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ENGINEERING MECHANICS

1. Coplanar Force System

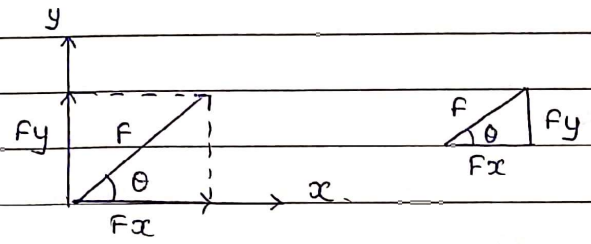
⊙ Types:
(i) Concurrent
(ii) Parallel & General

+ve →
-ve ←
+ve ↑
-ve ↓

"JISKA ANGLE USKA COS"

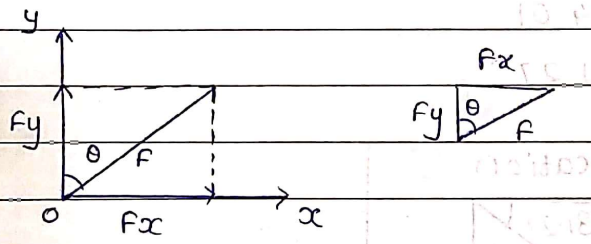
Resolution And Composition

⊙ θ → w.r.t x axis



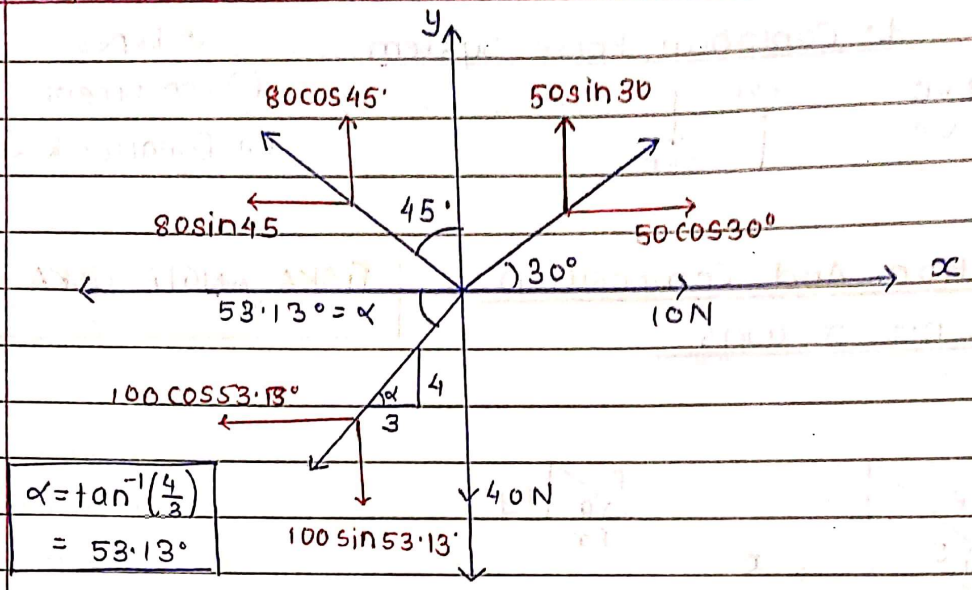
Resolution	Composition
$\sin\theta = Fy/F \quad \therefore Fy = F\sin\theta$	$F = \sqrt{Fx^2 + Fy^2}$
$\cos\theta = Fx/F \quad \therefore Fx = F\cos\theta$	$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right)$

⊙ θ → wrt Y axis



Resolution	Composition
$\sin\theta = Fx/F \quad Fx = F\sin\theta$	$F = \sqrt{Fx^2 + Fy^2}$
$\cos\theta = Fy/F \quad Fy = F\cos\theta$	$\theta = \tan^{-1}\left(\frac{Fx}{Fy}\right)$

(Q) Find Resultant and its direction



$\alpha = \tan^{-1}\left(\frac{4}{3}\right)$
$= 53.13°$

$\Sigma Fx = 10 + 50\cos 30° - 80\sin 45 - 100\cos 53.13°$

$\Sigma Fx = -63.26N \quad \Sigma Fx = 63.26N\ (\leftarrow)$

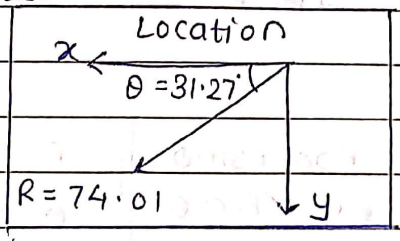
$\Sigma Fy = 50\sin 30 + 80\cos 45 - 100\sin 53.13 - 40$

$\Sigma Fy = -38.43\ N \quad \Sigma Fy = 38.43N\ (\downarrow)$

$R = \sqrt{\Sigma Fx^2 + \Sigma Fy^2} = 74.01$

$\theta = \tan^{-1}\left(\frac{\Sigma Fy}{\Sigma Fx}\right) = 31.27°$

wrt x axis



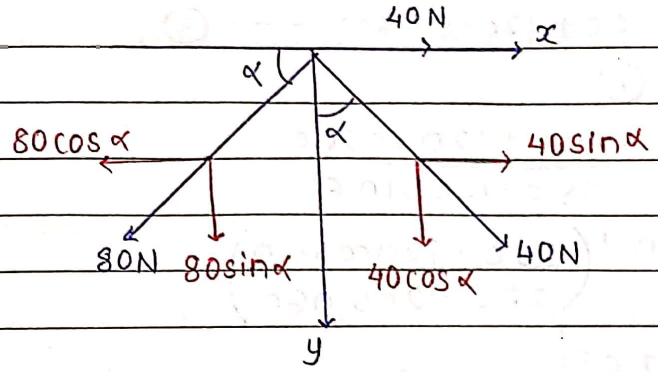
Imp Note

① If Resultant is Horizontal then $\Sigma Fx = R$
$\Sigma Fy = 0$

② If Resultant is vertical then $\Sigma Fx = 0$, $\Sigma Fy = R$

③ If Resultant is zero, $\Sigma Fx = 0$, $\Sigma Fy = 0$.

(Q) Find Resultant and angle α if resultant of given force system is vertical:



Resultant is vertical, $\therefore \Sigma Fx = 0$, $\Sigma Fy = R$

$\Sigma Fx = 40 + 40\sin\alpha - 80\cos\alpha$

$0 = 40 + 40\sin\alpha - 80\cos\alpha$

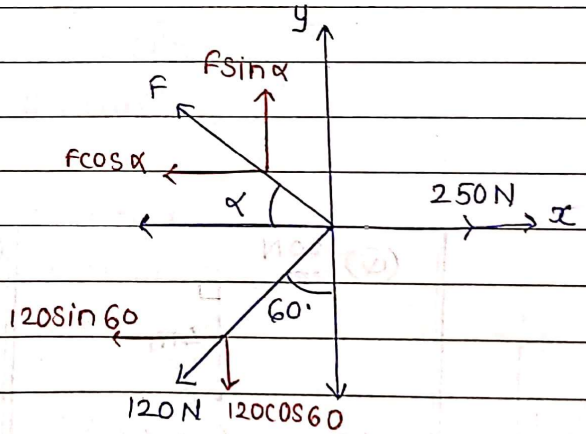
$\alpha = 36.86°$

$\Sigma Fy = -40\cos\alpha - 80\sin\alpha$

$R = -40\cos\alpha - 80\sin\alpha$

$R = -79.99\ N$ $\quad \boxed{R = 79.99\ N\ (\downarrow)}$

(Q)



Find F & α if Resultant of given force system is 100N acting in vertical direction

⇒ Resultant is vertical

$\therefore \Sigma Fx = 0$, $\Sigma Fy = R = 100N$

$\therefore \Sigma Fx = 250 - F\cos\alpha - 120\sin 60$

$\therefore 0 = 250 - F\cos\alpha - 120\sin 60°$

$F\cos\alpha = 250 - 120\sin 60$ —— (1)

$\Sigma F_y = F\sin\alpha - 120\cos 60$

$100 = F\sin\alpha - 120\cos 60$

$F\sin\alpha = 100 + 120\cos 60° \quad \text{——} (2)$

eqn (2) ÷ (1)

$$\frac{F\sin\alpha}{F\cos\alpha} = \frac{100 + 120\cos 60}{250 - 120\sin 60}$$

$$\alpha = \tan^{-1}\left(\frac{100 + 120\cos 60}{250 - 120\sin 60}\right)$$

$$\boxed{\alpha = 47.6°}$$

From eq (2) $F = \dfrac{100 + 120\cos 60}{\sin(47.6°)} = 216.64\,N$

MOMENT (M)

$M = F \times d$ (Nm)

F → Force (N)

d → ⊥ distance

↻ = -ve

↺ = +ve

(1) $M_A = -(10 \times 3)$

$M_A = -30\,Nm$

A ---- 3m ---- ↓ 10N

$\boxed{M_A = 30\,Nm\ (↻)}$

(2) 20N ←

4m ↓

B

$M_B = +(20 \times 4)$

$\boxed{M_B = 80\,Nm\ (↺)}$

(3) 3m C

20N →

$M_C = 20 \times 0$

$\boxed{M_C = 0}$

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Imp Note:
If force is passing through moment center then moment of that force will be zero.

VARIGNON'S THEOREM

Summation of moment of all forces wrt any point is equal to moment of resultant of that forces wrt same point.

$$\Sigma M_A^F = M_A^R$$

PARALLEL AND GENERAL

Step I : Resolution

Step II : ΣFx

Step III : ΣFy

Step IV : $R = \sqrt{\Sigma Fx^2 + \Sigma Fy^2}$

Step V : $\theta = \tan^{-1}\left(\frac{\Sigma Fy}{\Sigma Fx}\right)$ (wrt x axis)

Step VI : location

Step VII : varignon's Th^m

Step VIII : $\bar{x} = \frac{\Sigma M}{\Sigma Fy}$ $\quad \bar{y} = \frac{\Sigma M}{\Sigma Fx}$

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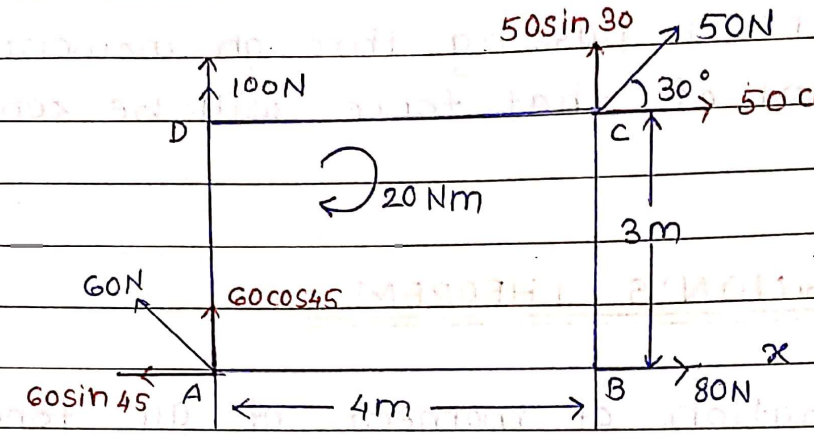
(Q) Find Resultant wrt 'A'

(I)



$\Sigma Fx = 80 + 50\cos 30' - 60\sin 45'$

(II) $\boxed{\Sigma Fx = 80.87\,N\ (\rightarrow)}$

$\Sigma Fy = 50\sin 30' + 100 + 60\cos 45$

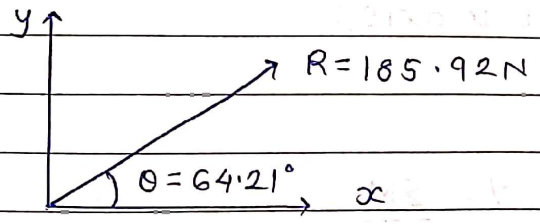
(III) $\boxed{\Sigma Fy = 167.42\,N\ (\uparrow)}$

(IV) $R = \sqrt{\Sigma Fx^2 + \Sigma Fy^2}$ $\therefore$ $\boxed{R = 185.92\,N}$

(V) $\theta = \tan^{-1}\left(\frac{\Sigma Fy}{\Sigma Fx}\right)$ wrt x axis $\therefore$ $\boxed{\theta = 64.21°}$

Location

(VI)



(VII) Varignon's Theorem

$\Sigma M_A^F = M_A^R$

$LHS = \Sigma M_A^F$

$= -(50\cos 30' \times 3) + (50\sin 30 \times 4) - 20$

$= -49.9$

$\boxed{\Sigma M_A^F = 49.9\,Nm\ (\circlearrowleft)}$

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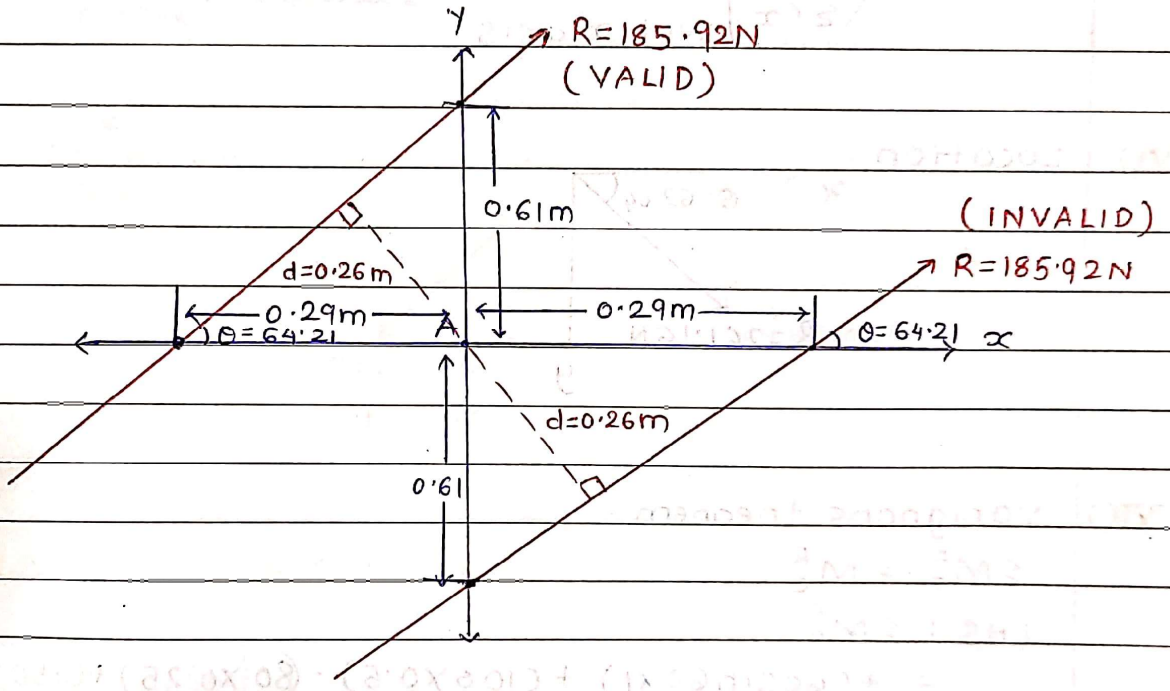
RHS: $M_A^R = R \times d = 185.92 \times d$

According to V.T,

$49.9 = 185.92 \times d$ $\therefore$ $\boxed{d = 0.26\,m}$

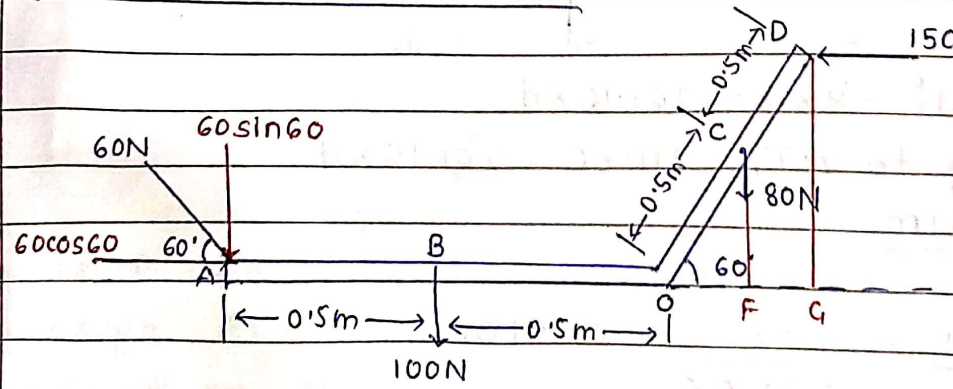
(VIII) $\bar{X} = \frac{\Sigma M}{\Sigma Fy} = \frac{49.9}{167.42}$ $\therefore$ $\boxed{\bar{X} = 0.29\,m}$

$\bar{Y} = \frac{\Sigma M}{\Sigma Fx} = \frac{49.9}{80.87}$ $\therefore$ $\boxed{\bar{Y} = 0.61\,m}$



(Q) Find resultant wrt 'O'

(I)



$\cos 60° = \frac{OF}{0.5}$
$OF = 0.25\,m$
$\sin 60 = \frac{GD}{1}$
$GD = 1 \sin 60$

(II) $\Sigma Fx = 60\cos 60 - 150$

$\Sigma Fx = -120\,N$ $\quad \boxed{\Sigma Fx = 120\,N\ (\leftarrow)}$

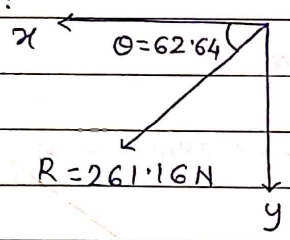
(III) $\Sigma Fy = -60\sin 60 - 100 - 80$

$\Sigma Fy = -231.96\,N$ $\quad \boxed{\Sigma Fy = 231.96\,N\ (\downarrow)}$

(IV) $R = \sqrt{\Sigma Fx^2 + \Sigma Fy^2}$ $\quad \therefore \boxed{R = 261.16\,N}$

(V) $\theta = \tan^{-1}\left(\dfrac{\Sigma Fy}{\Sigma Fx}\right)$ wrt x axis $\quad \boxed{\theta = 62.64^\circ}$

(VI) Location :



(VII) Varignons theorem :

$\Sigma M_o^F = M_o^R$

LHS : ΣM_o^F

$= +(60\sin 60 \times 1) + (100 \times 0.5) - (80 \times 0.25) + (150 \times 1\sin 60)$

$\boxed{\Sigma M_o^F = 211.86\,Nm\ (\circlearrowleft)}$

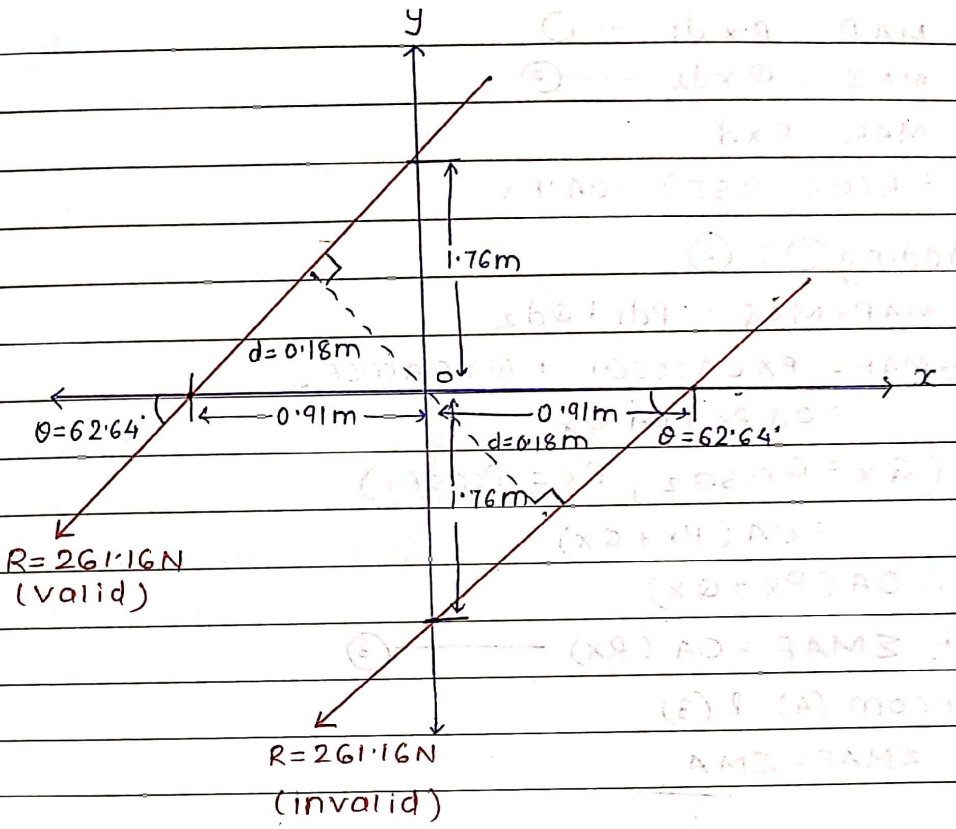
RHS : $M_o^R = R \times d = 261.16 \times d$

According to V.T, $211.86 = 261.16 \times d$

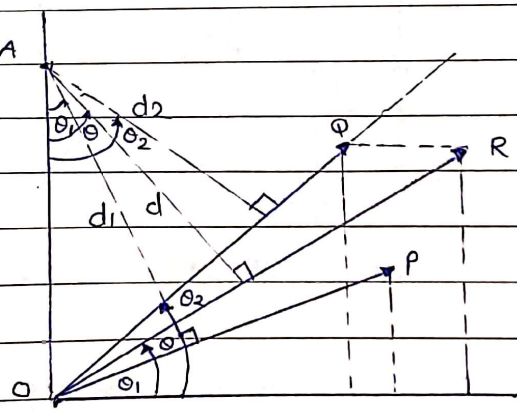
$\therefore d = 0.81\,m$

(VIII) $\bar{X} = \dfrac{\Sigma M}{\Sigma Fy} = \dfrac{211.86}{231.96} = 0.91\,m$

$$\bar{Y} = \frac{\Sigma M}{\Sigma Fx} = \frac{211.86}{120} = 1.76\,m$$



* VARIGNON'S THEOREM PROOF



Let P and Q be two concurrent forces at o making angle θ_1 and θ_2 with x axis

Let R be the resultant making angle θ

Let A be point on Y axis about which we shall

find the moments of P and Q and also resultant R

d_1, d_2, d be moment arm of P, Q, R from A.

$\therefore MAP = P \times d_1$ —— ①

$\therefore MAQ = Q \times d_2$ —— ②

$\therefore MAR = R \times d$

$= R(OA \cdot \cos\theta) = OA \cdot Rx$

Adding ① & ②

$\therefore MAP + MAQ = Pd_1 + Qd_2$

$\Sigma MAF = P \times OA\cos\theta_1 + Q \times OA\cos\theta_2$

$= OA \cdot Px + OA \cdot Qx$

$(Qx = Q\cos\theta_2,\ Px = P\cos\theta_1)$

$= OA(Px + Qx)$

$\therefore OA(Px + Qx)$

$\therefore \Sigma MAF = OA(Rx)$ —— ③

From (4) & (3)

$\Sigma MAF = \Sigma MA$

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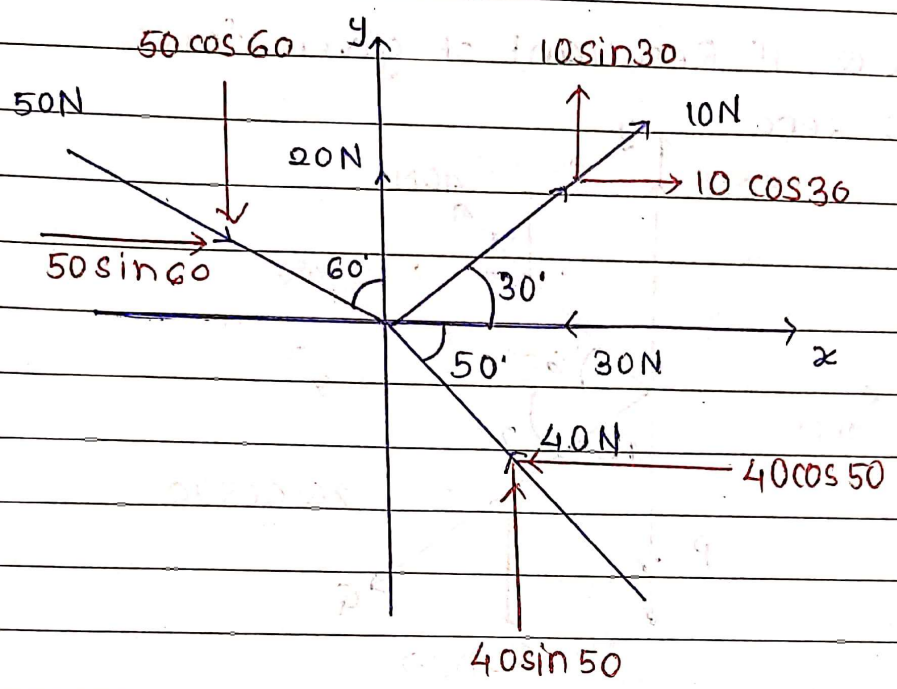
COPLANAR FORCE SYSTEM

(Q)①



$$\Sigma Fx = -40\cos 50 - 30 + 10\cos 30 + 50\sin 60$$
$$= +9.95 - 3.74 = 3.74\ (\leftarrow)$$

$$\Sigma Fy = 40\sin 50 + 10\sin 30 + 20 - 50\cos 60$$
$$= 30.64\ (\uparrow)$$

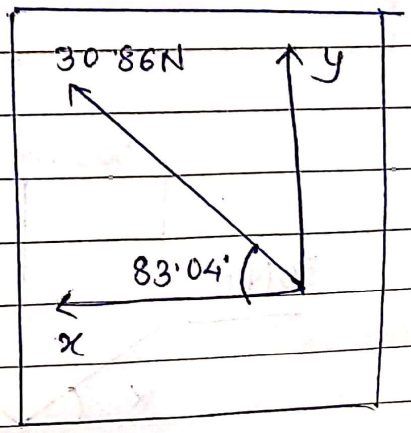
$$\therefore R = \sqrt{\Sigma Fx^2 + \Sigma Fy^2}$$

$$= \sqrt{(3.74)^2 + (30.64)^2} = 30.86\,N$$

$$\theta = \tan^{-1}\left(\frac{Fy}{Fx}\right)$$

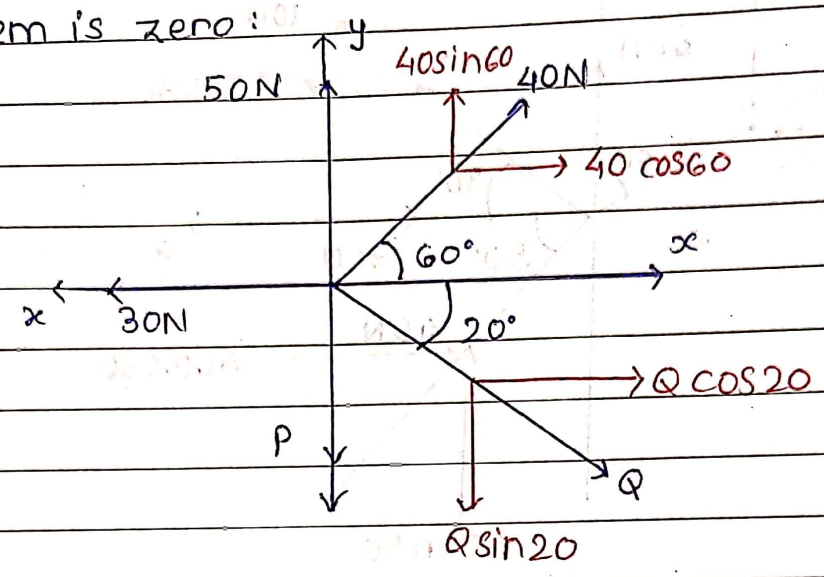
$$= \tan^{-1}\left(\frac{30.64}{3.74}\right)$$

$$= 83.04°$$



(2) Find resultant of given force system.

(3) Find P & Q if Resultant of given force system is zero:



$\Sigma Fx = Q\cos 20 + 40\cos 60 - 30$

$\therefore Q\cos 20 + 40\cos 60 - 30 = 0$

$\therefore \boxed{Q = 10.64}$

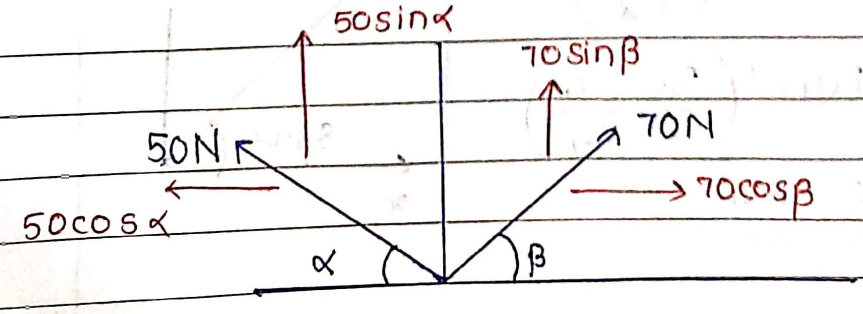
$\Sigma Fy = -P - Q\sin 20 + 40\sin 60 + 50$

$= -P - 10.64 \times \sin 20 + 40\sin 60 + 50$

$\therefore \boxed{P = 81.00}$

$\therefore \boxed{P = 81.00\ N \ \&\ \ Q = 10.64 N}$

(4)



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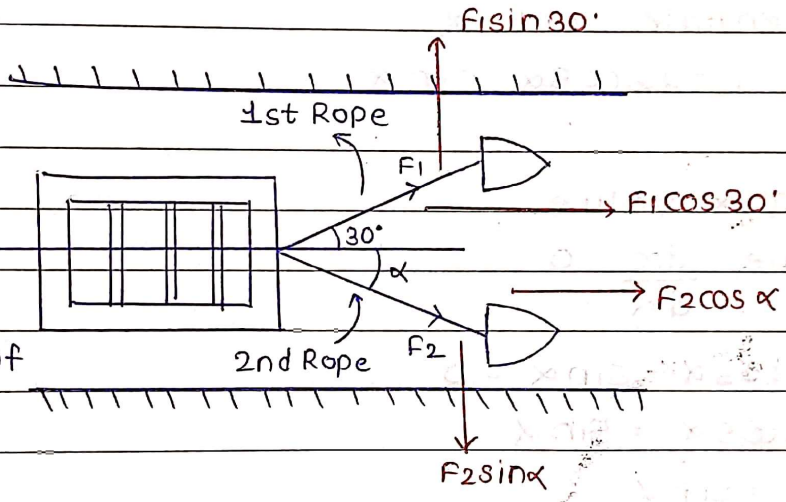
Find α and β if Resultant of given force system is 90N acting in vertical direction.

(Q) A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is 5000N force directed along the axis of the barge, determine the value of α for which the tension in rope 2 is minimum

⇒



R is horizontal $\therefore \Sigma F_x = R = 5000, \quad \Sigma F_y = 0$

$\therefore \Sigma F_x = F_1\cos 30° + F_2\cos\alpha$

$\therefore 5000 = F_1\cos 30° + F_2\cos\alpha$ ——— (1)

$\Sigma F_y = F_1\sin 30 - F_2\sin\alpha$

$0 = F_1\sin 30° - F_2\sin\alpha$ ——— (2)

From eqn (2)

$F_2\sin\alpha = F_1\sin 30°$

$F_1 = \frac{F_2\sin\alpha}{\sin 30}$ ——— (3)

Put eqn (3) into (1)

$5000 = F_1\cos 30° + F_2\cos\alpha$

$5000 = \frac{F_2\sin\alpha}{\sin 30°}\cdot\cos 30° + F_2\cos\alpha$

$$\frac{\cos 30^\circ}{\sin 30^\circ} = \cot 30^\circ = \frac{1}{\tan 30^\circ} = 1.732$$

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$$5000 = \frac{F_2 \sin\alpha}{\tan 30^\circ} + F_2 \cos\alpha$$

$$5000 = 1.732 F_2 \sin\alpha + F_2 \cos\alpha$$

$$5000 = (1.732 \sin\alpha + \cos\alpha) F_2$$

$$F_2 = \frac{5000}{1.732 \sin\alpha + \cos\alpha} \longrightarrow x$$

$$F_2 = \frac{5000}{x}$$

$$x = 1.732 \sin\alpha + \cos\alpha$$

Differentiate wrt α

$$\frac{dx}{d\alpha} = 1.732 \cos\alpha - \sin\alpha$$

For max value

Assume $\frac{dx}{d\alpha} = 0$

$$1.732 \cos\alpha - \sin\alpha = 0$$

$$1.732 \cos\alpha = \sin\alpha$$

$$1.732 = \frac{\sin\alpha}{\cos\alpha}$$

$$\therefore \alpha = \tan^{-1}(1.732)$$

$$\boxed{\alpha = 60^\circ}$$

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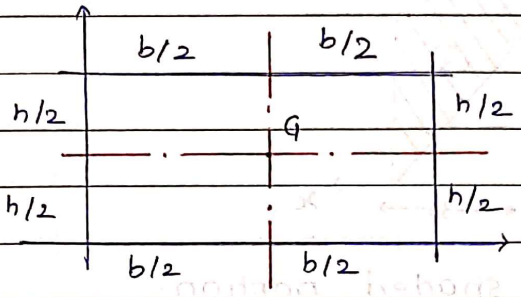
CENTROID (8M)

(1) Rectangle :

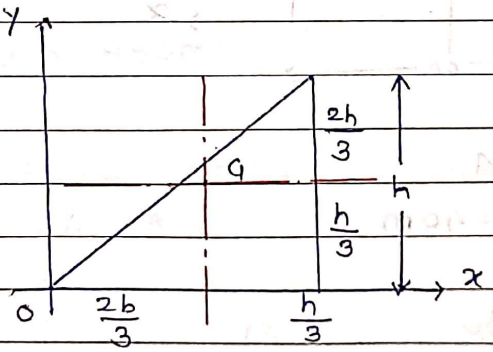


AOS → Axis of Symmetry

* b/2 & h/2 wrt 90° corner
* x and y wrt origin
* $x = b/2$ $y = h/2$

$A = b \times h$

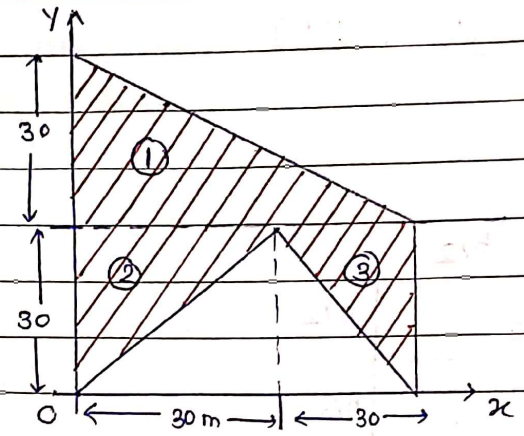
(2) Triangle :



* $\frac{b}{3}$ and $\frac{h}{3}$ wrt 90° corner
* x and y wrt origin
* $x = \frac{2b}{3}$, $y = \frac{h}{3}$

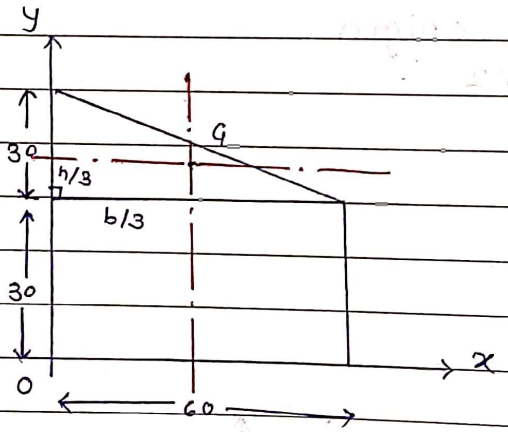
$A = \frac{1}{2} \times b \times h$

(Q)



Find centroid of shaded portion:

① Triangle:



$b = 60, \quad h = 30$

$x_1 = b/3 = 20M$

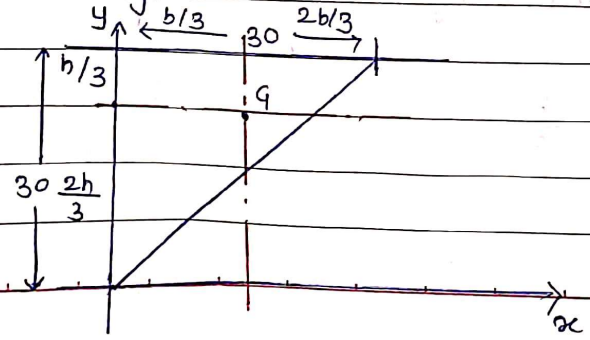
$y_1 = 30 + h/3 = 40m$

$A_1 = 1/2 \times b \times h$

$= 1/2 \times 60 \times 30$

$= 900 m^2$

② Triangle:



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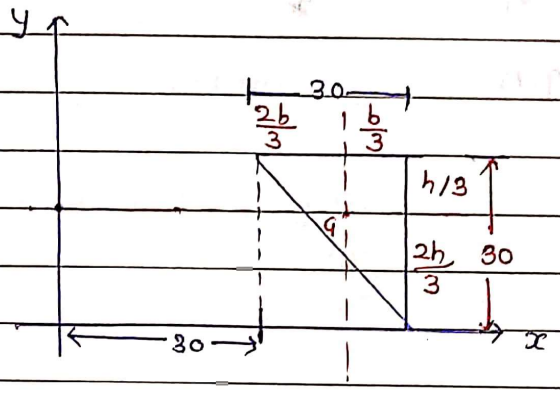
$b = 30$, $h = 30$

$x_2 = b/3 = 10\,m$

$y_2 = 2h/3 = 20\,m$

$A_2 = 1/2 \times b \times h = 450\,m^2$

(3) Triangle



$b = 30$, $h = 30$

$x_3 = \frac{30 + 2b}{3} = 50\,m$

$y_3 = \frac{2h}{3} = 20\,m$

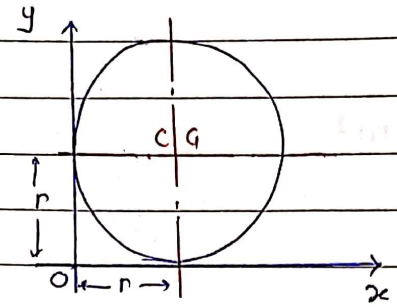
$A_2 = 1/2 \times b \times h = 450\,m^2$

$\bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3} = 25\,m$

$\bar{Y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = 30\,m$

$G(\bar{x}, \bar{y}) = (25, 30)$

(3) Circle :-



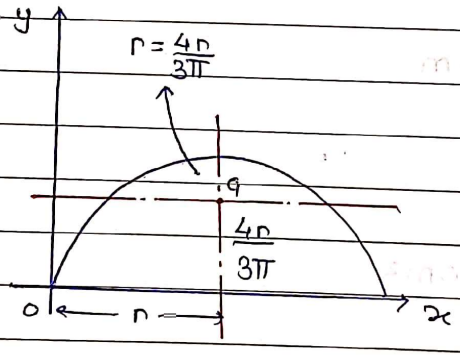
x and y wrt origin

$x = r$

$y = r$

$A = \pi r^2$

(4) Semi-circle :-



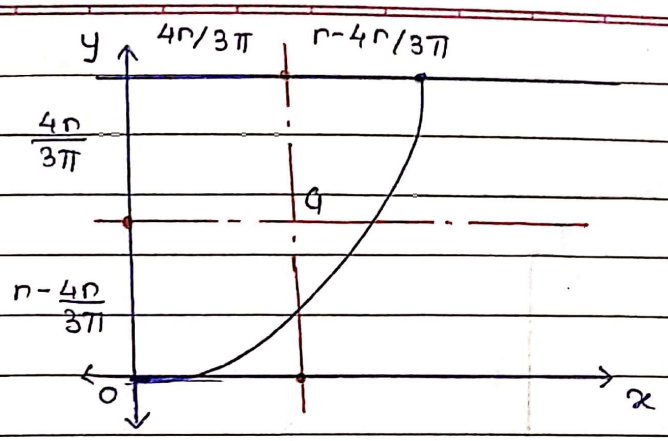
* $\frac{4r}{3\pi}$ wrt center

* x and y wrt origin

$x = r$, $y = \frac{4r}{3\pi}$

$A = \pi r^2/2$

(5) Quarter-circle



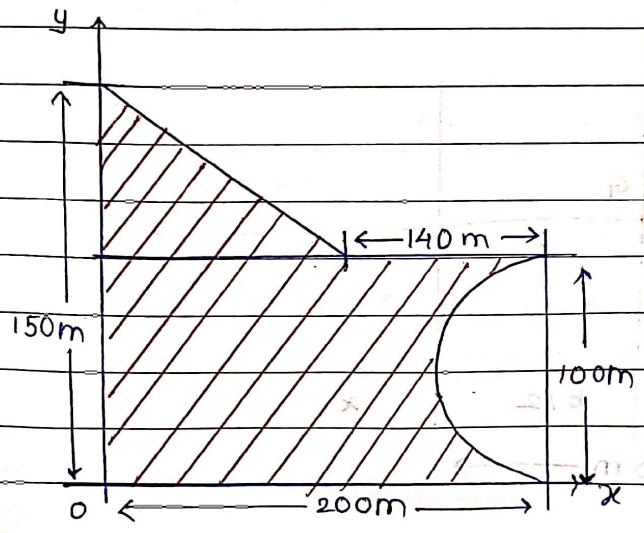
* $\frac{4r}{3\pi}$ wrt center

* x and y wrt origin

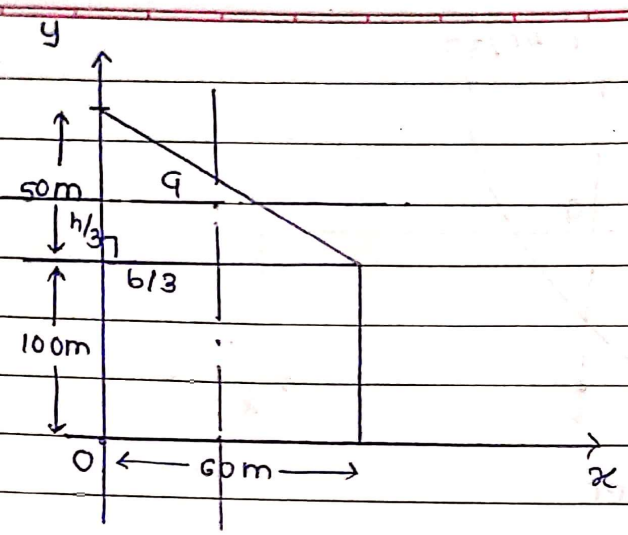
* $x = \frac{4r}{3\pi}$ $y = r - \frac{4r}{3\pi}$

$A = \frac{\pi r^2}{4}$

(Q) Find centroid of the shaded portion



① Triangle



$b = 60, h = 50$

$x_1 = b/3 = 20m$

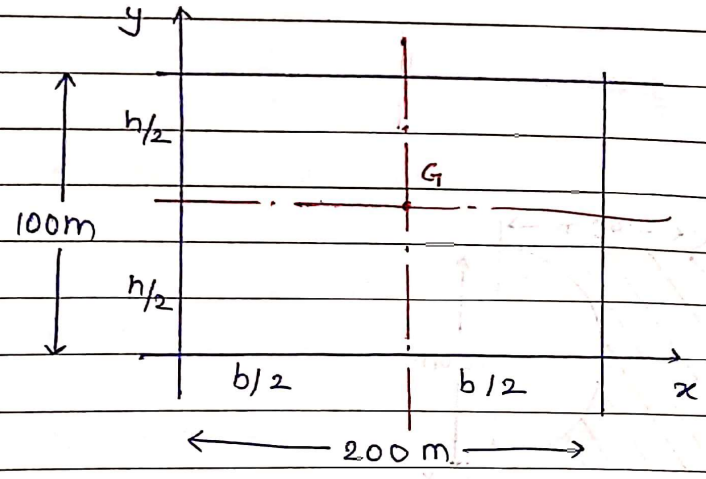
$y_1 = 100 + h/3 = 116.66\ m$

$A1 = 1/2 \times b \times h$

$= 1/2 \times 60 \times 50$

$= 1500 m^2$

(2) Rectangle



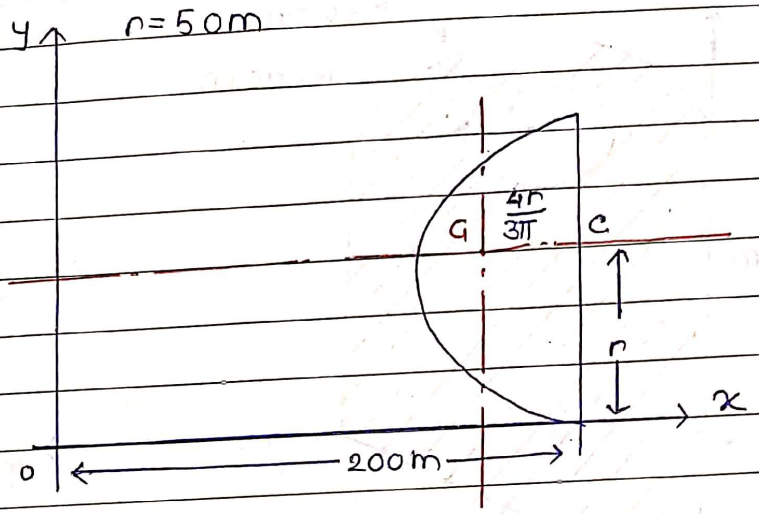
$b = 200, h = 100$

$x_2 = b/2 = 100m$

$y_2 = h/2 = 50\ m$

$A_2 = b \times h = 20000\ m^2$

③ Semicircle :



$\bar{x}_3 = 200 - \frac{4r}{3\pi} = 178.77\,m$

$y_3 = r = 50m$

$A_3 = \frac{\pi r^2}{2} = \frac{\pi \times 50^2}{2} = 3926.99\,m^2$

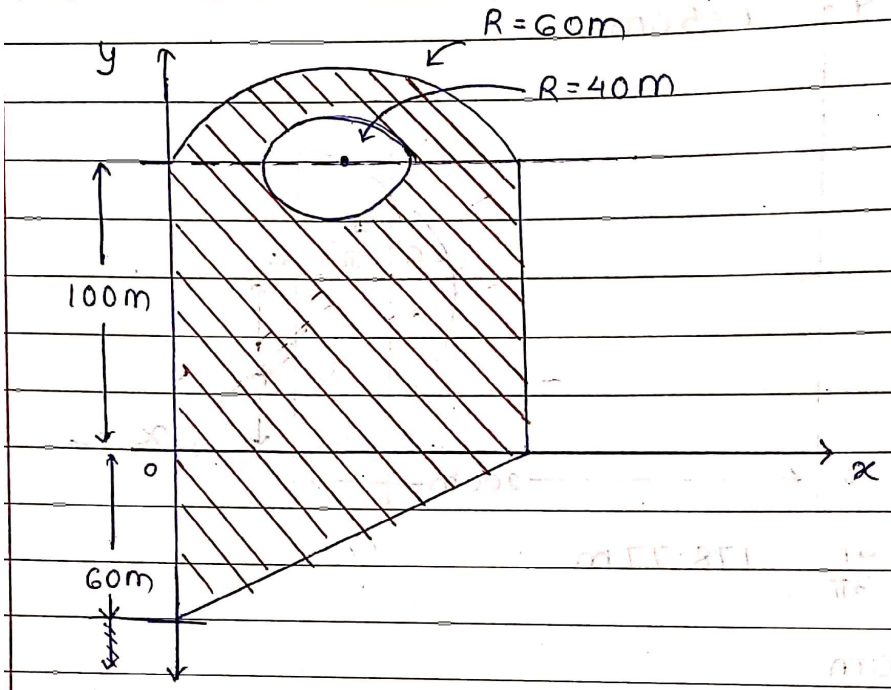
$\bar{x} = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 - A_3} = 75.56\,m$

