ELEC 207 Part B

Control Theory Lecture 13: Frequency Design (1)

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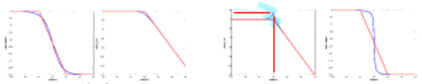
UNIVERSITY OF LIVERPOOL

Bode Plots for Second Order Systems - Revisited



$\zeta = 1$ $\qquad$ $\zeta = 0.1$

$$P(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$|P(j\omega)| = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}}$$

Differentiating with respect to ω^2:

$$\frac{1}{2}\frac{\omega_n^2}{\left((\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2\right)^{\frac{3}{2}}} \times \left(2(\omega_n^2 - \omega^2) + 4\zeta^2\omega_n^2\right) = 0$$

$$2(\omega_n^2 - \omega^2) + 4\zeta^2\omega_n^2 = 0$$

$$\omega^2 = \omega_n^2(1 - 2\zeta^2)$$

$$\omega = \omega_n\sqrt{1 - 2\zeta^2}$$

$$|P(j\omega)|_{\omega = \omega_n\sqrt{1-2\zeta^2}} = \frac{\omega_n^2}{\sqrt{(\omega_n^2 - \omega_n^2(1 - 2\zeta^2))^2 + 4\zeta^2\omega_n^2\omega_n^2(1 - 2\zeta^2)}}$$

$$= \frac{1}{2\zeta\sqrt{1 - \zeta^2}}$$

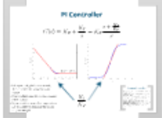
Remember: $\frac{\%OS}{100} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}}$

Frequency Response of PI

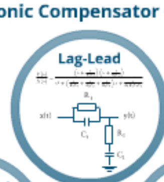
Consider a plant, P(s), and controller, C(s).
Open loop magnitude and phase responses are then:

$$20\log_{10}\left|C(j\omega)P(j\omega)\right| = 20\log_{10}\left|C(j\omega)\right| + 20\log_{10}\left|P(j\omega)\right|$$

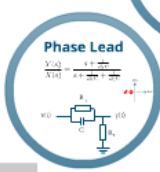
$$\angle C(j\omega)P(j\omega) = \angle C(j\omega) + \angle P(j\omega)$$

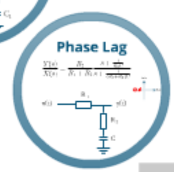


Electronic Compensator Circuits



- Passive components
- Need to be isolated using an Op-Amp with a gain of -1





This lecture covers:

- Combining Controller and Process Frequency Response
- Frequency Response of PI
- Phase-lead and Phase-lag compensators

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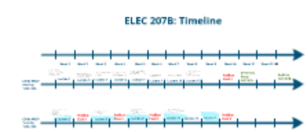
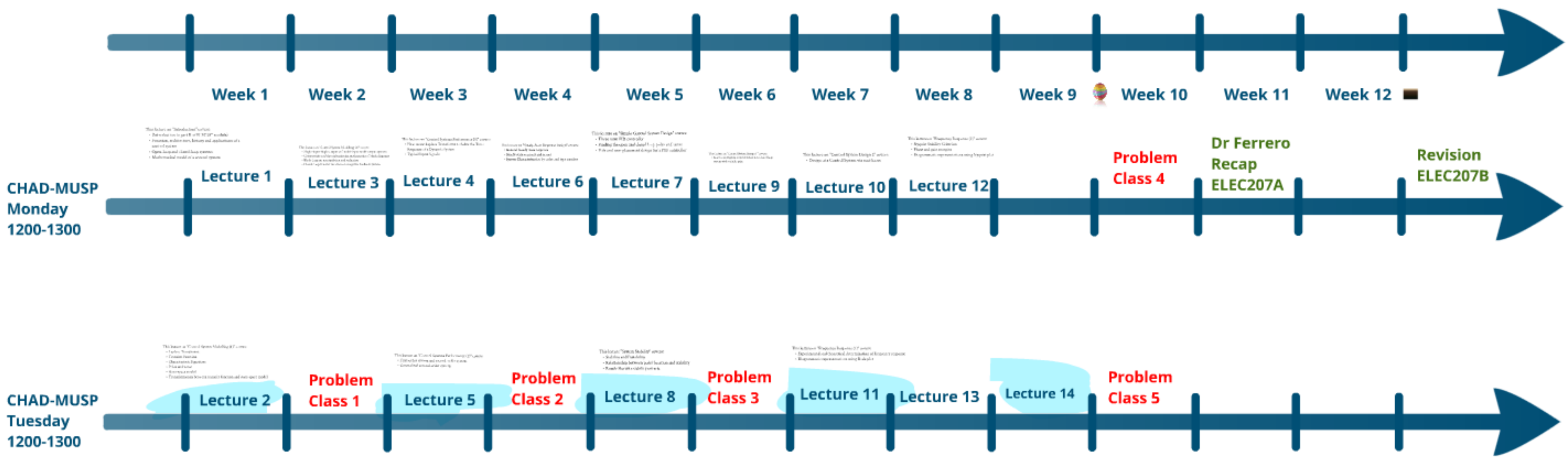
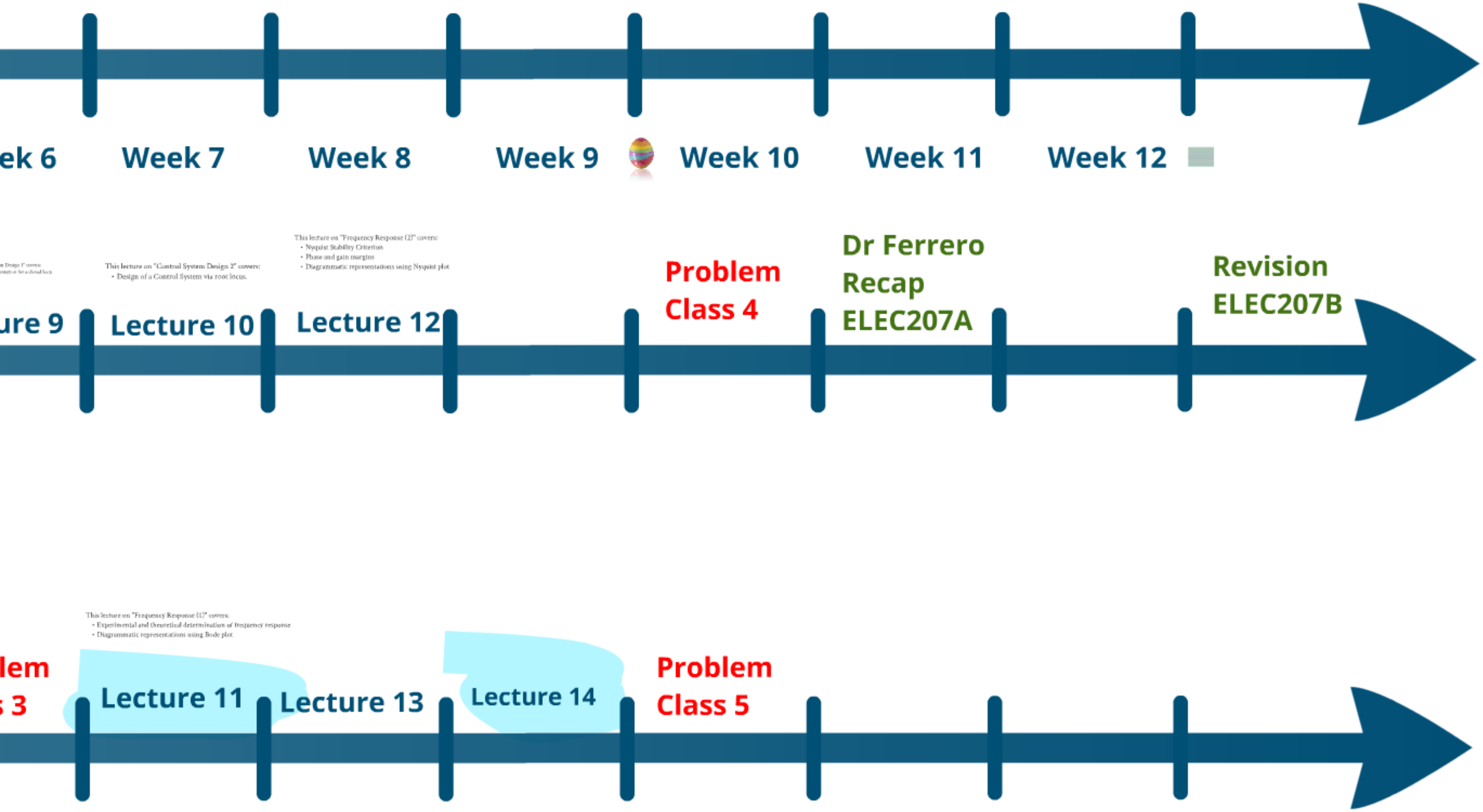


