ELEC 207 Part B

Control Theory Lecture 5: Control System Performance (2)

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0151 794 4573

UNIVERSITY OF LIVERPOOL

First and Second Order Systems

First-Order System

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$t\,u(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$\sin(\omega t)\,u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)\,u(t)$	$\frac{s}{s^2+\omega^2}$

Changing parameter will just change the speed of (exponential) response and stability

Second-Order System

$$\frac{c}{s^2+bs+c}$$

Changing parameters can change the type of response

Poles are at: $s = \frac{-b \pm \sqrt{b^2-4c}}{2}$

Generalised Second-Order System

$$G(s) = \frac{\underbrace{\omega^2}_{\text{natural frequency}}}{s^2 + 2\underbrace{\zeta}_{\text{damping ratio}}\omega s + \omega^2}$$

Changing parameters can change the type of response

Poles are at: $s = -\zeta\omega \pm \sqrt{\zeta^2 - 1}\,\omega$

This lecture covers:

- First-order system and second-order system
- Generalized second-order system

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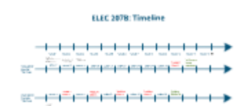
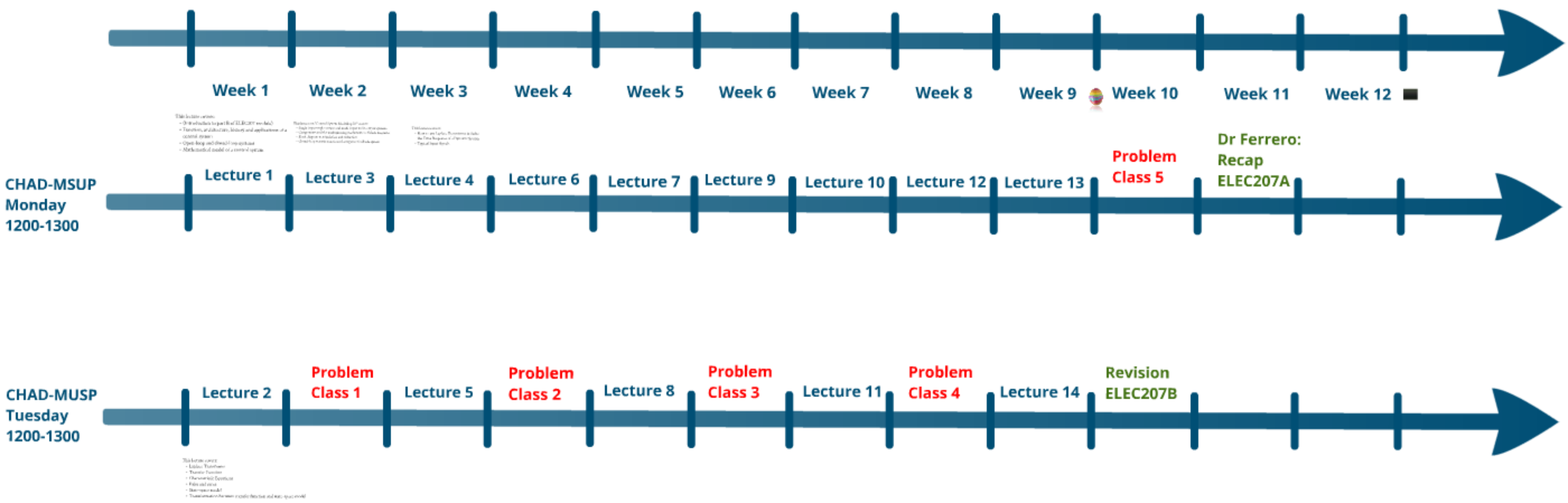


ELEC 207B: Timeline



Week 1

This lecture covers:

- (Introduction to part B of ELEC207 module)
- Function, architecture, history and applications of a control system
- Open-loop and closed-loop systems
- Mathematical model of a control system

Lecture 1

This lecture on "Control S

- Single-input single-ou
- Components and the u
- Block diagram manipu
- Closed-loop transfer fu

Lec

Lecture 2

Problem Class 1

This lecture covers:

- Laplace Transforms
- Transfer Function
- Characteristic Equations
- Poles and zeros
- State-space model
- Transformation between transfer function and state-space model

module)
pplications of a

tem

Week 2

This lecture on "Control System Modelling (2)" covers:

- Single-input single-output and multi-input multi-output systems
- Components and the underpinning mathematics of block diagrams
- Block diagram manipulation and reduction
- Closed-loop transfer function of a negative feedback system

This lecture covers:

- How to use Lap
  the Time Respo
- Typical Input Si

e 1

Lecture 3

Lectu

vers:
lti-output systems
cs of block diagrams

dback system

3

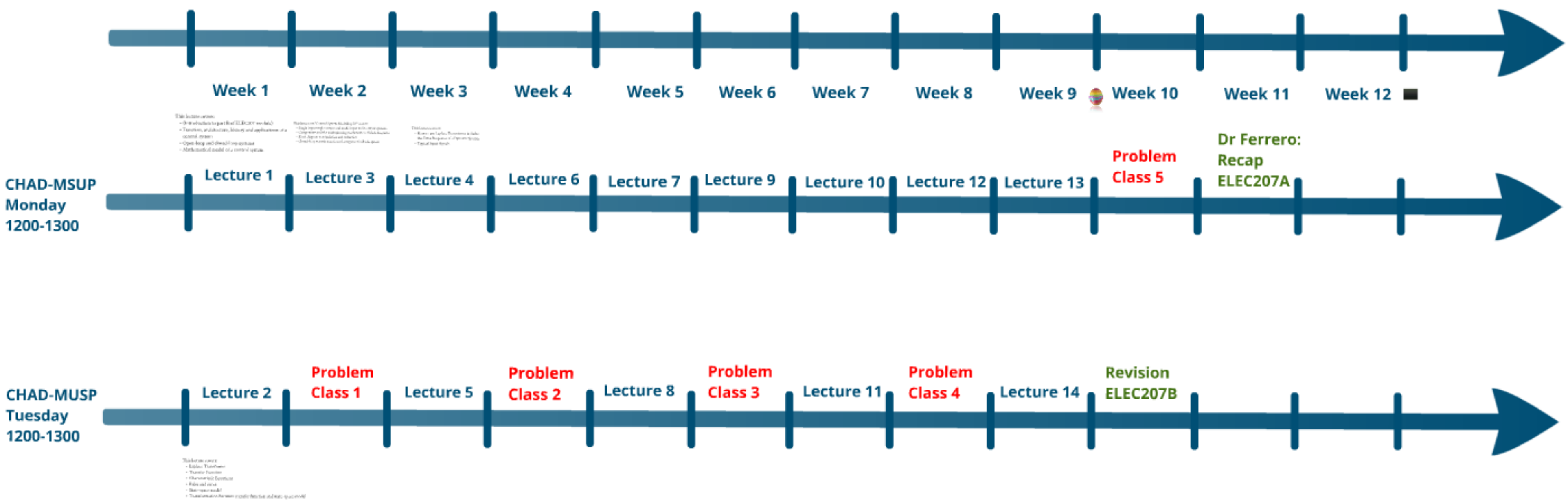
This lecture covers:

- How to use Laplace Transforms to Solve the Time Response of a Dynamic System
- Typical Input Signals

Lecture 4

Lect

ELEC 207B: Timeline



This lecture covers:

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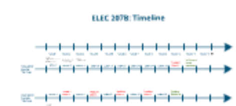
- Generalized second-order system



First and Second Order Systems

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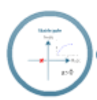
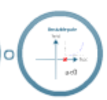
Changing parameter will just change the speed of (exponential) response and stability



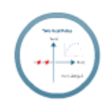


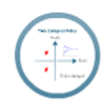
Second-Order System

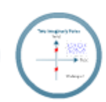
$$\frac{c}{s^2 + bs + c}$$

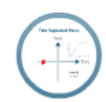
Changing parameters can change the type of response

Poles are at: $s = \dfrac{-b \pm \sqrt{b^2 - 4c}}{2}$









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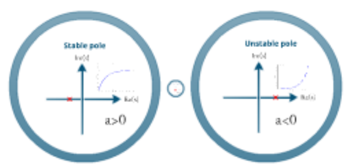
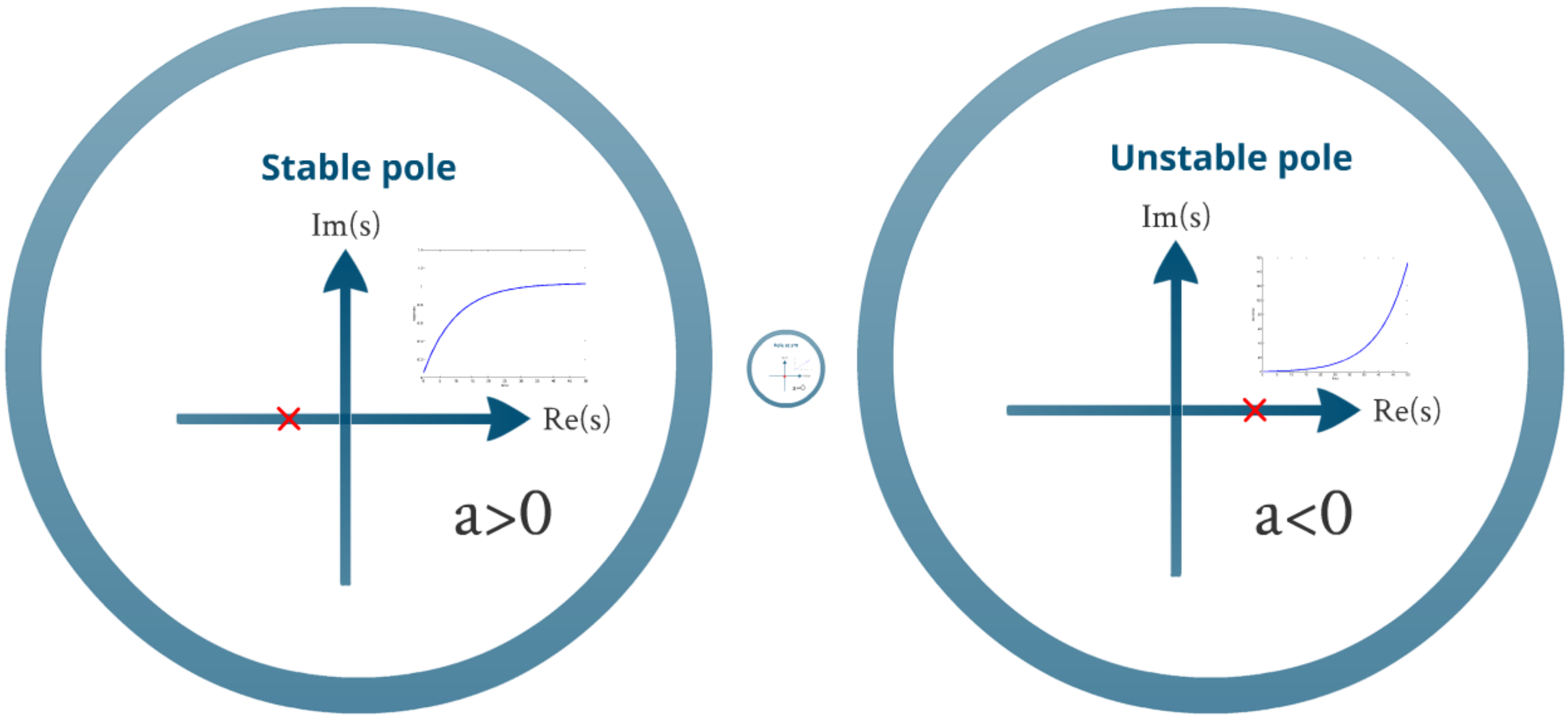
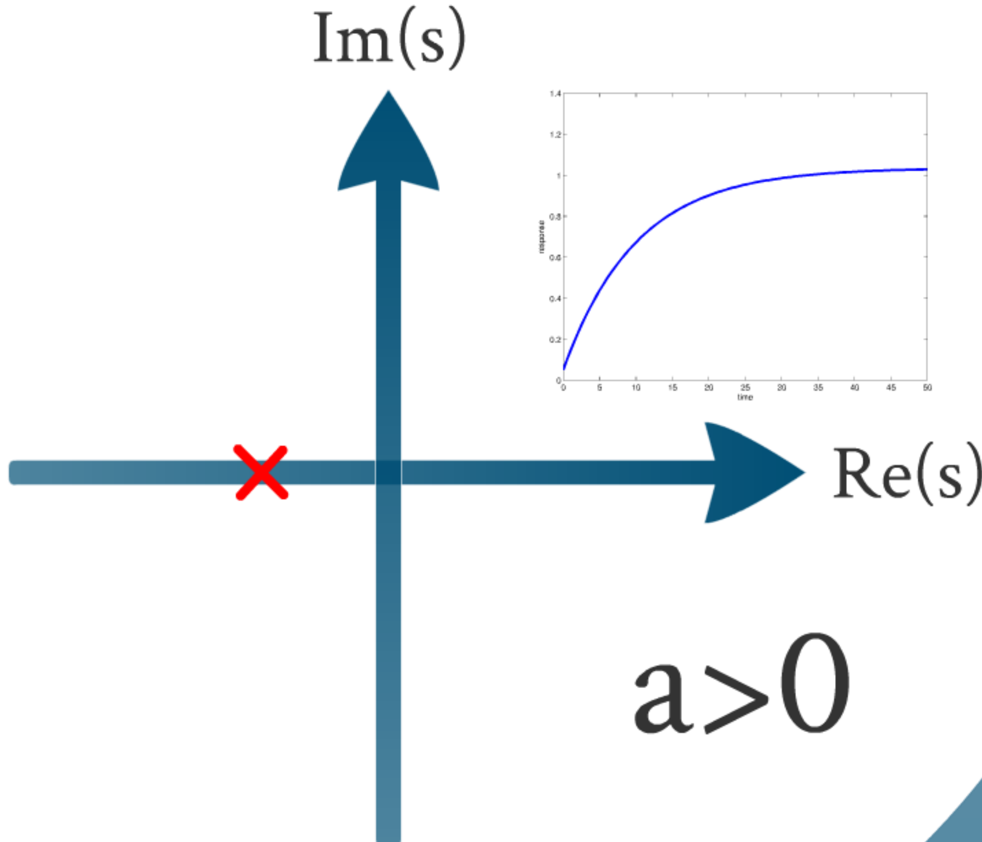


