

# ELEC 207 Part B

## Control Theory Lecture 8: System Stability

Prof Simon Maskell  
CHAD-G68  
s.maskell@liverpool.ac.uk  
0151 794 4573

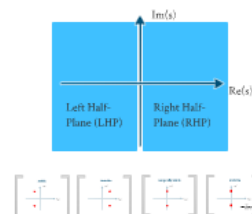


## Stability and Instability

**Bounded Input Bounded Output**  
definition of stability:

- A system is **stable** if every bounded input yields a bounded output.
- Systems that aren't stable are **unstable** (and are pretty useless).

## Relationship Between Poles' Location and Stability



## Example 2: Numeric

$$s^4 + 6s^3 + 13s^2 + 12s + 4$$

$s^4$	1	13	4
$s^3$	6	12	0
$s^2$	$-\frac{1}{6}$	$\frac{1}{6} \cdot 13 = 11$	$-\frac{1}{6} \cdot 4 = -\frac{2}{3}$
$s^1$	$-\frac{1}{11}$	$\frac{1}{11} \cdot 12 = \frac{12}{11}$	0
$s^0$	$-\frac{11}{108}$	$\frac{11}{108} \cdot 4 = \frac{11}{27}$	0

## Example 5: Parameterised

$s^2$	1	K
$s^1$	0	0
$s^0$	0	0

$s^2$	1	K
$s^1$	0	0
$s^0$	0	0

## Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$

$$r_1 > 0 \quad \forall s \quad a_1 > 0 \quad \forall s$$

Find the (equivalent) closed-loop transfer function

Produce the initial layout of the Routh table

Write the coefficients of the characteristic equation in descending order

Complete the remaining part of the Routh table until all terms generated are zero

Produce the stability test for the system of the Routh table all terms must be positive and non-zero

Then produce the test for the system

Interpret the results

The number of sign changes in the first column of the Routh table is the number of poles in the right half plane

If there are no sign changes in the first column of the Routh table the system is stable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

If there are sign changes in the first column of the Routh table the system is unstable

## Example 3: Zero First Column

$s^2$	1	0	0
$s^1$	0	0	0
$s^0$	0	0	0

$s^2$	1	0	0
$s^1$	0	0	0
$s^0$	0	0	0

$s^2$	1	0	0
$s^1$	0	0	0
$s^0$	0	0	0

$s^2$	1	0	0
$s^1$	0	0	0
$s^0$	0	0	0

This lecture covers:  
• Stability and instability  
• Relationship between poles' location and stability  
• Routh-Hurwitz stability criteria

# ELEC 207 Part B

## Control Theory Lecture 8: System Stability

Prof Simon Maskell

CHAD-G68

s.maskell@liverpool.ac.uk

0151 794 4573



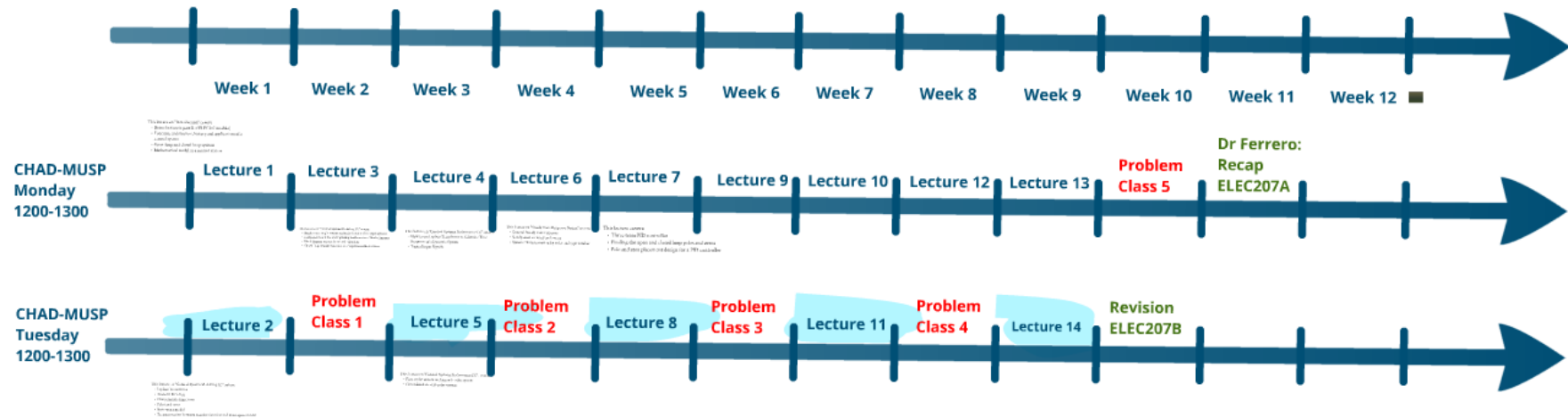
UNIVERSITY OF  
LIVERPOOL

This lecture covers:

- Stability and Instability
- Relationship between poles' location and stability
- Routh-Hurwitz stability criteria



# ELEC 207B: Timeline



This lecture on "Introduction" covers:

- (Introduction to part B of ELEC207 module)
- Function, architecture, history and applications of a control system
- Open-loop and closed-loop systems
- Mathematical model of a control system

# Lecture 1

A decorative graphic consisting of a horizontal blue bar and a vertical blue bar that intersect to form a cross-like shape, positioned at the bottom of the slide.

# Lecture 2

This lecture on "Control System Modelling (1)" covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model

1

# Lecture 3

Lect

This lecture on "Control System Modelling (2)" covers:

- Single-input single-output and multi-input multi-output systems
- Components and the underpinning mathematics of block diagrams
- Block diagram manipulation and reduction
- Closed-loop transfer function of a negative feedback system

This lecture on "Control Sy

- How to use Laplace Tra
- Response of a Dynamic
- Typical Input Signals

e 3

# Lecture 4

L

covers:  
multi-output systems  
atics of block diagrams  
feedback system

This lecture on "Control Systems Performance (1)" covers:

- How to use Laplace Transforms to Solve the Time Response of a Dynamic System
- Typical Input Signals

This lect

- Gene
- Stead
- Syste



em  
1

# Lecture 5

Pro  
Clas

This lecture on "Control Systems Performance (2)" covers:

- First-order system and second-order system
- Generalized second-order system

# Lecture 6

Lec

This lecture on "Steady-State Response Design" covers:

- General Steady-state response
- Steady-state accuracy and errors
- System Characterization by order and type number

This lecture c

- Three ter
- Finding t
- Pole and

6

# Lecture 7

Lec

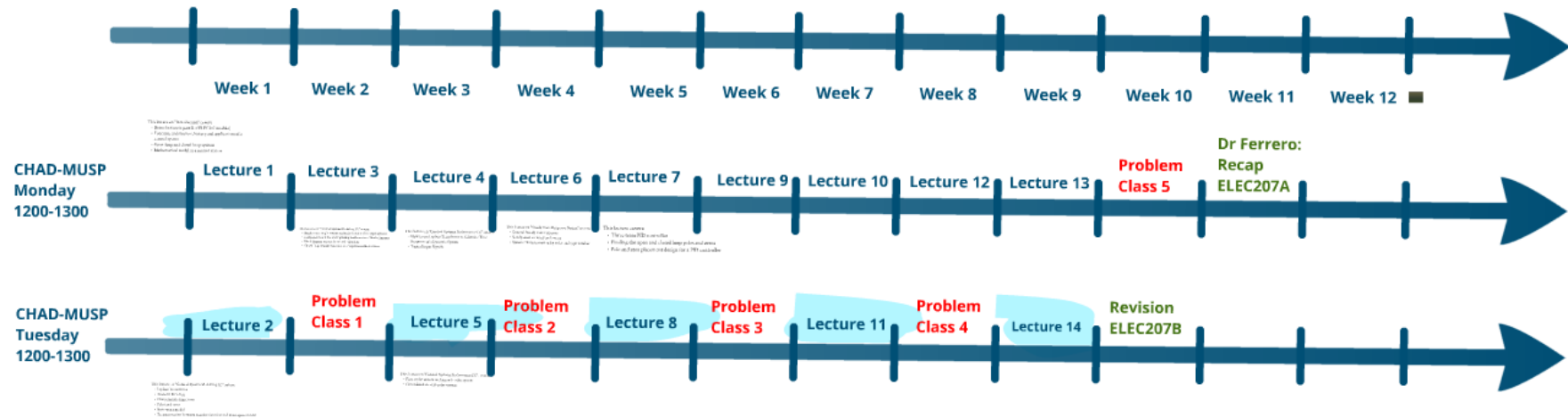
gn" covers:

be number

This lecture covers:

- Three term PID controller
- Finding the open and closed loop poles and zeros
- Pole and zero placement design for a PID controller

# ELEC 207B: Timeline



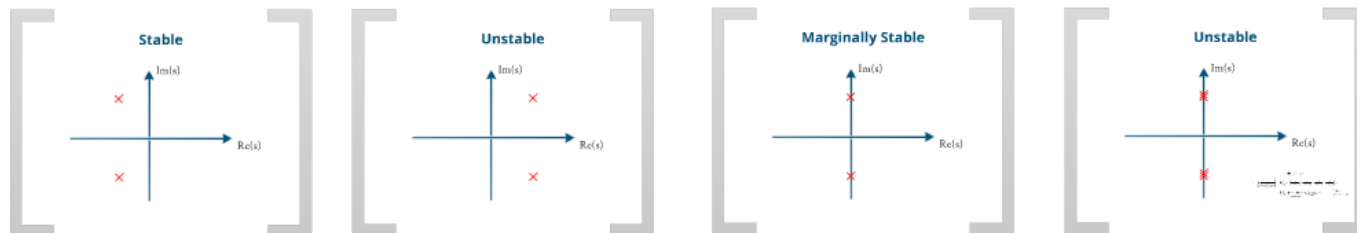
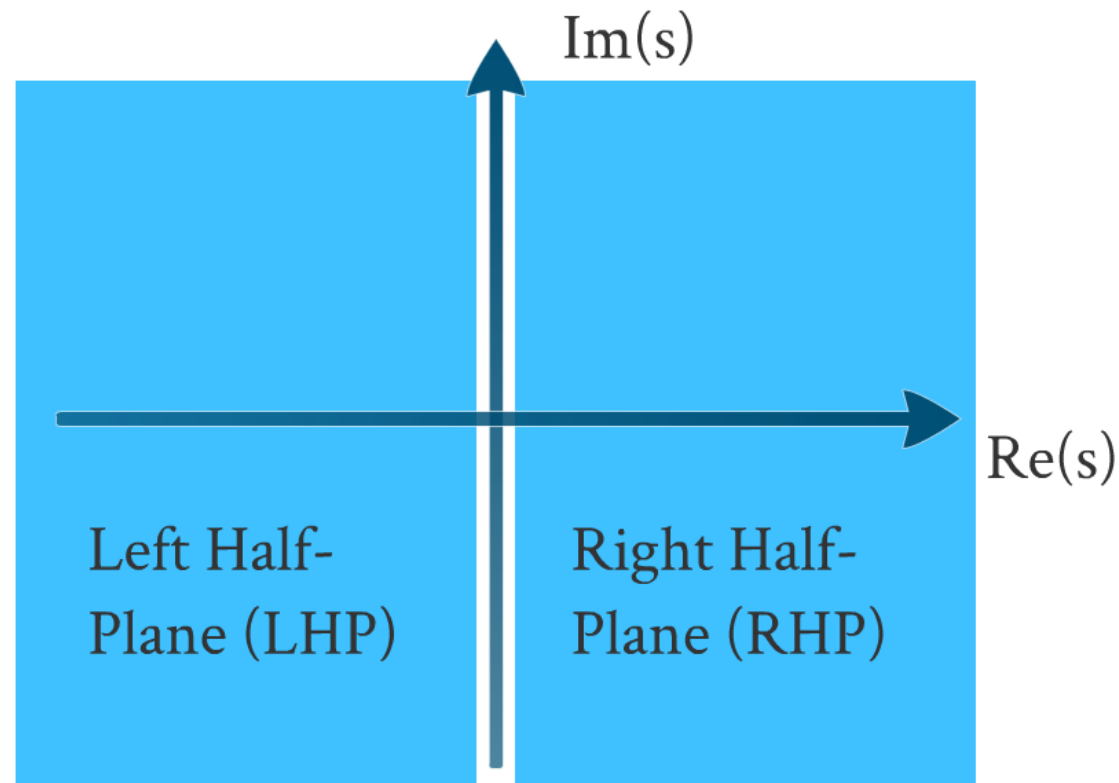
# Stability and Instability