$\bar{y} = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3} = 55.68\,m$

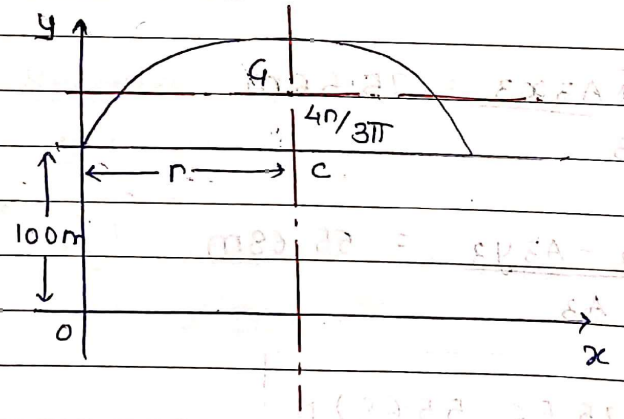
$\therefore G(\bar{x}, \bar{y}) = (75.56, 55.68)\,m$

Date

Find centroid of the shaded portion



① Semicircle



r = 60m

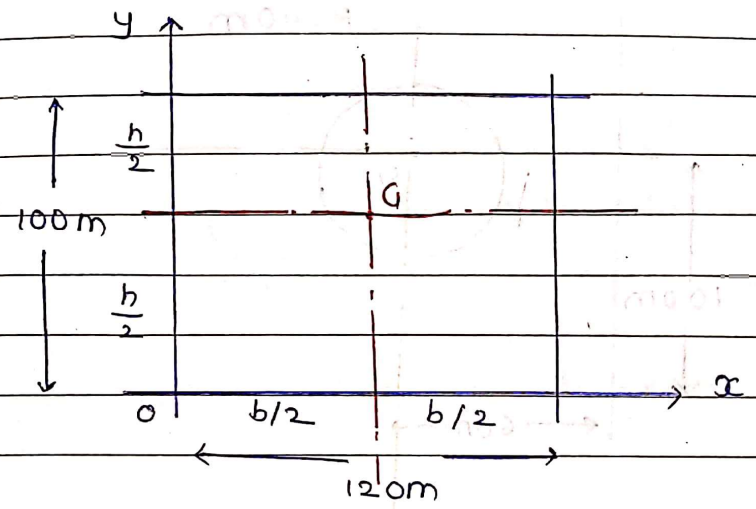
$x_1 = r = 60m$

$y_1 = 100 + 4r/3\pi = 125.47m$

$A_1 = \pi r^2/2 = \pi \times 60^2/2$

$= 5654.86 m^2$

(2) Rectangle



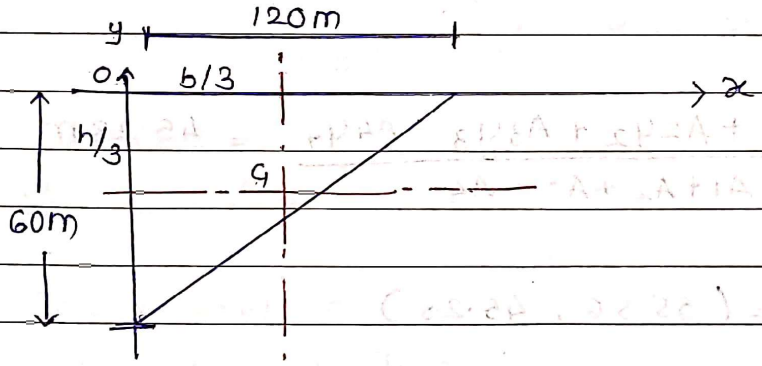
$b = 120, h = 100$

$x_2 = b/2 = 60m$

$y_2 = h/2 = 50m$

$A_2 = b \times h = 12000 m^2$

(3) Triangle



$b = 120\ m, h = 60m$

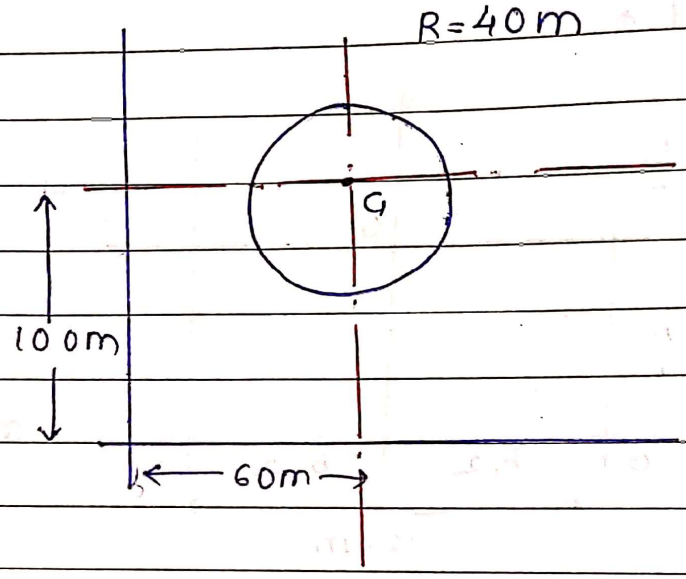
$x_3 = b/3 = 40\ m$

$y_3 = -h/3 = -20\ m$

$A_3 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 120 \times 60$

$= 3600 m^2$

④ Circle:



$x_4 = 60m$

$y_4 = 100m$

$A_4 = \pi r^2$

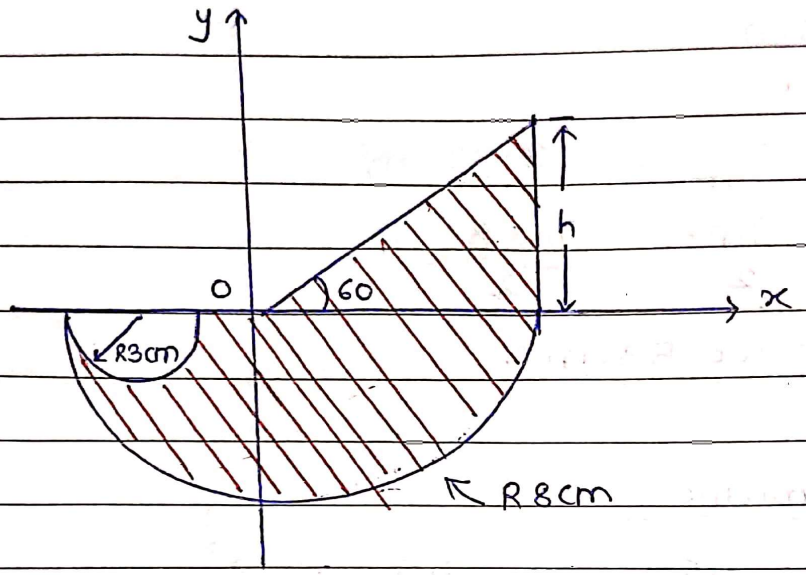
$= 5026.54 m^2$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 - A_4x_4}{A_1 + A_2 + A_3 - A_4} = 55.56\ m$$

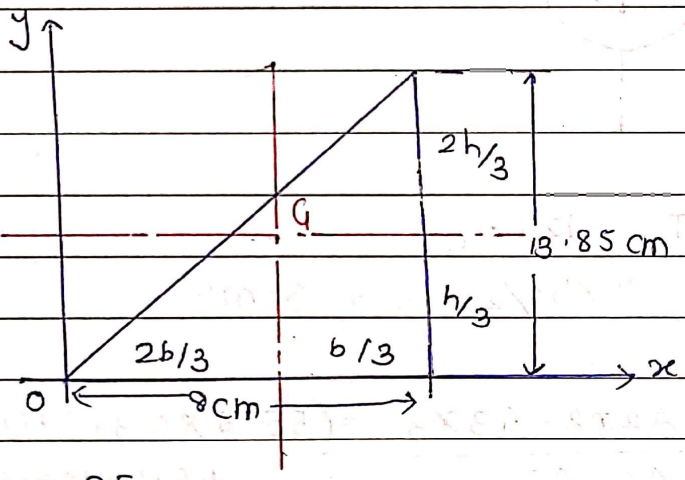
$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 - A_4y_4}{A_1 + A_2 + A_3 - A_4} = 45.28\ m$$

$(\bar{x}, \bar{y}) = (55.56, 45.28)$

(①) Find the centroid of the shaded portion



① Triangle



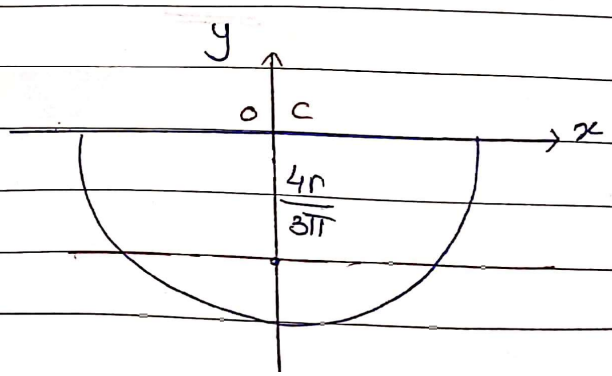
$b = 8, \quad h = 13.85$

$x_1 = \frac{2b}{3} = 5.33 \text{ cm}$

$y_1 = h/3 = 4.61 \text{ cm}$

$A_1 = 1/2 \times b \times h = 55.4 \text{ cm}^2$

② Semicircle :



$r = 8\,cm$

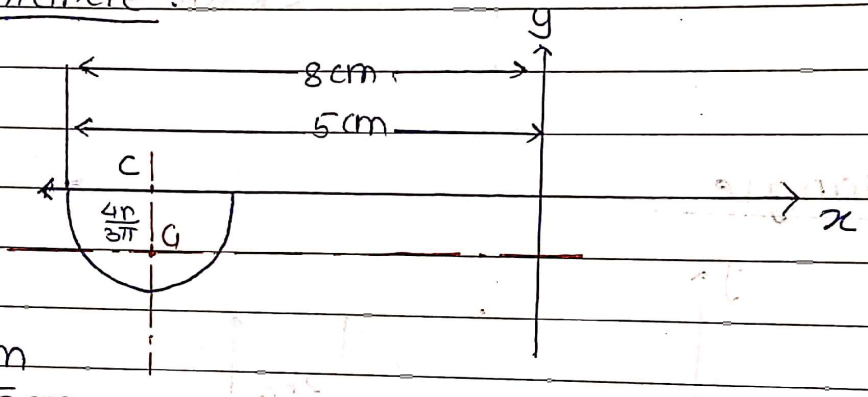
$x_2 = 0$

$y_2 = \frac{-4r}{3\pi} = -3.39\,cm$

$A_2 = \frac{\pi r^2}{2} = \frac{\pi \times 8^2}{2}$

$= 100.53\,cm^2$

③ Semicircle :



$r = 3\,cm$

$x_3 = -5\,cm$

$y_3 = -4r/3\pi = -1.27\,cm$

$A_3 = \pi r^2/2 = \pi \times 3^2/2 = 14.13\,cm^2$

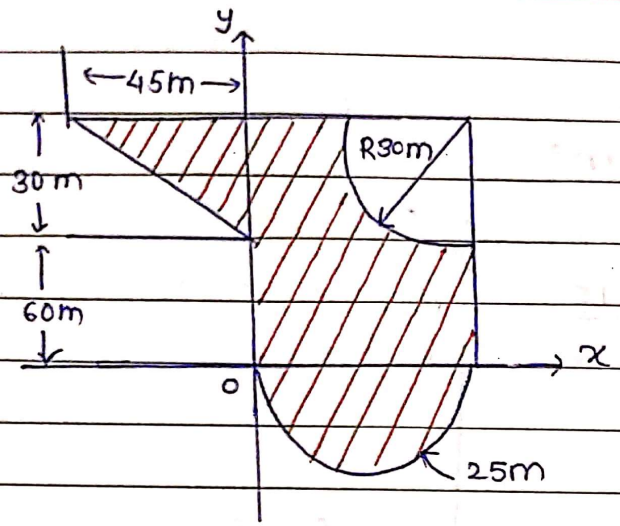
$\bar{x} = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 - A_3} = \frac{(55.4 \times 5.33) + (100.53 \times 0) - (14.13 \times -5)}{55.4 + 100.53 - 14.13}$

$\bar{x} = 2.58\,cm$

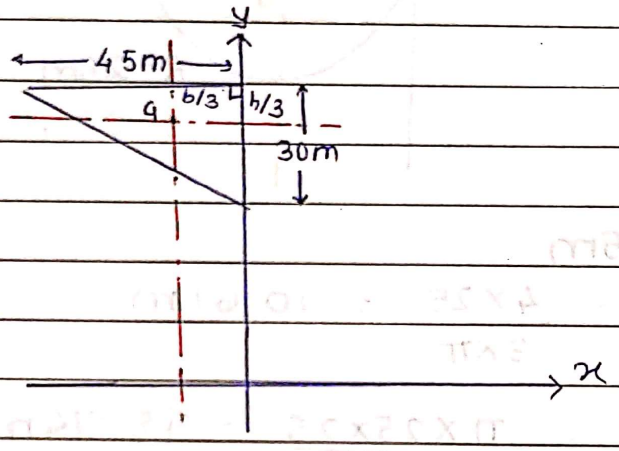
$\bar{y} = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3} = -0.47\,cm$

$G(\bar{x}, \bar{y}) = (2.58, -0.47)\,cm$

(Q)



① Triangle



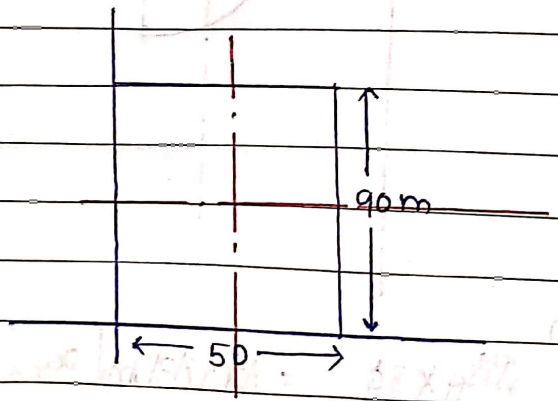
$b = 45m$, $h = 30m$

$x_1 = -b/3 = -45/3 = -15$

$y_1 = 90 - h/3 = 90 - 30/3 = 80$

$A_1 = 1/2 \times b \times h = 1/2 \times 45 \times 30 = 675\ m^2$

② Rectangle :



$b = 50m$, $h = 90m$

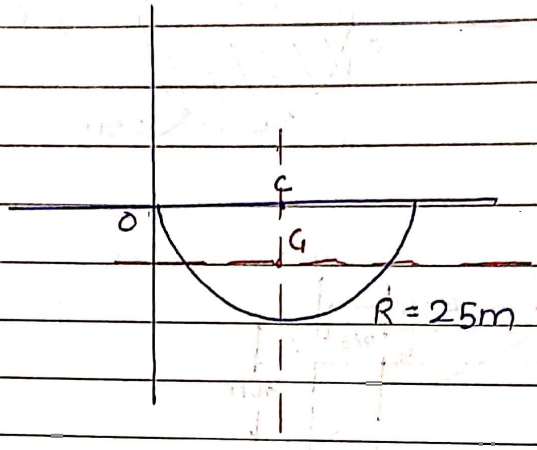
$x_2 = b/2 = 50/2 = 25m$, $y_2 = h/2 = 90/2 = 45m$

$A_2 = b \times h$

$= 50 \times 90$

$= 4500 m^2$

③ Semicircle



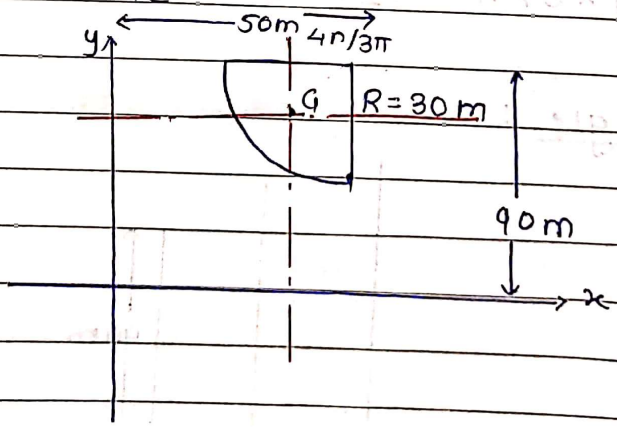
$r = 25m$

$x_3 = r = 25m$

$y_3 = \frac{-4r}{3\pi} = \frac{-4 \times 25}{3 \times \pi} = -10.61 m$

$A_3 = \frac{\pi r^2}{2} = \frac{\pi \times 25 \times 25}{2} = 981.74 m^2$

④ Quarter circle



$r = 30 m$

$x_4 = 50 - \frac{(4 \times 30)}{3 \times \pi} = 37.26 m$

$y_4 = 90 - \frac{4r}{3\pi} = 77.26 m$

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$A_4 = \frac{\pi r^2}{4} = 706.85$

$$\bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3 - A_4x_4}{A_1 + A_2 + A_3 - A_4}$$

$$\bar{Y} = \frac{A_1y_1 + A_2y_2 + A_3y_3 - A_4y_4}{A_1 + A_2 + A_3 - A_4}$$

$= 35.13$

$= 18.45$

$\bar{Y} = 35.13, \quad \bar{X} = 18.45$

9 $(\bar{X}, \bar{Y}) = (18.45, 35.13)$

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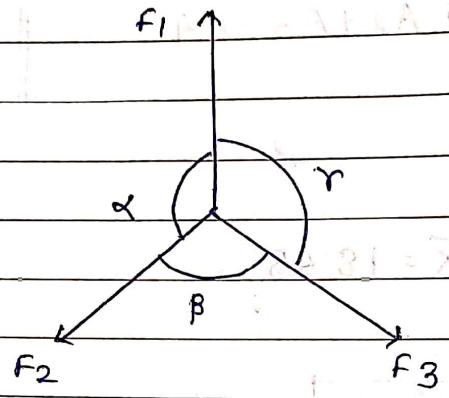
10/10/22

EQUILIBRIUM (20M)

Conditions on Equilibrium (COE)

[$\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$]

LAMI'S THEOREM



Conditions :

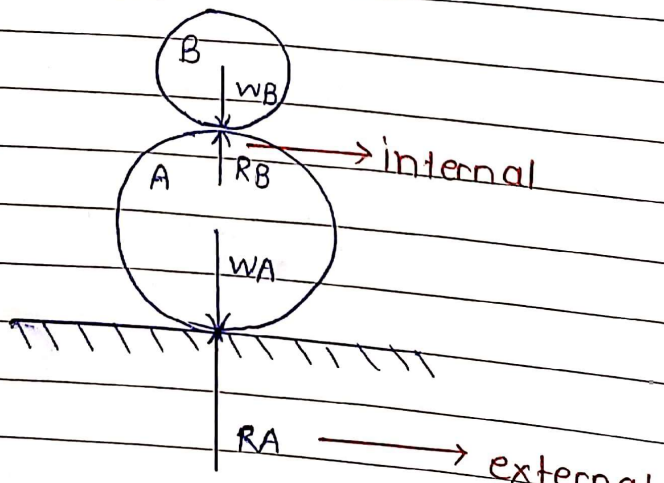
(i) There should be 3 forces

(ii) All 3 forces should be concurrent

(iii) All the 3 forces should be either incoming or outgoing.

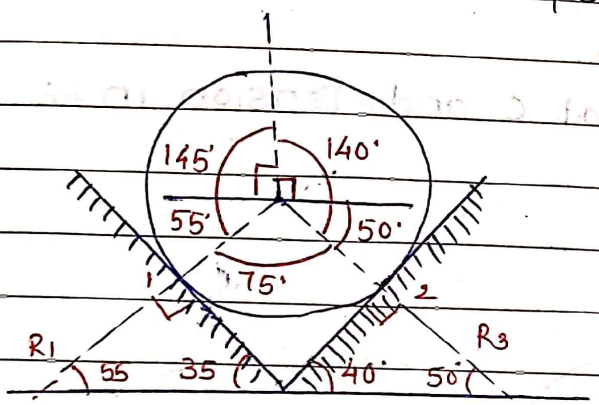
$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\gamma} = \frac{F_3}{\sin\alpha}$$

* Types of Reaction

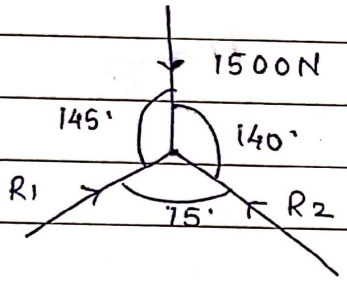


(Ex 1) Find Reactions at all contact points :

W = 1500N



FBD



Lami's Theorem

$$\frac{R_1}{\sin 140^\circ} = \frac{R_2}{\sin 145^\circ} = \frac{1500}{\sin 75^\circ}$$

$$\therefore R_1 = \frac{1500 \sin 140^\circ}{\sin 75^\circ} = 998.19\ N$$

$$\therefore R_2 = \frac{1500 \sin 145^\circ}{\sin 75^\circ} = 890.71\ N$$

Reaction: Towards Body
Tension: Away from Body

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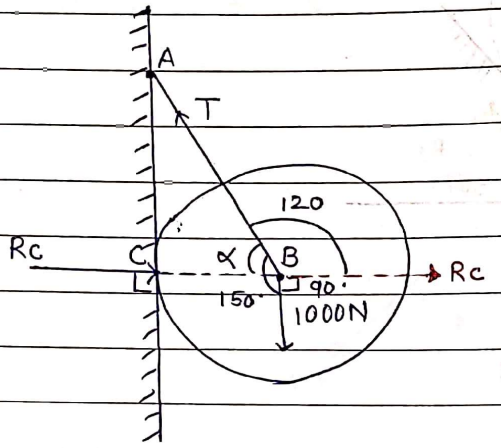
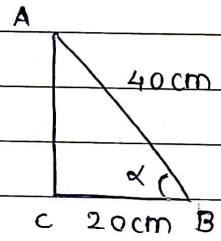
Q(2) W = 1000N, diameter = 40 cm, length of cable AB = 40cm.

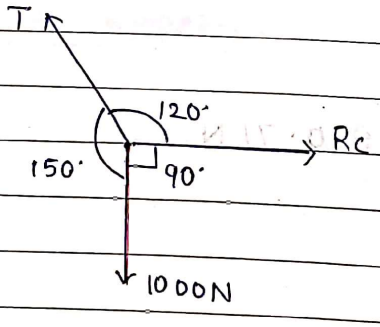
Find Reaction at C and Tension in AB





$\cos\alpha = \frac{20}{40}$ $\therefore \alpha = \cos^{-1}\left(\frac{20}{40}\right) = 60°$

F.B.D

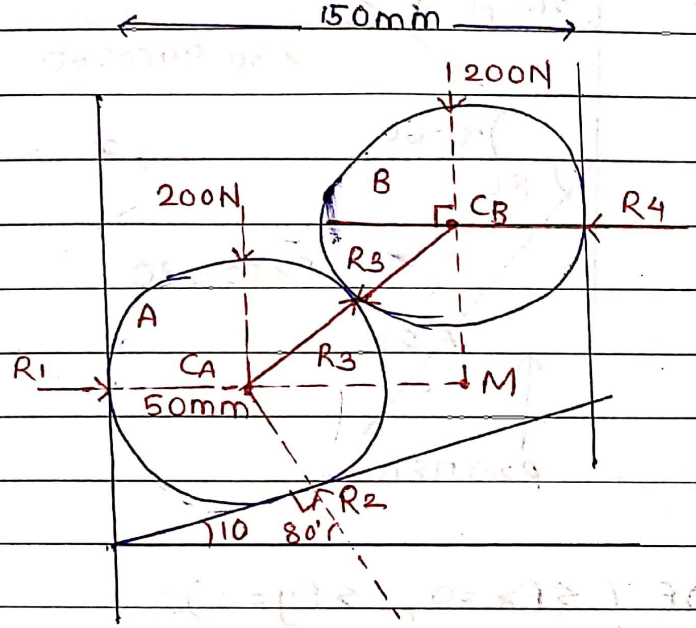


$\frac{T}{\sin 90} = \frac{Rc}{\sin 150} = \frac{1000}{\sin 120°}$

$T = \frac{1000 \sin 90°}{\sin 120°} = 1154.7N$

$Rc = \frac{1000 \sin 150°}{\sin 120°} = 577.35N$

Q(3) Two identical cylinders diameter 100mm and weight 200N. Find Reactions at all contact points.

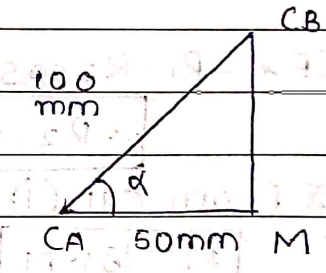


$CA \cdot CB = r_A + r_B = 100\,mm$

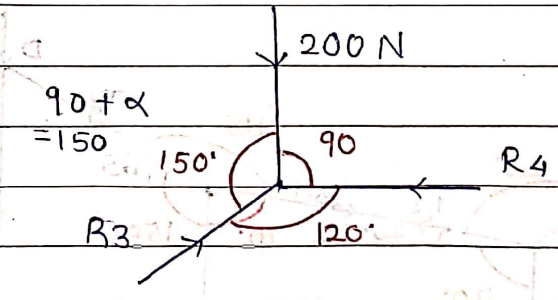
$CAM = 150 - r_A - r_B = 50\,mm$

$\cos\alpha = \frac{50}{100}$

$\boxed{\alpha = 60°}$



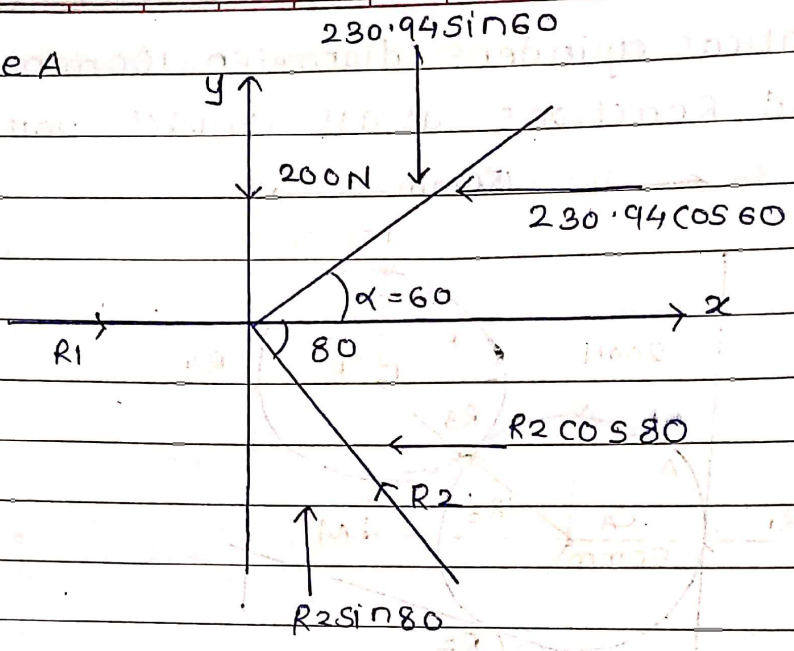
Sphere B



$\frac{R3}{\sin 90} = \frac{R4}{\sin 150} = \frac{200}{\sin 120}$ $\therefore R3 = 230.94N$

$R4 = 115.47N$

Sphere A



Applying COE ($\Sigma F_x = 0$, $\Sigma F_y = 0$)

$$\Sigma F_x = R_1 - R_2 \cos 80 - 230 \cdot 94 \cos 60 = 0 \quad \text{— (1)}$$

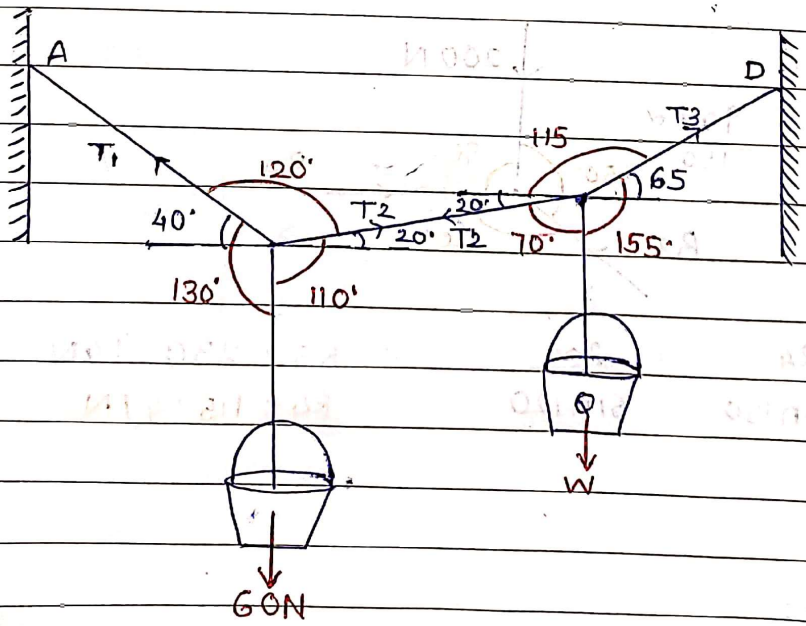
$$\therefore R_2 = 406 \cdot 17 \text{ N}$$

& From eqn (1)

$$R_1 = 186 \text{ N}$$

11|10|

(4)



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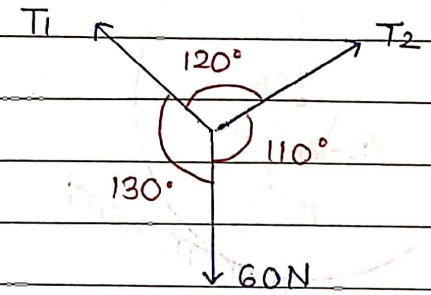
If cords suspend the two buckets in eqbm position shown in fig. Determine weight of bucket Q if bucket P has a weight of 60N [M-19]

⇒ Point B :



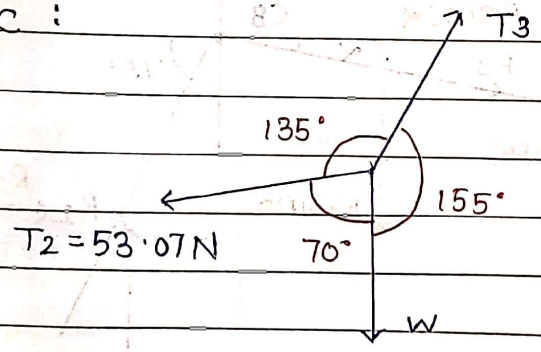
Lami's Theorem :

$$\frac{T_1}{\sin 110^\circ} = \frac{T_2}{\sin 130^\circ} = \frac{60}{\sin 120^\circ}$$

$T_1 = 65.1\,N$

$T_2 = 53.07\,N$

Point C :



$$\frac{T_2}{\sin 155} = \frac{T_3}{\sin 70} = \frac{W}{\sin 135}$$

$\therefore T_3 = 118\,N$

$W = 88.79\,N$

IF horizontal dimension is not given, α cannot be found, here consider full body and apply COE

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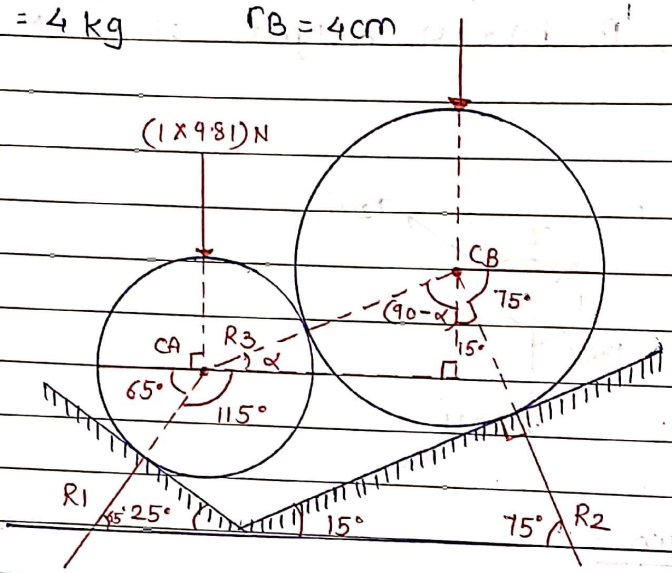
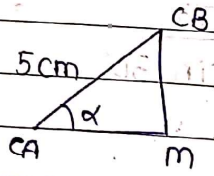
(3) ★ Find Reactions of all contact points

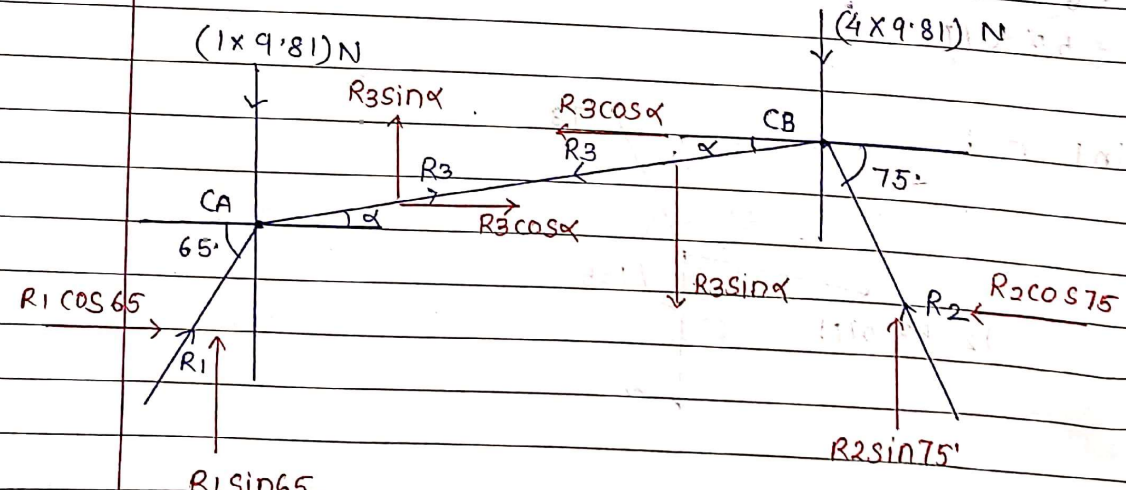
$W_A = 1kg$ $r_A = 1cm$ $(4\times9.81)N$

$W_B = 4kg$ $r_B = 4cm$



$CA\,CB = r_A + r_B = 5\,cm$





Applying C·O·F ($\Sigma F_x = 0$, $\Sigma F_y = 0$)

$\Sigma F_x = R_1\cos65 + R_3\cos\alpha - R_3\cos\alpha - R_2\cos75 = 0$

$\therefore R_1\cos65 - R_2\cos75 = 0$ ——— ①

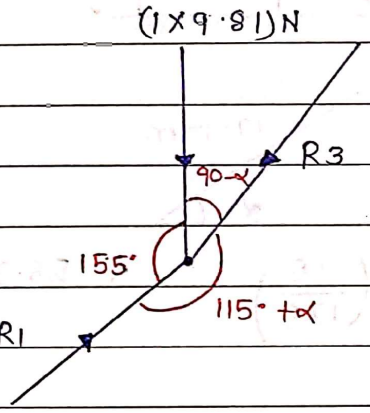
$\Sigma F_y = R_1 \sin 65° + R_3 \sin\alpha - R_3 \sin\alpha + R_2 \sin 75° - 1 \times 9.81 - 4 \times 9.81 = 0$

$R_1 \sin 65° + R_2 \sin 75° = 9.81 + (4 \times 8.1)$ ——— (2)

$R_1 = 19.75\ N$

$R_2 = 32.24\ N$

Sphere A :



By Lami's Theorem,

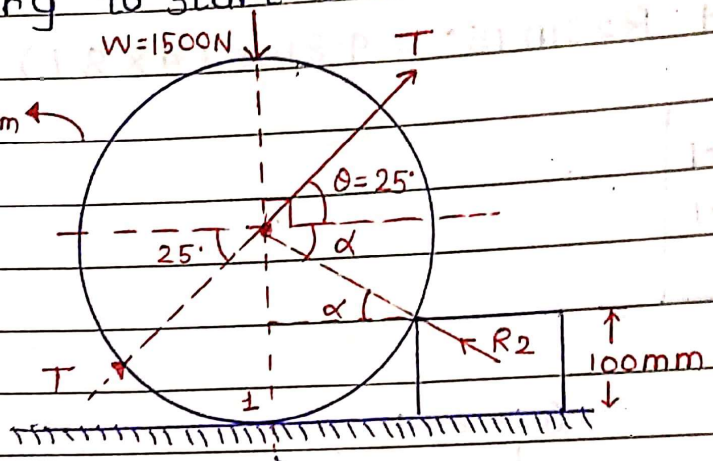
$$\frac{R_1 = 19.75}{\sin(90-\alpha)} = \frac{R_3}{\sin 155°} = \frac{9.81}{\sin(115°+\alpha)}$$

$$\therefore \frac{19.75}{\sin(90-\alpha)} = \frac{9.81}{\sin(115°+\alpha)}$$

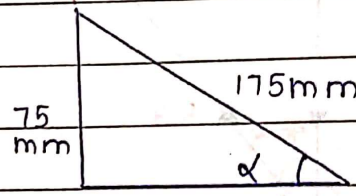
$\therefore \alpha = 10000124.1$

$$\therefore R_3 = \frac{9.81 \sin 155°}{\sin(115°+\alpha)} = R_3 = 11.62\ N$$

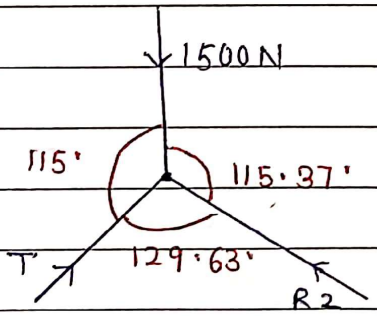
(4) The roller shown in fig is of 1500N. what force T is necessary to start over the block, if $\theta = 25°$



Radius = 175mm



$\sin\alpha = \frac{75}{175}$ $\therefore \alpha = \sin^{-1}\left(\frac{75}{175}\right)$ $\therefore \alpha = 25.37°$



$\frac{1500}{\sin 129.63} = \frac{R_2}{\sin 115°} = \frac{T}{\sin 115.37°}$

$R_2 = 1765.12N$

$T = 1759.77N$

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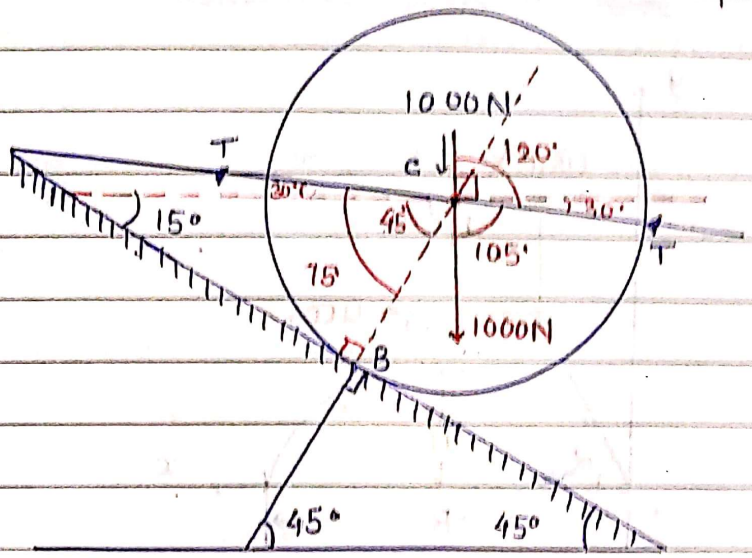
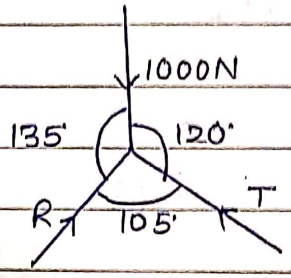
⑤ Find Tension in Cable and reaction at point of contact





Lami's Theorem

$$\frac{1000}{\sin 105^\circ} = \frac{R}{\sin 120^\circ} = \frac{T}{\sin 135^\circ}$$

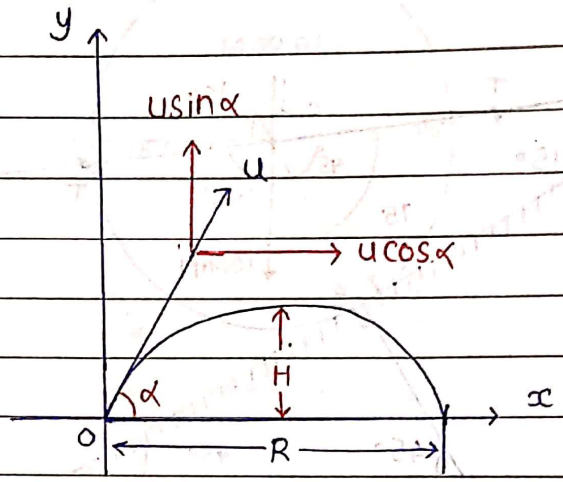
$R = 896.57\ N$

$T = 732.05\ N$

11/10/22

KINEMATICS OF PARTICLE (24 M)

PROJECTILE MOTION (6M)



- **Equation of path :**

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

- $\alpha \longrightarrow$ wrt x axis

- $s = ut + \frac{1}{2}at^2$
  $v^2 = u^2 + 2as$
  $v = u + at$

[when starting point and ending point of projectile are not in same level we cannot use H, T, R formulas. Here, we use these 3 formulas (HM, VM calculation]

- $H = \frac{u^2\sin^2\alpha}{2g}$, $T = \frac{2u\sin\alpha}{g}$, $R = \frac{u^2\sin 2\alpha}{g}$

- **IMP NOTE :**

(1) Time 't' is same for both H·M and V·M

(2) Acceleration for H·M = 0 , V·M = $-9.81\ m/s^2$

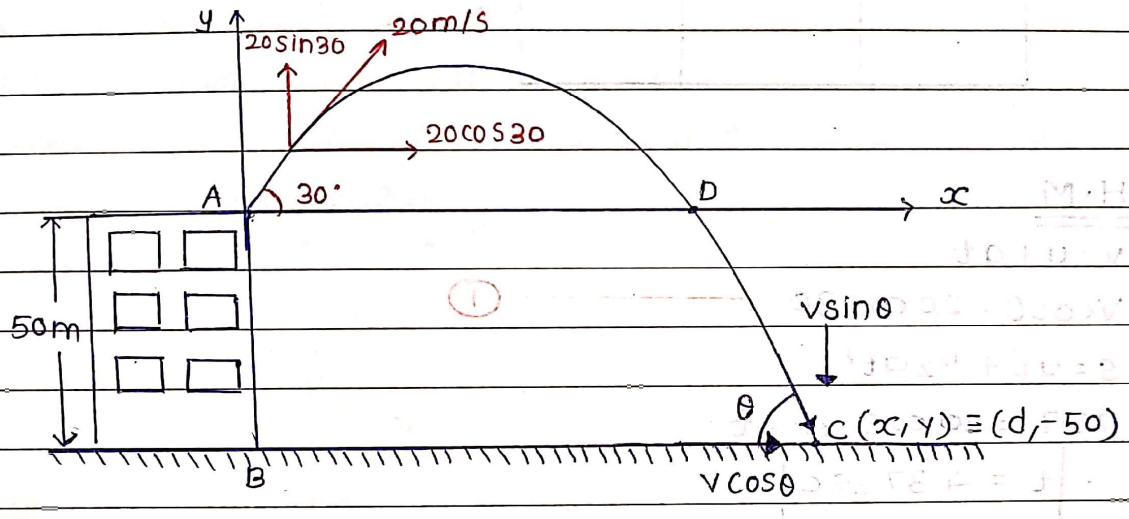
(1) A particle is projected from the top of a tower of height 50m with velocity 20m/s with angle of 30° to the horizontal

Find (i) Horizontal distance BC it travel from foot of tower

(ii) Velocity

(iii) Time

(iv) Max height



Soln: $x = d$

$y = -50m$

$u = 20m/s$

$\alpha = 30°$

Eqn of path,

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$\therefore -50 = d\tan 30° - \frac{9.81 \times d^2}{2 \times 20^2 \times \cos^2(30)}$$

$$\therefore \boxed{d = 75.7m}$$

Date

	H·M	V·M
u	$20\cos 30$	$20\sin 30$
v	$v\cos\theta$	$-v\sin\theta$
s	75·7	−50
a	0	−9·81
t	t	t

H·M

$v = u + at$

$v\cos\theta = 20\cos 30$ ——— (1)

$s = ut + \frac{1}{2}at^2$

$75.7 = 20\cos 30 \times t$

$\therefore$ **$t = 4.37$ sec**

V·M

$v = u + at$

$-v\sin\theta = 20\sin 30 - (9.81 \times 4.37)$

$\therefore -v\sin\theta = -32.86$

$\therefore v\sin\theta = 32.86$ ——— (2)

Eqn (2) ÷ (1)

$\frac{v\sin\theta}{v\cos\theta} = \frac{32.86}{20\cos 30}$ $\therefore$ **$\theta = 62.2°$**

From eqn (1)

$v = 37.13$ m/s

Trajectory A to D,

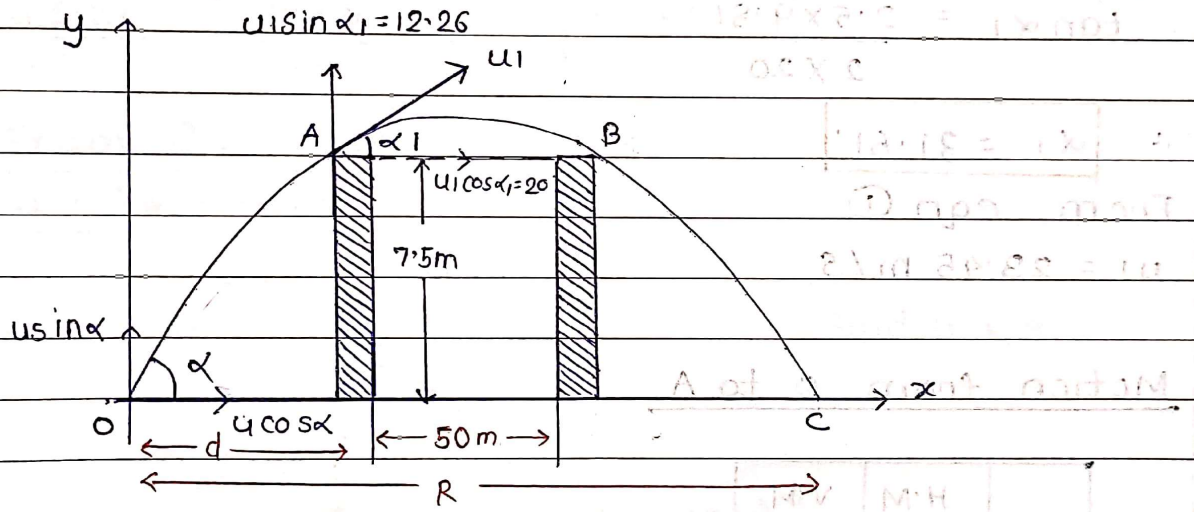
Date

$$H = \frac{u^2 \sin^2\alpha}{2g} = \frac{(20)^2 \times \sin^2(30)}{2 \times 9.81}$$

$\therefore H = 5.09\,m$

$\therefore$ Max height $= 50 + 5.09 = \underline{\underline{55.09\,m}}$

(2) An object is projected so that it just clears two obstacles each of 7.5m height, which are situated 50m from each other. If the time passing b/w obstacles is 2.5 sec. Calculate complete range.



