Project One Template

MAT350: Applied Linear Algebra

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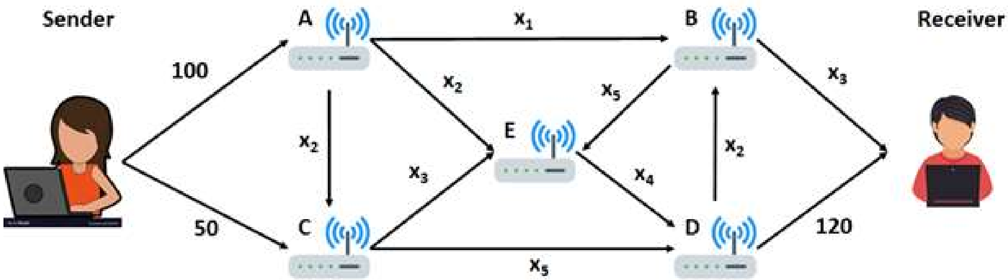
Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.

Solution:

Put your math/explanation here..



A system of equations must be derived that summarizes the node traffic for each node. The equation's initial setup is "Flow Output = Flow Input". Following setup, simplification of the equation to place unknowns to the left-side and constants to the right-side.

Node A:

$x_1 + 2x_2 = 100$

Node B:

$x_3 + x_5 = x_1 + x_2$

$-x_1 + -x_2 + x_3 + x_5$ =0

Node C:

$x_3 + x_5 = 50 + x_2$

$-x_2 + x_3 + x_5 = 50$

Node D:

$x_2 + 120 = x_4 + x_5$

$-x_2 + x_4 + x_5 = 120$

Node E:

$x_4 = x_2 + x_3 + x_5$

$x_2 + x_3 - x_4 + x_5 = 0$

A = [

1 2 0 0 0;

-1 -1 1 0 1;

0 -1 1 0 1;

0 -1 0 1 1;

0 1 1 -1 1

]

x=[

x_1;

x_2;

x_3;

x_4;

x_5;

]

b = [

100;

0;

50;

120;

0

]

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function**.** Write out your **reduced matrix and identify the free and basic variables of the system.**

Solution:

```
%code
A = [
    1 2 0 0 0;
    -1 -1 1 0 1;
    0 -1 1 0 1;
    0 -1 0 1 1;
    0 1 1 -1 1
    ]
```

```
A = 5×5

     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
b = [
    100;
    0;
    50;
    120;
    0
    ]
```

```
b = 5×1

   100
     0
    50
   120
     0
```

```
Ab = [A b]
```

```
Ab = 5×6
```

```
     1     2     0     0     0   100
    -1    -1     1     0     1     0
     0    -1     1     0     1    50
     0    -1     0     1     1   120
     0     1     1    -1     1     0
```

```
ech_Ab = rref(Ab)
```

```
ech_Ab = 5×6
     1     0     0     0     0    50
     0     1     0     0     0    25
     0     0     1     0     0    30
     0     0     0     1     0   100
     0     0     0     0     1    45
```

```
%A basic variable is a variable with a leading coefficient 1 , whereas a
%free variable is a variable without a pivot. There are no free variables
%in this system since every column preceding constants is a pivot column.
```

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x.** Solve the system of equations L**y** = **b** for the unknown vector **y.**

Solution:

```
%code
%Initiate the coefficient matrix A & column of constants matrix b
A = [
    1 2 0 0 0;
    -1 -1 1 0 1;
    0 -1 1 0 1;
    0 -1 0 1 1;
    0 1 1 -1 1
    ]
```

```
A = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
b = [
    100;
    0;
    50;
    120;
    0
    ]
```

```
b = 5×1
   100
     0
    50
   120
     0
```

```
%use LU command
[L, U] = lu(A)
```

```
L = 5×5
```

```
    1.0000         0         0         0         0
   -1.0000    1.0000         0         0         0
         0   -1.0000    1.0000         0         0
         0   -1.0000    0.5000    1.0000         0
         0    1.0000         0   -1.0000    1.0000

U = 5×5
     1     2     0     0     0
     0     1     1     0     1
     0     0     2     0     2
     0     0     0     1     1
     0     0     0     0     1
```

```
%solve using system of linear equations Ax=b. store solutions x & y
y = L\b
```

```
y = 5×1
   100
   100
   150
   145
    45
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
%code
inv(U)
```

```
ans = 5×5
    1.0000   -2.0000    1.0000         0         0
         0    1.0000   -0.5000         0         0
         0         0    0.5000         0   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y.**

Solution:

```
%code
x = inv(U)*y
```

```
x = 5×1
    50
    25
    30
   100
    45
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
%code
%checking all answers to be thorough :)
A = [
    1 2 0 0 0;
    -1 -1 1 0 1;
```

```
    0 -1 1 0 1;
    0 -1 0 1 1;
    0 1 1 -1 1
    ]
```

```
A = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
b = [
    100;
    0;
    50;
    120;
    0
    ]
```

```
b = 5×1
   100
     0
    50
   120
     0
```

```
%Initialize the matrices A1, A2, and A3 as matrix A.
A1 = A
```

```
A1 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A2 = A
```

```
A2 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A3 = A
```

```
A3 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A4 = A
```

```
A4 = 5×5
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
A5 = A
```

```
A5 = 5×5
```

```
     1     2     0     0     0
    -1    -1     1     0     1
     0    -1     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
%Replace the appropriate columns in A1, A2, and A3 with the column vector of constants b.
A1(:,1)=b
```

```
A1 = 5×5
   100     2     0     0     0
     0    -1     1     0     1
    50    -1     1     0     1
   120    -1     0     1     1
     0     1     1    -1     1
```

```
A2(:,2)=b
```

```
A2 = 5×5
     1   100     0     0     0
    -1     0     1     0     1
     0    50     1     0     1
     0   120     0     1     1
     0     0     1    -1     1
```

```
A3(:,3)=b
```

```
A3 = 5×5
     1     2   100     0     0
    -1    -1     0     0     1
     0    -1    50     0     1
     0    -1   120     1     1
     0     1     0    -1     1
```

```
A4(:,4)=b
```

```
A4 = 5×5
     1     2     0   100     0
    -1    -1     1     0     1
     0    -1     1    50     1
     0    -1     0   120     1
     0     1     1     0     1
```

```
A5(:,5)=b
```

```
A5 = 5×5
     1     2     0     0   100
    -1    -1     1     0     0
     0    -1     1     0    50
     0    -1     0     1   120
     0     1     1    -1     0
```

```
%Find the solution for x1, x2, and x3 using ratios of determinants.
x1 = det(A1)/det(A)
```

```
x1 = 50
```

```
x2 = det(A2)/det(A)
```

```
x2 = 25
```

```
x3 = det(A3)/det(A)
```

```
x3 = 30.0000
```

```
x4 = det(A4)/det(A)
```

```
x4 = 100
```

```
x5 = det(A5)/det(A)
```

```
x5 = 45
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

Fill out the table in the original project document and export your table as an image. Then, use the ***Insert*** *tab in the MATLAB editor to insert your table as an image.*

Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x_1	60	50	No Change	Network link x_1 is 10 Mbps below recommended capacity. Which allows room for network throughput increase.
x_2	50	25	No Change	Network link x_2 is 25 Mbps below recommended capacity. This allows room for network throughput increase.
x_3	100	30	No Change	Network link x_3 is 70 Mbps below recommended capacity. See x_4 network recommendation for utilizing this available throughput.
x_4	100	100	Upgrade Link	Network link x_4 is at throughput recommended capacity. If additional activity is routed through this piece of equipment user experience will suffer. You can pursue 2 distinct solutions. 1. Upgrade this link so it has a higher recommended capacity. 2. Re-route 30-50% of this network's capacity through network link x_3 .
x_5	50	45	No Change	Network link x_5 is 10 Mbps below recommended capacity. Which allows room for network throughput increase.