## Bounded Input Bounded Output

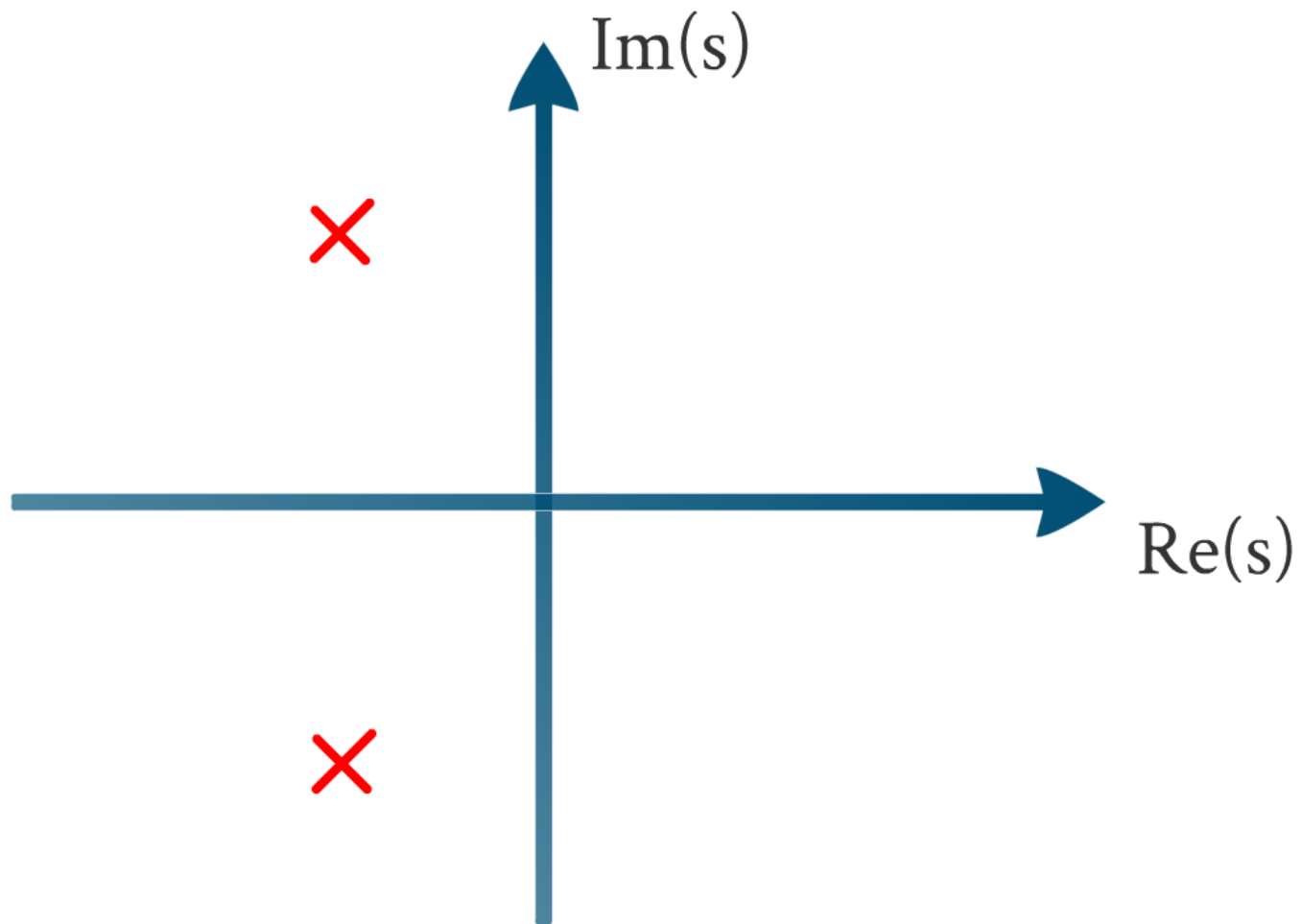
definition of stability:

- A system is **stable** if every bounded input yields a bounded output.
- Systems that aren't stable are **unstable** (and are pretty useless).

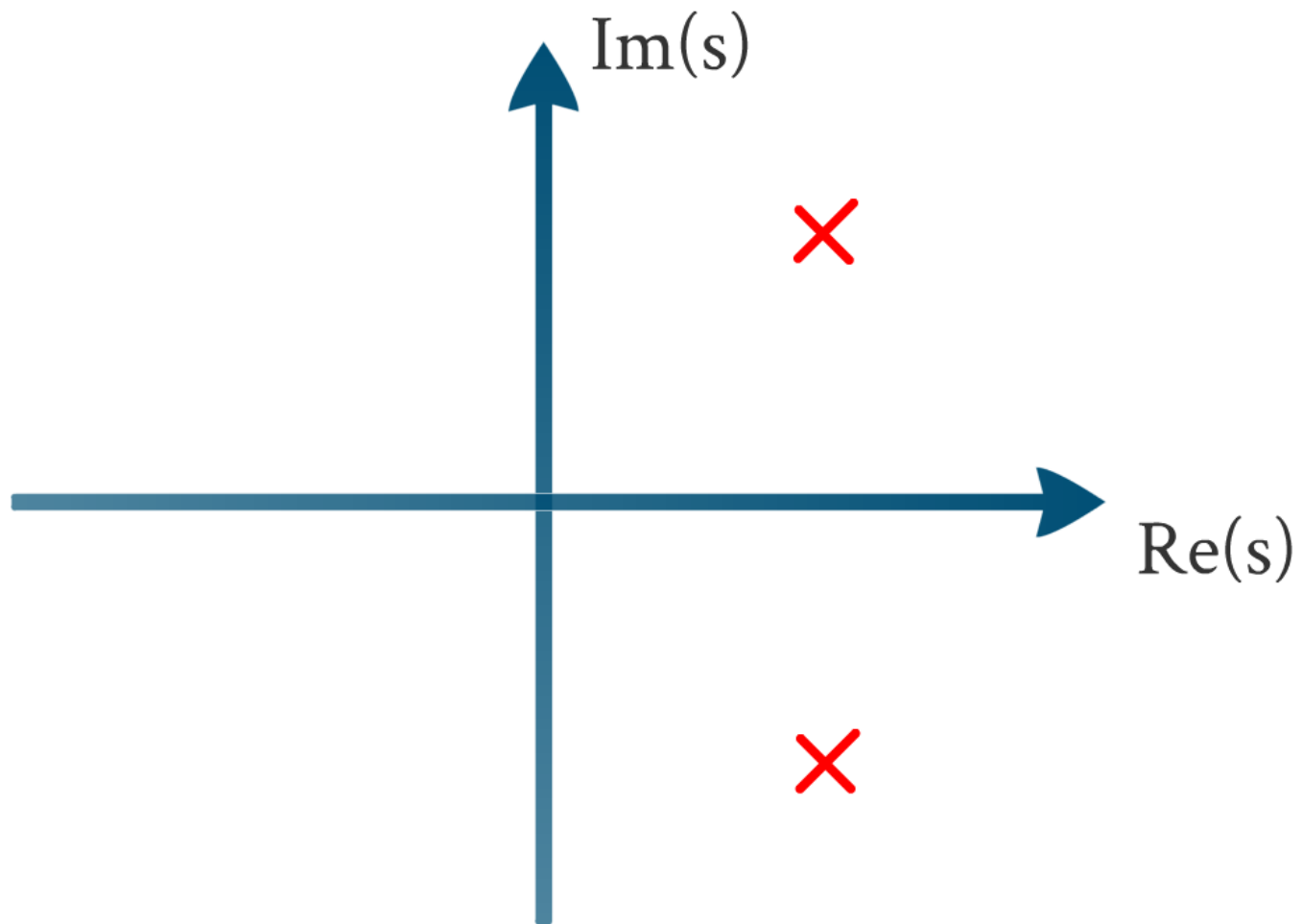
# Relationship Between Poles' Location and Stability