Ans: Motion from A to B,

$R = 50\,m$, $T = 2.5\,sec$

$$R = \frac{u^2 \sin 2\alpha}{g}$$

$$\therefore 50 = \frac{u_1^2 \sin 2\alpha_1}{9.81}$$

$$\therefore 50 \times 9.81 = u_1^2 \sin 2\alpha_1$$

$$\therefore \frac{50 \times 9.81}{2} = u_1 \sin\alpha_1 \, u_1 \cos\alpha_1$$

$$\therefore \frac{50 \times 9.81}{2} = \frac{2.5 \times 9.81}{2} \cdot u_1 \cos\alpha_1$$

$\therefore 20 = u_1 \cos\alpha_1$ ——— (1)

$$T = \frac{2u\sin\alpha}{g}$$

$$2.5 = \frac{2u_1\sin\alpha_1}{9.81}$$

$$\therefore \frac{2.5 \times 9.81}{2} = u_1\sin\alpha_1 \quad \text{——— (2)}$$

$$\therefore u_1\sin\alpha_1 = 12.25$$

Eqn (2) ÷ (1)

$$\frac{u_1\sin\alpha_1}{u_1\cos\alpha_1} = \frac{2.5 \times 9.81 / 2}{20}$$

$$\therefore \tan\alpha_1 = \frac{2.5 \times 9.81}{2 \times 20}$$

$$\therefore \boxed{\alpha_1 = 31.51^\circ}$$

From eqn (1)

$u_1 = 23.45$ m/s

Motion from O to A

	H.M	V.M
u	$u\cos\alpha$	$u\sin\alpha$
v	20	12.26
s	d	7.5
a	0	-9.81
t	t	t

HM:

$v = u + at$

$20 = u\cos\alpha$ ——— (1)

$s = ut + \frac{1}{2}at^2$

$d = u\cos\alpha \cdot t$ ——— (2)

Date

V·M

$v = u + at$

$12.26 = u\sin\alpha - 9.81 \times t$ ——— (3)

$s = ut + 1/2\ at^2$

$7.5 = u\sin\alpha \cdot t - 1/2 \times 9.81 \times t^2$ ——— (4)

$v^2 = u^2 + 2as$

$(12.26)^2 = u^2\sin^2\alpha - 2 \times 9.81 \times 7.5$

$\therefore u\sin\alpha = 17.24$ ——— (5)

Eqn (5) ÷ (1)

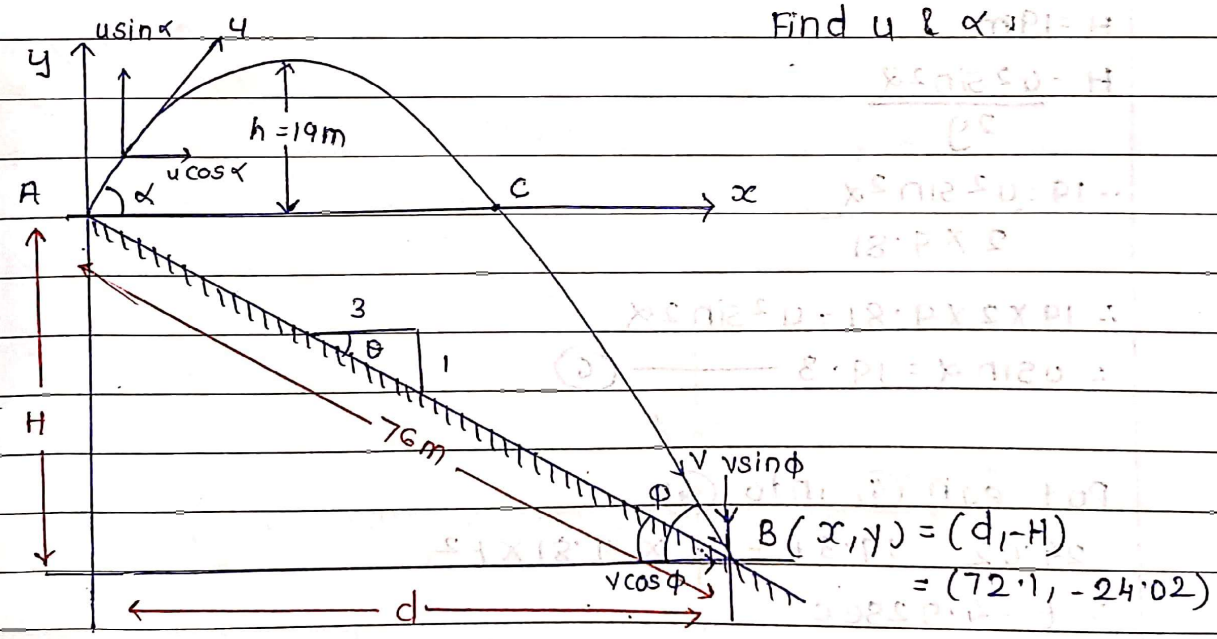
$\frac{u\sin\alpha}{u\cos\alpha} = \frac{17.24}{20}$ $\therefore$ $\boxed{\alpha = 40.76°}$

From (1),

$\boxed{u = 26.4\ m/s}$

(3)

Find u & α :



	H·M	V·M
u	$u\cos\alpha$	$u\sin\alpha$
v	$v\cos\phi$	$-v\sin\phi$
s	72.1	-24.05
a	0	-9.81
t	t	t

Ans :

$\tan\theta = 1/3$ $\therefore$ $\boxed{\theta = 18.43°}$

$\sin\theta = \frac{H}{76}$ $\therefore$ $H = 76\sin 18.43$

$\therefore H = 24.02m$

$\therefore \cos\beta = \frac{d}{76}$ $\therefore$ $\boxed{d = 72.1m}$

H.M :

$v = u + at$

$v\cos\phi = u\cos\alpha$ ——— (1)

$s = ut + \frac{1}{2}at^2$

$72.1 = u\cos\alpha \cdot t$ ——— (2)

V.M :

$v = u + at$

$-v\sin\phi = u\sin\alpha - 9.81 \times t$ ——— (3)

$s = ut + 1/2\,at^2$

$-24.02 = u\sin\alpha \cdot t - 1/2 \times 9.81 \times t^2$ ——— (4)

$v^2 = u^2 + 2as$

$(-v\sin\phi)^2 = (u\sin\alpha)^2 - 2 \times 9.81 \times (-24.02)$ ——— (5)

Motion from A to C,

$H = 19\,m$

$H = \frac{u^2\sin^2\alpha}{2g}$

$\therefore 19 = \frac{u^2\sin^2\alpha}{2 \times 9.81}$

$\therefore 19 \times 2 \times 9.81 = u^2\sin 2\alpha$

$\therefore u\sin\alpha = 19.3$ ——— (6)

Put eqn (6) into (4)

$-24.02 = 19.3t - 1/2 \times 9.81 \times t^2$

$\therefore t = 4.92\,sec$

From eqn (2)

$72.1 = u\cos\alpha \times 4.92$

$\therefore u\cos\alpha = 14.65$ ——— (7)

Eqn (6) ÷ (7)

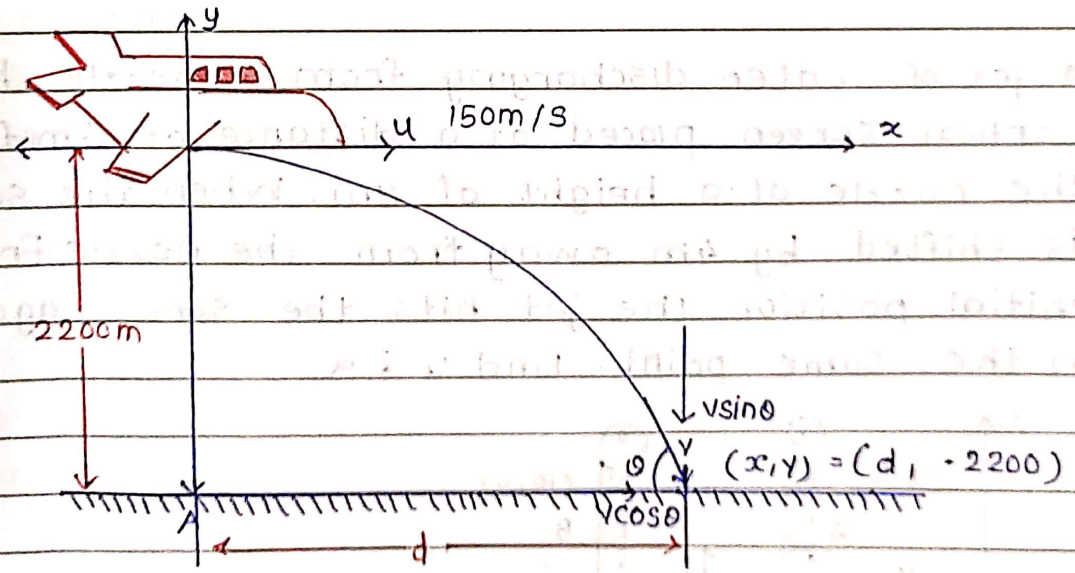
$$\frac{u\sin\alpha}{u\cos\alpha} = \frac{19.3}{14.05}$$

$\therefore \alpha = 52.79°$

From eqn (4)

$u = 24.23$ m/s

(4) An aeroplane is flying in Horizontal direction with velocity of 540 km/hr and at height of 2200m. When it is vertically above the point A on the ground, a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB. Find velocity at B and time taken to reach B.



Ans :

	HM	VM
u	150	0
v	vcosθ	-vsinθ
s	3176.75	-2200
a	0	-9.81
t	t	t

$\alpha = 0$, $u = 150$ m/s, $x = d$, $y = -2200$ m

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$-2200 = d\tan(0) - \frac{9.8 \times d^2}{2 \times 150^2 \times \cos^2(0)}$$

$\therefore$ $d = 3176.75$ m

H.M

$V = u + at$

$V\cos\theta = 150$ ——— (1)

$S = ut + \frac{1}{2}at^2$

$3176.75 = 150 \times t$

$\therefore t = 21.17 \text{ sec}$

V.M

$v = u + at$

$-V\sin\theta = 0 - 9.81 \times 21.17$

$V\sin\theta = 207.67$ ——— (2)

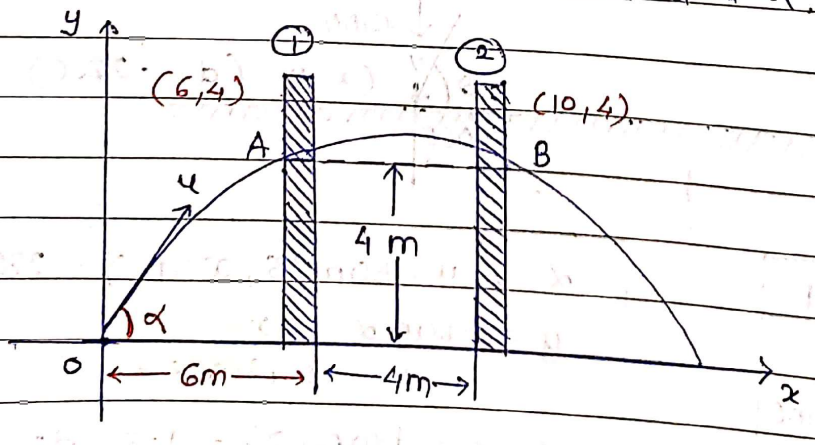
Eqn (1) ÷ (2)

$\theta = 54.15°$

From eqn (1)

$V = \frac{150}{\cos\theta} = 256.11 \text{ m/s}$

(5) A jet of water discharging from a nozzle hits a vertical screen placed at a distance of 6m from the nozzle at a height of 4m. When the screen is shifted by 4m away from the nozzle from its initial position the jet hits the screen again at the same point. Find u & α.



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Equation of path (Point A)

$$y = x\tan\alpha - \frac{9x^2}{2u^2\cos^2\alpha}$$

$$\therefore 4 = 6\tan\alpha - \frac{9.81 \times 6^2}{2u^2\cos^2\alpha}$$

$$\therefore 4 = 6\tan\alpha - \frac{176.5}{u^2\cos^2\alpha}$$

$\tan\alpha = p$, $\frac{1}{u^2\cos^2\alpha} = q$

$$\therefore 4 = 6p - 176.5q \quad \text{——— (1)}$$

$$\tan\alpha = 1.063$$

$$\therefore \boxed{\alpha = 46.74^\circ}$$

Point B

$$4 = \tan\alpha \times 10 - \frac{9.81 \times 10^2}{2u^2\cos^2\alpha}$$

$$\therefore 4 = 10\tan\alpha - \frac{490.5}{u^2\cos^2\alpha}$$

$$\therefore 4 = 10p - 490.5q \quad \text{——— (2)}$$

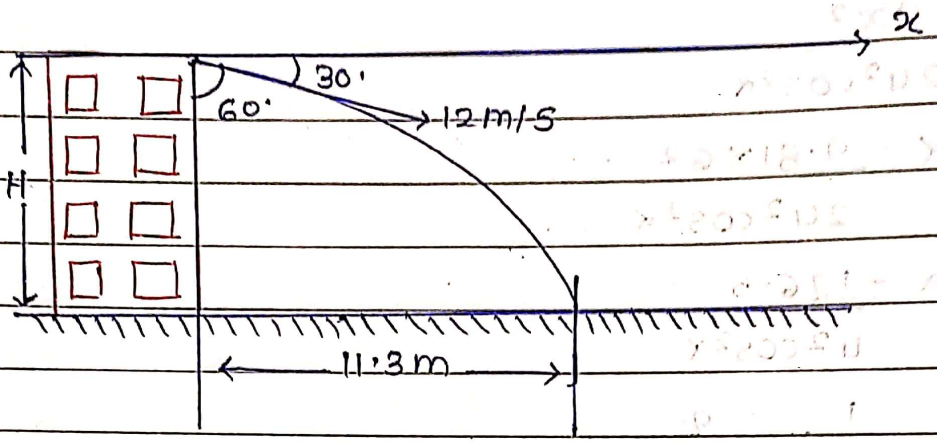
$$\therefore p = 1.063, \quad q = 0.013$$

$$\therefore \frac{1}{u^2\cos^2\alpha} = 0.013$$

$$\therefore u = \sqrt{\frac{1}{0.013 \times \cos^2(46.74^\circ)}}$$

$$\therefore \boxed{u = 12.789 \text{ m/s}}$$

(6) Find Height of Tower :



Ans : $u = 12\,m/s,\ \alpha = -30,\ x = 11.3\,m,\ y = -H$

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

$$\therefore -H = 11.3\tan(-30^\circ) - \frac{9.81 \times 11.3^2}{2 \times 12^2\cos^2(-30)^\circ}$$

$$\therefore \boxed{H = 12.32\,m}$$

Kinematics of Rigid Body (ICR) (12M)

Formulas:

(1) $v = r \times \omega$

Velocity $\rightarrow$ v (m/sec)

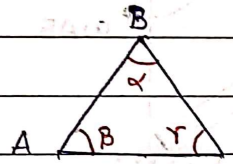
Distance between fixed point and moving point $\rightarrow$ r

Angular velocity $\rightarrow$ ω (rad/sec)

(2) $\omega = \frac{2\pi N}{60}$

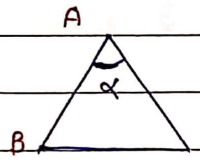
No. of revolutions $\rightarrow$ N (rpm)

(3) Sine rule:



$$\frac{AB}{\sin\gamma} = \frac{BC}{\sin\beta} = \frac{AC}{\sin\alpha}$$

(3) Cosine Rule:



$$BC = \sqrt{AB^2 + AC^2 - 2AB\,AC\cos\alpha}$$

* Instantaneous center of rotation:

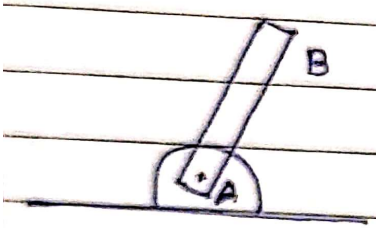
(1) It is a fixed point i.e. zero velocity point.

(2) It is an Imaginary Point

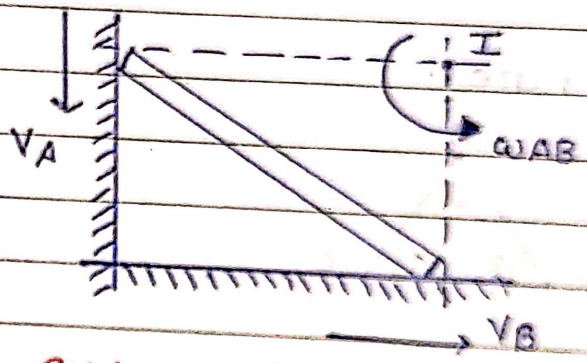
It may be lies within the body or outside the body

TYPES OF LINKS

- Rotating (ICR → Inside the Body)
- Sliding (ICR → Outside the body)

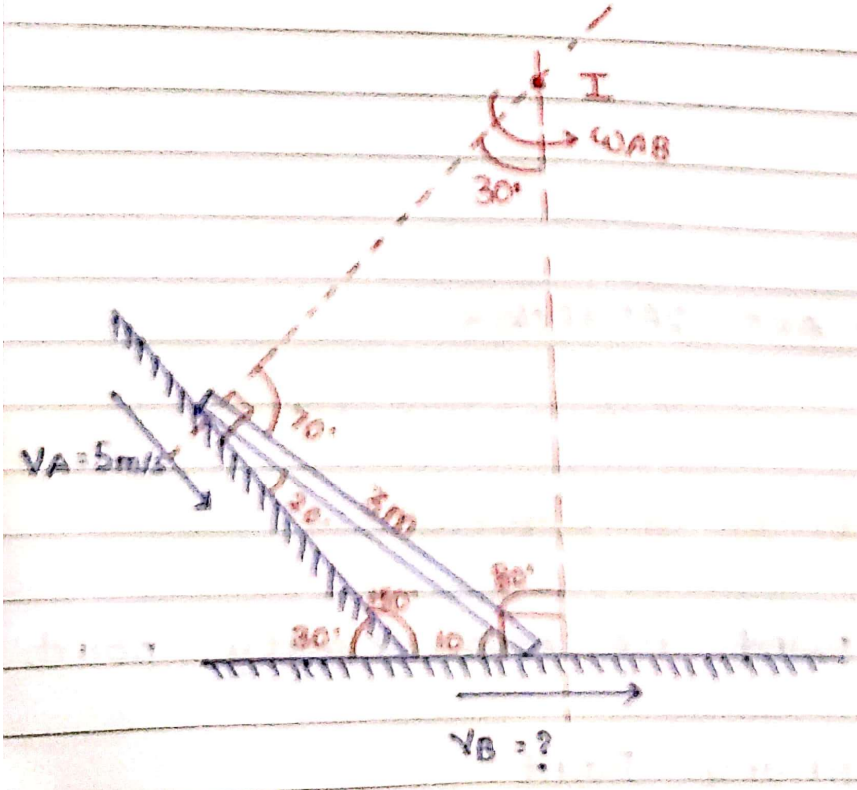


Rotating Link



Sliding Link

TYPE 1: ONE SLIDING LINK



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Find ω_{AB} and V_B

⇒ Link AB (sliding)

Pt I is an ICR

$V_A = AI \times \omega_{AB}$

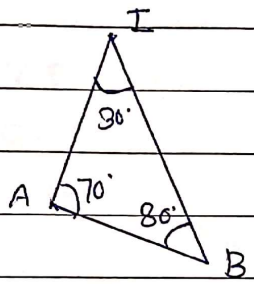
$5 = 5.9 \times \omega_{AB}$

$\omega_{AB} = 0.84$ r/sec

$V_B = BI \times \omega_{AB}$

$= 5.63 \times 0.84$

$V_B = 4.72$ m/sec

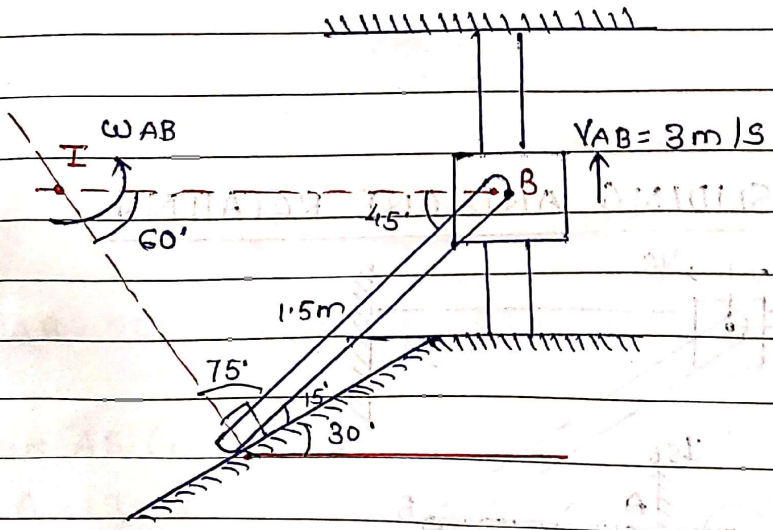


Sine Rule,

$$\frac{AI}{\sin 80^\circ} = \frac{BI}{\sin 70^\circ} = \frac{3}{\sin 30^\circ}$$

$$\therefore AI = \frac{3 \sin 80^\circ}{\sin 30^\circ} = 5.9\text{m}, \quad BI = \frac{3 \sin 70^\circ}{\sin 30^\circ} = 5.63\text{m}$$

(Q) Find ω_{AB} & V_A



No. of links = No. of ICR = No. of angular velocity

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Link AB (Sliding)

Pt I is an ICR

$V_B = BI \times \omega_{AB}$

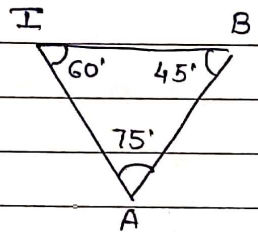
$3 = 1.67 \times \omega_{AB}$

$\omega_{AB} = 1.79$ m/s

$V_A = I \times \omega_{AB}$

$V_A = 1.22 \times 1.79$

$V_A = 2.18$ m/sec



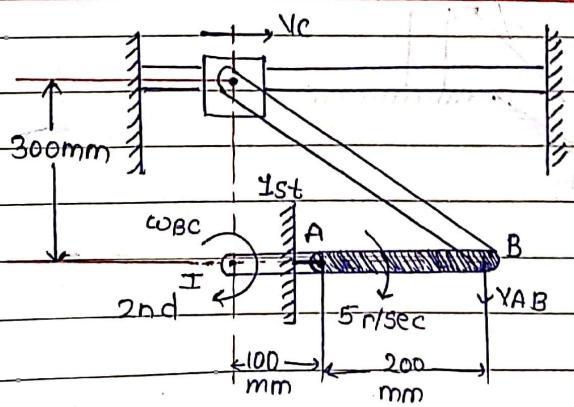
Sine Rule:

$$\frac{BI}{\sin 75^\circ} = \frac{AI}{\sin 45} = \frac{1.5}{\sin 60^\circ}$$

$$BI = \frac{1.5 \sin 75}{\sin 60^\circ} = 1.67 \text{ m}$$

$$AI = \frac{1.5 \sin 45}{\sin 60^\circ} = 1.22 \text{ m}$$

* TYPE 2: ONE SLIDING AND ONE ROTATING



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Link AB (Rotating)

Pt A is an ICR

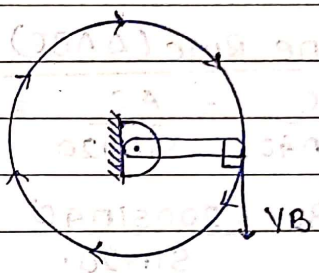
$V_A = 0$

$V_B = AB \times \omega_{AB}$

$= 200 \times 5$

$V_B = 1000 \text{ mm/sec}$

Pt I is an ICR



$V_B = BI \times \omega_{BC}$

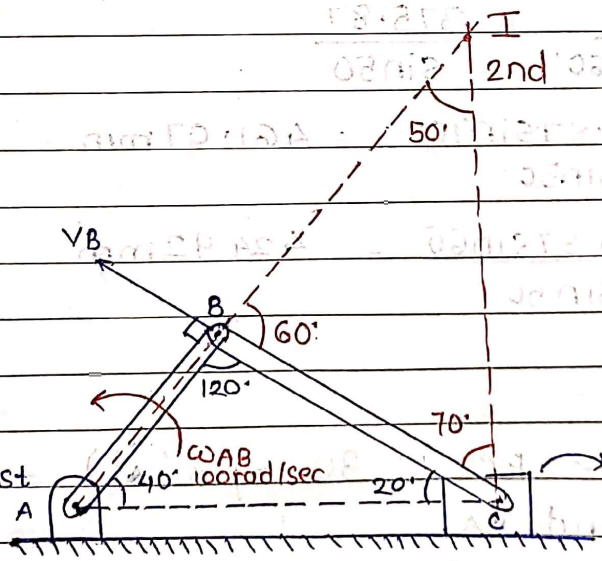
$1000 = 300 \times \omega_{BC}$

$\omega_{BC} = \frac{1000}{300}$ r/sec

$V_C = CI \times \omega_{BC}$

$= 300 \times \frac{10}{3}$ $\therefore V_C = 1000 \text{ m/sec}$

(Q)



(Q) AB = 200 mm, Find ω_{BC} and V_C

⇒ Link AB (rotating)

Pt A is an ICR

$V_A = 0$

$V_B = AB \times \omega_{AB}$

$= 200 \times 100 = 20000 \text{ mm/sec}$

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Link BC (Sliding)

Pt I is an ICR

$V_B = BI \times \omega_{BC}$

$20000 = 461.07 \times \omega_{BC}$

$\omega_{BC} = 43.37$ rad/sec

$V_C = CI \times \omega_{BC}$

$= 424.92 \times 43.37$

$= 18428.7$ mm/sec

Sine Rule (ΔABC)

$$\frac{BC}{\sin 40} = \frac{AC}{\sin 120} = \frac{200}{\sin 20}$$

$$\therefore BC = \frac{200 \sin 40}{\sin 20}$$

$BC = 375.87$ mm

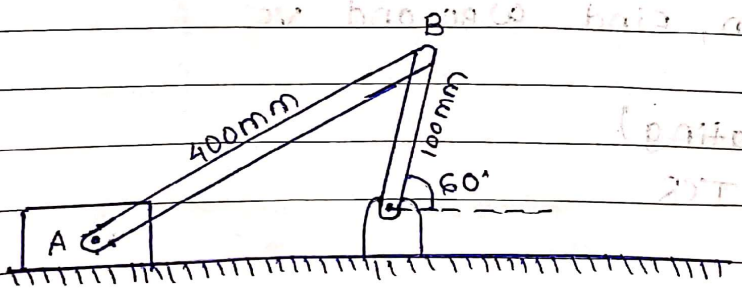
Sine Rule (ΔBIC)

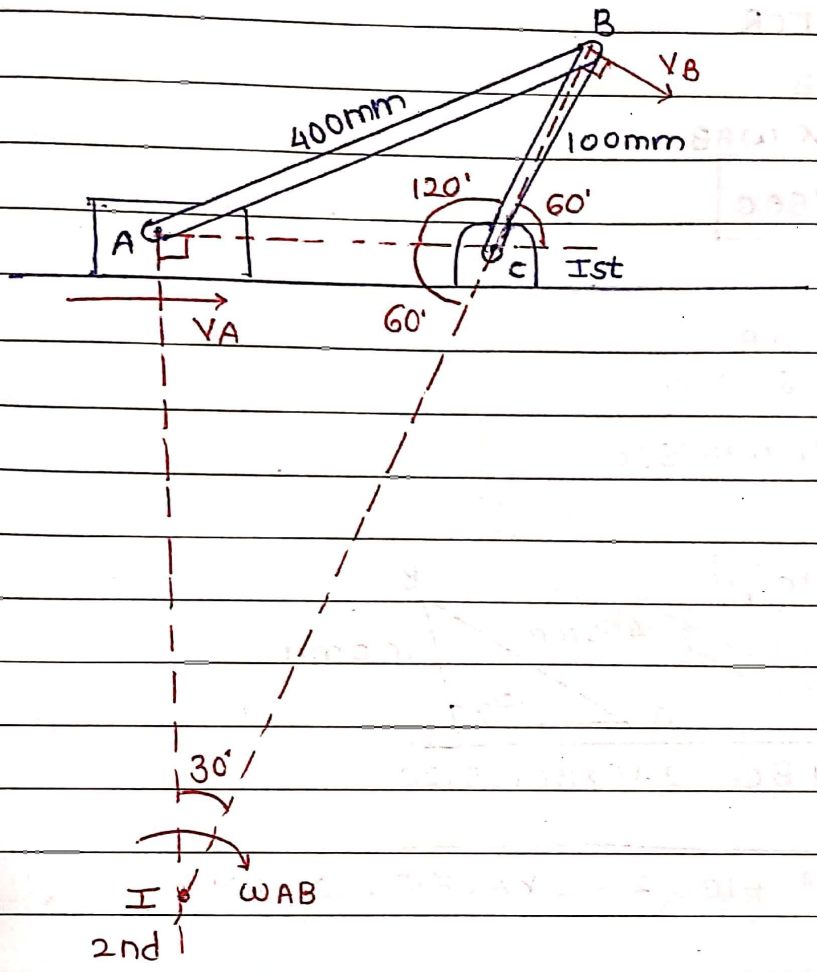
$$\frac{BI}{\sin 70} = \frac{CI}{\sin 60} = \frac{375.87}{\sin 50}$$

$$\therefore BI = \frac{375.87 \sin 70}{\sin 50} = 461.07 \text{ mm}$$

$$\therefore CI = \frac{375.87 \sin 60}{\sin 50} = 424.92 \text{ mm}$$

(3) Speed of link BC is 30 rpm (?)
Find ω_{AB} and V_A





⇒ Link (BC) [Rotating]

Pt C is an ICR

$V_C = 0$

$V_B = BC \times \omega_{BC}$

$= 100 \times 3.14$

$= 314$ mm/sec

Angular velocity (ω_{BC})

$\omega_{BC} = \frac{2\pi N}{60} = \frac{2\pi \times 30}{60}$

$\omega_{BC} = 3.14$ r/sec

Sine Rule (ΔAIC)

$\frac{AI}{\sin 60} = \frac{CI}{\sin 90} = \frac{340.51}{\sin 30^\circ}$

$\therefore AI = \frac{340.51 \sin 60}{\sin 30} = 589.78$

$\therefore CI = \frac{340.51 \sin 90^\circ}{\sin 30^\circ} = 681.02$

$BI = BC + CI = 100 + 681.02$

$= 781.02$

Link AB (Sliding)

Pt I is on ICR

Pt I is an ICR

$V_B = BI \times \omega_{AB}$

$314 = 781.02 \times \omega_{AB}$

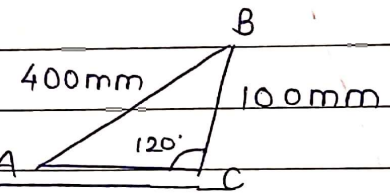
$\boxed{\omega_{AB} = 0.4 \text{ rad/sec}}$

$V_A = AI \times \omega_{AB}$

$= 589.78 \times 0.4$

$V_A = 235.91 \text{ mm/sec}$

Cosine Rule,



$AB = \sqrt{AC^2 + BC^2 - 2AC \times BC \cos 120°}$

$400 = \sqrt{AC^2 + 100^2 - 2 \times AC \times 100 \cos 120°}$

$\therefore \boxed{AC = 340.51 \text{ mm}}$

$\therefore 400^2 = AC^2 + 100^2 - 2 \times AC \times 100 \cos 120°$

$\therefore 0 = AC^2 - 2 \times 100 \cos 120° \times AC + 100^2 - 400^2$

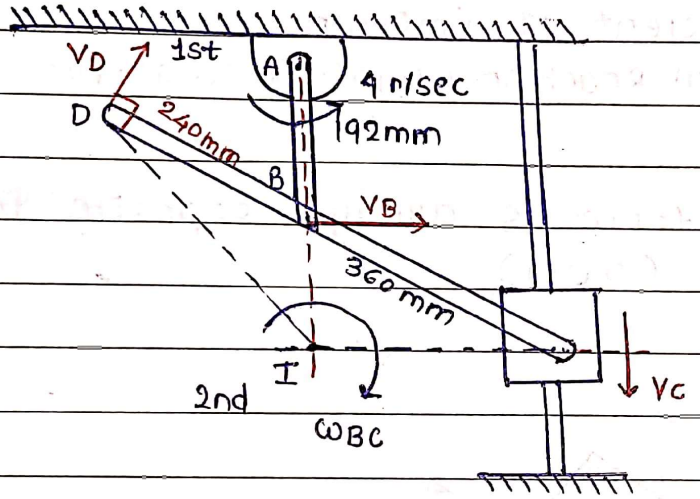
$AC = -440.51$ ✗ $\qquad AC = 340.51$ ✓

$\therefore \boxed{AC = 340.51 \text{ mm}}$

(3) TYPE 3:

ONE SLIDING AND ONE ROTATING AND ONE FREE END

(Q) Find V_C and V_D



⇒ **Link AB (Rotating)**

Pt A is an ICR

$V_A = 0$

$V_B = AB \times \omega_{AB}$

$= 192 \times 4$

$V_B = 768$ mm/sec

$\sin 30^\circ = \frac{BI}{360}$

$BI = 180$ mm

$\cos 30^\circ = \frac{CI}{360}$

$CI = 311.76$ mm

Link BC (Sliding)

Pt I is an ICR

$V_B = BI \times \omega_{BC}$

$768 = 180 \times \omega_{BC}$

$\omega_{BC} = 4.26$ r/sec

$V_C = CI$

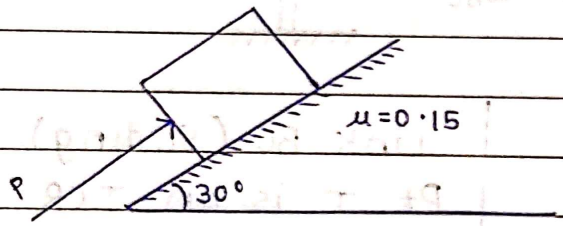
FRICTION (20m)

$$F = \mu N$$

F → Frictional Force
μ → coefficient of Friction
N → Normal Reaction / Surface Reaction

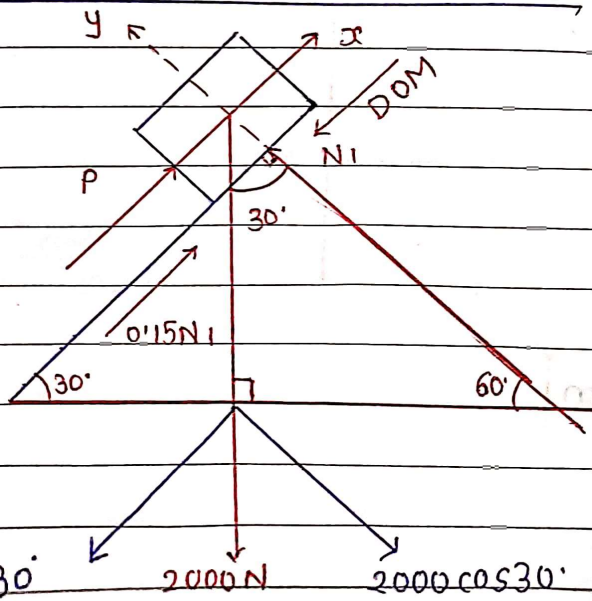
Imp: Frictional force is always opposite to the direction of motion (DOM)

(Q)



A block of weight 2000N is kept on a rough inclined surface. Find out the range of P for which the block will be in equilibrium.

⇒ Case I: P → To stop the motion



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Applying COE ($\Sigma F_x = 0$, $\Sigma F_y = 0$)

$\Sigma F_x = P + 0.15N_1 - 2000\sin 30^\circ = 0$

$\Sigma F_y = N_1 - 2000\cos 30^\circ = 0$

$N_1 = 1732.05\ N$

From eqn ①

$P = 740.19N$

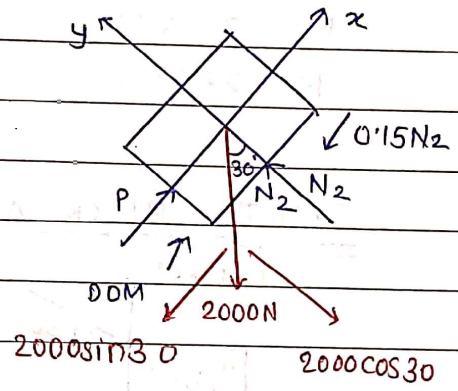
Case 2 :- P → To start the motion

$\Sigma F_y = N_2 - 2000\cos 30^\circ = 0$

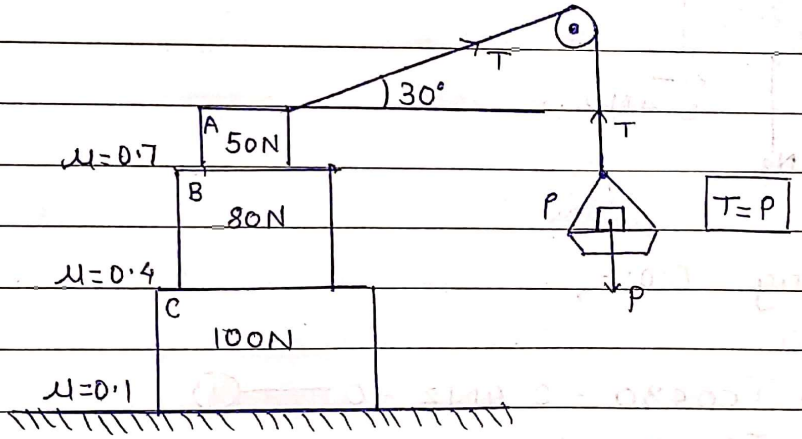
$N_2 = 1732.05\ N$

$\Sigma F_x = P - 0.15N_2 - 2000\sin 30^\circ = 0$

$\boxed{P = 1259.8\ N}$

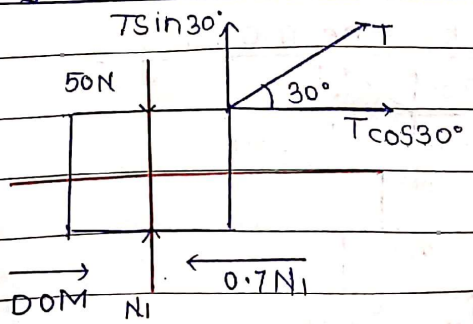


(Q)



Find maximum value of weight P which move the blocks / block

⇒ Case I: only BLOCK A is moving



Applying COE,

$\Sigma F_x = T\cos 30° - 0.7N_1 = 0$ ——— (1)

$\Sigma F_y = T\sin 30° + N_1 - 50 = 0$

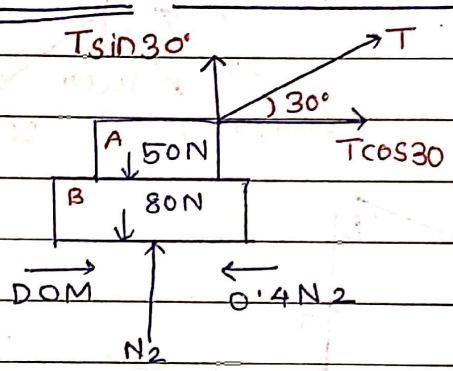
$T\sin 30° + N_1 = 50$ ——— (2)

$T = 28.78\ N$

$N_1 = 35.6\ N$

$P = 28.78\ N$

Case II: Block A and B both are moving



Applying C.O.F,

$\Sigma F_x = 0$

$\therefore \Sigma F_x = T\cos 30 - 0.4N_2 = 0$ ——— (1)

$\therefore \Sigma F_y = T\sin 30 + N_2 - 50 - 80 = 0$

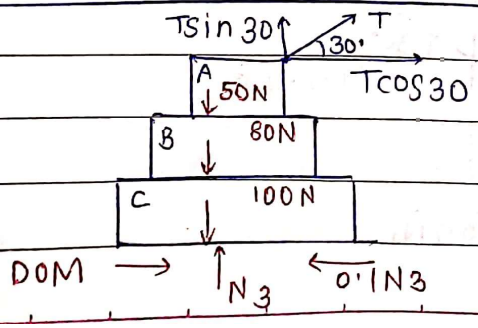
$T\sin 30° + N_2 = 130$ ——— (2)

$\therefore T = 48.77\ N$

$N_2 = 105.61\ N$

$P = 48.77\ N$

Case III : All blocks are moving



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Applying COE,

$\Sigma Fx = T\cos 30 - 0.1N_3 = 0$ ——— (1)

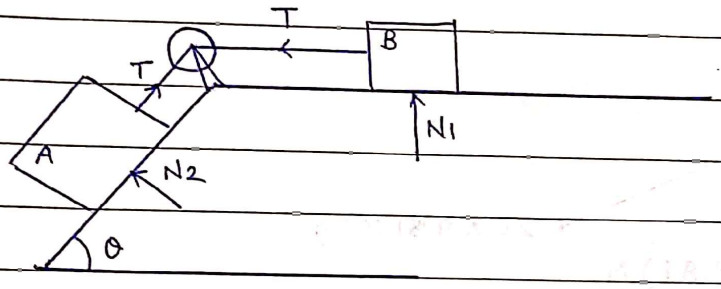
$\Sigma Fy = T\sin 30° + N_3 - 50 - 80 - 100 = 0$

$T\sin 30° + N_3 = 230$ ——— (2)

$T = 25.1N \qquad N_3 = 217.4N$

$P = 25.1N$

(Q)

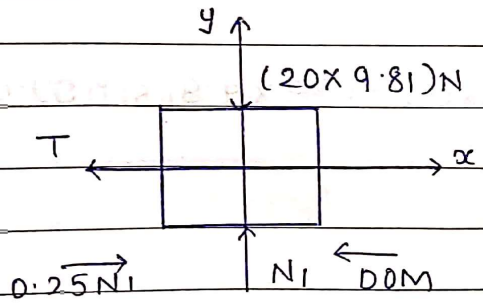


$m_A = 20kg$, $\mu_A = 0.25$

$m_B = 20kg$, $\mu_B = 0.25$

Find the value of θ if the blocks A and B shown in fig have impending motion

⇒ Block B



Applying C.O.E

$\Sigma Fx = -T + 0.25N_1 = 0$ ——— (1)

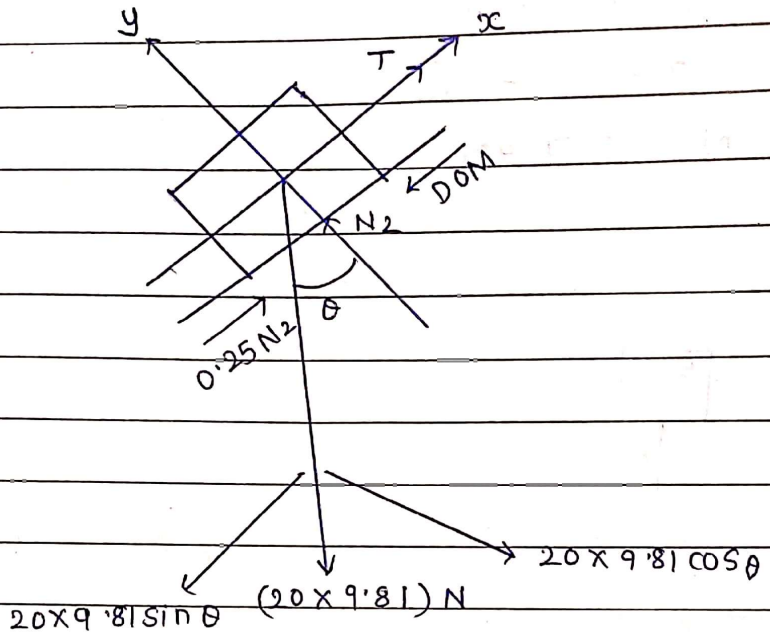
$\Sigma Fy = N_1 - 20 \times 9.81 = 0$

$N_1 = 196.2N$

From eqn (1)

$T = 49.05\ N$

Block A



Applying COE,

$\Sigma Fx = T + 0.25N_2 - 20\times9.81\sin\theta = 0$ ——— (1)

$\Sigma Fy = N_2 - 20\times9.81\cos\theta = 0$ ——— (2)

$N_2 = 20\times9.81\cos\theta$

From eqn (1)

$T + (0.25\times20\times9.81\cos\theta) - (20\times9.81\sin\theta) = 0$

49.05 ←

$\therefore \boxed{\theta = 28.07°}$

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KINEMATICS OF PARTICLE

(MOTION CURVE)

- Displacement (x or s)

  $\Delta x = x_2 - x_1$ (m)

- Velocity (v)

  $v = \frac{dx}{dt}$ (m/sec)

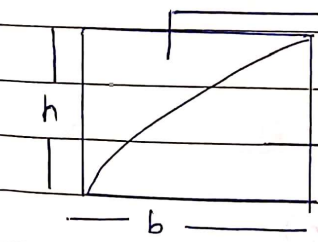
- Acceleration (a)

  $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$ (m/sec^2)

$$x \xrightarrow[\leftarrow]{d/dt \;\; \int} v \xrightarrow[\leftarrow]{d/dt \;\; \int} a$$

$x = t$ (linear)	$x = t^2$ (Parabola)	$x = t^3$ (cubic)
$v = \frac{dx}{dt} = 1$ (constant)	$v = \frac{dx}{dt} = 2t$ (linear)	$v = \frac{dx}{dt} = 3t^2$ (Parabola)
$a = \frac{dv}{dt} = 0$ (zeno)	$a = \frac{dv}{dt} = 2$ (constant)	$a = \frac{dv}{dt} = 6t$ (linear)

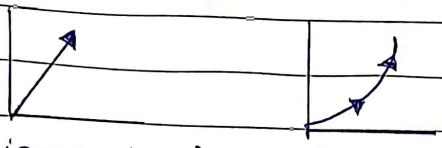
Imp ★



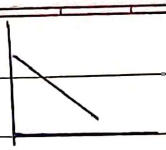
convex $\left(\frac{1}{3} \times b \times h\right)$

concave $\left(\frac{2}{3} \times b \times h\right)$

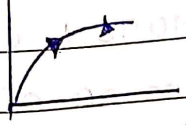
Imp ★



linear (increasing) Parabola (Anticlockwise)



linear (decreasing)



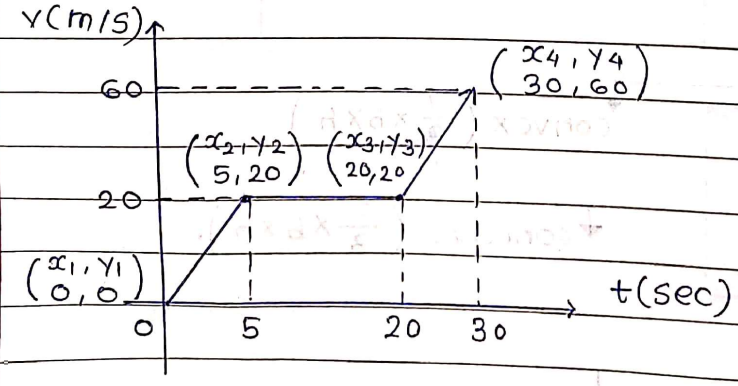
Parabola (clockwise)

Imp Formulas

• **Displacement** Change in displacement within time interval	= Area under v-t graph
• **Velocity** Change in velocity within time interval	= Area under a-t graph
• **Acceleration** Acceleration within time interval	= Slope of v-t graph

Questions:

Draw x-t and a-t graph:



	0-5	5-20	20-30
v-t	linear (increas.)	constant	linear (increas.)
x-t	Parabola (ACW)	linear (increas.)	Parabola (ACW
a-t	constant	zero	constant

displacement :

$x_5 - x_0 = 1/2 \times 5 \times 20$

$x_5 - x_0 = 50$ (Assume $x_0 = 0$)

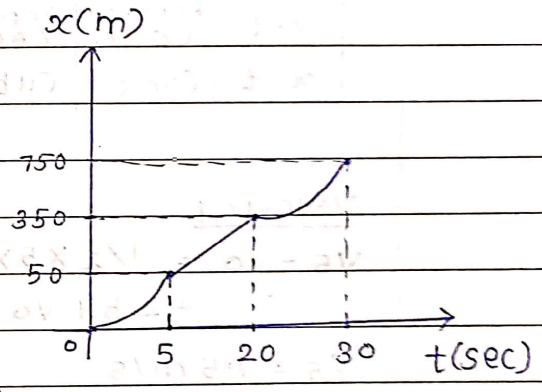
$x_5 = 50 + x_0$

$x_5 = 50$

$x_{20} - x_5 = 15 \times 20 = 300$

$x_{20} = 300 + x_5$

$x_{20} = 350\ m$



Graph : x vs t

$x_{30} - x_{20} = (10 \times 20) + (1/2 \times 10 \times 40)$

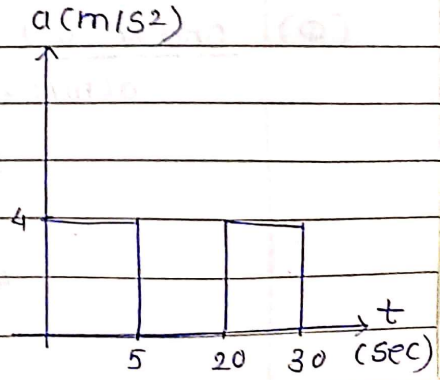
$x_{30} = 750\ m$

Acceleration :

