Writer's Notebook: Numbering Systems 13 June 2013

This is just going to be a quick write-up explaining numbering systems of different bases, because my entire portfolio is numbered in hexadecimal and I can imagine that that might anger a few people who aren't smart enough to figure it out for themselves.

You might not have known this, but humans tend to have ten fingers. Those fingers are pretty handy. We can use them to grab things, to mash things, and, importantly, to count. Using my fingers, I can count all the way up to ten!

Example: I set up an altar where people can sacrifice tacos to me. Jim hands me a taco. I hold up one finger on my right hand because now I have one taco. Then George gives me another taco. I hold up a second finger on my right hand because now I have another taco. If I'm ever unsure as to how many tacos I have, I can look at the number of fingers I have and the answer will be right there!

Three more people give me tacos. Now I have five tacos (I know, because I looked at my hand to check). But what about when Ginette gives me another taco? I don't have six fingers on my right hand!

I can hold up a finger on my left hand. Now I have a total of six fingers held up.

That's all fine and dandy, but what happens when I get my *eleventh* taco? I can't just give the taco back to the person who gave it to me. I want to *eat* that taco!

We have invented the digits 0-9 to store how many tacos we have at any given time. If we take a pencil and paper and write one of these digits, we can come back later and see what number we wrote down. But we can't have a sigit for every number. That would be silly!

So, after we get past nine tacos, we put a '1' to the left of a '0'. The '1' means we have 'one ten', and the '0' means we have 'zero ones'. If we have twelve tacos, we write '12'; that's one ten, and two ones. If we have twenty-three tacos, we write '23'; that's *two* tens, and three ones. That's a lot of tacos!

I'm going to cut the silly baby-talk now, if that's okay with you. It was getting more annoying for me than it was for you.

Each time we add a digit to the right, we're multiplying that digit's value by ten to the power of the digit number. The third digit, counting from the right, in the number '254', is worth 2 times 10^{3-1}. The second is worth 5 times 10^{2-1}, and the first is worth 4 times 10^{1-1}. This continues such that we can represent any number.

In exponents, the big number that has smaller numbers attached to it is called the 'base'. In numbering systems, we still refer to this as the base. For this reason, the numbering system that we use, called 'decimal', is a 'base-10' numbering system.

There are other numbering systems, though. Let's look at binary, which is a base-2 numbering system.

The number '1101' actually means

$1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0$, which evaluates to 13.

Hexadecimal is just a base-16 numbering system, with digits 0-F.

I bet you can figure out the rest on your own. To give you a reference point, this page would be page 36 in decimal.