ELEC 207B: Timeline





ek 6
Week 7
Week 8
Week 9
Week 10
Week 11
Week 12
This lecture on "Control System Design 2" covers:
• Design of a Control System via root locus.
This lecture on "Frequency Response (2)" covers:
• Nyquist Stability Criterion
• Phase and gain margins
• Diagrammatic representations using Nyquist plot
ure 9
Lecture 10
Lecture 12
Problem Class 4
Dr Ferrero Recap ELEC207A
Revision ELEC207B
This lecture on "Frequency Response (1)" covers:
• Experimental and theoretical determination of frequency response
• Diagrammatic representations using Bode plot
lem
s 3
Lecture 11
Lecture 13
Lecture 14
Problem Class 5

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- Combining Controller and Process Frequency Response
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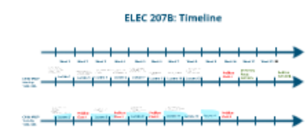
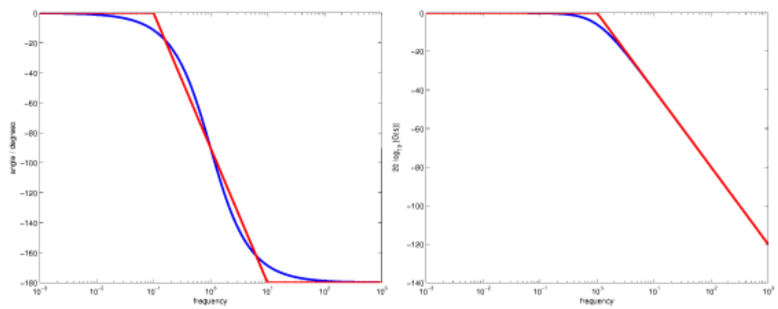
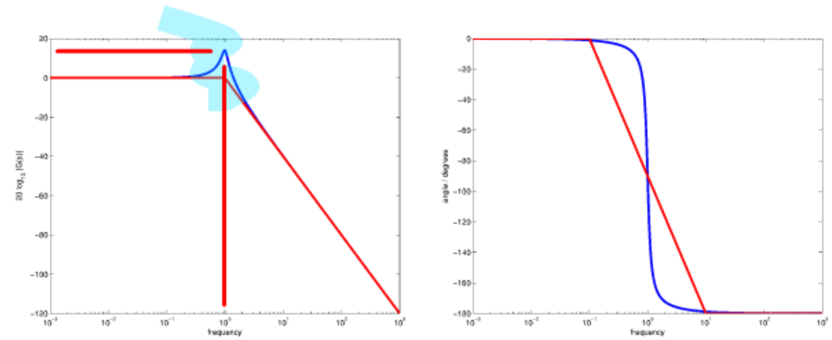


