

This lecture covers
• Design of a feedback control system
• Design of a cascade control system

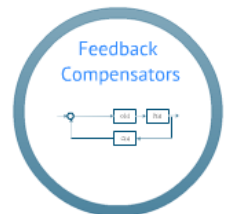
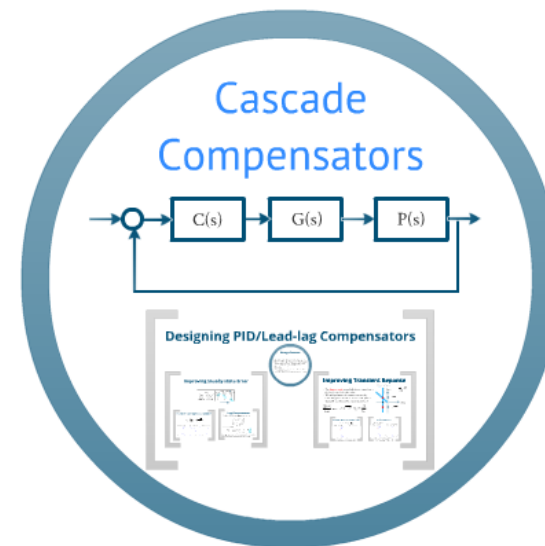
ELEC 207 Part B

Control Theory Lecture 10: Control System Design (2)

Prof Simon Maskell
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Compensators



This lecture covers:
• Design of a Control System via root locus.

ELEC 207 Part B

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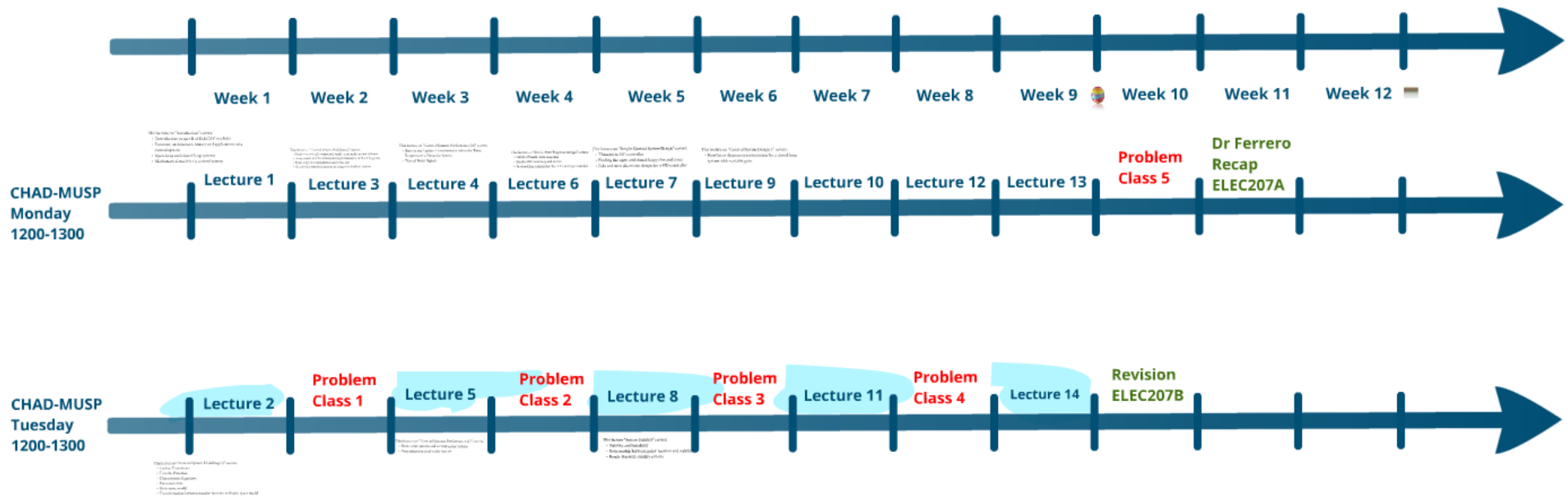
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LIVERPOOL

This lecture covers:

- Design of a Control System via root locus.



ELEC 207B: Timeline



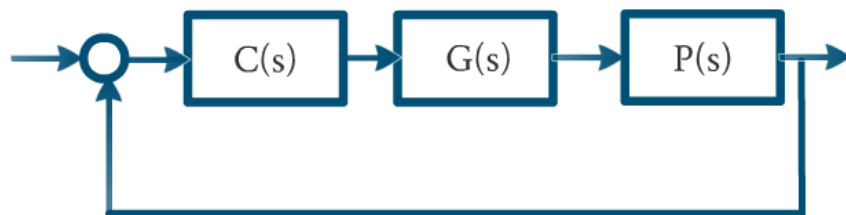
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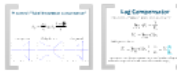
Compensators