**Stable**

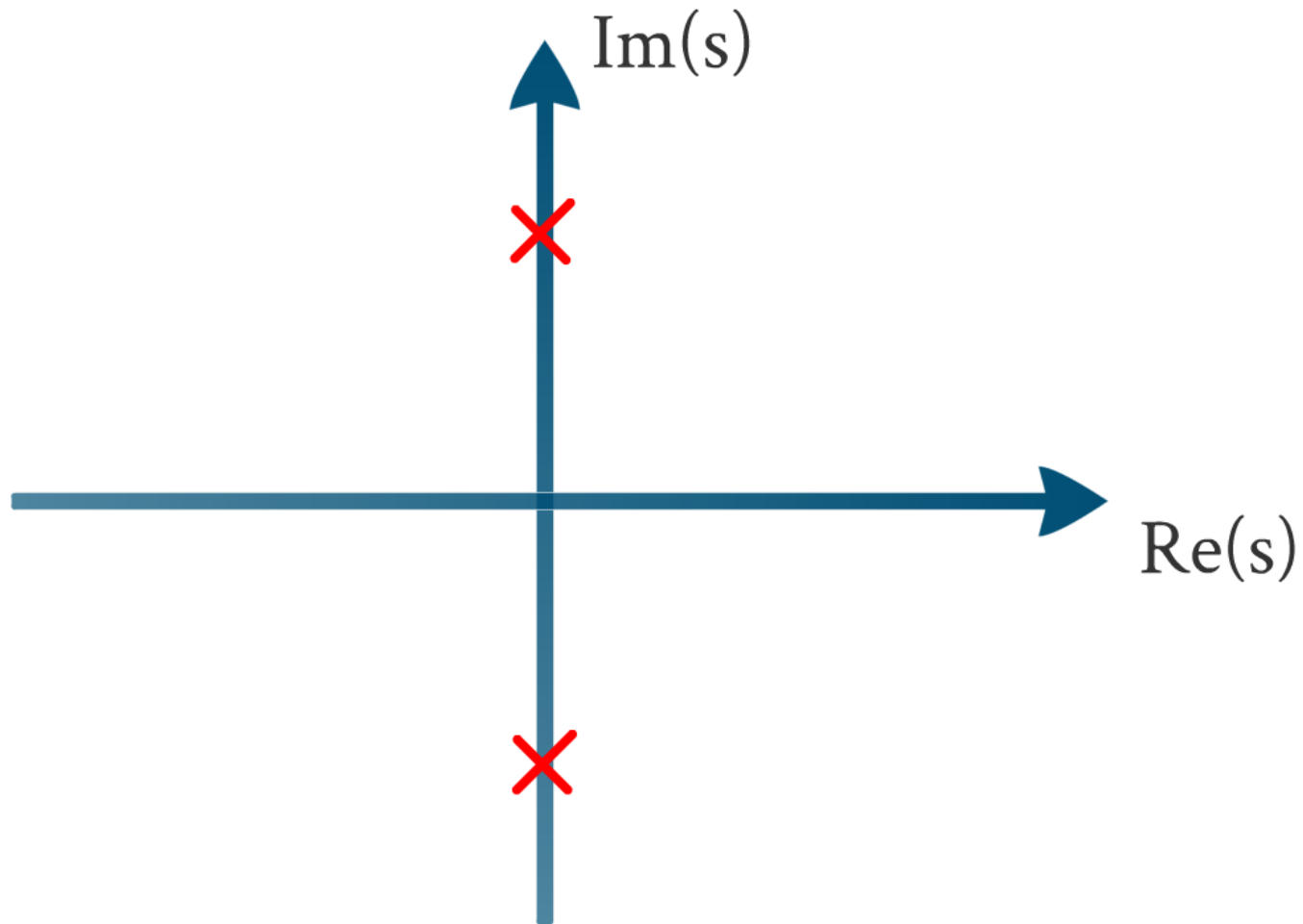


# Unstable

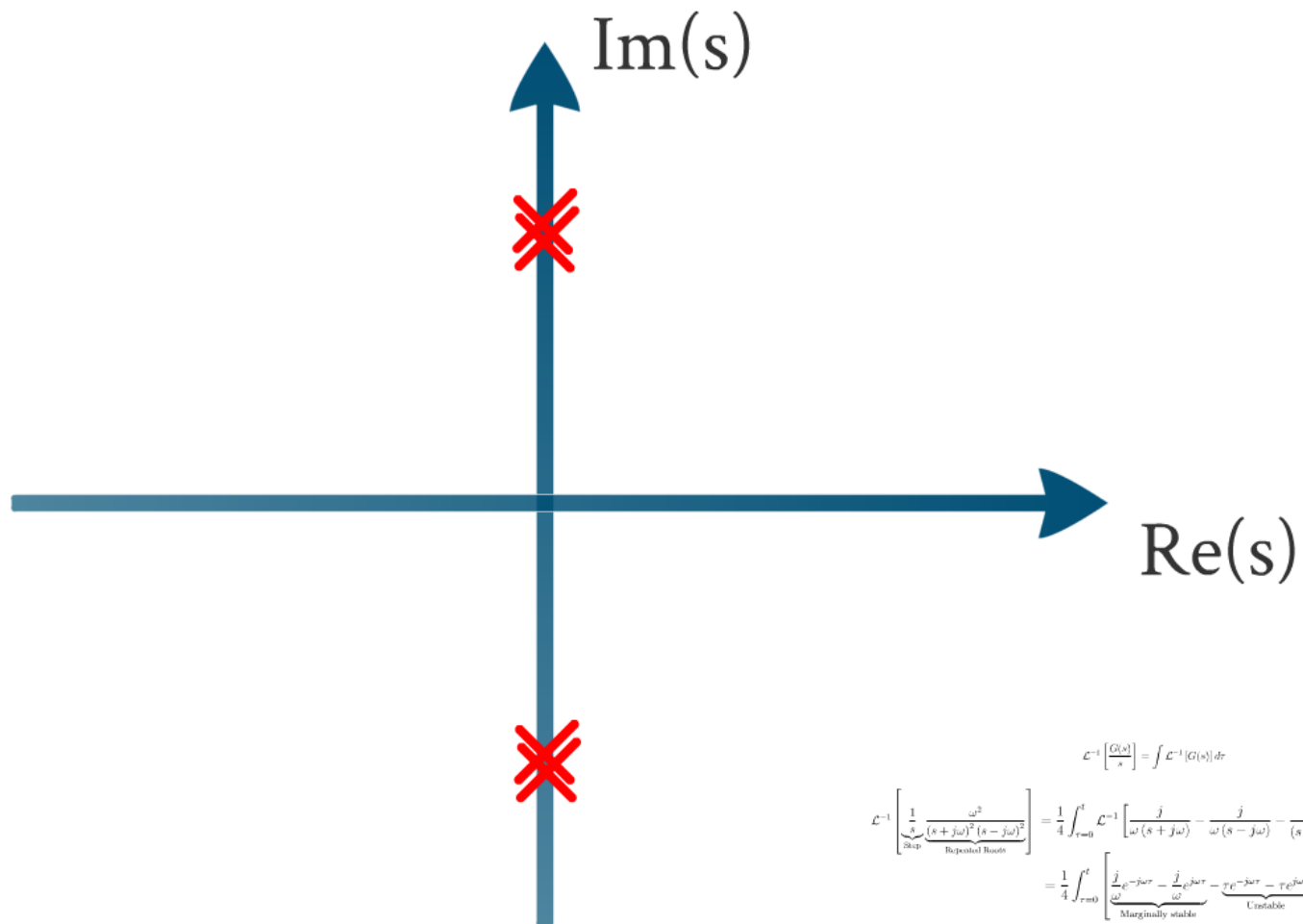




# Marginally Stable



# Unstable



$$\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right] = \int \mathcal{L}^{-1}[G(s)] d\tau$$

$$\mathcal{L}^{-1}\left[\frac{1}{\omega_{\text{dp}}^2} \frac{\omega^2}{(s+j\omega)^2(s-j\omega)^2}\right] = \frac{1}{4} \int_{\tau=0}^t \mathcal{L}^{-1}\left[\frac{j}{\omega(s+j\omega)} - \frac{j}{\omega(s-j\omega)} - \frac{1}{(s+j\omega)^2} - \frac{1}{(s-j\omega)^2}\right] d\tau$$

$$= \frac{1}{4} \int_{\tau=0}^t \left[ \underbrace{\frac{j}{\omega} e^{-j\omega\tau}}_{\text{Marginally stable}} - \underbrace{\frac{j}{\omega} e^{j\omega\tau}}_{\text{Unstable}} - \tau e^{-j\omega\tau} - \tau e^{j\omega\tau} \right] d\tau \quad \mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = t$$

$$\mathcal{L}^{-1}[G(s-a)] = e^{at} \mathcal{L}^{-1}[G(s)]$$

$$\mathcal{L}^{-1} \left[ \frac{G(s)}{s} \right] = \int \mathcal{L}^{-1} [G(s)] d\tau$$

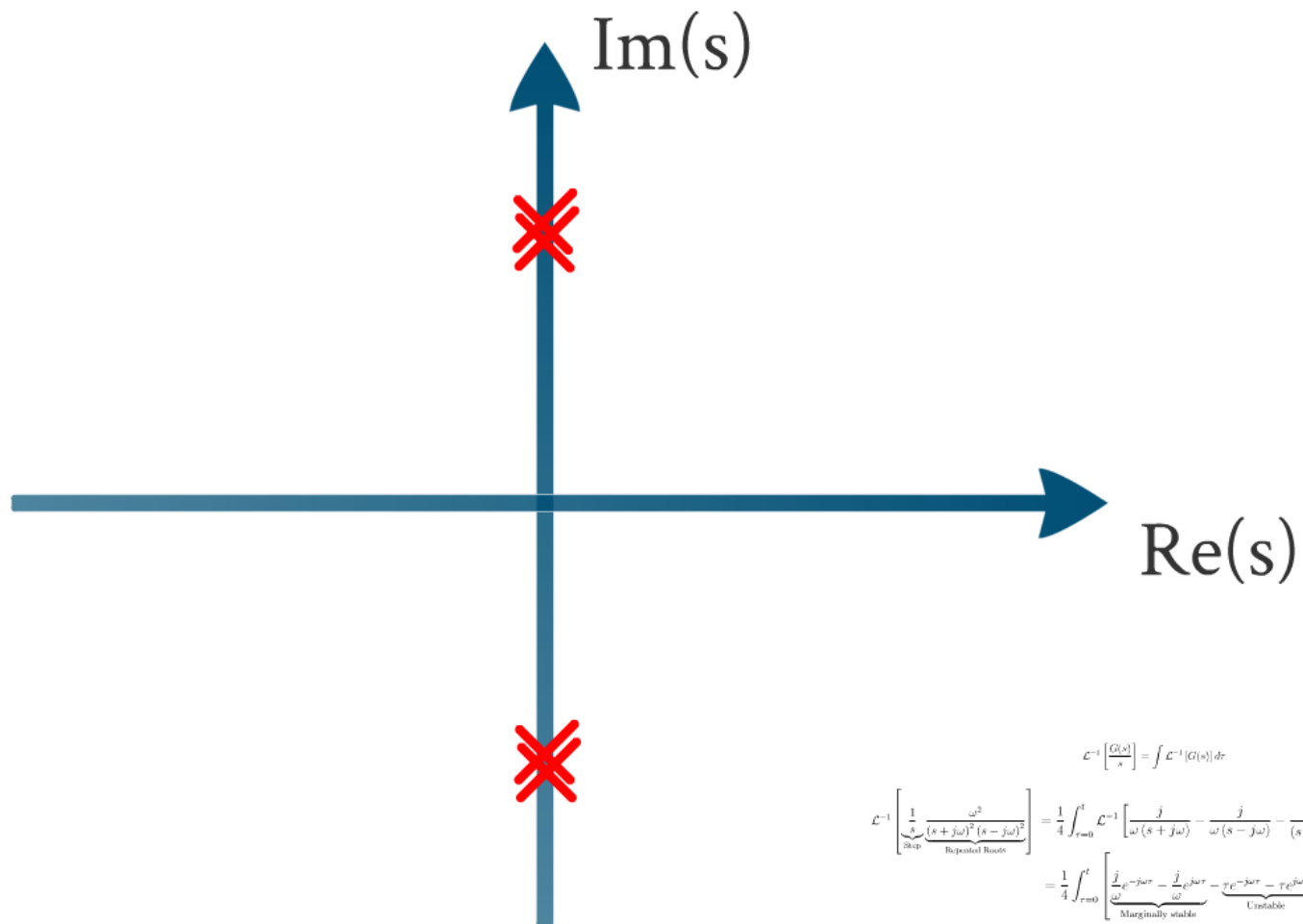
$$\mathcal{L}^{-1} \left[ \underbrace{\frac{1}{s}}_{\text{Step}} \underbrace{\frac{\omega^2}{(s+j\omega)^2(s-j\omega)^2}}_{\text{Repeated Roots}} \right] = \frac{1}{4} \int_{\tau=0}^t \mathcal{L}^{-1} \left[ \frac{j}{\omega(s+j\omega)} - \frac{j}{\omega(s-j\omega)} - \frac{1}{(s+j\omega)^2} - \frac{1}{(s-j\omega)^2} \right] d\tau$$

$$= \frac{1}{4} \int_{\tau=0}^t \left[ \underbrace{\frac{j}{\omega} e^{-j\omega\tau} - \frac{j}{\omega} e^{j\omega\tau}}_{\text{Marginally stable}} - \underbrace{\tau e^{-j\omega\tau} - \tau e^{j\omega\tau}}_{\text{Unstable}} \right] d\tau$$

