ELEC 207 Part B

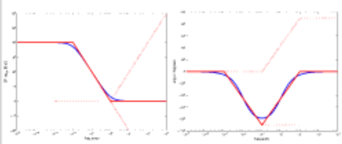
Control Theory Lecture 12: Frequency Response (2)

Prof Simon Maskell
CHAD-G68
s.maskell@liverpool.ac.uk
0151 794 4573

Frequency Response Techniques

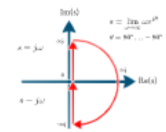
Bode Plot

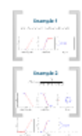
$$G(s) = \frac{s+1}{s+\frac{1}{100}}$$



Nyquist Plot

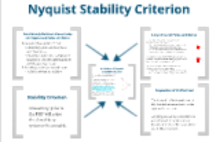
Bode plot shows magnitude and phase of G(s)
Nyquist plot shows G(s) on the complex plane





Phase and Gain Margins

Nyquist Stability Criterion



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If there are purely imaginary poles,
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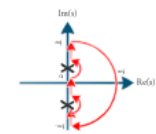

This lecture covers:

- Nyquist Stability Criterion
- Phase and gain margins
- Diagrammatic representations using Nyquist plot

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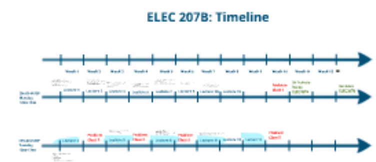
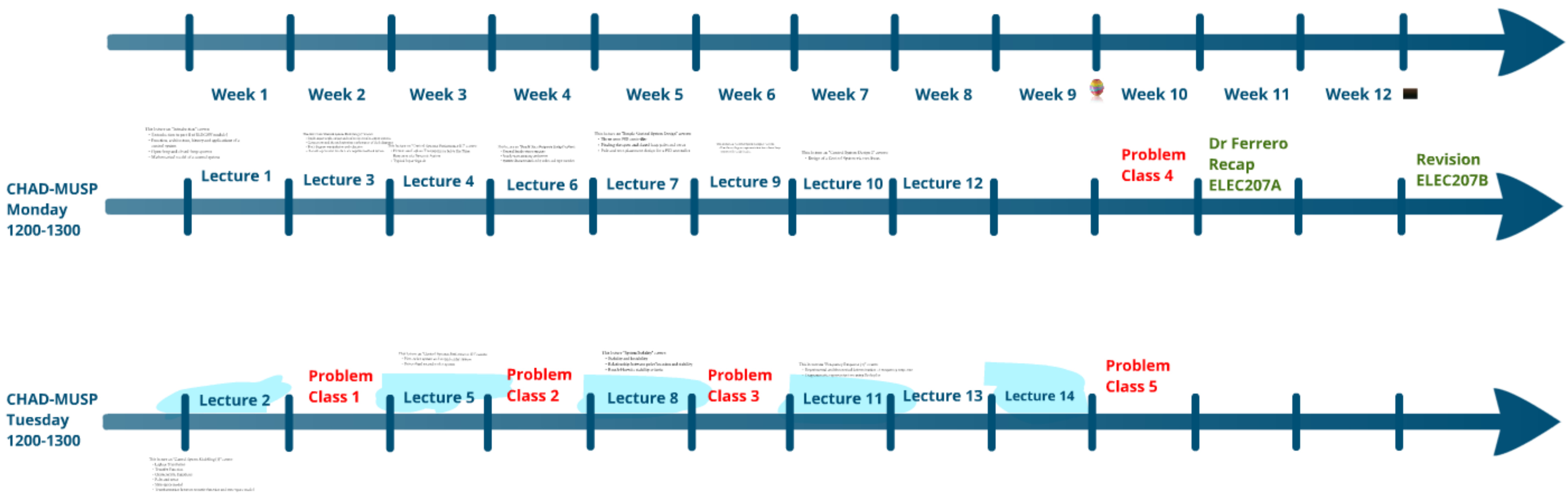
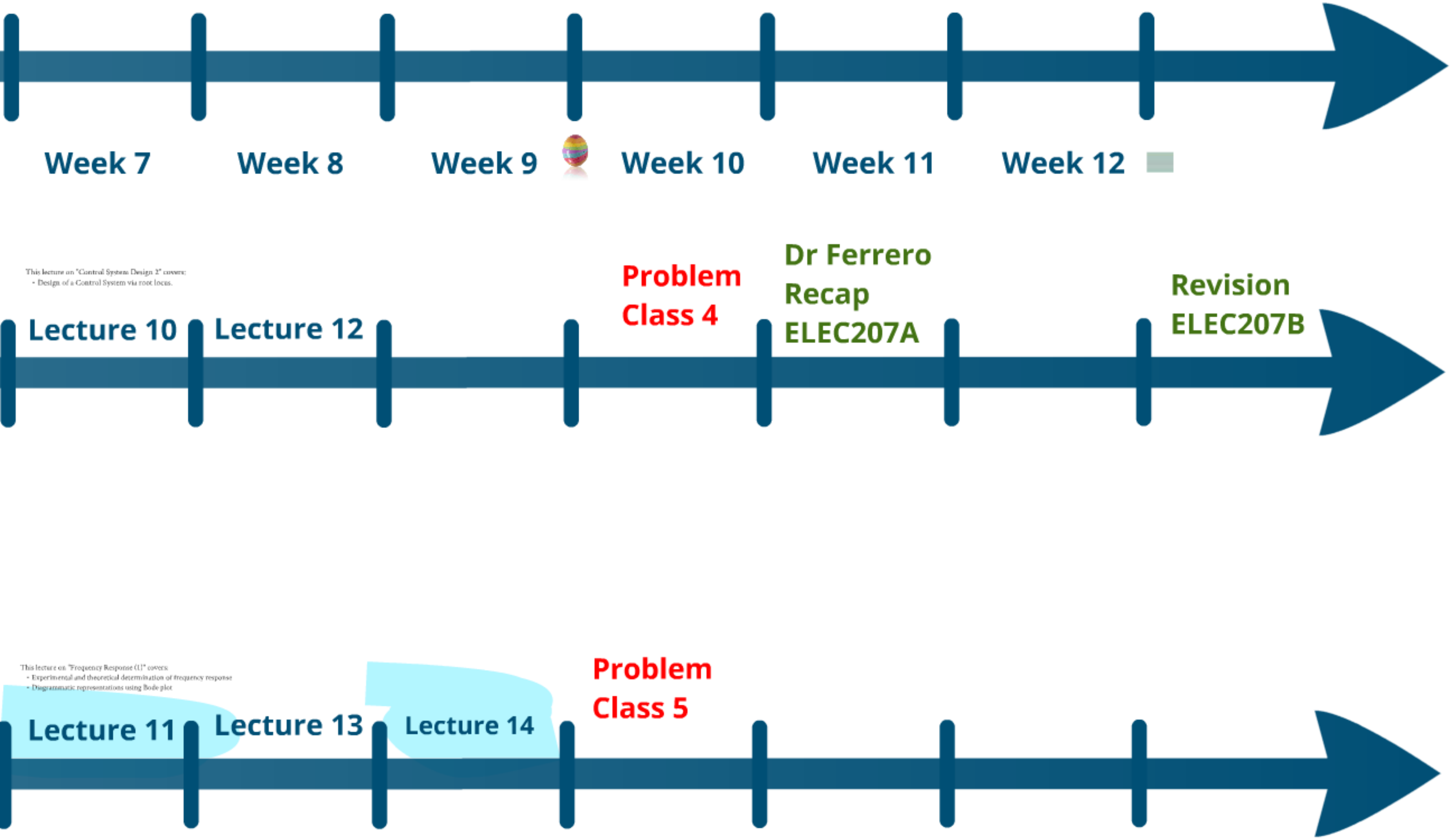


ELEC 207B: Timeline




Week 7
Week 8
Week 9
Week 10
Week 11
Week 12
This lecture on "Control System Design 2" covers:
• Design of a Control System via root locus.
Lecture 10
Lecture 12
Problem Class 4
Dr Ferrero Recap ELEC207A
Revision ELEC207B
This lecture on "Frequency Response (1)" covers:
• Experimental and theoretical determination of frequency response
• Diagrammatic representations using Bode plot
Lecture 11
Lecture 13
Lecture 14
Problem Class 5


CSC

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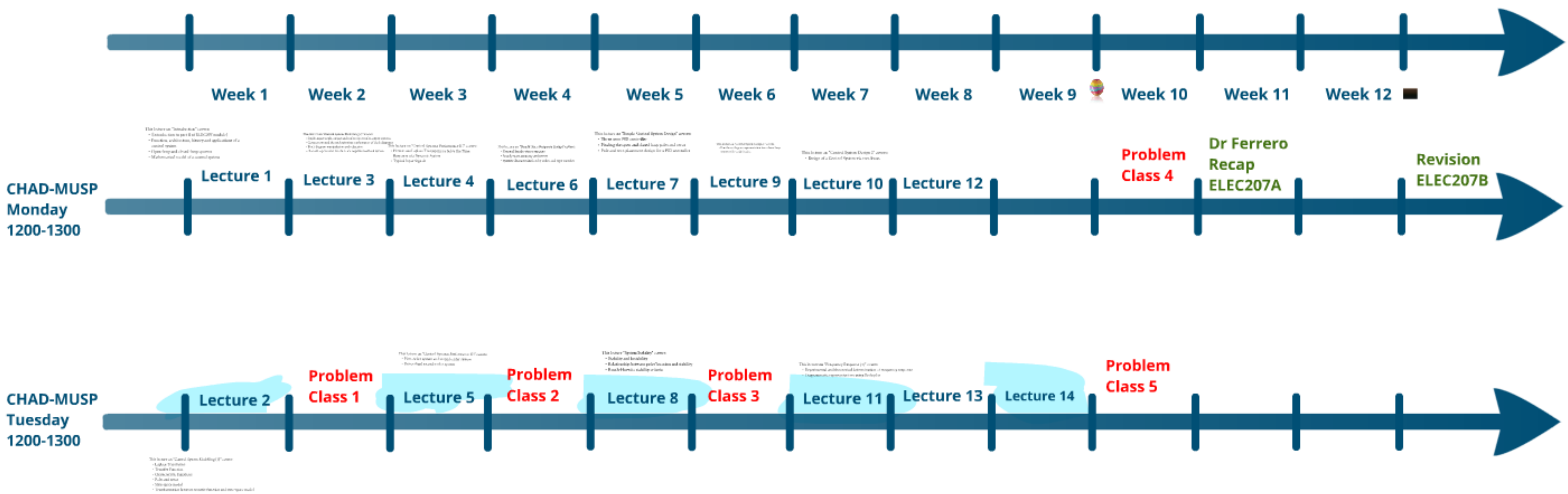

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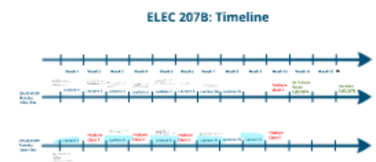

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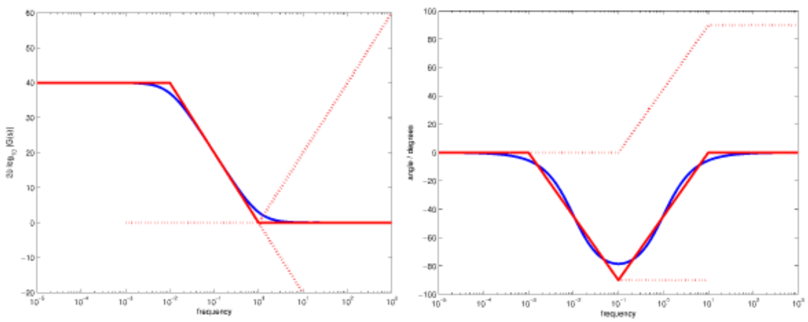

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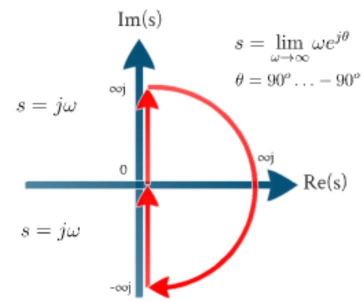

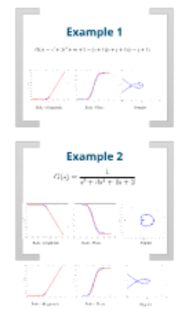

Phase and Gain Margins

We can be interested in relative stability, ie how close a system is to going unstable

- Phase Margin is the extra phase required to make the phase -180° at a gain of zero dB [or $-20\log_{10}(K)$ if K≠1];
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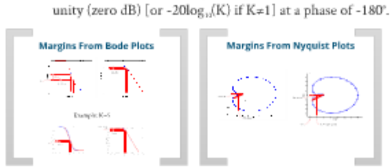

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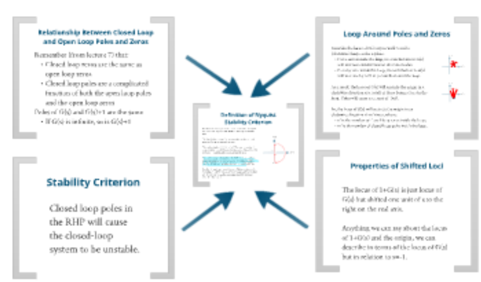

