ELEC 207
Instrumentation and Control

Example – Signal Processing

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Current measurement

Filtering, sampling, A/D conversion, data transmission

A **current transducer** is employed to measure a current waveform containing several frequency components, in the range from 0 to 400 Hz.

A 16-bit **analog-to-digital converter (ADC)** is then used to convert the analog voltage output of the transducer into a digital signal. The sampling frequency of the converter is 20 kHz.

The digital signal is then **transmitted** to a remote receiver, using Hamming code.

The transducer and the digital transmission line are placed in a **noisy environment**, where electromagnetic interference affects the measurement signal by adding noise in the frequency range above 1 kHz.

Problem definition

Questions (1)

A **signal processing element** has to be designed to be connected to the transducer output, in order to meet the following specifications:

1) A **first-order low-pass (anti-aliasing) RC filter** is connected to the transducer output to remove high-frequency noise which would cause significant aliasing errors:
   - Based on the ADC sampling frequency, calculate the **filter time constant** $\tau = RC$ required to have a cut-off frequency one decade below the bandwidth limit of the converter input signal;

2) Calculate the **relative error** affecting the amplitude of the highest-frequency components in the measurement signal, due to the RC filter designed above;

Problem definition

Questions (2)

3) The full measurement range of the current transducer is 0…±55 A, and the ADC is designed in such a way that its range corresponds exactly to the transducer range:

   - Calculate the **resolution** of the digital current measurement after the analog signal is converted by the ADC;

4) Each 16-bit word generated by the ADC is transmitted using the Hamming code:

   - Calculate the required number of **additional bits** that must be added to the 16-bit word in order to detect and correct single bit errors in the transmission.

Solution of Q1

RC filter design

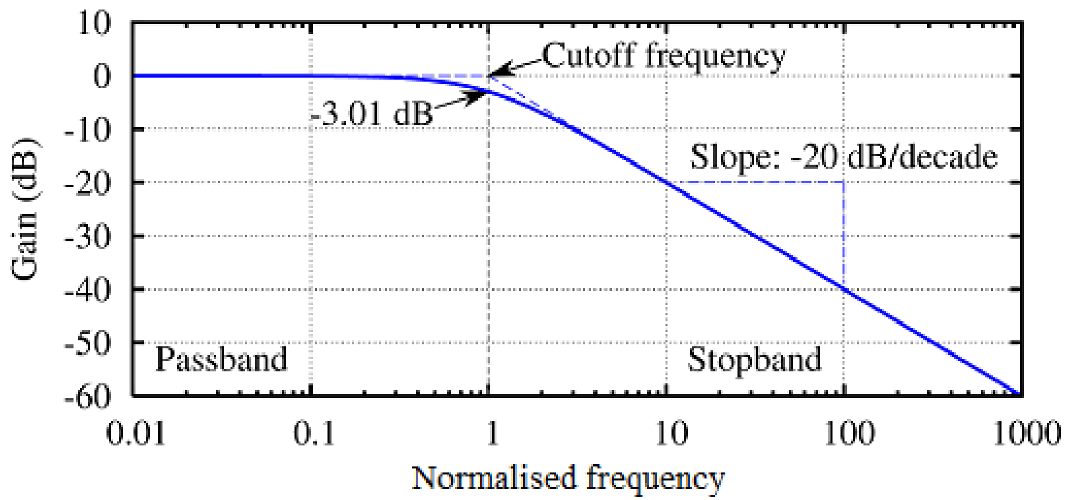
The **bandwidth limit** of a signal that can be correctly sampled at a sampling frequency f_s = 20 kHz is:

$$f_{max} = f_s/2 = 10\text{ kHz} \qquad (\text{so the bandwidth is } f < f_{max})$$