$$\mathcal{L}^{-1} \left[ \frac{1}{s^2} \right] = t$$

$$\mathcal{L}^{-1} [G(s-a)] = e^{at} \mathcal{L}^{-1} [G(s)]$$

# Unstable



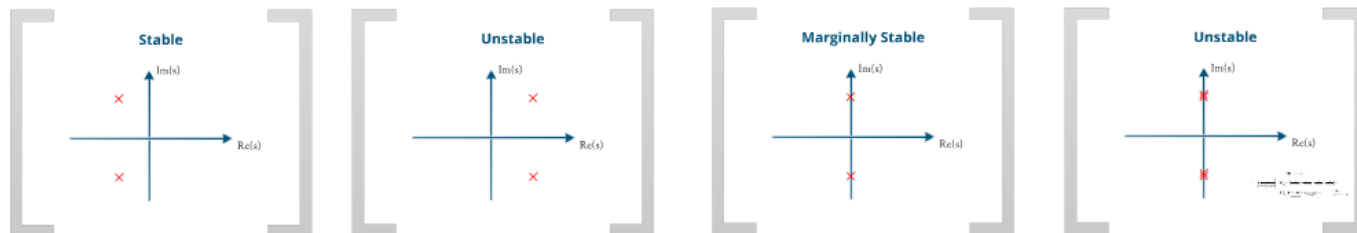
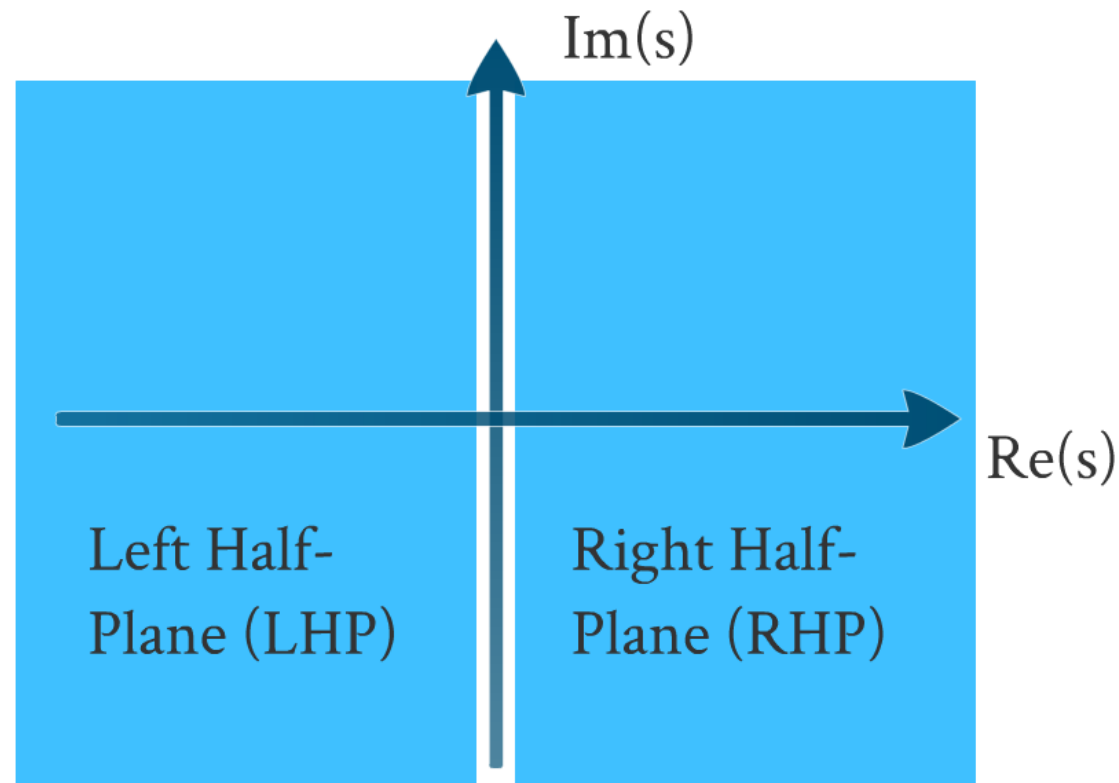
$$\mathcal{L}^{-1}\left[\frac{G(s)}{s}\right] = \int \mathcal{L}^{-1}[G(s)] d\tau$$

$$\mathcal{L}^{-1}\left[\frac{1}{\underbrace{s}_{\text{DC}} \underbrace{(s+j\omega)^2 (s-j\omega)^2}_{\text{Repeated Roots}}}\right] = \frac{1}{4} \int_{\tau=0}^t \mathcal{L}^{-1}\left[\frac{j}{\omega(s+j\omega)} - \frac{j}{\omega(s-j\omega)} - \frac{1}{(s+j\omega)^2} - \frac{1}{(s-j\omega)^2}\right] d\tau$$

$$= \frac{1}{4} \int_{\tau=0}^t \left[ \underbrace{\frac{j}{\omega} e^{-j\omega\tau}}_{\text{Marginally stable}} - \underbrace{\frac{j}{\omega} e^{j\omega\tau}}_{\text{Unstable}} - \tau e^{-j\omega\tau} - \tau e^{j\omega\tau} \right] d\tau \quad \mathcal{L}^{-1}\left[\frac{1}{s^2}\right] = t$$

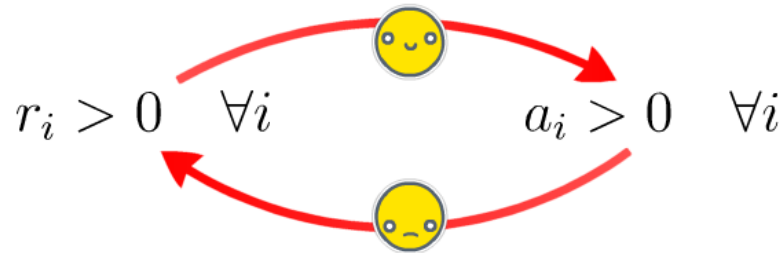
$$\mathcal{L}^{-1}[G(s-a)] = e^{at} \mathcal{L}^{-1}[G(s)]$$

# Relationship Between Poles' Location and Stability



# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

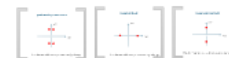
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

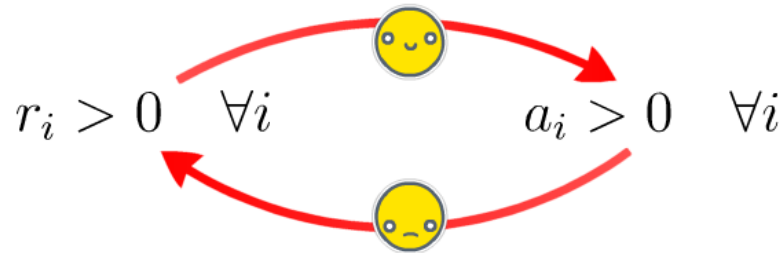
- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

$$1 + \dots + a_1 s + a_0 = 0$$

$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

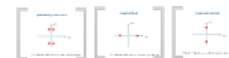
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.



## Example 1

$$a_6s^6 + a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s^1 + a_0s^0$$

$\mathfrak{N}^6$	$\alpha_6$	$\alpha_4$	$\alpha_2$	$\alpha_0$
$\mathfrak{N}^5$	$\alpha_5$	$\alpha_3$	$\alpha_1$	$\emptyset$
$\mathfrak{N}^4$				
$\mathfrak{N}^3$				
$\mathfrak{N}^2$				
$\mathfrak{N}^1$				
$\mathfrak{N}^0$				

$\alpha^0$	$\alpha_0$	$\alpha_1$				$\alpha_2$	$\alpha_3$
$\alpha^1$	$\alpha_5$	$\alpha_3$				$\alpha_1$	0
$\alpha^4$	$b_3 = -\frac{1}{\alpha_5}$	$\frac{\alpha_1}{\alpha_5}$	$\frac{\alpha_2}{\alpha_5}$	$b_2 = -\frac{1}{\alpha_5}$	$\frac{\alpha_1}{\alpha_5}$	$\frac{\alpha_2}{\alpha_5}$	0
$\alpha^5$	$c_1 = -\frac{1}{b_3}$	$\frac{\alpha_5}{b_3}$	$\frac{\alpha_2}{b_3}$	$c_2 = -\frac{1}{b_3}$	$\frac{\alpha_5}{b_3}$	$\frac{\alpha_2}{b_3}$	0
$\alpha^3$							
$\alpha^2$							

$s^6$	$a_6$	$a_5$	$a_2$	$a_0$
$s^5$	$a_5$	$a_3$	$a_1$	$\emptyset$
$s^4$	$b_1 = -\frac{1}{a_1} \begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$			
$s^3$				
$s^2$				
$s^1$				
$s^0$				

$x^k$	$a_0$	$a_1$	$a_2$	$a_3$
$x^1$	$b_1 = -\frac{1}{a_1}$	$\frac{a_0}{a_1}$	$\frac{a_2}{a_1}$	$\frac{a_3}{a_1}$
$x^2$	$c_1 = -\frac{1}{b_1}$	$\frac{a_0}{b_1}$	$\frac{a_2}{b_1}$	$\frac{a_3}{b_1}$
$x^3$	$d_1 = -\frac{1}{c_1}$	$\frac{a_0}{c_1}$	$\frac{a_2}{c_1}$	$\frac{a_3}{c_1}$
$x^4$				
$x^5$				

$\sigma^0$	$a_6$	$a_4$	$a_2$	$a_0$
$\sigma^1$	$a_5$	$a_3$	$a_1$	0
$\sigma^4$	$b_1 = -\frac{1}{a_5} \begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5} \begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$		
$\sigma^3$				
$\sigma^2$				
$\sigma^5$				
$\sigma^6$				

$\alpha^A$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_8$
$\alpha^B$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$\alpha_5$	$\alpha_6$	$\alpha_7$	$\alpha_8$
$\alpha^1$	$b_1 = -\frac{1}{\alpha_1}$	$\alpha_2$	$\alpha_3$	$\alpha_4$	$b_2 = -\frac{1}{\alpha_1}$	$\alpha_5$	$\alpha_6$	$\alpha_7$
$\alpha^2$	$c_1 = -\frac{1}{\alpha_1}$	$\alpha_2$	$\alpha_3$	$b_2$	$c_2 = -\frac{1}{\alpha_1}$	$b_3$	$\alpha_6$	$\alpha_7$
$\alpha^3$	$d_1 = -\frac{1}{\alpha_1}$	$b_3$	$b_5$	$b_2$	$d_2 = -\frac{1}{\alpha_1}$	$b_1$	$b_5$	$\alpha_7$
$\alpha^4$	$e_1 = -\frac{1}{\alpha_1}$	$c_2$	$c_3$	$c_4$	$e_2 = -\frac{1}{\alpha_1}$	$c_1$	$\alpha_6$	$\alpha_7$
$\alpha^5$	$f_1 = -\frac{1}{\alpha_1}$	$d_2$	$d_3$	$d_4$	$f_2 = -\frac{1}{\alpha_1}$	$d_1$	$d_5$	$\alpha_7$
$\alpha^6$	$g_1 = -\frac{1}{\alpha_1}$	$e_2$	$e_3$	$e_4$	$g_2 = -\frac{1}{\alpha_1}$	$e_1$	$e_5$	$\alpha_7$
$\alpha^7$	$h_1 = -\frac{1}{\alpha_1}$	$f_2$	$f_3$	$f_4$	$h_2 = -\frac{1}{\alpha_1}$	$f_1$	$f_5$	$\alpha_7$
$\alpha^8$	$i_1 = -\frac{1}{\alpha_1}$	$g_2$	$g_3$	$g_4$	$i_2 = -\frac{1}{\alpha_1}$	$g_1$	$g_5$	$\alpha_7$

[illegible]