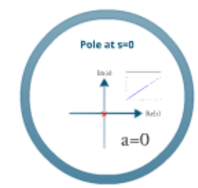


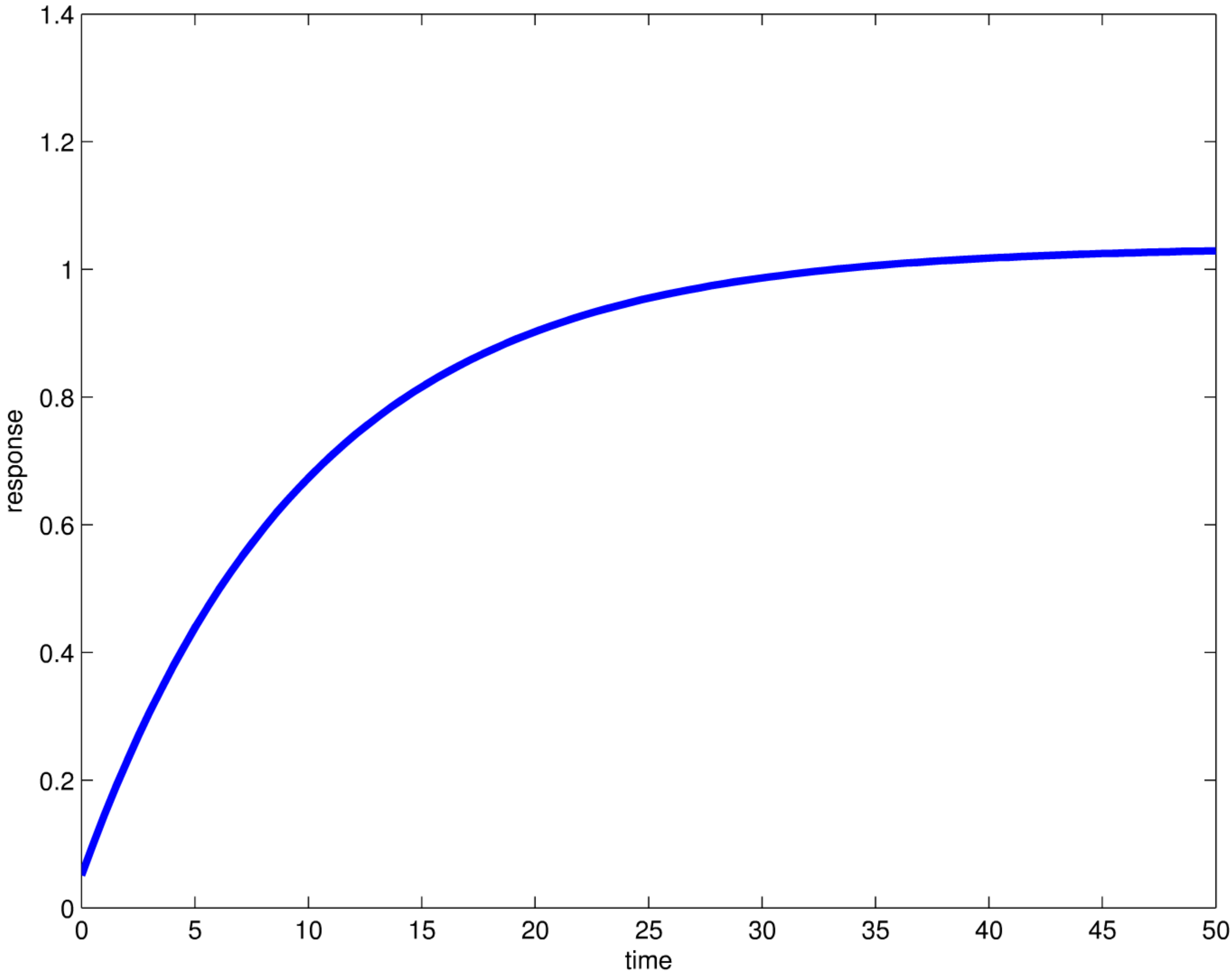
Stable pole
Im(s)
Re(s)
a>0
Unstable pole
Im(s)
Re(s)
a<0

Stable pole



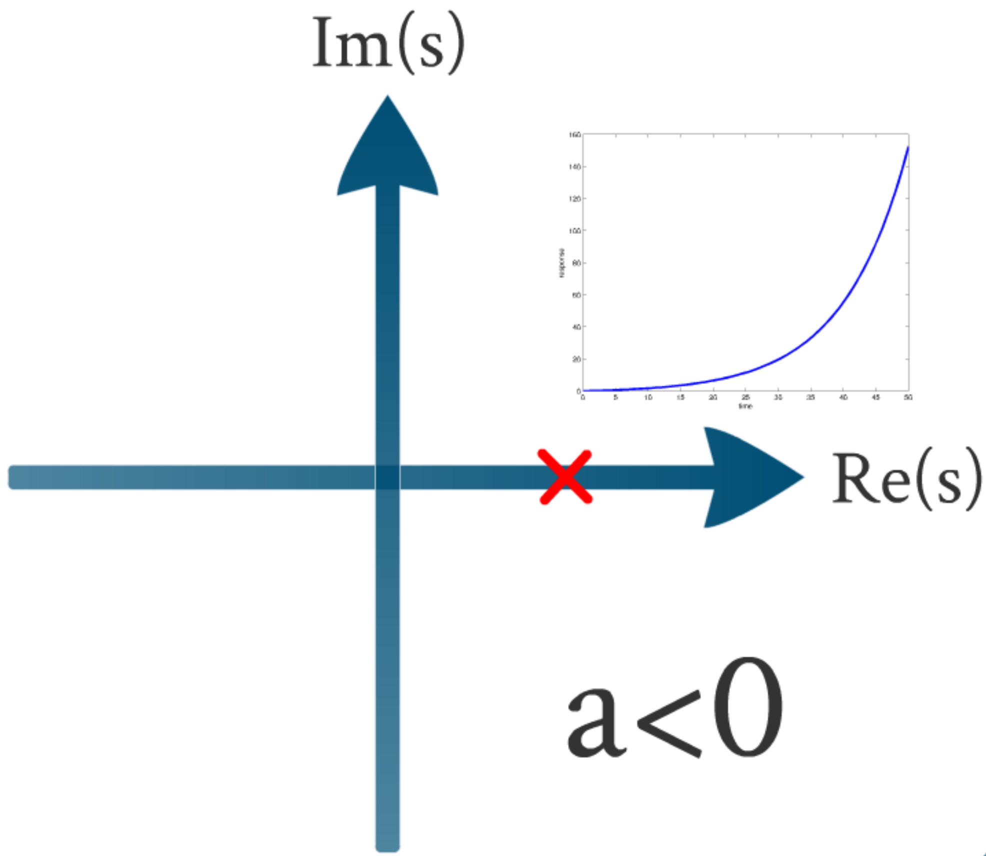
a>0

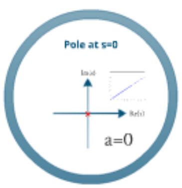


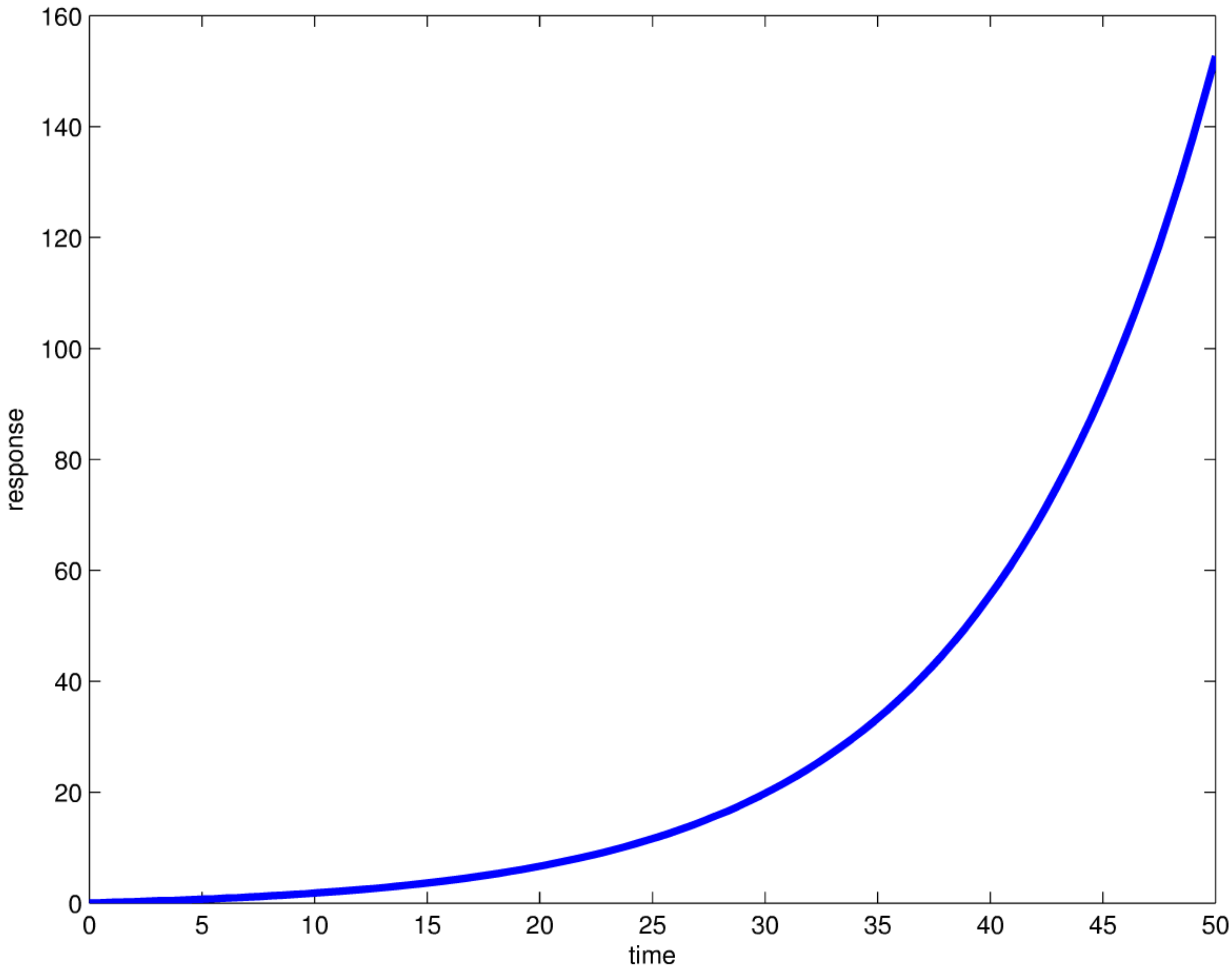


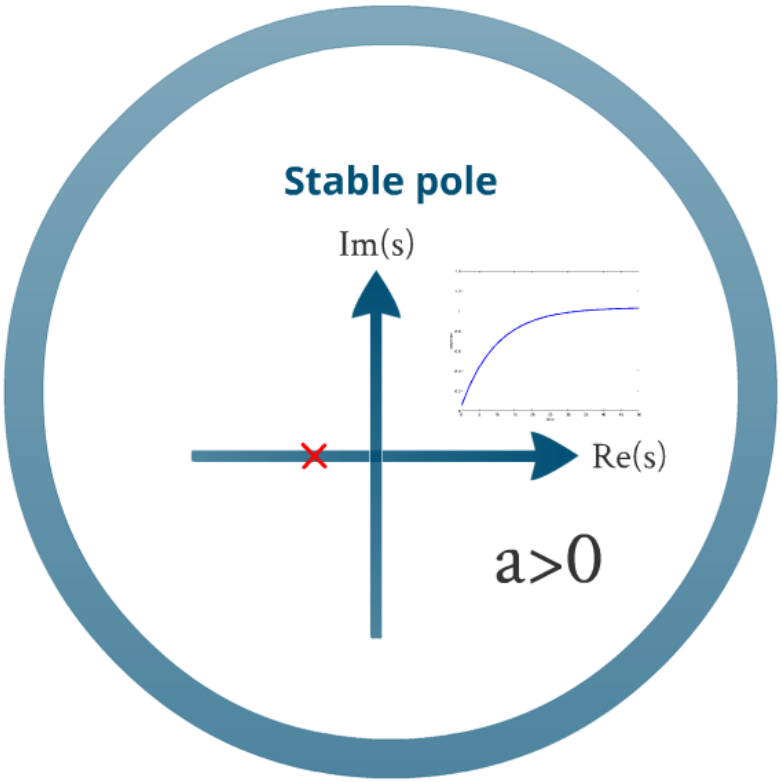
response
time
0
0.2
0.4
0.6
0.8
1
1.2
1.4
0
5
10
15
20
25
30
35
40
45
50

