ELEC 207
Instrumentation and Control

Example – Uncertainty Calculation

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Current measurement

Transducer specifications (1)

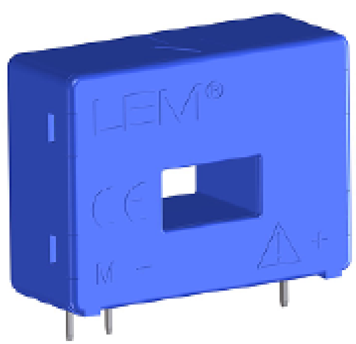
A DC current is measured by the following **current transducer**:

LEM

Current Transducer LA 25-P

I_{PN} = 25 A

For the electronic measurement of currents: DC, AC, pulsed..., with galvanic separation between the primary circuit and the secondary circuit.



Electrical data

I_{PN}	Primary nominal rms current	25	A
I_{PM}	Primary current, measuring range	0 .. ± 55	A

Current measurement

Transducer specifications (2)

The following **static characteristics** are provided in the data sheet:

I_{SN}	Secondary nominal rms current		25		mA
K_N	Conversion ratio		1 : 1000		
U_C	Supply voltage (± 5 %)		± 12 .. 15		V
I_C	Current consumption		10 (@ ± 15 V) + I_S		mA
	Accuracy - Dynamic performance data				
X	Accuracy @ I_{PN}, T_A = 25 °C	@ ± 15 V (± 5 %)	± 0.95		%
		@ ± 12 .. 15 V (± 5 %)	± 1.25		%
ε_L	Linearity error		< 0.15		%
			Typ	Max	
I_O	Offset current @ I_P = 0, T_A = 25 °C			± 0.2	mA
I_{OM}	Magnetic offset current [1] @ I_P = 0 and specified R_M, after an overload of 3 × I_{PN}			± 0.3	mA
I_{OT}	Temperature variation of I_O	0 °C .. + 70 °C	± 0.1	± 0.5	mA
		- 25 °C .. + 85 °C	± 0.1	± 0.6	mA

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Problem definition

Questions (1)

Based on the transducer specifications, answer the following questions:

1) Calculate the **transducer accuracy** when the transducer is used in the following conditions:
   - Supply voltage: $\pm$15 V;
   - Operating temperature: 25 °C;
   - Measured current: approx. 5 A.

2) Convert the accuracy calculated in question 1) into a **standard uncertainty**, assuming that the accuracy corresponds to a maximum error and that the error is distributed over the whole range $\pm a$ with uniform probability.

Problem definition

Questions (2)

3) Repeated current measurements were carried out in the same conditions, and they provided the following results, whose variation can only be attributed to random errors: 5.2 A, 5.0 A, 4.8 A, 5.1 A, 4.7 A, 5.3 A, 5.2 A. In these conditions, calculate the best estimate of the current value and the **standard uncertainty** contribution of a single measurement, arising from random errors.

4) Combine this uncertainty with the contribution arising from the instrument accuracy calculated in question 2), in order to calculate the **overall (combined) uncertainty** of the current measurement.

5) Assuming a Gaussian probability distribution for the current values, calculate the confidence interval (expanded uncertainty) corresponding to a **confidence level** of 95.5%.

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Solution of Q1

Accuracy calculation

In the transducer data sheet, the accuracy is provided as a percentage of the nominal current (25 A).

Accuracy @ I_{PN}, T_A = 25 °C	@ ± 15 V (± 5 %)	± 0.95	%
	@ ± 12 .. 15 V (± 5 %)	± 1.25	%

- In the specified conditions ($\pm$15 V power supply), the **accuracy** is therefore:

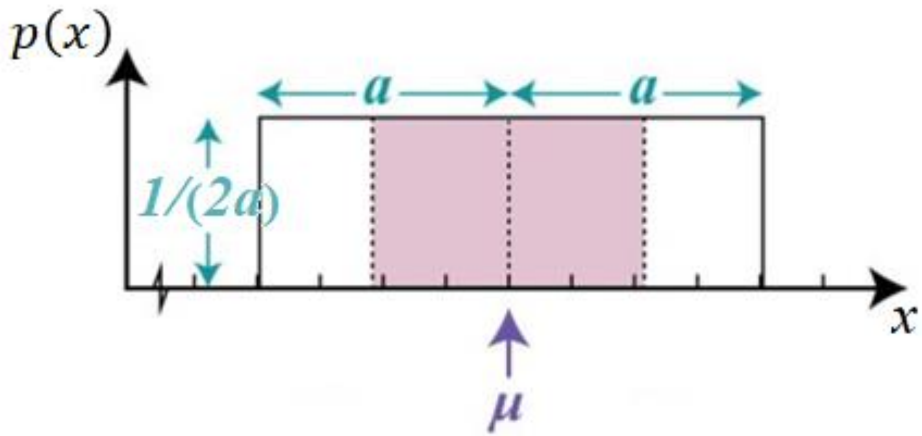
$$a = 0.95\% \cdot \ 25 \text{ A} = 0.238 \text{ A}$$

NOTE: This estimate of the accuracy does not depend on the measured current, but in other instrument specifications it may depend on the measured value.

Solution of Q2

Uncertainty corresponding to the accuracy

The accuracy is interpreted as a maximum error, and the error is assumed to be distributed over the interval ±a with uniform probability:



$$\sigma = \sqrt{\int_{\mu-a}^{\mu+a} (x-\mu)^2 p(x)\mathrm{d}x} = \sqrt{\frac{a^2}{3}}$$

- Therefore the **standard uncertainty corresponding to the accuracy** is:

$$u_{acc} = \frac{a}{\sqrt{3}} = \frac{0.238\ \mathrm{A}}{\sqrt{3}} = 0.137\ \mathrm{A}$$

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Solution of Q3

Uncertainty estimate from repeated measurements

The **mean value** of all measurement results represents the best estimate of the measurand value:

$$\bar{x} = \mu = \frac{1}{N}\sum_{i=1}^{N} x_i = 5.043\ \mathrm{A}$$

The **standard deviation** of all measurement results represents the best estimate of the uncertainty of each measurement due to random errors alone:

$$u_{ran} = \sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \mu)^2} = 0.223\ \mathrm{A}$$

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Note on Q3

Uncertainty of each measurement and of the average measurement

<ins>NOTE</ins>: The standard deviation calculated in the previous slide is an estimate of the uncertainty of **a single measurement** x_i, not of the average $\bar{x}$.

- E.g., the result of the first measurement is:

$$x_1 = 5.2 \text{ A } \pm 0.223 \text{ A}$$

The uncertainty of **the average measurement** $\bar{x}$ is calculated as follows:

$$u(\bar{x}) = \frac{u(x_i)}{\sqrt{N}} = \frac{\sigma}{\sqrt{N}} = 0.084 \text{ A}$$

- The average measurement result is therefore:

$$\bar{x} = 5.043 \text{ A } \pm 0.084 \text{ A}$$

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Solution of Q4 & Q5

Combined uncertainty and confidence level

The two uncertainty contributions (instrument accuracy and random errors) can be **combined** in order to obtain the overall uncertainty:

$$u = \sqrt{u_{acc}^2 + u_{ran}^2} = \sqrt{(0.137\ \mathrm{A})^2 + (0.223\ \mathrm{A})^2} = 0.262\ \mathrm{A}$$

Assuming a Gaussian distribution, a 95.5% **confidence level** is achieved by choosing an interval equal to ±2σ:

- The **expanded uncertainty** is then:

$$U = 2u = 0.524\ \mathrm{A}$$