$\alpha^k$	$\alpha_1$			$\alpha_2$			$\alpha_3$
$\alpha^k$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_3$
$\alpha^1$	$b_1 = -\frac{1}{\alpha_1}$	$b_2 = \frac{\alpha_1}{\alpha_2}$	$b_3 = \frac{\alpha_1}{\alpha_3}$	$b_4 = -\frac{1}{\alpha_1}$	$b_5 = \frac{\alpha_1}{\alpha_2}$	$b_6 = -\frac{1}{\alpha_1}$	$b_7 = \frac{\alpha_1}{\alpha_3}$
$\alpha^2$	$c_1 = -\frac{1}{\alpha_1}$	$c_2 = \frac{\alpha_1}{\alpha_2}$	$c_3 = \frac{\alpha_1}{\alpha_3}$	$c_4 = -\frac{1}{\alpha_1}$	$c_5 = \frac{\alpha_1}{\alpha_2}$	$c_6 = \frac{\alpha_1}{\alpha_3}$	$c_7 = \frac{\alpha_1}{\alpha_3}$
$\alpha^3$	$d_1 = -\frac{1}{\alpha_1}$	$d_2 = \frac{\alpha_1}{\alpha_2}$	$d_3 = \frac{\alpha_1}{\alpha_3}$	$d_4 = -\frac{1}{\alpha_1}$	$d_5 = \frac{\alpha_1}{\alpha_2}$	$d_6 = \frac{\alpha_1}{\alpha_3}$	$d_7 = \frac{\alpha_1}{\alpha_3}$
$\alpha^4$	$e_1 = -\frac{1}{\alpha_1}$	$e_2 = \frac{\alpha_1}{\alpha_2}$	$e_3 = \frac{\alpha_1}{\alpha_3}$	$e_4 = -\frac{1}{\alpha_1}$	$e_5 = \frac{\alpha_1}{\alpha_2}$	$e_6 = \frac{\alpha_1}{\alpha_3}$	$e_7 = \frac{\alpha_1}{\alpha_3}$
$\alpha^5$	$f_1 = -\frac{1}{\alpha_1}$	$f_2 = \frac{\alpha_1}{\alpha_2}$	$f_3 = \frac{\alpha_1}{\alpha_3}$	$f_4 = -\frac{1}{\alpha_1}$	$f_5 = \frac{\alpha_1}{\alpha_2}$	$f_6 = \frac{\alpha_1}{\alpha_3}$	$f_7 = \frac{\alpha_1}{\alpha_3}$

$\kappa^0$	$a_6$	$a_4$	$a_2$	$a_0$
$\kappa^1$	$a_5$	$a_3$	$a_1$	0
$\kappa^2$	$b_3 = -\frac{1}{a_6} \frac{a_0}{a_5} \frac{a_4}{a_3}$	$b_2 = -\frac{1}{a_6} \frac{a_6}{a_5} \frac{a_2}{a_3}$	$b_1 = -\frac{1}{a_6} \frac{a_0}{a_5} \frac{a_1}{a_3}$	0
$\kappa^3$				
$\kappa^4$				
$\kappa^5$				

$s^6$	$a_6$	$a_4$	$a_2$	$a_0$
$s^5$	$a_5$	$a_3$	$a_1$	0
$s^4$				
$s^3$				
$s^2$				
$s^1$				
$s^0$				

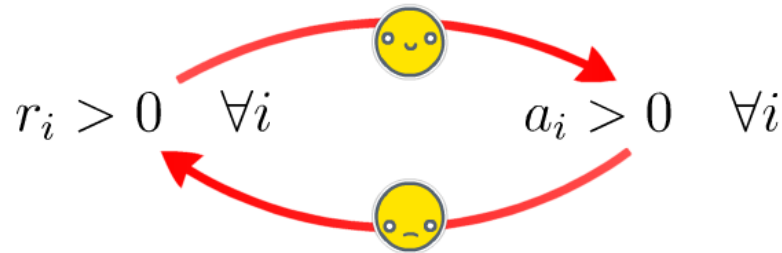


You Tube

$s^6$	$a_6$	$a_4$	$a_2$	$a_0$
$s^5$	$a_5$	$a_3$	$a_1$	0
$s^4$				
$s^3$				
$s^2$				
$s^1$				
$s^0$				

# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

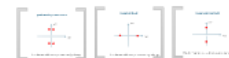
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

## Example 1

$$a_6s^6 + a_5s^5 + a_4s^4 + a_3s^3 + a_2s^2 + a_1s^1 + a_0s^0$$

$\chi_1^6$	$a_6$	$a_8$	$a_2$	$a_0$
$\chi_1^5$	$a_5$	$a_3$	$a_1$	$\emptyset$
$\chi_1^4$				
$\chi_1^3$				
$\chi_1^2$				
$\chi_1^1$				
$\chi_1^0$				

$s^6$	$a_5$		$a_4$		$a_2$	$a_3$
$s^5$	$a_5$		$a_3$		$a_1$	0
$s^4$	$b_2 = -\frac{1}{a_5} \frac{a_4}{a_5} \frac{a_4}{a_5}$	$b_2 = -\frac{1}{a_5} \frac{a_4}{a_5} \frac{a_2}{a_1}$	$b_3 = -\frac{1}{a_5} \frac{a_4}{a_5} \frac{a_3}{a_1}$	$b_3 = -\frac{1}{a_5} \frac{a_4}{a_5} \frac{a_3}{a_1}$	$b_3 = -\frac{1}{a_5} \frac{a_4}{a_5} \frac{a_3}{a_1}$	0
$s^3$	$c_1 = -\frac{1}{b_2} \frac{a_5}{a_5} \frac{a_3}{a_1}$	$c_2 = -\frac{1}{b_2} \frac{a_5}{a_5} \frac{a_3}{a_1}$	$c_3 = -\frac{1}{b_2} \frac{a_5}{a_5} \frac{a_3}{a_1}$	$c_3 = -\frac{1}{b_2} \frac{a_5}{a_5} \frac{a_3}{a_1}$	$c_3 = -\frac{1}{b_2} \frac{a_5}{a_5} \frac{a_3}{a_1}$	0
$s^2$						
$s^1$						
$s^0$						

$s^6$	$a_6$	$a_5$	$a_2$	$a_0$
$s^5$	$a_5$	$a_3$	$a_1$	$\emptyset$
$s^4$	$h_1 = -\frac{1}{a_1} \begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$			
$s^3$				
$s^2$				
$s^1$				
$s^0$				

$x^k$	$a_0$	$a_1$	$a_2$	$a_3$	$a_4$	$a_5$
$x^1$	$b_1 = -\frac{1}{a_1}$	$\frac{a_0}{a_1}$	$\frac{a_2}{a_1}$	$\frac{a_3}{a_1}$	$\frac{a_4}{a_1}$	$\frac{a_5}{a_1}$
$x^2$	$c_1 = -\frac{1}{a_2}$	$\frac{a_0}{a_2}$	$\frac{a_1}{a_2}$	$\frac{a_3}{a_2}$	$\frac{a_4}{a_2}$	$\frac{a_5}{a_2}$
$x^3$	$d_1 = -\frac{1}{a_3}$	$\frac{a_0}{a_3}$	$\frac{a_1}{a_3}$	$\frac{a_2}{a_3}$	$\frac{a_4}{a_3}$	$\frac{a_5}{a_3}$
$x^4$	$e_1 = -\frac{1}{a_4}$	$\frac{a_0}{a_4}$	$\frac{a_1}{a_4}$	$\frac{a_2}{a_4}$	$\frac{a_3}{a_4}$	$\frac{a_5}{a_4}$
$x^5$						

$\sigma^0$	$a_6$	$a_4$	$a_2$	$a_0$
$\sigma^1$	$a_5$	$a_3$	$a_1$	0
$\sigma^4$	$b_1 = -\frac{1}{a_5} \begin{vmatrix} a_6 & a_4 \\ a_5 & a_3 \end{vmatrix}$	$b_2 = -\frac{1}{a_5} \begin{vmatrix} a_6 & a_2 \\ a_5 & a_1 \end{vmatrix}$		
$\sigma^5$				
$\sigma^6$				
$\sigma^7$				
$\sigma^8$				

$\alpha^A$	$\alpha_1$			$\alpha_2$			$\alpha_0$
	$\alpha_5$	$\alpha_4$	$\alpha_3$	$\alpha_1$	$\alpha_0$	$\alpha_0$	
$\alpha^1$	$b_1 = -\frac{1}{\alpha_5}$	$a_4$	$a_3$	$b_2 = -\frac{1}{\alpha_5}$	$a_3$	$a_2$	0
$\alpha^3$	$c_1 = -\frac{1}{c_5}$	$a_3$	$a_2$	$c_2 = -\frac{1}{c_5}$	$a_3$	$a_1$	0
$\alpha^2$	$d_1 = -\frac{1}{d_5}$	$b_3$	$b_2$	$d_2 = -\frac{1}{d_5}$	$b_3$	$b_0$	0
$\alpha^4$	$e_1 = -\frac{1}{e_5}$	$c_3$	$c_2$	$e_2 = -\frac{1}{e_5}$	$c_3$	$c_1$	0
$\alpha^0$		$a_1$	$a_0$		$b_1$	$b_0$	

[illegible]

$\alpha^k$	$\alpha_1$			$\alpha_2$			$\alpha_3$
	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_4$
$\alpha^1$	$b_1 = -\frac{1}{\alpha_1}$	$\alpha_2$	$\alpha_3$	$b_2 = -\frac{1}{\alpha_1}$	$\alpha_2$	$\alpha_3$	$b_3 = -\frac{1}{\alpha_1}$
$\alpha^2$	$c_1 = -\frac{1}{\alpha_1}$	$b_1$	$\alpha_3$	$c_2 = -\frac{1}{\alpha_1}$	$b_1$	$\alpha_3$	0
$\alpha^3$	$d_1 = -\frac{1}{\alpha_1}$	$b_1$	$b_2$	$d_2 = -\frac{1}{\alpha_1}$	$b_1$	$b_2$	0
$\alpha^4$	$c_1 = -\frac{1}{\alpha_1}$	$c_2$	$c_3$	$d_2 = -\frac{1}{\alpha_1}$	$c_2$	0	0
$\alpha^5$	$f_1 = -\frac{1}{\alpha_1}$	$d_1$	$d_2$	0	0	0	0
$\alpha^6$	$f_1 = -\frac{1}{\alpha_1}$	$d_1$	$d_2$	0	0	0	0

$\kappa^0$	$a_6$	$a_4$	$a_2$	$a_0$
$\kappa^1$	$a_5$	$a_3$	$a_1$	0
$\kappa^2$	$b_3 = -\frac{1}{a_6} \frac{a_0}{a_5} \frac{a_4}{a_3}$	$b_2 = -\frac{1}{a_6} \frac{a_6}{a_5} \frac{a_2}{a_3}$	$b_1 = -\frac{1}{a_6} \frac{a_0}{a_5} \frac{a_1}{a_3}$	0
$\kappa^3$				
$\kappa^4$				
$\kappa^5$				

$s^6$	$a_6$			$a_4$	$a_2$	$a_0$
$s^5$	$a_5$			$a_3$	$a_1$	0
$s^4$	$b_1 = -\frac{1}{a_5}$	$a_6$	$a_4$			
		$a_5$	$a_3$			
$s^3$						
$s^2$						
$s^1$						
$s^0$						

$s^6$	$a_6$			$a_4$	$a_2$	$a_0$
$s^5$	$a_5$			$a_3$	$a_1$	0

$s^6$	$a_6$			$a_4$			$a_2$	$a_0$
$s^5$	$a_5$			$a_3$			$a_1$	0
$s^4$	$b_1 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_4$ $a_3$	$b_2 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_2$ $a_1$		
$s^3$								
$s^2$								
$s^1$								
$s^0$								

