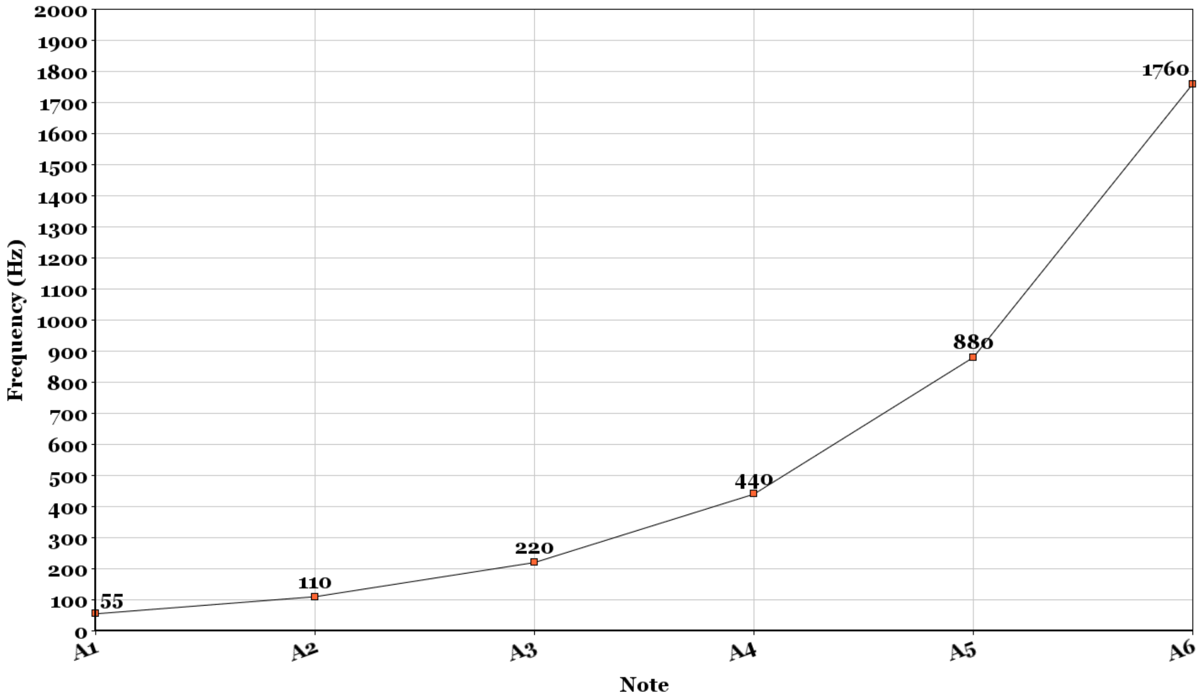

Note frequencies increase exponentially.

When you hear the same note in three consecutive octaves, you'd probably be willing to say that the not in the middle is "in between" the other two, and *sounds* in between the other two. In reality, the middle note is only a quarter of the way from the first note to the third note, in terms of frequency.

Now, it's just math. There are twelve half-steps in an octave. When you move up on half-step, you multiply the frequency by 1.059463094... Why, though?

1.059463094... is the twelfth root of two. It can be written mathematically as $\sqrt[12]{2}$, or $2^{1/12}$. That means that, when you multiply 1.059463094... by itself twelve times, you'll get 2.

From this point, I will define t = $\sqrt[12]{2}$ so that I don't have to keep writing it out.

If you multiply a frequency, like 440Hz, by t^{12}, you'll double it. Twelve represents the number of half-steps from the initial note. Multiplying a frequency by t^1, then, would produce the frequency one half-step up. Multiplying it by t^4 would bump up the frequency by a perfect third. It works in the opposite direction, too. Multiply the frequency by t^{-1} to move it down a half-step.

It is possible, then, to find any note's frequency given one starting point (a popular one is A_4 = 440Hz).

$$f = 440 * t^s$$

where f is the final frequency, t is $\sqrt[12]{2}$, and s is the half steps from A_4.

Give it a try. 440 * t^{-9} should be C_4, or middle C; and, indeed, it comes out to the 261.63 that we all know and love.