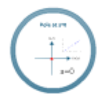
Unstable pole

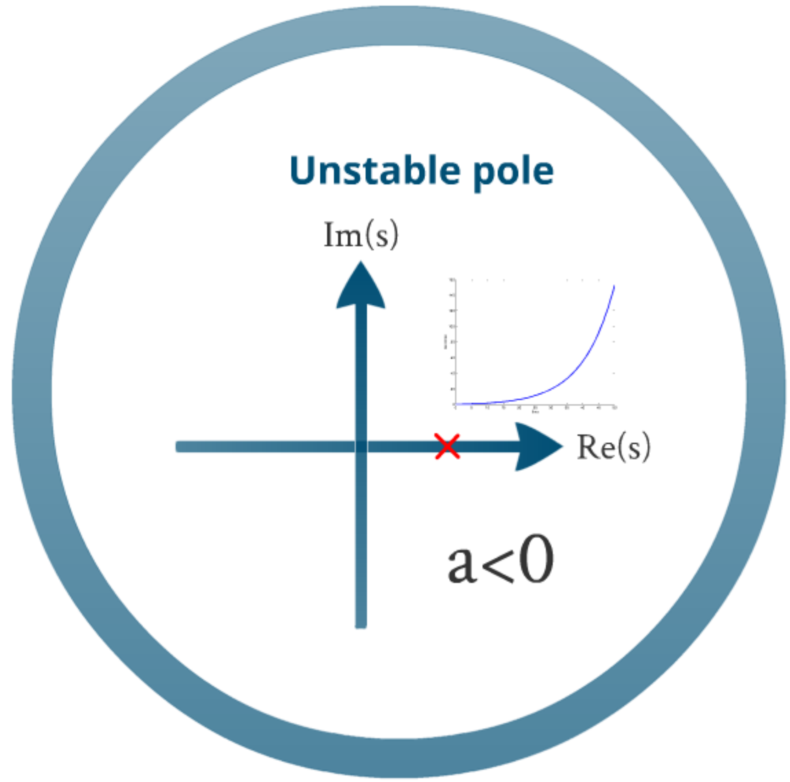




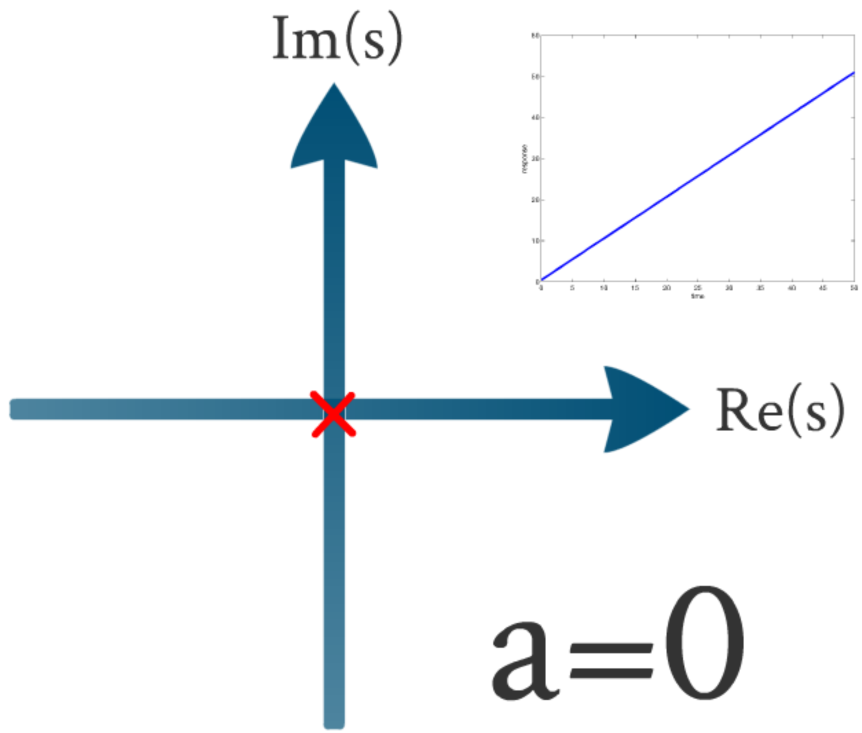

160
140
120
100
80
60
40
20
0
response
0
5
10
15
20
25
30
35
40
45
50
time

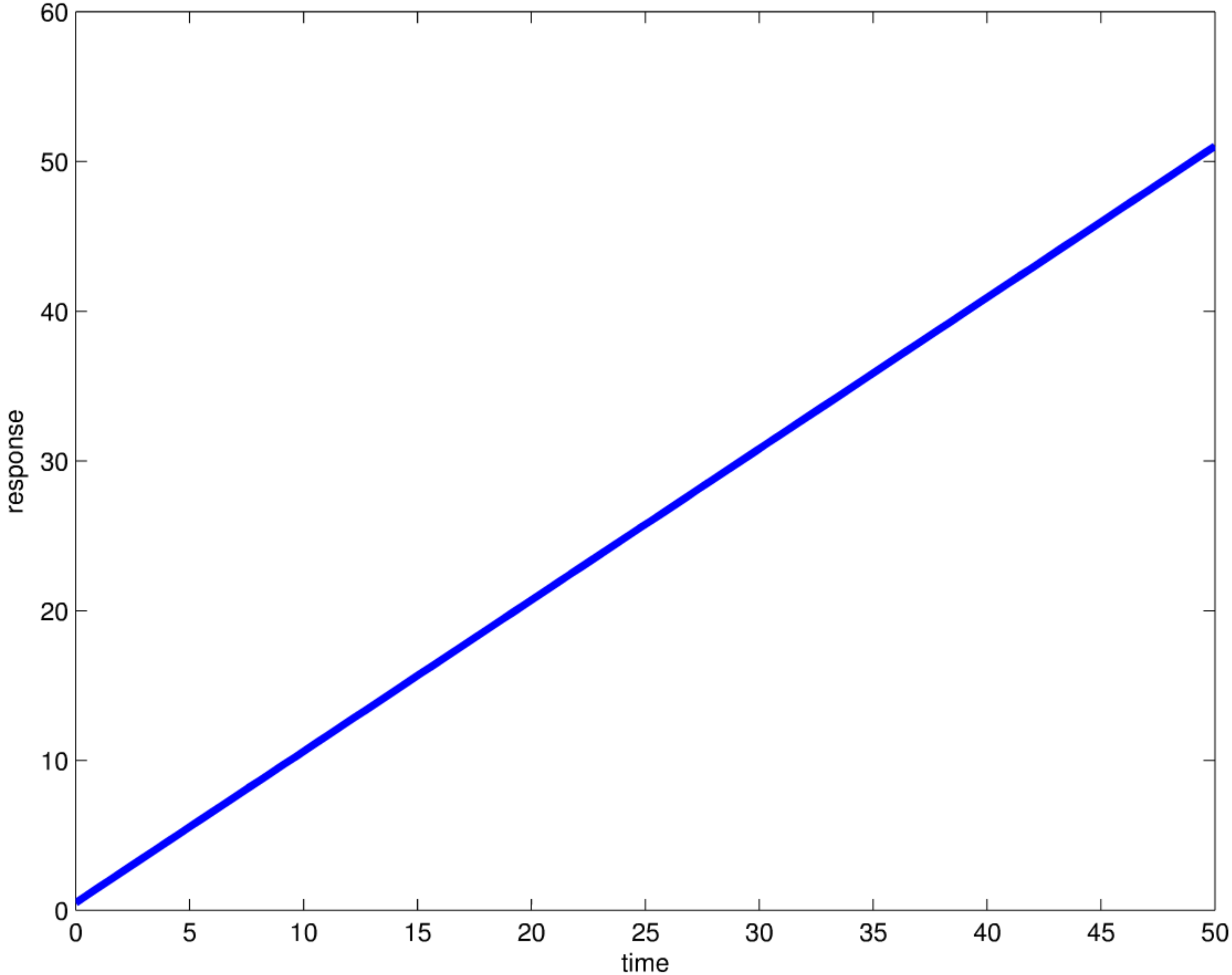

Stable pole
Im(s)
Re(s)
a>0



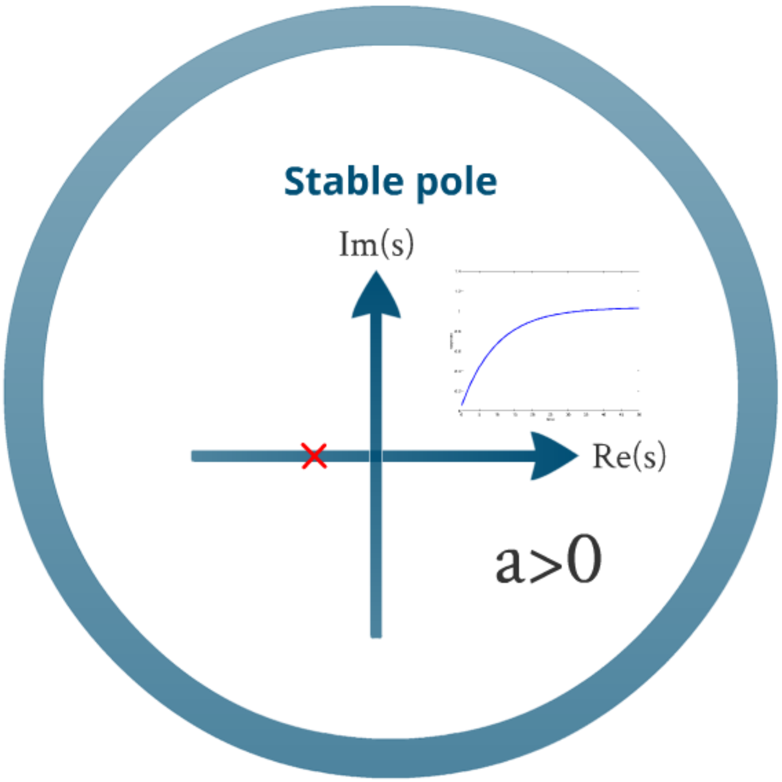

Unstable pole
Im(s)
Re(s)
a<0

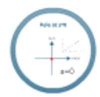
Pole at s=0

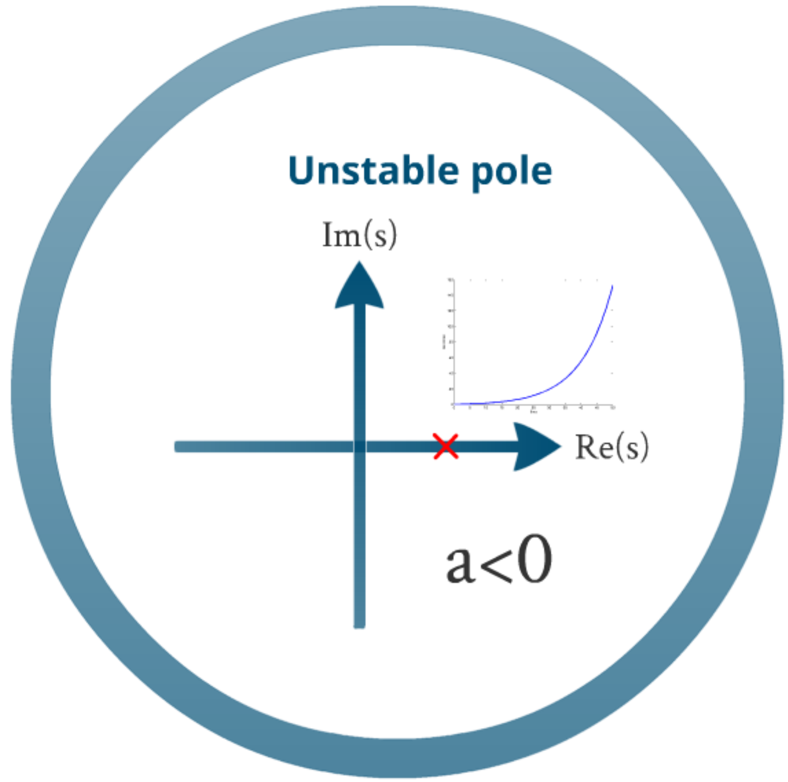




60
50
40
30
20
10
0
response
0
5
10
15
20
25
30
35
40
45
50
time


Stable pole
Im(s)
Re(s)
a>0

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Re(s)
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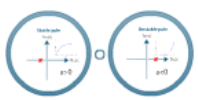

Second-Order System

$$\frac{c}{s^2 + bs + c}$$

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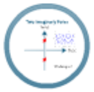
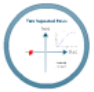
Poles are at: $s = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$





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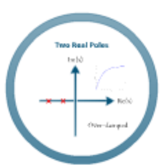
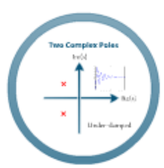
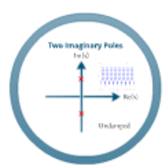
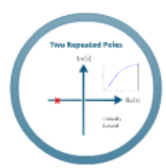
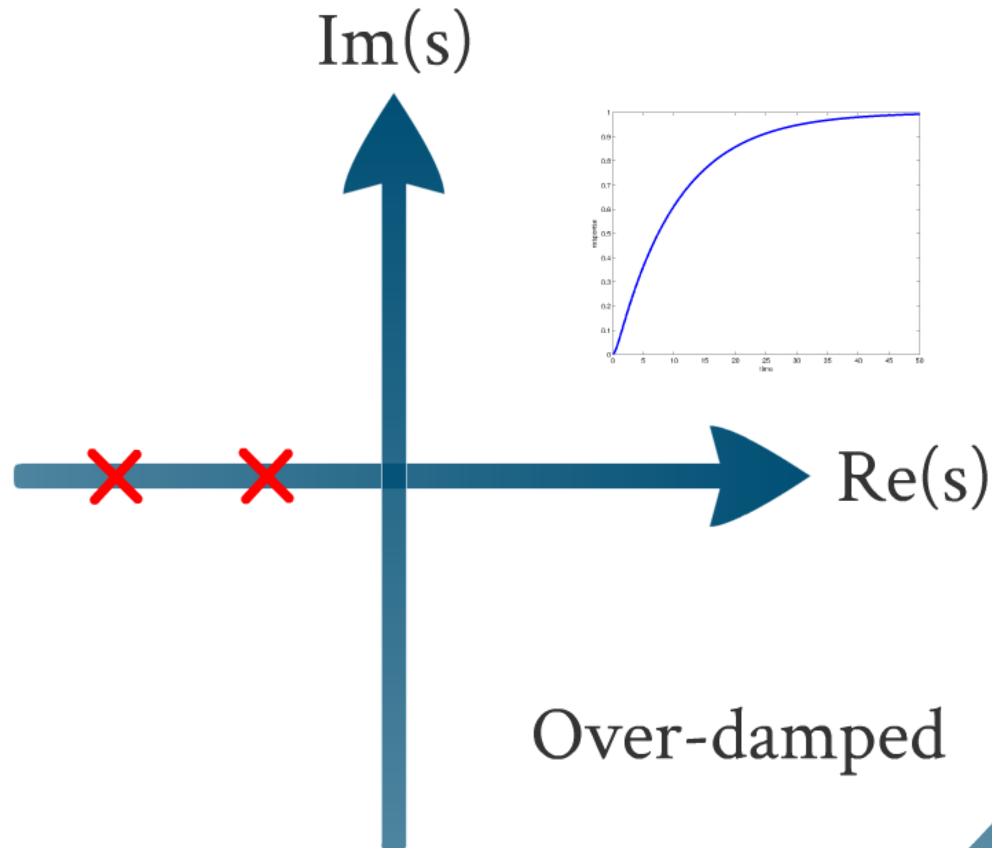