Cascade Compensators



Designing PID/Lead-lag Compensators

Improving Steady-state Error



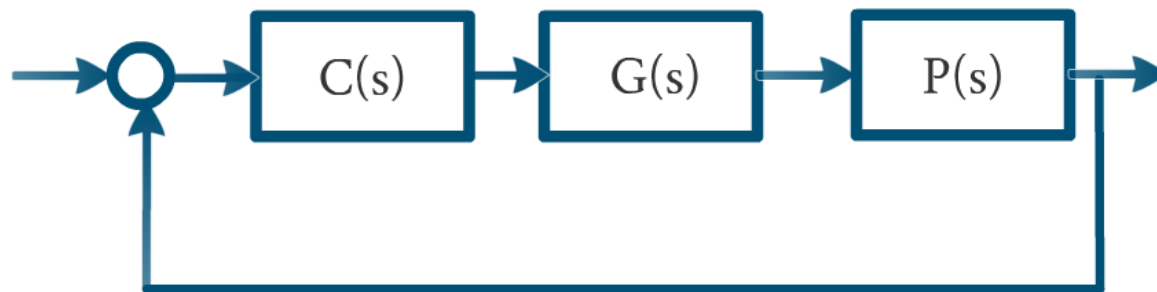
Improving Transient Response



Feedback Compensators



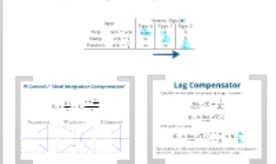
Cascade Compensators



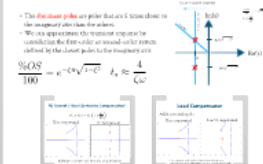
Designing PID/Lead-lag Compensators

Design Process

Improving Steady-state Error

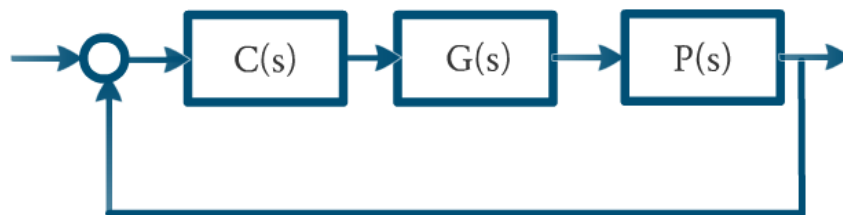


Improving Transient Response



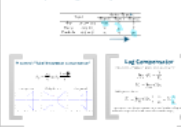
Compensators

Cascade Compensators

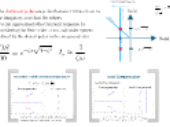


Designing PID/Lead-lag Compensators

Improving Steady-state Error



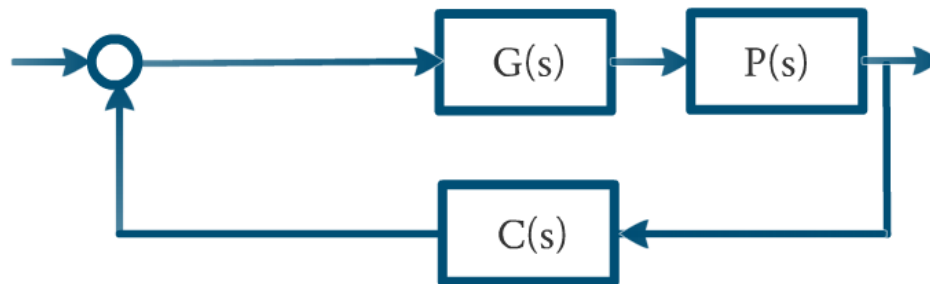
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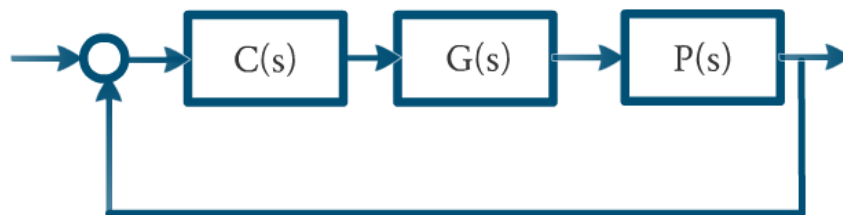


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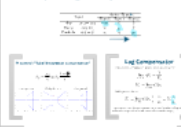
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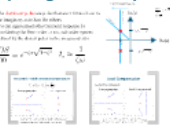


Designing PID/Lead-lag Compensators

Improving Steady-state Error



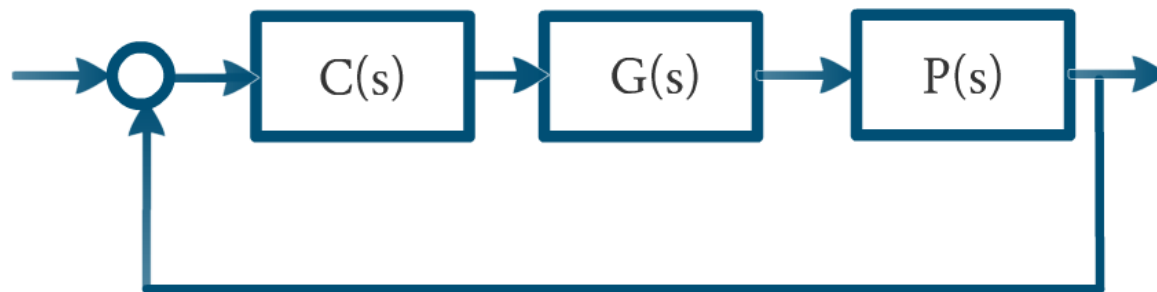
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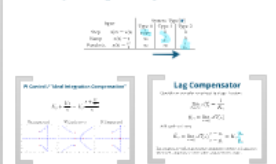


Cascade Compensators



Designing PID/Lead-lag Compensators

Improving Steady-state Error



Design Process

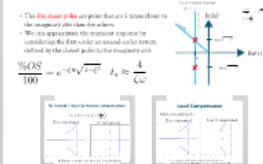
1. Determine the desired closed-loop system response (e.g., settling time, overshoot, steady-state error).

2. Determine the open-loop system response (e.g., poles, zeros, steady-state error).


3. Design a compensator that meets the desired response (e.g., PID, lead-lag, notch filter).

4. Verify the design (e.g., root locus, Bode plot, time-domain simulation).

Improving Transient Response



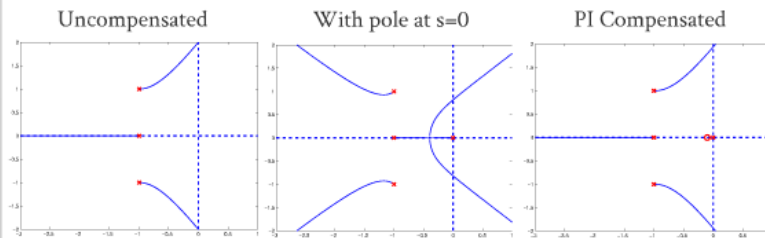
Improving Steady-state Error

Input		System Type 		
		Type-0	Type-1	Type-2
Step	$x(t) = u(t)$	$\frac{1}{1+K_p}$	0	0
Ramp	$x(t) = t$	∞	$\frac{1}{K_v}$	0
Parabola	$x(t) = \frac{t^2}{2}$	∞	∞	$\frac{1}{K_a}$



PI Control / "Ideal Integration Compensation"

$$K_p + \frac{K_I}{s} = K_p \frac{s + \frac{K_I}{K_p}}{s}$$



Lag Compensator

Consider an exemplar ramp input to a type-1 system:

$$\lim_{t \rightarrow \infty} e(t) = \frac{1}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)$$

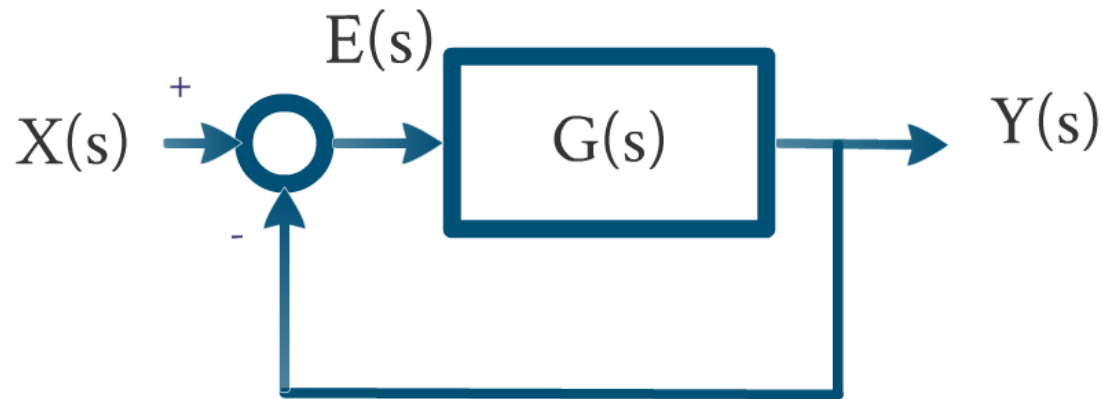
Add a pole and zero:

$$\tilde{K}_v = \lim_{s \rightarrow 0} sG(s) \frac{s - z_c}{s - p_c} = K_v \frac{z_c}{p_c}$$

Lag compensators only require passive components (resistors and capacitors) whereas PI compensators require active components (Op Amps).


