ELEC 207
Instrumentation and Control

10 – Instrument Frequency Response

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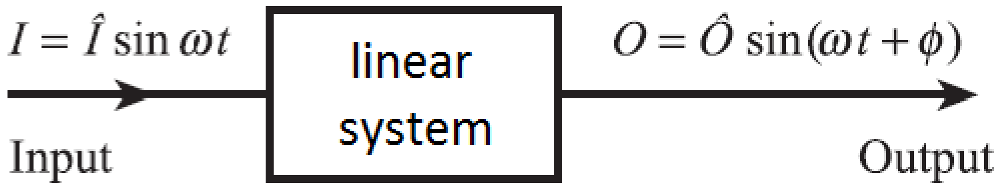
Dynamic characteristics of instruments

Frequency response

The dynamic response of a linear time-invariant system can be described also in the **frequency domain**:

- The response to a sinusoidal input is also sinusoidal, at the same frequency;



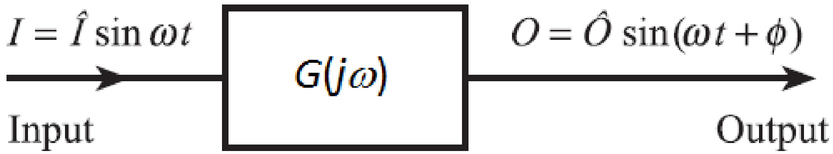
- So the response can be fully described by the change in amplitude and the change in phase, for different frequencies of the input signal.

Frequency response

Mathematical definition

The frequency response function is a **complex function** $G(j\omega)$, defined as the ratio between the Fourier transforms of the output and input signals (similar to phasors in AC circuit analysis):

$$G(j\omega) = \frac{O(j\omega)}{I(j\omega)}$$



- The **magnitude** of $G(j\omega)$ describes the change in the signal amplitude (e.g., amplification);
- The **phase** of $G(j\omega)$ describes the change in the signal phase.

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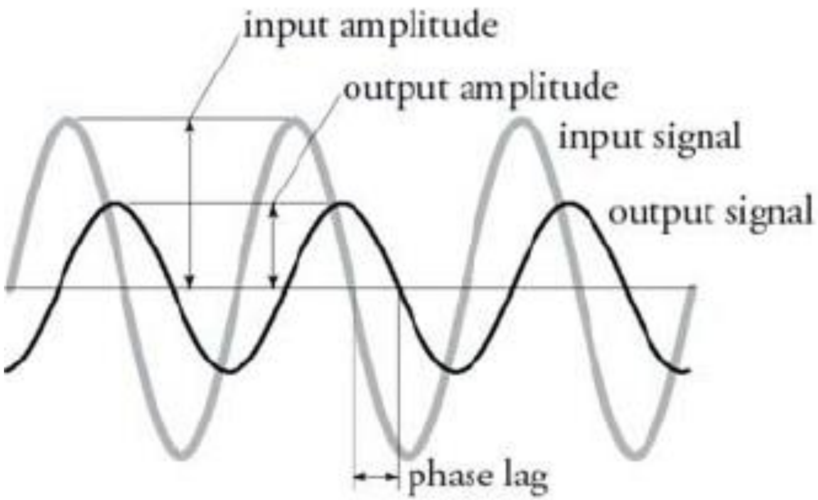
Frequency response

Physical interpretation

The information about the system provided in terms of frequency response is equivalent to the information provided in terms of step response:

- Higher input frequency means faster-changing signal:
  - Unless the system is a zero-order system (instantaneous response), the output will not follow exactly the input at high frequencies.