$s^6$	$a_6$			$a_4$			$a_2$	$a_0$
$s^5$	$a_5$			$a_3$			$a_1$	0



$s^4$				
$s^1$				
$s^0$				

$s^6$	$a_6$			$a_4$			$a_2$			$a_0$
$s^5$	$a_5$			$a_3$			$a_1$			0
$s^4$	$b_1 = -\frac{1}{a_5}$	$a_6$	$a_4$	$b_2 = -\frac{1}{a_5}$	$a_6$	$a_2$	$b_3 = -\frac{1}{a_5}$	$a_6$	$a_0$	
	$a_5$	$a_3$		$a_5$	$a_1$		$a_5$	0		
$s^3$										
$s^2$										
$s^1$										
$s^0$										

$s^6$	$a_6$			$a_4$			$a_2$			$a_0$
$s^5$	$a_5$			$a_3$			$a_1$			0
$s^4$	$b_1 = -\frac{1}{a_5}$	$a_6$	$a_4$	$b_2 = -\frac{1}{a_5}$	$a_6$	$a_2$	$b_3 = -\frac{1}{a_5}$	$a_6$	$a_0$	0

$s$				
$s^1$				
$s^0$				

$s^6$	$a_6$			$a_4$			$a_2$			$a_0$
$s^5$	$a_5$			$a_3$			$a_1$			0
$s^4$	$b_1 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_4$ $a_3$	$b_2 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_2$ $a_1$	$b_3 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_0$ 0	0
$s^3$										
$s^2$										
$s^1$										
$s^0$										

$s^6$	$a_6$			$a_4$			$a_2$			$a_0$
$s^5$	$a_5$			$a_3$			$a_1$			0
$s^4$	$b_1 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_4$ $a_3$	$b_2 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_2$ $a_1$	$b_3 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_0$ 0	0
$s^3$	$c_1 = -\frac{1}{b_1}$	$a_5$ $b_1$	$a_3$ $b_2$	$c_2 = -\frac{1}{b_1}$	$a_5$ $b_1$	$a_1$ $b_3$	0			0
$s^2$										
$s^1$										
$s^0$										

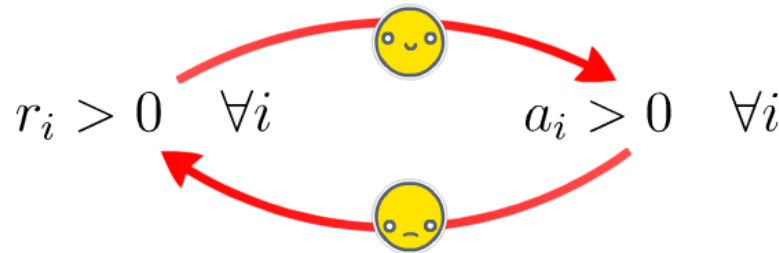
$s^6$	$a_6$			$a_4$			$a_2$			$a_0$			
$s^5$	$a_5$			$a_3$			$a_1$			0			
$s^4$	$b_1 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_4$ $a_3$		$b_2 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_2$ $a_1$		$b_3 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_0$ 0		0
$s^3$	$c_1 = -\frac{1}{b_1}$	$a_5$ $b_1$	$a_3$ $b_2$		$c_2 = -\frac{1}{b_1}$	$a_5$ $b_1$	$a_1$ $b_3$		0				0
$s^2$	$d_1 = -\frac{1}{c_1}$	$b_1$ $c_1$	$b_2$ $c_2$		$d_2 = -\frac{1}{c_1}$	$b_1$ $c_1$	$b_3$ 0		0				0
$s^1$													
$s^0$													

$s^6$	$a_6$			$a_4$			$a_2$			$a_0$
$s^5$	$a_5$			$a_3$			$a_1$			0
$s^4$	$b_1 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_4$ $a_3$	$b_2 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_2$ $a_1$	$b_3 = -\frac{1}{a_5}$	$a_6$ $a_5$	$a_0$ 0	0
$s^3$	$c_1 = -\frac{1}{b_1}$	$a_5$ $b_1$	$a_3$ $b_2$	$c_2 = -\frac{1}{b_1}$	$a_5$ $b_1$	$a_1$ $b_3$	0			0
$s^2$	$d_1 = -\frac{1}{c_1}$	$b_1$ $c_1$	$b_2$ $c_2$	$d_2 = -\frac{1}{c_1}$	$b_1$ $c_1$	$b_3$ 0	0			0
$s^1$	$e_1 = -\frac{1}{d_1}$	$c_1$ $d_1$	$c_2$ $d_2$	0			0			0
$s^0$										

$s^6$	$a_6$			$a_4$			$a_2$			$a_0$
$s^5$	$a_5$			$a_3$			$a_1$			0
$s^4$	$b_1 = -\frac{1}{a_5}$	$\frac{a_6}{a_5}$	$\frac{a_4}{a_3}$	$b_2 = -\frac{1}{a_5}$	$\frac{a_6}{a_5}$	$\frac{a_2}{a_1}$	$b_3 = -\frac{1}{a_5}$	$\frac{a_6}{a_5}$	$\frac{a_0}{0}$	0
$s^3$	$c_1 = -\frac{1}{b_1}$	$\frac{a_5}{b_1}$	$\frac{a_3}{b_2}$	$c_2 = -\frac{1}{b_1}$	$\frac{a_5}{b_1}$	$\frac{a_1}{b_3}$	0			0
$s^2$	$d_1 = -\frac{1}{c_1}$	$\frac{b_1}{c_1}$	$\frac{b_2}{c_2}$	$d_2 = -\frac{1}{c_1}$	$\frac{b_1}{c_1}$	$\frac{b_3}{0}$	0			0
$s^1$	$e_1 = -\frac{1}{d_1}$	$\frac{c_1}{d_1}$	$\frac{c_2}{d_2}$	0			0			0
$s^0$	$f_1 = -\frac{1}{e_1}$	$\frac{d_1}{e_1}$	$\frac{d_2}{0}$	0			0			0

# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

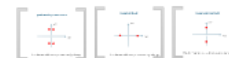
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

## Example 4: Row of Zeros

$s^5$	1	2	1
$s^4$	1	2	1
$s^3$			
$s^2$			
$s^1$			
$s^0$			

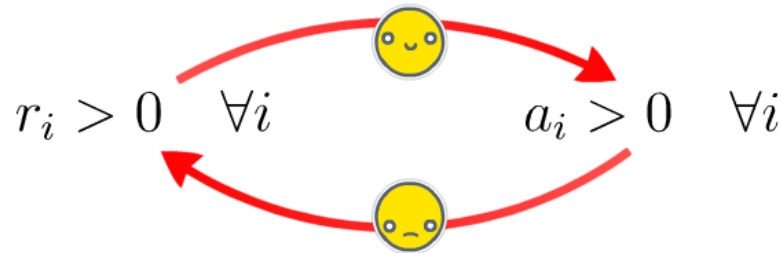
$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2} = 0 \rightarrow 4$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1} = 0 \rightarrow 4$	0
$s^2$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4} = 1$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{0} = 1$	0
$s^1$							
$s^0$							



$s^6$	$a_6$			$a_4$			$a_2$			$a_0$
$s^5$	$a_5$			$a_3$			$a_1$			0
$s^4$	$b_1 = -\frac{1}{a_5}$	$\frac{a_6}{a_5}$	$\frac{a_4}{a_3}$	$b_2 = -\frac{1}{a_5}$	$\frac{a_6}{a_5}$	$\frac{a_2}{a_1}$	$b_3 = -\frac{1}{a_5}$	$\frac{a_6}{a_5}$	$\frac{a_0}{0}$	0
$s^3$	$c_1 = -\frac{1}{b_1}$	$\frac{a_5}{b_1}$	$\frac{a_3}{b_2}$	$c_2 = -\frac{1}{b_1}$	$\frac{a_5}{b_1}$	$\frac{a_1}{b_3}$	0			0
$s^2$	$d_1 = -\frac{1}{c_1}$	$\frac{b_1}{c_1}$	$\frac{b_2}{c_2}$	$d_2 = -\frac{1}{c_1}$	$\frac{b_1}{c_1}$	$\frac{b_3}{0}$	0			0
$s^1$	$e_1 = -\frac{1}{d_1}$	$\frac{c_1}{d_1}$	$\frac{c_2}{d_2}$	0			0			0
$s^0$	$f_1 = -\frac{1}{e_1}$	$\frac{d_1}{e_1}$	$\frac{d_2}{0}$	0			0			0

# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

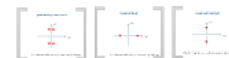
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

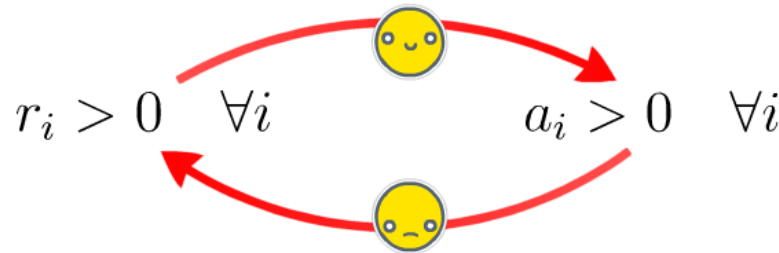
## Example 2: Numeric

$$s^4 + 6s^3 + 13s^2 + 12s + 4$$

$s^4$	1			13			4
$s^3$	6			12			0
$s^2$	$-\frac{1}{6}$	1 6	13 12	$=11$	$-\frac{1}{6}$	1 6	4 0 $=4$
$s^1$	$-\frac{1}{11}$	6 11	12 4	$=\frac{108}{11}$	0		0
$s^0$	$-\frac{11}{108}$	11 $\frac{108}{11}$	4 0	$=4$	0		0

# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

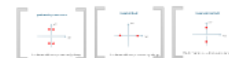
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

e completed the  $s^0$  row, working out

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

er of roots in the RHP.

n of Routh table implies the system

# Example 3: Zero First Column

$s^5$	1			3			5
$s^4$	2			6			3
$s^3$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{6} = \epsilon$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{3} = \frac{7}{2}$	0
$s^2$							
$s^1$							
$s^0$							

$s^5$	1			3			5
$s^4$	2			6			3
$s^3$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{6} = \epsilon$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{3} = \frac{7}{2}$	0
$s^2$	$\lim_{\epsilon \rightarrow 0} -\frac{1}{\epsilon}$	$\frac{2}{\epsilon}$	$\frac{6}{\frac{7}{2}} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}$	$\frac{2}{\epsilon}$	$\frac{3}{0} = 3$	0
$s^1$	$\lim_{\epsilon \rightarrow 0} \frac{\epsilon}{7}$	$\frac{\epsilon}{-\frac{7}{\epsilon}}$	$\frac{\frac{7}{2}}{3} = \frac{7}{2}$	0			0
$s^0$							

$s^5$	1			3			5
$s^4$	2			6			3
$s^3$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{6} = \epsilon$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{3} = \frac{7}{2}$	0
$s^2$	$\lim_{\epsilon \rightarrow 0} -\frac{1}{\epsilon}$	$\frac{2}{\epsilon}$	$\frac{6}{\frac{7}{2}} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}$	$\frac{2}{\epsilon}$	$\frac{3}{0} = 3$	0
$s^1$							
$s^0$							