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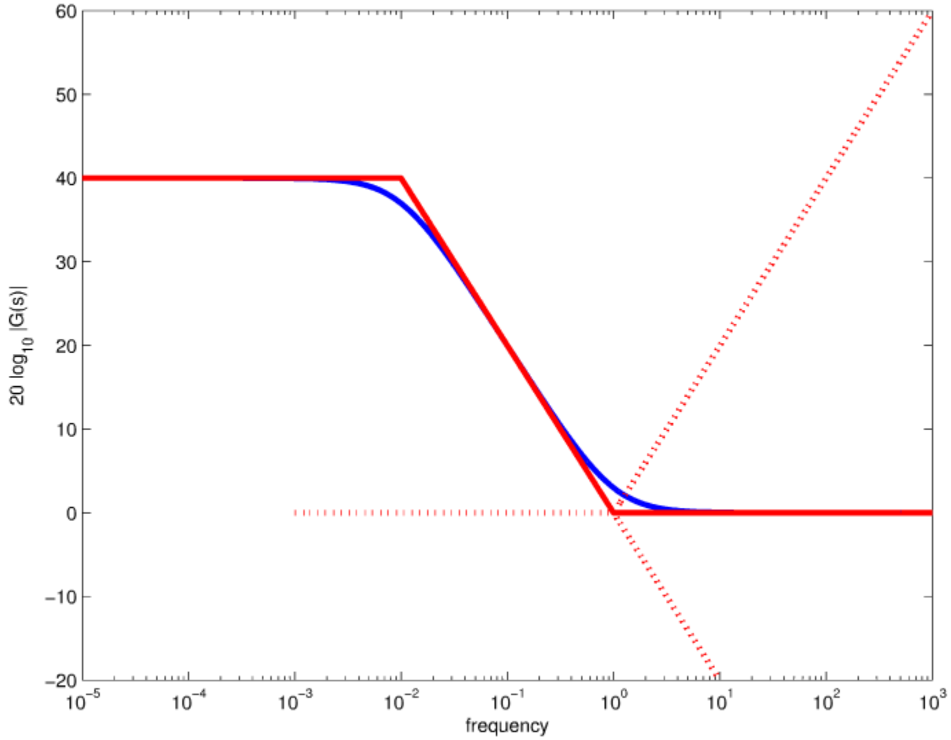

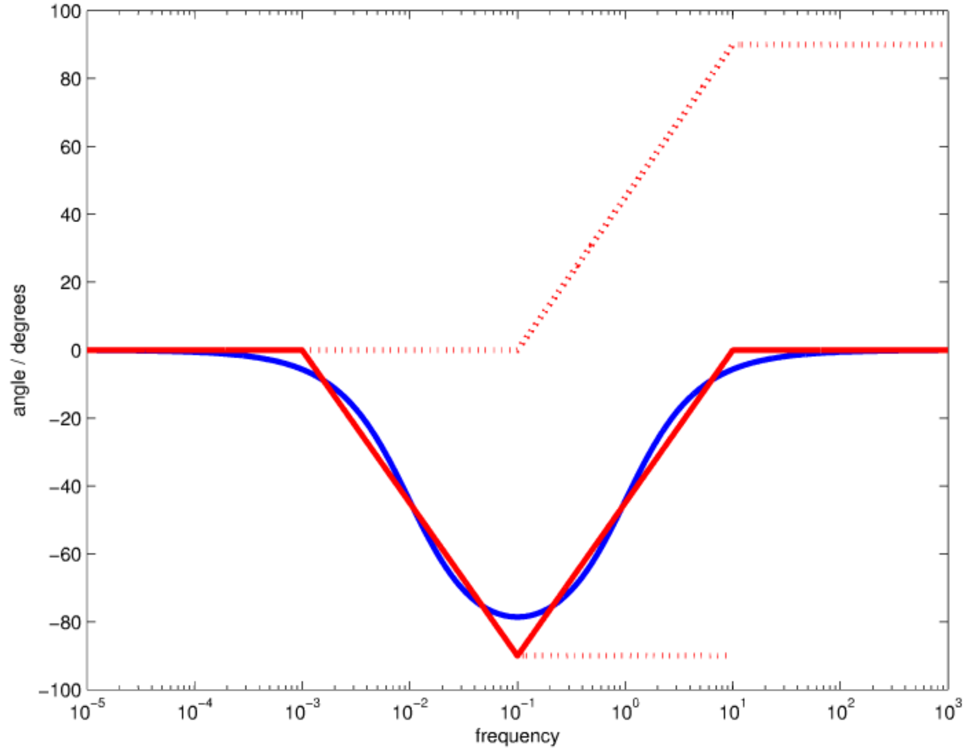

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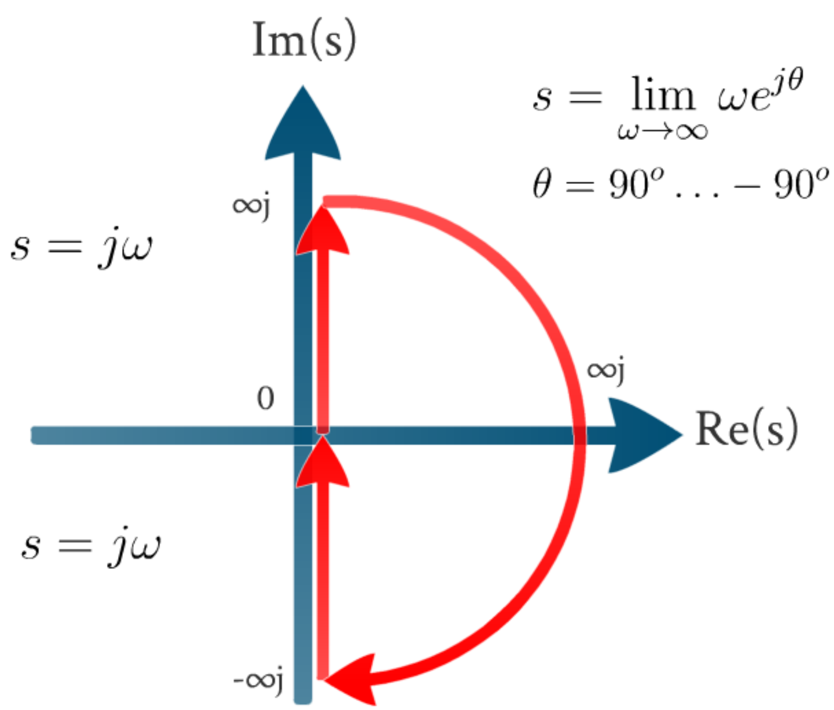

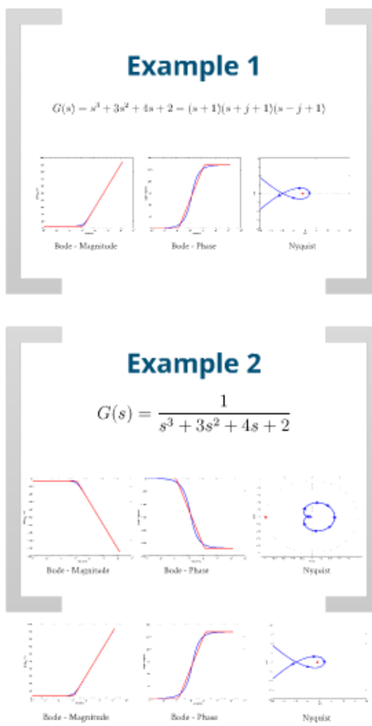

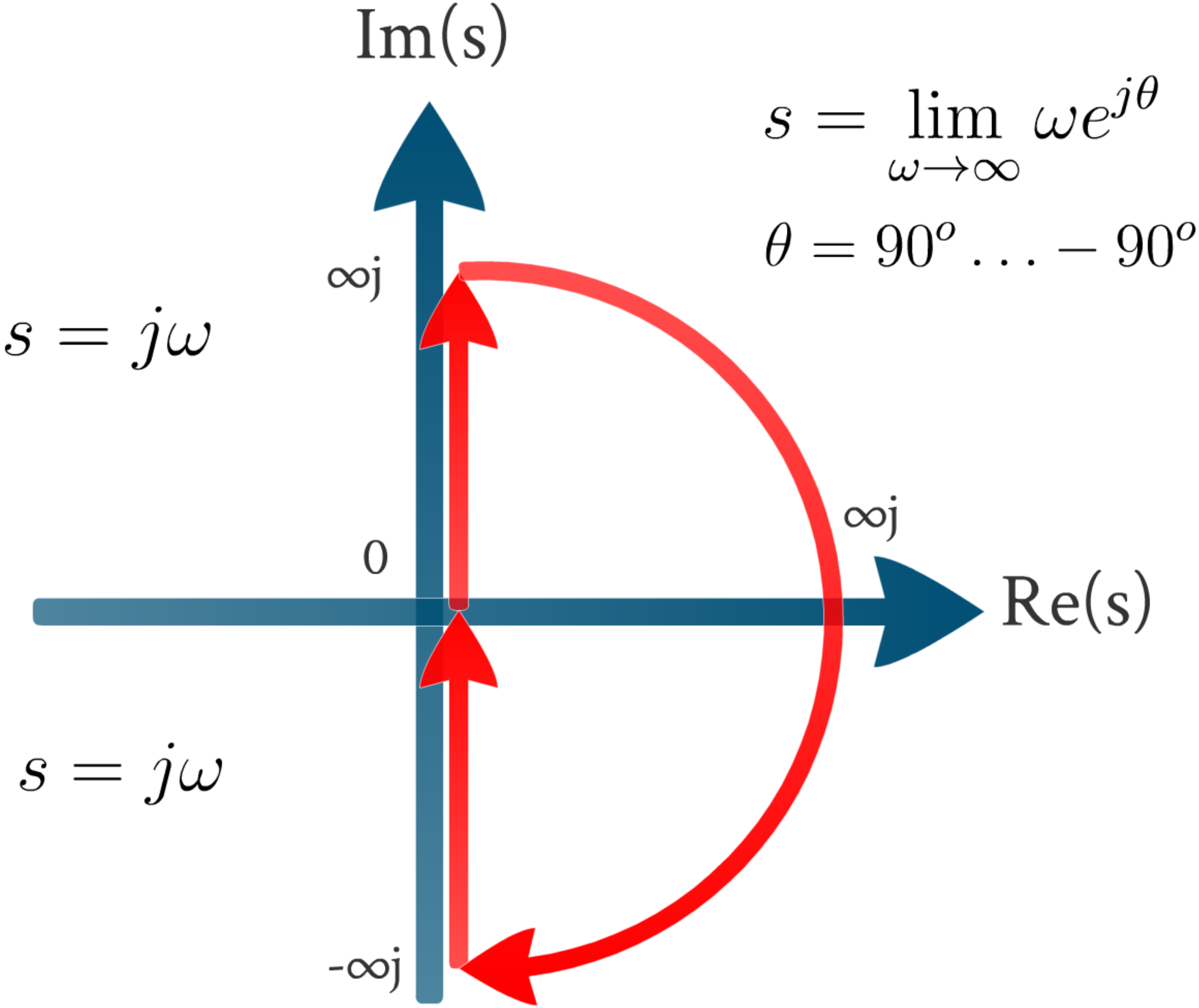
Im(s)
$s = \lim_{\omega\to\infty} \omega e^{j\theta}$
$\theta = 90^o \ldots - 90^o$
∞j
$s = j\omega$
∞j
0
Re(s)
$s = j\omega$
-∞j

Nyquist Plot

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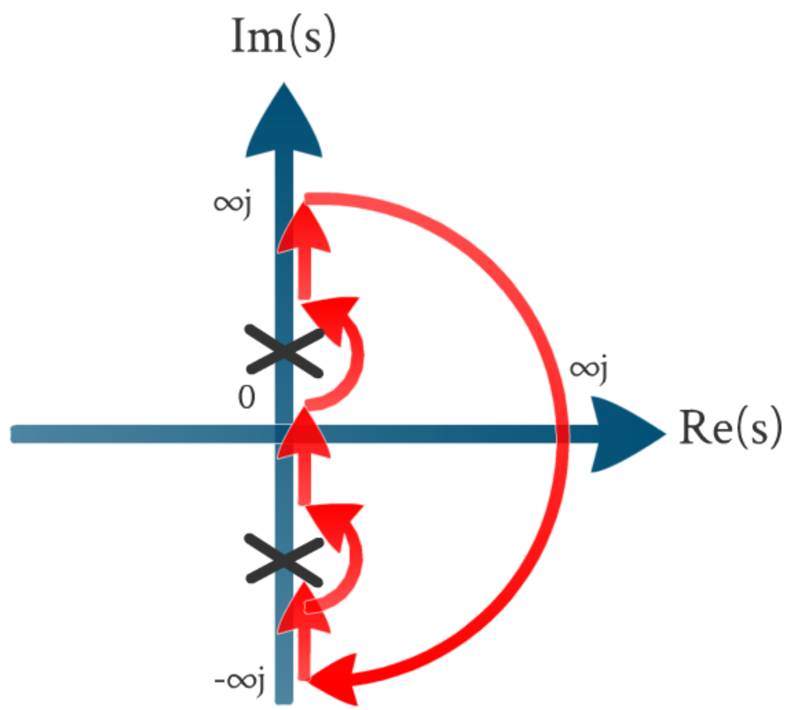