$a_{0-5} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 0}{5 - 0} = 4\ m/s^2$

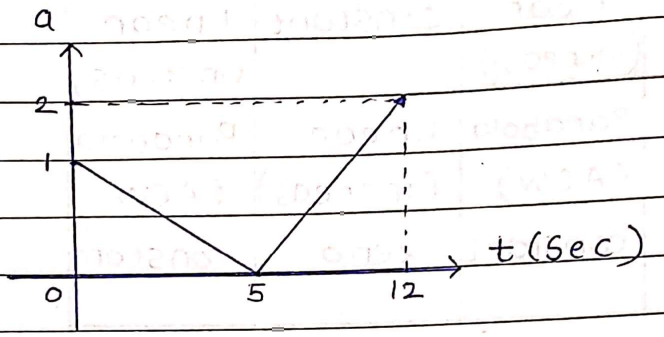
$a_{5-20} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{20 - 20}{20 - 5} = 0$

$a_{20-a_{30}} = \frac{y_4 - y_3}{x_4 - x_3} = \frac{60 - 20}{30 - 20} = 4\ m/s^2$



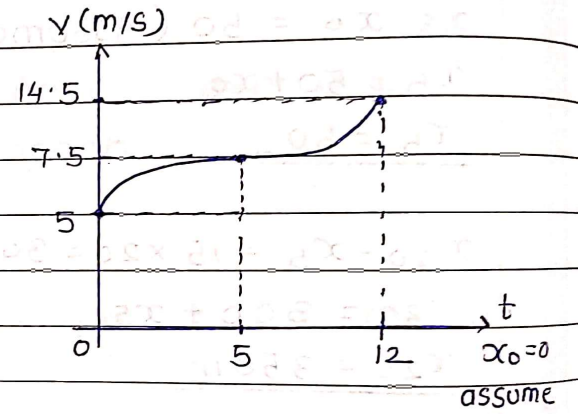
Graph : a vs t

(Q) Draw v-t graph



Initial velocity is 5 m/s

	0-5	5-12
a-t	linear (dec)	linear (incr)
v-t	Parabola (CW)	Parabola (ACW)
x-t	Cubic	Cubic



Velocity

$V_5 - V_0 = 1/2 \times 5 \times 1$

$= 2.5 + V_0 \quad (V_0 = 5)$

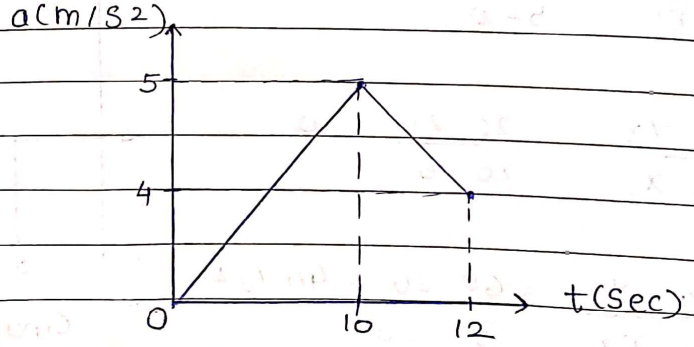
$V_5 = 7.5$ m/s

$V_{12} - V_5 = 1/2 \times 7 \times 2 = 7$

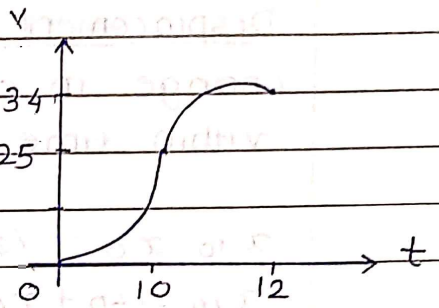
$V_{12} = 7 + V_5$

$V_{12} = 14.5$ m/s

(Q) Draw v-t graph



	0-10	10-12
a-t	linear (incn.)	linear (decn)
v-t	Parabola (ACW)	Parabola (CW)
x-t	cubic	cubic



Velocity

$v_{10} - v_0 = 1/2 \times 10 \times 5 = 25$

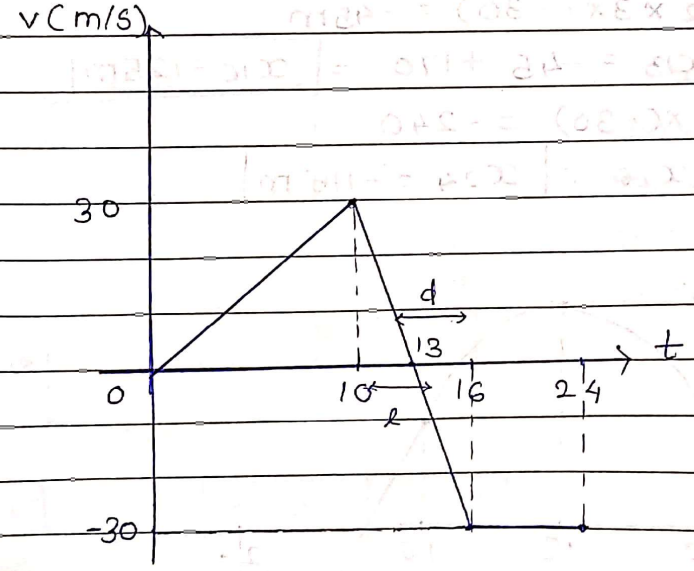
$v_{10} = 25 \text{ m/s}$

$v_{12} - v_{10} = (2 \times 4) + (1/2 \times 2 \times 1)$

$= 8 + 1 = 9$

$v_{12} = 9 + 25 = 34 \text{ m/s}$

(Q) Draw x-t and a-t graph



$\tan\theta = \frac{30}{\ell}$ | $\tan\theta = \frac{30}{d}$

$\therefore \frac{30}{\ell} = \frac{30}{d}$ $\therefore \ell = d$

	0-10	10-13	13-16	16-24
v-t	Linear (incr)	Linear (decr)	Linear (decr)	constant
x-t	Parabola (ACW)	Parabola (CW)	Parabola (CW)	Linear (decr)
a-t	constant	constant	constant	Zero

Displacement

Change in displacement = Area under v-t graph within time interval

$x_{10} - x_0 = 1/2 \times 10 \times 30 = 150$

$x_{10} = 150 + x_0$ ($x_0 = -25$ m given)

$x_{10} = 150 + x_0$

$x_{10} = 125$ m

$x_{13} - x_{10} = 1/2 \times 3 \times 30 = 45$

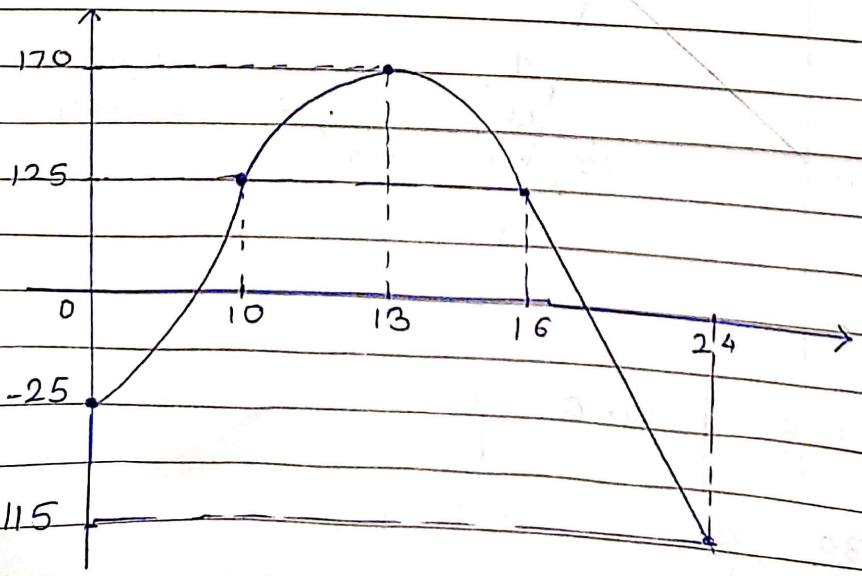
$x_{13} = 45 + x_{10} = 170$ m $\boxed{x_3 = 170\text{m}}$

$x_{16} - x_{13} = 1/2 \times 3 \times (-30) = -45$m

$x_{16} = -45 + x_{13} = -45 + 170 = \boxed{x_{16} = 125\text{m}}$

$x_{24} - x_{16} = 8 \times (-30) = -240$

$x_{24} = -240 + x_{16} \therefore \boxed{x_{24} = -115\text{ m}}$



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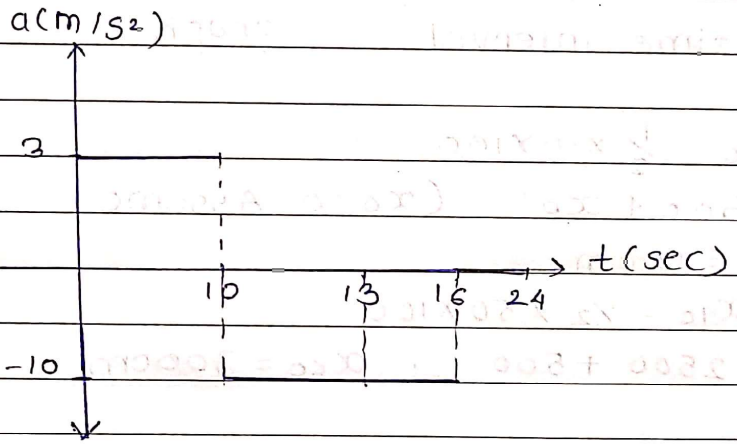
Acceleration : (slope of v-t graph)

$$a_{0-10} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{30-0}{10-0} = 3\ m/s$$

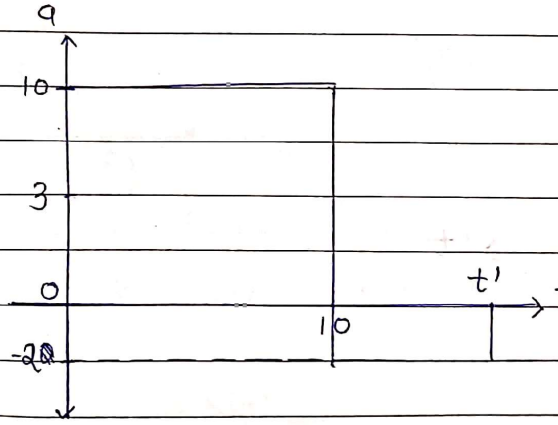
$$a_{10-13} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{0-30}{13-10} = -10\ m/s^2$$

$$a_{183-a16} = \frac{y_4 - y_3}{x_4 - x_3} = \frac{-30-0}{16-13} = -10\ m/s^2$$

$$a_{16-a24} = \frac{y_5 - y_4}{x_5 - x_4} = \frac{-30+30}{24-16} = 0$$



(Q)



A car starts from rest and travels along a straight track. Draw v-t graph and determine t' needed to stop the car.

⇒

	0-10	10-t'
a-t	const	const
v-t	linear (incr)	linear (decr)
x-t	Parabola (ACW)	Para. (CW)

Velocity :

Change in velocity within time interval = Area under a-t graph

Date

$v_{10} - v_0 = 10 \times 10$

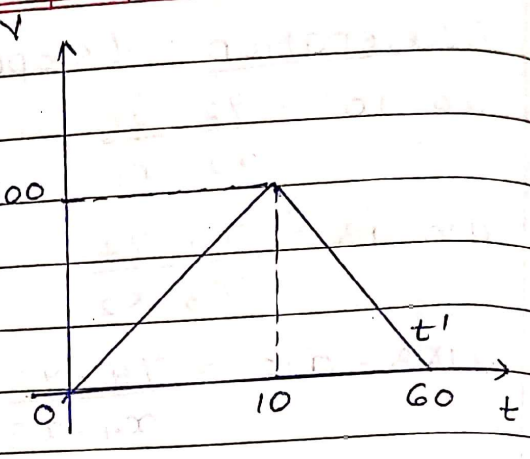
$v_{10} = 100 + v_0$ ($v_0 = 0$ given)

$v_{10} = 100 \text{ m/s}$

($v_{t'} = 0$ given)

$-100 = -2(t' - 10)$

$50 = t' - 10 \quad \therefore \boxed{t' = 60 \text{ sec}}$



Displacement

Change in displacement within time interval = Area under v-t graph

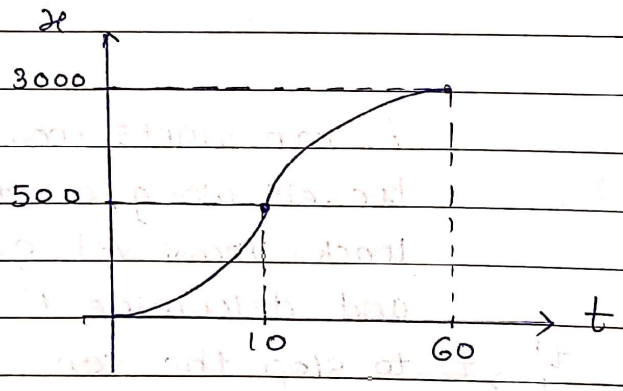
$x_{10} - x_0 = \frac{1}{2} \times 10 \times 100$

$x_{10} = 500 + x_0 \quad (x_0 = 0$ Assume)

$\therefore x_{10} = 500 \text{ m}.$

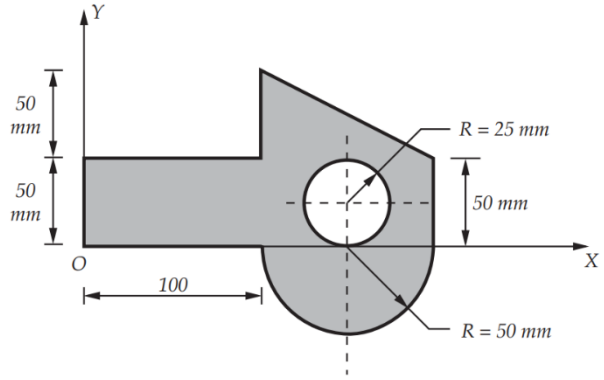
$x_{60} - x_{10} = 1/2 \times 50 \times 100$

$x_{60} = 2500 + 500 \quad \therefore x_{60} = 3000 \text{ m}$

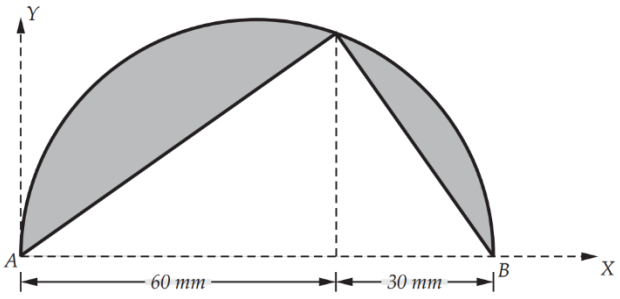


Centroid

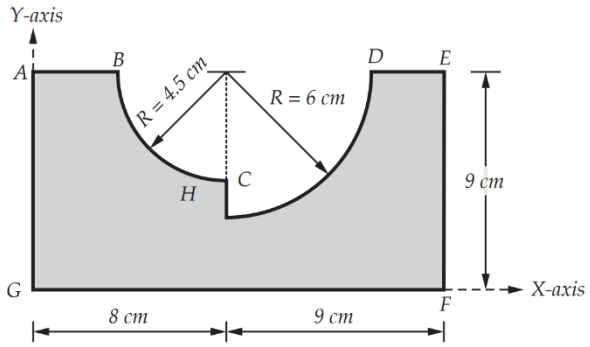
Q.1 Find centroid of the shaded portion.



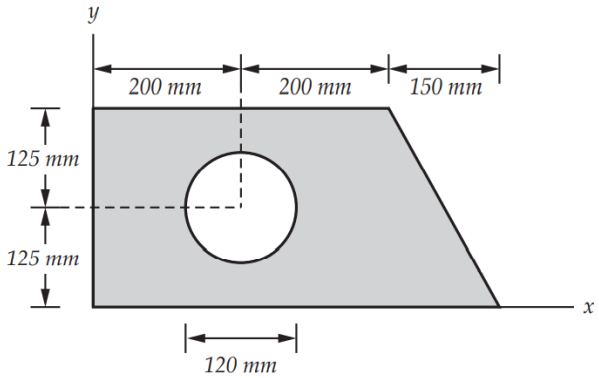
Q.2 Find resultant of the given force system.



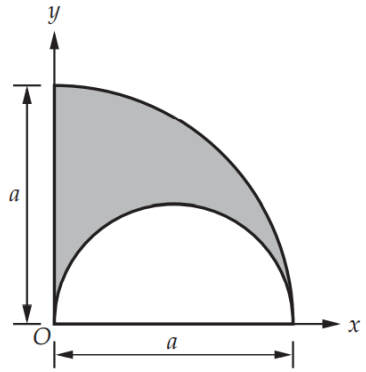
Q.3 Find centroid of the shaded portion.



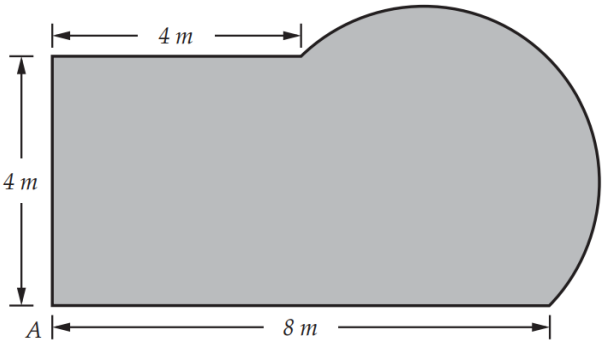
Q.4 Find centroid of the shaded portion.



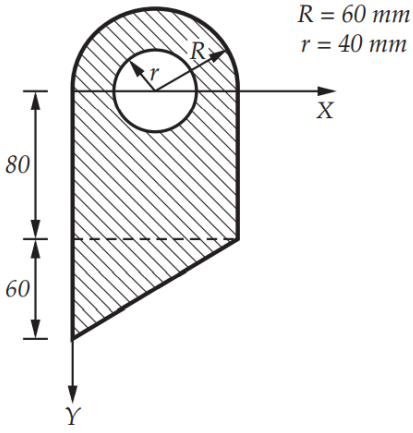
Q.5 Find centroid of the shaded portion.



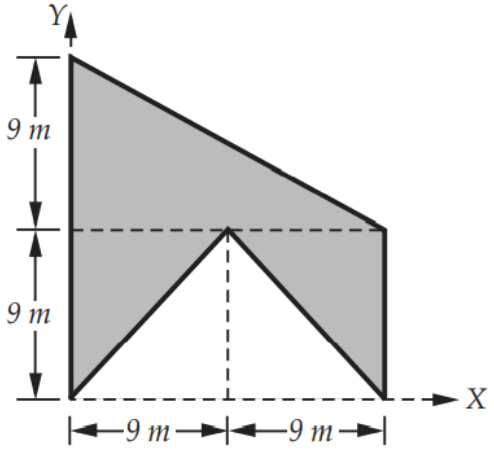
Q.6 Find centroid of the shaded portion.



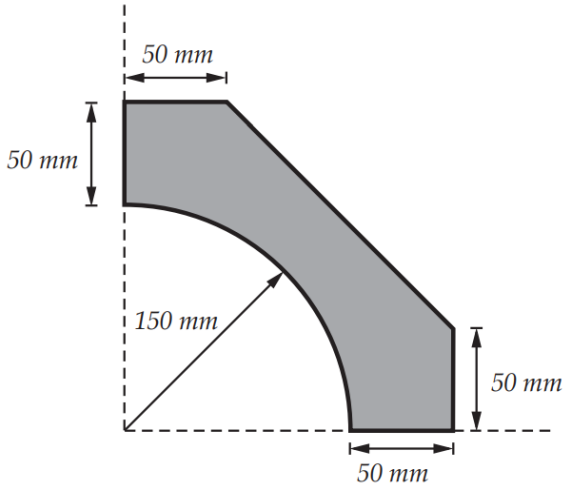
Q.7 Find centroid of the shaded portion.



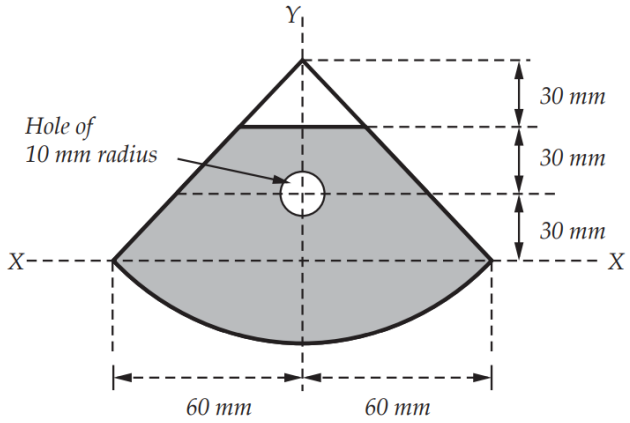
Q.8 Find centroid of the shaded portion.



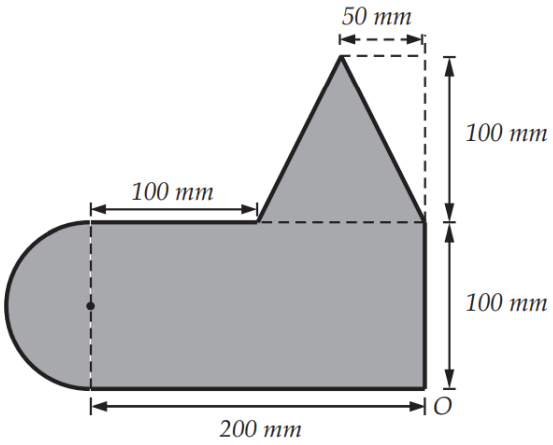
Q.9 Find centroid of the shaded portion.



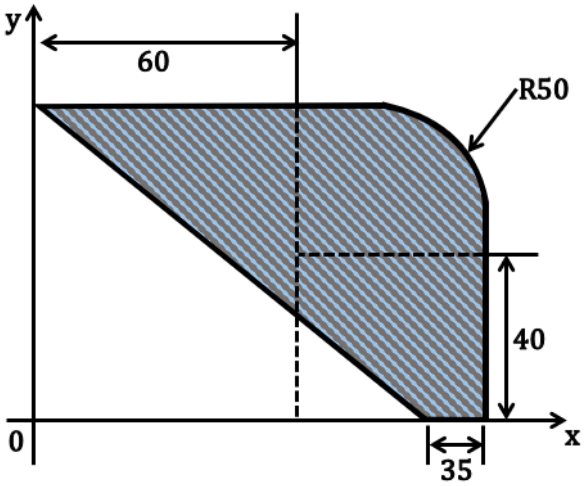
Q.10 Find centroid of the shaded portion.



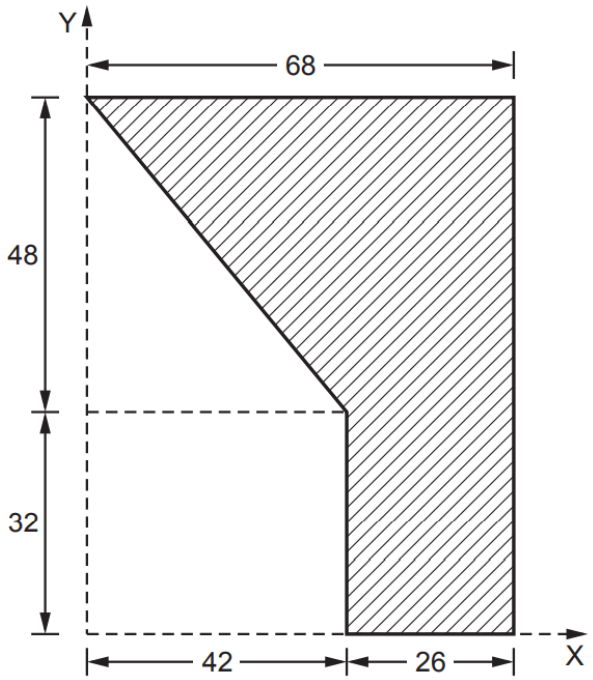
Q.11 Find centroid of the shaded portion.



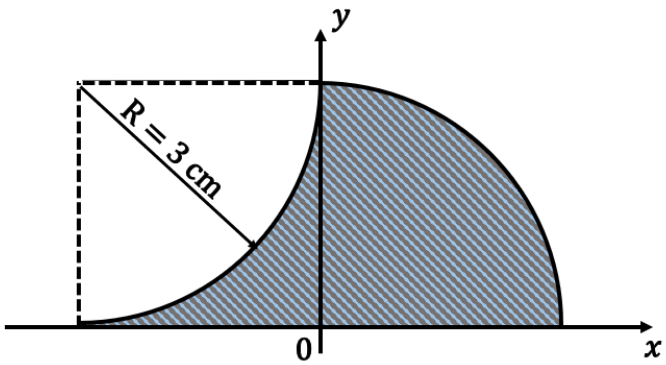
Q.12 Find centroid of the shaded portion.



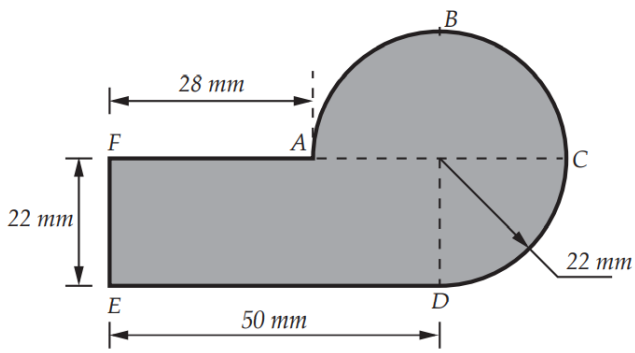
Q.13 Find centroid of the shaded portion.



Q.14 Find centroid of the shaded portion.



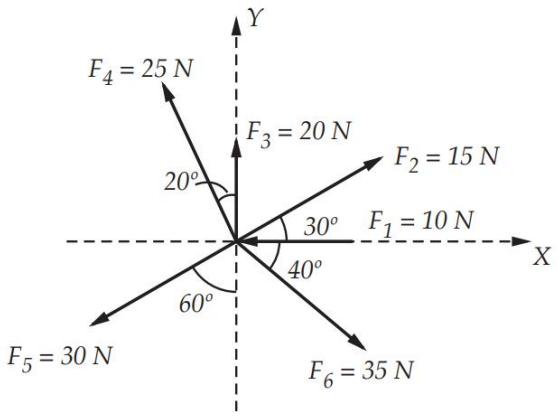
Q.15 Find centroid of the shaded portion.



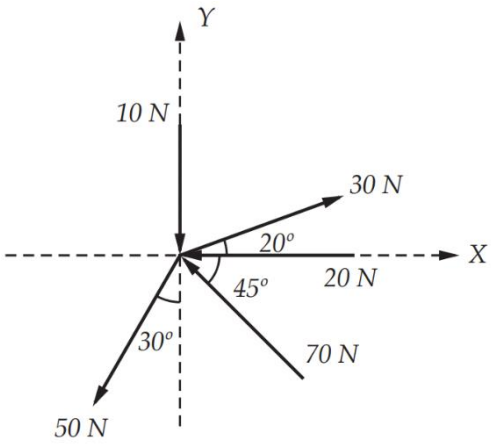
Coplanar Force System

Type 1: Concurrent Force System

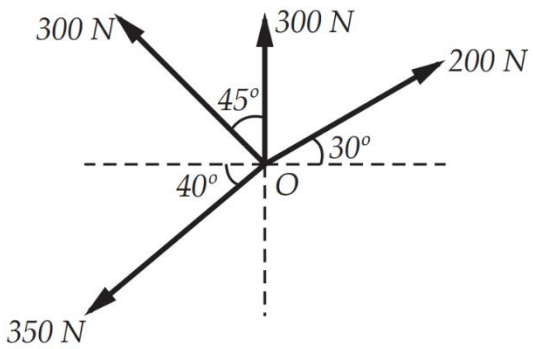
Q.1 Find resultant of the given force system.



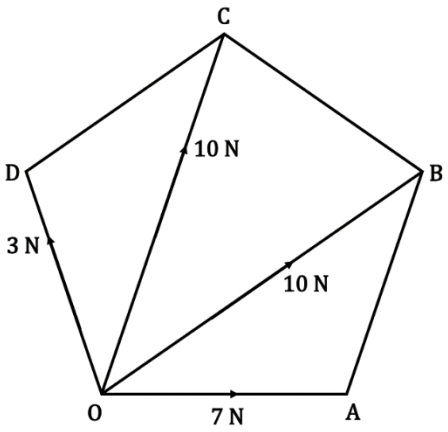
Q.2 Find resultant of the given force system.



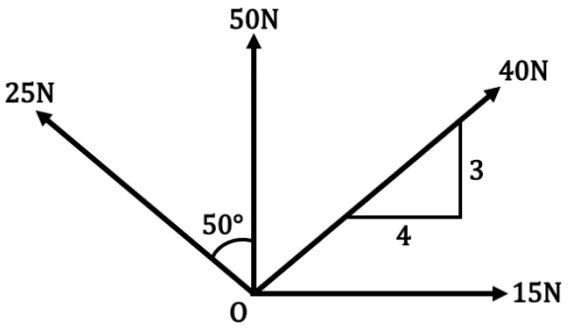
Q.3 Find resultant of the given force system.



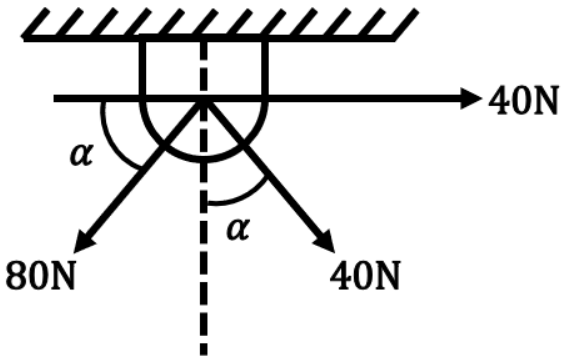
Q.4 Find resultant of the given force system.



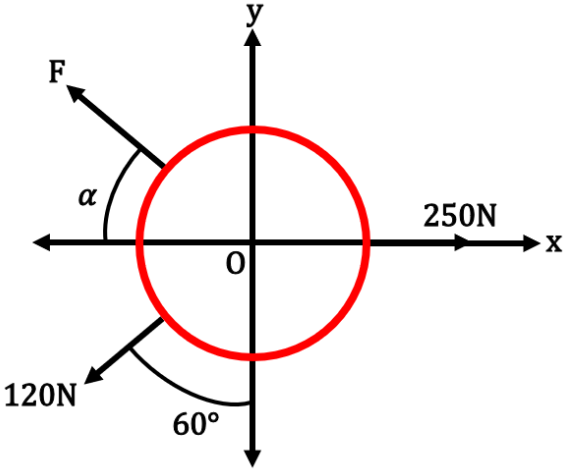
Q.5 Find resultant of the given force system.



Q.6 Find angle α and resultant of given system such that resultant of given system will be vertical.

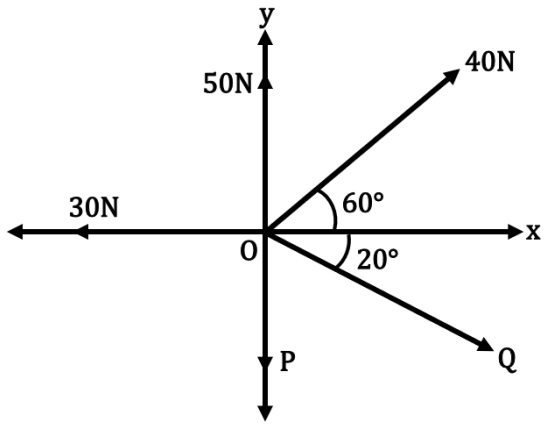


Q.7

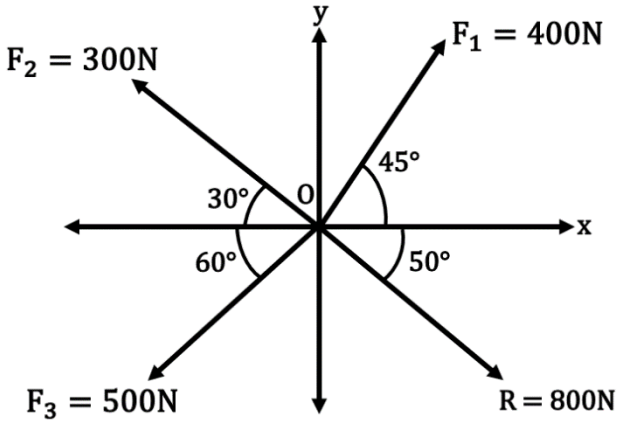


A ring is pulled by three forces as shown in figure. Find the force F and the angle α if resultant of these three forces is 100N acting in vertical direction.

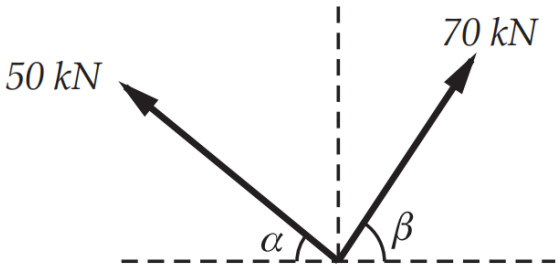
Q.8 Find P and Q if the resultant of given force system is Zero.



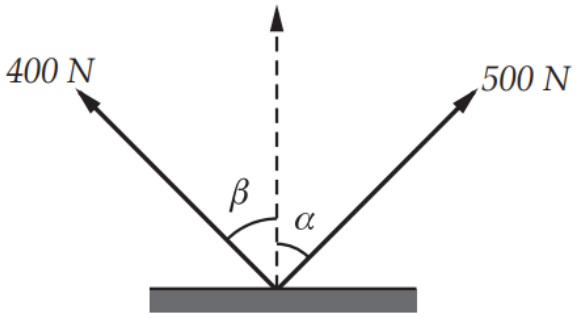
Q.9 Find force F4 so as to give the resultant of the system of forces as shown in the figure.



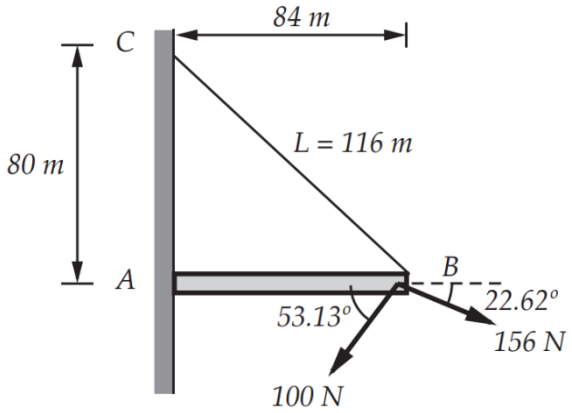
Q.10 Find α and β if resultant of given force system is 90 kN and acting in vertical direction.



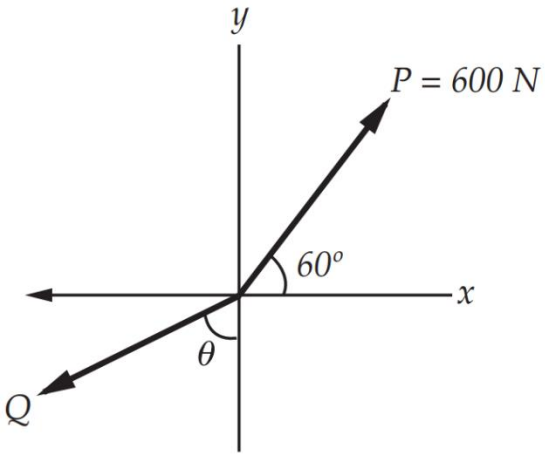
Q.11 Find α and β if resultant of given force system is 700 N and acting in vertical direction.



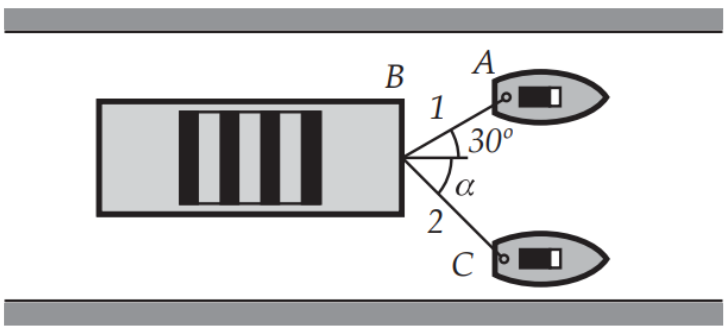
Q.12 Knowing that the tension in the cable BC is 145N, determine resultant of the three forces exerted at the three forces at point B of beam AB as shown in figure.



Q.13 Resultant of two forces P and Q is 1200 N acting in horizontal direction. Find the inclined force Q and its corresponding inclination $\boldsymbol{\theta}$ for the system of force as shown in figure.



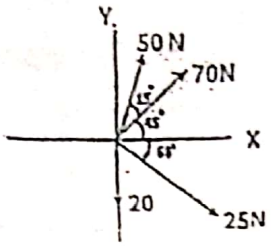
Q.14 A barge is pulled by two tugboats. If the resultant of the forces exerted by the tugboats is a 5000 N force directed along the axis of the barge, deermine the value of α for which the tension in rope 2 is minimum.



Module-1

Coplanar force System & Centroid

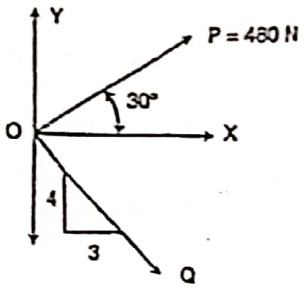
Q.1) Determine the resultant of the forces acting as given in figure below. Find the angle which the resultant makes with the positive x-axis. [M-16]



[Ans.: θ = 30.45°]

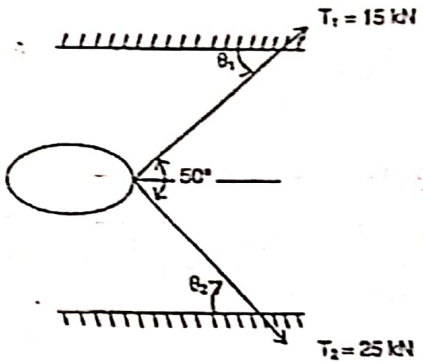
Q.2) Two concurrent forces P and Q acts at O such that their resultant acts along x-axis. Determine the magnitude of Q and hence the resultant. [M-14]

[Ans.: Q = 300 N, R = 595.69 N (→)]

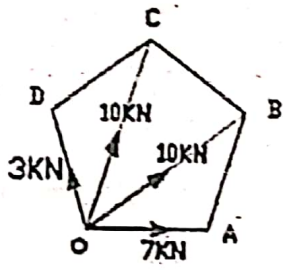


Q.3) Two electromotives in the opposite banks of a canal pull a vessel moving parallel to banks by means of two horizontal ropes as shown in fig. Find the resultant pull on the vessel and the angle between each rope and the sides of the canal.

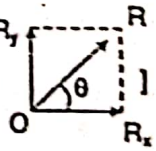
[Ans.: $\theta_1 = 31.8°$, $\theta_2 = 18.2°$, R = 36.5 KN]



Q.4) Find the resultant of forces as shown in fig. [M-19]



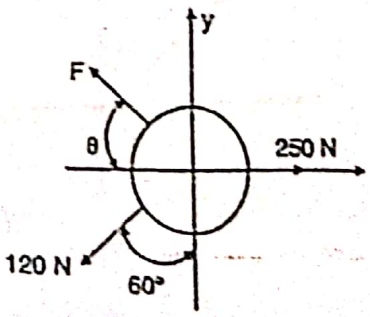
[Ans.: R = 25.1 kN, θ = 46.59°,



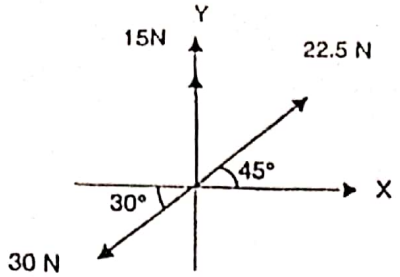
]

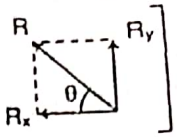
Q.5) A ring is pulled by three forces as shown in figure. Find the force F and the angle θ if resultant of these three forces is 100 N acting in vertical direction. [D-13]

[Ans.: θ = 47.6°, F = 216.65 N]



Q·6) Find resultant of the force system. [D-12]

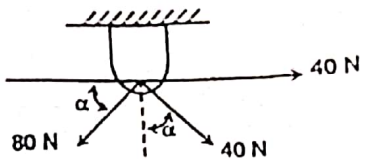


[Ans.:
$R = \sqrt{R_x^2 + R_y^2} = 18.83$ N
$\theta = \tan^{-1}(R_y/R_x) = 57.67°$


]

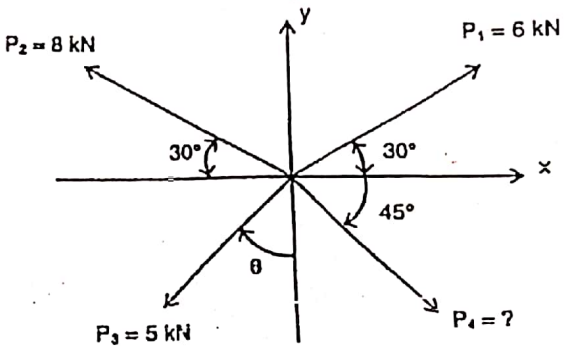
Q·7) Three coplanar forces act at a point on a bracket as shown. Determine the value of the angle α such that the resultant of the three forces will be vertical. Also find the magnitude of the resultant.

[Ans.: α = 36.87°, R = 80 N (↓)]

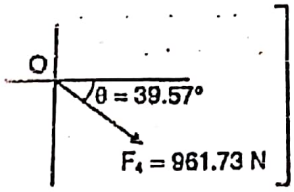


Q·8) Find out the magnitude of the force P_4 and the direction of force P_3 if the resultant of four coplanar concurrent forces P_1, P_2, P_3 & P_4 as shown in fig. is zero.

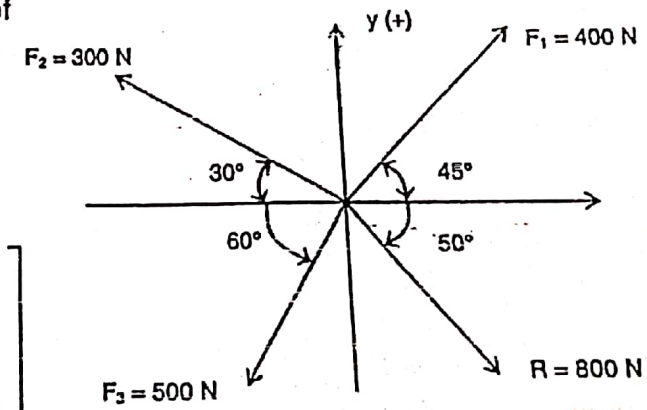
[Ans.: P_4 = 2.84 kN, θ = 3.16°]



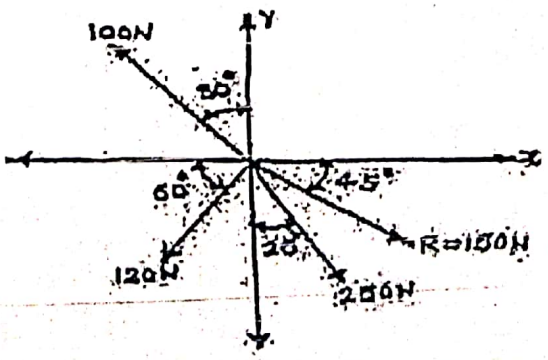
Q·9) Find force F_4 completely so as to give the resultant of the system of forces as shown in the sketch.

[Ans.:


]



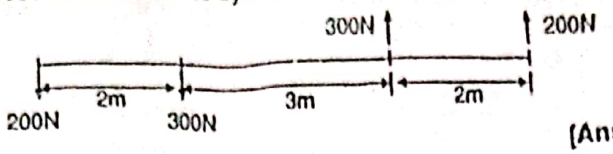
Q·10) Find fourth force (F4) completely so as to give the resultant of the system of force as shown in figure. [M-18]



[Ans.: F_4 = 176.60 N, θ = 51.30°]

Parallel Force System

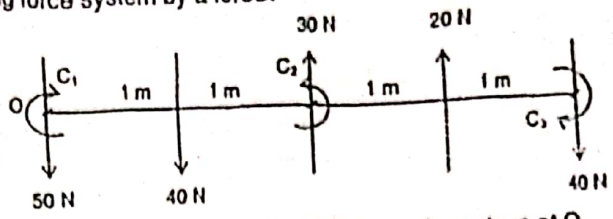
Q.11) Find the resultant of the force system shown in fig. [D-17]



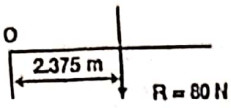
[Ans.: 2300 N-m (↺)]

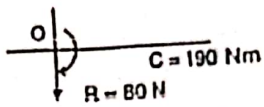
(a) Replace the following force system by a force.

C_1 = 85 N. m.
C_2 = 65 N. m.
C_3 = 90 N. m.



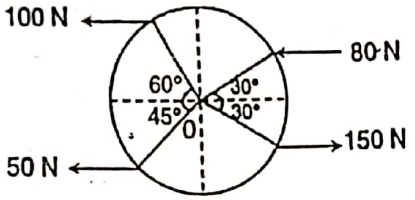
(b) Replace the above force system by a equivalent force couple system at O.

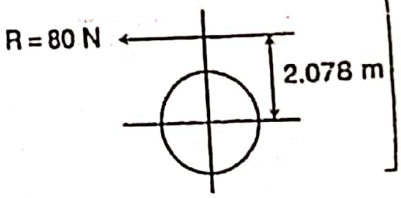
[Ans.: (a) 

(b) 

]

Q.12) Determine resultant of the following parallel forces and locate w.r.t. 'O' radius is 1 m.