System Type

Unity Negative Feedback (Representation of) System



$$\begin{aligned} G(s) &= \frac{1}{(s-3)(s-0)(s-4)(s-5)(s-0)} \\ &= \frac{1}{s^2(s-3)(s-4)(s-5)} \end{aligned}$$

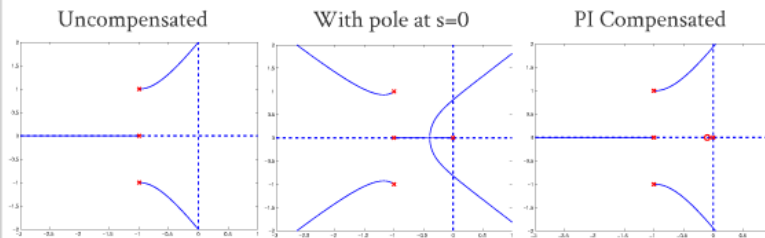
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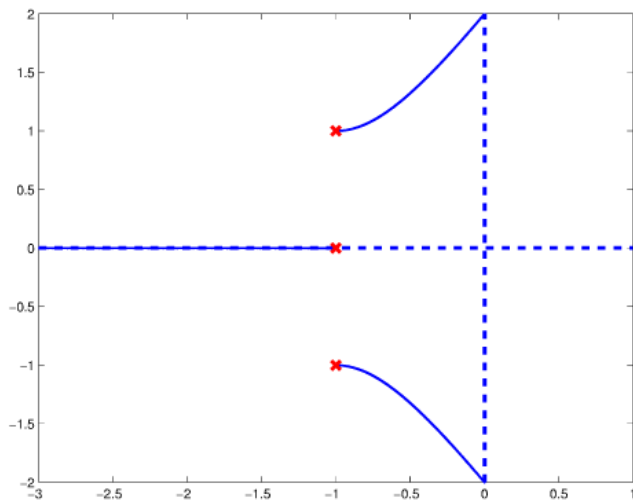
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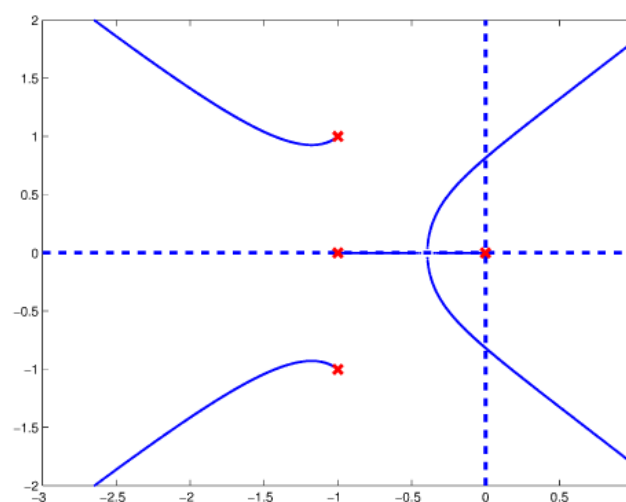
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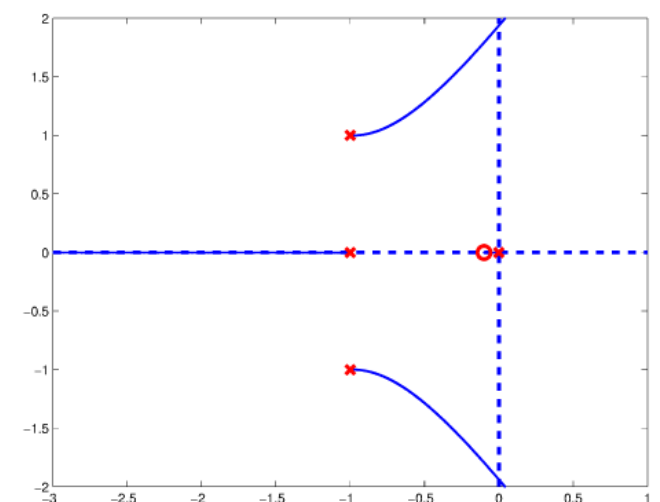
Uncompensated




With pole at s=0



PI Compensated



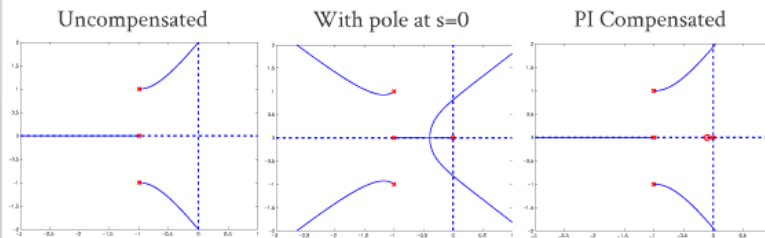
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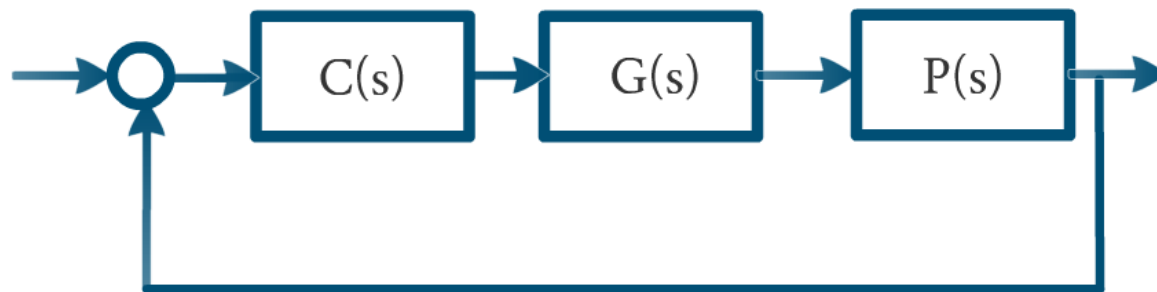
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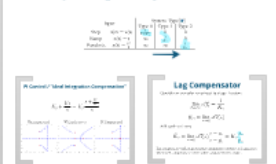
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Cascade Compensators



Designing PID/Lead-lag Compensators

Improving Steady-state Error



Design Process

1. Determine the desired system response (e.g., steady-state error, transient response).

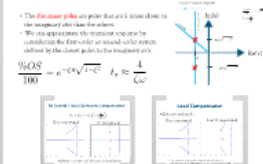
2. Analyze the system's current response (e.g., steady-state error, transient response).