Bode Plots for Second Order Systems - Revisited



$\zeta = 1$



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Differentiating with respect to ω^2:

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$\sqrt{1-2\zeta^2}$ must exist for
their to be a peak frequency
(ie you won't get a peak if $\zeta > \frac{1}{\sqrt{2}}$

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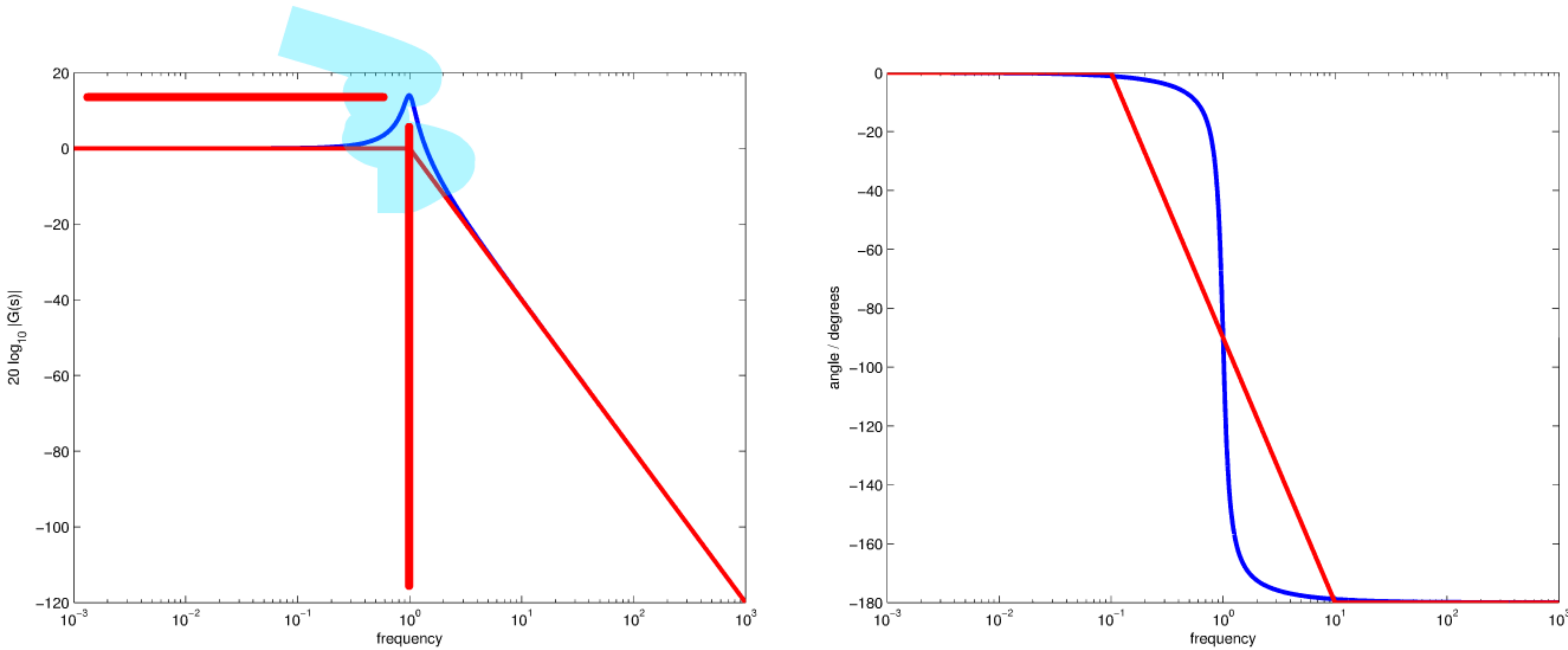

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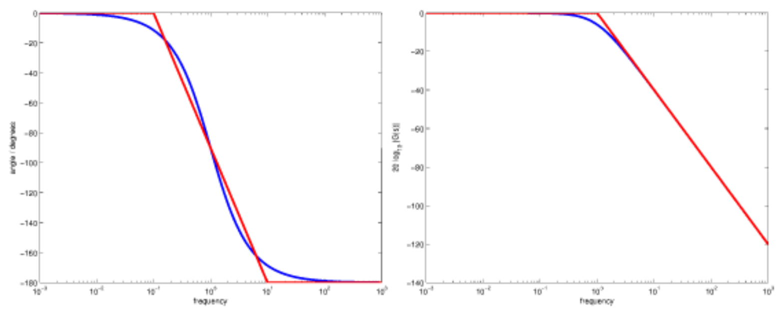

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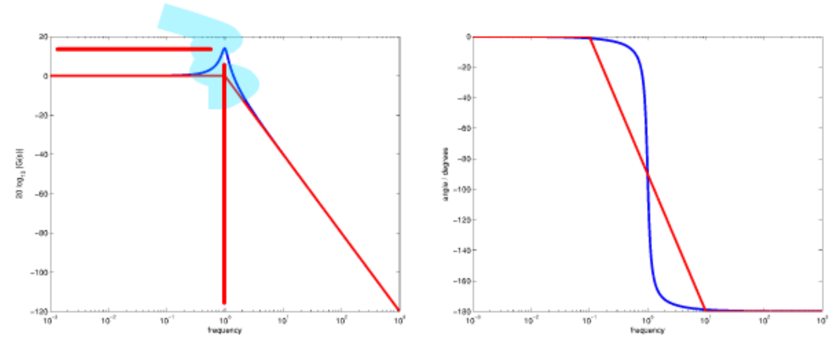