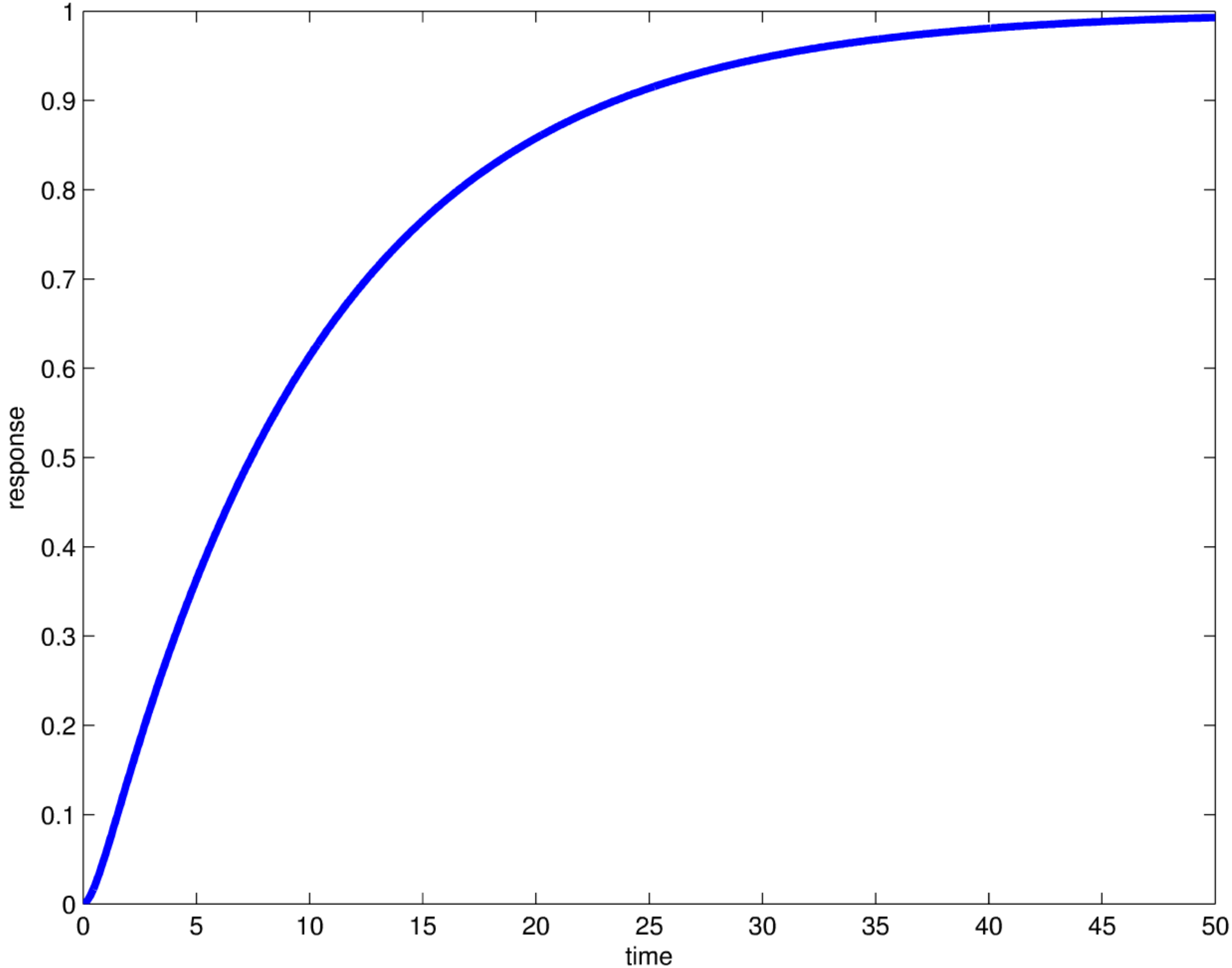






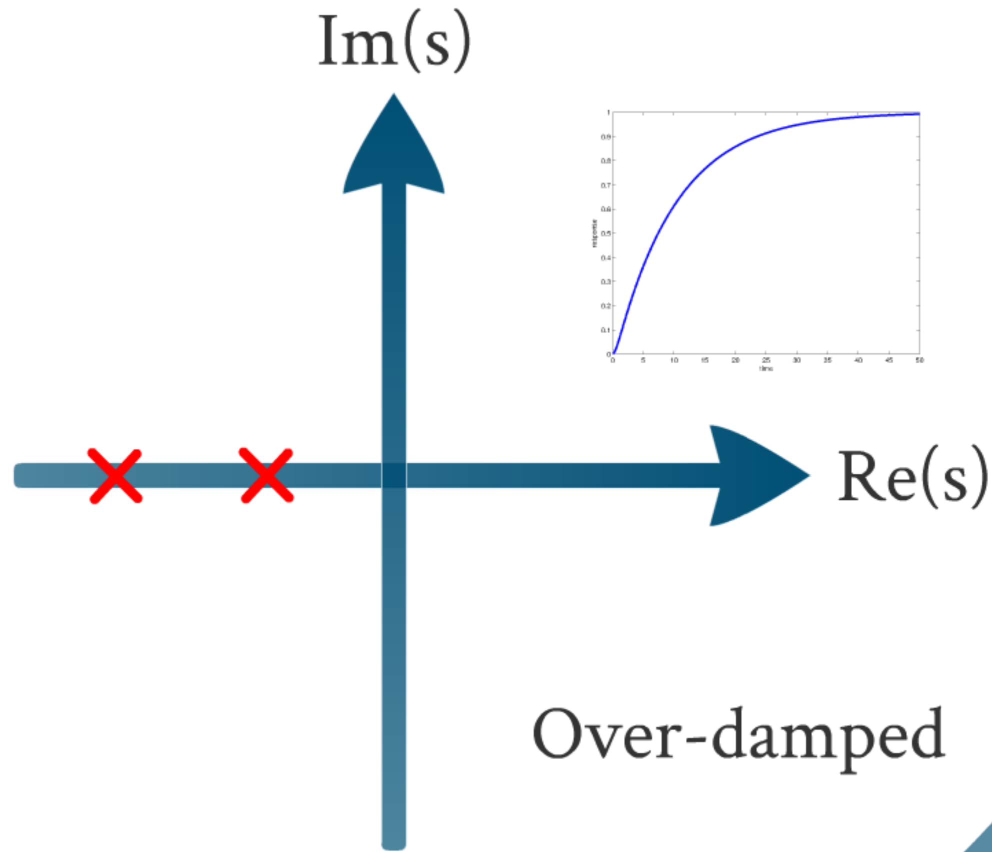
Two Real Poles

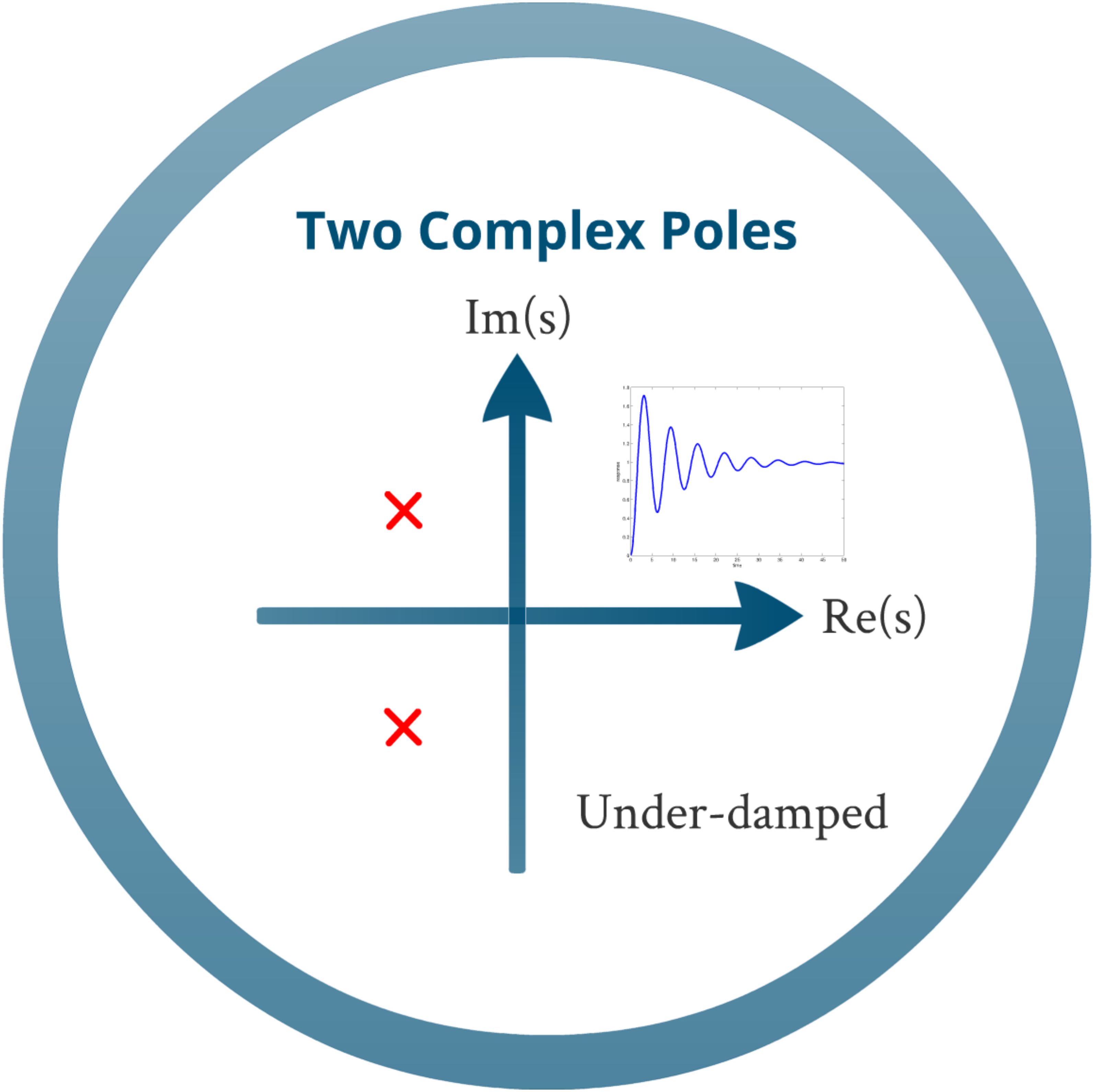


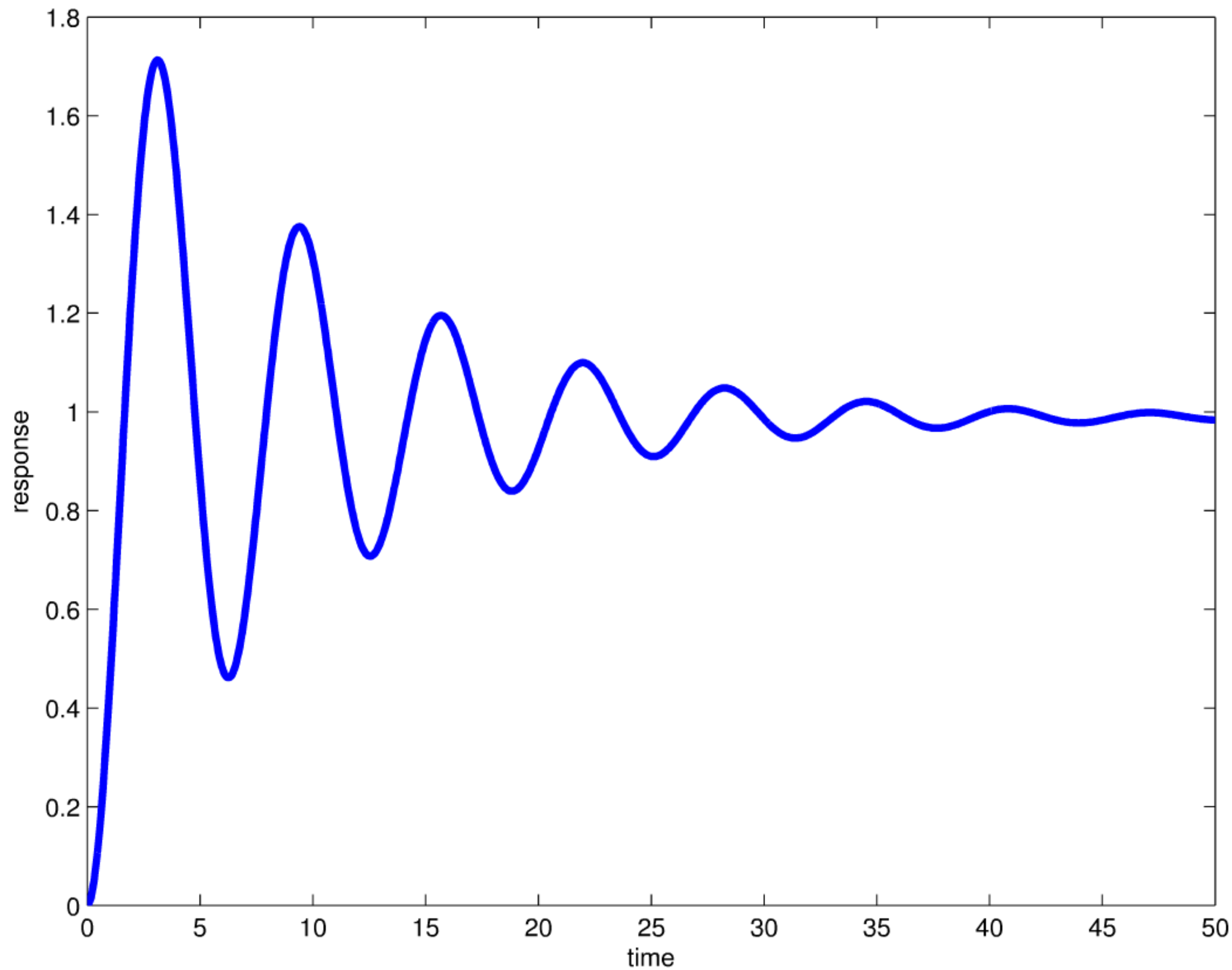


response
time
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40
45
50

Two Real Poles