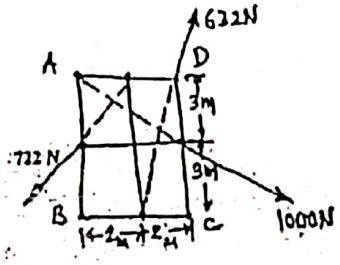


[Ans.: 

]

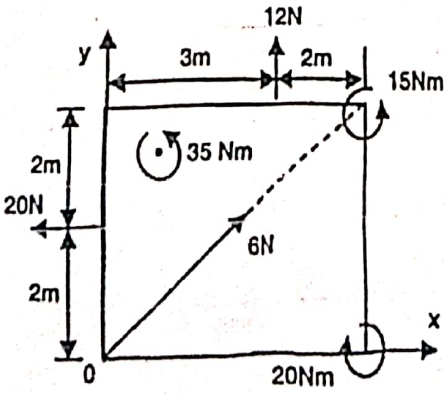
General Force System

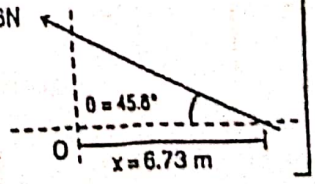
Q.13) Compute the resultant of the three forces acting on the plate shown in fig. Locate its intersection with AB and BC. [M-17; D-18]



[Ans.: x = 4 m, y = 4 m]

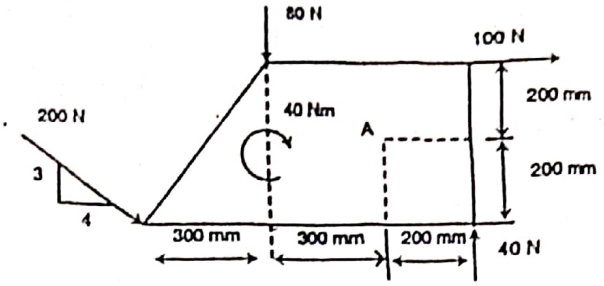
Q.14) Replace the system of forces and couples by a single force and locate the point on the x–axis through which the line of action of the resultant passes. [D-12]



[Ans.:
R = 21.96N


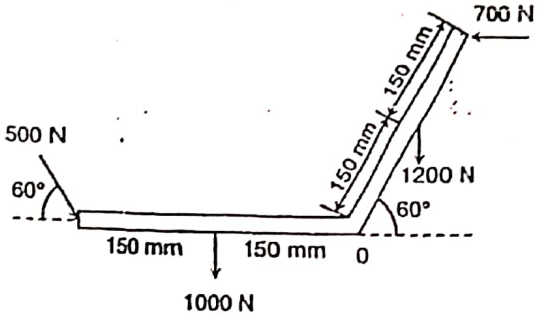
]

Q·15) Four forces and a couple are acting on a plate as shown in figure. Determine the resultant force and locate it with respect to point A. [D-15]



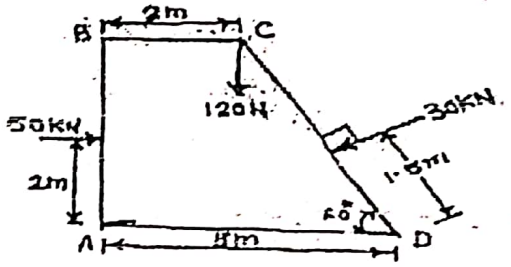
[Ans.: R = 305.28N, θ = 31.6°, d = 0.248 m, $\bar{x}$ = 0.475 m, $\bar{y}$ = 0.292 m]

Q·16) A system of forces acting on a bell crank is as shown. Determine the magnitude, direction and the point of application of the resultant w.r.t 'O'. [M-14]



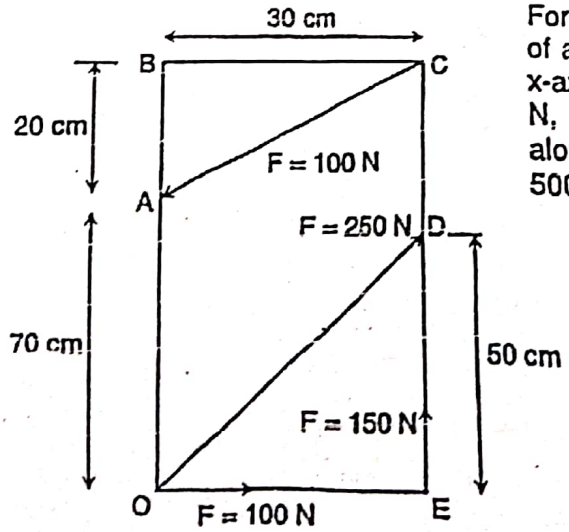
[Ans.: R = 2671.19 N, θ = 80.31°, (↻+ve) ΣM_0 = 371.77 Nm (↻), x = 0.141 m, y = 0.826 m]

Q·17) A dam is subjected to three forces as shown in fig. Determine the single equivalent force and locate its point of intersection with base AD. [M-18]



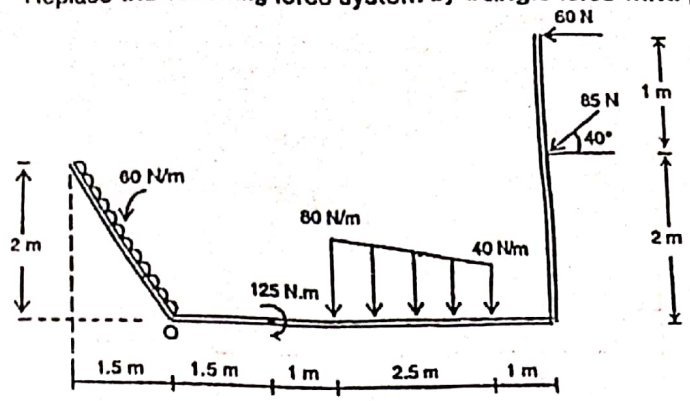
[Ans.: R = 137.12 kN, θ = 79.91°, x = 2.74 m]

Q·18) For given system find resultant and its point of application with respect to point O on the x-axis (X intercept). Force, along CA = 100 N, along OD = 250 N, along ED = 150 N, along OE = 100 N. An clockwise moment of 5000 N-cm is also acting at the point O. [D-14]

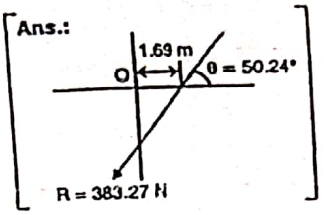


[Ans.: R = 341.42 N, θ = 64.79°, x = 17.24 cm]

Q.19) Replace the following force system by a single force w.r.t. point O.



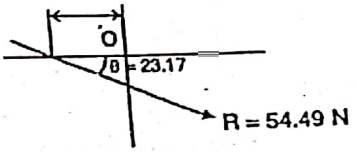
Ans.:



Q.20) Determine the resultant of the general force systems. F is in Newtons and the co-ordinates are in m.

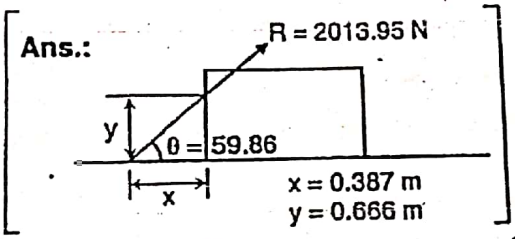
F	20	30	50	10
θ_x	45°	120°	190°	270°
Co-ordinates	(1, 3)	(4, –5)	(5, 2)	(–2, –4)

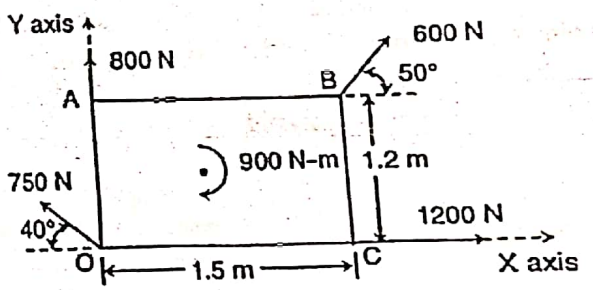
Ans.:



Q.21) Find the resultant of the force system acting on a body OABC as shown in figure. Also find the points where the resultant will cut the X and Y axis.

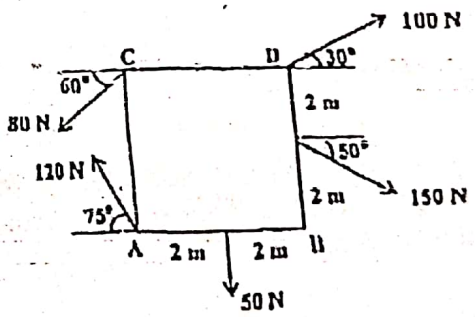
Ans.:



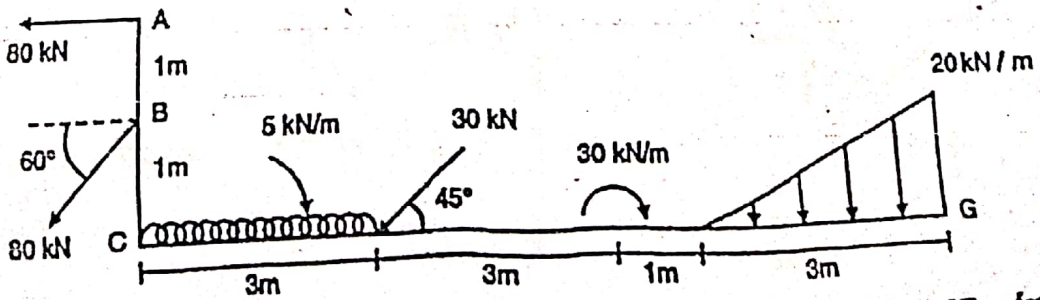


Q.22) Determine the resultant of the system of forces shown in figure. Locate the point where the resultant cuts the base AB.

[Ans.: R = 131.14 N, θ = 31.38°, X = 10.82 m (right of A)]

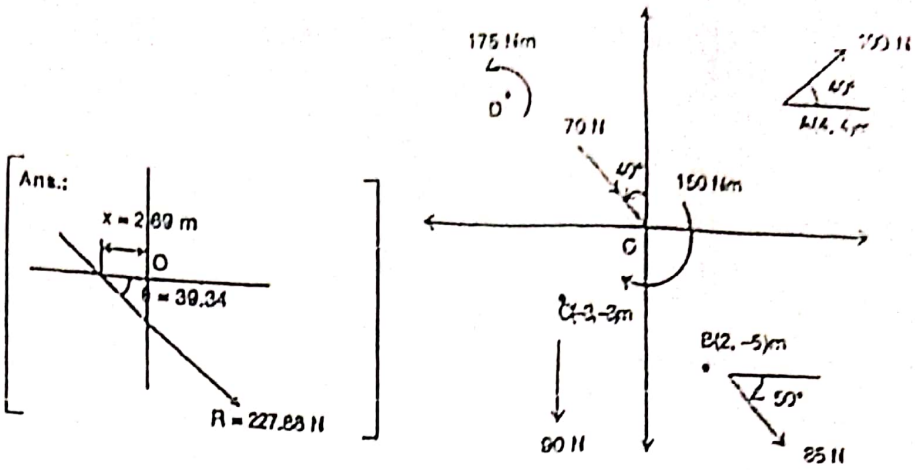


Q.23) Determine the resultant.

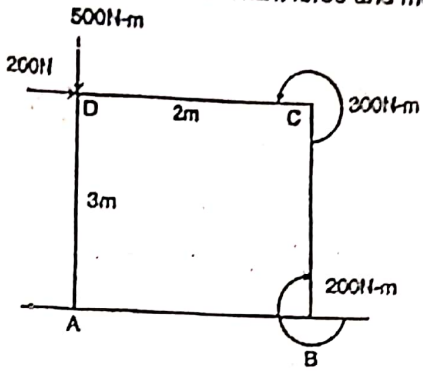


[Ans.: R = 195.7 kN 43.81° at 1.37 m from 'C']

Q. 24) Find the resultant of following force system.

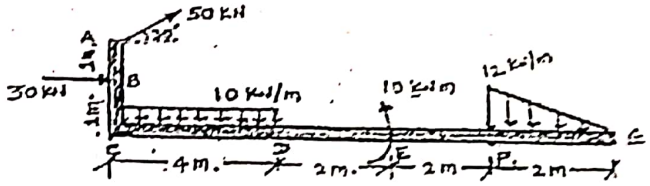


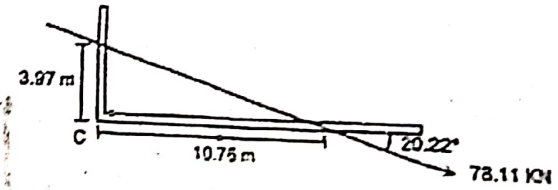
Q. 25) For the figure shown, find resultant force and moment at point A.



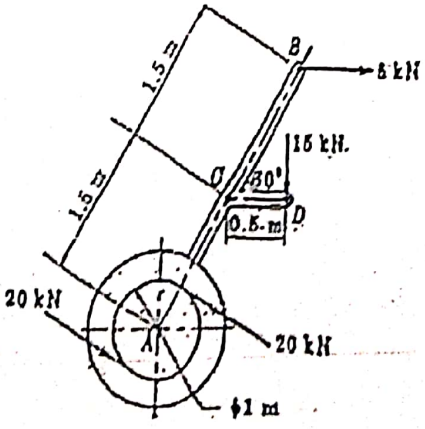
[Ans.: R = 538.5 N, θ = 63.19°, C = 500 Nm (clockwise)]

Q. 26) Replace the force system (Figure) by a single force w. r. t point C.

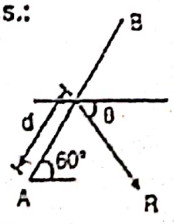




Q. 27) A machine part is subjected to forces as shown. Find the resultant of force in magnitude and direction. Also locate the point where resultant cuts the centre line of the bar AB. [D-16]



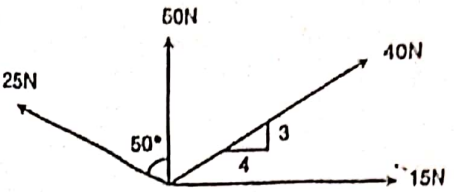
[Ans.:



Ans.: R = 15.81 kN, θ = 71.55, d = 0.992 m]

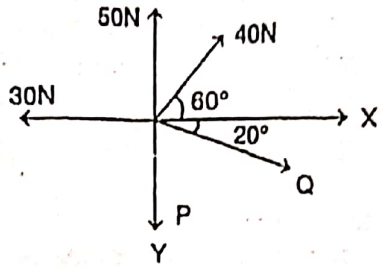
Q.28) Four concurrent forces act at a point as shown. Find their resultant. [D-14]

[Ans.: R = 94.28 N, θ = 72.8°]

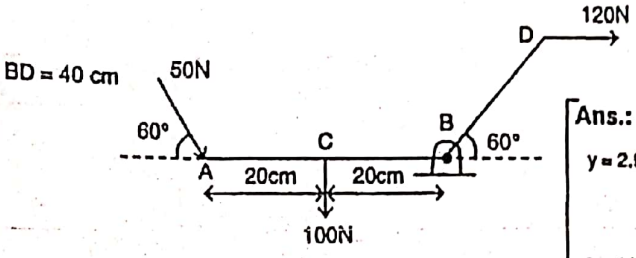


Q.29) Find forces P and Q such that resultant of given system is zero. [M-13]

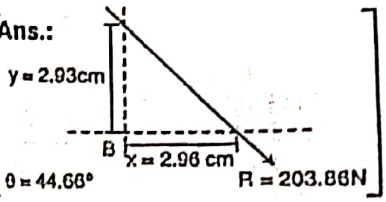
[Ans.: P = 81 N (↓), Q = 10.64 N (↘)]



Q.30) Find out resultant of given (lever) force system w.r.t. "B".

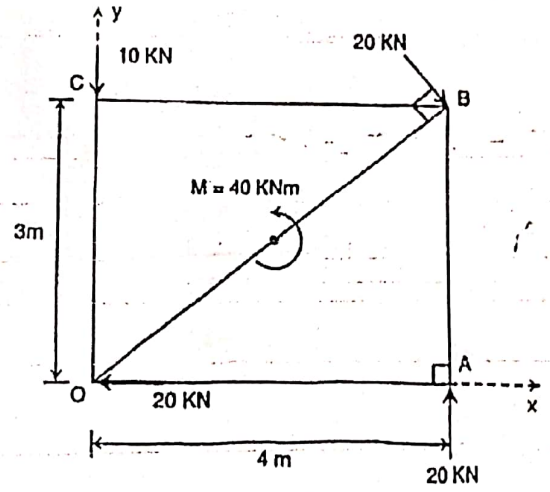


[Ans.:

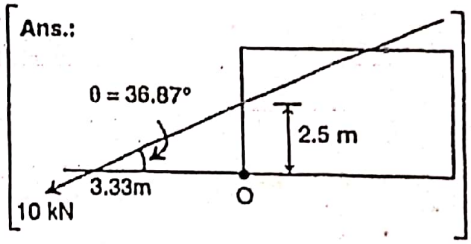


]

Q.31) Find the resultant of the force system acting on a body OABC, shown in figure. Also find the points where the resultant will cut the x and y axis. What is the distance of resultant from O.

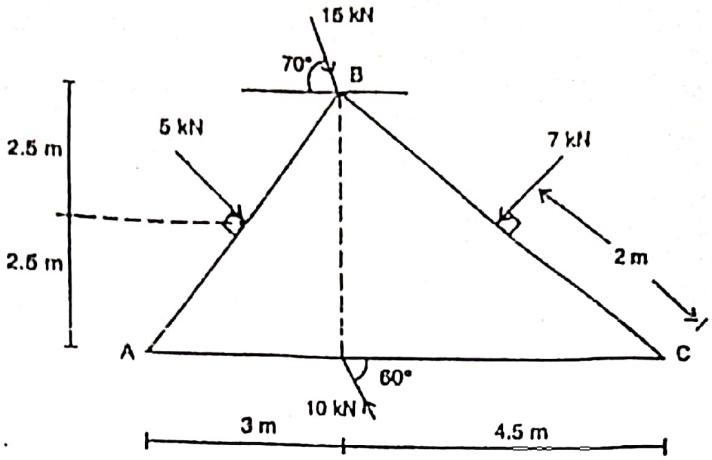


[Ans.:

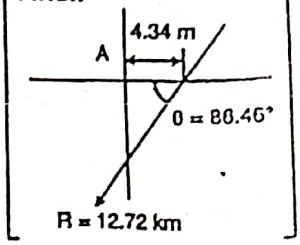


]

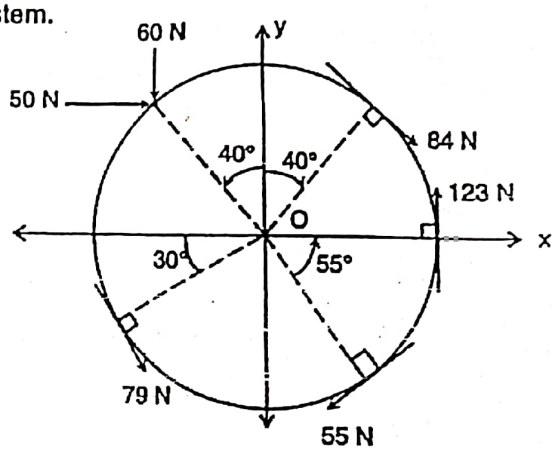
Q.32) A triangular plate is subjected to four coplanar forces as shown in fig. Find the resultant completely.



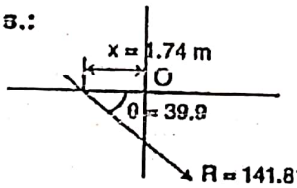
Ans.:



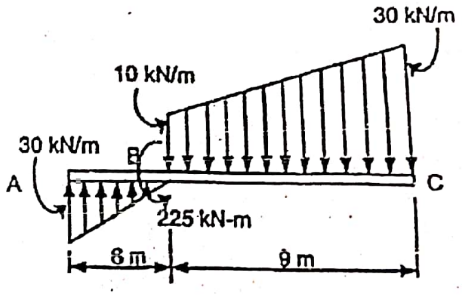
Q.33) Find the resultant of following force system. Radius = 2.5 m.



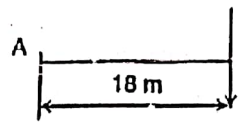
Ans.:



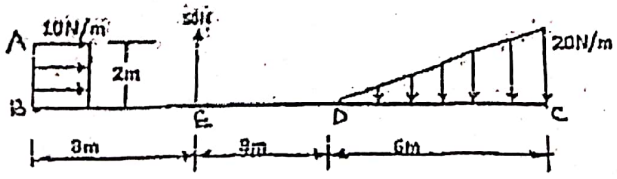
Q.34) A member ABC is loaded by distributed load and pure moment as shown in the figure. Find the (i) magnitude and (ii) position along AC of the resultant. [D-13]



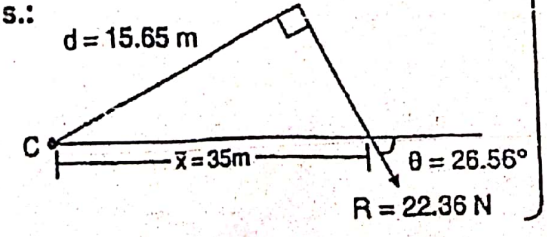
Ans.:



Q.35) Replace the force system by a single force w.r.to point C.

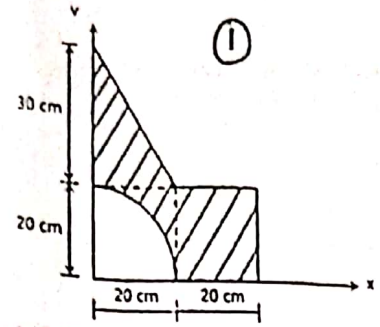


Ans.:



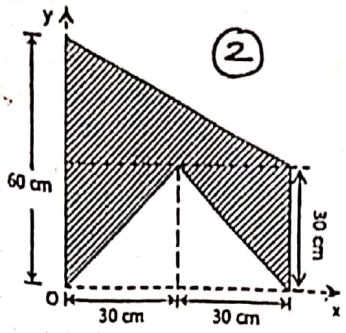
Centroid

1



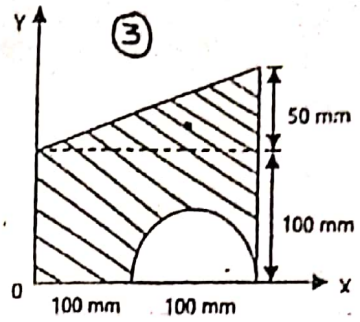
[Ans.: $\bar{x}$ = 19.51 cm ; $\bar{y}$ = 18.24 cm]

2



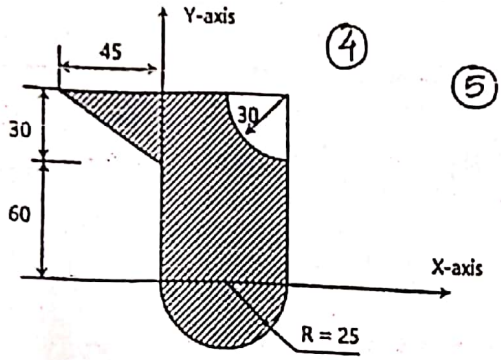
[Ans.: $\bar{x}$ = 25 cm, $\bar{y}$ = 30 cm]

3



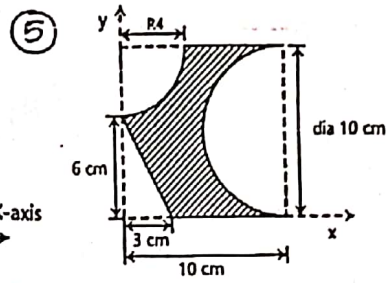
[Ans.: $\bar{x}$ = 98.59 mm, $\bar{y}$ = 71.18 mm]

4



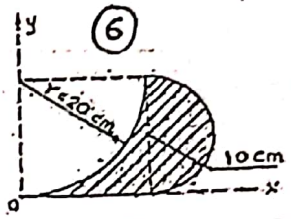
[Ans.: G = $(\bar{x}, \bar{y})$ = (18.45, 35.13)]

5



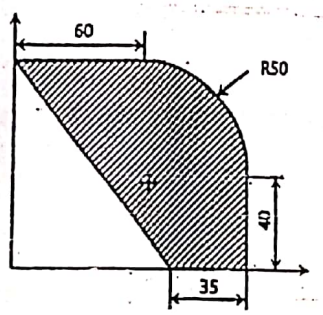
[Ans.: $\bar{x}$ = 4.093 cm, $\bar{y}$ = 4.63 cm]

6



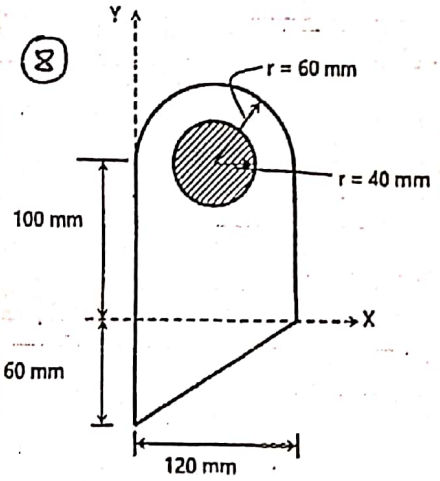
[Ans.: G = $(\bar{x}, \bar{y})$ = (21.16, 8.05)]

7



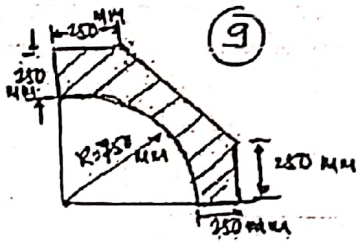
[Ans.: $\bar{x}$ = 67.98 mm, $\bar{y}$ = 50.42 mm]

8



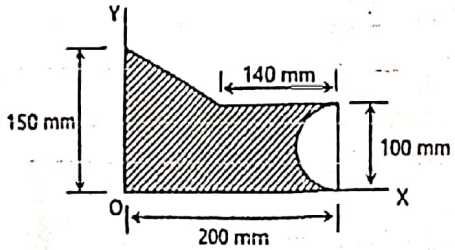
[Ans.: centroid G = $(\bar{x}, \bar{y})$ = (55.56, 45.28)m]

9



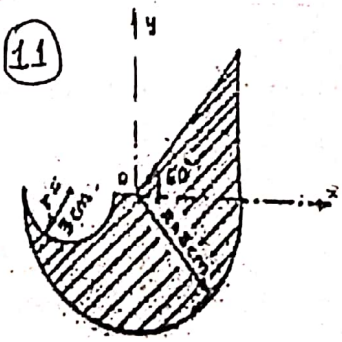
[Ans.: 535.95 mm]

10



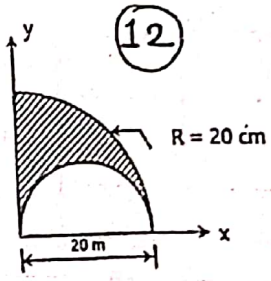
[Ans.: $\bar{X}$ = 75.57 mm, $\bar{Y}$ = 55.69 mm]

11



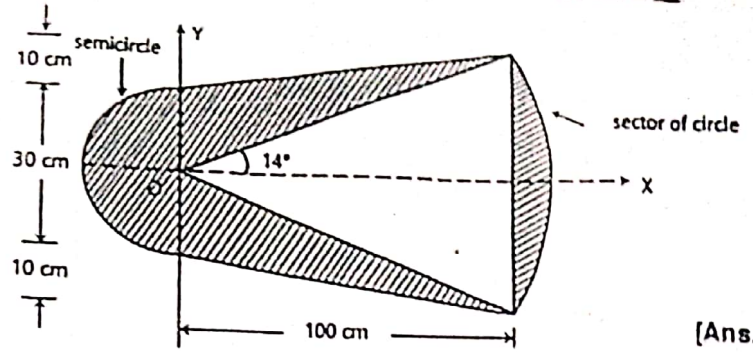
[Ans.:Centroid $\bar{x}$ = 2.5813 cm, $\bar{y}$ = –0.474 cm]

12



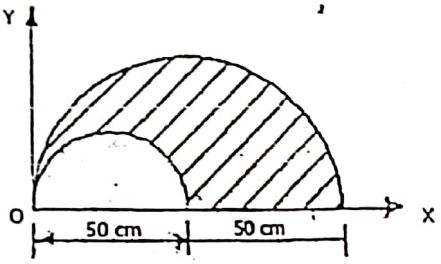
$(\bar{x}, \bar{y})$ = (6.98, 12.73) cm]

(13)



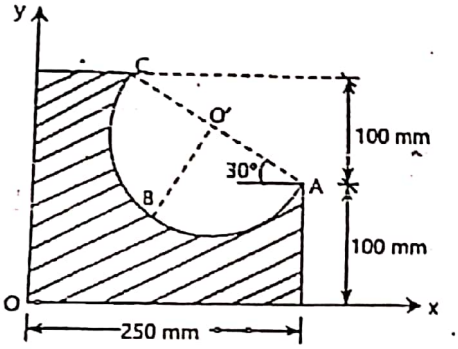
[Ans.: $\bar{x}$ = 29.62 cm]

(14)



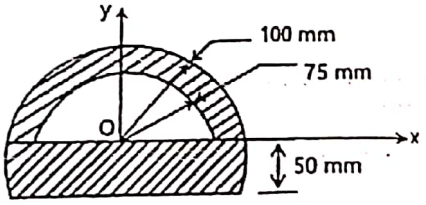
[Ans.: $\bar{x}$ = 58.33 cm, $\bar{y}$ = 24.76 cm]

(15)



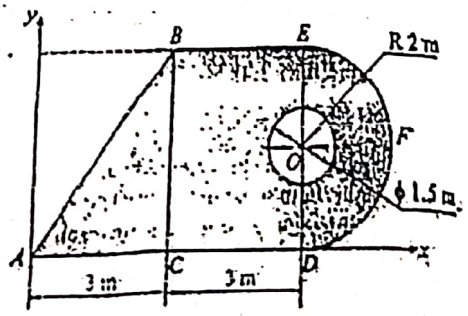
[Ans.: $\bar{x}$ = 91.175 mm,
$\bar{y}$ = 69.361 mm]

(16)



[Ans.: $\bar{y}$ = 9.238]

(17)



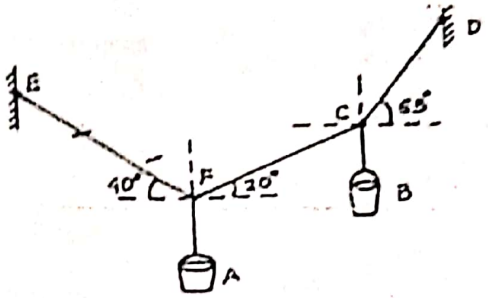
[Ans.: $\bar{X}$ = 4.37 m, $\bar{Y}$ = 1.821 m]

(10)

Module-2

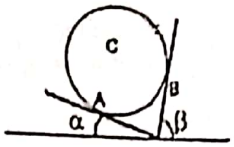
Equilibrium

Q.1) If the cords suspend the two buckets in equilibrium position shown in Fig. Determine weight of bucket B if Bucket A has a weight of 60N. [M-19]



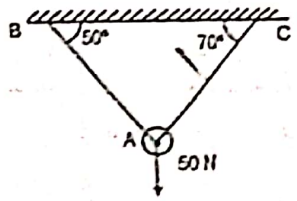
[Ans.: W_B = 88.79 N]

Q.2) A smooth circular cylinder of weight W and radius R rests in a V shape groove whose sides are inclined at angles α and β to the horizontal as shown. Find the reactions R_A and R_B at the points of contact. [D-12]



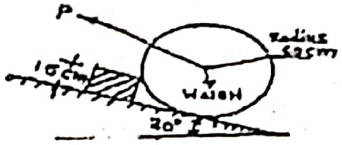
$$\left[\text{Ans.: } R_A = \frac{w\sin\beta}{\sin(\alpha+\beta)},\ \text{and } R_B = \frac{w\sin\alpha}{\sin(\alpha+\beta)}\right]$$

Q.3) Sphere A is supported by two wires AB, AC. Find out tension in wire AC: [M-13]
(i) before AB is cut
(ii) just after AB is cut.



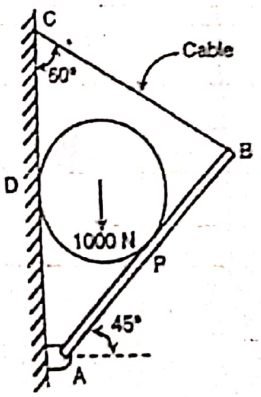
[Ans.: (i) T_{AC} = 37.11 N,
(ii) T_{AC} = 46.98 N]

Q.4) Determine the magnitude and direction of the smallest force 'P' required to start the wheel W = 10 N over the block. [M-18]



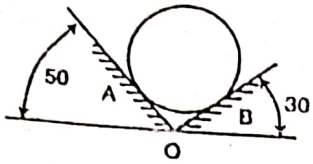
[Ans.: P = 9.47 N, θ = 41.4°]

Q.5) A cylinder weighing 1000 N and 1.5 m diameter is supported by a beam AB of length 6 m and weight 400 N as shown. Neglecting friction at the surface of contact of the cylinder, determine,
(i) Wall reaction at D
(ii) Tension in the cable BC and
(iii) Hinged reaction at support A



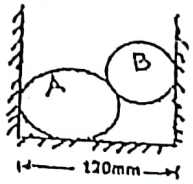
[Ans.: R_D = 1000 N, T = 588.08 N,
H_A = 490.71 (←),
V_A = 1106.18 N (↑)]

Q.6) A cylinder of weight 500 N is kept on two inclined planes as shown in the figure. Determine the reactions at the contact points A and B. [D-14]



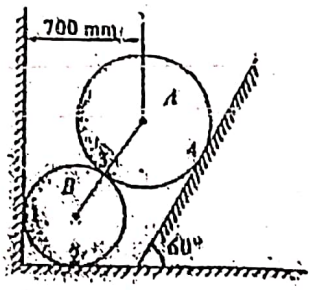
[Ans.: $R_A = 253.85$ N, $R_B = 388.93$ N]

Q.7) Two spheres A and B are kept in a horizontal channel. Determine the reactions coming from all the contact surfaces. Consider the radius of A and B are 40 mm and 30 mm respectively. Take $W_A = 500$ N and $W_B = 200$ N. [M-16]

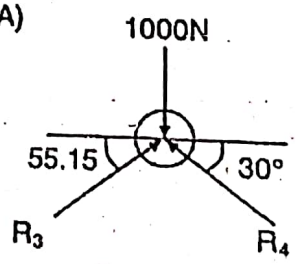


[Ans.: $R_1 = 204.09$ N, $R_2 = 285.75$ N
$R_3 = 204.09$ N, $R_4 = 700$ N]

Q.8) Two spheres A and B of weight 1000N and 750N respectively are kept as shown in fig. Determine the reactions at all contact points 1, 2, 3 and 4. Redius of A is 400 mm and Radius of B is 300 mm. [D-16; M-19]

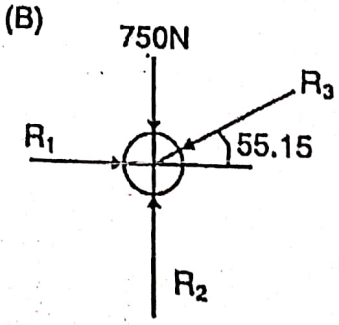


Ans.: (A)



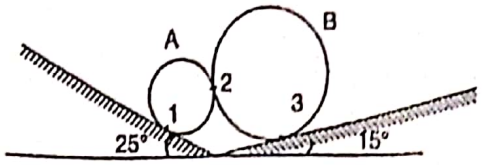
$R_3 = 869.14$ N
$R_4 = 573.48$ N

(B)



$R_1 = 496.65$ N
$R_2 = 1463.26$ N

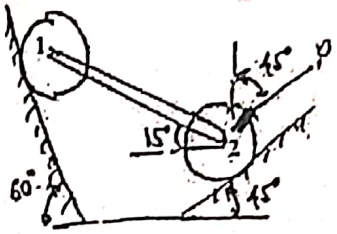
Q.9) Determine the reaction at points of contact 1, 2 and 3. Assume smooth surfaces. [D-17]



$W_A = 1$ kg, $r_A = 1$cm
$W_B = 4$ kg, $r_B = 4$cm

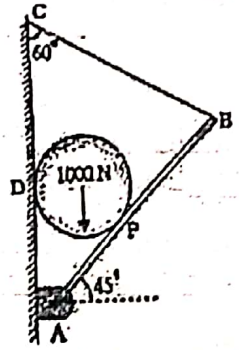
[Ans.: $R_1 = 19.750$ N; $R_3 = 32.249$ N; $R_2 = 11.61$N; $\alpha = 44.09°$]

Q.10) Two cylinders 1 and 2 are connected by a rigid bar of negligible weight hinged to each cylinder and left to rest in equilibrium in the position shown under the application of force 'P' applied at the center of cylinder 2. Determine the magnitude of force 'P'. If the weights of the cylinders 1 and 2 are 100N and 50 N respectively. [M-17]



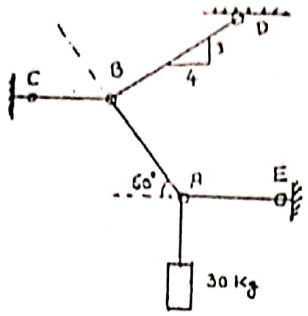
[Ans.: P = 25.88 N]

Q.11) A cylinder weighing 1000 N and 1.5 m diameter is supported by a beam AB of length 6 m and weight 400 N as shown in figure. Neglecting friction at the surfaces of contacts, determine (i) Wall reaction at D. (ii) Tension in the cable BC and (iii) Hinged reaction at support A. [D-13; M-18]



[Ans. : $R_D = 1000$ N (→), $R_P = 1414.21$ N ($\angle 45°$), T = 588.08 N,
$H_A = -490.71 = 490.71$ N (←), $V_A = 1105.96$ N (↑)]

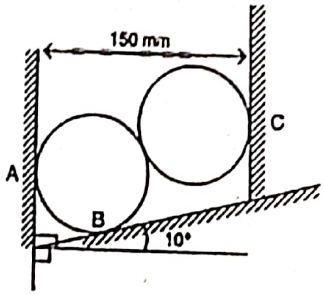
Q.12) A 30 kg pipe is supported at 'A' by a system of five chords. Determine the force in each chord for equilibrium.



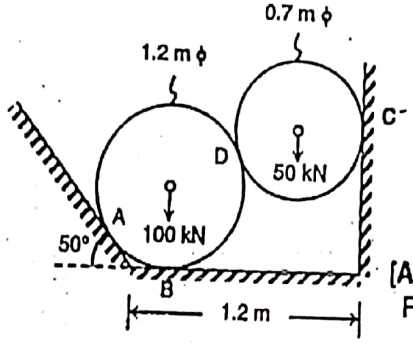
[Ans.: $T_{BC} = 562.29N$, $T_{BD} = 490.48N$, $T_{AE} = 169.19N$, $T_{AB} = 339.82N$]

Q.13) Two identical cylinders diameter 100 mm weight 200 N are placed as shown. All contacts are smooth. Find out reactions at A, B and C. [M-13]

[Ans.: $R_C = 115.47N$ (←), $R_B = 406.17$ N (↑), $R_A = 186$ N (→)]

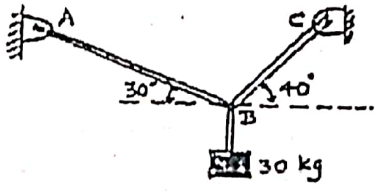


Q.14) Two cylinders are kept in a channel as shown in figure. Determine the reactions at all the contact points A, B, C and D. Assume all surfaces smooth. [M-14]



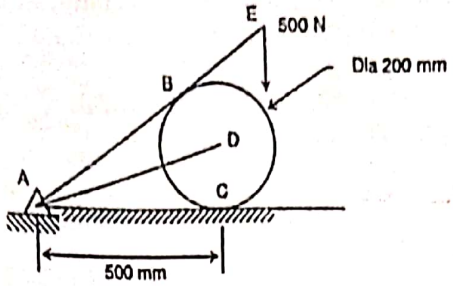
[Ans.: $x' = 0.28$ m, $\alpha = 53.11°$, $R_A = 48.96$ kN (↗), $R_B = 118.52$ kN (↑), $R_C = 37.5$ kN, $R_D = 62.5$ kN]

Q.15) Determine the tensions in cords AB & BC for equilibrium of 30 kg block (Figure). [M-15]



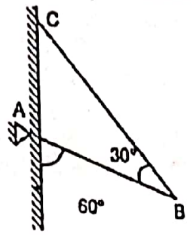
[Ans.: $T_{AB} = 239.92$ N, $T_{BC} = 271.23$ N]

Q.16) A Cylinder of weight 300N is held in equilibrium as shown in figure given below. Determine the tension in the string AD and reaction at C and B. The length of AE = 750 mm. [D-14]



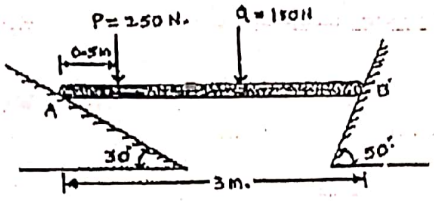
[Ans.: T = 271.54 N, R_B = 692.28]

Q.17) A prismatic bar AB of length 6m and weight 3 KN is hinged to a wall and supported by a cable BC. Find hinge reaction and tension in cable BC. [D-15]



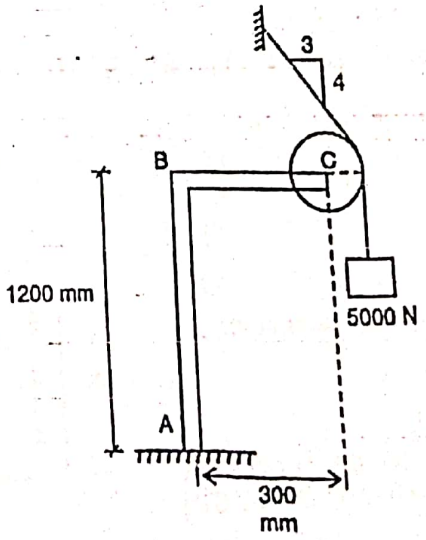
[Ans.: T_{BC} = 2.598 N, R_A = 1.5 N]

Q.18) A bar of 3 m. length & negligible weight rests in horizontal position on two smooth inclined planes (Figure). Determine the distance x at which the load Q = 150 N should be placed from point B to keep the bar horizontal. [M-15]



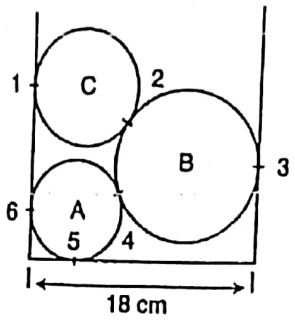
[Ans.: x = 1.22 m]

Q.19) A weight 5000 N is suspended by a cable and pulley attached at pin C as shown in fig. Find the support reaction H_A, V_A & M_A at the point A of the column ABC. Take radius of the pulley as 150mm. Neglect weight of pulley, column and cable.



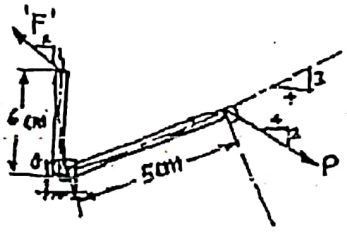
[Ans.: H_A = 3000N (→), V_A = 1000N (↑), $M_A = 33 \times 10^5$ N-mm (↻)]

Q. 20) Three right circular cylinders A, B, C are piled up in a rectangular channel as shown in figure. Determine the reactions at point 3 between the cylinder A and vertical wall of the channel. [D-15]
(Cylinder A : radius = 4cm, mass = 15kg, Cylinder B : radius = 6cm, mass = 40kg. Cylinder C : radius = 5 cm, mass = 20 kg.)



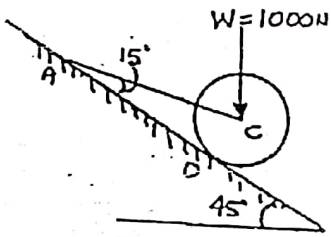
[Ans.: 784.84 N]

Q. 21) In the rocket arm shown in fig the moment of 'F' about 'O' balances that 'P' = 250 N find 'F'. [M-17]



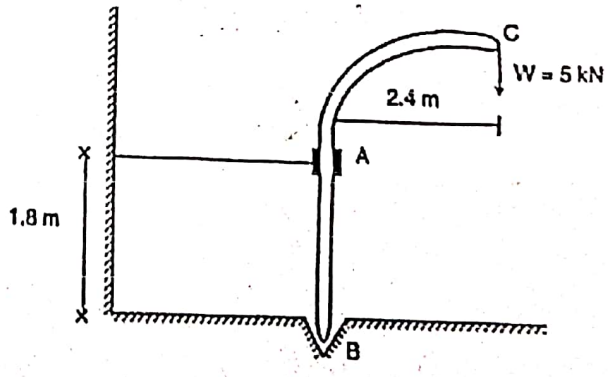
[Ans.: F = 208.33 N]

Q. 22) A roller of weight W = 1000 N rest on a smooth incline plane. It is kept from rolling down the plane by a string AC. Find the tension in the string and reaction at the point of contact D.



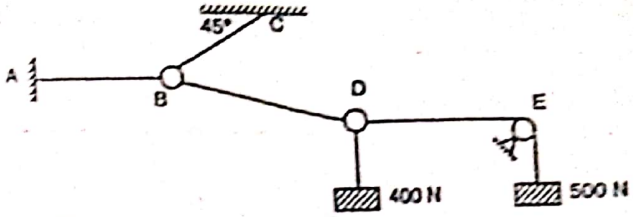
[Ans.: T = 732.04N, R_D = 896.575N]

Q. 23) A crane is pivoted at the end B and is supported by guide at A. Determine the reaction produced at A and B by a vertical load
W=5kN applied at C.
Assume guide to be smooth.



[Ans.: R_A = 6.6 kN, R_B = 8.2 kN, θ = 37.81°]

Q.24) Find force transmitted by cable "BC" shown. E is a friction less pulley, where B and D are weightless rings.

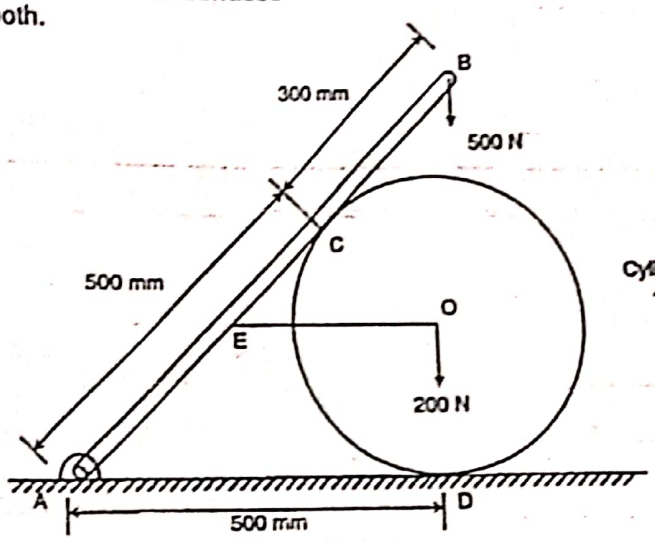


[Ans.: F_{BC} = 565.69 N]

Q.25) A bar is hinged at A and rests on cylinder at C. AC = 500 mm, CB = 300 mm, diameter of cylinder is 400 mm and its weight is 200 N. The centre of cylinder is connected to the bar by a horizontal wire OE as shown in figure. A weight of 500 N is suspended at B. Determine

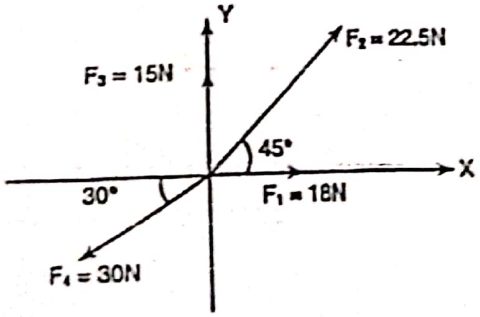
(I) Reaction of hinge A, (II) Tension in the wire,
(III) Reactions at C and D.

Neglect the weight of the bar and assume all surfaces smooth.

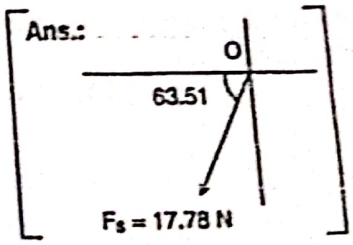


[Ans.: V_A = 79.3 N (↓), H_A = 0 ; T = 551.7 N, R_C = 800 N, R_D = 779.34 N]

Q.26) Under the action of 5 forces, following system is in equilibrium. Determine fifth force.



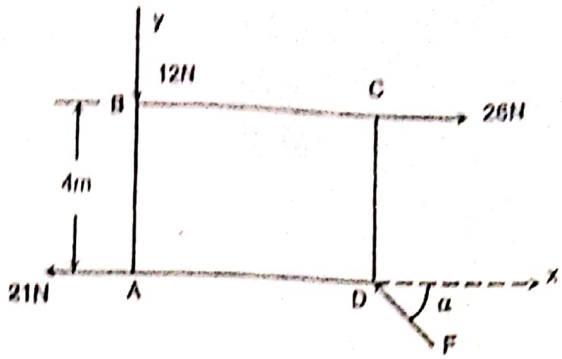
Ans.:



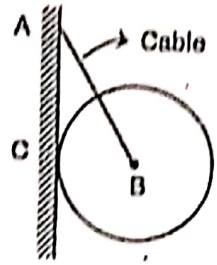
Q.27) Forces act on the plate ABCD as shown in Fig. The distance AB is 4 m. Given that the plate is in equilibrium find.