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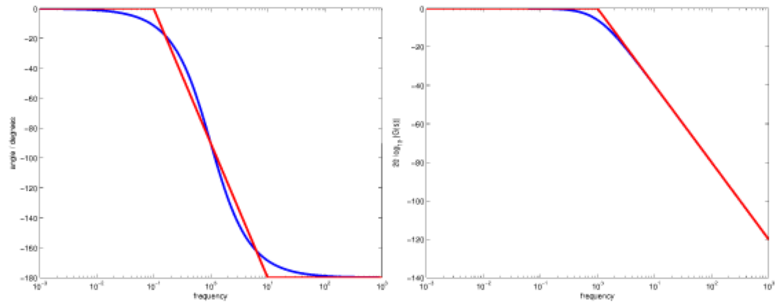

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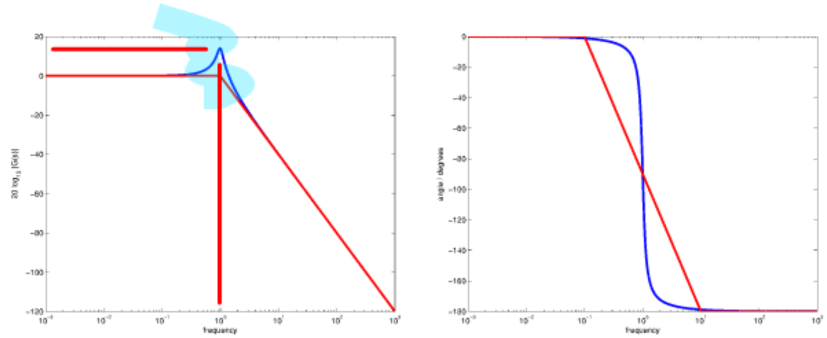

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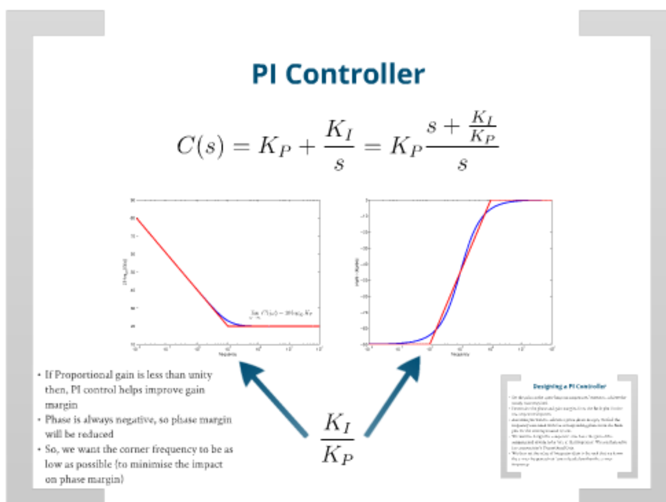
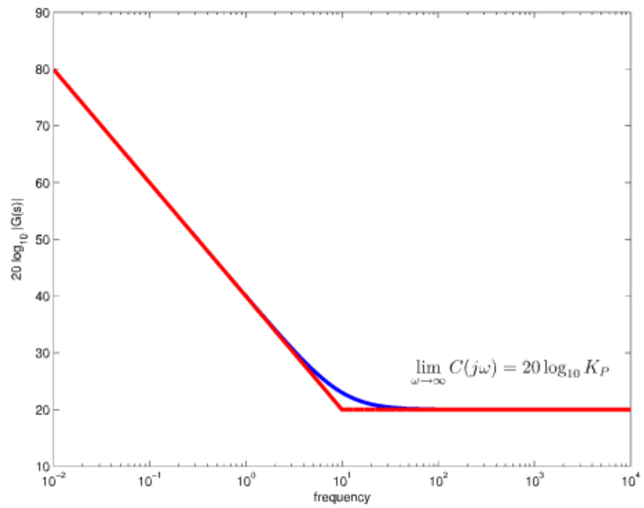
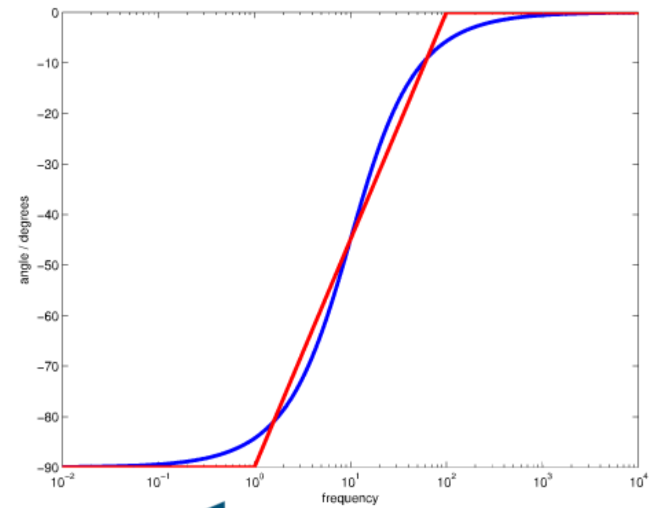