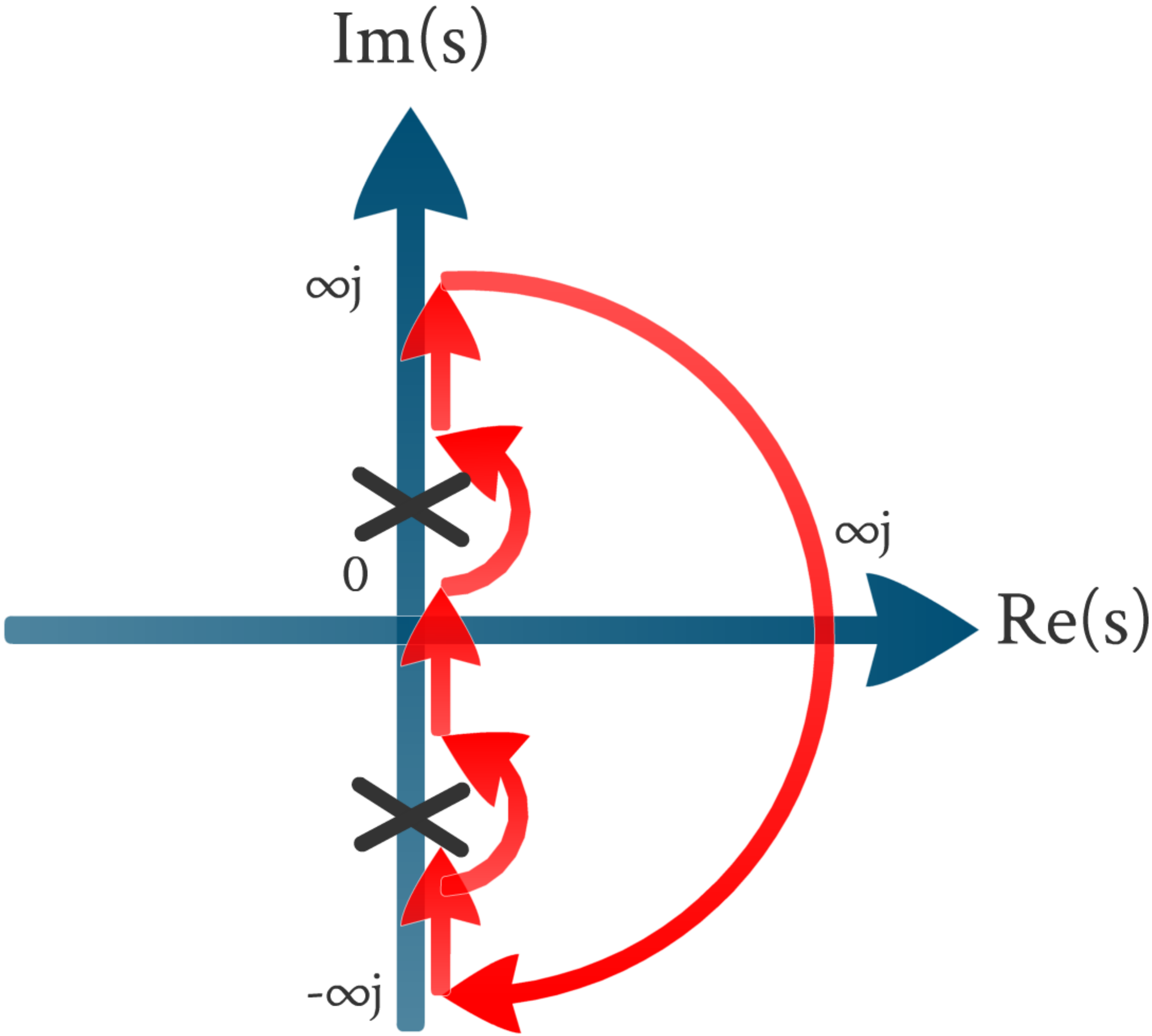

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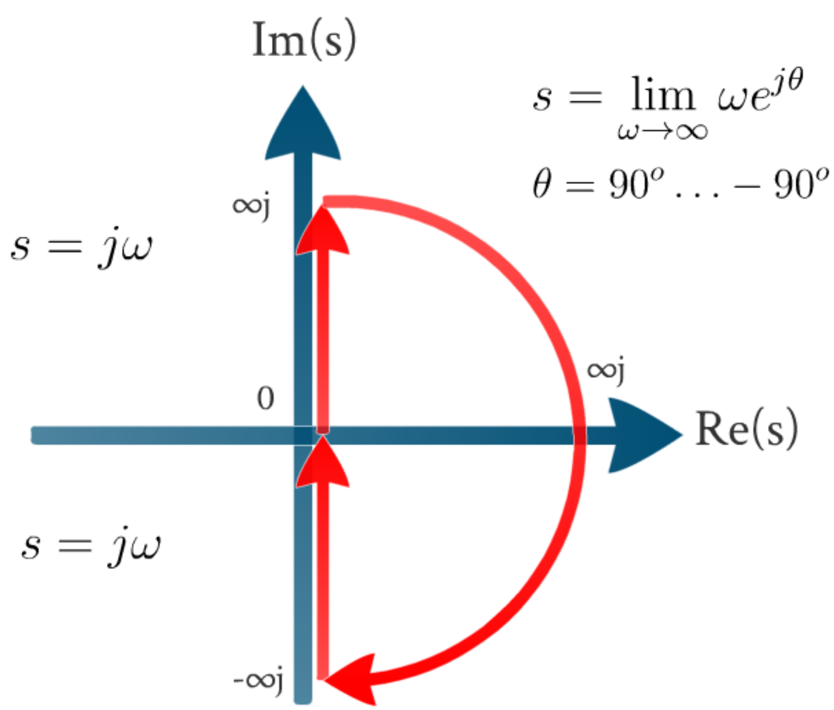
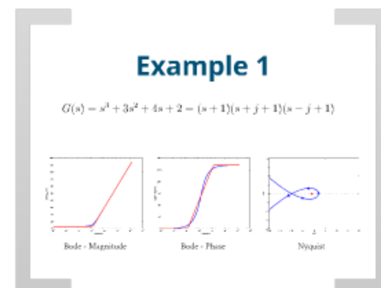
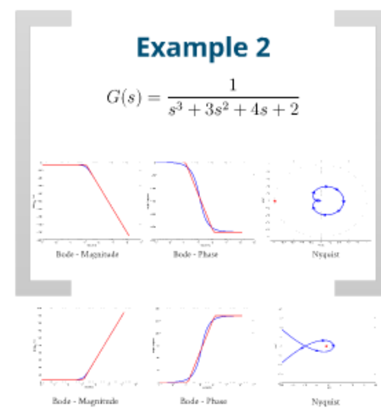
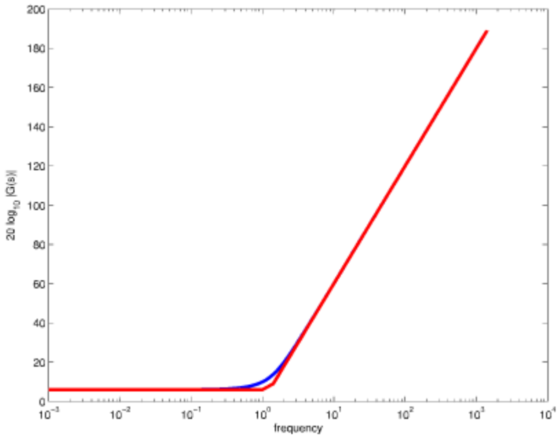




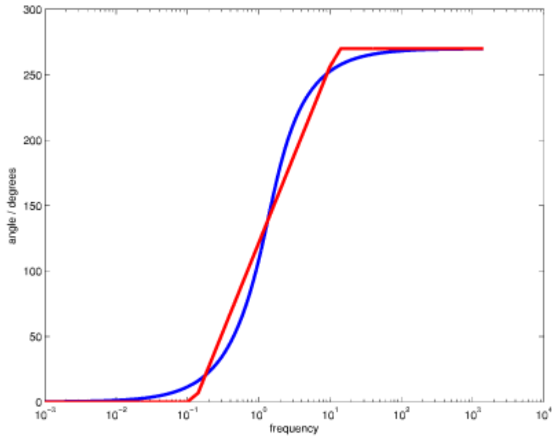


Example 1

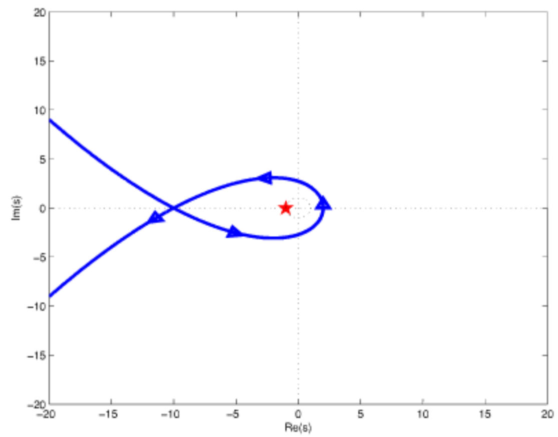
$$G(s) = s^3 + 3s^2 + 4s + 2 = (s+1)(s+j+1)(s-j+1)$$



Bode - Magnitude



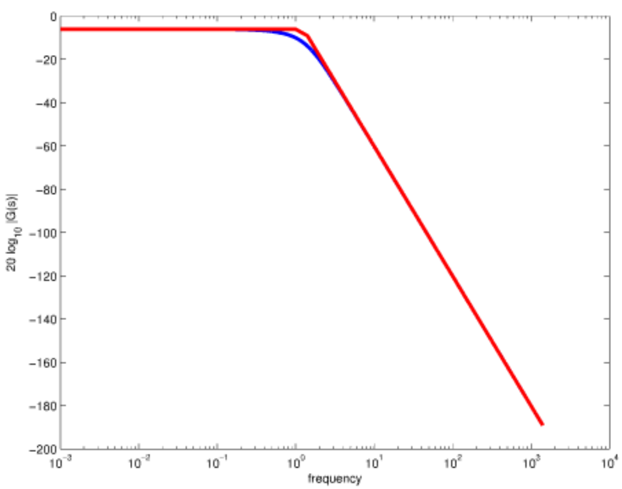
Bode - Phase



Nyquist

Example 2

$$G(s) = \frac{1}{s^3 + 3s^2 + 4s + 2}$$



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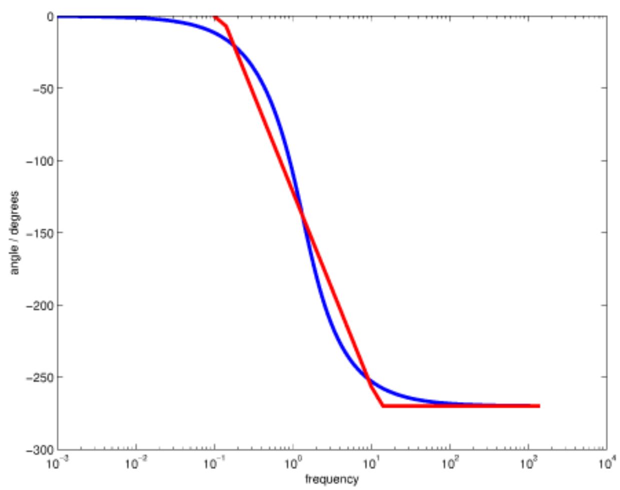

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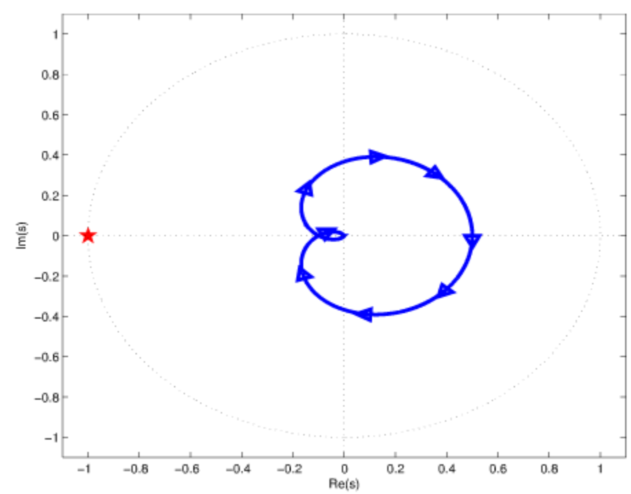
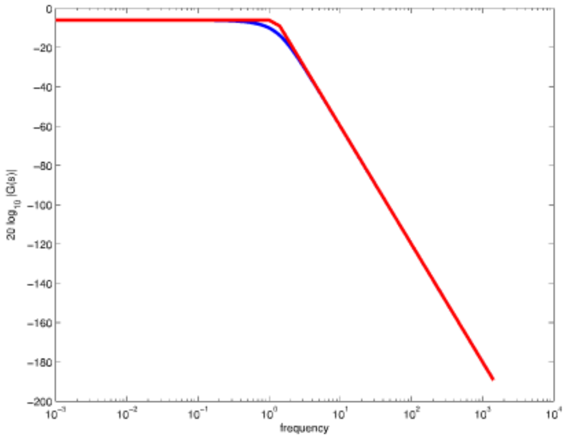


Nyquist



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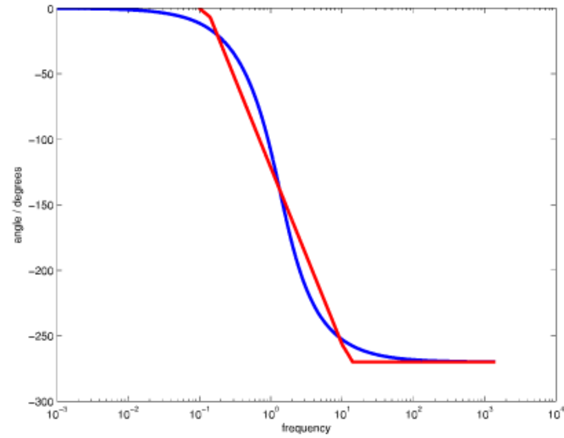

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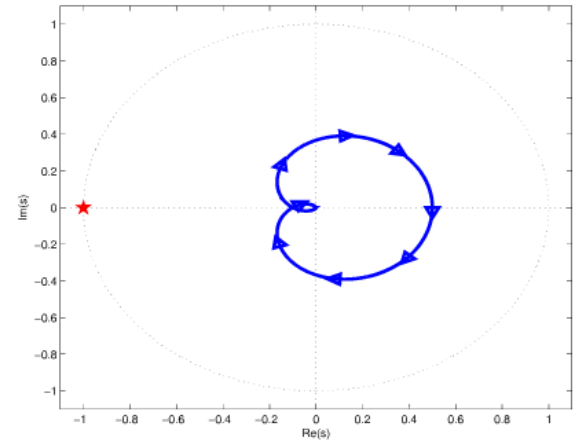

