MATH230: Tutorial Four

Test Revision

Key ideas:

- Revise the propositional logic topic.

Relevant content: Lectures 1 - 13 and Tutorials 1,2,3
Relevant reading: L∃∀N Chapters 1,2,3,5,6

Hand in exercises: **2022 test**

Discussion Questions

1. Use valuations to decide whether the following propositions are tautologies, contingent, or contradictions.

   (a) $(\neg P \wedge \neg Q) \rightarrow \neg(P \wedge Q)$

   (b) $\neg(P \wedge Q) \rightarrow (\neg P \wedge \neg Q)$

   (c) $\neg(P \vee Q) \rightarrow (P \wedge Q)$

2. Provide a natural deduction to prove $\vdash A \vee \neg A$.

3. Explain why there can be no derivations of the proposition $A \wedge \neg A$. Your answer should reference a metalogical theorem.

Tutorial Exercises

1. Work through the 2022 midterm test.

2. Prove each of the following claims of semantic consequence

   (a) $\neg P \ \wedge \neg Q \ \models \ \neg(P \vee Q)$
   (b) $\neg(P \vee Q) \ \models \ \neg P \ \wedge \neg Q$
   (c) $\neg P \ \vee \neg Q \ \models \ \neg(P \wedge Q)$
   (d) $\neg(P \wedge Q) \ \models \ \neg P \ \vee \neg Q$

3. Explain how we know there must be natural deductions proving each of the corresponding syntactic consequences. Your answer should appeal to an appropriate metalogical theorem.

4. Provide natural deductions for the following claims of syntactic consequence

   (a) $\neg P \ \wedge \neg Q \ \vdash \ \neg(P \vee Q)$
   (b) $\neg(P \vee Q) \ \vdash \ \neg P \ \wedge \neg Q$
   (c) $\neg P \ \vee \neg Q \ \vdash \ \neg(P \wedge Q)$
   (d) $\neg(P \wedge Q) \ \vdash \ \neg P \ \vee \neg Q$

   One of these natural deductions will *require* the use of RAA. Use the BHK interpretation of the propositional connectives to figure out which one.

5. Prove the following claim twice:

$$\not\models A \to \neg A$$

   (a) with an argument using a truth table, and
   (b) with an argument using a valuation function.