PI Controller

$$C(s) = K_P + \frac{K_I}{s} = K_P \frac{s + \frac{K_I}{K_P}}{s}$$



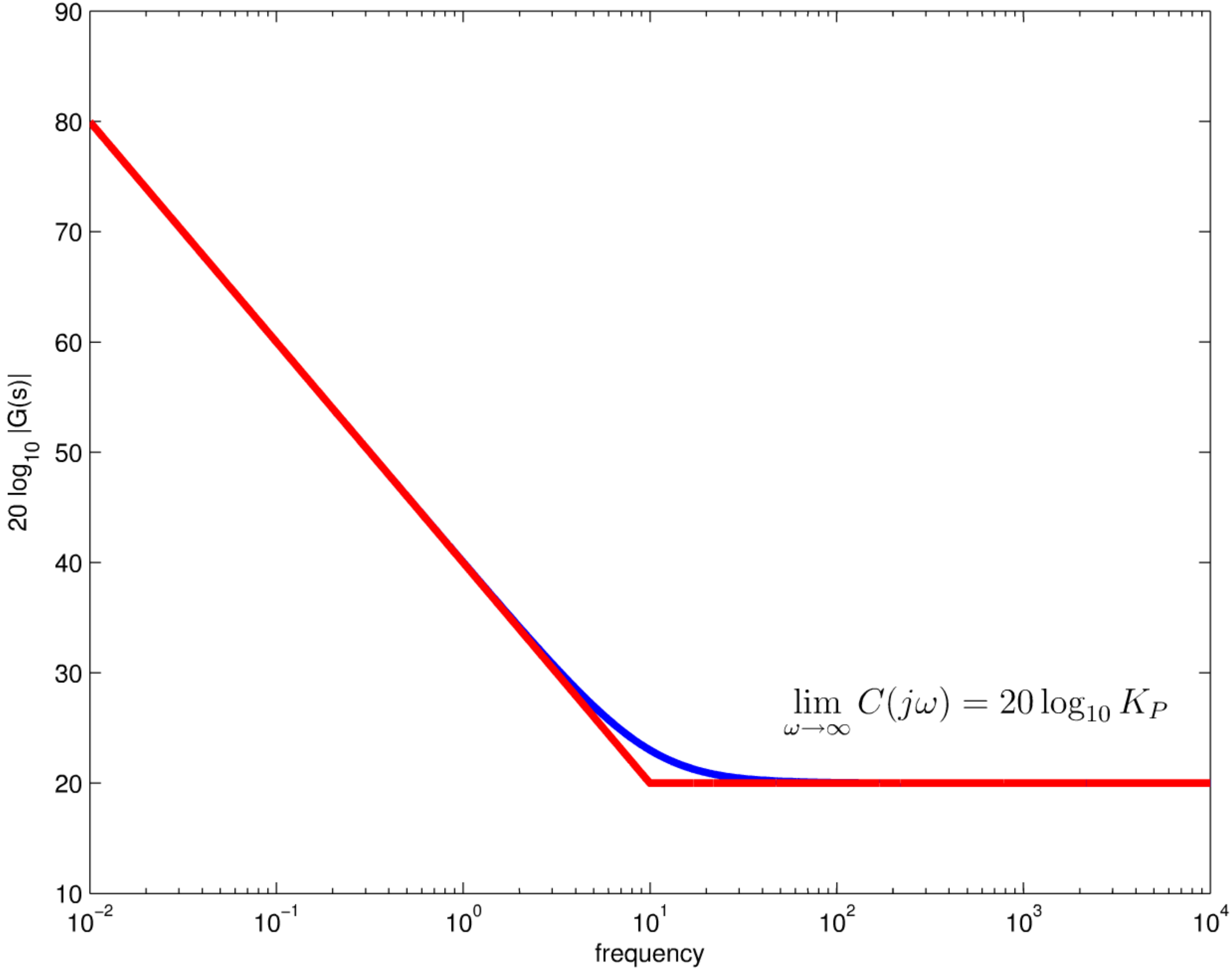


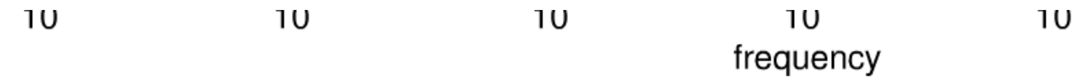
- If Proportional gain is less than unity then, PI control helps improve gain margin
- Phase is always negative, so phase margin will be reduced
- So, we want the corner frequency to be as low as possible (to minimise the impact on phase margin)

$$\frac{K_I}{K_P}$$

Designing a PI Controller

- Set the gain on the open-loop uncompensated system to achieve the steady-state required.
- Determine the phase and gain margins from the Bode plot for the uncompensated system.
- Assuming we want to achieve a given phase margin, we find the frequency associated with the corresponding phase from the Bode plot for the uncompensated system.
- We want to design the compensator to force the gain of the compensated system to be zero at that frequency. We can then solve for compensator's Proportional Gain
- We then set the value of Integrator Gain to be such that we know the corner frequency is at least a decade less than the corner frequency


90
80
70
60
50
40
30
20
10
20 $\log_{10}$ |G(s)|
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
frequency
$\lim_{\omega \to \infty} C(j\omega) = 20 \log_{10} K_P$



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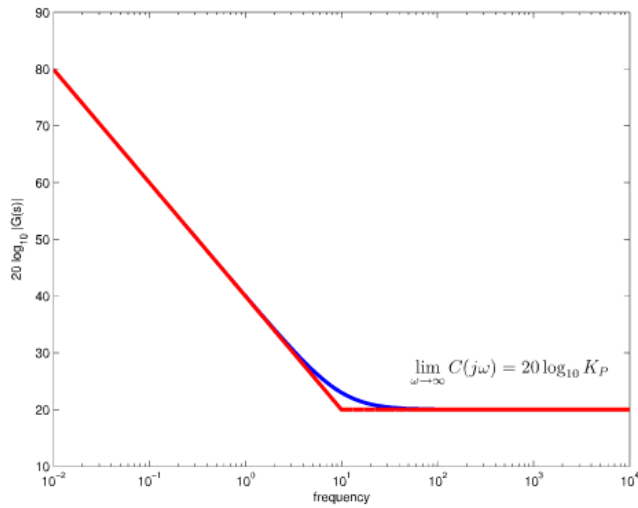