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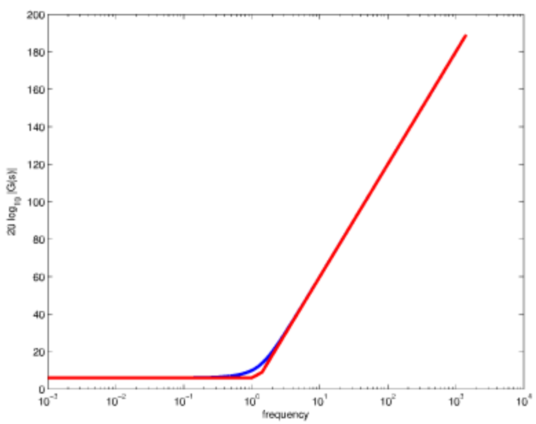

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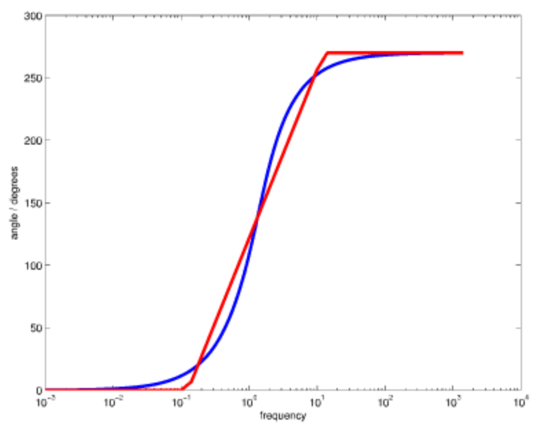

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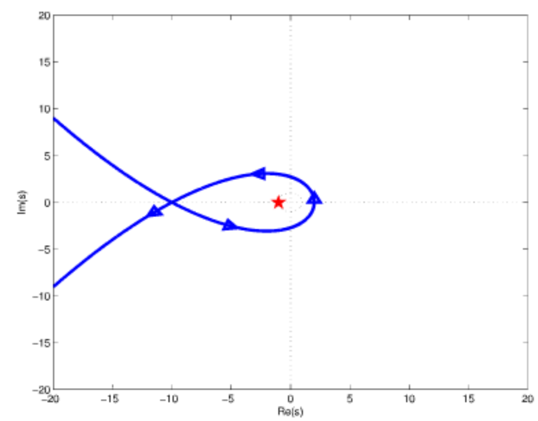

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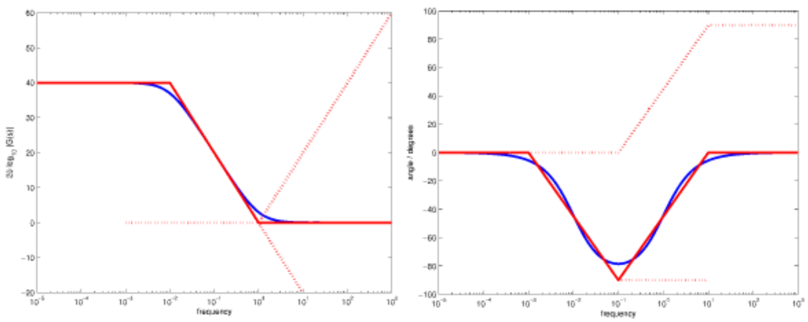

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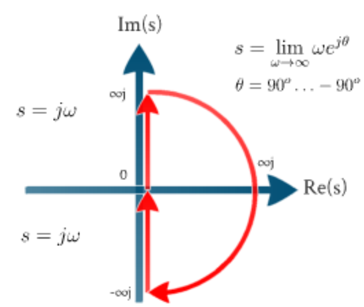

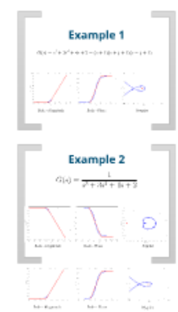

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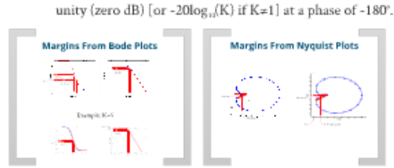
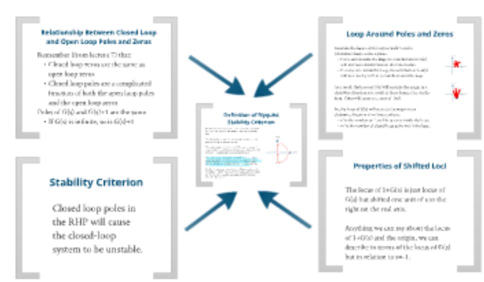


Nyquist Stability Criterion



Nyquist Stability Criterion

Relationship Between Closed Loop and Open Loop Poles and Zeros

Remember (from lecture 7) that:

- Closed loop zeros are the same as open loop zeros
- Closed loop poles are a complicated function of both the open loop poles and the open loop zeros

Poles of G(s) and G(s)+1 are the same

- If G(s) is infinite, so is G(s)+1

Stability Criterion

Closed loop poles in the RHP will cause the closed-loop system to be unstable.

Definition of Nyquist Stability Criterion

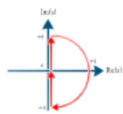
We define a loop that goes around the RHP. Note that the loop will enclose all open loop zeros and (unstable) open loop poles in the RHP.

The Nyquist plot is the exactly the locus of the open loop transfer function, G(s), as s goes round this loop.

The number of poles of G(s)+1 in the RHP minus the number of zeros of G(s)+1 in the RHP is equal to the number of times that G(s) encircles s = −1 anticlockwise as s goes round the loop.

The number of **open-loop poles** in the RHP minus the number of **closed-loop poles** in the RHP is equal to the **number of times that the Nyquist plot of G(s) encircles s = −1 anticlockwise**.

Note that if there is a parameterised gain of K, we are interested in properties of 1 + KG(s). We know this is the same as the properties of 1/K + G(s). So, we can look how many times G(s) encircles s = −1/K, rather than s = −1.

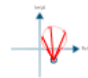


Loop Around Poles and Zeros

Consider the locus of G(s) as you walk round a (clockwise) loop on the s-plane.

- For a zero outside the loop, its contribution to G(s) will increase and decrease to the initial value.
- For any zero inside the loop, its contribution to G(s) will increase by 360° as you walk around the loop.

As a result, the locus of G(s) will encircle the origin in a clockwise direction as a result of there being a zero in the loop. Poles will cause rotations of -360°.



So, the locus of G(s) will encircle the origin in an clockwise direction n'-m' times, where:

- n' is the number of closed-loop zeros inside the loop;
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Properties of Shifted Loci

The locus of 1+G(s) is just locus of G(s) but shifted one unit of s to the right on the real axis.

Anything we can say about the locus of 1+G(s) and the origin, we can describe in terms of the locus of G(s) but in relation to s=-1.

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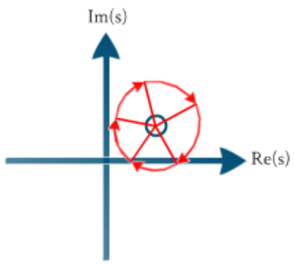

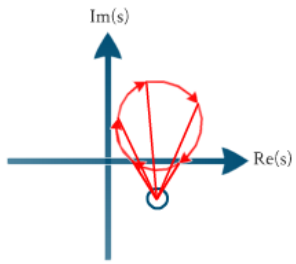

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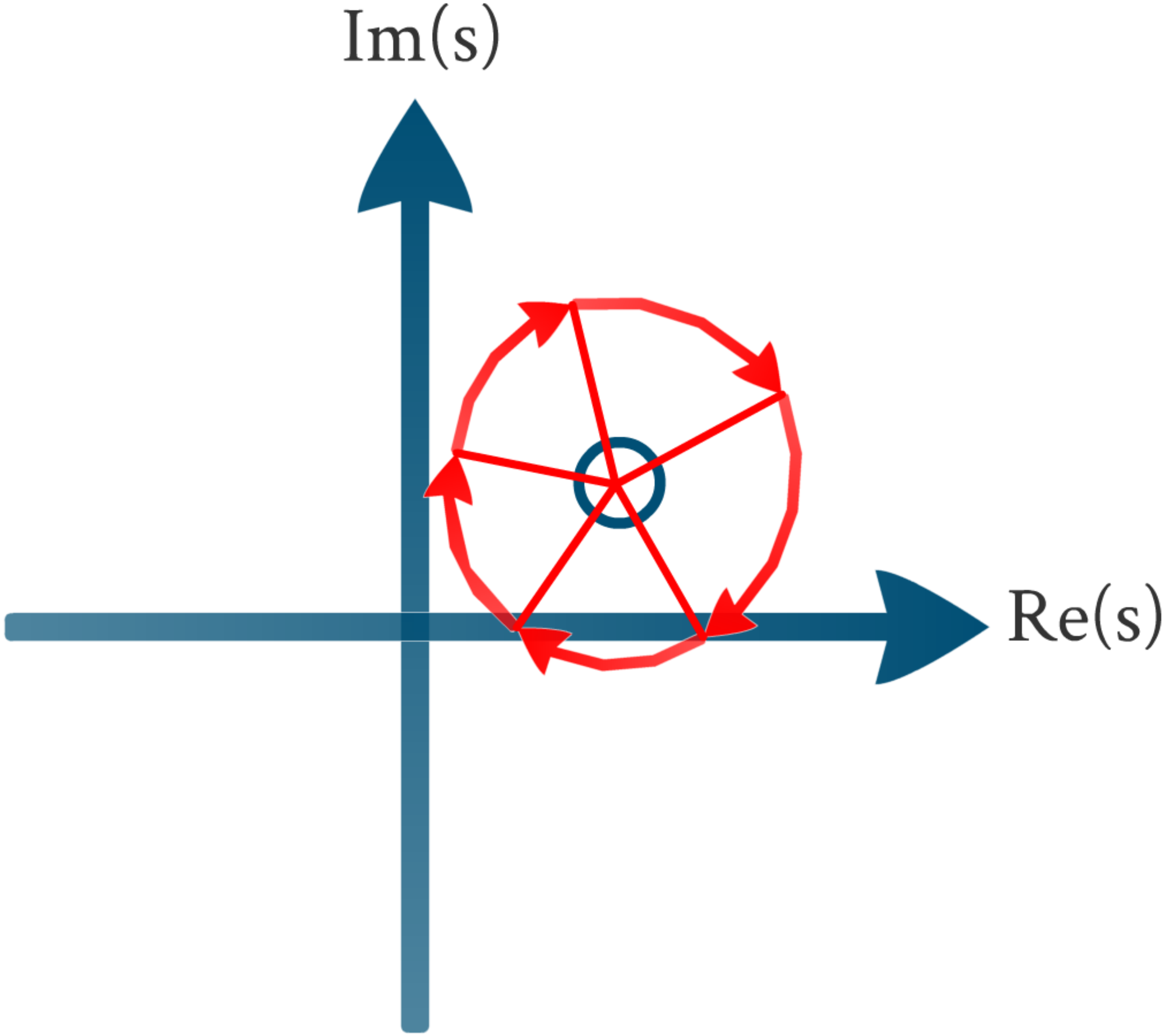
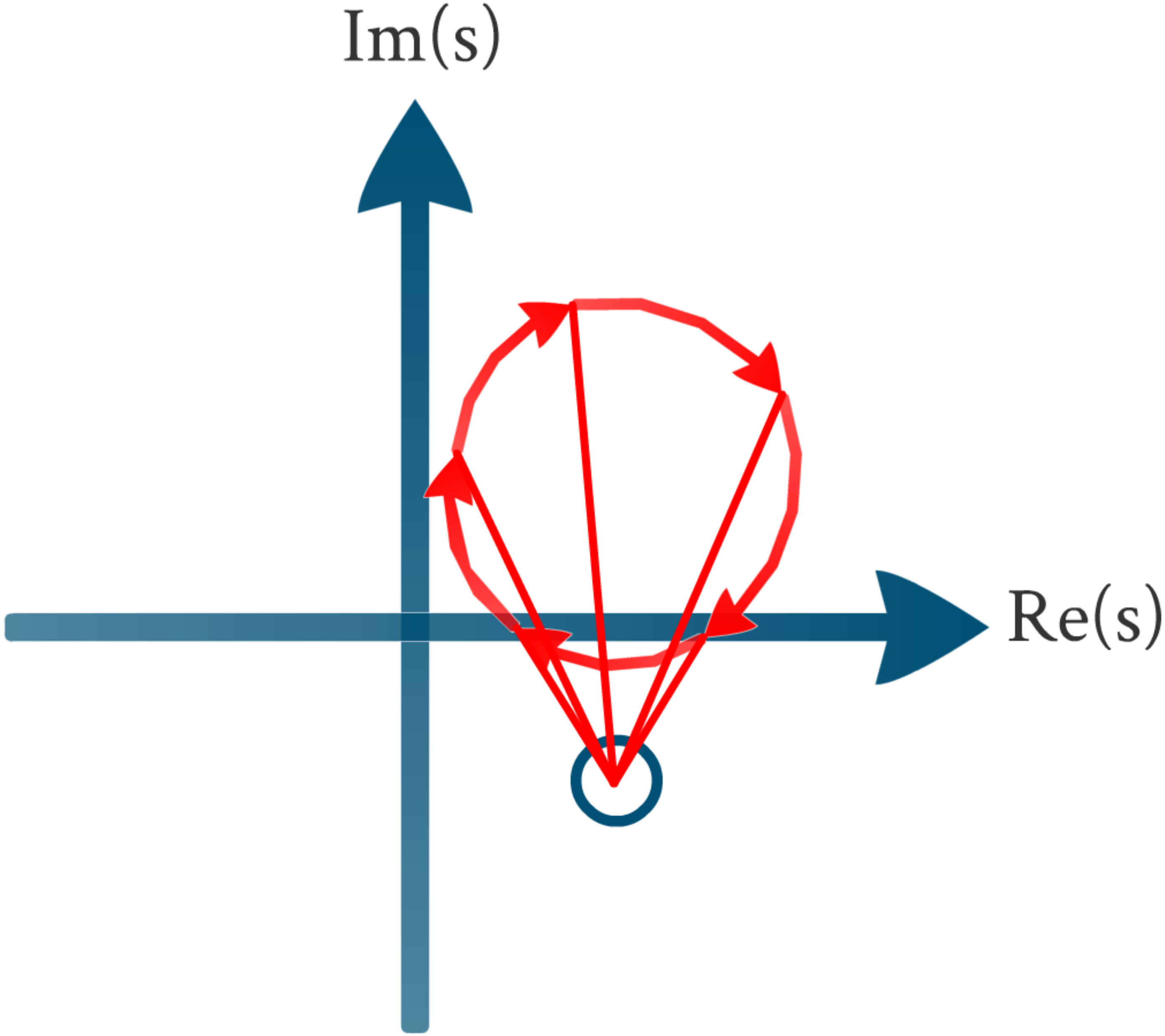

Im(s)
Re(s)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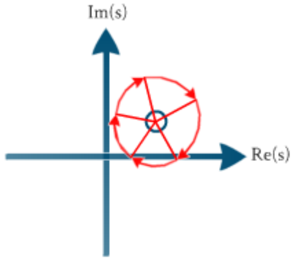

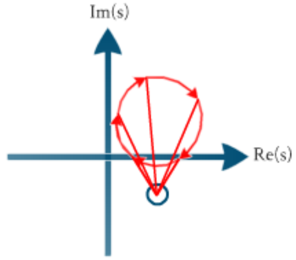

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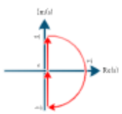
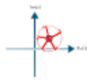


Loop Around Poles and Zeros

Consider the locus of G(s) as you walk round a (clockwise) loop on the s-plane.