3. Identify the system's deficiencies (e.g., steady-state error, transient response).

4. Design a compensator to address the deficiencies (e.g., steady-state error, transient response).

5. Verify the compensator's effectiveness (e.g., steady-state error, transient response).

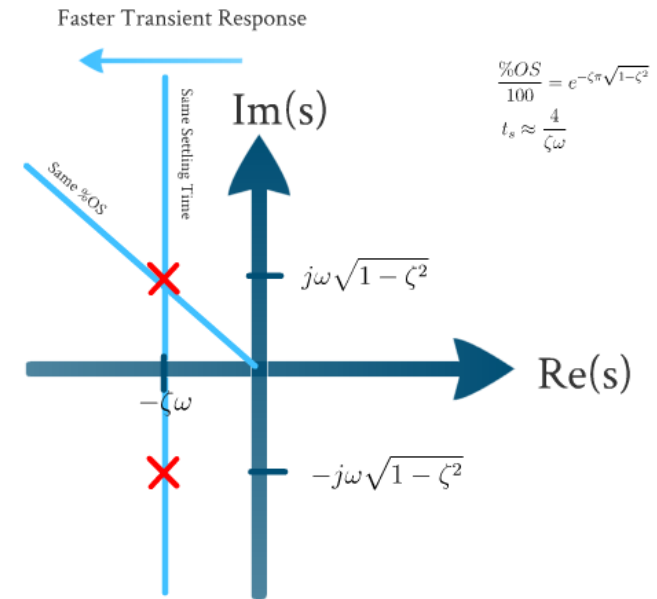
Improving Transient Response



Improving Transient Response

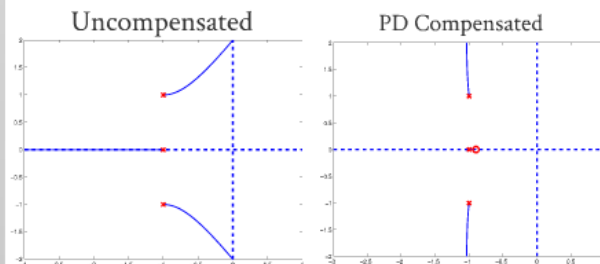
- The **dominant poles** are poles that are 5 times closer to the imaginary axis than the others.
- We can approximate the transient response by considering the first-order or second-order system defined by the closest poles to the imaginary axis

$$\frac{\%OS}{100} = e^{-\zeta\pi\sqrt{1-\zeta^2}} \quad t_s \approx \frac{4}{\zeta\omega}$$



PD Control / "Ideal Derivative Compensation"

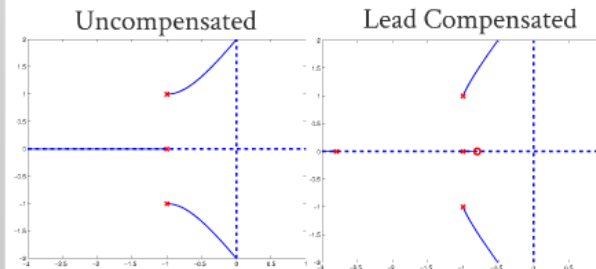
$$K_p + K_d s = K_d \left(s + \frac{K_p}{K_d} \right)$$



Adding PD control will be likely to alter (ie not necessary improve) the steady-state error.

Lead Compensator

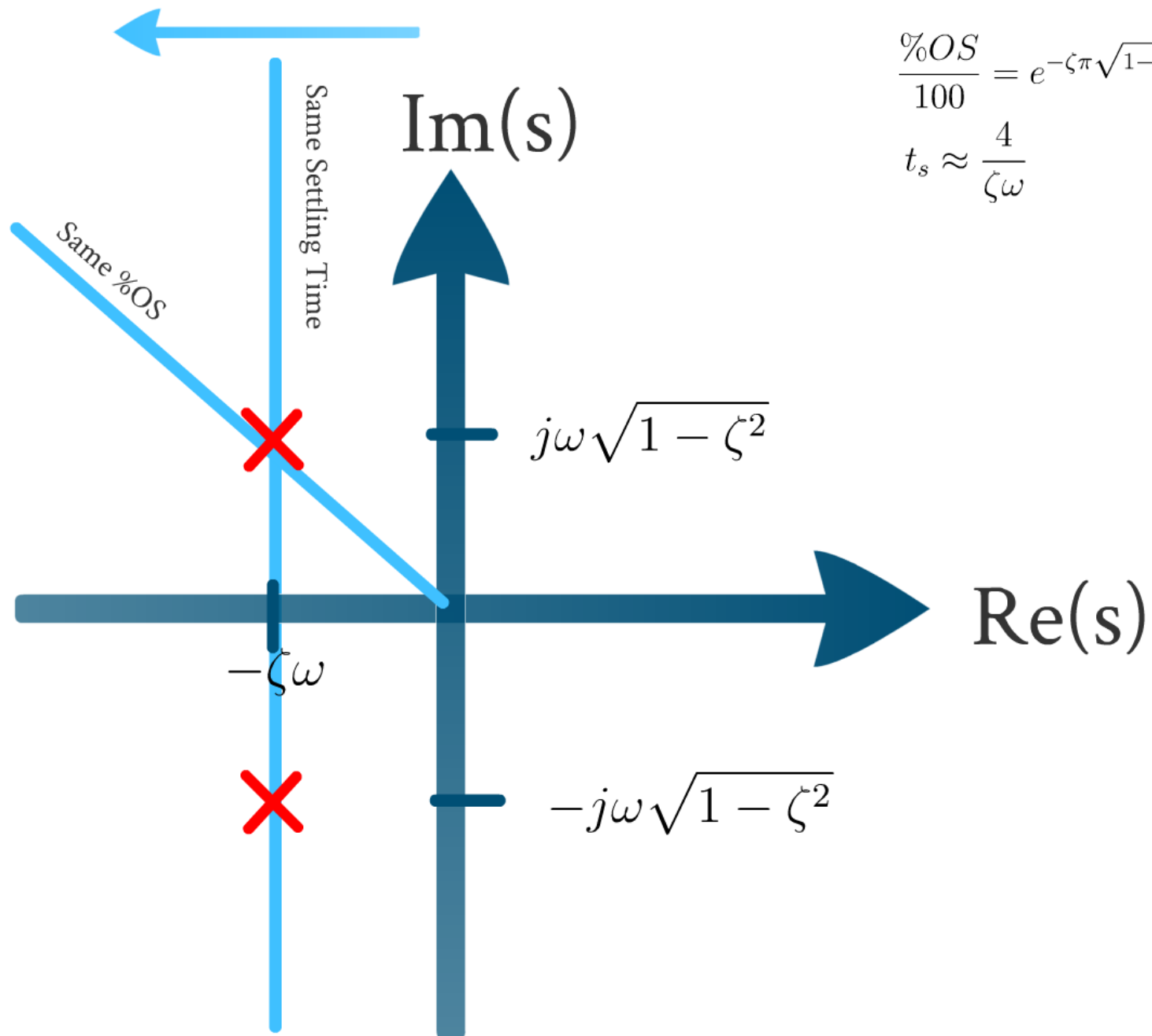
Add a zero and a pole:



- Lead compensators only require passive components (resistors and capacitors) whereas PD compensators require active components (Op Amps).
- A lead compensator also avoids amplification of high-frequency noise.