First-order system

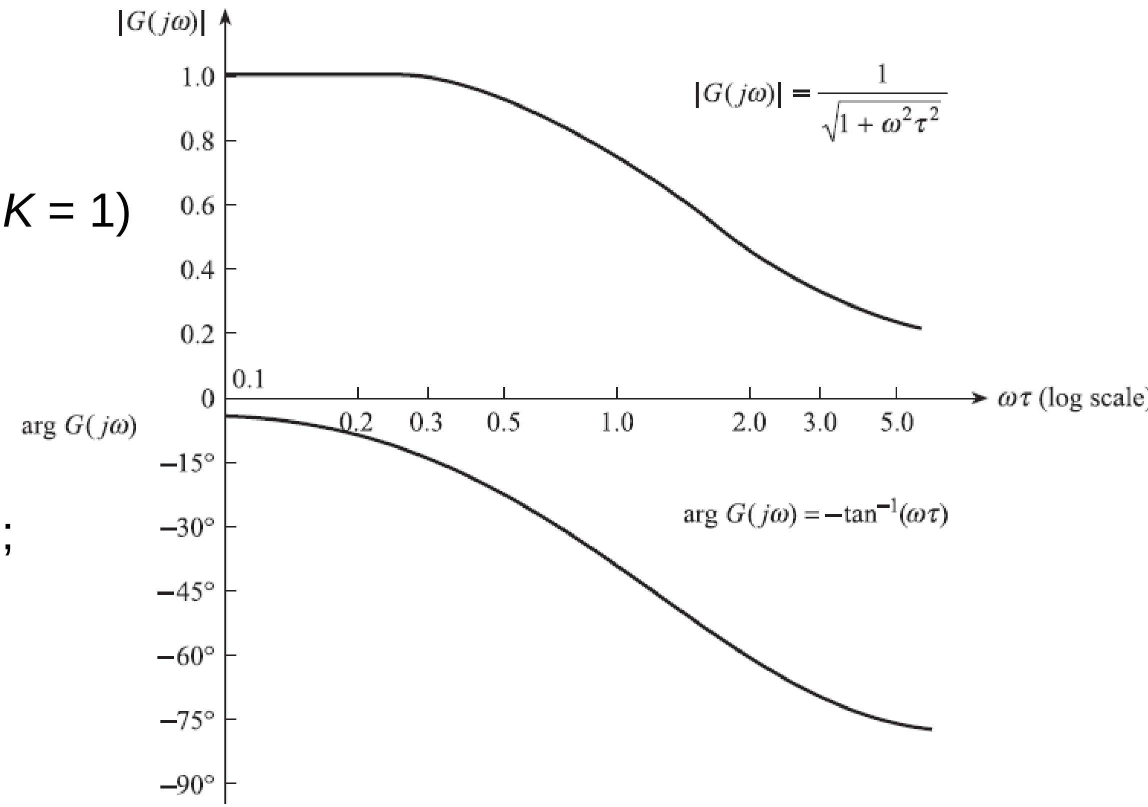
Frequency response

The response of a **first-order system** is characterised by its **time constant** τ:

$$G(j\omega) = \frac{1}{1 + j\omega\tau}$$

(assuming static sensitivity K = 1)

- At $\omega = 1/\tau$:
  - $|G(j\omega)| = 1/\sqrt{2}$ (-3dB);
  - $\arg(G(j\omega)) = -45^{\circ}$.



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First-order system

Physical interpretation

If an instrument has a first-order response with time constant τ, it means that:

- The instrument can be used at frequencies $\omega << 1/\tau$, **without any significant error** arising from the dynamic characteristics of the instrument;
- At the cut-off frequency $\omega = 1/\tau$, a significant error appears:
  - The output amplitude is $1/\sqrt{2}$ = 70.7% of the low-frequency amplitude;
  - The phase is shifted by 45^o;
- At frequencies $\omega >> 1/\tau$, the instrument is no longer able to follow the input signal and **the output tends to zero**.