- For a zero outside the loop, its contribution to G(s) will increase and decrease to the initial value.
- For any zero inside the loop, its contribution to G(s) will increase by 360° as you walk around the loop.



As a result, the locus of G(s) will encircle the origin in a clockwise direction as a result of there being a zero in the loop. Poles will cause rotations of -360°.

So, the locus of G(s) will encircle the origin in an clockwise direction n'-m' times, where:

- n' is the number of closed-loop zeros inside the loop;
- m' is the number of closed-loop poles inside the loop.

Properties of Shifted Loci

The locus of 1+G(s) is just locus of G(s) but shifted one unit of s to the right on the real axis.

Anything we can say about the locus of 1+G(s) and the origin, we can describe in terms of the locus of G(s) but in relation to s=-1.

Definition of Nyquist Stability Criterion

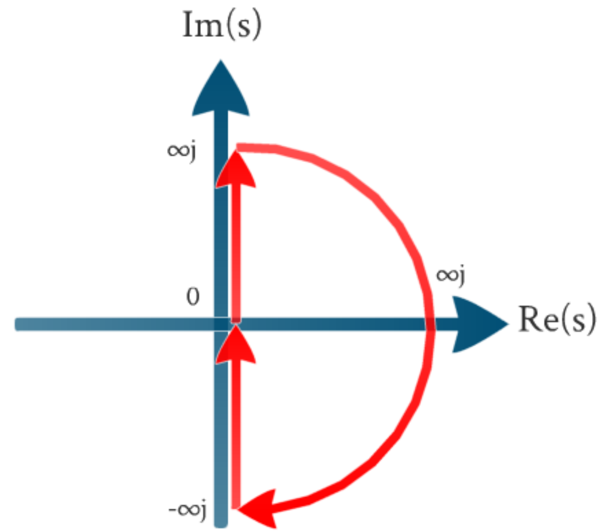
We define a loop that goes around the RHP. Note that the loop will enclose all open loop zeros and (unstable) open loop poles in the RHP.

The Nyquist plot is the exactly the locus of the open loop transfer function, $G(s)$, as s goes round this loop.

The number of poles of $G(s)+1$ in the RHP minus the number of zeros of $G(s)+1$ in the RHP is equal to the number of times that $G(s)$ encircles $s = -1$ anticlockwise as s goes round the loop.

The number of **open-loop poles** in the RHP minus the number of **closed-loop poles** in the RHP is equal to the **number of times that the Nyquist plot of G(s) encircles s = –1 anticlockwise**.

Note that if there is a parameterised gain of K, we are interested in properties of $1 + KG(s)$. We know this is the same as the properties of $1/K + G(s)$. So, we can look how many times $G(s)$ encircles $s = -1/K$, rather than $s = -1$.



Nyquist Stability Criterion

Relationship Between Closed Loop and Open Loop Poles and Zeros

Remember (from lecture 7) that:

- Closed loop zeros are the same as open loop zeros
- Closed loop poles are a complicated function of both the open loop poles and the open loop zeros

Poles of G(s) and G(s)+1 are the same

- If G(s) is infinite, so is G(s)+1

Loop Around Poles and Zeros

Consider the locus of G(s) as you walk round a (clockwise) loop on the s-plane.

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The number of **open-loop poles** in the RHP minus the number of **closed-loop poles** in the RHP is equal to the **number of times that the Nyquist plot of G(s) encircles s = −1 anticlockwise**.

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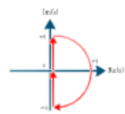


Stability Criterion

Closed loop poles in the RHP will cause the closed-loop system to be unstable.

Properties of Shifted Loci

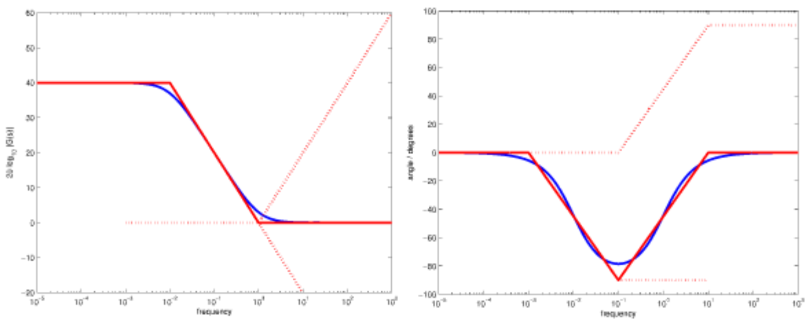
The locus of 1+G(s) is just locus of G(s) but shifted one unit of s to the right on the real axis.

Anything we can say about the locus of 1+G(s) and the origin, we can describe in terms of the locus of G(s) but in relation to s=-1.

Frequency Response Techniques

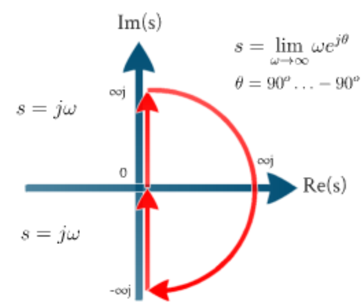
Bode Plot

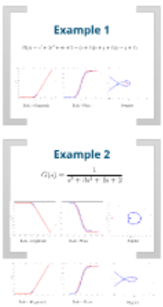
$$G(s) = \frac{s+1}{s+\frac{1}{100}}$$



Nyquist Plot

Bode plot shows magnitude and phase of G(s)
Nyquist plot shows G(s) on the complex plane

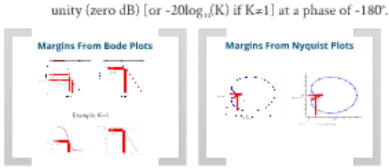




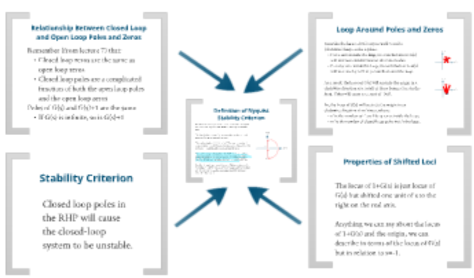
Phase and Gain Margins

We can be interested in relative stability, ie how close a system is to going unstable

- Phase Margin is the extra phase required to make the phase -180° at a gain of zero dB [or $-20\log_{10}(K)$ if K≠1];
- Gain Margin is the extra gain required to make the gain unity (zero dB) [or $-20\log_{10}(K)$ if K≠1] at a phase of -180°.



Nyquist Stability Criterion



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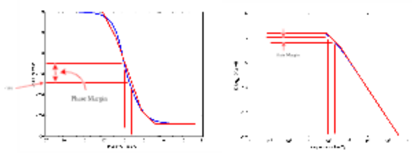
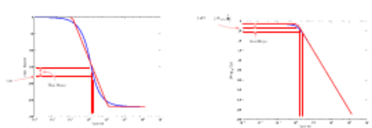
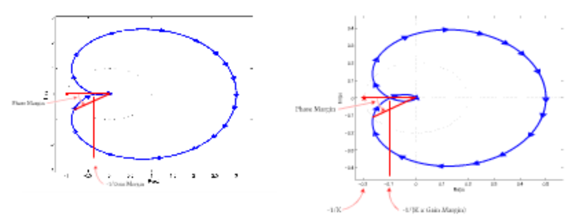
Margins From Bode Plots