Faster Transient Response

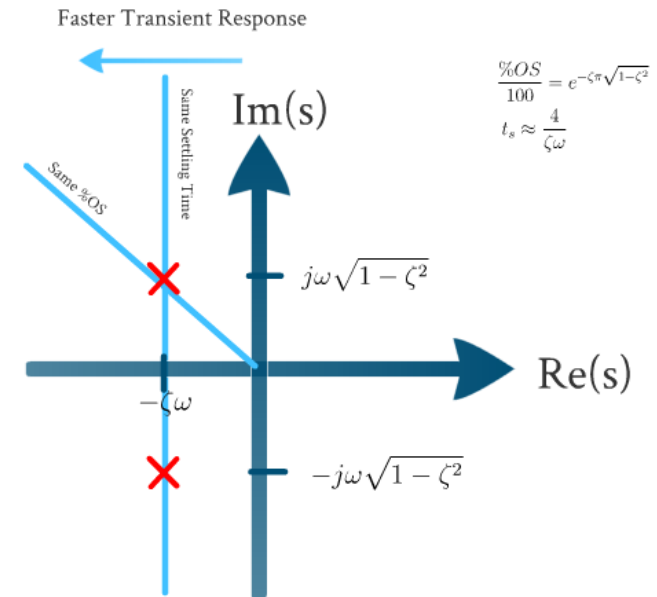
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Improving Transient Response

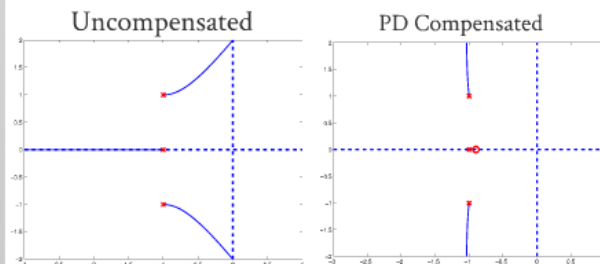
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PD Control / "Ideal Derivative Compensation"

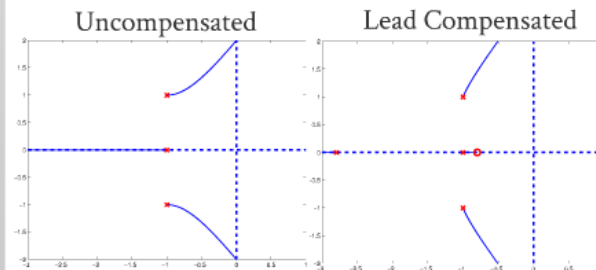
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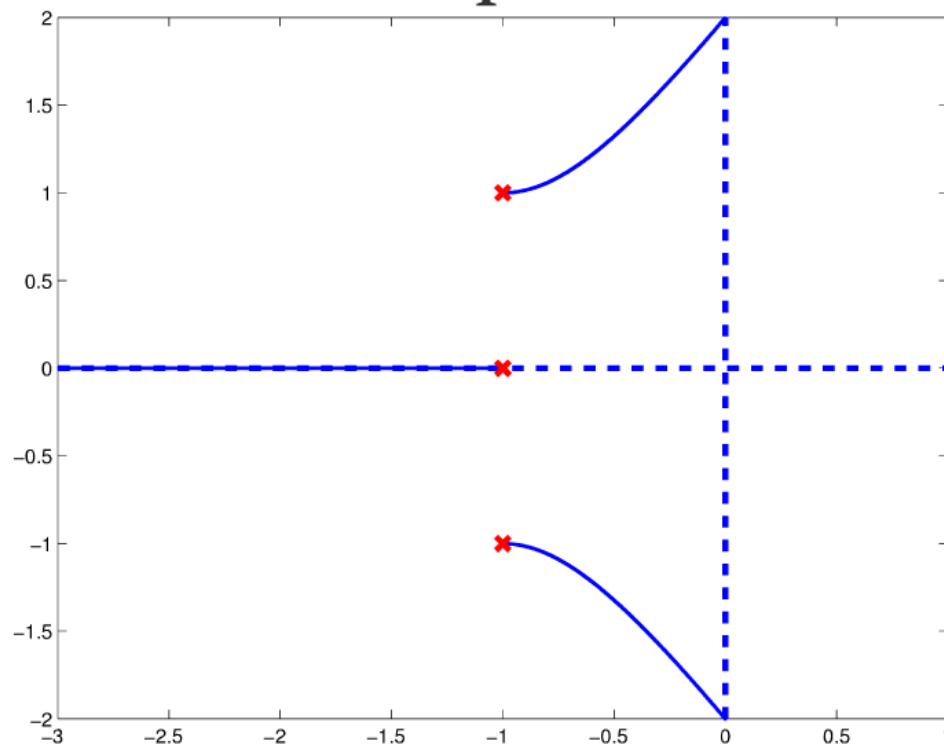


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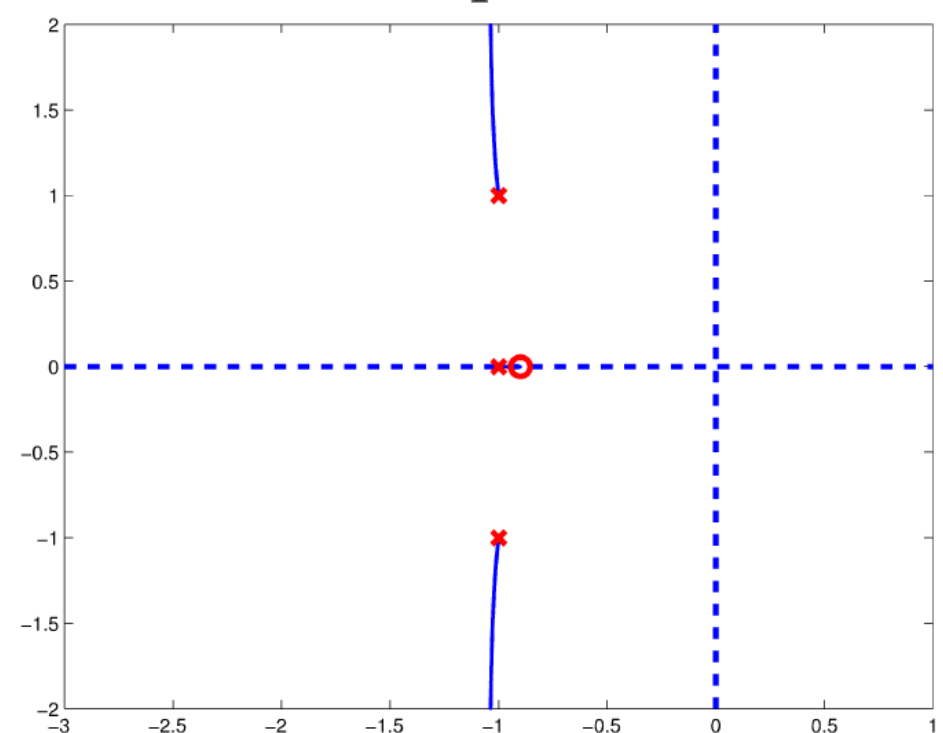
PD Control / "Ideal Derivative Compensation"

