1.1

Homgenious cordinates

a)

consider the point $\tilde{x} = \tilde{l}_1 \times \tilde{l}_2$

it is easy to show that

$$\tilde{l}_1^T \tilde{x} = \tilde{l}_1^T (\tilde{l}_1 \times \tilde{l}_2) = (\tilde{l}_1 \times \tilde{l}_2) \cdot \tilde{l}_1 = (\tilde{l}_1 \times \tilde{l}_1) \cdot \tilde{l}_2 = 0$$

$$\tilde{l}_2^T \tilde{x} = \tilde{l}_2^T (\tilde{l}_1 \times \tilde{l}_2) = (\tilde{l}_1 \times \tilde{l}_2) \cdot \tilde{l}_2 = (\tilde{l}_2 \times \tilde{l}_2) \cdot \tilde{l}_1 = 0$$

$\therefore$ $\tilde{x}$ falls on $\tilde{l}_1$ & $\tilde{l}_2$

$\therefore$ $\tilde{x}$ is the intersection of $\tilde{l}_1$ & $\tilde{l}_2$

b)

consider a line $\tilde{l} = \tilde{x}_1 \times \tilde{x}_2$

$$\tilde{l}^T x_1 = (\tilde{x}_1 \times \tilde{x}_2) \cdot \tilde{x}_1 = (\tilde{x}_1 \times \tilde{x}_1) \cdot \tilde{x}_2 = 0$$

$$\tilde{l}^T x_2 = (\tilde{x}_1 \times \tilde{x}_2) \cdot \tilde{x}_2 = (\tilde{x}_2 \times \tilde{x}_2) \cdot \tilde{x}_1 = 0$$

$\therefore$ $\tilde{l}$ pass through $\tilde{x}_1$ & $\tilde{x}_2$

c) (i) solving linear equations

$$\begin{cases} x + y + 3 = 0 \\ -x - 2y + 7 = 0 \end{cases}$$

$$\Rightarrow \begin{cases} x = -13 \\ y = 10 \end{cases}$$

(ii) cross product

$\tilde{l}_1^T = (1, 1, 3)$

$\tilde{l}_2^T = (-1, -2, 7)$

$$\tilde{x} = \tilde{l}_1 \times \tilde{l}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 3 \\ -1 & -2 & 7 \end{vmatrix} = 13\hat{i} - 10\hat{j} - \hat{k} = \begin{pmatrix} -13 \\ 10 \\ 1 \end{pmatrix}$$

both way result in the same point.

d)

$$\tilde{l}^T = \left(\frac{3}{5}, \frac{4}{5}, 3\right)$$

e)

$$n = \left(\frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}\right)$$

$$d = \frac{\sqrt{29}}{5} / \sqrt{29} = \frac{1}{5}$$

Transformations

a)

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix}$$

b)

$$E(T) = \sum_{i=1}^{N} \| T\bar{x}_i - y_i \|_2^2$$

$$= \sum_{i=1}^{N} (T\bar{x}_i - y_i)^T (T\bar{x}_i - y_i)$$

$$= \sum_{i=1}^{N} [(\bar{x}_i^T T^T - y_i^T)(T\bar{x}_i - y_i)]$$

$$= \sum_{i=1}^{N} (\bar{x}_i^T T^T T \bar{x}_i - 2 y_i^T T \bar{x}_i + y_i^T y_i)$$

$$\frac{\partial E(T)}{\partial t_1} = \sum_{i=1}^{N} (2t_1 + 2x_1^i - 2y_1^i) = 2\left(Nt_1 + \sum_{i=1}^{N} x_1^i - \sum_{i=1}^{N} y_1^i\right) = 0$$

$$\frac{\partial E(T)}{\partial t_2} = \sum_{i=1}^{N} (2t_2 + 2x_2^i - 2y_2^i) = 2\left(Nt_2 + \sum_{i=1}^{N} x_2^i - \sum_{i=1}^{N} y_2^i\right) = 0$$

$$\Rightarrow \begin{cases} t_1 = \frac{1}{N}\left(\sum_{i=1}^{N} x_1^i - \sum_{i=1}^{N} y_1^i\right) = \frac{1}{N} \sum_{i=1}^{N} (y_1^i - x_1^i) \\ t_2 = \frac{1}{N}\left(\sum_{i=1}^{N} x_2^i - \sum_{i=1}^{N} y_2^i\right) = \frac{1}{N} \sum_{i=1}^{N} (y_2^i - x_2^i) \end{cases}$$

$$T^* = \begin{pmatrix} 1 & 0 & t_1 \\ 0 & 1 & t_2 \end{pmatrix}$$ where t_1 & t_2 are the mean values of the offset in each component.