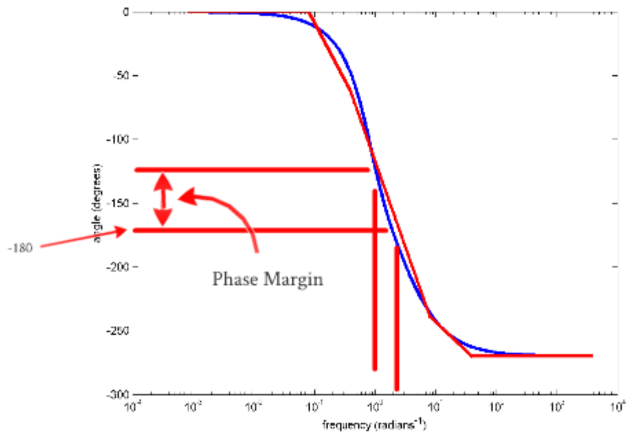
Example: K=5

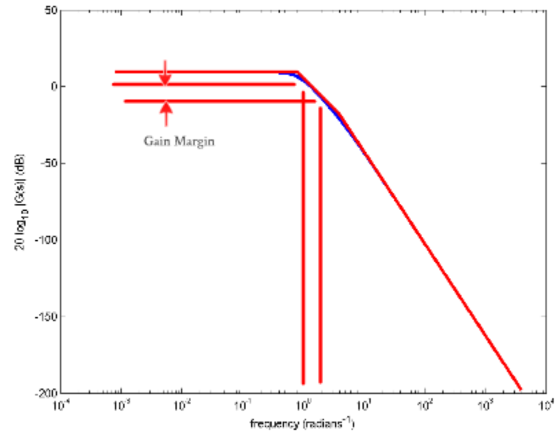


Margins From Nyquist Plots

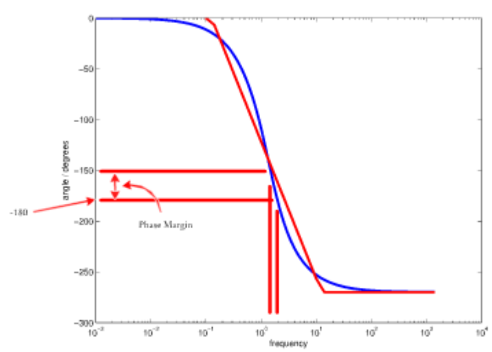


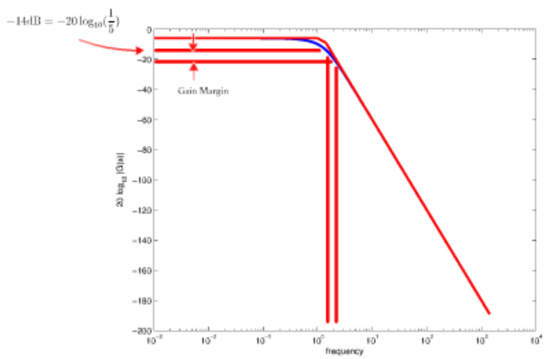
Margins From Bode Plots

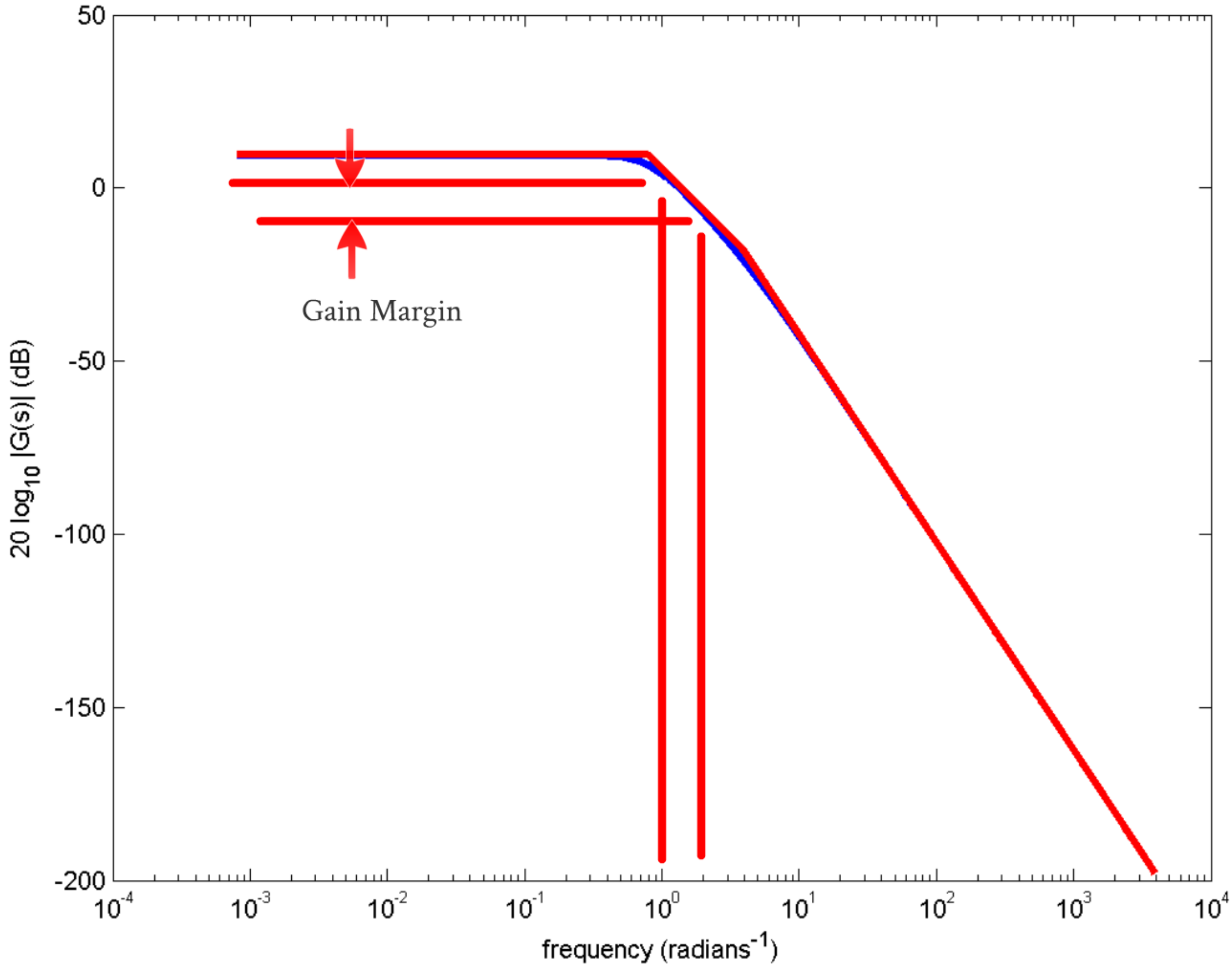


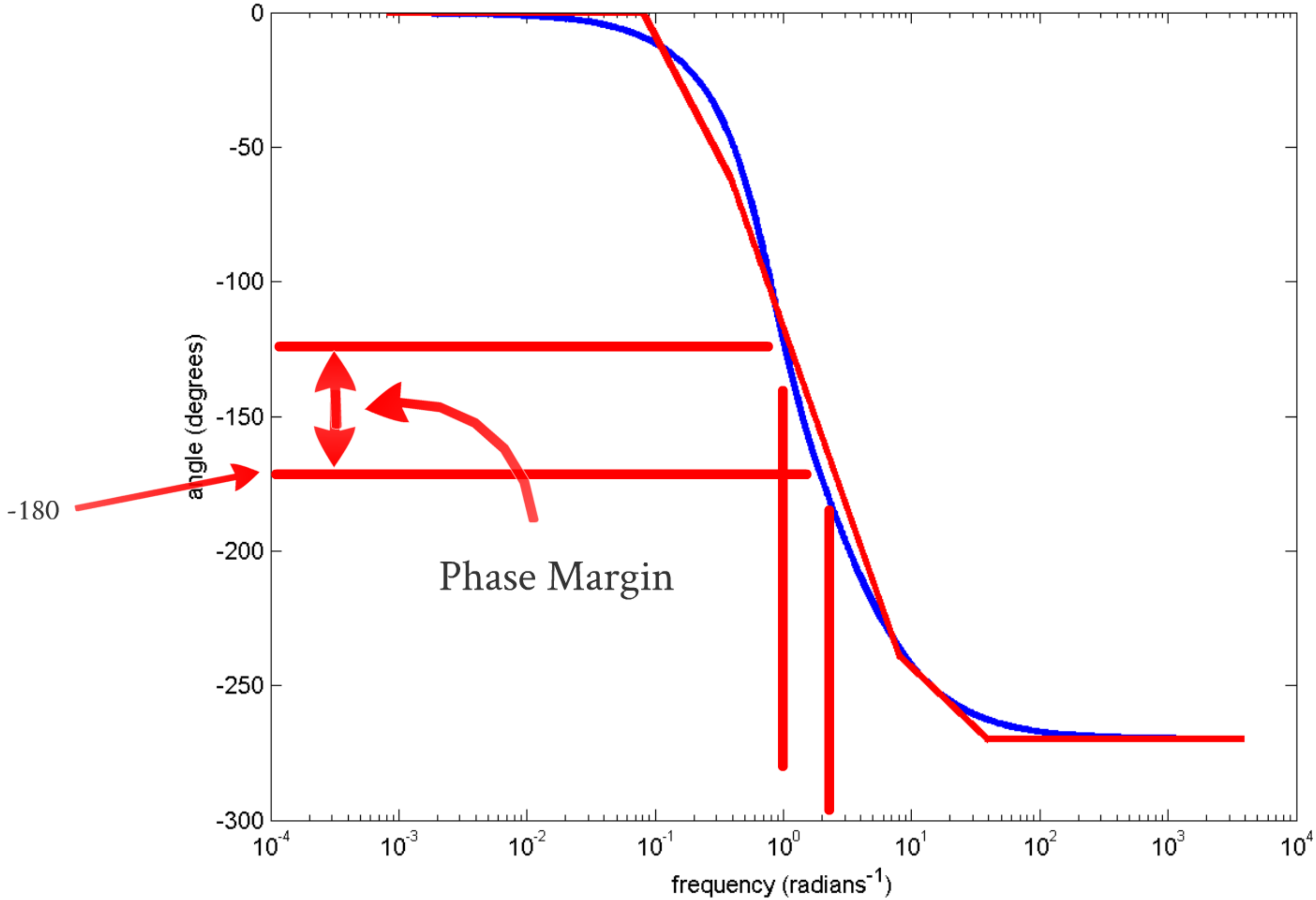


Example: K=5

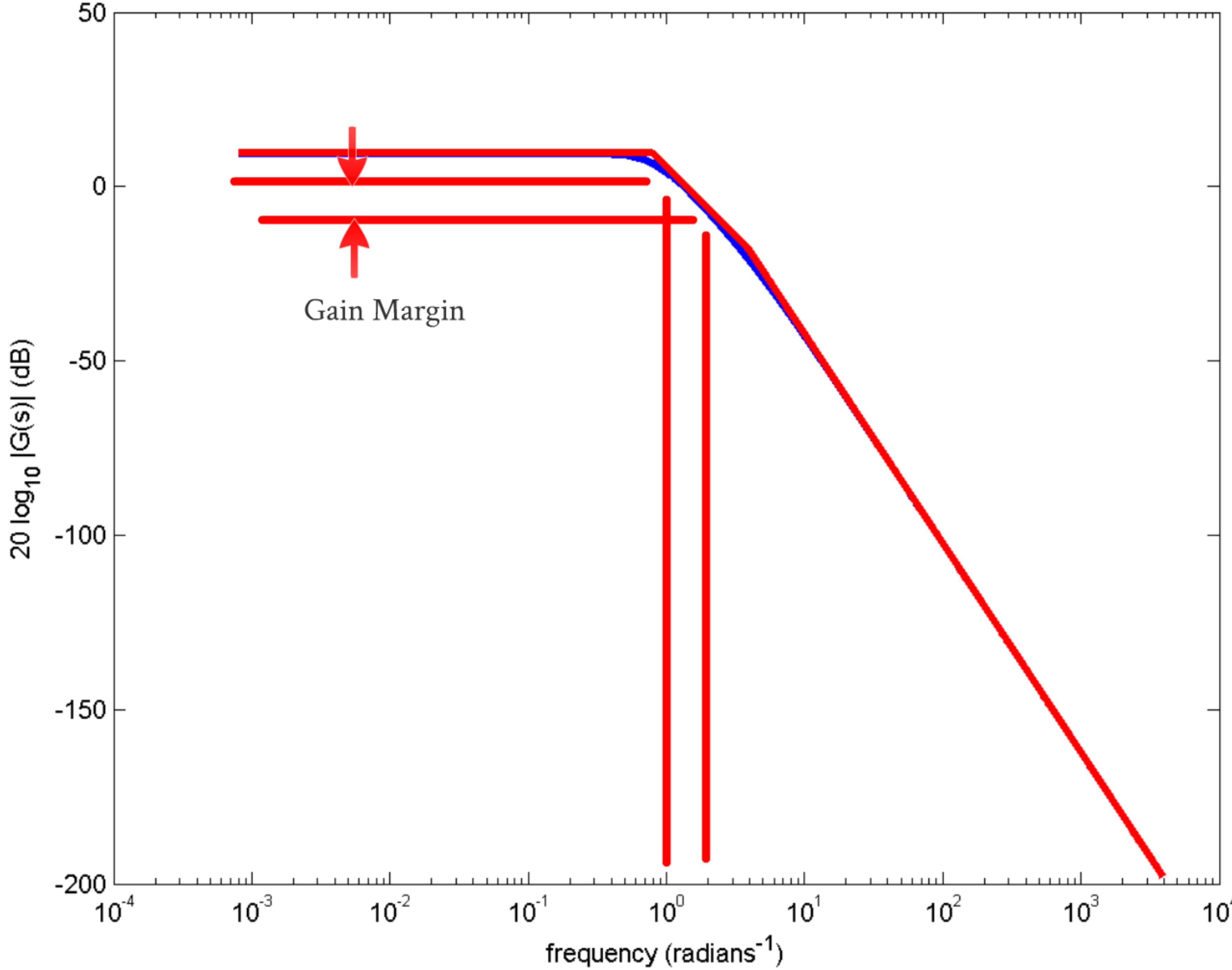





Gain Margin
20 log10 |G(s)| (dB)
50
0
-50
-100
-150
-200
10^-4
10^-3
10^-2
10^-1
10^0
10^1
10^2
10^3
10^4
frequency (radians^-1)