- The **cut-off frequency** of the RC filter is required to be one decade below the bandwidth limit to guarantee that all frequency components ≥ f_{max} are significantly attenuated:

$$f_c = \frac{f_{max}}{10} = 1\text{ kHz}$$

$$\tau = \frac{1}{\omega_c} = \frac{1}{2\pi f_c} = 159\ \mu s$$



Solution of Q2

Attenuation of signal components

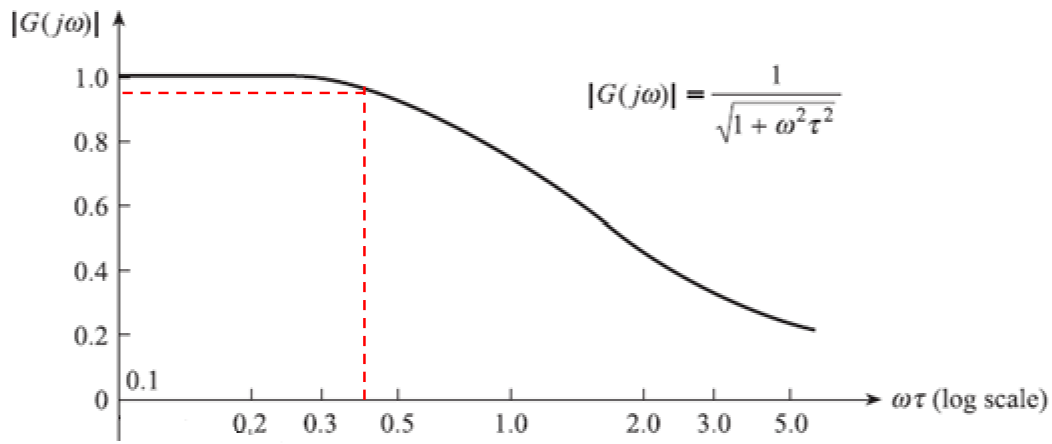
The low-pass filter attenuates also frequency components below the cut-off frequency:

- The measurement signal contains frequency components up to 400 Hz:

$$400 \text{ Hz} = 0.4\ f_c$$

- The **magnitude of the filter frequency response** at $0.4f_c$ is:

$$|G(j\omega)| = 0.93$$



- Therefore the components of the signal at the highest frequencies are affected by an **error in their amplitude** up to 7%.

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Solution of Q3

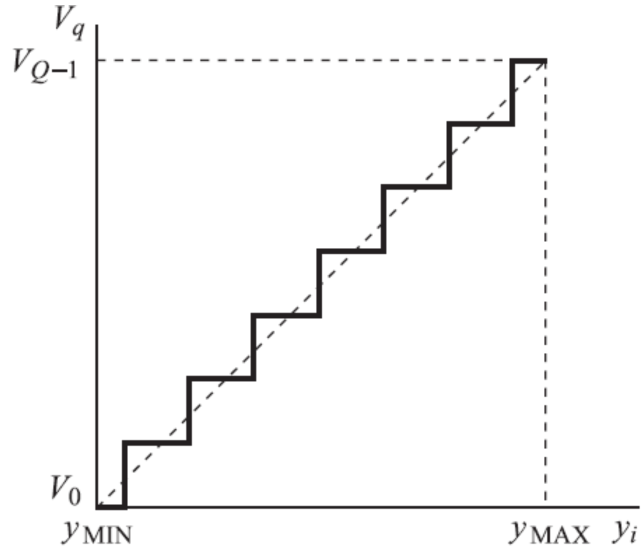
Resolution of the ADC

The ADC converts the analog signal into 16-bit digital numbers:

- The total number of **different available codes** is:

$$2^{16} = 65536$$

- These codes cover the whole measurement range of the transducer (from -55 A to +55 A), therefore the **resolution** of the digital current measurement is:



$$\frac{55\ \mathrm{A} - (-55\ \mathrm{A})}{2^{16}} = \frac{110\ \mathrm{A}}{65536} = 1.7\ \mathrm{mA}$$

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Solution of Q4

Hamming code

The required number of additional bits (*r*) in order to **correct single bit errors** in a word composed of 16 information bits (*k*) is calculated as:

$$2^r \geq n + 1 \quad \Longrightarrow \quad r \geq \log_2(n+1)$$

where *n* = *k*+*r*, with *k* = 16. Therefore:

$$r \geq \log_2(r+17)$$

- Since *r* is an integer number, the minimum value of *r* can be found by different trials:
  - In this case, the **minimum value of additional bits** is *r* = 5;
  - The whole transmitted word will be composed of 16+5 = 21 bits.