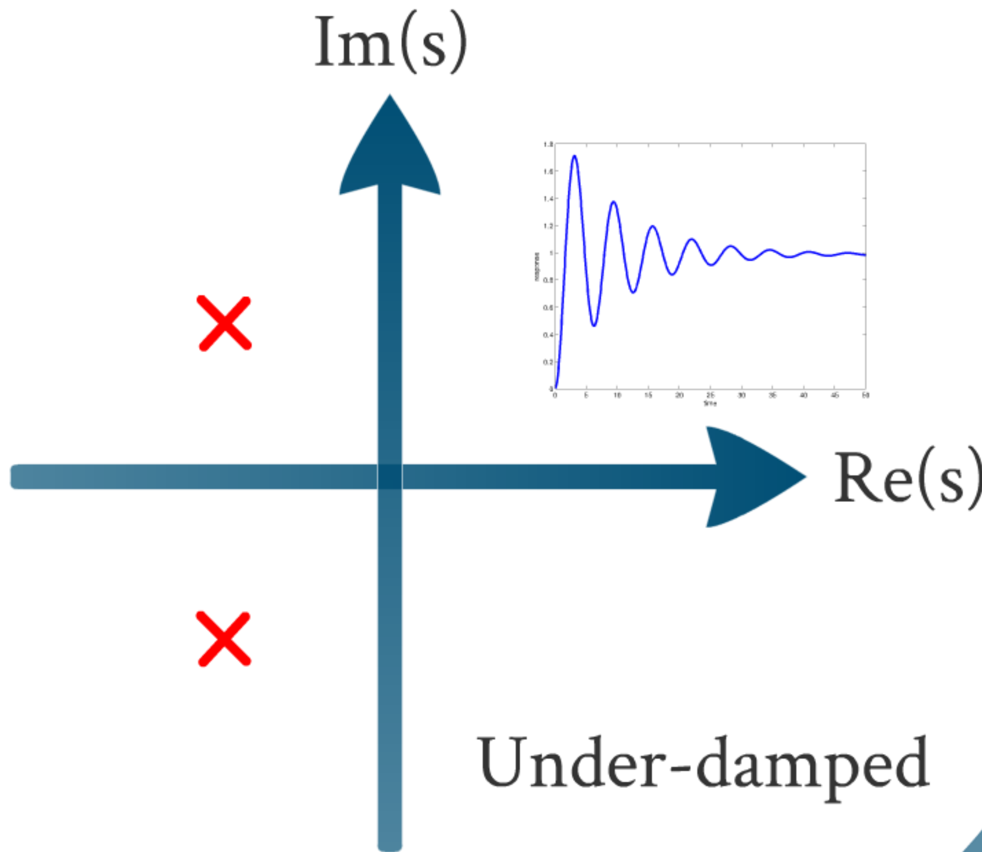


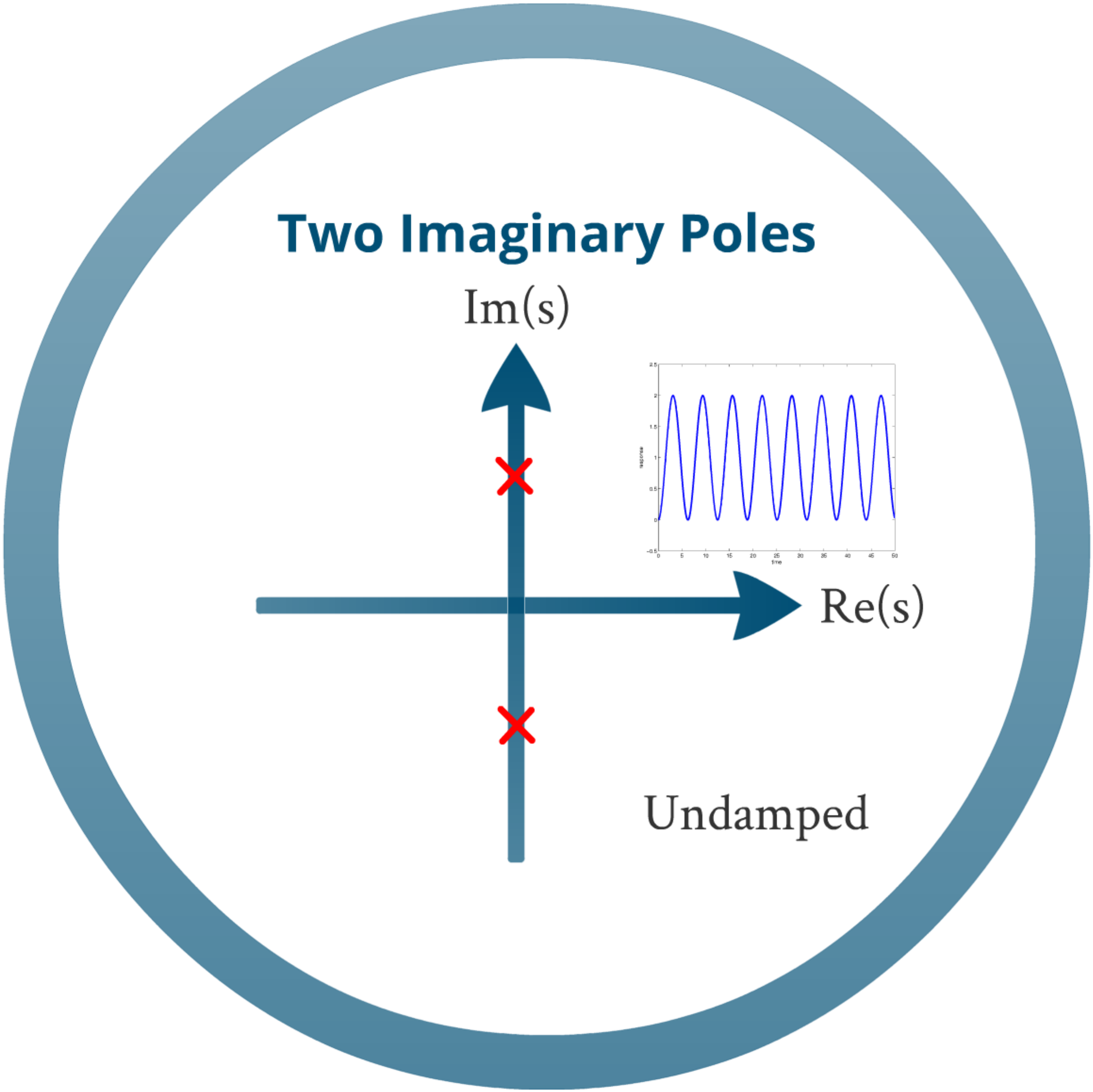

Two Complex Poles
Im(s)
Re(s)
Under-damped

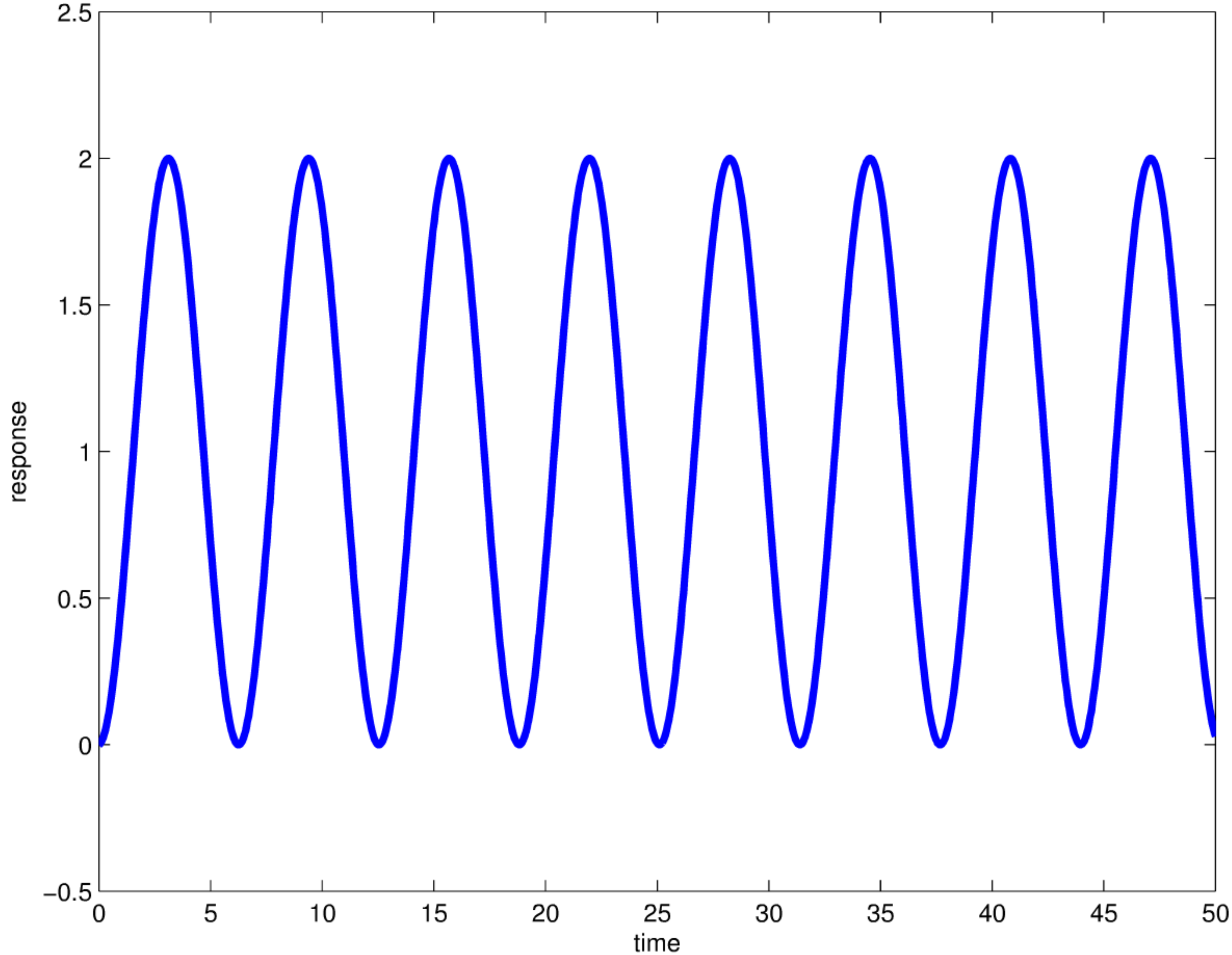


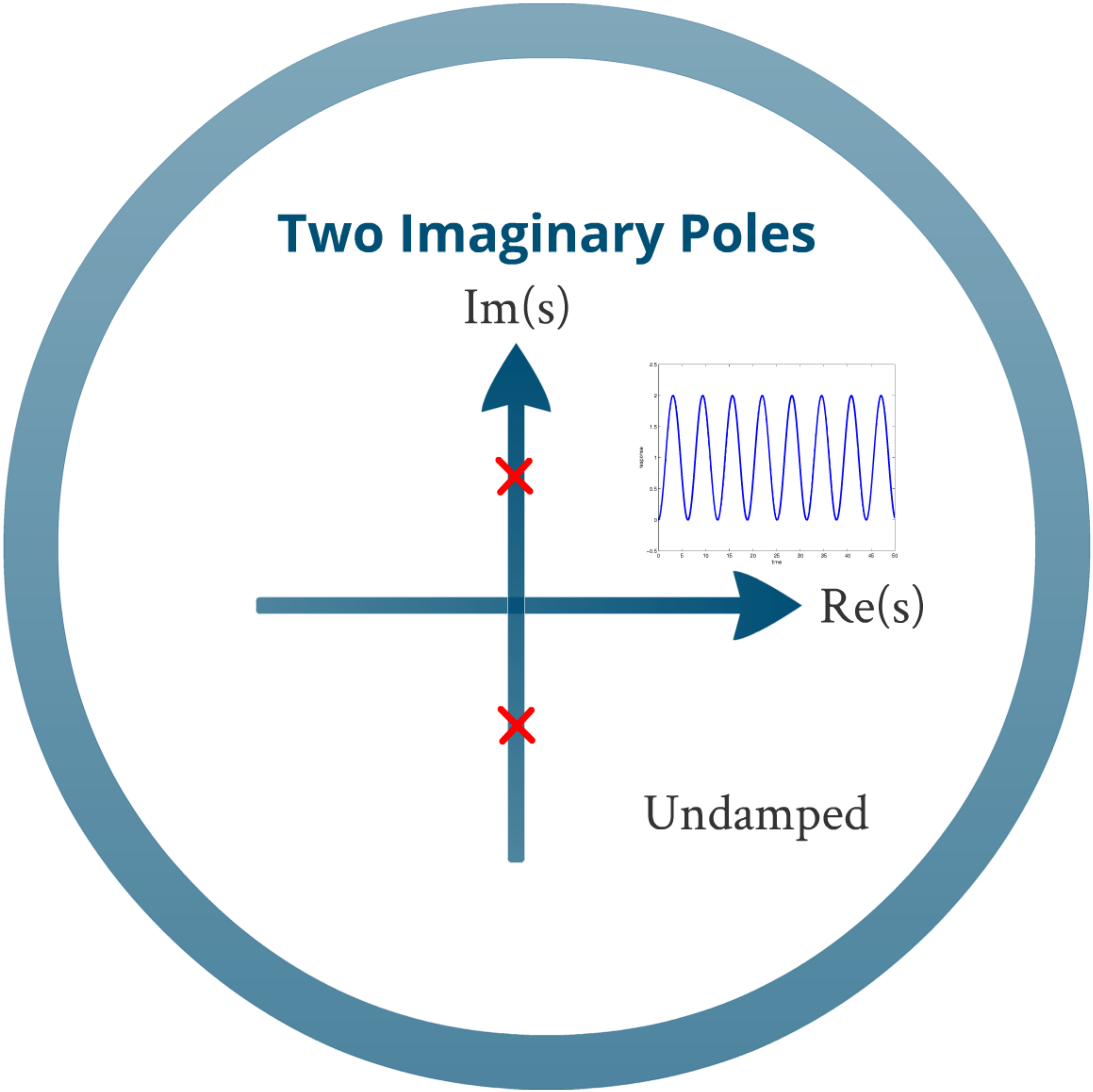
response
time
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
5
10
15
20
25
30
35
40
45
50