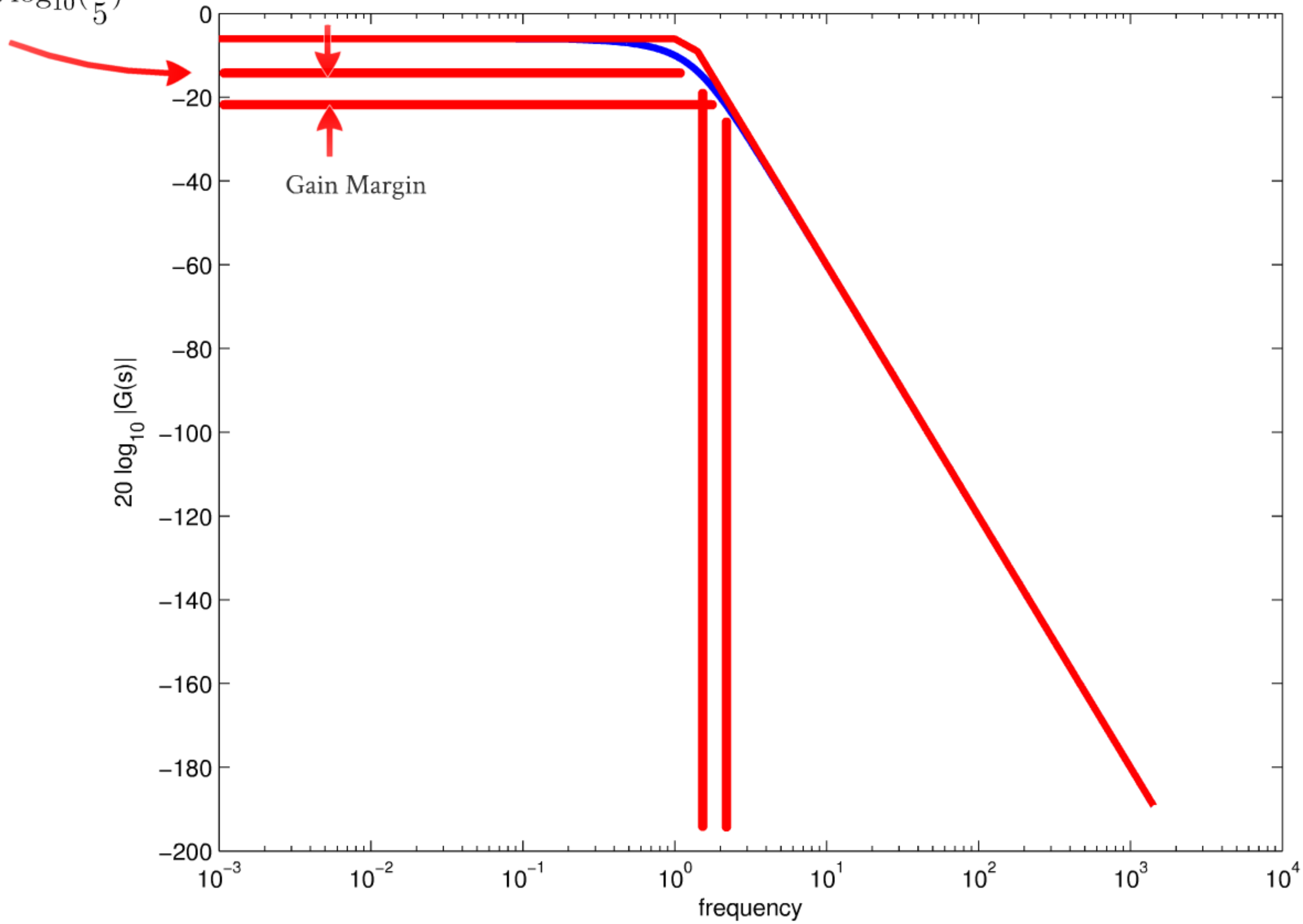


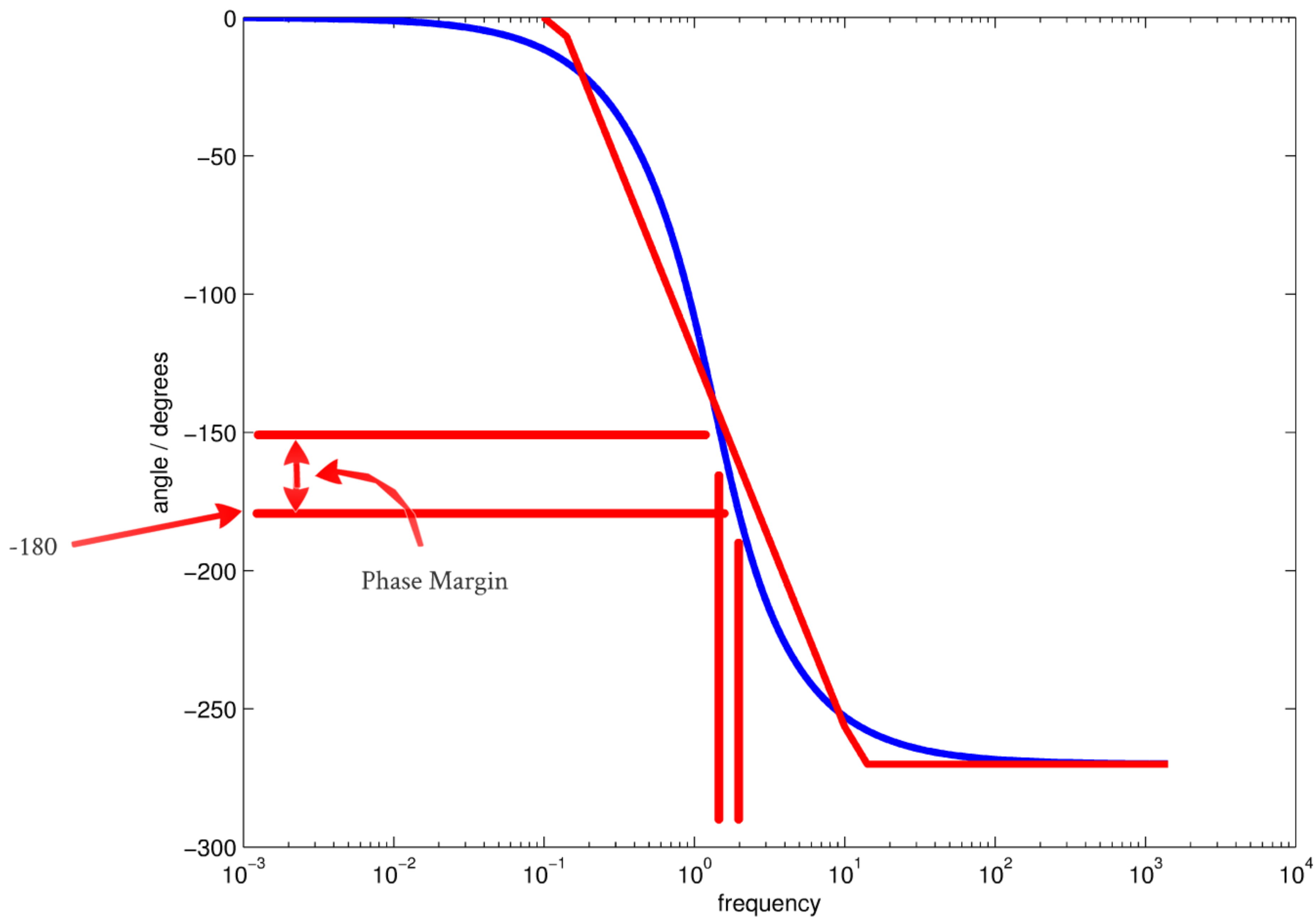
0
-50
-100
-150
-200
-250
-300
angle (degrees)
-180
Phase Margin
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
frequency (radians^{-1})



Gain Margin
50
0
-50
-100
-150
-200
$20 \log_{10} |G(s)|$ (dB)
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
frequency (radians^{-1})

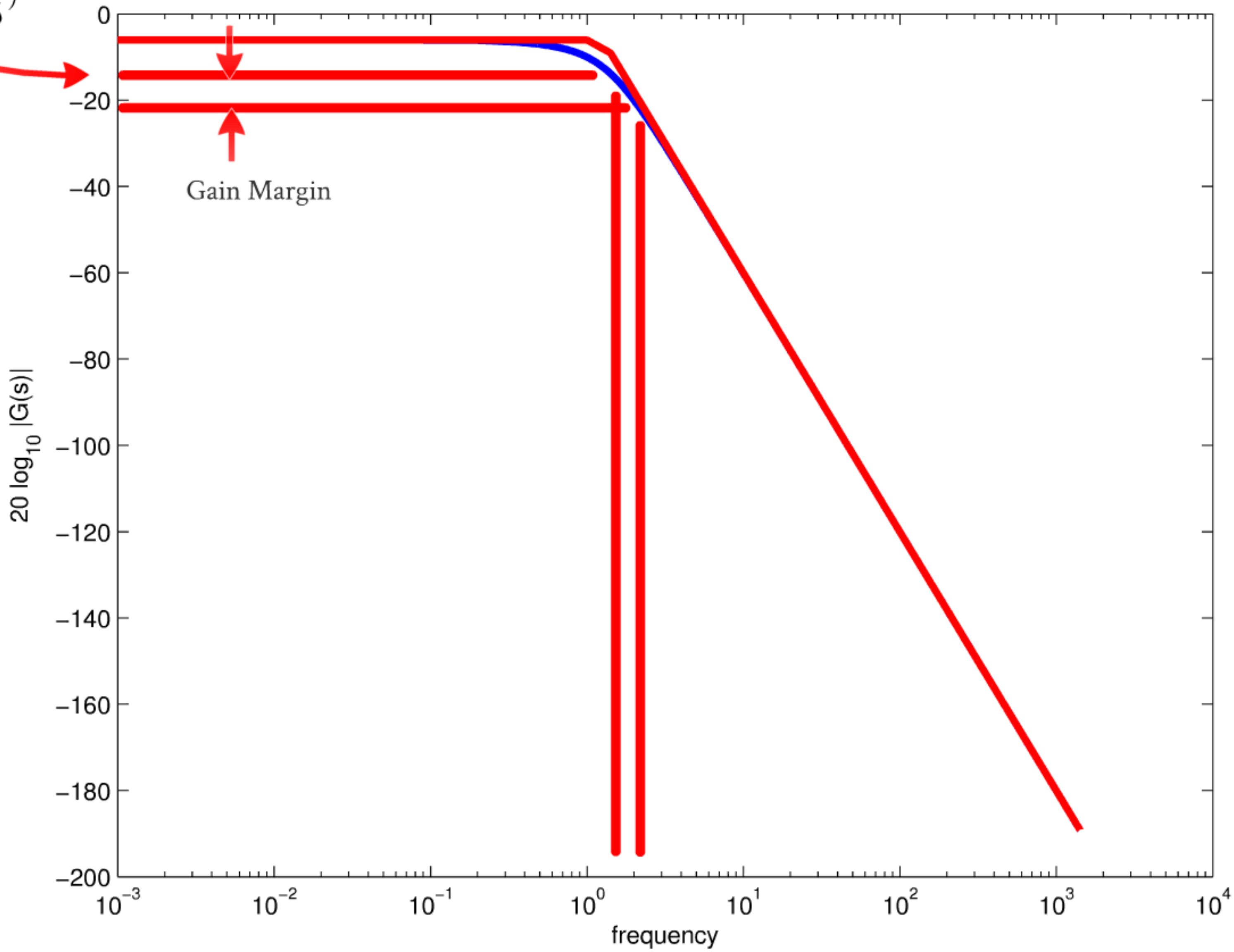
$$-14\text{dB} = -20\log_{10}(\frac{1}{5})$$



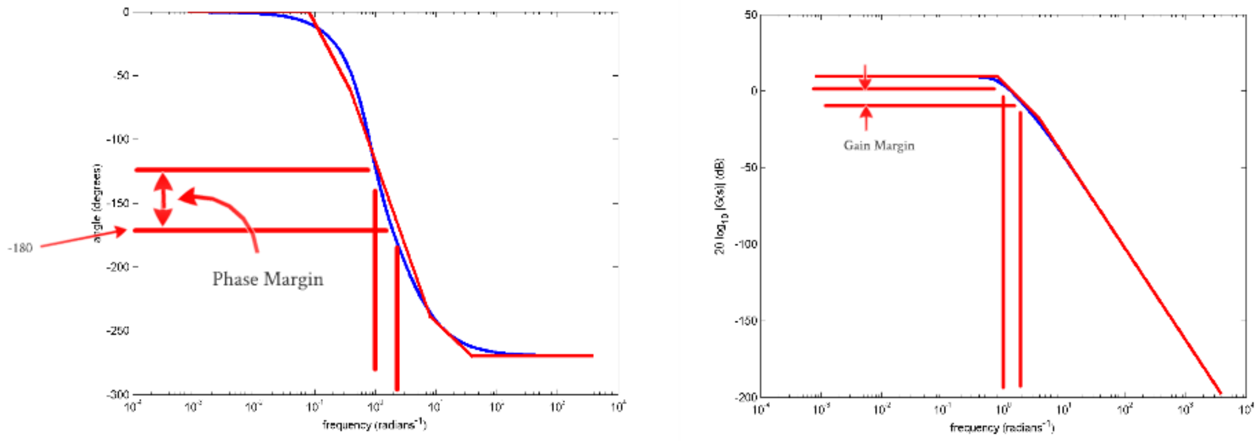


0
−50
−100
−150
−200
−250
−300
angle / degrees
-180
Phase Margin
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
frequency

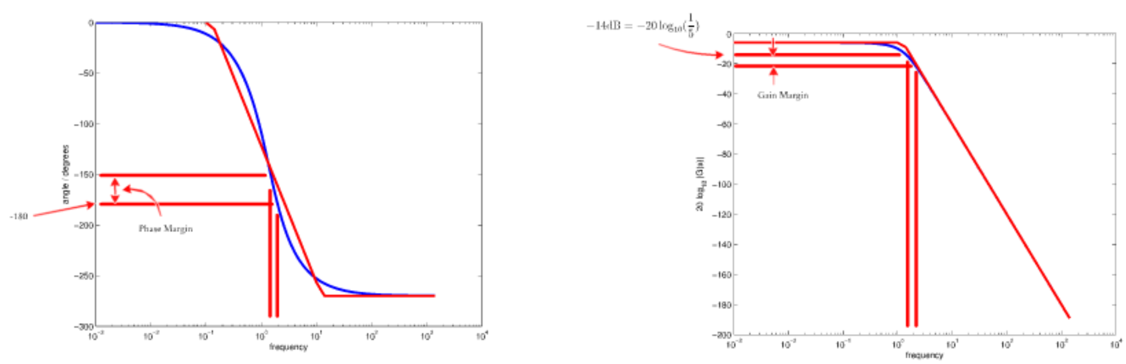
$$-14\text{dB} = -20\log_{10}\left(\frac{1}{5}\right)$$