$s^5$	1			3			5
$s^4$	2			6			3
$s^3$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{6} = \epsilon$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{3} = \frac{7}{2}$	0
$s^2$	$\lim_{\epsilon \rightarrow 0} -\frac{1}{\epsilon}$	$\frac{2}{\epsilon}$	$\frac{6}{\frac{7}{2}} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}$	$\frac{2}{\epsilon}$	$\frac{3}{0} = 3$	0
$s^1$	$\lim_{\epsilon \rightarrow 0} \frac{\epsilon}{7}$	$\frac{\epsilon}{-\frac{7}{\epsilon}}$	$\frac{\frac{7}{2}}{3} = \frac{7}{2}$	0			0
$s^0$	$\lim_{\epsilon \rightarrow 0} -\frac{2}{7}$	$\frac{-\frac{7}{\epsilon}}{\frac{7}{2}}$	$\frac{3}{0} = 3$	0			0

$s^5$	1			3			5
$s^4$	2			6			3
$s^3$	$-\frac{1}{2}$	$\begin{matrix} 1 & 3 \\ 2 & 6 \end{matrix}$	$= \epsilon$	$-\frac{1}{2}$	$\begin{matrix} 1 & 5 \\ 2 & 3 \end{matrix}$	$= \frac{7}{2}$	0
$s^2$							
$s^1$							
$s^0$							

$s^5$	1			3			5
$s^4$	2			6			3
$s^3$	$-\frac{1}{2}$	$\begin{matrix} 1 & 3 \\ 2 & 6 \end{matrix}$	$= \epsilon$	$-\frac{1}{2}$	$\begin{matrix} 1 & 5 \\ 2 & 3 \end{matrix}$	$= \frac{7}{2}$	0
$s^2$	$\lim_{\epsilon \rightarrow 0} -\frac{1}{\epsilon}$	$\begin{matrix} 2 & 6 \\ \epsilon & \frac{7}{2} \end{matrix}$	$= -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}$	$\begin{matrix} 2 & 3 \\ \epsilon & 0 \end{matrix}$	$= 3$	0
$s^1$							
$s^0$							

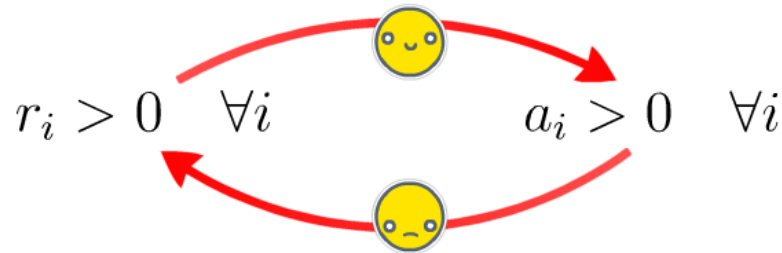


$s^5$	1			3			5
$s^4$	2			6			3
$s^3$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{6} = \epsilon$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{3} = \frac{7}{2}$	0
$s^2$	$\lim_{\epsilon \rightarrow 0} -\frac{1}{\epsilon}$	$\frac{2}{\epsilon}$	$\frac{6}{\frac{7}{2}} = -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}$	$\frac{2}{\epsilon}$	$\frac{3}{0} = 3$	0
$s^1$	$\lim_{\epsilon \rightarrow 0} \frac{\epsilon}{7}$	$\frac{\epsilon}{-\frac{7}{\epsilon}}$	$\frac{\frac{7}{2}}{3} = \frac{7}{2}$	0			0
$s^0$							

$s^5$	1			3			5
$s^4$	2			6			3
$s^3$	$-\frac{1}{2}$	$\begin{array}{cc} 1 & 3 \\ 2 & 6 \end{array}$	$= \epsilon$	$-\frac{1}{2}$	$\begin{array}{cc} 1 & 5 \\ 2 & 3 \end{array}$	$= \frac{7}{2}$	0
$s^2$	$\lim_{\epsilon \rightarrow 0} -\frac{1}{\epsilon}$	$\begin{array}{cc} 2 & 6 \\ \epsilon & \frac{7}{2} \end{array}$	$= -\frac{7}{\epsilon}$	$-\frac{1}{\epsilon}$	$\begin{array}{cc} 2 & 3 \\ \epsilon & 0 \end{array}$	$= 3$	0
$s^1$	$\lim_{\epsilon \rightarrow 0} \frac{\epsilon}{7}$	$\begin{array}{cc} \epsilon & \frac{7}{2} \\ -\frac{7}{\epsilon} & 3 \end{array}$	$= \frac{7}{2}$	0			0
$s^0$	$\lim_{\epsilon \rightarrow 0} -\frac{2}{7}$	$\begin{array}{cc} -\frac{7}{\epsilon} & 3 \\ \frac{7}{2} & 0 \end{array}$	$= 3$	0			0

# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

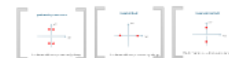
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

out negative of scaled determinant of m

Case 2: There is a zero across an entire row.

- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

m is **stable**.

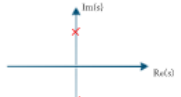
Quadrantal symmetric roots



Purely Real Roots

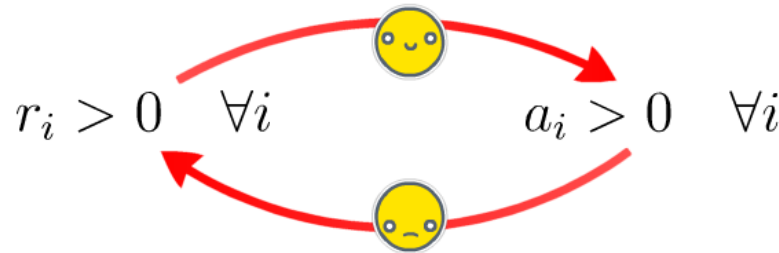


Purely Imaginary Roots



# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

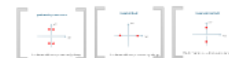
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

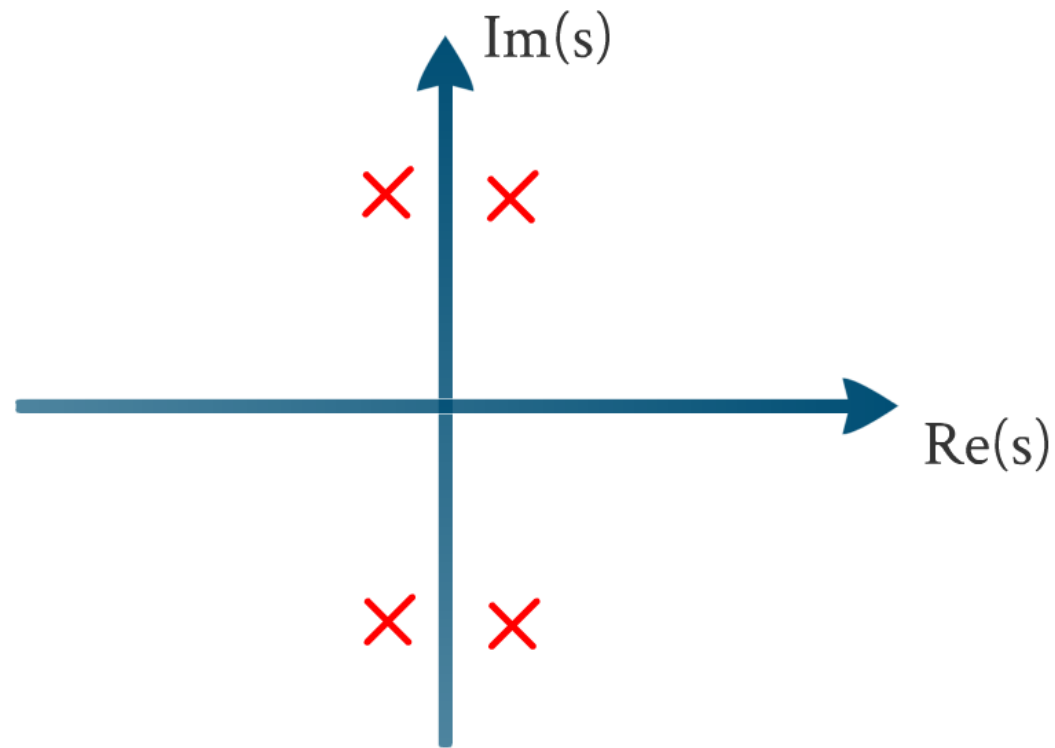
- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

## Example 4: Row of Zeros

$s^5$	1	2	1
$s^4$	1	2	1
$s^3$			
$s^2$			
$s^1$			
$s^0$			

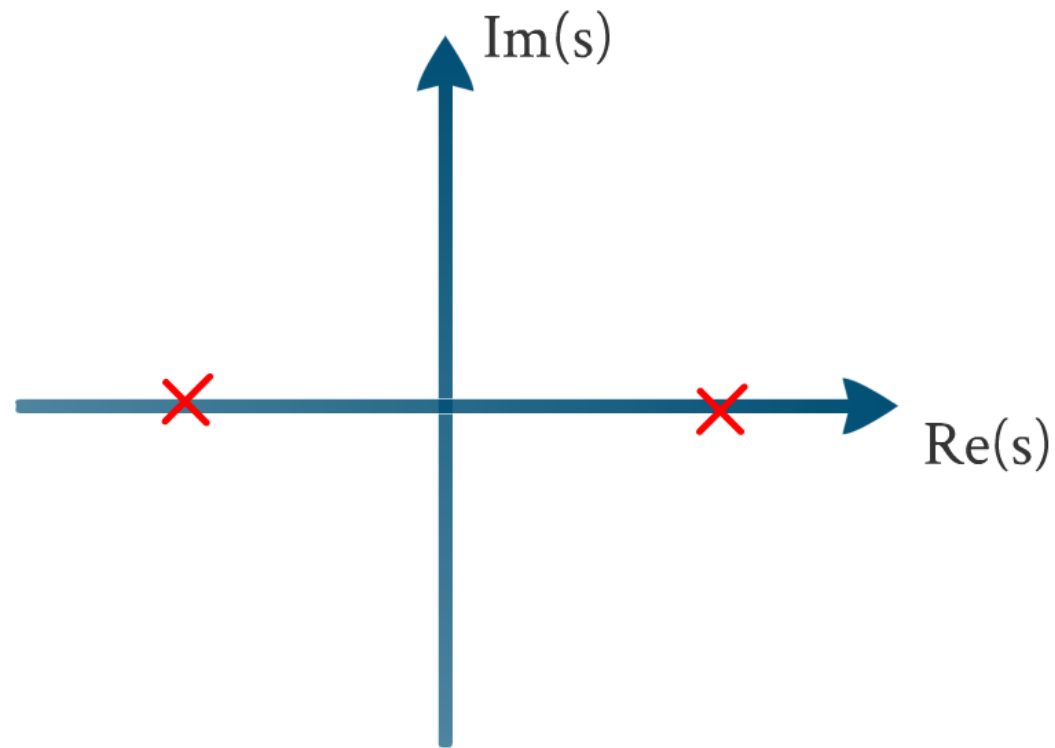
$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$= 0 \rightarrow 4$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1} = 0 \rightarrow 4$
$s^2$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$= 1$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{0} = 1$
$s^1$							
$s^0$							