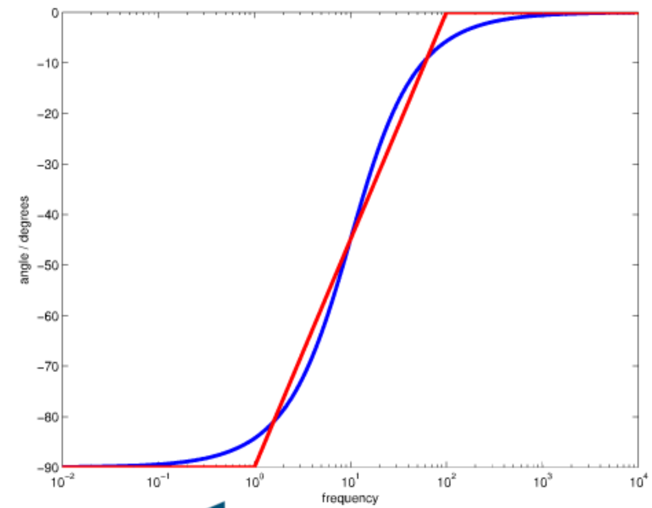

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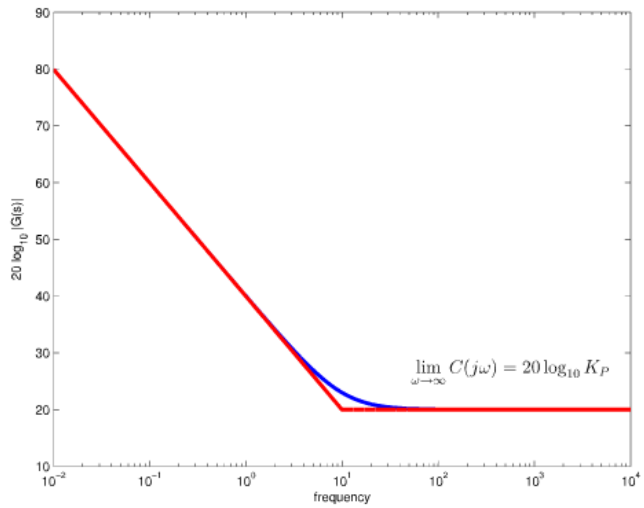

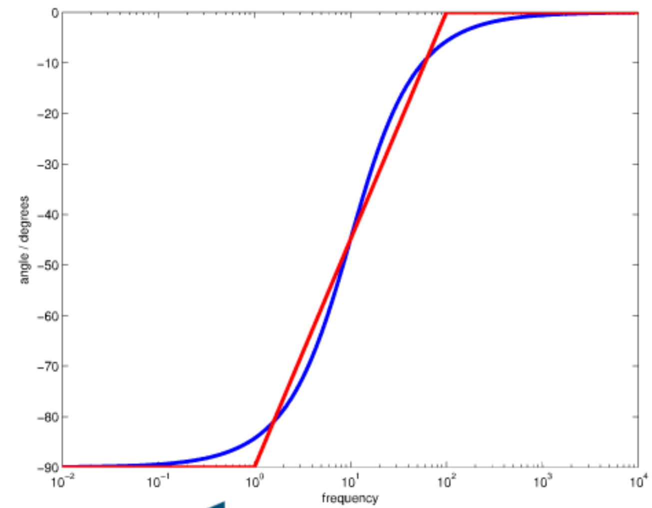

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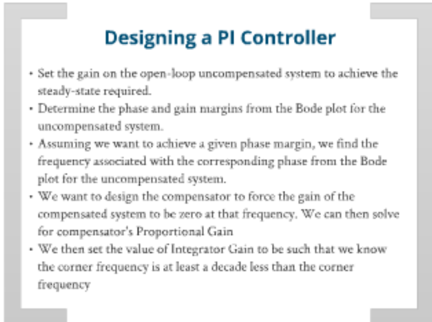

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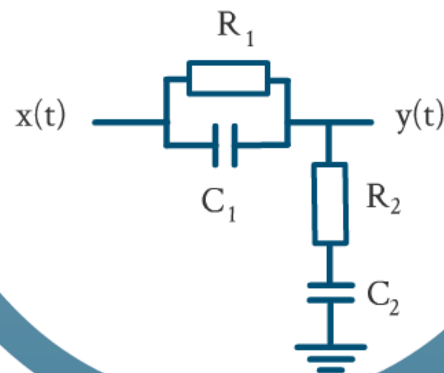
Electronic Compensator Circuits

Lag-Lead

$$\frac{Y(s)}{X(s)} = \frac{\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}\right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$



- Passive components
- Need to be isolated using an Op-Amp with a gain of -1

Phase Lead

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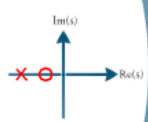
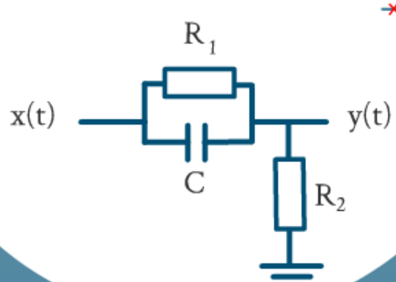
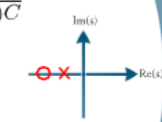
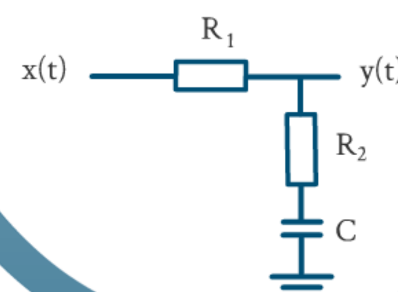




Phase Lag

$$\frac{Y(s)}{X(s)} = \frac{R_2}{R_1 + R_2} \frac{s + \frac{1}{R_2 C}}{s + \frac{1}{(R_1 + R_2) C}}$$