$$K_p + K_d s = K_d \left(s + \frac{K_p}{K_d} \right)$$

Uncompensated



PD Compensated

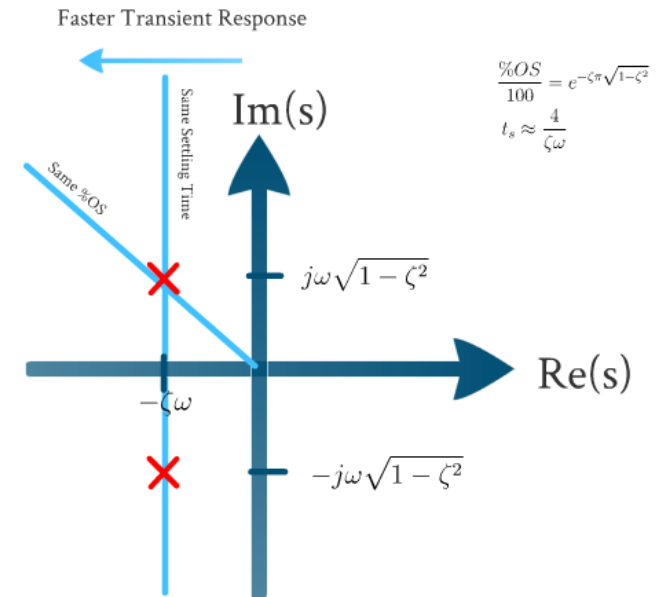


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Improving Transient Response

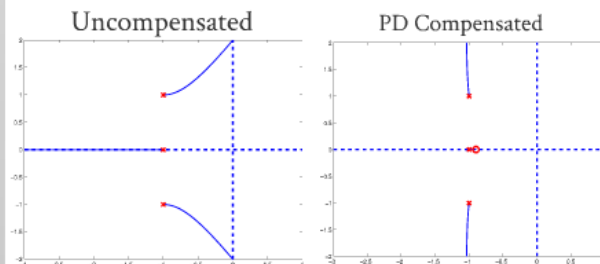
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PD Control / "Ideal Derivative Compensation"

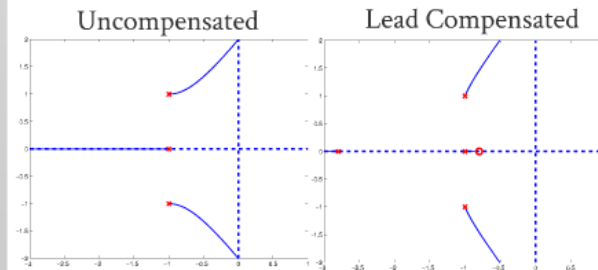
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Lead Compensator

Add a zero and a pole:

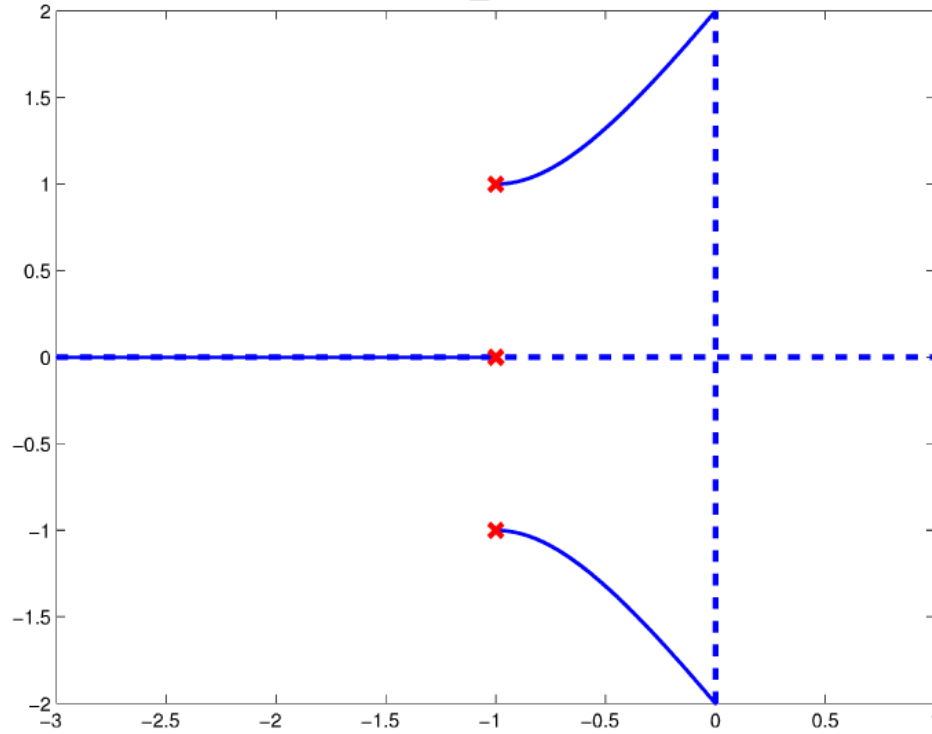


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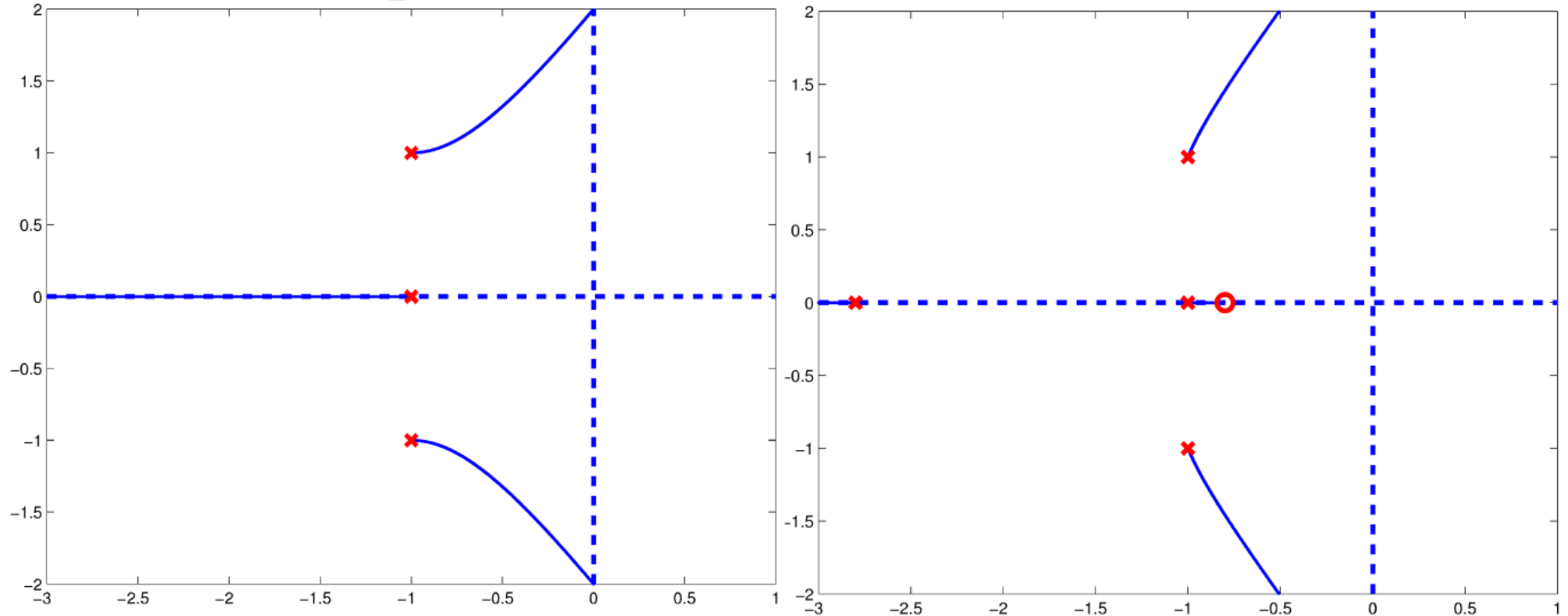
Lead Compensator

Add a zero and a pole:

Uncompensated



Lead Compensated

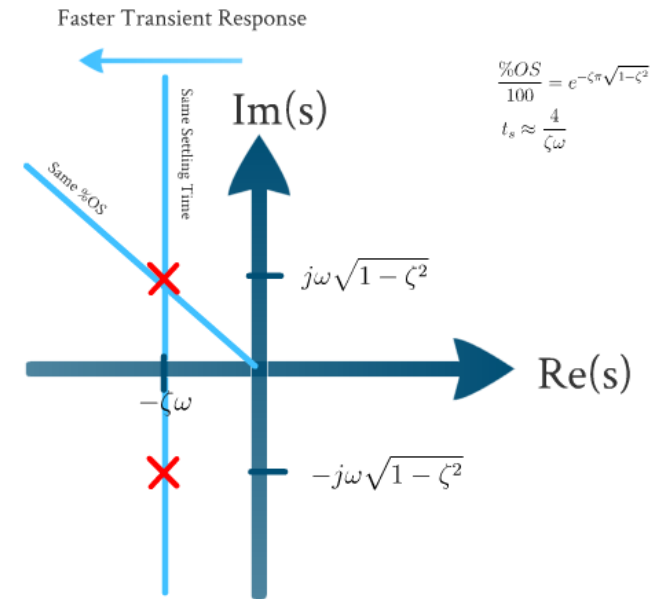


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Improving Transient Response

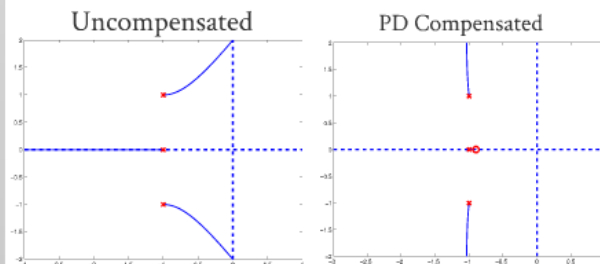
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PD Control / "Ideal Derivative Compensation"

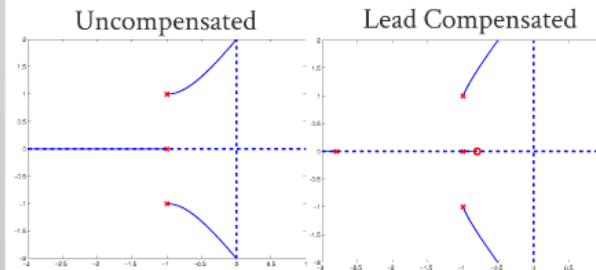
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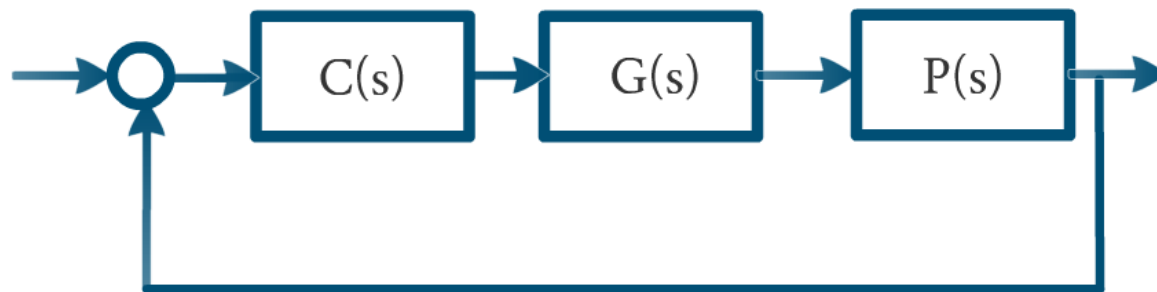
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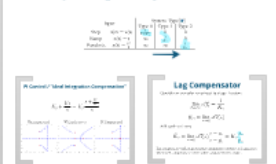
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Cascade Compensators



Designing PID/Lead-lag Compensators

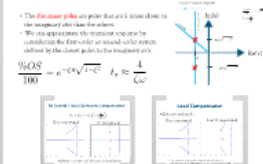
Improving Steady-state Error



Design Process

1. Determine the desired closed-loop poles based on the transient response specifications.
 2. Calculate the required compensator transfer function $C(s)$ to place the poles at the desired locations.
 3. Verify the steady-state error requirements are satisfied.
 4. Simulate the system to ensure all specifications are met.

Improving Transient Response



Designing PID/Lead-lag Compensators

Design Process

- Design the PD controller or lead compensator to meet the transient response specifications; this moves the root locus to where it needs to be.
- Verify that the transient response is as predicted and iterate if necessary.
 - The poles nearest the imaginary axis may not be dominant poles.
- Design the PI controller or lag compensator to achieve the required steady-state error.
 - Verify that the steady-state error and transient response are as predicted and iterate the design if necessary.
 - The "small" changes to the root locus may be larger than anticipated.