## Quadrantal symmetric roots



Includes unstable roots, so causes a sign change

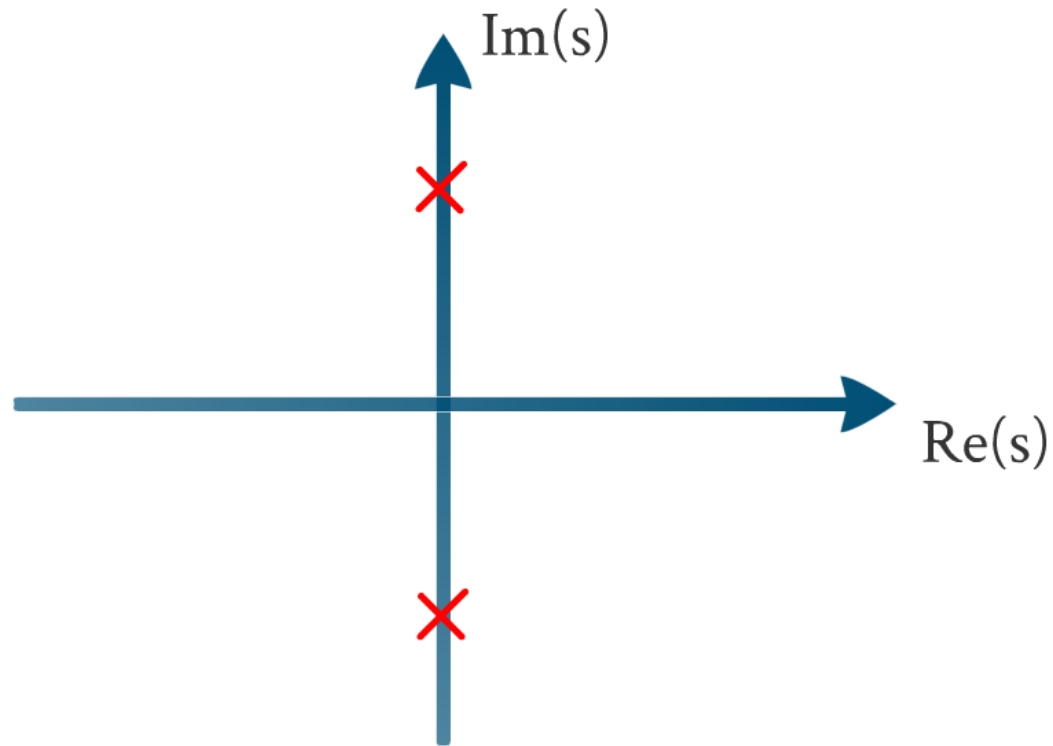
## Purely Real Roots



Includes unstable root, so causes a sign change



## Purely Imaginary Roots



Purely imaginary roots do not cause a sign change

# Example 4: Row of Zeros

$s^5$	1	2	1
$s^4$	1	2	1
$s^3$			
$s^2$			
$s^1$			
$s^0$			

$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$= 0 \rightarrow 4$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$
$s^2$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$= 1$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{0}$
$s^1$							
$s^0$							

$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$= 0$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$
$s^2$							
$s^1$							
$s^0$							

$s^5$	1				2				1
$s^4$	1				2				1
$s^3$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$= 0 \rightarrow 4$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$	$= 0 \rightarrow 4$	0
$s^2$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$= 1$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{0}$	$= 1$	0
$s^1$	$-\frac{1}{1}$	$\frac{4}{1}$	$\frac{4}{1}$	$= \epsilon$	0				0
$s^0$									

$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$= 0 \rightarrow 4$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$
$s^2$							
$s^1$							
$s^0$							

$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{2}{2}$	$= 0 \rightarrow 4$	$-\frac{1}{1}$	$\frac{1}{1}$	$\frac{1}{1}$
$s^2$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$= 1$	$-\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{0}$
$s^1$	$-\frac{1}{1}$	$\frac{4}{1}$	$\frac{4}{1}$	$= \epsilon$	0		0
$s^0$	$-\frac{1}{\epsilon}$	$\frac{1}{\epsilon}$	$\frac{1}{0}$	$= 1$	0		0

$$\frac{d}{ds} (s^4 + 2s^2 + 1) = 4s^3 + 4s$$

$s^5$	1	2	1
$s^4$	1	2	1
$s^3$			
$s^2$			
$s^1$			
$s^0$			

$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\begin{matrix} 1 & 2 \\ 1 & 2 \end{matrix}$	$= 0$	$-\frac{1}{1}$	$\begin{matrix} 1 & 1 \\ 1 & 1 \end{matrix}$	$= 0$	0
$s^2$							
$s^1$							
$s^0$							

$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\begin{array}{cc} 1 & 2 \\ 1 & 2 \end{array}$	$= 0 \rightarrow 4$	$-\frac{1}{1}$	$\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array}$	$= 0 \rightarrow 4$	0
$s^2$							
$s^1$							
$s^0$							

$$\frac{d}{ds} (s^4 + 2s^2 + 1) = 4s^3 + 4s$$

# How to Zeros

$s^5$	1			2			1
$s^4$	1			2			1
$s^3$	$-\frac{1}{1}$	$\begin{array}{cc} 1 & 2 \\ 1 & 2 \end{array}$	$= 0 \rightarrow 4$	$-\frac{1}{1}$	$\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array}$	$= 0 \rightarrow 4$	0
$s^2$	$-\frac{1}{4}$	$\begin{array}{cc} 1 & 2 \\ 4 & 4 \end{array}$	$= 1$	$-\frac{1}{4}$	$\begin{array}{cc} 1 & 1 \\ 4 & 0 \end{array}$	$= 1$	0
$s^1$							
$s^0$							

$s^5$	1			2			1
-------	---	--	--	---	--	--	---

$s$			
-----	--	--	--

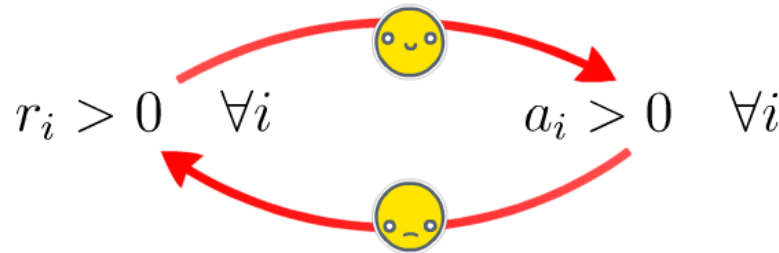
$s^5$	1				2				1
$s^4$	1				2				1
$s^3$	$-\frac{1}{1}$	$\begin{array}{cc} 1 & 2 \\ 1 & 2 \end{array}$	$= 0 \rightarrow 4$		$-\frac{1}{1}$	$\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array}$	$= 0 \rightarrow 4$		0
$s^2$	$-\frac{1}{4}$	$\begin{array}{cc} 1 & 2 \\ 4 & 4 \end{array}$	$= 1$		$-\frac{1}{4}$	$\begin{array}{cc} 1 & 1 \\ 4 & 0 \end{array}$	$= 1$		0
$s^1$	$-\frac{1}{1}$	$\begin{array}{cc} 4 & 4 \\ 1 & 1 \end{array}$	$= \epsilon$		0				0
$s^0$									

$s^5$	1				2				1
$s^4$	1				2				1
$s^3$	$-\frac{1}{1}$	$\begin{array}{c} 1 \\ 1 \end{array}$	$\begin{array}{c} 2 \\ 2 \end{array}$	$= 0 \rightarrow 4$	$-\frac{1}{1}$	$\begin{array}{c} 1 \\ 1 \end{array}$	$\begin{array}{c} 1 \\ 1 \end{array}$	$= 0 \rightarrow 4$	0
$s^2$	$-\frac{1}{4}$	$\begin{array}{c} 1 \\ 4 \end{array}$	$\begin{array}{c} 2 \\ 4 \end{array}$	$= 1$	$-\frac{1}{4}$	$\begin{array}{c} 1 \\ 4 \end{array}$	$\begin{array}{c} 1 \\ 0 \end{array}$	$= 1$	0
$s^1$	$-\frac{1}{1}$	$\begin{array}{c} 4 \\ 1 \end{array}$	$\begin{array}{c} 4 \\ 1 \end{array}$	$= \epsilon$	0				0
$s^0$	$-\frac{1}{\epsilon}$	$\begin{array}{c} 1 \\ \epsilon \end{array}$	$\begin{array}{c} 1 \\ 0 \end{array}$	$= 1$	0				0



# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

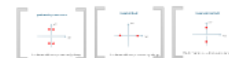
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

## Example 5: Parameterised

$s^2$	1	$K$
$s^1$	$0 \rightarrow 2$	$0 \rightarrow 0$
$s^0$		

$s^2$	1			$K$
$s^1$	$0 \rightarrow 2$			$0 \rightarrow 0$
$s^0$	$-\frac{1}{2}$	$\begin{matrix} 1 & K \\ 2 & 0 \end{matrix}$	$= K$	0

$s^2$	1	$K$
$s^1$	$0 \rightarrow 2$	$0 \rightarrow 0$
$s^0$		

$s^2$	1			$K$
$s^1$	$0 \rightarrow 2$			$0 \rightarrow 0$
$s^0$	$-\frac{1}{2}$	$\begin{matrix} 1 & K \\ 2 & 0 \end{matrix}$	$= K$	0

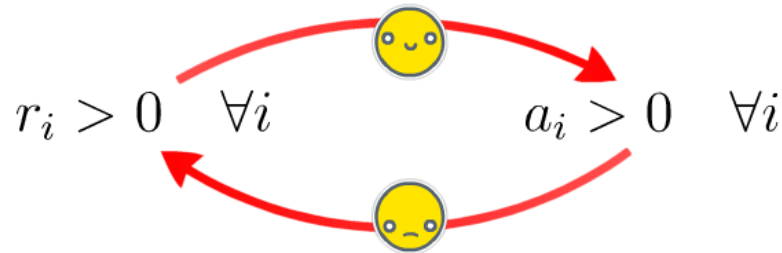
## Example 5: Parameterised

$s^2$	1	$K$
$s^1$	$0 \rightarrow 2$	$0 \rightarrow 0$
$s^0$		

$s^2$	1			$K$
$s^1$	$0 \rightarrow 2$			$0 \rightarrow 0$
$s^0$	$-\frac{1}{2}$	$\begin{matrix} 1 & K \\ 2 & 0 \end{matrix}$	$= K$	0

# Routh-Hurwitz Stability Criteria

$$(s + r_1)(s + r_2) \dots (s + r_n) = s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0$$



$$\underbrace{(s - 1 + 3j)(s - 1 - 3j)}_{\text{Roots in RHP}}(s + 3) = s^3 + s^2 + 4s + 30$$

Find the (equivalent) closed-loop transfer function

Produce the initial lay-out of the Routh table

Write the coefficients of the characteristic function across the first two rows.

Complete the remaining part of the Routh table until all terms generated are zero

Progress across and down until you have completed the  $s^0$  row, working out negative of scaled determinant of matrix of previously calculated values.

Two problematic cases can occur:

Case 1: There is a zero in the first column, but the rest of the row includes nonzero numbers.

- We can replace the zero with a small number,  $\epsilon$ , continue and reinterpret the results.

Case 2: There is a zero across an entire row.

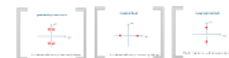
- Consider the **Auxiliary Equation**,  $A(s)$ , formed from the coefficients of the row just above the row of zeros.
- Replace the row of zeros with a row of coefficients from  $dA(s)/ds$  and continue Routh's tabulation.

Interpret the results

The number of sign changes is the number of roots in the RHP.

- So, no sign change in the first column of Routh table implies the system is **stable**.
- A sign change implies that the system is **unstable**.

If there were a row of zeros, then there are symmetric roots about the origin



If there are no sign changes, but an entire row of zeros was encountered in the tabulation process, then the possibility for repeated purely imaginary roots needs to be considered.

- The roots of the auxiliary equation are also roots of the original equation. So, you can solve each auxiliary equation for the corresponding roots.
- Repeated roots on the imaginary axis make the system **unstable**.
- Multiple distinct roots on the imaginary axis make the system **marginally stable**.

This lecture covers:

- Stability and Instability
- Relationship between poles' location and stability
- Routh-Hurwitz stability criteria