(I) force F
(II) angle α and
(III) the distance AD

[Ans.: (I) F = 13N; (II) α = 67.38°; (III) AD = 8.67 m.]



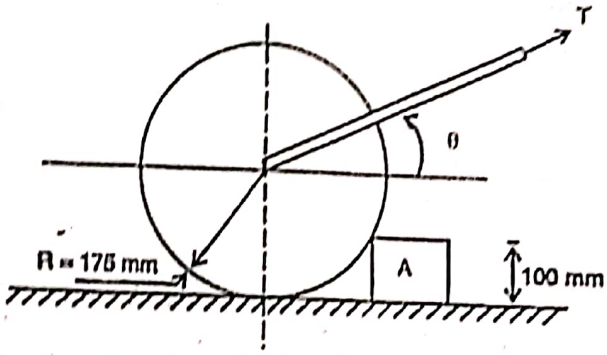
Q.28) A cylinder B, W_B = 1000 N, dia. 40 cm, hangs by a cable AB = 40 cm rests against a smooth wall. Find out reaction at C and TAB. [M-13]



[Ans.: T = 1154.7 N, R_C = 577.35 N]

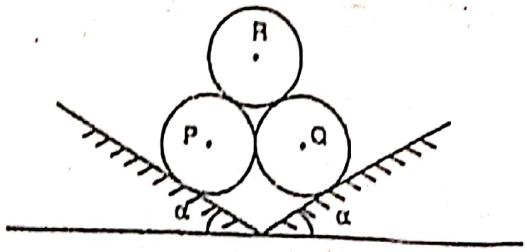
Q.29) The roller shown in fig. is of weight 1500 N. What force 'T' is necessary to start the roller over the block A, if θ = 25°.
Also find the minimum force 'T' required to start the roller over the block A.

[Ans.: T = 1759.34 N, T_{min} = 1355.23 N]



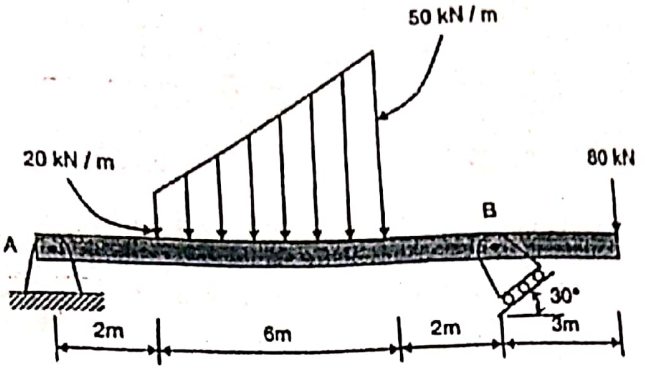
Q.30) Three identical spheres P, Q, R of weight 'W' are arranged on smooth inclined surface as shown in the figure. Determine the angle 'α' which will prevent the arrangement from collapsing.

[Ans.: θ = 10.94°]



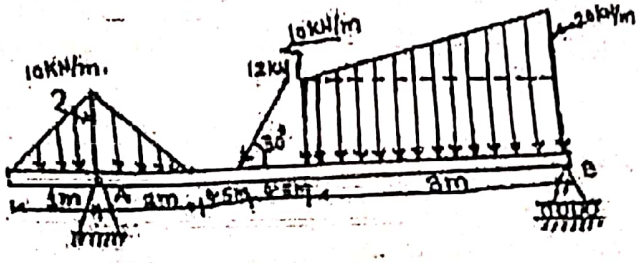
Support Reactions

1) Find the support reaction at A and B for the beam loaded as show in fig. [D-17]



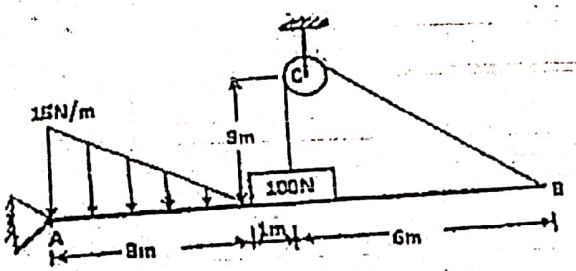
[Ans.: $R_B = 251.724$ kN; $H_A = 125.862$ kN(→); $V_A = 72$ kN(↑)]

2) Find the support reactions at point 'A' and 'B' of the given beam [M-18]



[Ans.: $H_A = 10.39$ kN(→); $V_A = 27.675$ kN(↑); $R_A = 29.56$ kN; $\theta = 69.41°$; $R_B = 38.325$ kN(↑)]

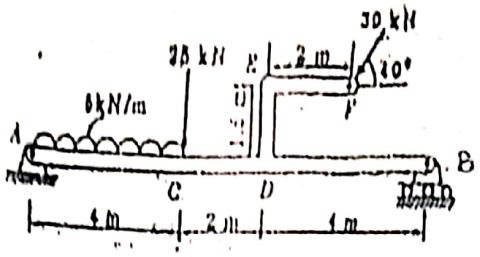
3) A uniform beam AB hinged at A is kept horizontal by supporting & setting a 100 N weight by using a string tied at B & passing over a smooth pulley at C The beam also loaded as shown in figure below. Find the reactions at A & C. [M-16]



[Ans.: $R_A = 67.25$ N, $\theta = 48.45°$, $R_C = 84.89$ N, $\alpha = 58.23°$]

4) Figure shows a beam AB hinged at A and roller supported at B. The L shaped portion is welded at D to the beam AB. For the loading shown, find the support reactions.

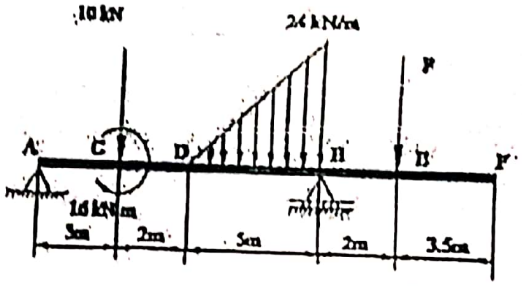
[D-16]



Ans.: $R_B = 25.08$ kN (↑)
$R_A = 44.66$
$\theta = 69.04°$

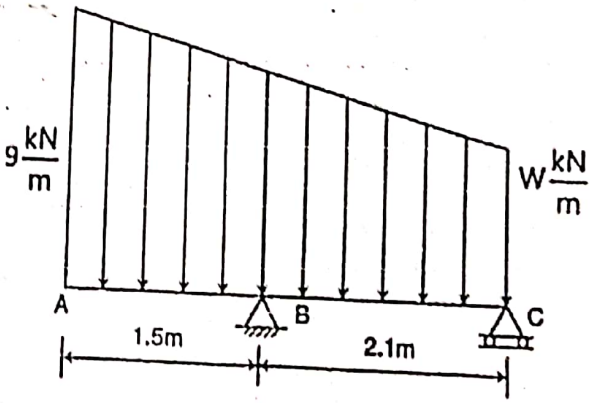
5) If the support reaction at A, for the beam shown in Figure, is zero, then find force 'P' and the support reaction at B.

[D-18]



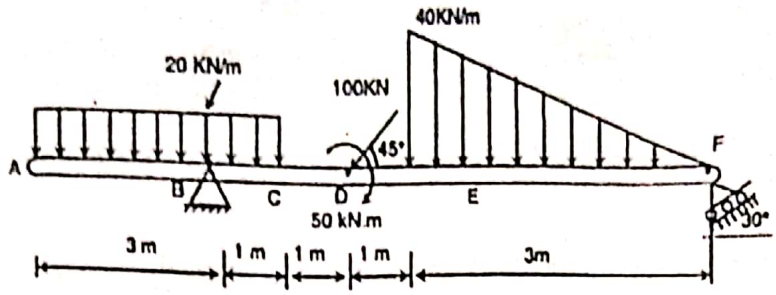
[Ans.: P = 77 kN, R_B = 147 kN]

6) Determine the intensity of distributed load W at the end C of the beam ABC for which the reaction at C is zero. Also calculate the reaction at B.



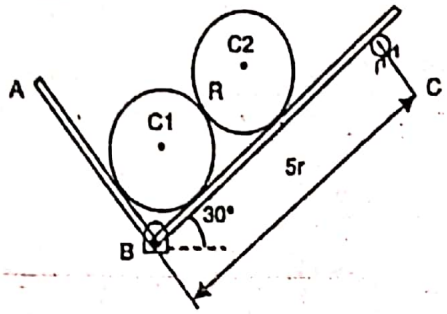
[Ans.: W = 3 kN/m,
R_B = 21.6 kN (↑)]

7) Find the support reactions for the beam loaded and supported as shown in fig. [D-12]



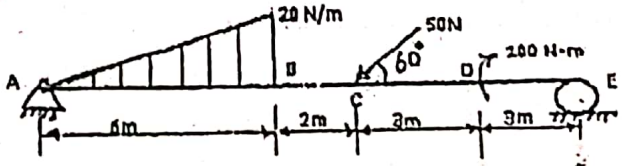
[Ans.: R_F = 67.63 kN (↖), H_B = 104.53 kN (→), V_B = 152.14 kN(↑)]

8) Two identical rollers each of weight 500N and radius r are kept on a right angle frame ABC having negligible weight. Assuming smooth surfaces, find the reactions induced at all contact surfaces. [D-12]



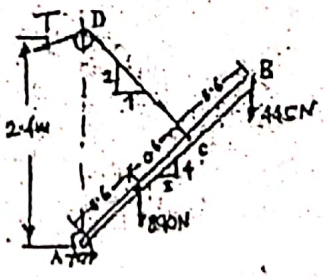
[Ans.: R_1 = 250N, R_2 = 433.01N, R_3 = 500 N(↗), R_4 = 433.01 N R_C = 246.41 N, H_B = 500 N(↗), V_B = 619.62 N(↖)]

9) Find the reactions at supports A and E for the beam loaded as shown in the figure below. [M-16]



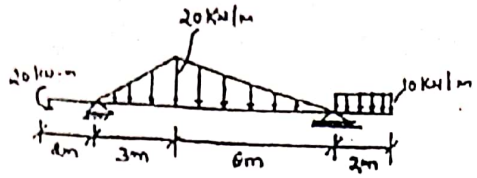
[Ans.: R_E = 49.03 N, R_A = 59.75 N, θ = 65.27°]

10) A boom AB is supported as Shawn in fig by a cable runs 'C' over a small smooth pulley at D. Compute the tension T in cable and reaction at A. Neglect the wt of the boom and size of the pulley. [M-17]



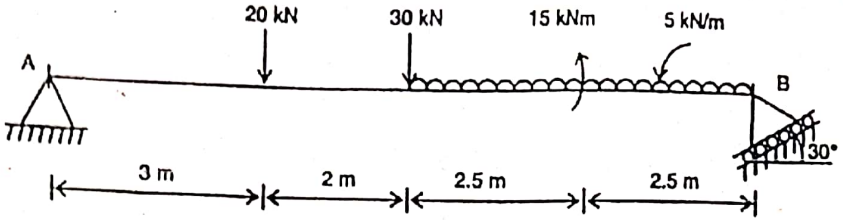
[Ans.: T = 678.43 N, RA = 788.92 N, θ = 67.38°]

11) Find the reactions at the supports of the beam loaded as shown in figure.



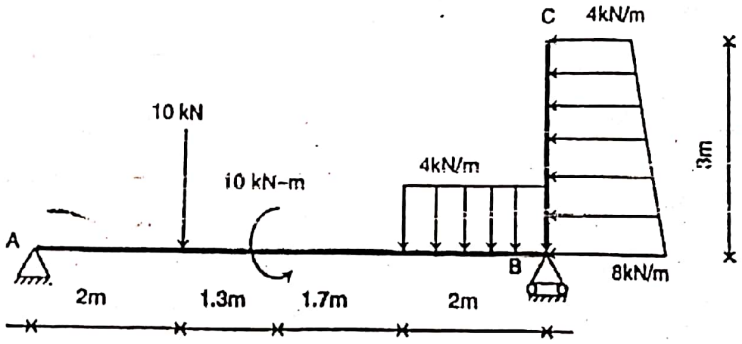
[Ans.: $R_B = 60$ kN(↑), $V_A = 50$ kN(↑), $H_A = 0$]

12) Find the reactions at the supports of the beam, applying conditions of equilibrium.



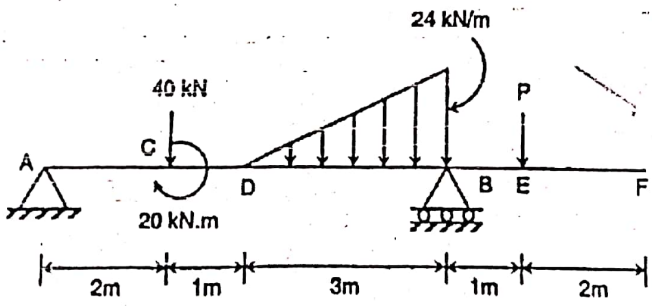
[Ans.: $H_A = 22.08$ kN (→), $V_A = 36.7$ kN (↑), $R_B = 44.16$ kN (60°)]

13) Find reactions at A and B for a bent beam ABC loaded is as shown in figure.



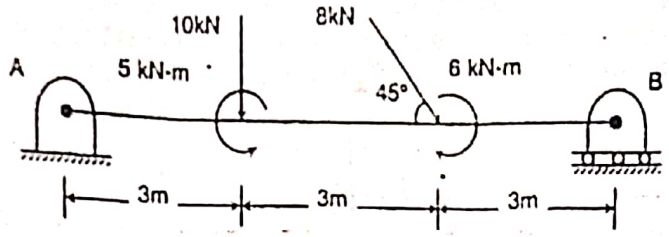
[Ans.: $H_A = 18$ kN, $V_A = 13.15$ kN, $R_B = 4.85$ kN]

14) Find the support reaction at B and the load P, for the beam shown in figure if the reaction at support A is zero. [M-14]



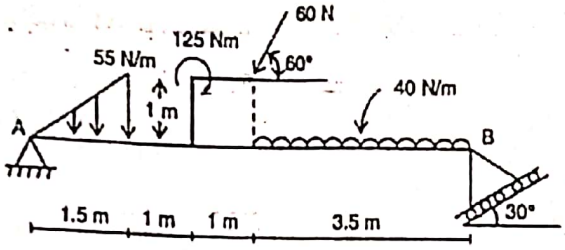
[Ans.: P = 176 kN (↓), $R_B = 252$ kN(↑)]

15) Find the reactions at the supports of the beam applying conditions of equilibrium.



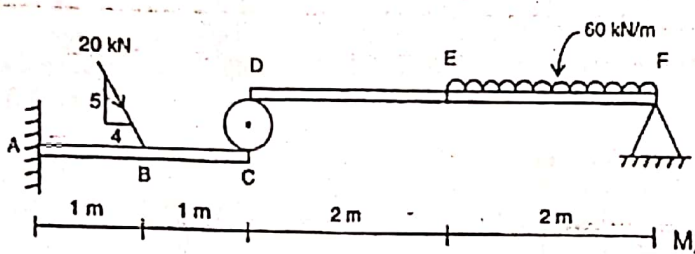
[Ans.: $R_A = 10.16$ kN, $\theta = 56.15°$, $R_B = 7.216$ kN (↑)]

16) Find support reaction of the beam as shown in fig.



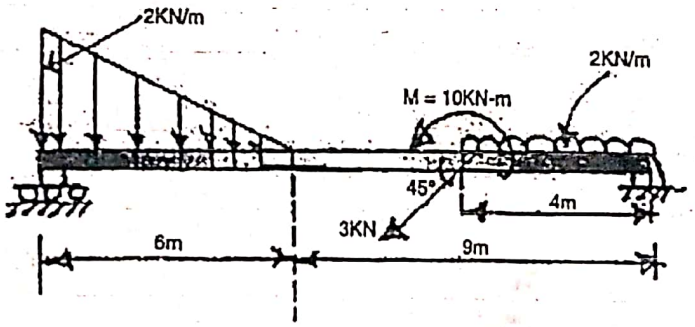
[Ans.: $H_A = 116.86$ N (→), $V_A = 82.77$ N (↑), $R_B = 173.72$ N (60°)]

17) Fig. shows two beams AC and DF. The end D of the beam DF rests on a roller which in turn rests on the end C of the beam AC. For the loads determine support reactions.



[Ans.: $H_F = 0$, $V_F = 90$ kN (↑), $H_A = 125$ kN (←), $V_A = 45.62$ kN (↑), $M_A = 75.62$ kNm (↺)]

18) Find the reactions at the supports of the Beam AB loaded as shown in the figure below.

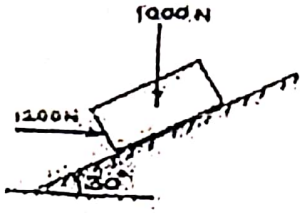


[Ans.: $R_B = 8.88$ kN, $\theta = 76.18°$ (∠), $R_A = 7.5$ kN (↑)]

Module-3

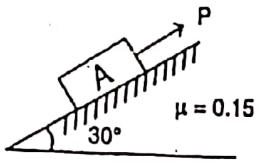
Friction

1) If a horizontal force of 1200N is applied to block of 1000N, then block will be held in equilibrium or slide down or move up? Take $\mu = 0.3$ **[M-18]**



[Ans.: Block will move up the plane]

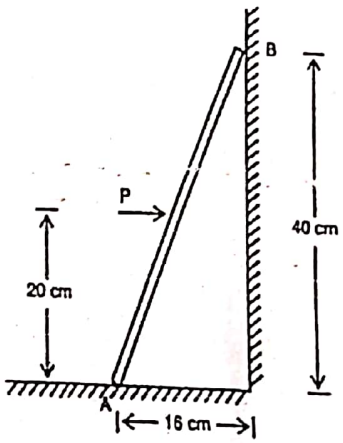
2) A block of weight 1000 N is kept on a rough inclined surface. Find out range of P for which the block will be in equilibrium. **[M-13]**



[Ans.: 370.1 ≤ P ≤ 629.9N]

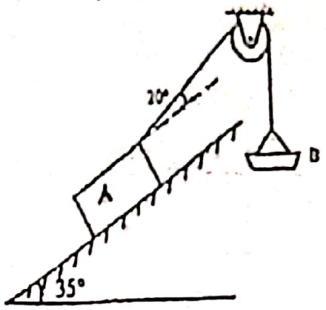
3) A uniform ladder weighing 100 N and 5 meters long has lower end B resting on the ground and upper end A resting against a vertical wall. The inclination of the ladder with horizontal is 60°. If the coefficient of friction at all surfaces of contact is 0.25, determine how much distance up along the ladder a man weighing 600 N can ascend without causing it to slip. **[Ans.: 2.3 m]**

4) A 100 N uniform rod AB is held in the position shown in figure. If coefficient of friction is 0.15 at A and B. Calculate range of values of P for which equilibrium is maintained. **[M-19]**



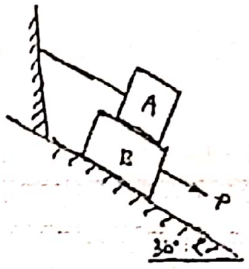
[Ans.: $P_{min} = 8.29$ N, $P_{max} = 80.6$ N]

5) Block A of weight 2000 N is kept on an inclined plane at 35°. It is connected to weight B by an inextensible string passing over smooth pulley. Determine the weight of pan B so that B just moves down. Assume $\mu = 0.2$. [D-16]



[Ans.: P = 1462.97 N]

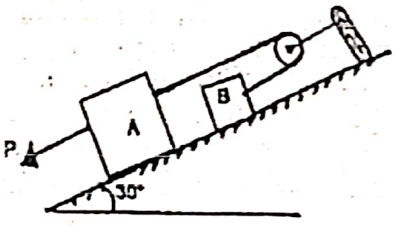
6) Ref to fig. If the coeff. of friction is 0.60 for all contact surfaces and $\theta = 30°$, what force 'P' applied to the block 'B' acting down and parallel to the incline will start motion and what will be the tension in the cord parallel to inclined plane attached to 'A'. Take $W_A = 120N$ and $W_B = 200N$. [M-17]



[Ans.: P = 128.63 N]

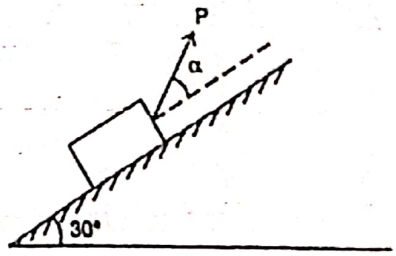
7) Determine the force 'P' to cause motion to impend. Take masses of blocks A and B as 8 kg and 4 kg respectively and the coefficient of sliding friction as 0.3. The force 'P' and rope are parallel to the inclined plane. Assume frictionless pulley.

[Ans.: P = 10.96 N]

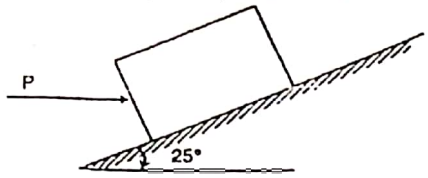


8) Determine the minimum value and the direction of a force P required to cause motion of a 100 Kg block to impend upon a 30° plane. The coefficient of friction is 0.20. (Refer fig.).

[Ans.: $\alpha = 11.31°$, $P_{min} = 647.59$ N]

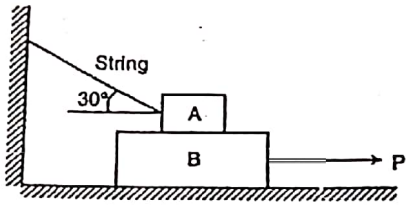


9) A block of weight 800N is acted upon by a horizontal force P as shown in figure. If the coefficient of friction between the block and incline are $\mu_s = 0.35$ and $\mu_k = 0.25$, determine the value of P for impending motion up the plane. [D-15]



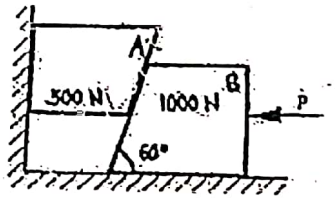
[Ans.: P = 780.4 N]

10) Find force requires to pull block B as shown. Coefficient of friction between A and B is 0.3 and between B and floor is 0.25. Mass of A = 40 kg and B = 60 kg. [D-15]



[Ans.: P = 331.07 N]

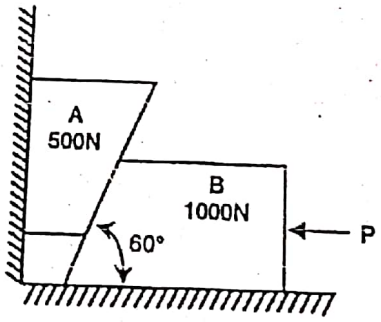
11) Two blocks A and B are resting against the wall and floor as shown in the figure. Find minimum value of P that will hold the system in equilibrium. Take $\mu = 0.25$ at the floor, $\mu = 0.3$ at the wall and $\mu = 0.2$ between the blocks. [D-16]



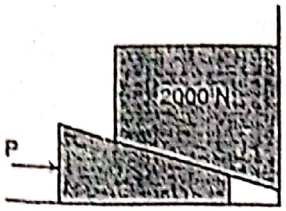
[Ans.: P = 81 N]

12) Assuming the values for $\mu = 0.25$ at the floor and 0.3 at the wall and 0.2 between the blocks, find the minimum value of horizontal force P applied to the lower block that will hold the system in equilibrium. [D-12]

[Ans.: P = 80.86 N]

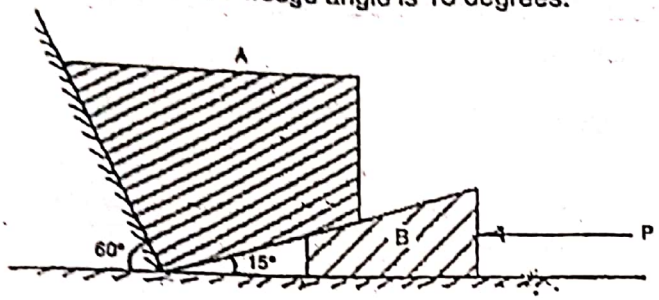


13) Find the necessary force to raise a heavy stone block of 2000 N. Take coefficient of friction as 0.25 for all surfaces. Neglect the weight of wedge. Take angle of wedge as 15°. **[D-13]**



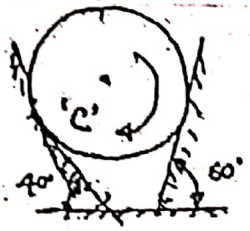
[Ans.: P = 1866.34 N]

14) Determine the force 10 required to move the block A of 5000N weight up the inclined plane, coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15 degrees. **[D-17]**



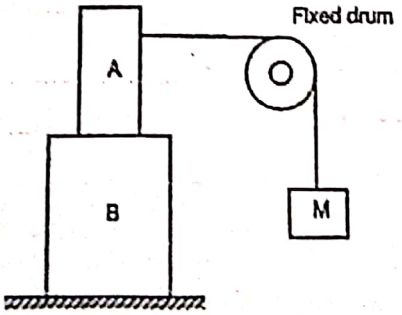
[Ans.: P = 3474 N]

15) A homogeneous cylinder 3m diameter and weighting 400N is resting on two rough inclined surface's shown. If the angle of friction is 15° find couple 'C' applied to the cylinder that will start it rotating clockwise. **[M-17]**



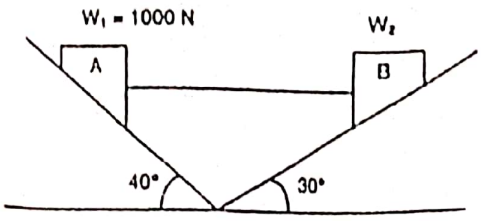
[Ans.: C = 240.66 Nm]

16) The mass of A is 23 Kg and mass of B is 36 Kg. The coefficient of friction are 0.4 between A and B, and 0.2 between ground and block B. Assume smooth drum. Determine the maximum mass of M at impending motion. **[M-14]**



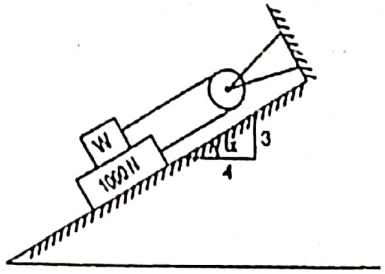
[Ans.: M = 9.2 Kg]

17) Two loads W_1 and W_2 resting on two inclined planes are connected by a horizontal bar AB as shown in fig. If W_1 equals 1000 N, determine the minimum and the maximum values of W_2 for the equilibrium can exist. The angle of limiting friction is 20° at all rubbing faces.



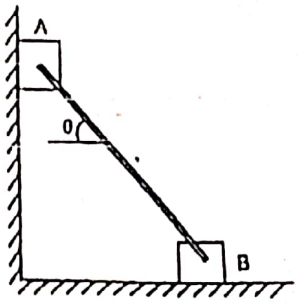
[Ans.: W_{min} = 300.63 N, W_{max} = 9984.47 N]

18) Determine the minimum weight W to prevent downward motion of the 1000 N body.
Take μ = 0.2 for all surfaces of contact.



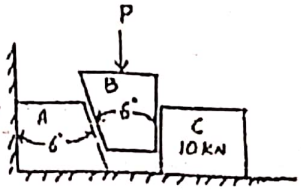
[Ans.: W = 175.3 N]

19) Two identical blocks A and B are connected by rod and rest against vertical and horizontal planes respectively as shown in the figure. If sliding impends when θ = 45°, determine the coefficient of friction μ, assuming it to be the same at both floor and wall.



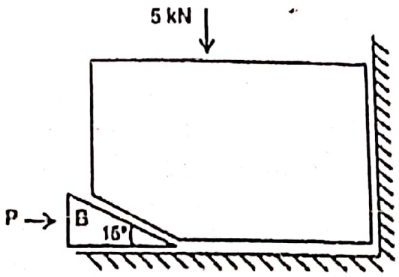
[Ans.: μ = 1/3]

20) Two 6° wedges are used to push a block horizontally as shown. Calculate the minimum force required to push the block of weight 10 kN. Take μ = 0.25 for all contact surfaces.

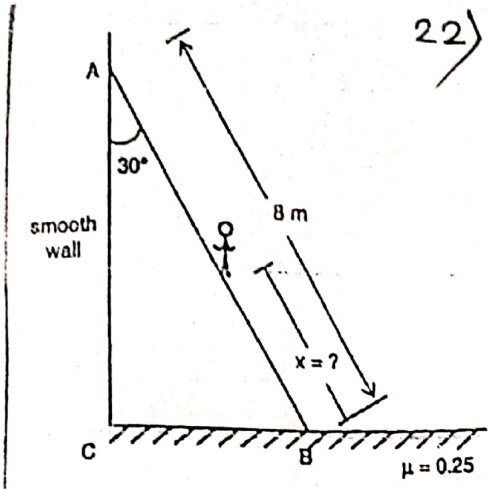


[Ans.: 1.629 kN]

21) The wedge B is used to raise the weight of 5 kN resting on a block A. What horizontal force P, is required to do this, if the coefficient of friction for all the surfaces of contact is 0.2 ?



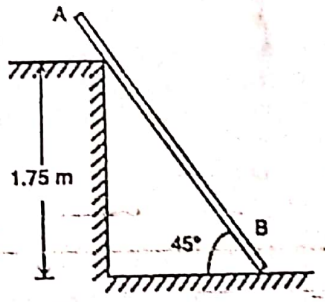
[Ans.: 3.86 kN]



22) A weightless ladder of length 8 m is resting against a smooth vertical wall and rough horizontal ground as shown in the figure. The coefficient of friction between ground and ladder is 0.25. A man of weight 500 N wants to climb up the ladder. Find how much distance along the ladder the man can climb without slip. A second person weighing 800 N wants to climb up the same ladder. Would he climb less than the earlier person? Find his distance covered.

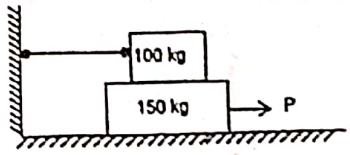
[Ans.: 3.46 m, same distance]

23) Determine minimum value of co-efficient of friction so as to maintain the position shown in figure. Length of Rod AB is 3.5 m and it weighs 250 N.



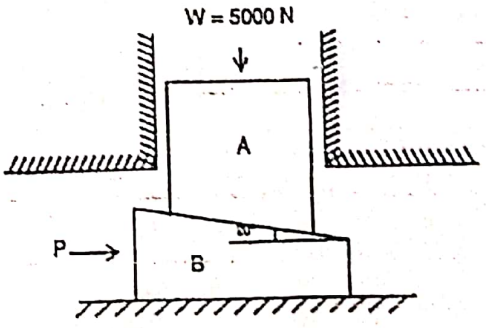
[Ans.: $\mu = 0.414$]

24) The coefficients of friction are $\mu_s = 0.3$ and $\mu_k = 0.25$ between all surfaces of contact. If a force of P = 900 N is applied as shown in figure, find the resultant of frictional force on 150 kg block.



[Ans.: R = 900 N (static),
R = 858.37 N (kinetic)]

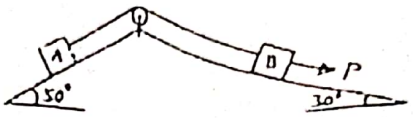
25) The block A as shown in figure, supports a load W = 5000 N and is to be raised by forcing the wedge B under it. The angle of friction for all surfaces in contact is $\phi = 15°$. Determine the force P which is necessary to start the wedge under the block. The block and wedge have negligible weight.



[Ans.: 5958.77 N]

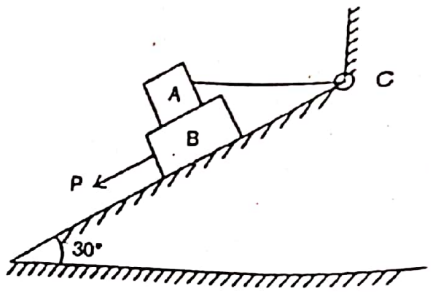
26) A uniform ladder weighing 100 N and 5 meters long has lower end B resting on the ground and upper end A resting against a vertical wall. The inclination of the ladder with horizontal is 60°. If the coefficient of friction at all surfaces of contact is 0.25, determine how much distance up along the ladder a man weighing 600 N can ascend without causing it to slip. [Ans.: 2.3 m]

27) Two blocks A and B of weight 500 N and 750 N respectively are connected by a cord that passes over a frictionless pulley as shown in figure. The coefficient of friction between the block A and the inclined plane is 0.4 and that between the block B and the inclined plane is 0.3. Determine the force P to be applied to block B to produce the impending motion of block B down the plane.



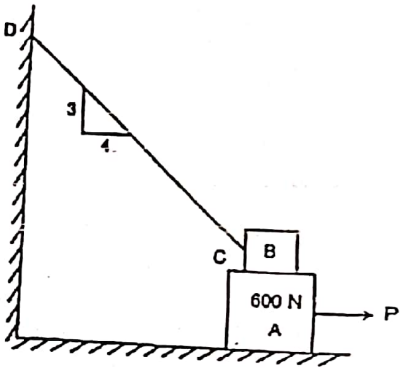
[Ans.: P = 331.4 N]

28) Block A of mass 30 Kg, rests on block B of mass 40 Kg. Block A is restrained from moving by a horizontal rope tied at point C, what force P applied parallel to the plane inclined at 30° with horizontal is necessary to start block B down the plane. Take coefficient of friction for all surfaces as 0.35.



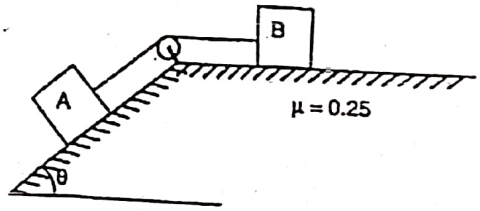
[Ans.: 225.14 N]

29) Calculate the force P required to cause the block A of weight 600 N to slide under the block B of weight 200 N as shown in figure. What will then be the tension in the string CD ? Assume the coefficient of friction for all surfaces of contact as 0.2.



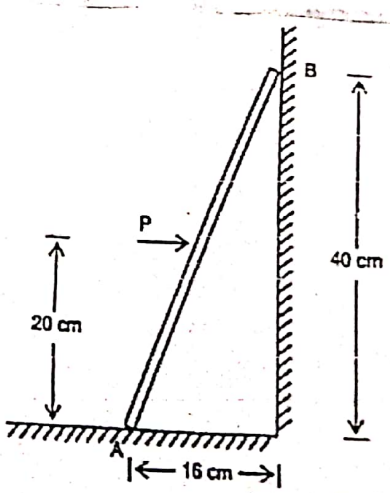
[Ans.: T = 43.47 N, P = 189.56 N]

30) Find the value of θ if the blocks A and B shown in figure have impending motion. Given Block A = 20 Kg, Block B = 20 Kg, $\mu_A = \mu_B = 0.25$.



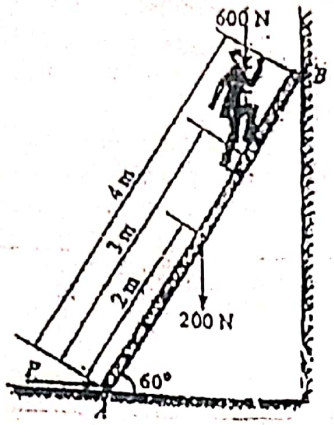
[Ans.: θ = 28.07°]

31) A 100 N uniform rod AB is held in the position shown in figure. If coefficient of friction is 0.15 at A and B. Calculate range of values of P for which equilibrium is maintained. [M-19]



[Ans.: P_{min} = 8.29 N, P_{max} = 80.6 N]

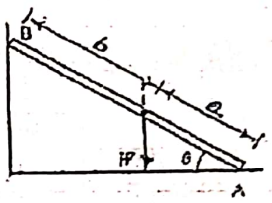
32) A ladder of 4 m length weighing 200 N is placed as shown in figure. $\mu_B = 0.25$ & $\mu_A = 0.35$. Calculate the minimum horizontal force to be applied at A to prevent slipping. **[M-15]**



[Ans.: P = 21.77 N(→)]

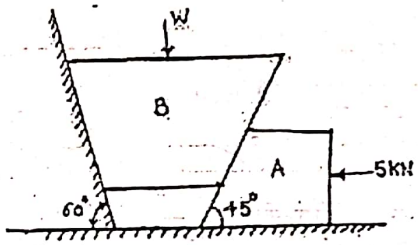
33) A heavy metal bar AB rests with its lower end A on a rough horizontal floor having coefficient of friction μ_F & the other end B on a rough vertical wall having coefficient of friction μ_w. If the centre of gravity of the bar is at distances a & b from the ends A & B respectively, show that at impending motion, the inclination of the bar with the horizontal will be : **[M-16]**

$$\theta = \tan^{-1}\left[\frac{1}{\mu_F} - \frac{(\mu F \mu_w + 1)b}{(a+b)\mu F}\right]$$



[Ans.: $\theta = \tan^{-1}\left[\frac{1}{\mu_F} - \frac{(\mu_F \omega + 1)b}{(a+b)\mu_F}\right]$**]**

34) A horizontal force of 5 KN is acting on the wedge as shown in fig. The coeffecient o friction at all rubbing surfaces is 0.25. Find the load 'W' which can be held in position. The weight of block 'B' may be neglected. **[M-18]**



[Ans.: W = 22.89 kN]

35) A ladder AB of length 3m and weight 25 kg is resting against a vertical wall and a horizontal floor. The ladder makes an angle 50 degrees with the floor. A man of weight 60 kg tries to climb the ladder. How much distance along the ladder he will be able to climb if the coefficient of friction between ladder and floor as 0.2 and that between ladder and wall as 0.3. Also find the angle the ladder should make with the horizontal such that the man can climb till the top of the ladder. **[D-14]**

[Ans.: x = 0.57 m, θ = 76.67°]

Module-4

Kinematics of Particle

Rectilinear Motion

Rectilinear Motion under uniform acceleration

1) A particle travels along a straight line path such that in 4 sec it moves from an initial position $S_A = -8$ m to position $S_B = +3$ m. Then in another 5 seconds it moves from S_B to $S_C = -6$ m. Determine the particles average velocity and average speed during 9 seconds interval. [Ans.: (i) 2/9 m/s; (ii) 20/9 m/s]

2) A particle starts moving along a straight line with initial velocity of 20 m/s, from O under a uniform acceleration of -2 m/s^2. Determine distance covered at (i) t = 6 sec. (ii) t = 16 sec (iii) t = 25 sec

[Ans.: (i) d = 84 m; (ii) d = 136 m; (iii) d = 325 m]

3) A stone is thrown vertically upwards and returns to the starting point at the ground in 6 sec. Find out max. height and initial velocity of stone. [M-13]

[Ans.: h = 44.145 m]

4) A motorist is travelling at 90 kmph, when he observes a traffic signal 250 m ahead of him turns red. The traffic signal is timed to stay red for 12 sec. If the motorist wishes to pass the signal without stopping just as it turns green. Determine (i) The required uniform deceleration of the motor. (ii) The speed of motor as it passes the signal. [D-13]

[Ans.: a = – 0.694 = 0.694 m/s^2 (←), V = 16.672 m/s (→)]

5) Car A starts from rest & accelerates uniformly on a straight road. Another car B starts from the same place 5 seconds later with initial velocity zero & it accelerates uniformly at 5 m/sec^2. If both the cars overtake at 500 m from the starting place, find the acceleration of car A. [M-16]

[Ans.: a = 2.73 m/s^2]

6) Two cars start towards each other from stop X & stop Y at 1:36 PM, the first car reaches stop Y, travelling 8 km path at 1:44 PM. Second car reaches stop X at 1:46 PM. If they move at uniform velocity, determine their time of meeting & their distance from stop X. [Ans.: t = 266.67 sec. & d = 4445.33 m] [M-15]

A stone is dropped from a balloon at an altitude of 600 metres. How much time is required for the stone to reach the ground if the balloon is

(a) ascending with a velocity of 10 m/sec.
(b) descending with a velocity of 10 m/sec.
(c) stationary
(d) ascending with a velocity of 10 m/sec and an acceleration of 1 m/s^2. (Neglect the air resistance)

[Ans.: (a) 12 sec; (b) 10 sec; (c) 10.95 sec; (d) 12 sec]

7) A stone falls freely from rest. Distance travelled by it in the last second of its motion is equal to distance covered in first three seconds of its motion. Determine time of flight.

[Ans.: Time of flight t = 5 sec.]

8) In Asian games, for 100 m event an athlete accelerates uniformly from the start to his maximum velocity in a distance of 4 m and runs the remaining distance with that velocity. If the athlete finishes the race in 10.4 sec, determine (i) his initial acceleration, (ii) his maximum velocity. [D-13]

[Ans.: (i) 12.5 m/s^2, (ii) v = 10 m/s]

Rectilinear Motion under variable acceleration

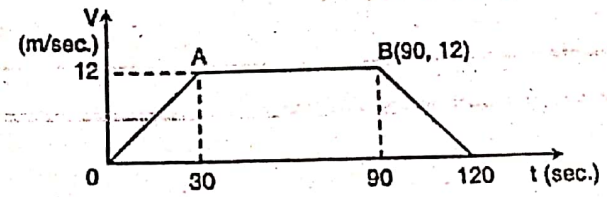
9) The car moves in a straight line such that for a short time its velocity is defined by $v = (9t^2 + 2t)$ m/s. Where t is in seconds. Determine its position and acceleration when t = 3 sec. [Ans.: $S_3 = 90$ m, $a_3 = 56$ m/s^2] [D-12]

10) A particle moving in the +ve x direction has an acceleration $a = 100 - 4v^2$ m/s^2. Determine the time interval and displacement of a particle when speed changes from 1 m/s to 3m/s. [D-12]
[Ans.: t = 0.025 sec., S = 0.051 m]

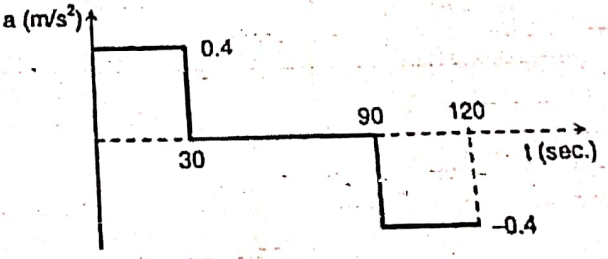
11) Acceleration of a particle moving along a straight line is represented by the relation $a = 30 - 4.5\ x^2$ m/s^2. The starts with zero initial velocity at x = 0. Determine (a) the velocity when x = 3 m (b) the position when the velocity is again zero (c) the position when the velocity is maximum. [D-14]
[Ans.: (i) V = 9.95 m/s, (ii) x = 4.47 m, (iii) x = 2.58 m]

12) A particle starts from rest from origin and its acceleration is given by, $a = \frac{k}{(x+4)^2}$ m/s^2. Knowing that V = 4 m/s when x = 8m, find (i) value of k and (ii) Position when V = 4.5 m/s. [D-16]
[Ans.: k = 48, x = 21.6 m]

13) Figure shows the v-t diagram for the motion of a train as it moves from station A to station B. Draw a-t graph & find the average speed of the train & the distance between the stations. [M-15]



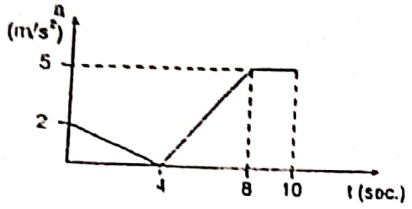
[Ans.:



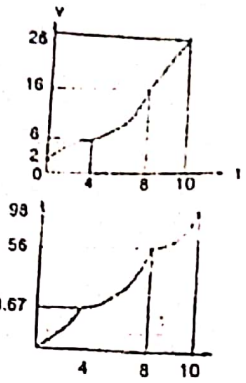
Avg. speed = 9 m/s, Distance = 1080 m]

Motion Curves

14) A particle is projected with an initial velocity of 2m/s along a straight line. The relation between acceleration and time is given in the diagram. Draw v-t and s-t diagram. [D-14]

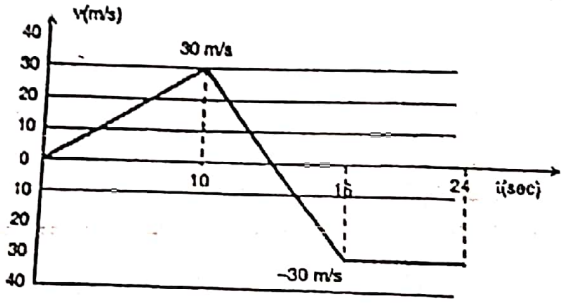


[Ans.:

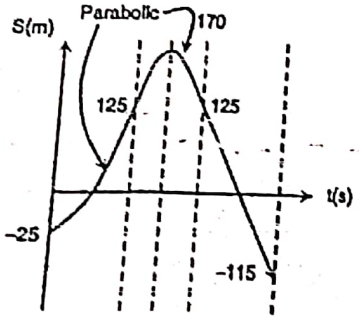


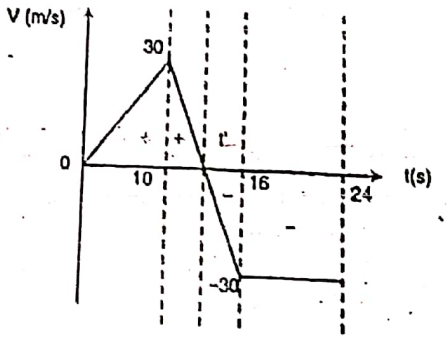
]

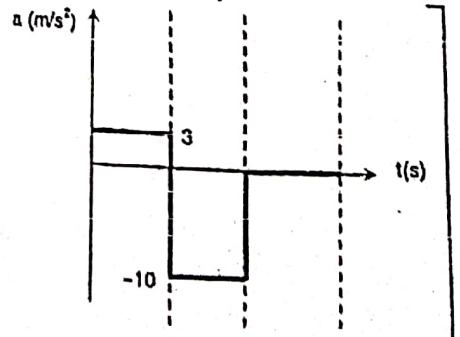
16) A particle moves in a straight line with a velocity-time diagram shown in figure. If S =–25m at t = 0, draw displacement-time and acceleration time diagrams for 0 to 24 seconds. [M-14]



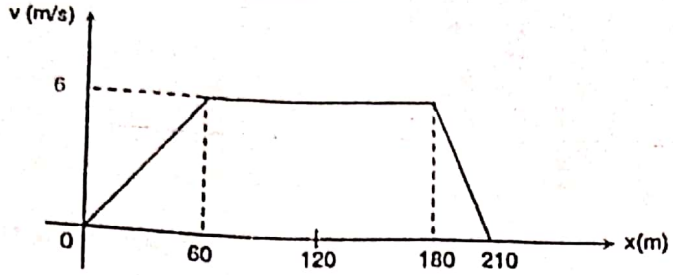
Ans.:







17) The V-X graph of a rectilinear moving particle is shown. Find acceleration of the particle at 20m, 80 m and 200 m. [D-17]



[Ans.: $a_{20} = 0.2\ m/s^2$, $a_{80} = 0$, $a_{200} = -0.4\ m/s^2$]

Curvilinear Motion

18) A particle is moving in X-Y plane and its position is defined by $\bar{r} = \left(\frac{3}{2}t^2\right)i + \left(\frac{2}{3}t^3\right)J$. Find ratio of curvature when t=2 sec. [D-17]

[Ans.: $\rho = 41.667m$]

19) The position vector of a particle which moves in the X-Y plane is given by $\bar{r} = (3t^3 - 4t^2)i + (0.5t^4)J\,m$. calculate velocity and acceleration at t=1 sec. [D-17]

The y coordinate of a particle is given by $y = 6t^3 - 5t$. If $a_x = 14t\ m/sec^2$ & $V_x = 4\ m/sec$ at $t = 0$, determine the velocity & acceleration of particle when $t = 1$ second. [Ans.: V = 17.029 m/s and $a = 38.62\ m/s^2$] [M-15]

20) A particle travels on a circular path, whose distance travelled is defined by $S = (0.5t^3 + 3t)m$. If the total acceleration is $10\ m/s^2$, at t = 2 sec, find its radius of curvature. [D-15]

[Ans.: $\rho = 10.125$ m]

21) A particle moves along a track which has a parabolic shape with a constant speed of 10 m/sec. The curve is given by $y = 5 + 0.3x^2$. Find the components of velocity and normal acceleration when x = 2m. [D-14]

[Ans.: $a_n = 15.742\ m/sec^2$, $V_x = 6.402$ m/sec (→), $V_y = 7.682$ m/sec (↑)]

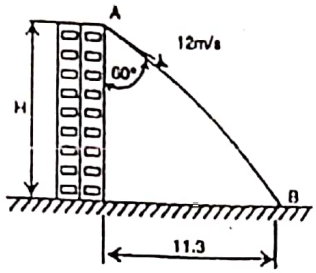
22) A point moves along a path $y = x^2/3$ with a constant speed of 8m/s. What are the x and y components of its velocity when x = 3? What is the acceleration of the point at this instant? [D-12]

[Ans.: $a = a_n = 3.816\ m/s^2$, $V_x = 3.578$ m/s, $V_y = 7.155$ m/s]

Projectile Motion

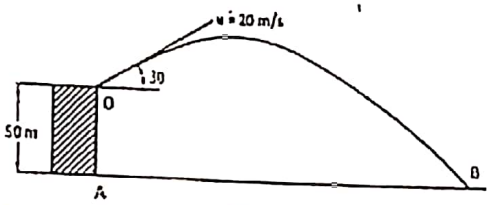
23) Derive the equation of the path of a projectile and hence shown that the path traced by a projectile is a parabolic curve. [D-17]

24) A ball thrown with speed of 12 m/s at an angle of 60° with a building strikes the ground 11.3 m horizontally from the foot of the building as shown. Determine the height of the building. [D-12]



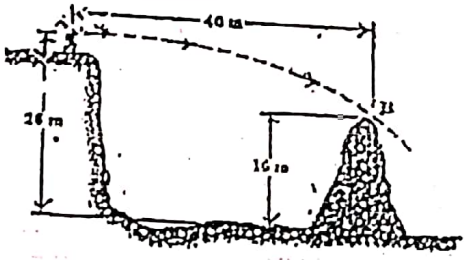
[Ans.: H = 12.32 m]

25) A particle is projected from the top of a tower of height 50 m with a velocity of 20 m/sec at an angle 30 degrees to the horizontal. Determine : [D-14]
(I) Horizontal distance AB it travel from the foot of the tower.
(II) The velocity with which it strikes the ground at B.
(III) Total time taken to reach point B.



