Writer's Notebook: The Twelfth Root of Two 15 March 2013

1.059304694... is a cool number. It's not like π or e, in that it's not a "special" or "natural" number; still, it's a very useful number to know about.

Let me provide some introduction. I'm not sure how much of this you already know, so I'll start at the beginning.

Music, of course, is produced by vibrations. Any sound you hear is a pattern of vibrations. Music is a collection of tones that are produced by those vibrations. If you've ever played with a piezo element, you might have some experience here. A piezo element is a small, round metal plate, sometimes encased in plastic. When electricity is sent to it, it makes a faint clicking noise. That's the noise of the plate vibrating.

By itself, a piezo element is essentially useless. Faint clicking noises hardly constitute music. Or... don't they? Music is made up of tones, and tones are made up of vibrations. Still, a single click is not a very good song. The piezo element, without external help, is not going to produce tones. That external help generally comes in the form of a microcontroller.

To make a tone, the piezo element needs to vibrate continuously. A microcontroller can send short pulses to the piezo element, and so will be able to create tones.

Tone pitches are measures in Hz (Hertz), which refers to the number of "clicks" per second. So, if I have a frequency that I want to turn into a tone (we'll use 440Hz in the example), I start by dividing one by the frequency (effectively taking the frequency's reciprocal). That will provide the delay between each click.

I use my microcontroller to continuously loop:

```
1     Forever, do:
2           set PIEZO_ELEMENT on
3           wait ( ( 1 / 440 ) / 2 )
3           set PIEZO_ELEMENT off
5           wait ( ( 1 / 440 ) / 2 )
```

The loop will click the element, then, half of the specified time later, will reset the element, wait the remaining time, and repeat.

That code will produce a 440Hz tone, which is generally known as an "A_4".

I'm getting to the relevance of 1.059363094..., don't worry.

In music, you can go up an octave ($A_4 \rightarrow A_5$) by doubling the frequency. A_5's frequency is 880Hz. Likewise, A_3's frequency is 220Hz. I'll make a nice little graph to display the significance of that fact.