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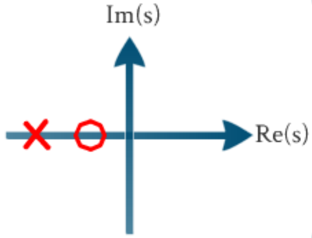
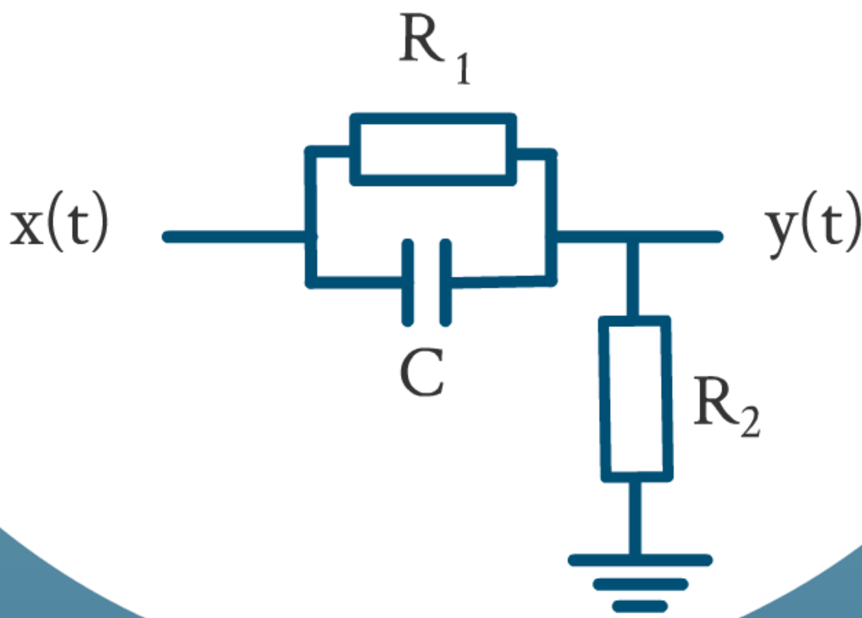
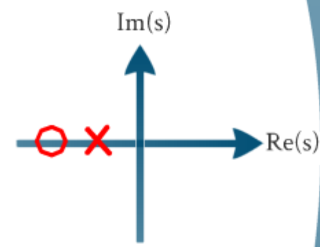
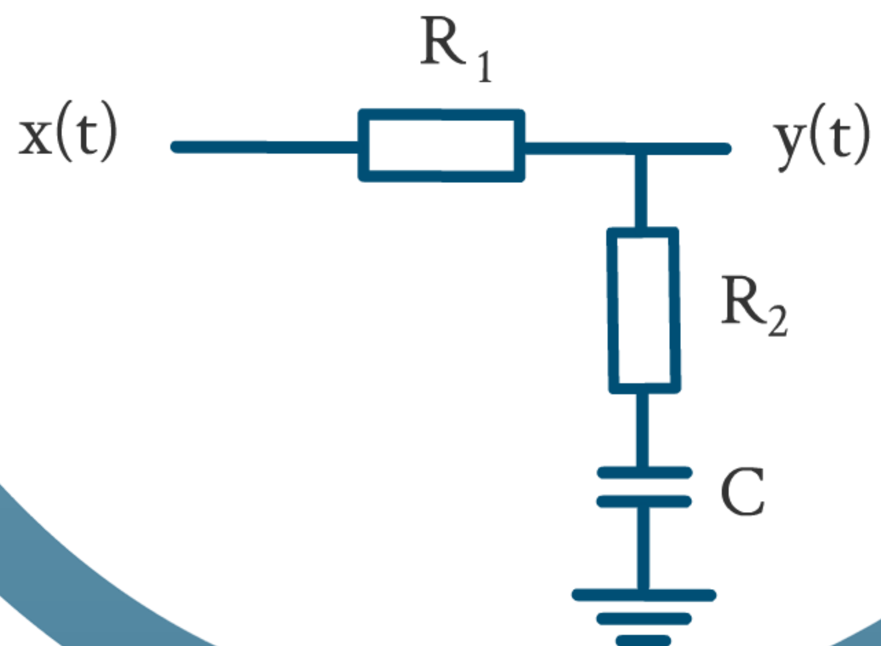




Phase Lag

$$\frac{Y(s)}{X(s)} = \frac{R_2}{R_1 + R_2} \frac{s + \frac{1}{R_2 C}}{s + \frac{1}{(R_1+R_2)C}}$$



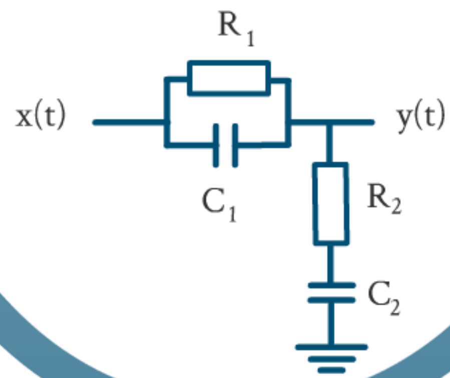


Electronic Compensator Circuits

- Passive components
- Need to be isolated using an Op-Amp with a gain of -1

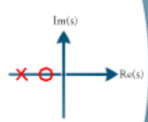
Lag-Lead

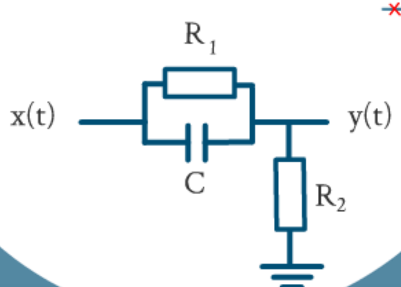
$$\frac{Y(s)}{X(s)} = \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$$