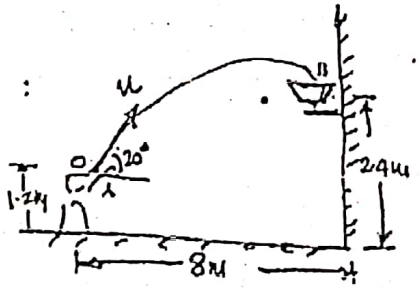
[Ans.: t = 4.371 sec,
AB = 75.71 m,
V_B = 37.163 m/sec,
θ_B = 62.22°]

26) With what minimum horizontal velocity (u) can a boy throw a rock at A & have it just clear the obstruction at B? Refer figure. [M-15]



[Ans.: u = 28.01 m/s]

27) Determine the speed at which the basket ball at 'A' must be thrown at an angle of 30° so that if makes it to the basket at B. Also find at what speed it passes through the hoop. [M-17]

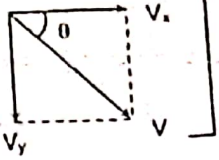


[Ans.: U_0 = 11.06 m/s, V_0 = 9.938 m/s]

28) A fighter plane moving horizontally with a constant velocity of 200 m/seconds releases a bomb from an altitude of 400 m. Find the velocity and direction of the bomb just before it strikes the ground. Also determine the distance travelled by the plane before the bomb just strikes the ground. [Ans.: V_B = 237.16 m/s] [M-16]

29) A gunman fires a bullet with a velocity of 100 m/s, 50° upwards from the top of a hill 300m high to hit a bird. The bullet misses its target and finally lands on the ground. Calculate (a) the maximum height reached by the bullet above the ground (b) total time of flight (c) velocity with which the bullet hits the ground. [M-14]

Ans.: H = 599.06 m, t = 18.86 sec.,
V = 126.04 m/s,
θ = 59.34°



Home Work Problems

Rectilinear Motion under uniform velocity and acceleration

30) A hot air balloon starts rising vertically up from the ground with an acceleration of 0.2 m/s^2. 12 seconds later the man sitting inside the balloon releases a stone. Find the time taken by the stone to hit the ground. [Ans.: t = 1.975 sec] [D-15]

31) A stone is thrown with a velocity (u) m/sec at an angle of 20° with horizontal from a point 2 m above the ground. The stone strikes the ground 5 m away from the original position. The motion of stone is subjected to gravitational acceleration & wind resistance of 0.82 m/sec^2, opposing the horizontal motion. Determine the time of flight of the stone. [Ans.: t = 0.896 sec] [M-16]

32) A radar equipped police car observes a truck travelling 110 kmph. The police car starts pursuit 30 sec. after the observation, accelerates to 160 kmph in 20 sec. Assuming the speeds are maintained constant on a straight road, how far from the observation point, will the chase end ? [Ans.: 3.909 km]

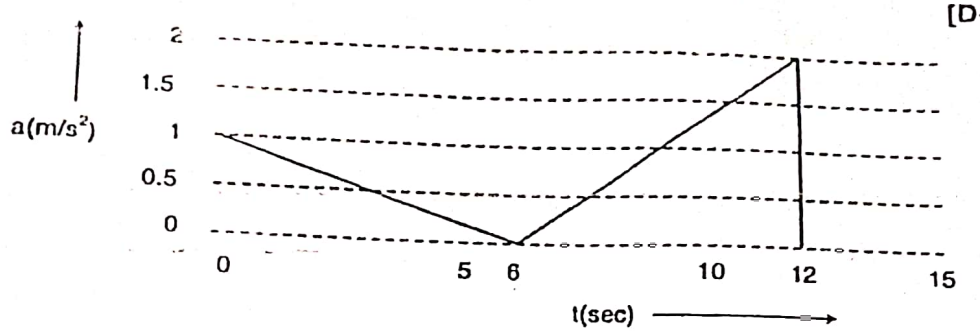
33) In a flood relief area a helicopter going vertically up with a constant velocity drops first batch of food packets which takes 4 seconds to reach the ground. No sooner this batch reaches the ground, second batch of food packets are released and this batch takes 5 seconds to reach the ground. From what height was the first batch is released ? Also determine the velocity with which the helicopter is moving up ? [Ans.: u = 9 m/s; h_1 = 44 m]

34) Water leaks from the ceiling 16 m high, at the rate of 5 drops per second. Find the distance between first and second drop when the first drop has just touched the ground. [Ans.: d = 3.347 m]

35) A stone is dropped from the top of a tower. When it has fallen a distance of 10 m, another stone is dropped from a point 38 m below the top of the tower. If both the stones reach the ground at the same time calculate :
(i) the height of the tower.
(ii) the velocity of the stones when they reach the ground.
[Ans.: h = 57.62 m, 33.63 m/s, 19.62 m/s]

36) A particle moves in a straight line with acceleration time diagram shown in figure. Construct velocity-time diagram for the motion assuming that the motion starts with initial velocity of 5 m/s from the starting point. Also determine its displacement at t = 12 seconds.

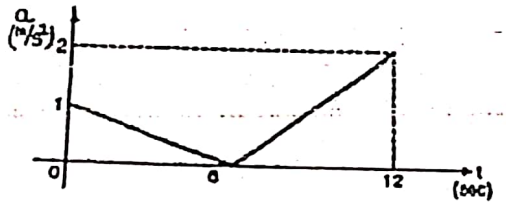
[D-15]



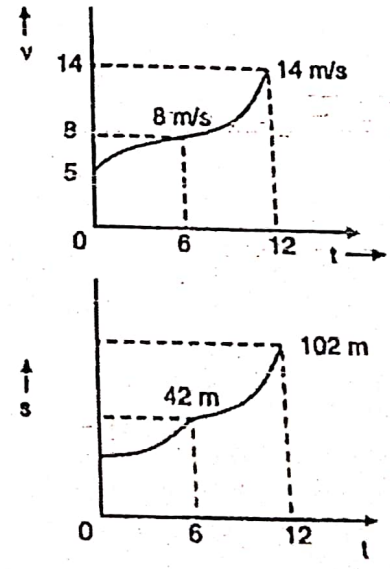
[Ans.: $s_{12} \cong 102m$]

37) The acceleration-time diagram for linear motion is shown. Construct velocity-time diagram and displacement-time diagram for the motion assuming that the motion starts with initial velocity of 5 m/s from starting point.

[D-16]



Ans.:



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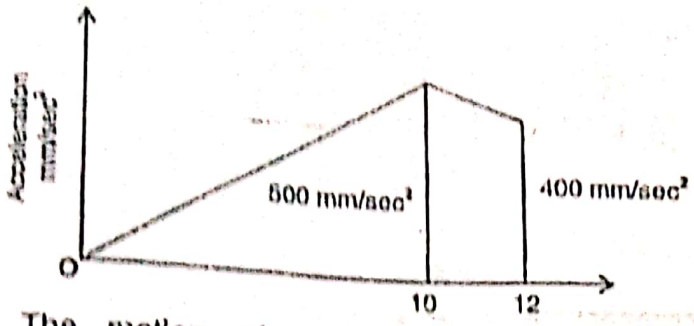
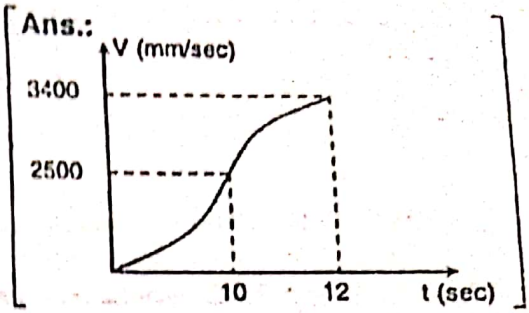
38) The motion of a particle from rest is given by the acceleration time diagram given in figure. Sketch velocity time diagram and hence calculate velocity at t = 12 sec.

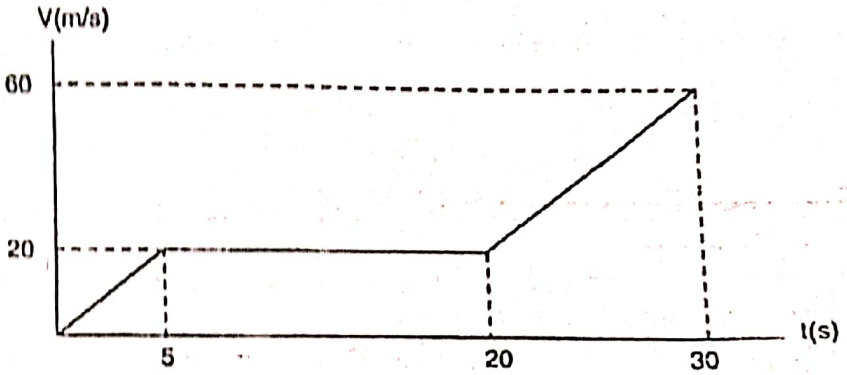


Ans.:



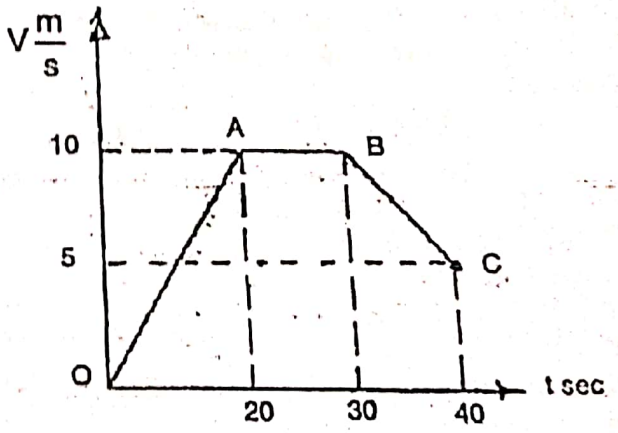
39) The motion of a jet plane while travelling along a runway is defined by the v–t graph shown. Construct the s–t and a–t graphs for the motion. The plane starts from rest.

[D-18; M-19]



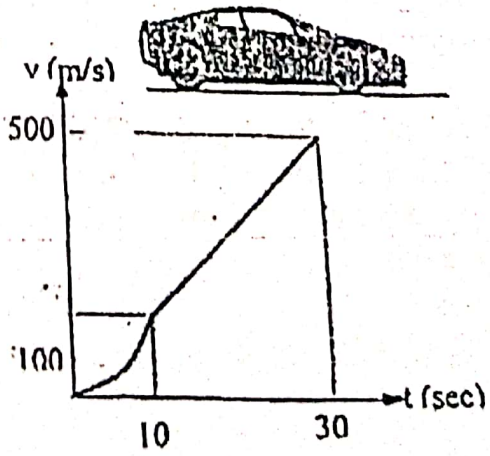
[Ans.: $a_n = 4\ m/s^2$, $s_5 = 50\ m$, $s_{20} = 350\ m$, $s_{30} = 750\ m$]

40) Velocity-Time Graph for a particle moving along a straight line is given below. Draw Displacement-Time and Acceleration-Time graphs. Also find the Maximum Displacement of the Particle.



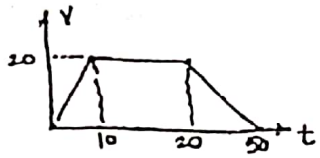
[Ans.: $a_{0-20} = 0.5\ m/s^2$, $a_{20-30} = 0$, $a_{30-40} = -0.5\ m/s^2$, $S_{20} = 100\ m$, $S_{30} = 200\ m$, $S_{40} = S_{max} = 275\ m$]

41) A car moves along a straight road such that its velocity is described by the graph shown in figure.
For the first 10 seconds the velocity variation is parabolic and between 10 seconds to 30 seconds the variation is linear. Construct the s-t and a-t graphs for the time period $0 \le t \le 30s$.



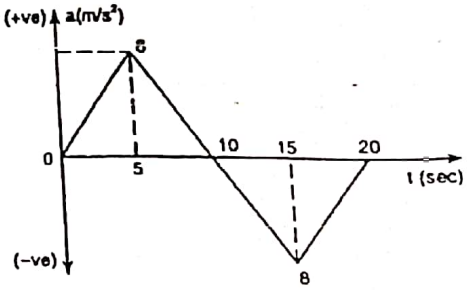
[Ans.: $S_{10} = 333.33\ m$, $a_{10} = 20\ m/s^2$, $S_{30} = 6333.33\ m$]

42) From (v–t) diagram find [M-17]
(i) distance travelled in 10 sec. (ii) total distance travelled in 50 sec.
(iii) Retardation.



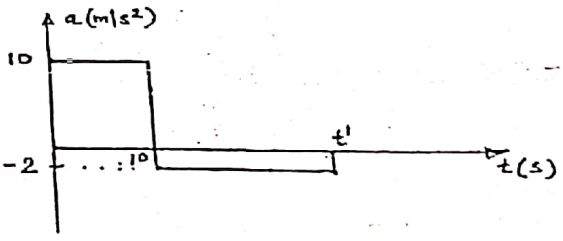
[Ans.: X_{10} = 100 m, ΔX = 600 m]

43) The acceleration–time diagram for the linear motion in figure. Construct velocity time and displacement time diagrams for the motion assuming that the motion starts from rest.



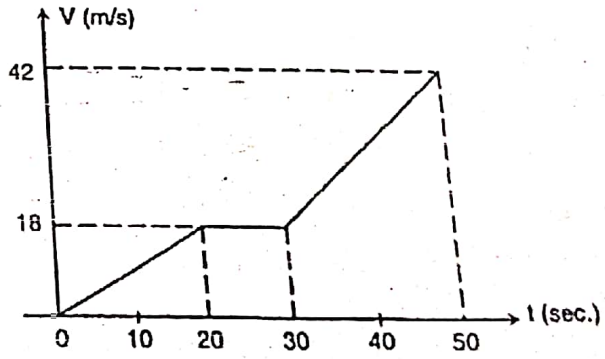
[Ans.: V_0 = 0m/s, V_5 = 20m/s, V_{10} = 40m/s, V_{15} = 20m/s, V_{20} = 0m/s, S_0 = 0m, S_5 = 33.33m, S_{10} = 200m, S_{15} = 366.67m, S_{20} = 400m]

44) The car starts from rest and travels along a straight track such that it accelerates at a constant rate for 10 seconds and then decelerates at a constant rate. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled ?



[Ans.: t = 60 sec, d = 3000 m]

45) The race car starts from rest and travels along a straight road until it reaches a speed of 42 m/s in 50 seconds as shown by v-t graph. Determine the distance travelled by race car in 50 seconds. Draw x-t and a-t graph.



[Ans.: d = 960 m]

46) Starting from rest at S = 0, a car travels in a straight line with an acceleration as shown by the a-s graph. Determine the cars speed when S = 20 m, S = 100m and S = 150 m. [M-18]

[Ans.: V_{20} = 6.324 m/s; V_{100} = 17.88 m/s; V_{150} = 20.736 m/s]

47) A particle falling under gravity travels 25 m in a particular second. Find the distance travelled by it in next three seconds. [D-16]

[Ans.: 132 m.]

48) The accl^n of the train starting from rest at any instant is given by the expression $a=\frac{8}{(v^2+1)}$ where V is the velocity of train in m/s. Find the velocity of the train when its displacement is 20m and its displacement when velocity is 64.8 kmph. [M-17]

[Ans.: V = 4.93 m/s, x = 3300.75 m]

49) The velocity of a particle travelling in a straight line is given by $v = 6t - 3t^2$ m/s. Where t is in seconds. If s = 0, when t = 0, determine the particle's deceleration and position when t = 3s. How far has the particle travelled during the 3 second time interval and what is its average speed ?

[Ans.: $S_3 = 0$, $a_3 = -12$ m/s^2, $d_3 = 8$m, Average speed = (8/3) m/s]

50) The motion of particle is defined by the relation $V = 4t^2 - 3t - 1$ where 'v' is in m/s & 't' is in sec. If the displacement is x = –4m at t = 0, determine

(i) The time at which particle reverses its sense of motion

(ii) At t=3 sec, (a) acceleration (b) Position (c) displacement (d) distance travelled.

[Ans.: (i) 1 sec (ii) (a) 21 m/s^2 ; (b) 15.5 m (c) 19.5 m (d) 21.82 m]

51) The motion of a particle moving in a straight line is given by the expression, $S = t^3 - 3t^2 + 2t + 5$, where 's' is the position in meters and 't' is time in seconds. Determine, (i) velocity and acceleration after 4 seconds (ii) maximum or minimum velocity and corresponding displacement and (iii) time at which velocity is zero.

[Ans.: (i) V_4 = 26 m/s, a_4 = 18 m/s^2; (ii) V_{min} = –1 m/s, displacement = 0, (iii) t = 1.577 sec or t = 0.422 sec]

52) A particle starting with an initial velocity and traveling in a straight line has an acceleration of (2t + 4) m/s^2 where t is time in sec from start. The distance covered in the first second measured from the starting point is 6.33 m.

Calculate : (i) Initial velocity (ii) the distance covered in the third second.

[Ans.: (i) u = 4 m/s; (ii) Distance covered in third second = 20.34 m]

53) The acceleration of a particle which moves with rectilinear translation is given by a = (t – 2) m/s^2 at t = 0, the displacement and velocity are zero. Find the velocity and displacement when t = 2 sec. and when t = 4 sec.

[Ans.: (i) v = –2 m/s, s = –2.66 m; (ii) v = 0, s = –5.33 m]

54) The acceleration of a particle is given by the relation $a = 90 - 6x^2$ where 'a' is expressed in cm/sec^2 and x in centimeters. If the particle starts with zero initial velocity at position x = 0, determine :
(i) the velocity when x = 5 cm.
(ii) the position where velocity is again zero
(iii) the position where the velocity is maximum.
[Ans.: (i) 20 cm/sec; (ii) 6.708 cm; (iii) 3.873 cm]

55) The velocity of a particle moving along a straight line is given by $v = 2t^3 + 5t^2$ where v is in m/sec and t (> 0) is in seconds. What distance does it travel while its velocity increases from 7 m/s to 99 m/s ?
[Ans.: 83.33 m]

56) The acceleration of the particle is defined by the relation $a = 25 - 3x^2$ mm/s². The particle starts with no initial velocity at the position x = 0. (i) Determine the velocity when x = 2 mm; (ii) the position when velocity is again zero; (iii) position where the velocity is maximum and the corresponding maximum velocity.
[Ans.: (i) V = ± 9.165 mm/s, (ii) x = ± 5 mm,
(iii) x = ± 2.887 mm & V_{max} = ± 9.809 mm/s]

57) The motion of the particle is defined by the relation $a = (0.8t)$ m/sec² where 't' is measured in sec. It is found that at X = 5m, V = 12m/sec when t = 2sec. Find the position and velocity at t = 6 sec. [M-18]
[Ans.: At t = 6 sec; V = 24.8 m/s; S = 74.33 m]

Curvilinear Motion

58) A curvilinear motion of a particle is defined by $v_x = 25 - 8t$ m/s and $y = 48 - 3t^2$ m. At t = 0, x = 0. Find out position, velocity and acceleration at t = 4 sec. [M-13]
[Ans.: V = 25 m/s, θ = 73.74°,
a = 10 m/s², α = 36.87°
Position (x, y) = (36, 0) m]

59) The motion of a particle is defined by the position vector, $\bar{r} = 6t\,i + 4t^2\,j$ where 'r' is in meters and 't' is in seconds. At the instant when t = 3 seconds, find
(i) Tangential and Normal components of accelerations
(ii) Radius of curvature. [Ans.: $a_t = 7.76$ m/s², $a_n = 1.94$ m/s², ρ = 315.47 m]

A particle moves along a circle of radius 20 cm so that $s = 20\pi t^2$ cm. Find its tangential and normal acceleration after it has completed a revolution.
[Ans.: $a_t = 125.66$ cm/s², $a_n = 1579.14$ cm/s²]

60) A rocket follows a path such that its acceleration is given by $\bar{a} = (4i + tj)$ m/s² at $\bar{r} = 0$, it starts from rest.
At t = 10 sec. Determine :
(i) Speed of the rocket
(ii) Radius of curvature of its path
(iii) Magnitude of normal and tangential components of acceleration.
[Ans.: (i) V = 64.03 m/s; (ii) R = 1312.64 m; (iii) $a_t = 10.3$ m/s², $a_n = 3.1233$ m/s²]

61) A particle moves in x–y plane and its position is given by $\bar{r} = (3t)\hat{i} + (4t - 3t^2)\hat{j}$ where r is the position vector of the particle measured in meters at time 't' seconds. Find the radius of curvature of its path and normal and tangential components of acceleration when it crosses x–axis again.

[Ans.: $\rho = 6.94$ m, $a_n = 3.6$ m/s^2, $a_t = 4.8$ m/s^2]

62) A bomb thrown from a plane flying at a height of 400 m moves along the path $\bar{r}(t) = 50t\bar{i} + 4t^2\bar{j}$ where t is in sec and distances are measured in m. The origin is taken as the point where the bomb is released and the y-axis is taken as pointing downwards. Find:
(i) the path of the bomb
(ii) the time taken to reach the ground
(iii) the horizontal distance traversed by the bomb.

[Ans.: (i) $y = (4x^2/2500)$; (ii) $t = 10$ sec; (iii) 500 m]

63) A rocket follows the path such that its acceleration is given by $\bar{a} = (4\bar{i} + \bar{t}\,\bar{j})$ m/s^2. At $t = 0$ it starts from rest. At $t = 10$ seconds, determine (i) speed of rocket, (ii) radius of curvature of its path.

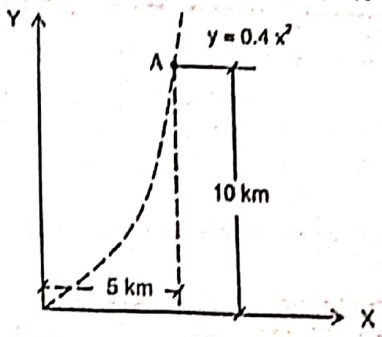
[Hint : Radius of curvature $= \left((V_x^2 + V_y^2)^{3/2} / V_x a_y - V_y a_x\right)$]

[Ans.: (i) $V_x = 40$, $V_y = 50$; (ii) R = 1312.6 m]

64) A particle moves in the x–y plane with velocity components $V_x = 8t - 2$ and $V_y = 2.0$. If it passes through the point (x, y) = (14, 4) at t = 2 seconds, determine the equation of the path traced by the particle. Find also the resultant acceleration at t = 2 seconds.

[Ans.: $x = y^2 - y + 2$; a = 8 (→)]

65) The Jet plane travels along a parabolic path. When it is at point A it has a speed of 200 m/s which is increasing at the rate of 0.8 m/s^2? Determine the magnitude of acceleration of the plane when it is at A.



[Ans.: a = 0.9211 m/s^2]

66) A point moves along the path $y = \frac{1}{3}x^2$ with a constant speed of 8 m/s. What are the x and y components of the velocity when x = 3? What is the acceleration of the point when x = 3? [Ans.: $V_x = 3.76$ m/s; $V_y = 7.152$ m/s; acc. = 3.84 m/s^2]

67) A particle at the position (4, 6, 3) at start, is accelerated at $\bar{a} = 4t\bar{i} - 10t^2\bar{j}$ m/s^2. Determine the acceleration, velocity and the displacement after 2 seconds.

[Ans.: $|\bar{a}| = 40.792$ m/s^2, $|\bar{v}| = 27.84$ m/sec, $|s| = 12.24$m]

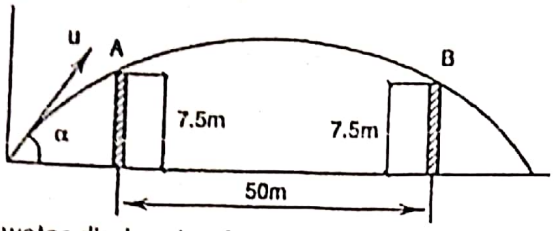
68) A particle moves in a plane with constant acceleration a = 4i m/s^2. At t = 0 the velocity of the particle was $v_0 = i + 1.732\,j$ m/s. Find velocity of the particle at t = 1 sec. [Ans.: v = 5.29 m/s, θ = 19.1°]

69) If x = 1 – t and $y = t^2$, where x and y are in meters and 't' is in seconds, determined x and y components of velocity and acceleration. Also write equation of the path.

[Ans.: $a_x = 0$, $a_y = 2$ m/s^2, $V_x = -1$ m/s, $V_y = 2t$ m/s, $y = (1 - x)^2$]

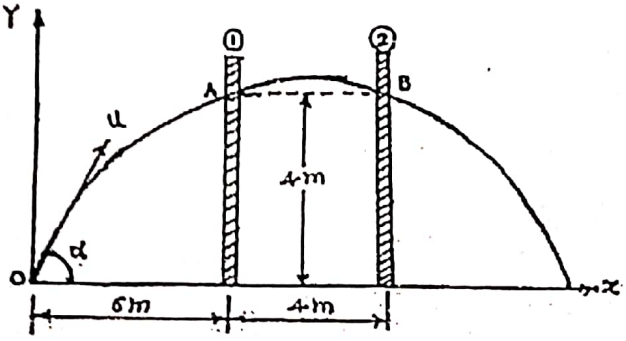
Projectile Motion

70) An object is projected so that it just clears two obstacles each of 7.5 m height, which are situated 50m from each other. If the time of passing between the obstacles is 2.5s, calculate the complete range of projection and the initial velocity of the projectile. [D-15]



[Ans.: u = 26.41 m/s, α = 40.78°, R = 70.32 m]

71) A jet of water discharging from a nozzle hits a vertical screen placed at a distance of 6 m from the nozzle at a height of 4m. When the screen is shifted by 4 m away from the nozzle from its initial position the jet hits the screen again at the same point. Find the angle of projection and velocity of projection of the jet at the nozzle. [M-18]

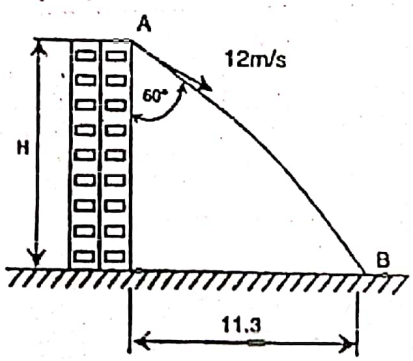


[Ans.: u = 12.54 m/s; θ = 46.85°]

72) A shot is fired with a bullet with an initial velocity of 20 m/s from a point 10 m infront of a vertical wall 5m high. Find the angle of projection with the horizontal to enable the shot to just clear the wall. Also find the range of shot where the bullet falls on the ground. [M-16]

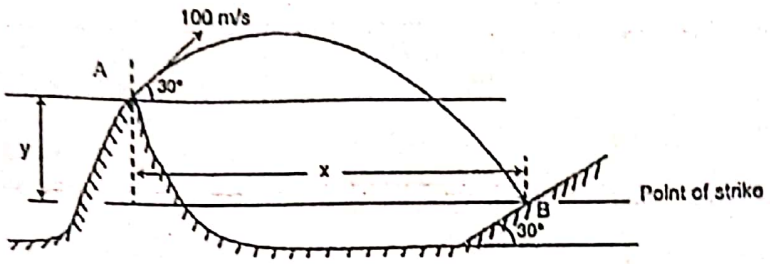
By what percentage the range of projectile is increased if initial velocity is increased by 5%? [Ans.: 10.25%]

73) A ball thrown with speed of 12 m/s at an angle of 60° with a building strikes the ground 11.3 m horizontally from the foot of the building as shown. Determine the height of the building. [D-12]



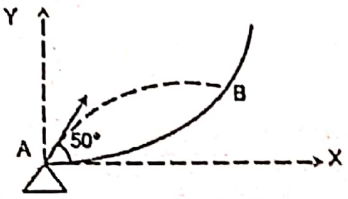
[Ans.: H = 12.32 m]

74) A ball is thrown upward from a high cliff with a velocity of 100 m/s at an angle of elevation of 30° with the horizontal. The ball strikes the inclined ground at right angles. If inclination of ground is 30° as shown, determine :
(i) Time after which the ball strikes the ground.
(ii) Velocity with it strikes the ground.
(iii) Co-ordinates (x, y) of a point of strike w.r.t. point of projection.



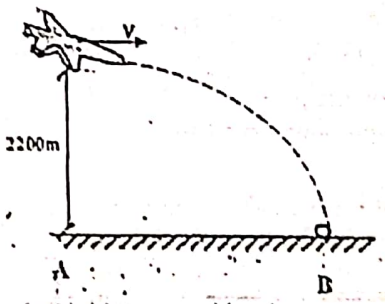
[Ans.: (i) 20.41 sec.; (ii) 173.2 m/sec. (60); (iii) (1.768, –1.021) km]

75) The water sprinkler positioned at the base of a hill releases a stream of water with a velocity of 6 m/s as shown. Determine the point B(x, y) where the water particles strike the ground on the hill. Assume that the hill is defined by the equation $y = 0.2\ x^2$ m, and neglect the size of the sprinkler.



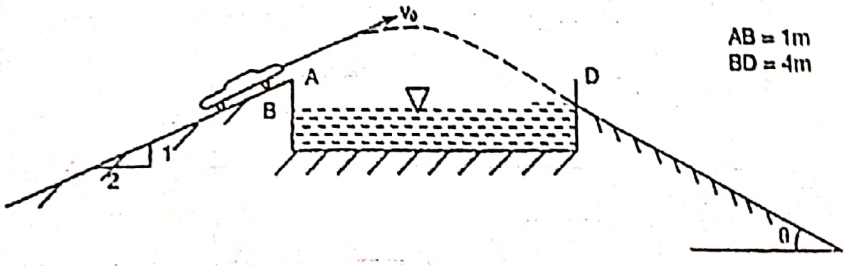
[Ans.: (2.25, 1.01) m]

76) An aeroplane is flying in horizontal direction with a velcoity of 540 km/hr and at a height of 2200 m. When it is vertically above the point A on the ground, a body is dropped from it. The body strikes the ground at point B. Calculate the distance AB (ignore air resistance). Also find velocity at B and time taken to reach B.



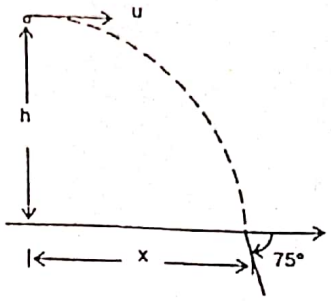
[Ans.: x = 3176.7 m, V_B = 256.25, θ_B = 54.17°, 21.178 sec]

77) An stunt-man wishes to jump across a water pool with his car and lands on a ramp at 'D' without jerk. Determine initial velocity and angle '0' ramp.



[Ans.: V_0 = 5.71 m/s, 0 = 45°]

78) A box released from a helicopter moving horizontally with constant velocity 'u' from a certain height 'h' from the ground takes 5 seconds to reach the ground hitting at an angle of 75° as shown in the figure. Determine (i) the horizontal distance 'x' (ii) the height 'h' and (iii) the velocity 'u'.



[Ans.: (i) x = 65.72 m; (ii) h = 122.63 m; (iii) u = 13.143 m/s (→)]

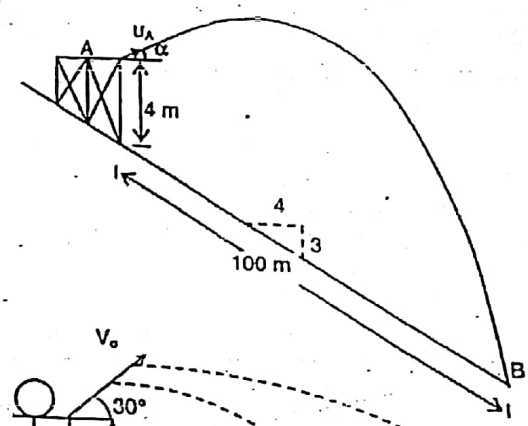
79) A particle is projected upwards at an angle of 53°8' to the horizontal with a velocity of 100 m/sec. What time will elapse before it is moving upwards at an angle of 45° to the horizontal? Find also the horizontal and the vertical distance from the starting point. [Ans.: t = 2 sec and 14 sec; (120, 140) m and (840, 140) m]

80) A particle projected from a point A, with the angle of projection equal to 15° falls short of a mark B on the horizontal plane through A by 22.5 m and when the angle of projection is 45° it falls beyond B by same distance. Show that for the particle to fall exactly at B, the angle of projection must be $\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)$, the velocity of projection being the same in all the three cases. Also determine the velocity of projection.

[Ans.: AB ≅ 67.5 m & velocity of projectile = 30 m/s]

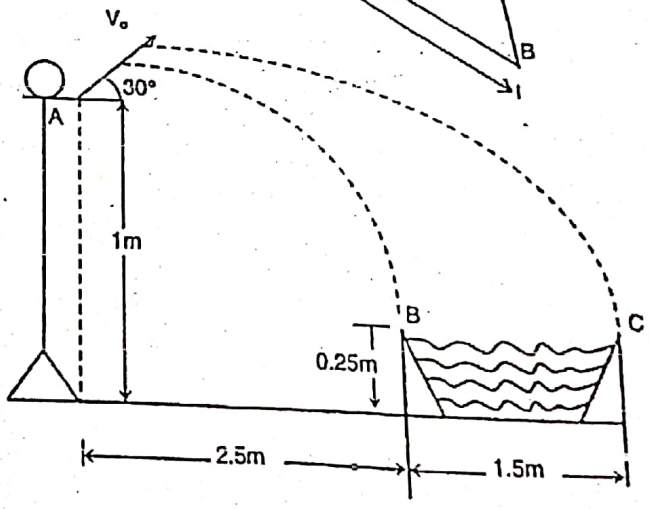
81) It is observed that a skier leaves the platform at A and then hits the ramp at B as shown in figure in 5 seconds.
Calculate the initial speed U_A and the launch angle α.

[Ans.: 19.82 m/sec., 36.18°]



82) A boy always throws two toys at an angle of 30° from point A as shown in figure. Determine the time between throws so that both toys strike the edges of pool B and C at the same instant, with what speed must he throw each toys.

[Ans.: 0.121 sec, 4.317 m/s, 5.848 m/s]

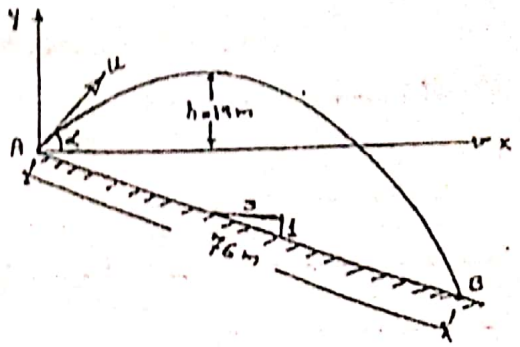


83) A boy throws a ball with an initial velocity 24 m/s. Knowing that the boy throws the ball from a distance of 30 m from the building, determine (i) the maximum height 'h' that can be reached by the ball and (ii) the corresponding angle α.

[Ans.: H_{max} = 22.5 m, α = 62.5°]

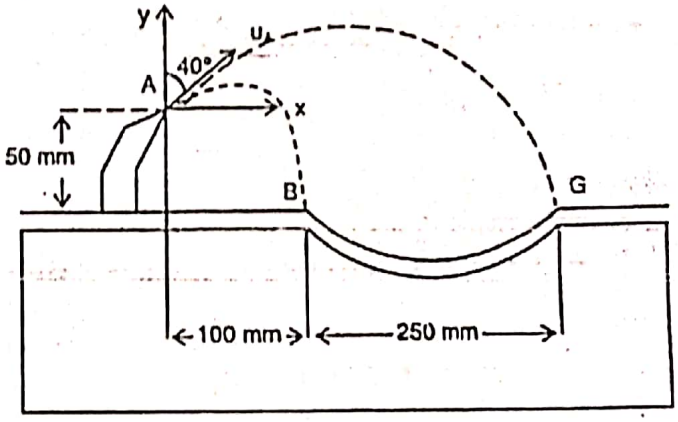
84) A ball rebounds at A and strikes the incline plane at point B at a distance 76 m as shown in figure. If the ball rises to a maximum height h = 19 m above the point of projection, compute the initial velocity and the angle of projection α.

[Ans.: α = 51.34, u = 24.72 m/s]



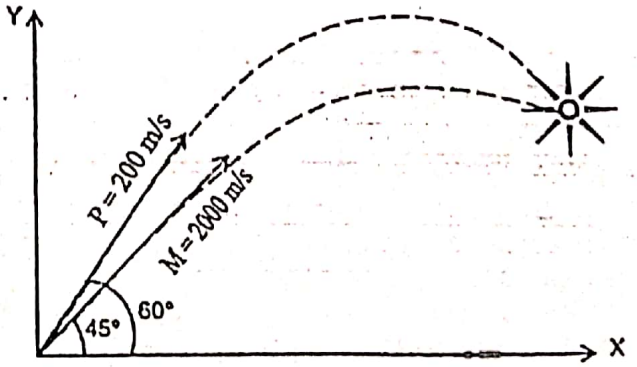
85) The drinking fountain is designed such that the nozzle is located from the edge of the basin as shown. Determine the maximum and minimum speed at which the water can be ejected from the nozzle so that it does not splash over the sides of the basin at B and G which are at same level.

[Ans.: u_{min} = 0.84 m/s, u_{max} = 1.76 m/s]



86) A projectile P is fired at a muzzle velocity of 200 m/s at an angle of elevation of 60°. After some time a missile M is fired at 2000 m/s muzzle velocity and at an angle of elevation of 45° from the same point to destroy the projectile P. Find :

(i) height
(ii) horizontal distance and
(iii) time with respect to P firing at which the destruction takes place.



[Ans.: (i) h = 1494.4 m; (ii) x = 1499.9 m; (iii) Time lag = 14 sec]

87) A missile is fired at 45° to horizontal on a level surface with initial velocity of 500 m/sec. After 10 seconds another missile is thrown at 5000 m/sec at such an angle so as to hit the earlier missile. Calculate the angle at which the second missile is thrown and locate the point where it hits the missile fired earlier. Neglect air resistance.

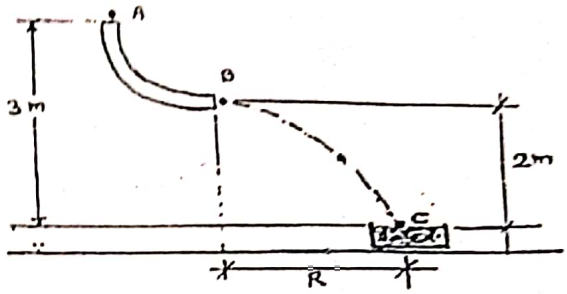
[Ans.: α_B = 40.18°, x = 3896 m, y = 3300 m]

88) A particle is projected from the ground at 50 m/s at an angle of $\tan^{-1}(4/3)$ to the horizontal. Determine the tangential and normal components of accelerations of the particle 1.5 sec. after the throw. Also determine the radius of curvature of the path then. State when and where the radius of curvature of the path is minimum and find its magnitude.

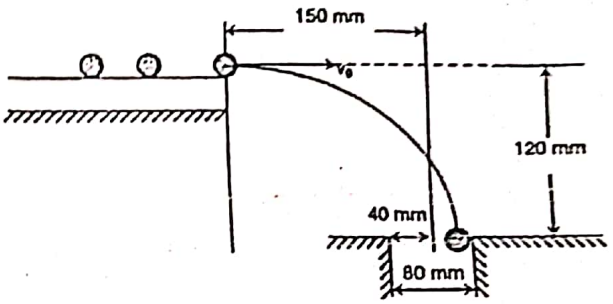
[Ans.: $a_n = 7.68$ m/s^2; $a_t = -6.42$ m/s^2 & $\rho = 200$ m, t = 4 sec; (120, 80) m; $\rho_{min} = 90$ m]

89) Marbles having a mass of 5 g fall from rest at A through the glass tube and accumulate in the can at 'C'. Determine the placement 'R' of the can from the end to the tube and the speed at which the marbles fall into the can. Neglect the size of the can.

[Ans.: R = 2.83 m]



90) Balls of 10 mm diameter (of ball bearings) leave the horizontal through with an initial horizontal velocity V_0 to fall through gap of 80 mm size as shown in figure. Calculate the permissible range of velocity V_0 that will enable the balls to enter the gap.
Take g = 9.81 m/s^2.



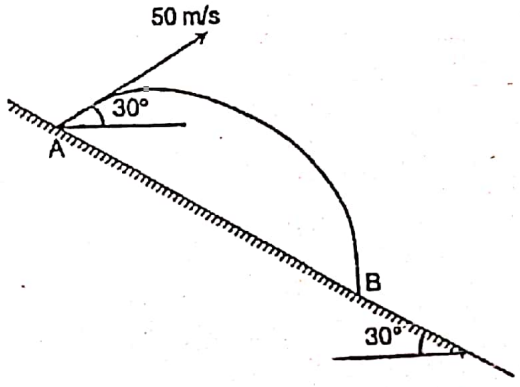
[Ans.: Permissible range of V_0 is from 735 mm/sec to 1183 mm/sec.]

91) A boy throws a ball so that if may just clear a wall 3.6 m high. The boy is at a distance of 4.8 m from the wall. The ball was found to hit the ground at a distance of 3.6 m on the other side of the wall. Find the velocity with which the ball can be thrown. [Ans.: u = 9.78 m/s, $\alpha = 60.24°$]

92) A ball is thrown, by a player with an initial velocity of 15 m/sec, from a point 1.5 m above ground. If the ceiling is 6 m high, determine the highest point on the wall at which the ball strikes the wall, 18 m away. [Ans.: h = 4.2 m]

93) A projectile is projected from position A on an inclined as shown in figure, with a velocity of 50 m/s at an angle of 30° to the horizontal. Incline is making an angle of 30° with horizontal. Find when and where it strikes the incline again.

[Ans.: R = 510 m, t = 10.2 sec.]

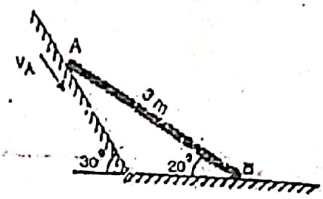


Module-5

Kinematics of Rigid Body

1) Rod AB of length 3m is kept on smooth planes as shown in fig. The velocity of end A is 5 m/s along the inclined plane. Locate the ICR and find the velocity of end B.

[D-16]

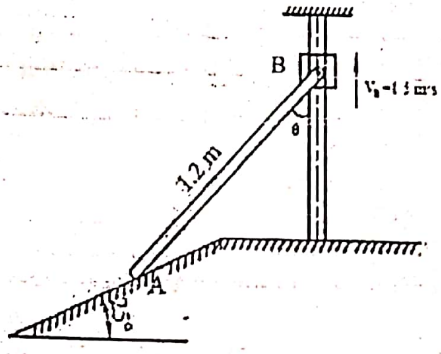


[Ans.: $V_B = 5.24$ m/s (→)]

2) Fig shows a collar B which moves upwards with constant velocity of 1.5 m/s. At the instant when $\theta = 50°$ determine [M-17; D-18]

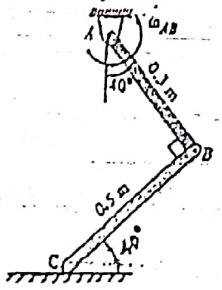
i) The Angular velocity of rod pinned at B and freely resting at A against 25° sloping ground and

ii) The velocity of end A of the rod.