Two Complex Poles

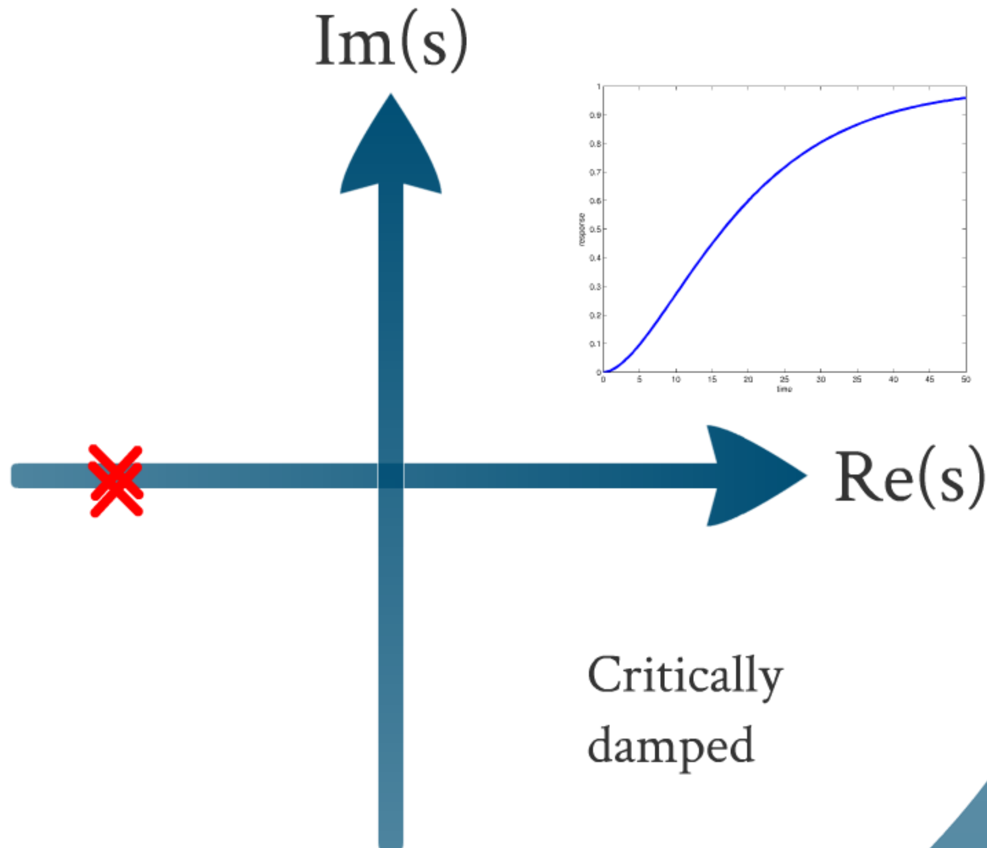



Two Imaginary Poles
Im(s)
Re(s)
Undamped

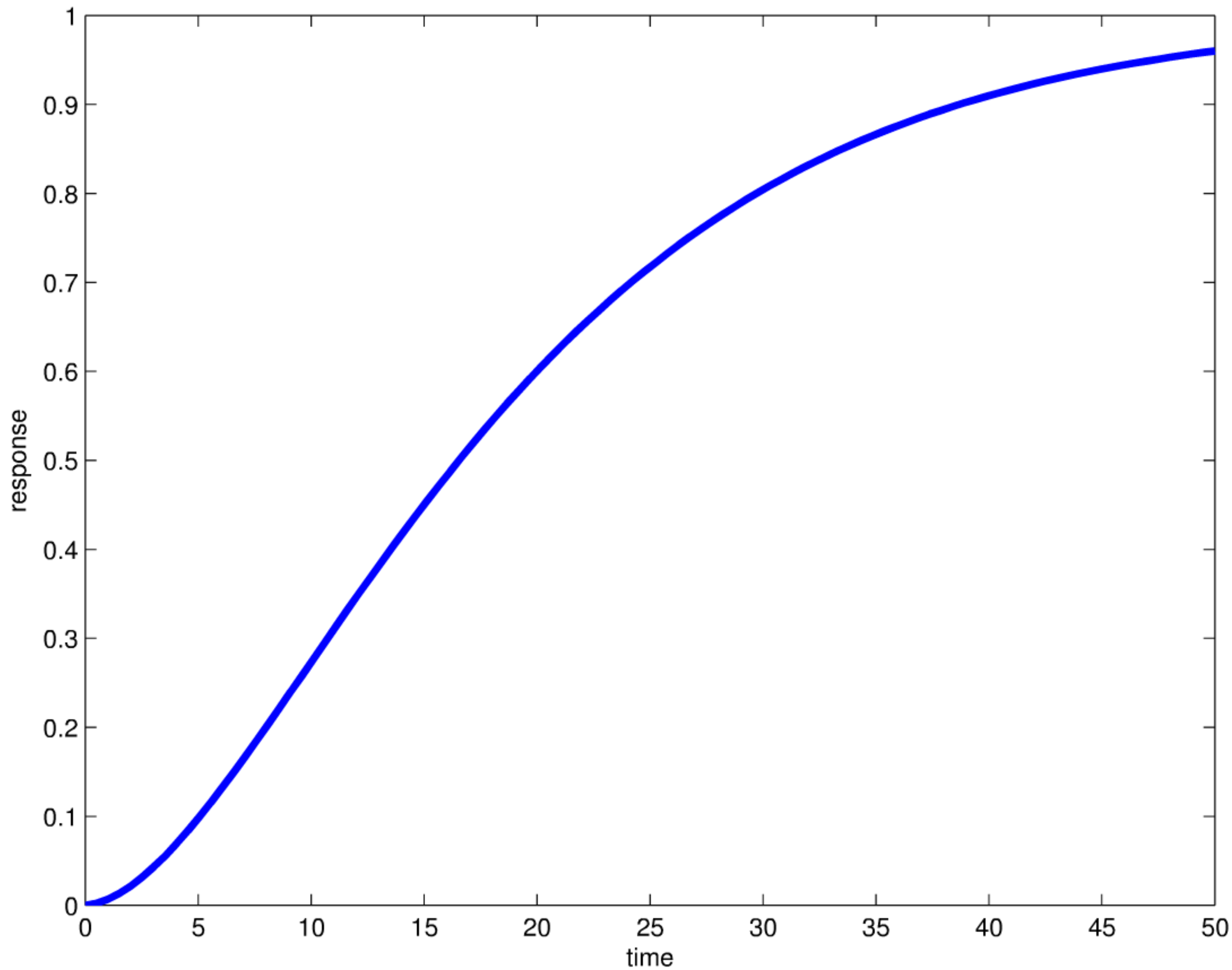

2.5
2
1.5
1
0.5
0
−0.5
response
0
5
10
15
20
25
30
35
40
45
50
time


Two Imaginary Poles
Im(s)
Re(s)
Undamped

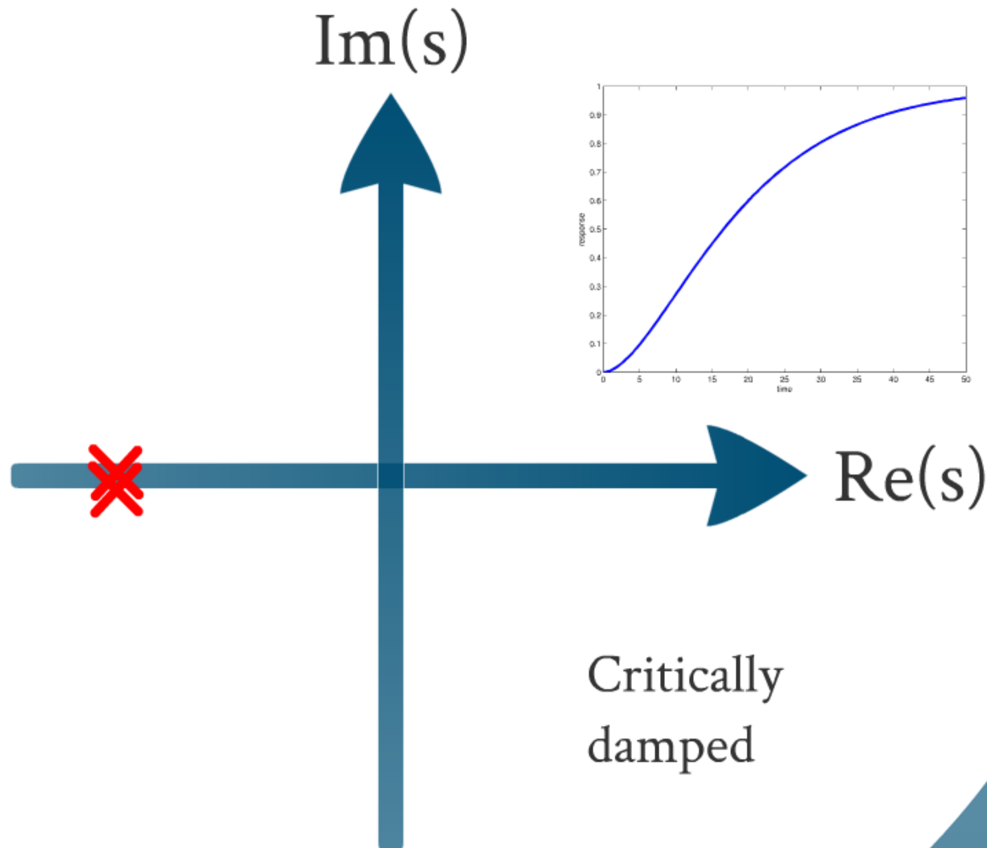
Two Repeated Poles



Critically damped


response
time
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
5
10
15
20
25
30
35
40
45
50

Two Repeated Poles

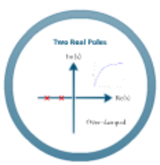


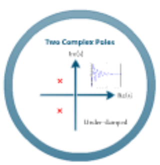
Second-Order System

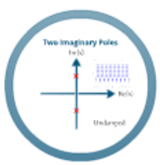
$$\frac{c}{s^2 + bs + c}$$

Changing parameters can change the type of response

Poles are at: $$s = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$$







Generalised Second-Order System

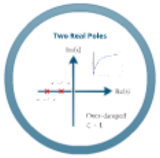
natural frequency

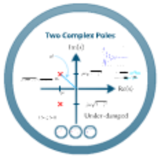
$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

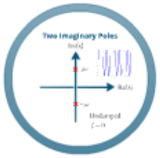
damping ratio

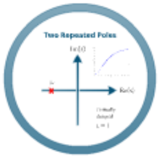
Changing parameters can change the type of response