Phase Lead

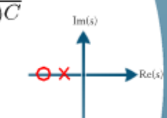
$$\frac{Y(s)}{X(s)} = \frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$$

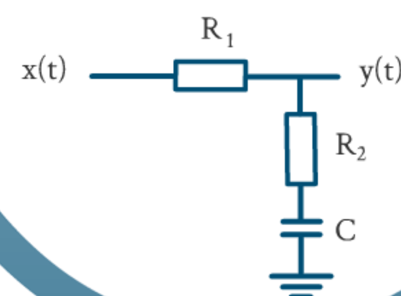




Phase Lag

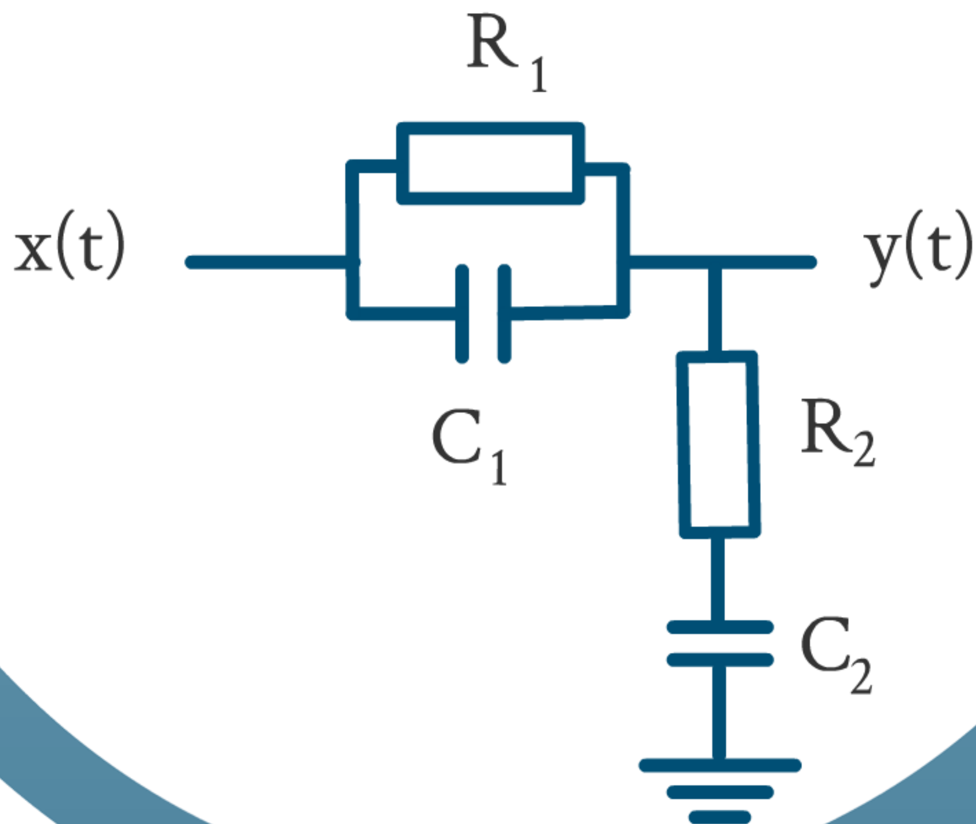
$$\frac{Y(s)}{X(s)} = \frac{R_2}{R_1+R_2}\,\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}}$$





Lag-Lead

$$\frac{Y(s)}{X(s)} = \frac{\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}\right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$

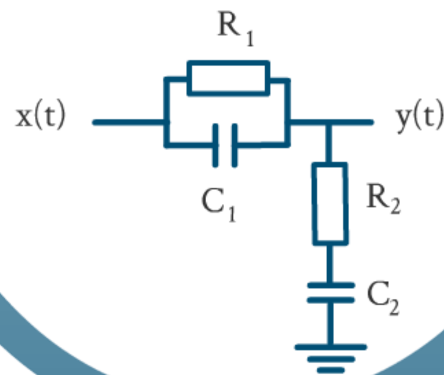


- Passive components
- Need to be isolated using an Op-Amp with a gain of -1

Electronic Compensator Circuits

Lag-Lead

$$\frac{Y(s)}{X(s)} = \frac{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}\right)s+\frac{1}{R_1R_2C_1C_2}}$$



- Passive components
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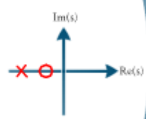
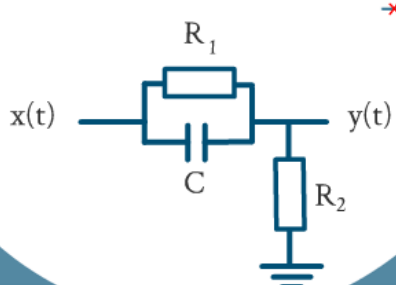
Phase Lead

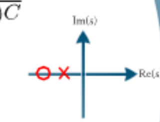
$$\frac{Y(s)}{X(s)} = \frac{s+\frac{1}{R_1C}}{s+\frac{1}{R_1C}+\frac{1}{R_2C}}$$

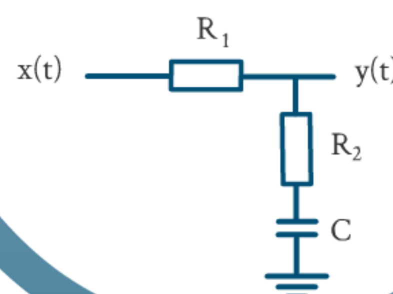




Phase Lag

$$\frac{Y(s)}{X(s)} = \frac{R_2}{R_1+R_2}\,\frac{s+\frac{1}{R_2C}}{s+\frac{1}{(R_1+R_2)C}}$$





This lecture covers:

- Combining Controller and Process Frequency Response
- Frequency Response of PI
- Phase-lead and Phase-lag compensators