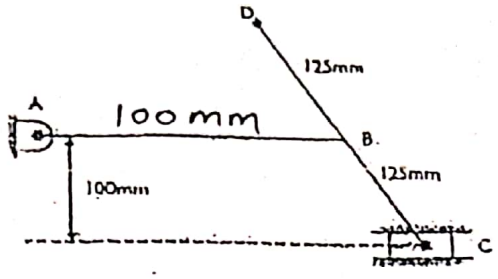
[Ans.: $W_{AB} = 1.173$ r/s (↻), $V_A = 1$ m/s]

3) A rod AB has an angular velocity of 2 rad/sec, counter clockwise as shown. End C of rod BC is free to move on a horizontal surface. Determine [D-16]
(i) Angular velocity of BC and (ii) Velocity of C



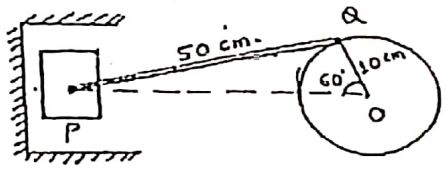
[Ans.: $\omega_{BC} = 1$ r/s(↻); $V_C = 0.778$ m/s(→)]

4) Find velocity of C and point D at the instant shown ω_{AD} = 3 rad/sec clockwise. AB= 100 mm. [M-16]



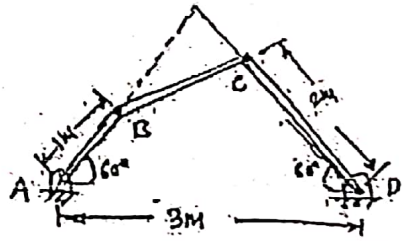
[Ans.: V_C = 400 mm/s (→), V_D = 721.12 mm/s (↙)]

5) For crank of concentric mechanism shown in figure, determine the instantaneous centre of rotation of connecting rod at position shown. The crank OQ rotates clockwise at 310 RPM. Crank length = 10 cm, connecting rod length = 50 cm. Also find the velocity of P & angular velocity of rod at that instant. [M-15]



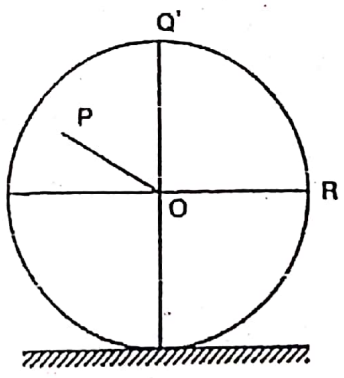
[Ans.: V_p = 310 cm/sec(→) ω_{PO} = 3.3 r/sec(↺)]

6) Angular velocity of connector BC is 4 r/s in clockwise direction. What are the angular velocities of cranks AB and CD. [M-17]



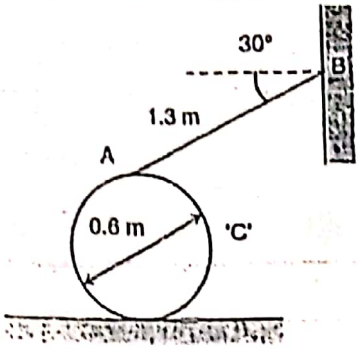
[Ans.: W_{AB} = 8 r/s (↻), W_{CD} = 2 r/s (↻)]

7) A wheel of 2m diameter rolls without slipping on a flat surface. The center of the wheel is moving with a velocity 4 m/s towards the right. Determine the angular velocity of the wheel and velocity of points P, Q and R on the wheel. [D-14]



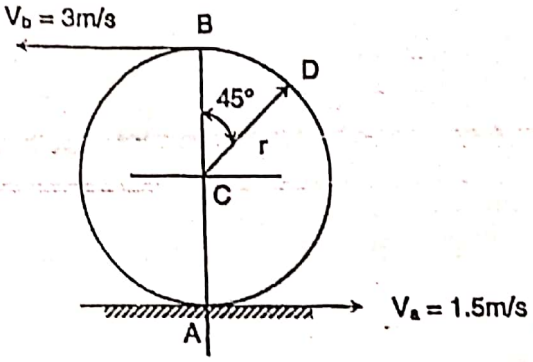
[Ans.: V_P = 5.6 m/sec ∠21.787°
V_Q = 8 m/sec (→)
V_R = 5.657 m/sec ∠45° V_R]

8) "C" is a uniform cylinder to which a rod 'AB' is pinned at 'A' and the other end of the rod 'B' is moving along a vertical wall as shown in figure. If the end 'B' of the rod is moving upward along the wall at a speed of 3.3 m/s find the angular velocity of the cylinder assuming that it is rolling without slipping. **[D-13; M-19]**



[Ans.: $\omega_1 = 3.17$ r/s (↻)]

9) Due to slipping, points A and B on the rim of the disk have the velocities as shown in figure. Determine the velocities of the centre point C and point D on the rim at this instant. Take radius of disk 0.24m. **[M-14]**

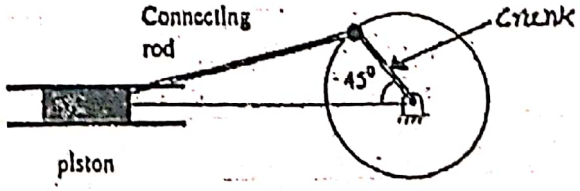


[Ans.:$V_C = 0.75$ m/s (←), $V_D = 2.8125$ m/s]

Home Work Problems

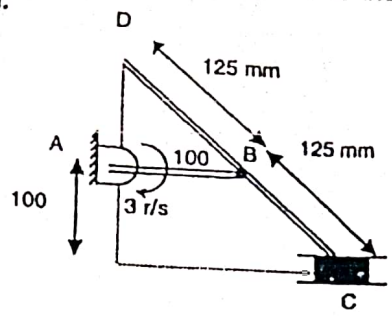
10) Explain Instantaneous centre of rotation. **[M-13]**

11) In a crank and connecting rod mechanism, the length of crank and the connecting rod are 300mm and 1200mm respectively. The crank is rotating at 180 rpm. Find the velocity of piston, when the crank is at an angle of 45° with the horizontal. **[D-12; M-18]**



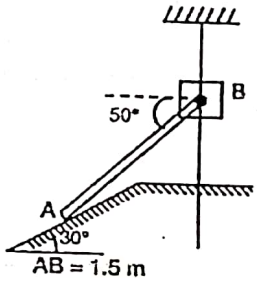
[Ans.: $V_A = 4.71$ m/s (→)]

12) At the position shown in figure the crank AB has Angular velocity of 3rad/sec clockwise. Find the velocity of slider C and the point D at the instant shown. [D-12]



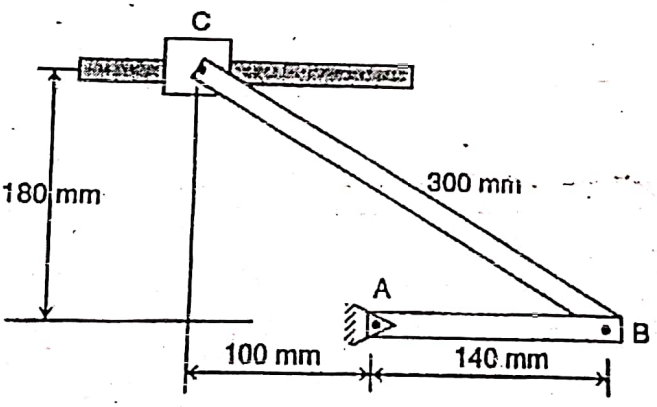
[Ans.: $V_C = 400$ mm/s (→), $V_D = 721.12$ mm/s (↙)]

13) Collor B moves up with constant velocity $V_B = 2$ m/s. Rod AB is pinned at B. Find out angular velocity of AB and velocity of A. [M-13]



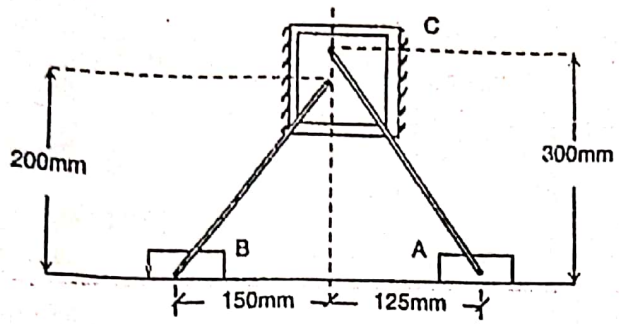
[Ans.: $\omega_1 = 1.229$ r/s (↻), $V_A = 1.63$ m/s (∠)]

14) In figure collar C slides on a horizontal rod. In the position shown rod AB is horizontal and has angular velocity of 0.6 rad/sec clockwise. Determine angular velocity of BC and velocity of collar C. [D-13]



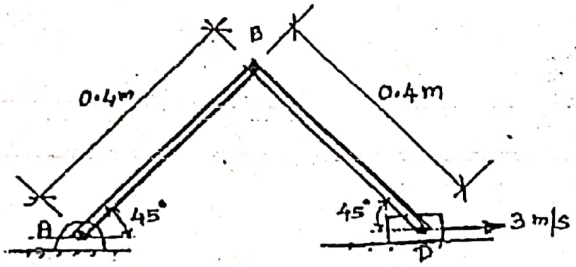
[Ans.: Rod AB = 84 mm/s (↓), Rod BC = 0.35 r/s (↻), $V_C = 63$ mm/s (→)]

15) In the mechanism shown in figure. Piston 'C' is constrained to move in a vertical slot. A and B moves on horizontal surface. Rods CA and CB are connected with smooth hinges. If V_A = 0.45 m/sec. to the right. Find velocity of C and B. Also find angular velocity of two rods.



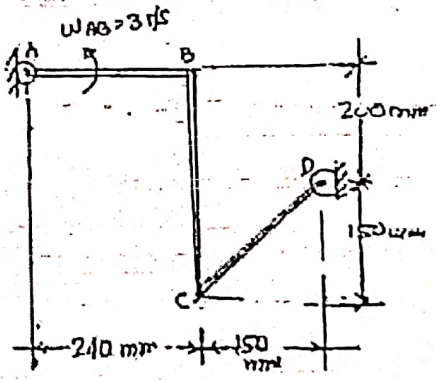
[Ans.: V_C = 0.187 m/s (↓), V_B = 0.249 m (←)
ω_{AC} = 1.5 (↻), ω_{BC} = 1.246 (↺)]

16) Block 'D' shown in figure moves with a speed of 3m/s. Determine the angular velocities of links BD and AB and the velocity of point B at the instant shown. Use method of instantaneous centre of zero velocity.



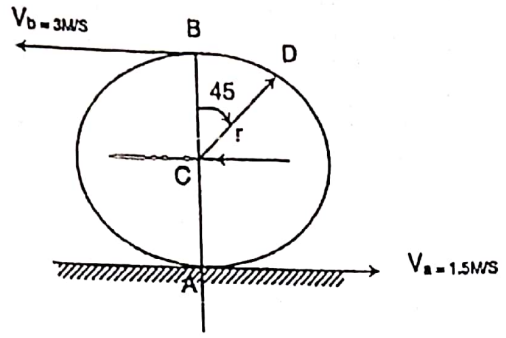
[Ans.: ω_{BD} = 5.3 r/s (↻), V_B = 2.12 m/s, ω_{AB} = 5.3 r/s (↺)]

17) In the position shown, bar AB has constant angular velocity of 3 rad/s anticlockwise, determine the angular velocity of bar CD.



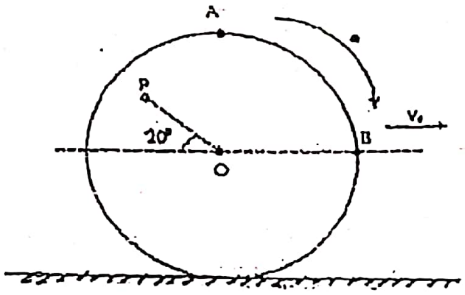
[Ans.: V_B = 720 mm/s (↑),
ω_{BC} = 2.05 r/s (↺), V_C = 1018.23 mm/s,
ω_{CD} = 4.8 r/s (↺)]

18) Due to slipping, points A and B on the rim of the disk have the velocities V_a = 1.5 m/s to the right and V_b = 3 m/s to the left as shown in figure. Determine the velocities of the centre point C and point D on the rim at this instant. Take radius of disk 0.24m. [D-15]



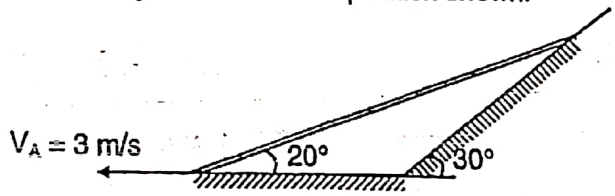
[Ans.: ω_{BC} = 0.386 r/s (↺), ω_{roller} = 1.89 r/s(↺), V_B = 3.49 m/s ∠22.5°]

19) A wheel is rolling along a straight path without slipping. Determine velocity of points A, B and P. OP = 600 mm, ω = 4 rad/sec, V_o = 4 m/s. [M-16]



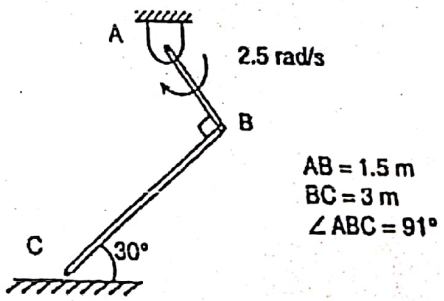
[Ans.: V_p = 5.32 m/s]

20) A bar AB 2m long slides down the plane as shown. The end A slides on the horizontal floor with a velocity of 3 m/s. Determine the angular velocity of the rod AB and the velocity of end B for the position shown. [D-17]



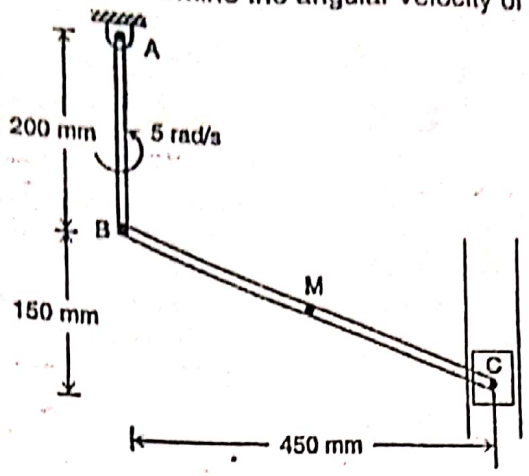
[Ans.: W_{AB} = 0.7615 rad/sec, V_B = 20.862 m/s]

21) At the instant shown in the figure, the rod AB is rotating clockwise at 2.5 rad/sec. If the end C of the rod BC is free to move on a horizontal surface find the angular velocity of rod BC and the velocity of its end point C.



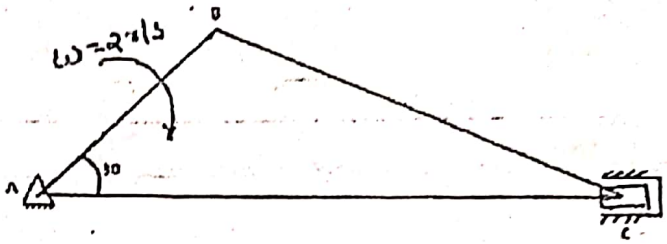
[Ans.: V_C = 4.33 m/s (←), ω_{BC} = 0.722 r/s (↻)]

22) In the mechanism shown the angular velocity of link AB is 5 rad/sec anticlockwise. At the instant shown, determine the angular velocity of link BC and velocity of piston C. [M-14]



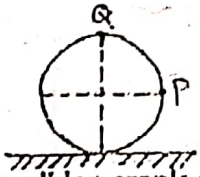
[Ans.: $\omega_2 = 6.67$ r/s (↻), $V_C = 3$ m/s (↓)]

23) Figure shows the crank and connecting rod mechanism. The crank AB rotates with an angular velocity of 2 rad/sec in clockwise direction. Determine the angular velocity of Connecting Rod BC and the velocity of Piston C using ICR method. AB = 0.3 m and BC = 0.8 m. [D-14]



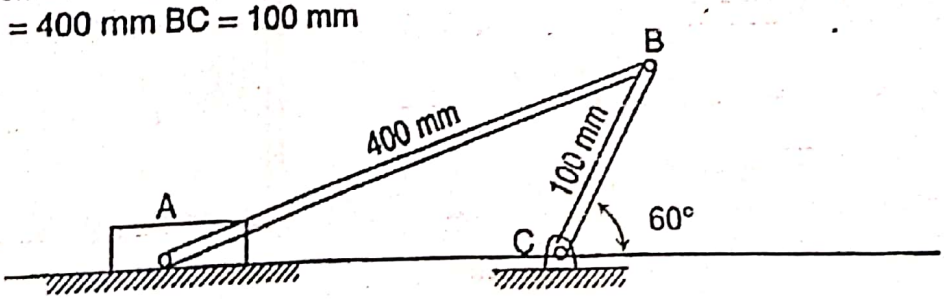
[Ans.: $\omega_{BC} = 0.661$ r/sec (↺), $V_C = 0.398$ m/sec.]

24) A wheel of radius 0.75 m rolls without slipping on a horizontal surface to the right. Determine the velocities of the points P and Q shown in figure when the velocity of centre of the wheel is 10 m/s towards right.



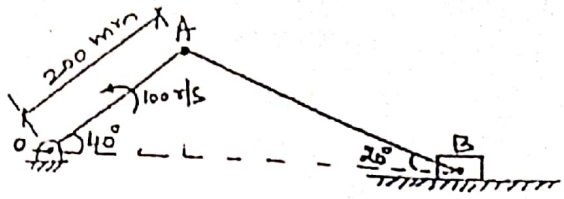
[Ans.: $V_P = 14.14$ m/s, $V_Q = 20$ m/s]

25) The crank BC of a slider crank mechanism is rotating at constant speed of 30 rpm clockwise. Determine the velocity of the piston A at the given instant. [D-15]
AB = 400 mm BC = 100 mm



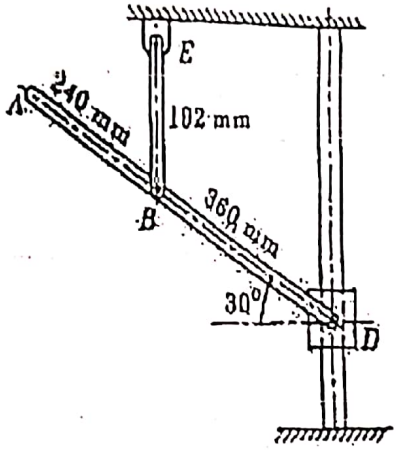
[Ans.: $V_A = 0.237$m/s (→)]

26) A slider crank mechanism is shown in figure. The crank OA rotates anticlockwise at 100 rad/s. Find the angular velocity of rod AB and the velocity of the slider at B. [M-19]



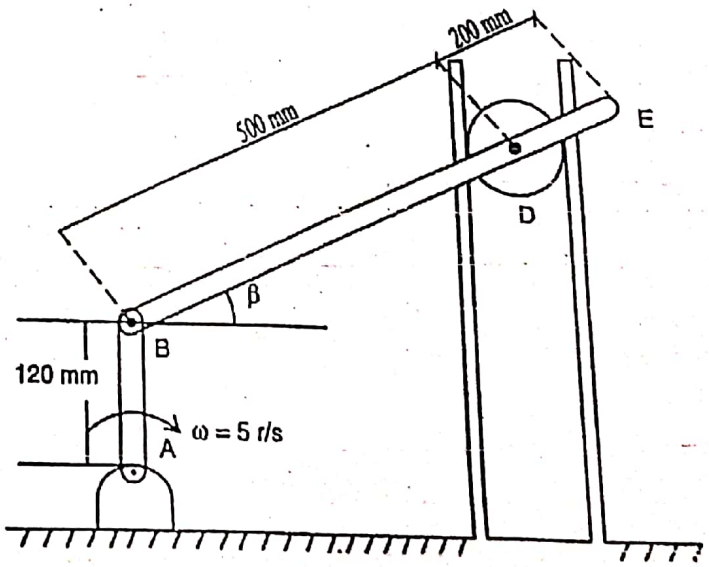
[Ans.: ω_{AB} = 43.38 rad/s, V_B = 18.437 m/s]

27) Rod EB in the mechanism shown in fig. has angular velocity of 4 rad/sec at the instant shown in counter clockwise direction. Calculate (i) angular velocity of rod AD (ii) velocity of collar 'D' (iii) Velocity of point 'A' [M-18]



[Ans.: W_{AD} = 4.266 r/s↻ ; V_D = 1.32 m/s; V_A = 1.557 m/s]

28) Rod BDE is partially guided by a roller at D which moves in a vertical track. Knowing that at the instant shown the angular velocity of AB is 5 rad/sec clockwise and β=25°, determine:
(i) angular velocity of rod BE
(ii) velocity of point E.



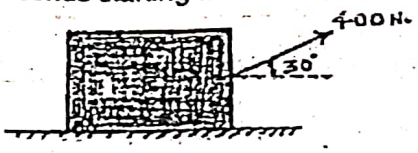
[Ans.: (i) ω_{BE} = 2.84 r/s (↺), (ii) V_E = 1817 mm/s]

Module-6

Kinetics of Particle

Force And Acceleration

1) The 550 N box (Figure) rests on a horizontal plane for which the coefficient of kinetic friction $\mu_k = 0.32$. If the box is subjected to a 400 N towing force as shown, find the velocity of the box in 4 seconds starting from the rest. **[M-15]**



[Ans.: $V_4 = 16.72$ m/s →]

2) A 50 kg block kept on a 15° inclined plane is pushed down the plane with an initial velocity of 20 m/s. If $\mu_k = 0.4$, determine the distance traveled by the block and the time it will take as it comes to rest. **[Ans.: d = 159.835 m, t = 15.98 sec] [D-13]**

3) Two blocks A and B are separated by 10m as shown in Figure on a 20° incline plane. If the blocks start moving, find the time t when the blocks collide and distance travelled by each block. Assume μk = 0.3 for block A and plane and μk = 0.10 for block B and plane. **[M-19]**

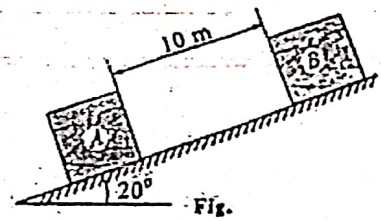
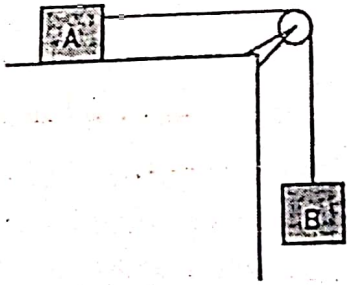


Fig.

[Ans.: t = 3.297 sec, $d_{\to 10} = 13.207$ m]

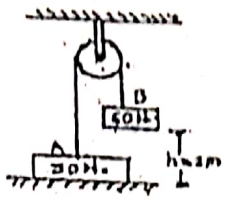
4) A vertical lift of total mass 750kg acquires an upward velocity of 3m/s over a distance of 4m moving with constant acceleration starting from rest. Calculate the tension in the cable. **[Ans.: T = 8201.25 N] [D-12]**

5) A body of mass 25 kg resting on a horizontal table is connected by string passing over a smooth pulley at the edge of the table to another body of mass 3.75 kg and hanging vertically as shown. Initially, the friction between the mass A and the table is just sufficient to prevent the motion. If an additional 1.25 kg is added to the 3.75 kg mass, find the acceleration of the masses. **[D-13]**



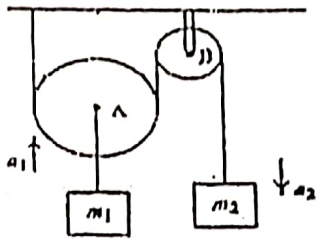
[Ans.: T = 47 N, a = 0.40875 m/s^2]

6) Two masses of 60 N &. 30 N. are positioned over frictionless & massless pulley (Figure). If the 60 N. mass is released from rest, find the speed at which the 60 N. mass will hit the ground. [M-15]



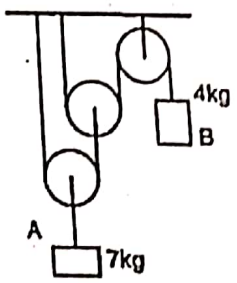
[Ans.: V = 3.617 m/s(↓)]

7) Two masses are interconnected with the pulley system Neglecting Inertial & frictional effect of pulleys & cord, determine the acceleration of the mass m_2. [M-16,18]
Take m_1 = 50 kg & m_2 = 40 kg



[Ans.: $a_2 = 2.803$ m/s^2]

8) Determine the tension developed in chords attached to each block and the accelerations of the blocks when the system shown is released from rest. Neglect the mass of the pulleys and chords.

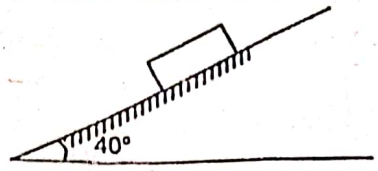


[Ans.: $T_A = 77.36$ N, $a_A = 1.244$ m/s^2 (↑)
$T_B = 19.34$ N, $a_B = 4.976$ m/s^2 (↑)]

9) There blocks A, B and C cf masses 3 kg, 2 kg and 7 kg respectively are connected as shown. Determine the acceleration of A, B and C. Also find the tension in the strings. [D-16]

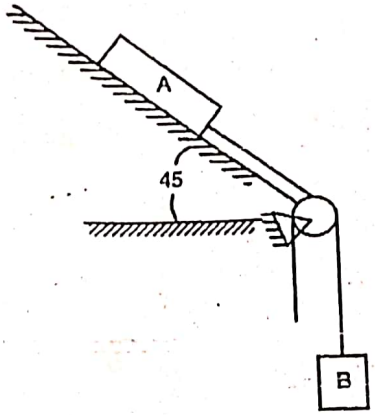
[Ans.: T = 27.93 N, $a_A = 0.4988$ m/s^2 (↓),
$a_B = 4.155$ m/s^2 (↑), $a_C = 1.83$ m/s^2 (↓)]

10) A block of mass 5 kg is released from rest along a 40 degree inclined plane. Determine the acceleration of the block when it travels a distance of 3 m using D'Alemberts principle. Take coefficient of friction as 0.2. [D-14]



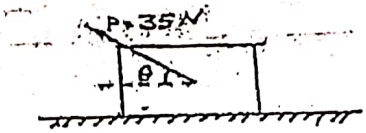
[Ans.: a = 4.803 m/s^2]

11) Two blocks A and B connected as shown in the diagram. The string is inextensible. Mass of A and B are 3 kg and 5 kg respectively. If the coefficient of friction between A and inclined plane is 0.25. Determine the tension on the strings and accelerations of A and B. [D-14]



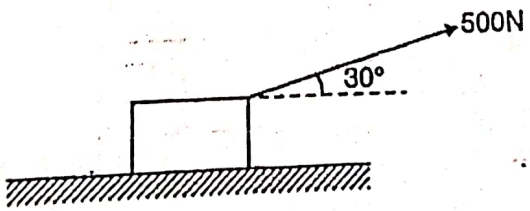
[Ans.: T = 8.639 N, a = 8.082 m/sec^2]

12) Find the angle the force P makes with horizontal such that the block of mass 4 kg has an acceleration of 10 m/sec^2, when it is subjected to a force of 35 N. $\mu s = 0.7$, $\mu k = 0.6$. [M-16]



[Ans.: $\theta = 35.26°$]

13) A force of 500N is acting on a black of 50 Kg mass resting on a horizontal surface as shown in fig. Determine the velocity after the block has travelled a distance of 10 m. Coeff. of kinetic friction = 0.5. [D-17]



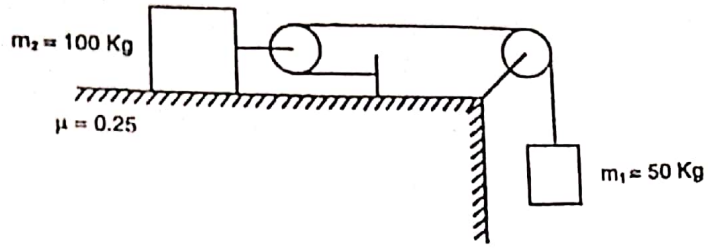
[Ans.: V = 11.185 m/s]

14) A 500N Crate kept on the top of a 15° sloping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu s = 0.5$ and $\mu k = 0.4$. Determine the distance travelled by the block and the time it will take as it comes to rest. [D-17]

[Ans.: s = 159.83 m, a = –1.251 m/s^2, t = 15.98 sec]

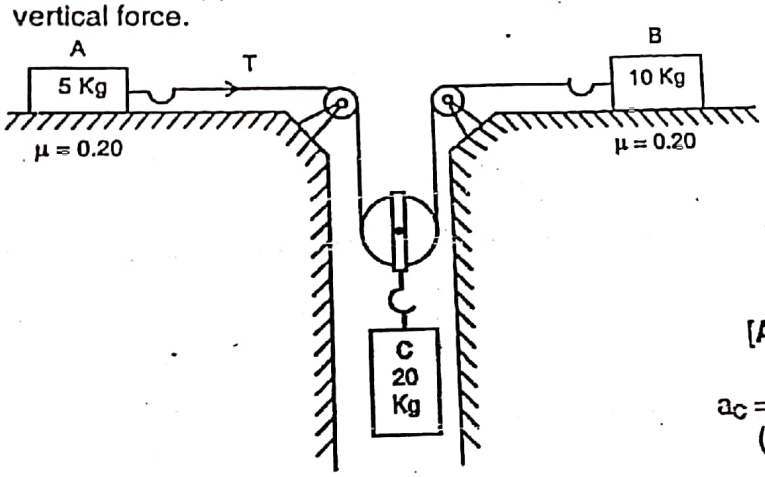
Home Work Problems

15) Two blocks, shown in figure start from rest. If the chord is in extensible, friction and inertia of pulley are negligible, calculate acceleration of each block and tension in each chord.



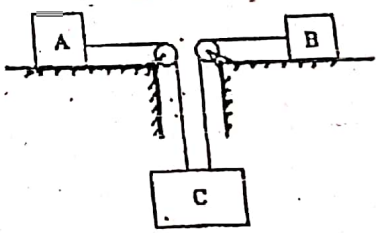
[Ans.: $T_1 = 245.25$ N, $T_2 = 490.5$ N, $a_1 = 4.905$ m/sec^2, $a_2 = 2.453$ m/sec^2]

16) Masses A (5 kg), B(10 kg), C(20 kg) are connected as shown in the figure by inextensible cord passing over massless and frictionless pulleys. The coefficient of friction for mass A and B and ground is 0.20. If the system is released from rest, find (i) the acceleration a_A, a_B, a_C and tension T in the cord. Present your answers in tabular form. (ii) Acceleration of blocks A & B if the block 'C' is replaced by 200 N vertical force.



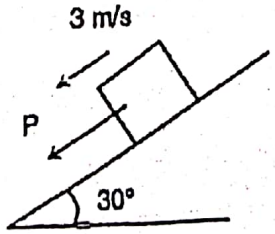
[Ans.: (i) $a_A = 7.46$ m/s^2 (→), $a_B = 2.75$ m/s^2 (←), $a_C = 5.10$ m/s^2 (↓), T = 47.09 N. (ii) $a_A = 18$ m/s^2; $a_B = 8$ m/s^2]

17) Masses A (5 kg), B (10 kg), C (20 kg) are connected as shown in figure by inextensible cords passing over massless and frictionless pulleys. The coefficient of friction for masses A and B with ground is 0.2. If the system is released from rest, find the acceleration of the blocks and tension in the cords.



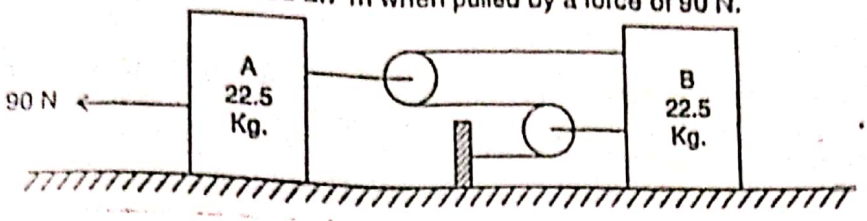
[Ans.: $a_A = a_B = a_C = 4.765$ m/s^2, $T_1 = 33.63$ N, $T_2 = 67.27$ N]

18) The block shown in the fig. has a weight of 500 N & is acted upon by a variable force having magnitude P = 200t. Compute its velocity 2 sec after 'P' is applied. The block has a initial velocity of 3 m/s down the plane. Also find the distance travelled by the block at t = 2 sec. take μk = 0.3



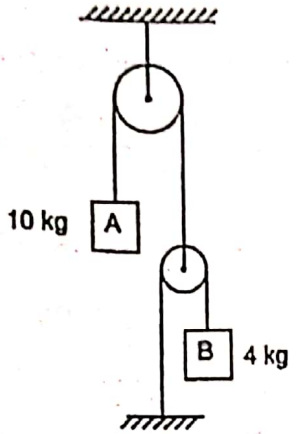
[Ans.: v = 15.6 m/s, s = 15.94 m]

19) A system shown in figure is at rest initially. Neglecting friction determine velocity of block A after it has moved 2.7 m when pulled by a force of 90 N.



[Ans.: $V_A = 3.867$ m/s^2]

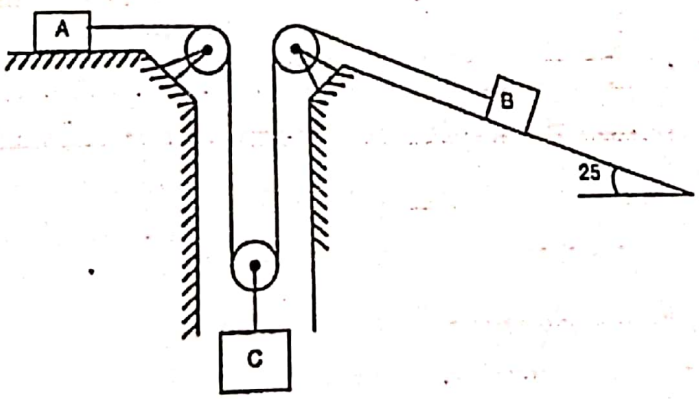
20) Determine the tension developed in the two cords and the acceleration of each block. Neglect the mass of the pulleys and cords.



[Ans.: $a_A = 0.76$ m/s^2 (↓), $a_B = 1.51$ m/s^2 (↑), $T_{(lower)} \approx 45.3$ N, $T_{(upper)} = 90.6$ N]

21) A vertical lift of weight 10 kN moving from rest with constant acceleration acquires an upward velocity of 4 m/s over a distance of 5 m. Determine the tension in the cables supporting the lift. [Ans.: 11631 N]

22) Find acceleration of block A, B and C shown in the figure when the system is released from rest. Mass of block A, B and C is 5 kg, 10 kg and 50 kg respectively. Co-efficient of friction for block A and B is 0.3. Neglect weight of pulley and rope friction.



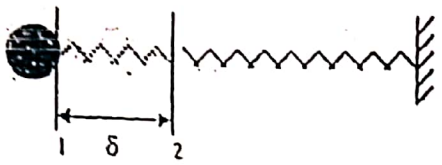
[Ans.: $a_A = 12.52$ m/s^2, $a_B = 0.92$ m/s^2, $a_C = 6.72$ m/s^2]

23) A motorist travelling at a speed of 90 kmph suddenly applies the brakes and comes to rest after skidding 100m. Determine the time required for the car to stop and coefficient of kinetic friction between the tires and the road. [M-14]

[Ans.: $a = 3.125$ m/s^2 (←), $t = 8$ sec., $\mu_K = 0.3186$]

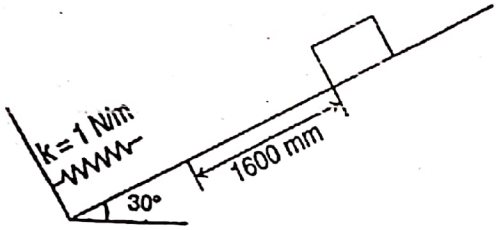
Work And Energy

24) A spring of stiffness k is placed horizontally and a ball of mass m strikes the spring with a velocity v. Find the maximum compression of the spring. (Take m = 5kg, k = 500N/m, v = 3m/s). **[D-12]**



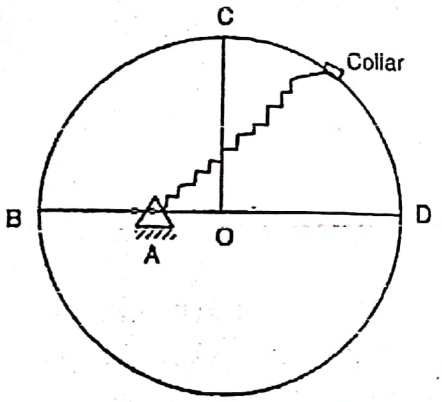
[Ans.: δ = 0.3 m]

25) A 30N block is released from rest. It slides down a rough incline having coefficient of friction 0.25. Determine the maximum compression of the spring. **[M-14]**



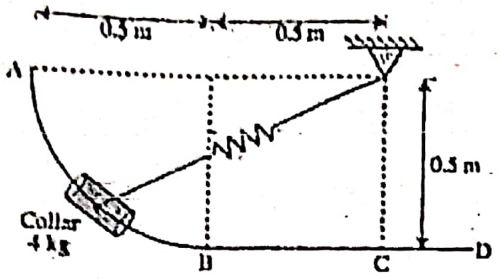
[Ans.: N_1 = 25.98 N, d = 18.48 m]

26) A collar of mass 1 kg is attached to a spring and slides without friction along a circular rod which lies in a horizontal plane. The spring is undeformed when the collar is at B, knowing that the collar is passing through the point D with a speed of 1.8 m/s, determine the speed of the collar when it passes through point C and B. Take stiffness of the spring, k = 250 N/m, Radius of the circular path = 300 mm and distance OA = 125 mm. **[D-14; M-16]**



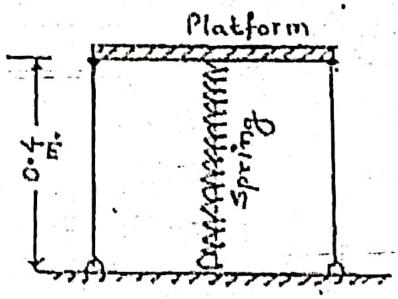
[Ans.: V_C = 3.63 m/s, V_B = 4.34 m/s]

27) A 4kg collar is attached to a spring, slides on a smooth bent rod ABCD. The spring has constant k = 500 N/m and is undeformed when the collar is at 'C'. If the collar is released from rest at A. Determine the velocity of collar, when it passes through 'B' and 'C'. Also find the distance moved by collar beyond 'C' before it comes to rest again. Refer Fig. [M-19]



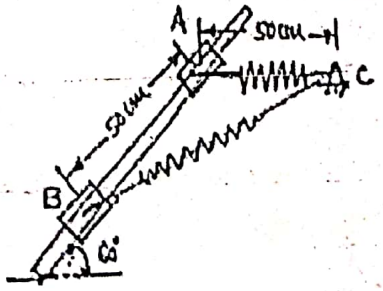
[Ans.: C_D = 0.9495 m]

28) The platform P (figure) has negligible mass & is tied down so that the 0.4 m long cords keep a 1 m long spring compressed to 0.6 m, when nothing is on the platform. If 4 kg block is placed on the platform & released from rest after the platform is pushed down 0.1 m., find the maximum height 'h' the block rises in the air, measured from the ground. Use Work & Energy Principle. (Assume k = 200 N/m) [M-15]



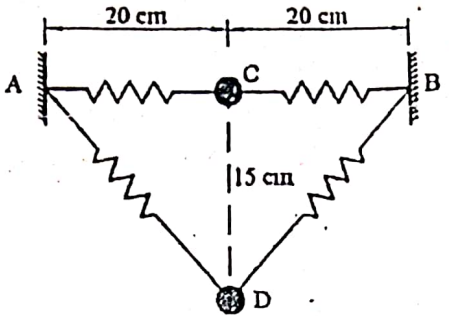
[Ans.: h = 0.631 m]

29) A 50N collar slides without friction along a smooth rod which is kept inclined at 60° to the horizontal. The spring attached to the collar and the support 'C'. The spring is unstretched when the collar is at 'A' (AC is horizontal) Determine the value of spring constant 'K' given that the collar has a velocity of 2.5 m/s when it has moved 0.5m along the rod as shown in fig. [M-17]



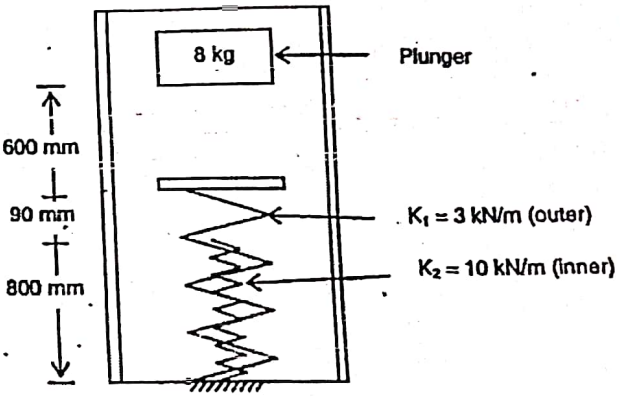
[Ans.: K = 85.4 N/m]

30) Two springs, each having stiffness of 0.6 N/cm and length 20 cm are connected to a ball B of weight 50 N. The initial tension developed in each spring is 1.6 N. The arrangement is initially horizontal, as shown in Figure. If the ball is allowed to fall is allowed to fall from rest, what will be its velocity at D, after it has fallen through a height of 15 cm? **[D-18]**



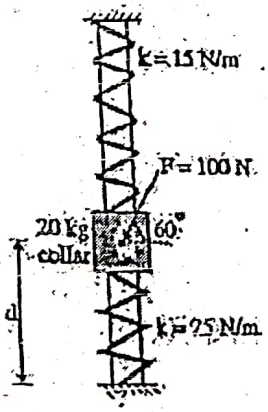
[Ans.: V_D = 1.68 m/s]

31) An 8 kg plunger is released from rest in the position shown in the figure and is stopped by two nested springs. The constant of the Outer spring is k_1 = 3 kN/m and constant of the inner spring k_2 = 10 kN/m. Determine the maximum deflection of the outer spring.



[Ans.: x = 0.157 m]

32) Fig. shows a collar of mass 20 kg which is supported on the smooth rod. The attached sprigs are both compressed 0.4m when d = 0.5 m. Determine the speed of the collar after the applied force F = 100 N causes it to be displaced so that d = 0.3m. Knowing that collar is at rest when d = 0.5 m. **[M-18]**

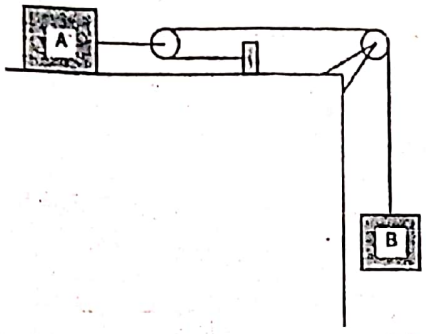


[Ans.: speed of collar = 2.36 m/s]

Home Work Problems

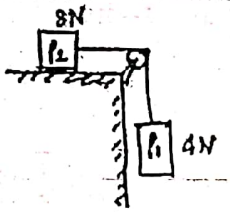
33) Explain work energy principle. [M-13]

34) Two blocks m_A = 10 kg and m_B = 5 kg are connected with cord and pulley system as shown in figure. Determine the velocity of each block when system is started from rest and block B gets displacement by 2m. Take μ_K=0.2 between block A and Horizontal surface. [D-13]



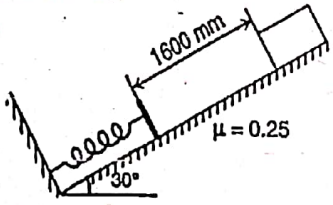
[Ans.: $V_A = 2.29$ m/s (→), $V_B = 2V_A = 4.58$ m/s (↓)]

35) Blocks P_1 and P_2 are connected by inextensible string. Find velocity of block P_1, if it falls by 0.6m starting from rest. The coefficient of friction is 0.2, pulley is friction less. [M-17]



[Ans.: $V_2 = 1.534$ m/s]

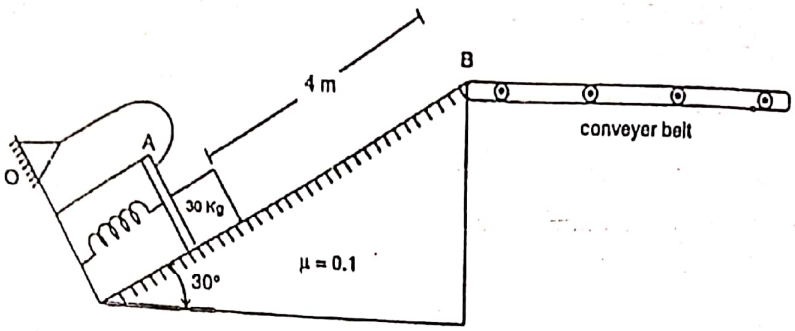
36) A 30N block is released from rest. It slides down a rough incline having coefficient of friction 0.25. Determine the maximum compression of the spring. [D-17]



[Ans.: l = 0.174 m]

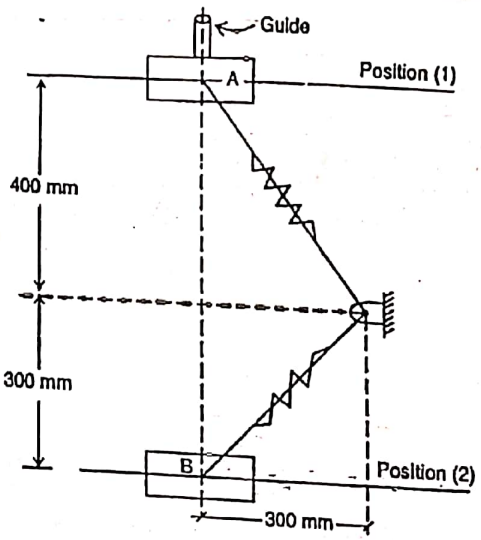
37) A 5 kg mass drops 2 m upon a spring whose modulus is 10 N/mm. What will be the speed of the block when the spring is deformed 100 mm ? [Ans.: V = 4.604 m/s]

38) A pre-compressed spring compressed by 0.2 m is held by a latch mechanism OA as shown in Fig. When the latch is released the spring propels a 30 Kg machine part which is being heat treated at A up the inclined plane onto a conveyer belt at B. The coefficient of friction between machine part and incline is 0.1. The desired speed of machine part when it reaches the top of incline is 5 m/s. Determine the spring constant K in kN/m that engineer must use. Angle of inclination of plane is 30° with horizontal.



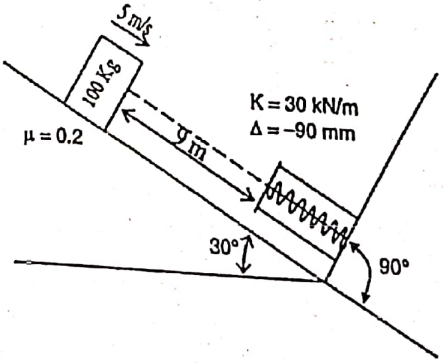
[Ans.: K = 53.85 kN/m]

39) A collar A of mass 10 Kg moves in vertical guide as shown in figure. Neglecting the friction between the guide and the collar, find its velocity when it passes through position (2) after starting from rest in position (1). The spring constant is 200 N/m and the free length of the spring is 200 mm.



[Ans.: v = 3.85 m/s]

40) A spring is used to stop a 100 kg package which is moving down a 30° incline. The spring has a constant k = 30 kN/m and is held by cables so that it is initially compressed 90 mm. If the velocity of package is 5m/s when it is 9 m from the spring, determine the maximum additional deformation of spring in bringing the package to rest. Assume coefficient of friction as 0.2.

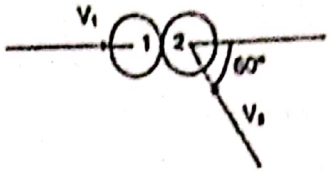


[Ans.: x = 0.452 m]

Impulse And Momentum

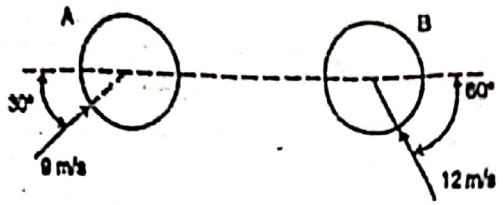
41) A ball is dropped on to a smooth horizontal floor from a height of 4m. On the second bounce it attains a height of 2.25 m. Find the coefficient of Restitution between the Ball and Floor. [Ans.: e = 0.866]

42) Two billard balls of equal masses collide with velocities V_1 = 1.5 m/s and V_2 = 2 m/s. Find velocities of balls after impact and percentage loss in kinetic energy. Coefficient of restitution is 0.9.



[Ans.: For ball No. (1) Vel. = 0.875 m/s and for ball No. (2) Vel. 2.21 m/s, 51.15; 9%]

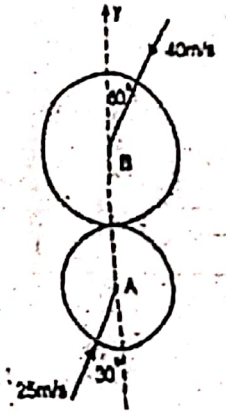
43) Two identical balls of 120 gm collide when they are moving with velocities as shown in figure. Determine the velocities of ball A and B completely after the impact. Take e = 0.8.



[Ans.: V_A = 8.49 m/s, θ_A = 44.25°, V_B = 12.21 m/s, θ_B = 58.33°]

44) Two balls with masses 20 kg and 30 kg are moving towards each other with velocities 10 m/s and 5 m/s respectively. If after impact the ball having mass 30 kg reverses its direction of motion and moves with velocity 6 m/s, then determine the coefficient of restitution between the two balls. [Ans.: e = 0.833] [D-15]

45) Two smooth balls A (mass 3 kg) and B (mass 4 kg) are moving with velocities 25 m/s and 40 m/s respectively. Before Impact, the directions of velocity of two balls are 30° and 60° with the line joining the centers as shown in Fig. If e = 0.8, find the magnitude and direction of velocities of the balls after impact. [M-19]



[Ans.: 24.602 m/s, θ_A = 59.463°, V_B = 36.703 m/s, θ_B = 19.298°]

46) Two balls of 0.12kg collide when they are moving with velocities 2m/sec and 6 m/sec perpendicular to each other as shown in fig. If coefficient of restitution between 'A' and 'B' is 0.8 determine the velocity of 'A' and 'B' after impact. [M-18]