c) $t_1 = \frac{1}{3}(3+2+1) = 2$

$t_2 = \frac{1}{3}(-6+(-1)+(-5)) = -4$

$\therefore T^* = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -4 \end{pmatrix}$

Camera projections

a)

(1) $R = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

这里似乎题目没讲是谁相对于谁

$t = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

(2)、(3)

$K = \begin{pmatrix} 100 & 0 & 25 \\ 0 & 100 & 25 \\ 0 & 0 & 1 \end{pmatrix}$

$$P = \begin{pmatrix} K & 0 \\ 0^T & 1 \end{pmatrix} \begin{pmatrix} R & t \\ 0^T & 1 \end{pmatrix} = \begin{pmatrix} 100 & 0 & 25 & 0 \\ 0 & 100 & 25 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 100 & 25 & 0 & 150 \\ 0 & 25 & -100 & 50 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

b) $\bar{x}_s = \begin{pmatrix} 25 \\ 50 \\ 1 \\ 0.25 \end{pmatrix}$

$\tilde{x}_s = \begin{pmatrix} 100 \\ 200 \\ 4 \\ 1 \end{pmatrix}$

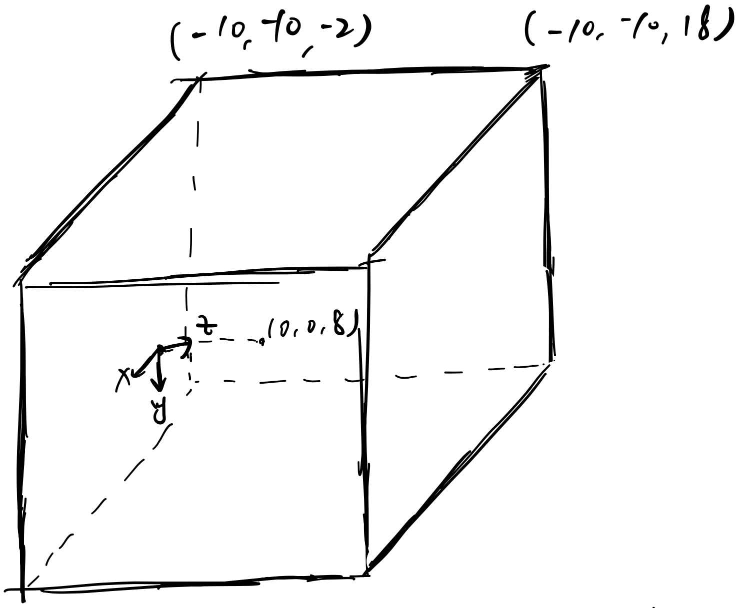
$\tilde{x}_s = P x_w$

$x_w = P^{-1} \tilde{x}_s = \begin{pmatrix} -1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$

c)

i)

$$K = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{pmatrix}$$



20 - 10

$$C_1 = \begin{pmatrix} -10 \\ -10 \\ -2 \end{pmatrix} \qquad \tilde{C}_{p_1} = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -10 \\ -10 \\ -2 \end{pmatrix} = \begin{pmatrix} -70 \\ -70 \\ -2 \end{pmatrix}$$

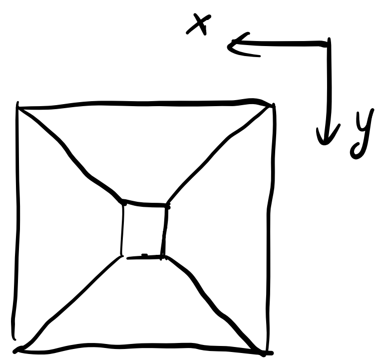
$$\bar{C}_{p_1} = \begin{pmatrix} 35 \\ 35 \\ 1 \end{pmatrix}$$

$$C_2 = \begin{pmatrix} -10 \\ -10 \\ 18 \end{pmatrix} \qquad \tilde{C}_{p_2} = \begin{pmatrix} 5 & 0 & 10 \\ 0 & 5 & 10 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -10 \\ -10 \\ 18 \end{pmatrix} = \begin{pmatrix} 130 \\ 130 \\ 18 \end{pmatrix}$$

$$\bar{C}_{p_2} = \begin{pmatrix} 7.2 \\ 7.2 \\ 1 \end{pmatrix}$$

the rest of these points are processed in the same way

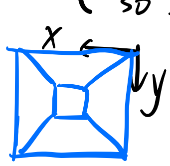
After projection:



ii) $$C_1' = \begin{pmatrix} -10 \\ -10 \\ 10 \end{pmatrix} \qquad K' = \begin{pmatrix} 10 & 0 & 10 \\ 0 & 10 & 10 \\ 0 & 0 & 1 \end{pmatrix} \qquad \tilde{C}_{p_1}' = K' C_1' = \begin{pmatrix} 0 \\ 0 \\ 10 \end{pmatrix} \qquad \bar{C}_{p_1}' = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$C_2' = \begin{pmatrix} -10 \\ -10 \\ 30 \end{pmatrix} \qquad \tilde{C}_{p_2}' = \begin{pmatrix} 200 \\ 200 \\ 30 \end{pmatrix} \qquad \bar{C}_{p_2}' = \begin{pmatrix} 6.7 \\ 6.7 \\ 1 \end{pmatrix}$$