Poles are at: $s = -\zeta\omega \pm \sqrt{\zeta^2 - 1}\omega$

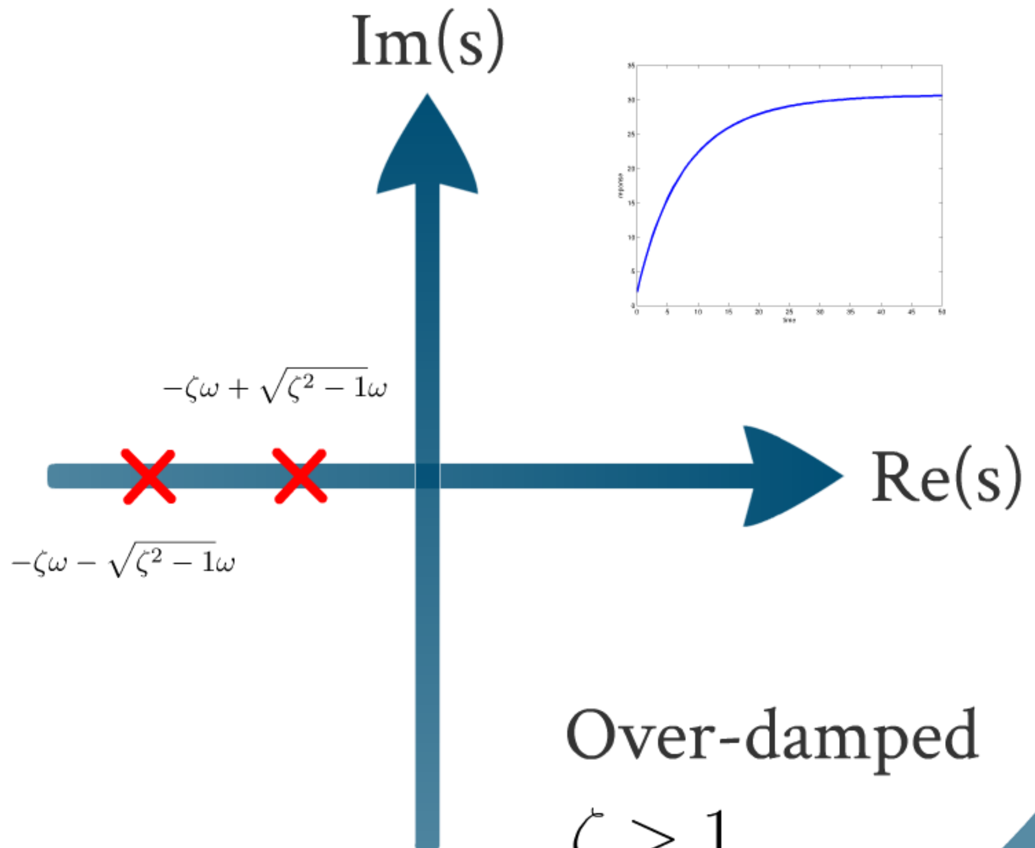








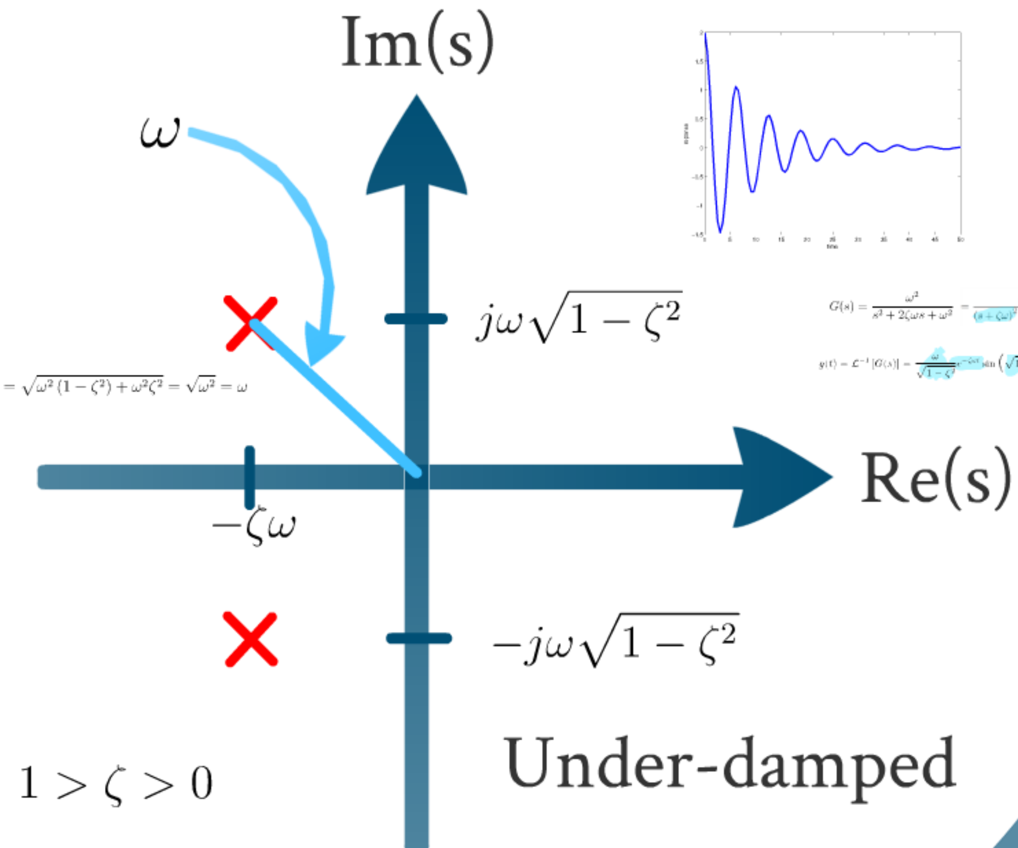
Two Real Poles



Over-damped

$\zeta > 1$

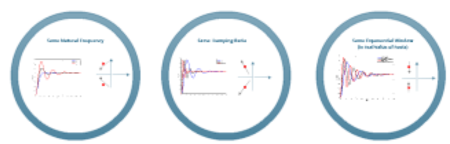
Two Complex Poles



$$\sqrt{(\omega\sqrt{1-\zeta^2})^2 + (\omega\zeta)^2} = \sqrt{\omega^2\left(1-\zeta^2\right) + \omega^2\zeta^2} = \sqrt{\omega^2} = \omega$$

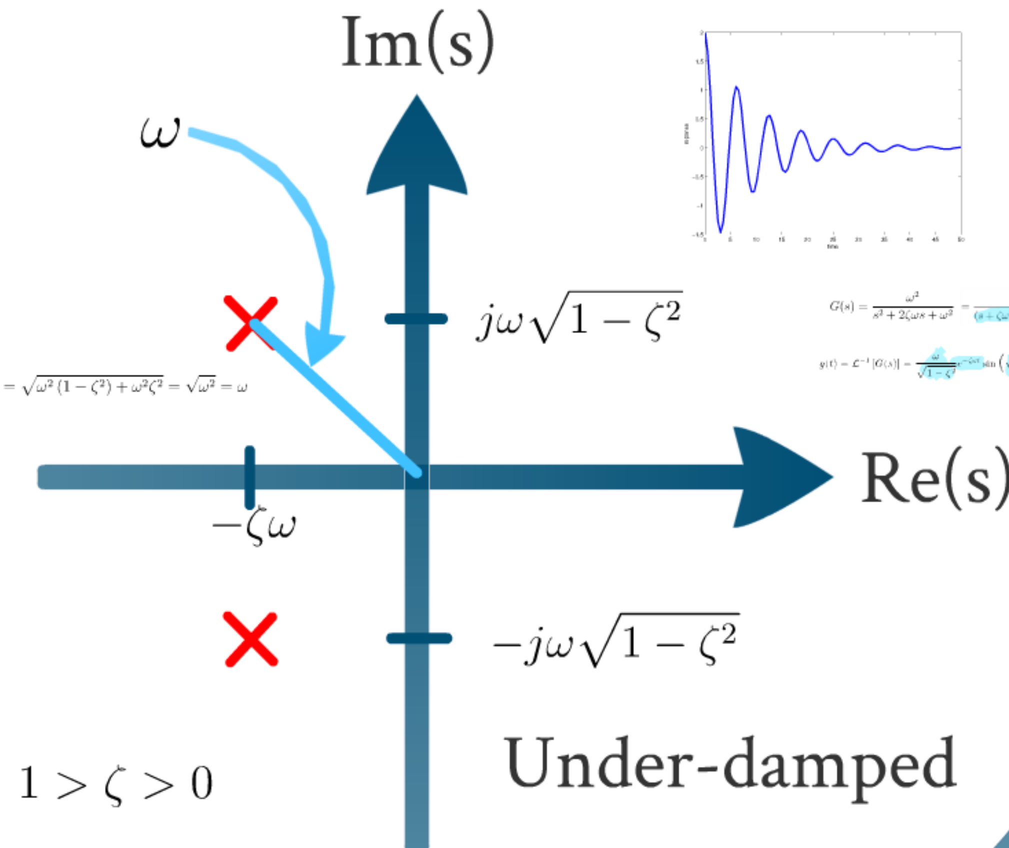
$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} = \frac{\omega^2}{(s+\zeta\omega)^2 + (1-\zeta^2)\omega^2}$$

$$y(t) = \mathcal{L}^{-1}\left[G(s)\right] = \frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \sin\left(\sqrt{1-\zeta^2}\,\omega t\right)$$



$$\sqrt{(\omega\sqrt{1-\zeta^2})^2+(\omega\zeta)^2}=\sqrt{\omega^2\left(1-\zeta^2\right)+\omega^2\zeta^2}=\sqrt{\omega^2}=\omega$$

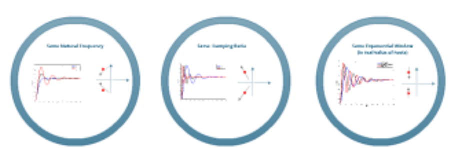
Two Complex Poles



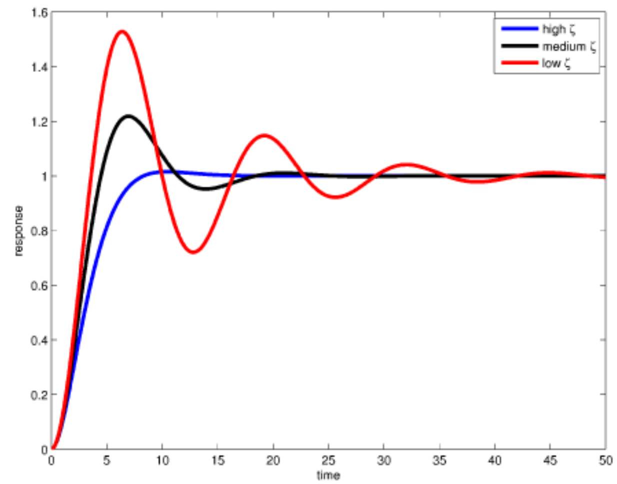
$$\sqrt{(\omega\sqrt{1-\zeta^2})^2 + (\omega\zeta)^2} = \sqrt{\omega^2(1-\zeta^2) + \omega^2\zeta^2} = \sqrt{\omega^2} = \omega$$

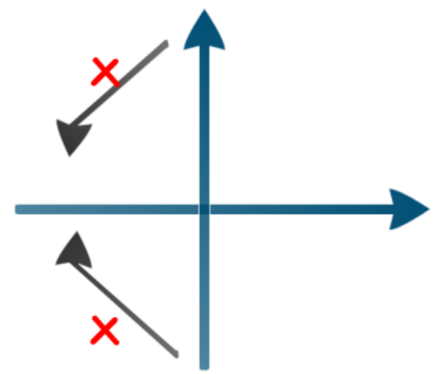
$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} = \frac{\omega^2}{(s+\zeta\omega)^2 + (1-\zeta^2)\omega^2}$$

$$y(t) = \mathcal{L}^{-1}[G(s)] = \frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \sin\left(\sqrt{1-\zeta^2}\,\omega t\right)$$