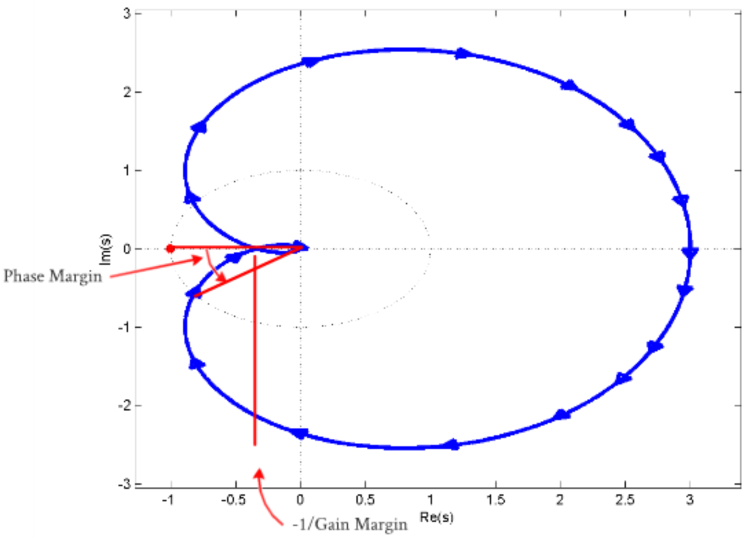
Margins From Bode Plots

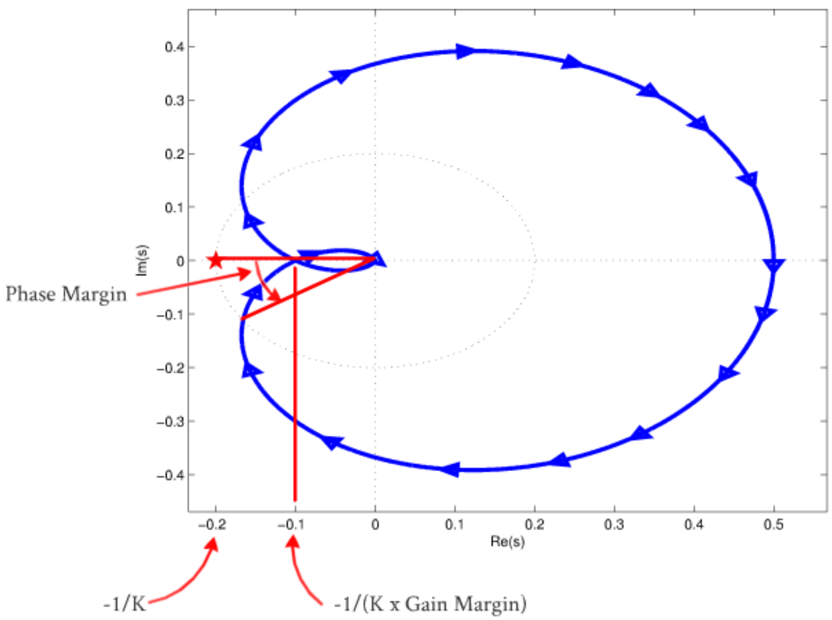


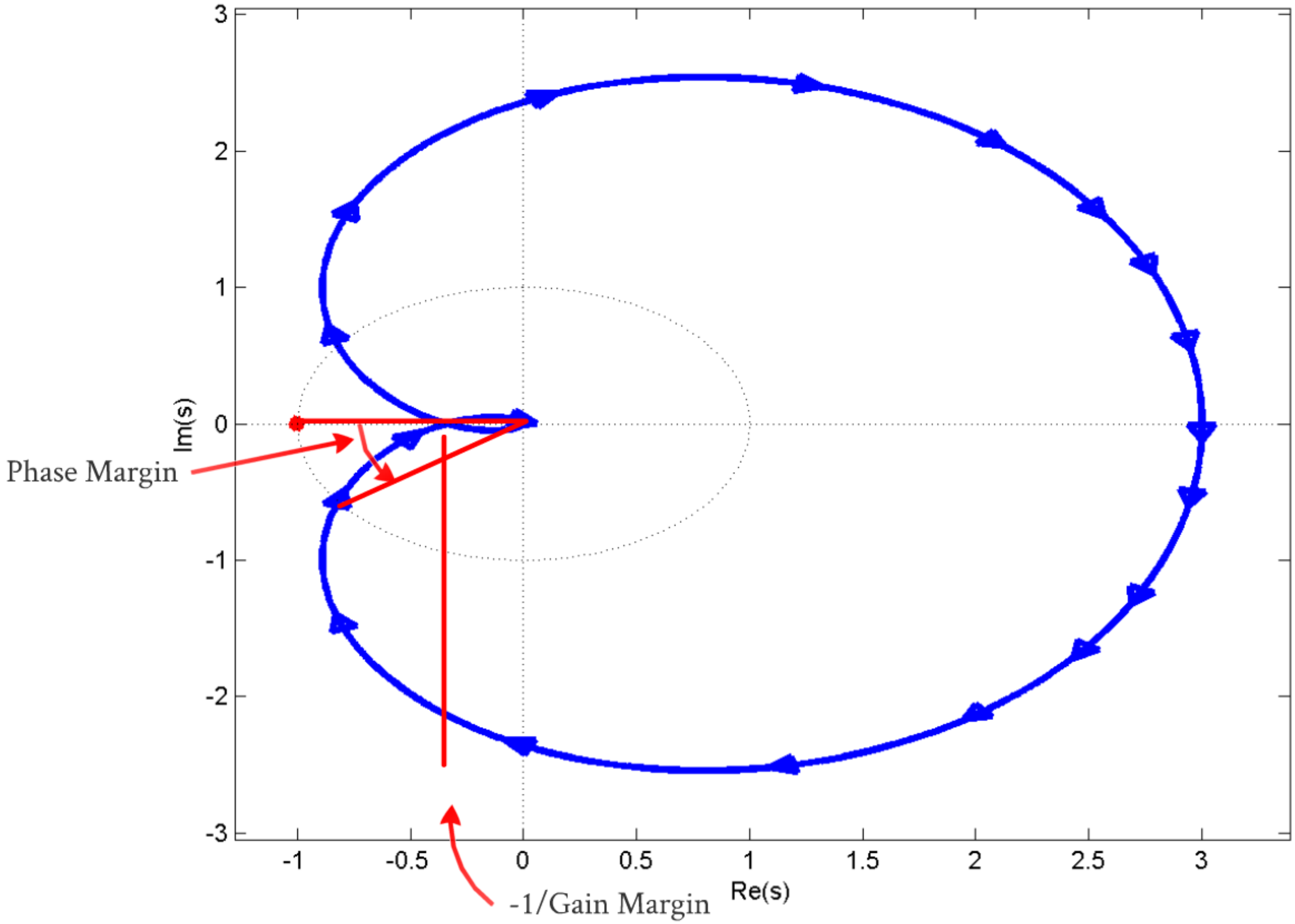
Example: K=5

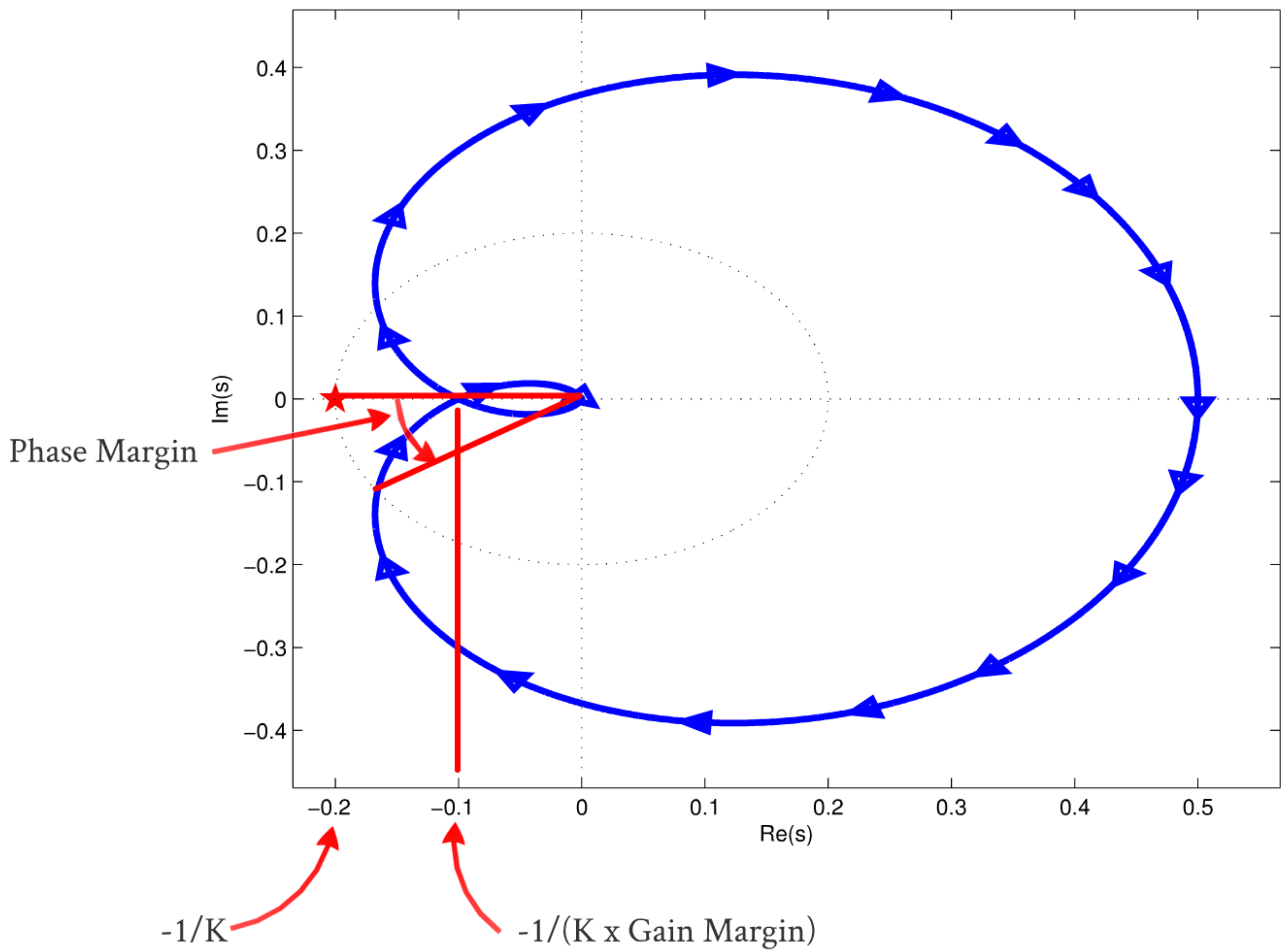


Margins From Nyquist Plots



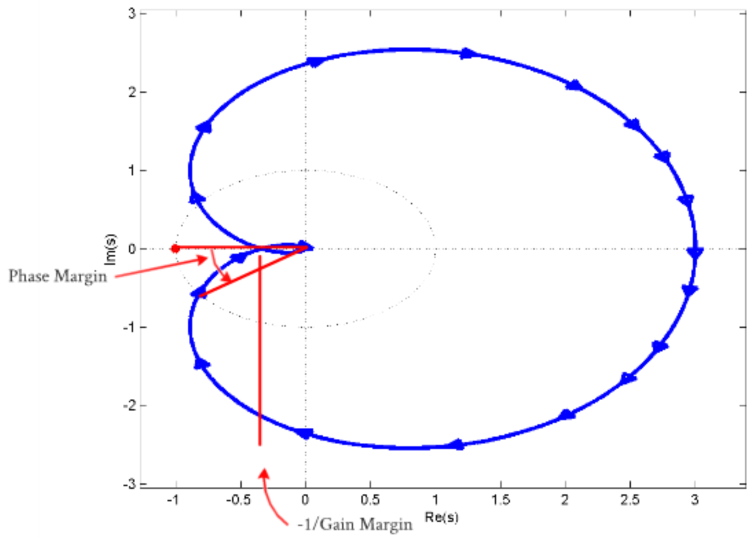


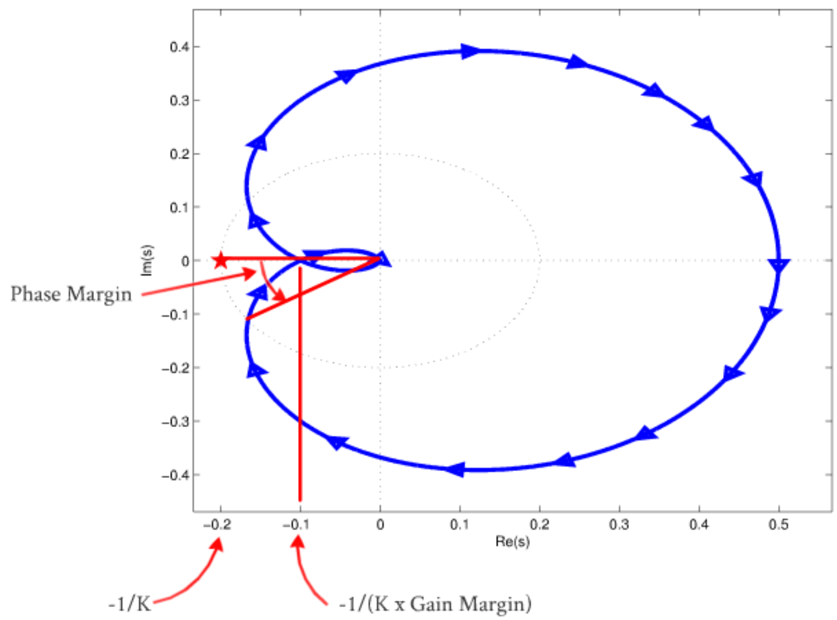

Phase Margin
-1/Gain Margin
Im(s)
Re(s)
Ph



0.4
0.3
0.2
0.1
0
−0.1
−0.2
−0.3
−0.4
Im(s)
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.5
Re(s)
Phase Margin
-1/K
-1/(K x Gain Margin)

Margins From Nyquist Plots





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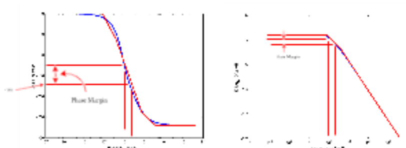

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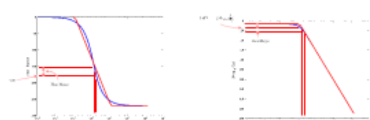
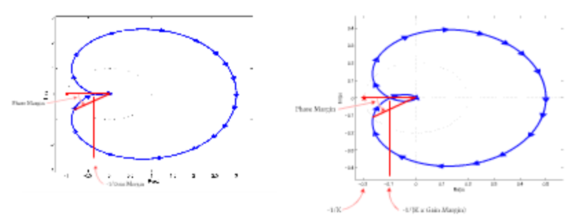


Margins From Nyquist Plots



Frequency Response Techniques

Bode Plot

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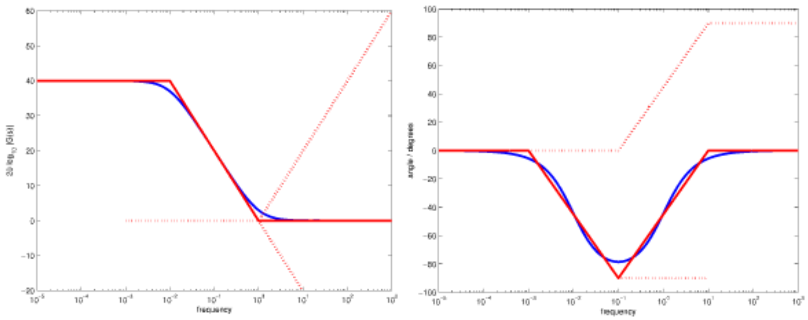
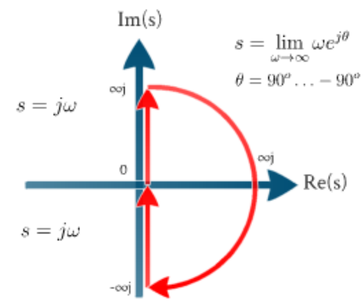
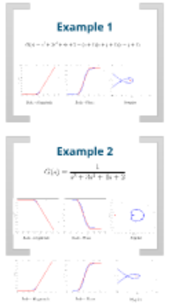


Nyquist Plot

Bode plot shows magnitude and phase of G(s)
Nyquist plot shows G(s) on the complex plane





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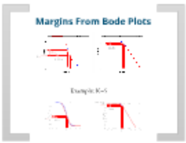
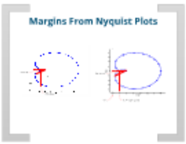
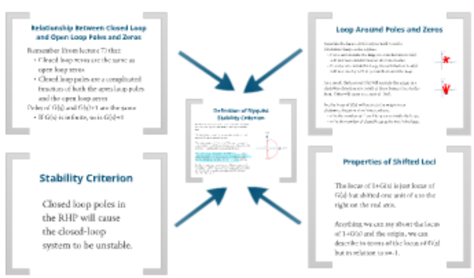




Nyquist Stability Criterion



This lecture covers:

- Nyquist Stability Criterion
- Phase and gain margins
- Diagrammatic representations using Nyquist plot