{Unearity}} \quad \underset{\mathcal{L}\left[s[t]\right] = sF(s)}{\mathcal{L}\left[s[t]\right] - sF(s)}$ $\mathcal{L}\left[\frac{df(t)}{s}\right] = sF(s) - f(0-s)$

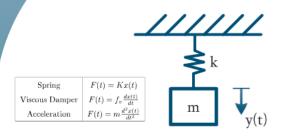
Examples

Electronics



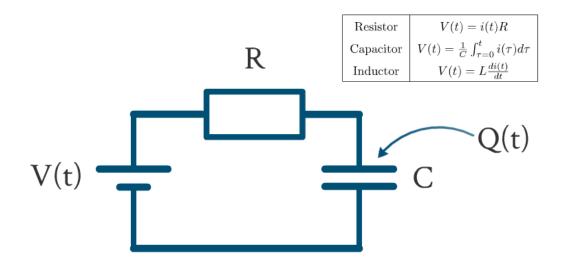
$$V(t) = \underbrace{\frac{dQ(t)}{dt}}_{\text{Current}} R + \frac{1}{C}Q(t)$$

Mechanics



$$mg(t) - ky(t) = m \underbrace{\frac{d^2y(t)}{dt^2}}_{\text{Acceleration}}$$

Electronics



$$V(t) = \underbrace{\frac{dQ(t)}{dt}}_{\text{Current}} R + \frac{1}{C}Q(t)$$

6

Resistor

Capacitor

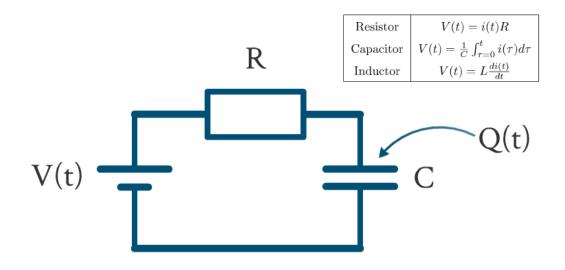
Inductor

$$V(t) = i(t)R$$

$$V(t) = \frac{1}{C} \int_{\tau=0}^{t} i(\tau) d\tau$$

$$V(t) = L \frac{di(t)}{dt}$$

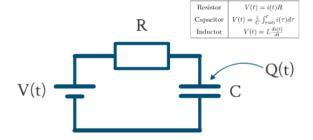
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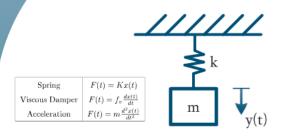
Examples

Electronics



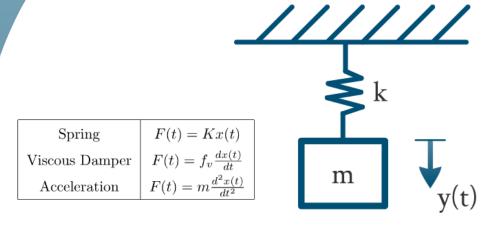
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Mechanics



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Spring

Viscous Damper

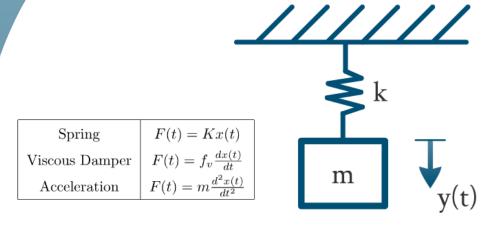
Acceleration

$$F(t) = Kx(t)$$

$$F(t) = f_v \frac{dx(t)}{dt}$$

$$F(t) = m \frac{d^2 x(t)}{dt^2}$$

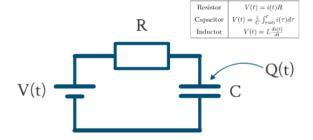
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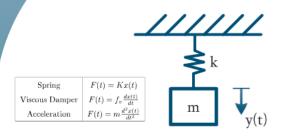
Examples

Electronics

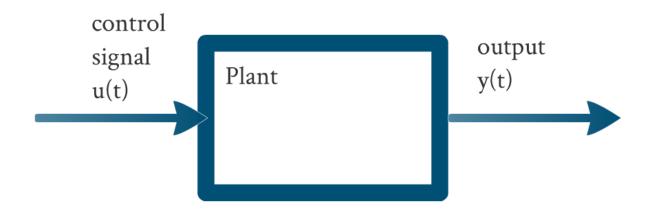


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Solve:

Linear Time Invariant (LTI) Order n

$$\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \ldots + b_0 u(t)$$





Laplace Transforms!

$$C[f(t)] = \int_{0}^{t} f(t)e^{-tt}dt = F(s)$$

$$\sum_{i=0}^{t} \sum_{s=0}^{t} \sum_{s=0}^{$$

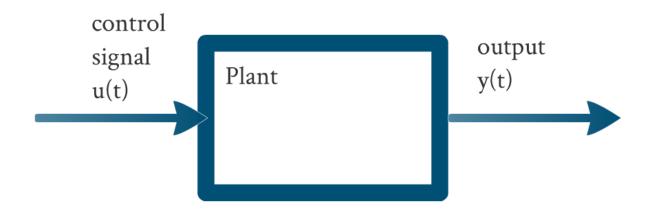
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 Characteristic function $s^n + a_{n-1} s^{n-1} + \ldots + a_0 = 0$ POLE $b_m s^m + \ldots + b_0 = 0$ ZERO

olve:

Linear Time Invariant (LTI)
Order n



Solve:

Linear Time Invariant (LTI) Order n

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$$\mathcal{L}\left[f(t)\right] = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$$\mathcal{L}[f(t) + g(t)] = F(s) + G(s)$$
$$\mathcal{L}[af(t)] = aF(s)$$

$$\mathcal{L} \left| \frac{df(t)}{dt} \right| = sF(s) - f(0-)$$

$$\mathcal{L}\left[\frac{d^2f(t)}{dt^2}\right] = s^2F(s) - sf(0-) - f'(0-)$$
 Not relevant (ie assumed to be zero)

$$\mathcal{L}\left[\frac{d^n f(t)}{dt^n}\right] = s^n F(s) - \sum_{k=1}^n s^{n-k} f^{k-1}(0-1)$$

No	Theorem	
1	$\mathcal{L}[f(t)]$	$= F(s) = \int_{0-}^{\infty} f(t) e^{-st} dt$
2	$\mathcal{L}\left[kf\left(t\right)\right]$	= kF(s)
3	$\mathcal{L}[f_1(t) + f_2(t)]$	
4		-F(s+a)
5		$=e^{-sT}F\left(s\right)$
6	$\mathcal{L}[f(at)]$	$=\frac{1}{a}F\left(\frac{s}{a}\right)$
7	$\mathcal{L}\left[\frac{df(t)}{dt}\right]$	= sF(s) - f(0-)
8	$\mathcal{L}\left[\frac{d^2f(t)}{dt^2}\right]$	$-s^{2}F(s) - sf(0-) - \dot{f}(0-)$
9	$\mathcal{L}\left[\frac{d^{n}f(t)}{dt^{n}}\right]$	$= s^n F(s) - \sum_{k=1}^n s^{n-k} f^{k-1}(0 -$
10	$\mathcal{L}\left[\int_{0-}^{t} f(\tau) d\tau\right]$	$=\frac{F(s)}{s}$
11	$f(\infty)$	$= \lim_{s\to 0} sF(s)$
12	f (0+)	$= \lim_{s \to \infty} sF(s)$

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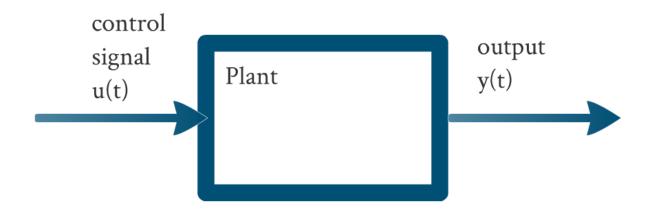
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No	Theorem	
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2	$\mathcal{L}\left[kf\left(t ight) ight]$	$=kF\left(s\right)$
3	$\mathcal{L}\left[f_1\left(t\right) + f_2\left(t\right)\right]$	$=F_{1}\left(s\right) +F_{2}\left(s\right)$
4	$\mathcal{L}\left[e^{-at}f\left(t\right)\right]$	$=F\left(s+a\right)$
5	$\mathcal{L}\left[f\left(t-T\right)\right]$	$=e^{-sT}F\left(s\right)$
6	$\mathcal{L}\left[f\left(at ight) ight]$	$=\frac{1}{a}F\left(\frac{s}{a}\right)$
7	$\mathcal{L}\left[rac{df(t)}{dt} ight]$	$= sF\left(s\right) - f\left(0 - \right)$
8	$\mathcal{L}\left[rac{d^2f(ec{t})}{dt^2} ight]$	$=s^{2}F\left(s\right) -sf\left(0-\right) -\dot{f}\left(0-\right)$
9	$\mathcal{L}\left[rac{d^n f(t)}{dt^n} ight]$	$= s^{n} F(s) - \sum_{k=1}^{n} s^{n-k} f^{k-1}(0-)$
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Solve:

Linear Time Invariant (LTI) Order n

$$\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \ldots + b_0 u(t)$$





Laplace Transforms!

$$C[f(t)] = \int_{0}^{t} f(t)e^{-tt}dt = F(s)$$

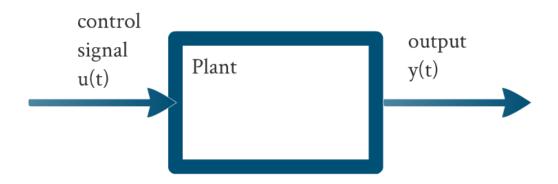
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 Characteristic function $s^n + a_{n-1} s^{n-1} + \ldots + a_0 = 0$ POLE $b_m s^m + \ldots + b_0 = 0$ ZERO

Mathematical Modelling of Plant



Solve:

Linear Time Invariant (LTI) Order n

$$\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \ldots + b_0 u(t)$$





Laplace Transforms!

$$L\begin{bmatrix} \frac{\partial (f)}{\partial x} \\ -\frac{\partial (f)}{\partial x} \end{bmatrix} = d^{2}(x) - \frac{f(x)}{2}$$

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SOIVE.

Linear Time Invariant (LTI) Order n

$$\frac{a^{n}y(t)}{dt^{n}} + a_{n-1}\frac{a^{n-1}y(t)}{dt^{n-1}} + \dots + a_{0}y(t) = b_{m}\frac{a^{m}u(t)}{dt^{m}} + \dots + b_{0}u(t)$$



Laplace Transforms!
$$\mathcal{L}[f(t)] = \int_{0}^{\infty} f(t)e^{-st}dt = F(s)$$

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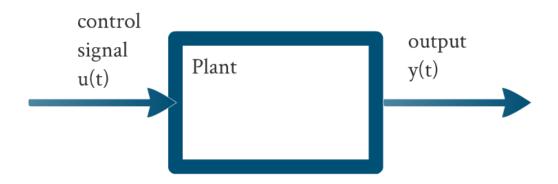
$$\mathcal{L}\left[\frac{ff(t)}{dt}\right] = s^{2}F(s) - \frac{f(t)}{dt} = \frac{s^{2}}{2}\frac{s^{2}}{2}\frac{s^{2}}{2}$$

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Mathematical Modelling of Plant



Solve:

Linear Time Invariant (LTI) Order n

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$$rac{Y(s)}{U(s)} = rac{b_m s^m + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$
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function:

$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$

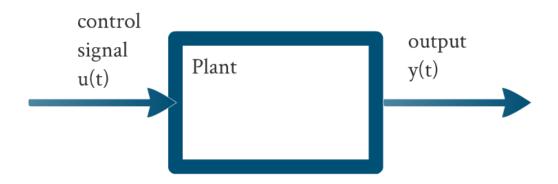
$$s^n + a_{n-1}s^{n-1} + \dots + a_0 = 0$$
 POLE

Characteristic

Equation

$$b_m s^m + ... + b_0 = 0$$
 ZERO

Mathematical Modelling of Plant



Solve:

Linear Time Invariant (LTI) Order n

$$\frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_0 y(t) = b_m \frac{d^m u(t)}{dt^m} + \ldots + b_0 u(t)$$





Laplace Transforms!

$$L\begin{bmatrix} \frac{\partial (f)}{\partial x} \\ -\frac{\partial (f)}{\partial x} \end{bmatrix} = d^{2}(x) - \frac{f(x)}{2}$$

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$$s^{n}Y(s) + a_{n-1}s^{n-1}Y(s) + \dots + a_{0}Y(s) = b_{m}s^{m}U(s) + \dots + b_{0}U(s)$$
$$\left(s^{n} + a_{n-1}s^{n-1} + \dots + a_{0}\right)Y(s) = \left(b_{m}s^{m} + \dots + b_{0}\right)U(s)$$



$$rac{Y(s)}{U(s)} = rac{b_m s^m + \ldots + b_0}{s^n + a_{n-1} s^{n-1} + \ldots + a_0}$$
 Characteristic function $s^n + a_{n-1} s^{n-1} + \ldots + a_0 = 0$ POLE $b_m s^m + \ldots + b_0 = 0$ ZERO

This lecture covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model