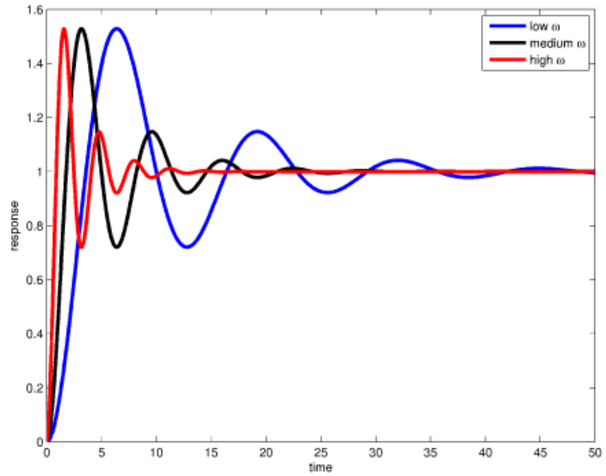


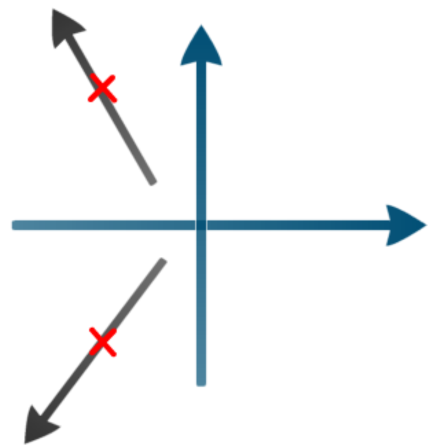
Same Natural Frequency



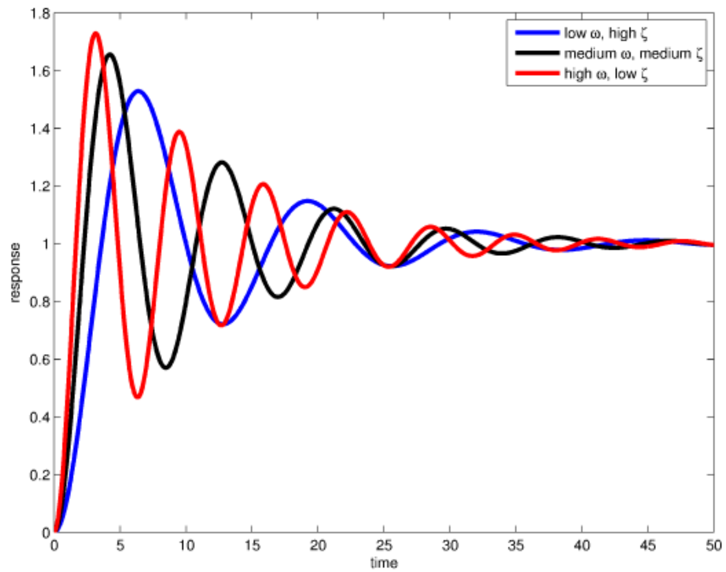


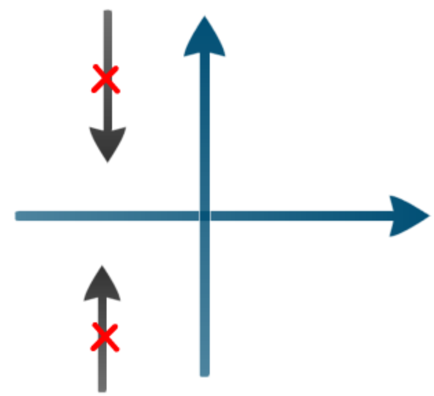
Same Damping Ratio



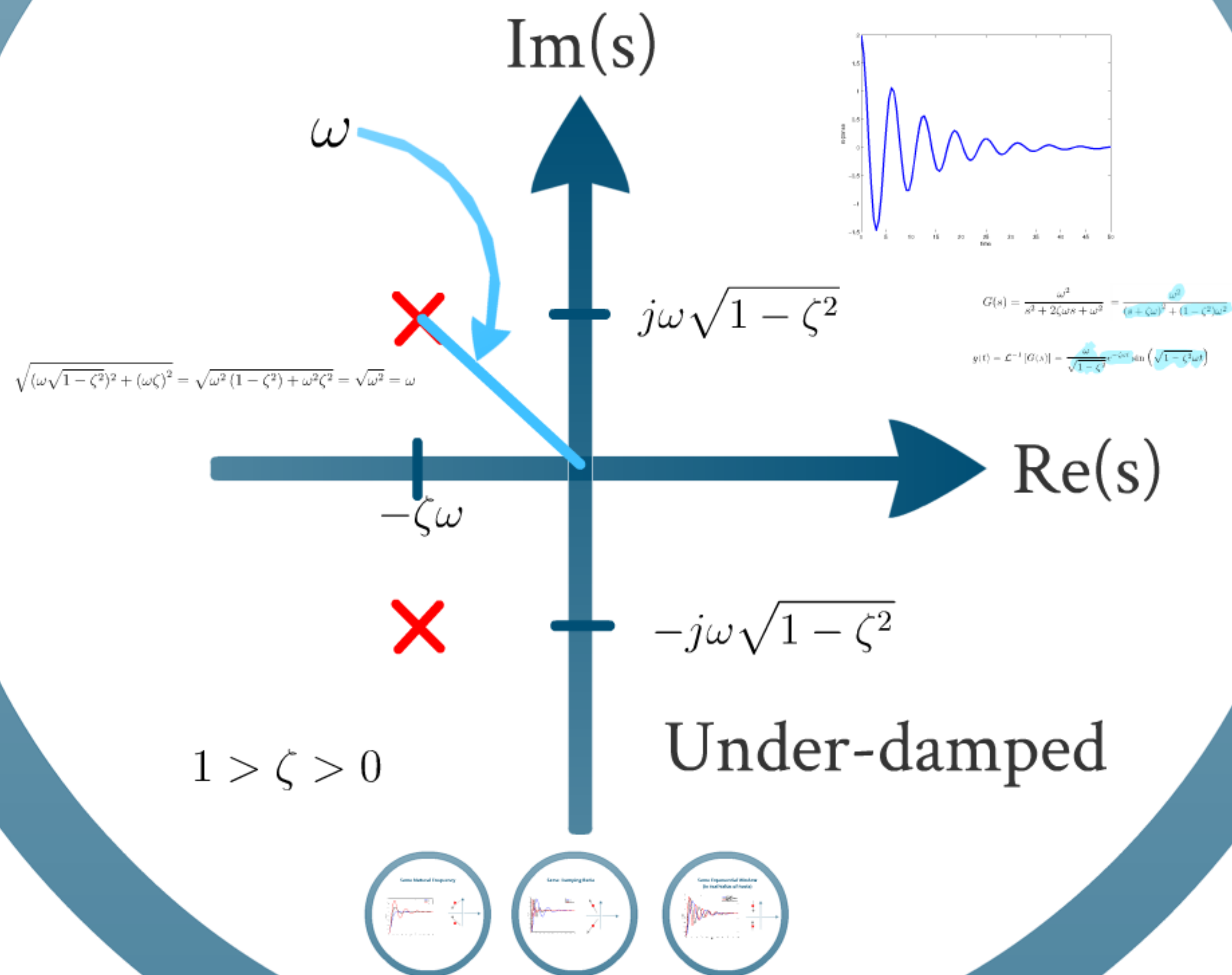


Same Exponential Window
(ie real value of roots)





Two Complex Poles

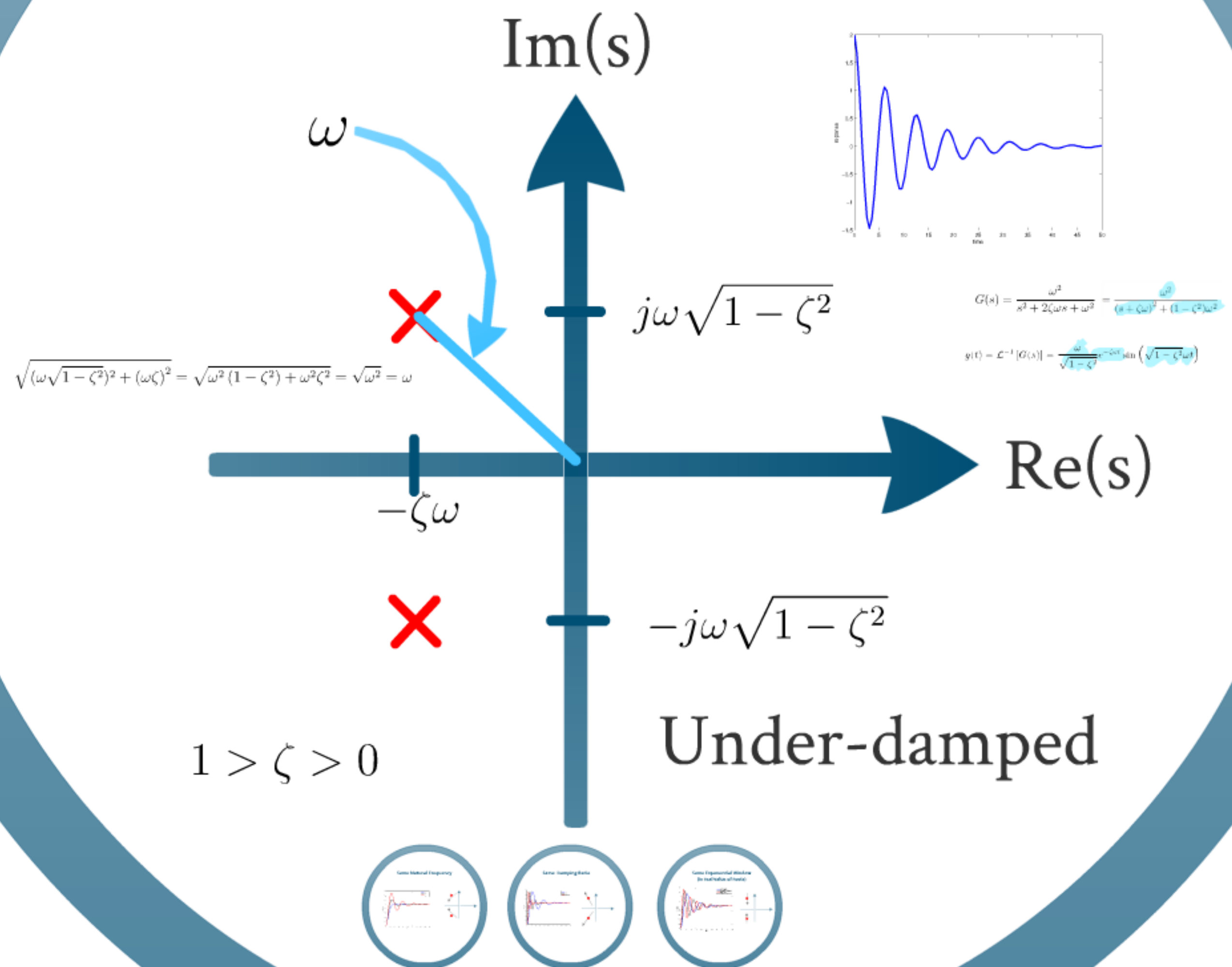


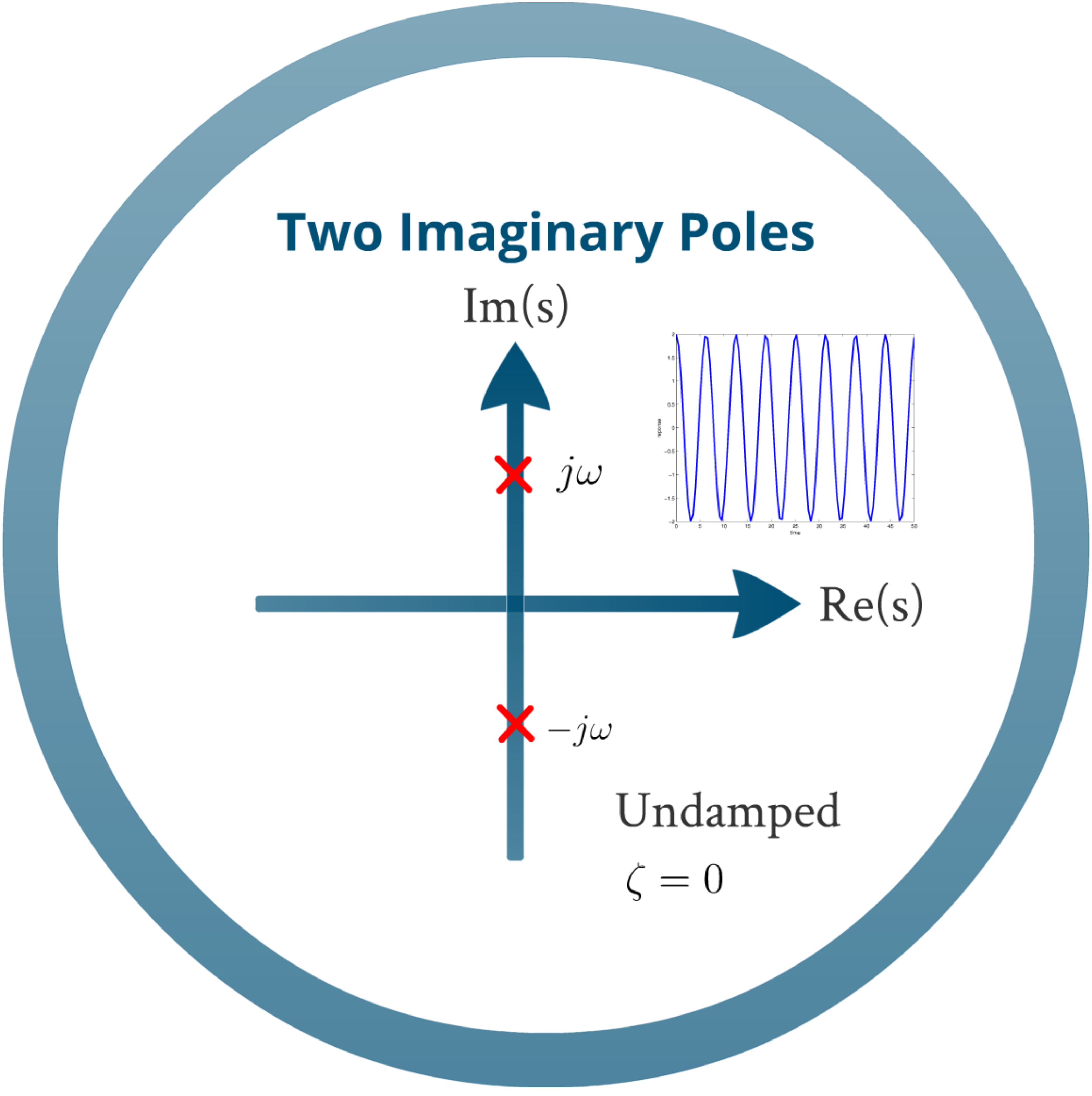
25 30 35 40 45 50
time

$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} = \frac{\omega^2}{(s+\zeta\omega)^2 + (1-\zeta^2)\omega^2}$$

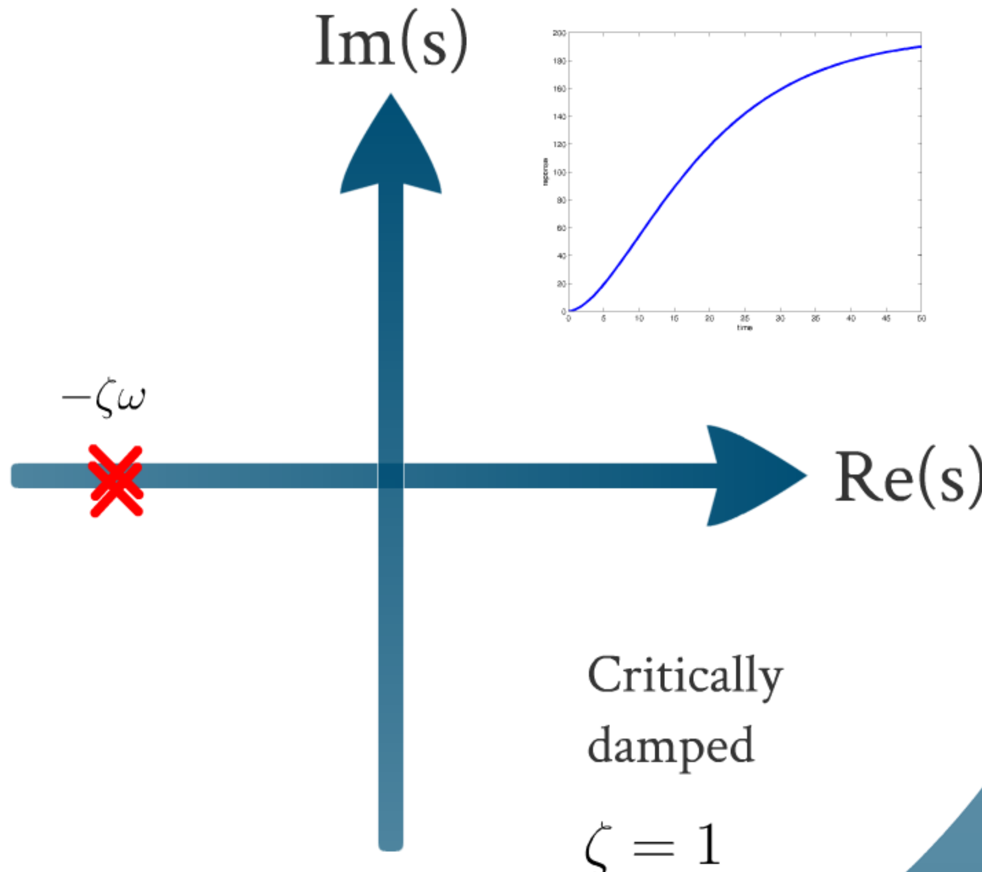
$$g(t) = \mathcal{L}^{-1}\left[G(s)\right] = \frac{\omega}{\sqrt{1-\zeta^2}} e^{-\zeta\omega t} \sin\left(\sqrt{1-\zeta^2}\,\omega t\right)$$

Two Complex Poles




Two Imaginary Poles
Im(s)
Re(s)
$j\omega$
$-j\omega$
Undamped
$\zeta = 0$

Two Repeated Poles



First and Second Order Systems

First-Order System

f(t)	F(s)
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$\sin(\omega t)\,u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)\,u(t)$	$\frac{s}{s^2+\omega^2}$

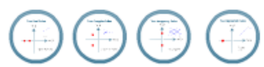
Changing parameter will just change the speed of (exponential) response and stability

Second-Order System

$$\frac{c}{s^2 + bs + c}$$

Changing parameters can change the type of response

Poles are at: $s = \frac{-b \pm \sqrt{b^2 - 4c}}{2}$



Generalised Second-Order System

natural frequency

$$G(s) = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}$$

damping ratio

Changing parameters can change the type of response

Poles are at: $s = -\zeta\omega \pm \sqrt{\zeta^2 - 1}\omega$

This lecture covers:

- First-order system and second-order system
- Generalized second-order system