Second-order system

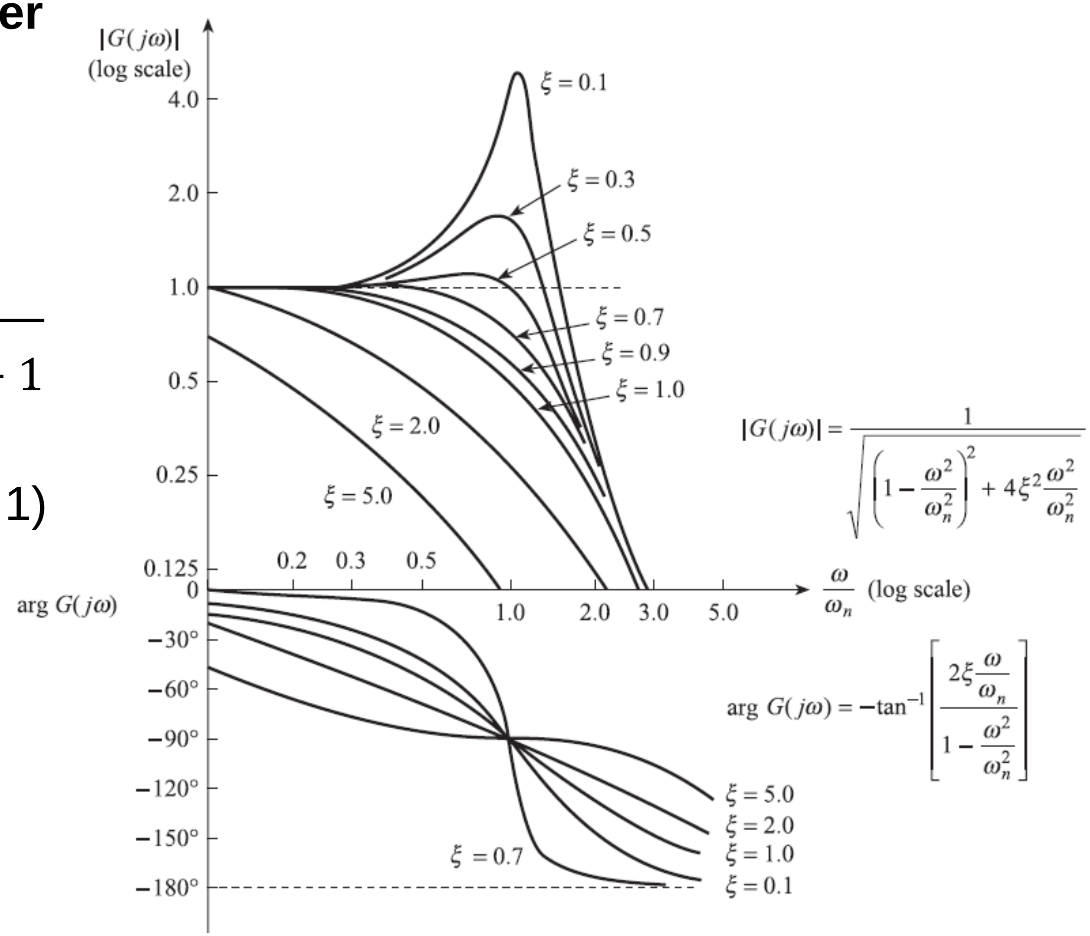
Frequency response

The response of a **second-order system** is characterised by its **natural frequency** ω_n and **damping ratio** ξ:

$$G(j\omega) = \frac{1}{\frac{1}{\omega_n^2}(j\omega)^2 + \frac{2\xi}{\omega_n}(j\omega) + 1}$$

(assuming static sensitivity $K = 1$)

- At $\omega = \omega_n$:
  - $|G(j\omega)| = 1/(2\xi)$;
  - $\arg(G(j\omega)) = -90^o$.



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Second-order system

Physical interpretation (1)

- If the damping ratio is smaller than 1/√2 (~0.7), the magnitude of the frequency response is greater than 1 at some frequencies:
  - The maximum magnitude is:

$$|G(j\omega)|_{max} = \frac{1}{2\xi\sqrt{1-\xi^2}}$$

  - The corresponding frequency is called **resonance frequency**:

$$\omega_r = \omega_n\sqrt{1-2\xi^2}$$

  - The resonance frequency tends to the natural frequency when ξ tends to zero; in this case, the maximum magnitude of $G(j\omega)$ tends to infinity.

Second-order system
Physical interpretation (2)

- If the damping ratio is greater than $1/\sqrt{2}$ (~0.7), the magnitude of the frequency response is never greater than 1:
  - There are **no resonance** frequencies;
  - The amplitude of the output signal is always lower or equal to the low-frequency amplitude;
  - However, the response becomes much **slower** for high values of ξ;
- Optimal values of ξ are usually around 0.7:
  - No large output amplitudes;
  - Fast response.

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References

Textbook: Principles of Measurement Systems, 4th ed.

For further explanation about the points covered in this lecture, please refer to the following chapters and sections in the **Bentley** textbook:

- Chapter 4, Sec. 4.2.2: **Sinusoidal response of first- and second-order elements**.

NOTE: Topics not covered in the lecture are not required for the exam.