MATH230: Tutorial One

Minimal Logic

Key ideas

- Identify the propositional structure of an argument,
- Translate natural language to propositional logic,
- Identify when to use temporary hypotheses,
- Write natural deduction proofs using minimal logic.

Relevant lectures: Lectures x,y, and z
Relevant reading: L∃∀N Chapters 3,4

Hand in exercises:
Due Friday @ 5pm to the tutor, or to lecturer.

Discussion Questions

1. Translate the following English argument into the formal language for propositional logic. Clearly state the atomic propositions, hypotheses, and the conclusion of the argument.

   I will either go to Matukituki or Rakiura. If I go to Matukituki, then I will go hiking. If I go to Rakiura, then I will go hiking. Therefore, I will go hiking.

2. Show $P \to Q \vdash P \to (P \land Q)$.

3. Show $P \vdash \neg\neg P$.

4. Show $(P \land Q) \lor R \ \vdash \ (P \lor R) \land (Q \lor R)$.

Tutorial Exercises

1. Translate the following English arguments into the formal language for propositional logic. Clearly state the atomic propositions, hypotheses, and the conclusion of the argument.

   (a) Moriarty knows Irene is either at work, or at home. He has heard from others that she is not at home. Therefore, he concludes she must be at work.

   (b) If Lestrade observes, then he will solve the crime. If Lestrade does not observe, then he calls for Holmes. As ever, Lestrade sees, but does not observe. Therefore he must call Holmes.

   (c) If Robert rushes, then he will blunder his queen. If Robert does not rush, then he will blunder his queen. Therefore, Robert will blunder his queen.

   (d) Either the vicar is a liar (L), or he shot the earl (V). For, either the vicar shot the earl or the butler did (B). And unless the vicar is a liar, the butler was drunk at nine o'clock (D). And if the butler shot the earl, then the butler wasn't drunk at nine o'clock.

   (e) We will win, for if they attack if we advance, then we will win, and we won't advance.

2. This exercise breaks the proof of the sequent

$$\vdash (P \to Q) \to (\neg Q \to \neg P)$$

   into steps to show what's happening when temporary hypotheses are used.

   (a) Using the deduction theorem (temporary hypotheses) move as many hypotheses as possible to the left of the turnstile $\vdash$ to get a new sequent. Proof of this new sequent will ultimately lead to the proof of the original sequent.

   (!) Remember $\neg P \equiv P \to \bot$.

   (b) Prove the following sequent

$$P \to Q, \neg Q, P \vdash \bot$$

   (c) Extend the proof above, through the use of implication introduction, to a proof of the original sequent.

3. **Minimal Logic.** Provide natural deduction proofs of the following sequents. These deductions require only the use of minimal logic. Some of these sequents have a double turnstile. This means you need to prove both directions.

   (a) $(P \land Q) \to R \dashv\vdash P \to (Q \to R)$

   (b) $\neg P \lor \neg Q \vdash \neg(P \land Q)$

   (c) $\neg(P \lor Q) \dashv\vdash \neg P \land \neg Q$

   (d) $P \to Q,\ Q \to R \vdash P \to R$

   (e) $P \lor Q,\quad P \to R,\quad Q \to D \vdash R \lor D$

   (f) $P \to R,\quad Q \to D,\quad \neg R \lor \neg D \vdash \neg P \lor \neg Q$

   (g) $P,\quad \neg P \vdash \neg Q$

   (h) $P \to Q,\ P \to \neg Q \vdash \neg P$