After projection

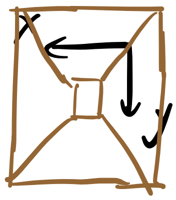


iii) $C_1'' = \begin{pmatrix} -10 \\ -10 \\ 80 \end{pmatrix}$ $K'' = \begin{pmatrix} 90 & 0 & 10 \\ 0 & 90 & 10 \\ 0 & 0 & 1 \end{pmatrix}$ $\tilde{C}_{p_1}'' = K'' C_1'' = \begin{pmatrix} -100 \\ -100 \\ 80 \end{pmatrix}$

$\bar{C}_{p_1}'' = \begin{pmatrix} -1.25 \\ -1.25 \\ 1 \end{pmatrix}$

$C_2'' = \begin{pmatrix} -10 \\ -10 \\ 100 \end{pmatrix}$ $\tilde{C}_{p_2}'' = \begin{pmatrix} 100 \\ 100 \\ 100 \end{pmatrix}$ $\bar{C}_{p_2}'' = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

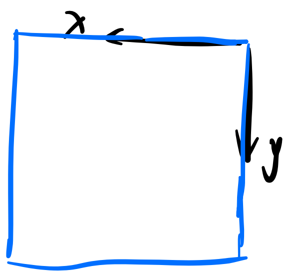
After projection



iv) $C_1 = \begin{pmatrix} -10 \\ -10 \\ 2 \end{pmatrix}$ $\tilde{C}_{p_1} = \begin{pmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -10 \\ -10 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \bar{C}_{p_1}$

$C_2 = \begin{pmatrix} -10 \\ -10 \\ 18 \end{pmatrix}$ $\tilde{C}_{p_2} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \bar{C}_{p_2}$

After projection

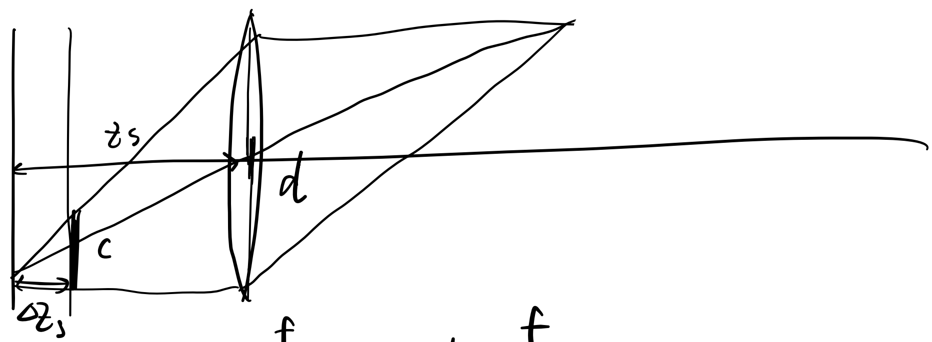


v) when the camera is far away from the object

Photometric Image Formation

a)
$$\frac{1}{z_c} + \frac{1}{z_s} = \frac{1}{f}$$
$$\Rightarrow z_c = \frac{z_s f}{z_s - f} = \frac{102 \times 100}{102 - 100} = 5100 \text{ mm} = 0.51\text{m}$$

b)



$$N = \frac{f}{d} \quad \Rightarrow d = \frac{f}{N}$$
$$\frac{c}{\delta z_s} = \frac{d}{z_s}$$
$$\Rightarrow c = d \cdot \frac{\delta z_s}{z_s} = \frac{f}{N} \cdot \frac{\delta z_s}{z_s}$$

c)
$$c_1 = \frac{35}{1.4} \times \frac{0.1}{40} = 1/16$$
$$c_2 = \frac{35}{1.4} \times \frac{0.03}{40} = 3/160$$

let pixel per size be PPS

$$PPS = 400 \times 400 / 64 = 2500 \text{ P/mm}^2$$

$$PPS \cdot \pi c_1^2 = 2500 \times \pi \times (\frac{1}{16})^2 = 30.6$$
$$PPS \cdot \pi c_2^2 = 2500 \times \pi \times (\frac{3}{160})^2 = 2.75$$