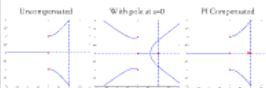
Improving Steady-state Error

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	Type-0	Type-1	Type-2
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Parabola $x(t) = \frac{t^2}{2}$	∞	∞	$\frac{1}{K_a}$



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$$K_p + \frac{K_I}{s} = K_p \frac{s + \frac{K_I}{K_p}}{s}$$



Lag Compensator

Consider an example ramp input to a type-1 system:

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$$K_v = \lim_{s \rightarrow 0} sG(s)$$

Add a pole and zero:

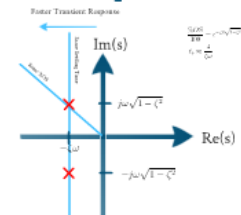
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Improving Transient Response

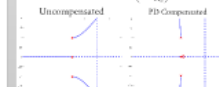
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$$\frac{\%OS}{100} = e^{-\zeta\pi\sqrt{1-\zeta^2}} \quad t_s \approx \frac{4}{\zeta\omega_n}$$



PD Control / "Ideal Derivative Compensation"

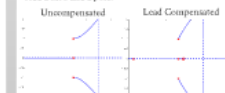
$$K_p + K_d s = K_p \left(1 + \frac{s}{\omega_n}\right)$$



Adding PD control will be likely to alter (ie not necessarily improve) the steady-state error.

Lead Compensator

Add a zero and a pole:



Lead compensators only require passive components (resistors and capacitors) whereas PD compensators require active components (Op-Amps).

Design Process

- Design the PD controller or lead-compensator to meet the transient response specifications; this moves the root locus to where it needs to be.
- Verify that the transient response is as predicted and iterate if necessary
 - The poles nearest the imaginary axis may not be dominant poles
- Design the PI controller or lag-compensator to achieve the required steady-state error.
- Verify that the steady-state error and transient response are as predicted and iterate the design if necessary
 - The "small" changes to the root locus may be larger than anticipated.

Designing PID/Lead-lag Compensators

Design Process

- Design the PD controller or lead compensator to meet the transient response specifications; this moves the root locus to where it needs to be.
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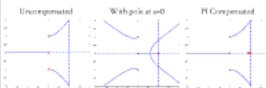
Improving Steady-state Error

Input	System Type*		
	Type-0	Type-1	Type-2
Step $x(t) = u(t)$	$\frac{1}{K_p K_v}$	0	0
Ramp $x(t) = t$	∞	$\frac{1}{K_v}$	0
Parabola $x(t) = \frac{t^2}{2}$	∞	∞	$\frac{1}{K_a}$



PI Control / "Ideal Integration Compensation"

$$K_p + \frac{K_I}{s} = K_p \frac{s + \frac{K_I}{K_p}}{s}$$



Lag Compensator

Consider an example ramp input to a type-1 system:

$$\lim_{t \rightarrow \infty} e(t) = \frac{1}{K_v}$$

$$K_v = \lim_{s \rightarrow 0} sG(s)$$

Add a pole and zero:

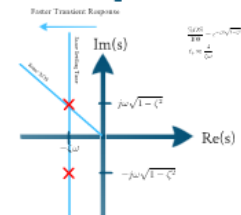
$$K_v = \lim_{s \rightarrow 0} sG(s) \frac{s - z_c}{s - p_c} = K_v \frac{z_c}{p_c}$$

Lag compensators only require passive components (resistors and capacitors) whereas PI compensators require active components (Op-Amps).

Improving Transient Response

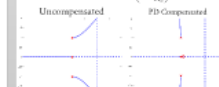
- The **dominant poles** are poles that are 5 times closer to the imaginary axis than the others.
- We can approximate the transient response by considering the first-order or second-order system defined by the closest poles to the imaginary axis

$$\frac{\%OS}{100} = e^{-\zeta\pi\sqrt{1-\zeta^2}} \quad t_s \approx \frac{4}{\zeta\omega_n}$$



PD Control / "Ideal Derivative Compensation"

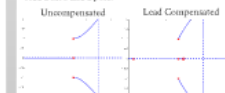
$$K_p + K_d s = K_d \left(s + \frac{K_p}{K_d} \right)$$



Adding PD control will be likely to alter (ie not necessarily improve) the steady-state error.

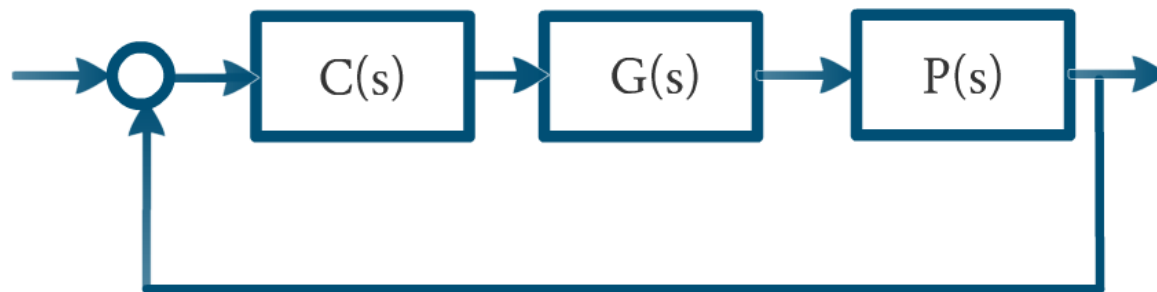
Lead Compensator

Add a zero and a pole:



Lead compensators only require passive components (resistors and capacitors) whereas PD compensators require active components (Op-Amps).

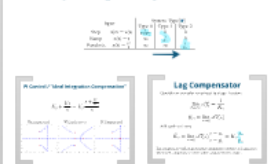
Cascade Compensators



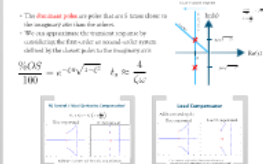
Designing PID/Lead-lag Compensators

Design Process

Improving Steady-state Error

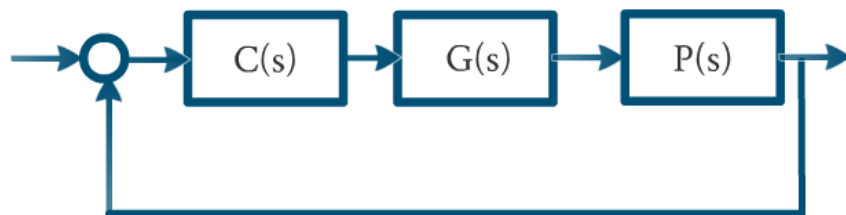


Improving Transient Response



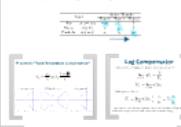
Compensators

Cascade Compensators

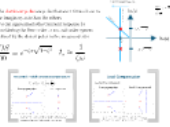


Designing PID/Lead-lag Compensators

Improving Steady-state Error



Improving Transient Response



Feedback Compensators



This lecture covers:

- Design of a Control System via root locus.

