MATH230: Tutorial Nine

Proofs-as-Programs: Minimal Logic in L∃∀N

Key ideas

- Learn L∃∀N syntax.
- Write explicit proof-terms in L∃∀N.
- Compare syntax across languages.

Relevant lectures:
Relevant reading: Theorem Proving in L∃∀N 4

Hand in exercises: 1
Sue Friday @ 5pm to the tutor, or to lecturer.

Discussion Questions

1. In a previous tutorial we wrote a proof-term witnessing the sequent

$$P \to Q \;\vdash\; P \to (P \wedge Q)$$

Translate this proof-term into the syntax for L∃∀N 4.

2. Write a proof-term in L∃∀N 4 to prove the following sequent

$$P \vdash \neg\neg P$$

3. Write a proof-term in L∃∀N 4 to prove the following sequent

$$(P \wedge Q) \vee R \;\vdash\; (P \vee R) \wedge (Q \vee R)$$

Tutorial Exercises

1. Throughout the course we have introduced rules of inference in logic, type constructors and destructors in the typed λ-calculus, and now we are to see how they are implemented in L∃∀N. As you write proof-terms for the sequents in Exercise 2 enter the missing components in the following table:

PL	$\boldsymbol{\lambda}$	**L∃∀N 4**
$\wedge I$		
$\wedge E_r$		
$\wedge E_l$		
$\to I$	$\lambda\_ : \_ .$	$\lambda\_ : \_ =>$
$\to E$		
$\vee I_r$		
$\vee I_l$		
$\vee E$		

Table 1: Syntax of logic, λ-calculus, and L∃∀N 4

2. For each of the sequents below, write proof-terms in L∃∀N 4. You have seen these proofs before, so it is a translating into L∃∀N exercise, rather than proving anything new.
   (a) $P \to Q, \neg Q \ \vdash \ \neg P$
   (b) $(P \wedge Q) \to R \dashv\vdash P \to (Q \to R)$
   (c) $\neg P \vee \neg Q \vdash \neg(P \wedge Q)$
   (d) $\neg(P \vee Q) \dashv\vdash \neg P \wedge \neg Q$
   (e) $P \to Q,\ Q \to R \vdash P \to R$
   (f) $P \vee Q, \quad P \to R, \quad Q \to S \vdash R \vee S$
   (g) $P \to R, \quad Q \to S, \quad \neg R \vee \neg S \vdash \neg P \vee \neg Q$
   (h) $P, \quad \neg P \vdash \neg Q$
   (i) $P \to Q,\ P \to \neg Q \vdash \neg P$

3. For each of the sequents below write proof-terms in L∃∀N. These exercises have not appeared in earlier tutorials. You could prove these by-hand first and then translate that into L∃∀N. Or you could write the proof straight into L∃∀N. Either way, they each have proofs in minimal logic.
   (a) $P \vdash \neg\neg P$
   (b) $\neg\neg\neg P \vdash \neg P$
   (c) $\vdash \neg\neg(P \vee \neg P)$