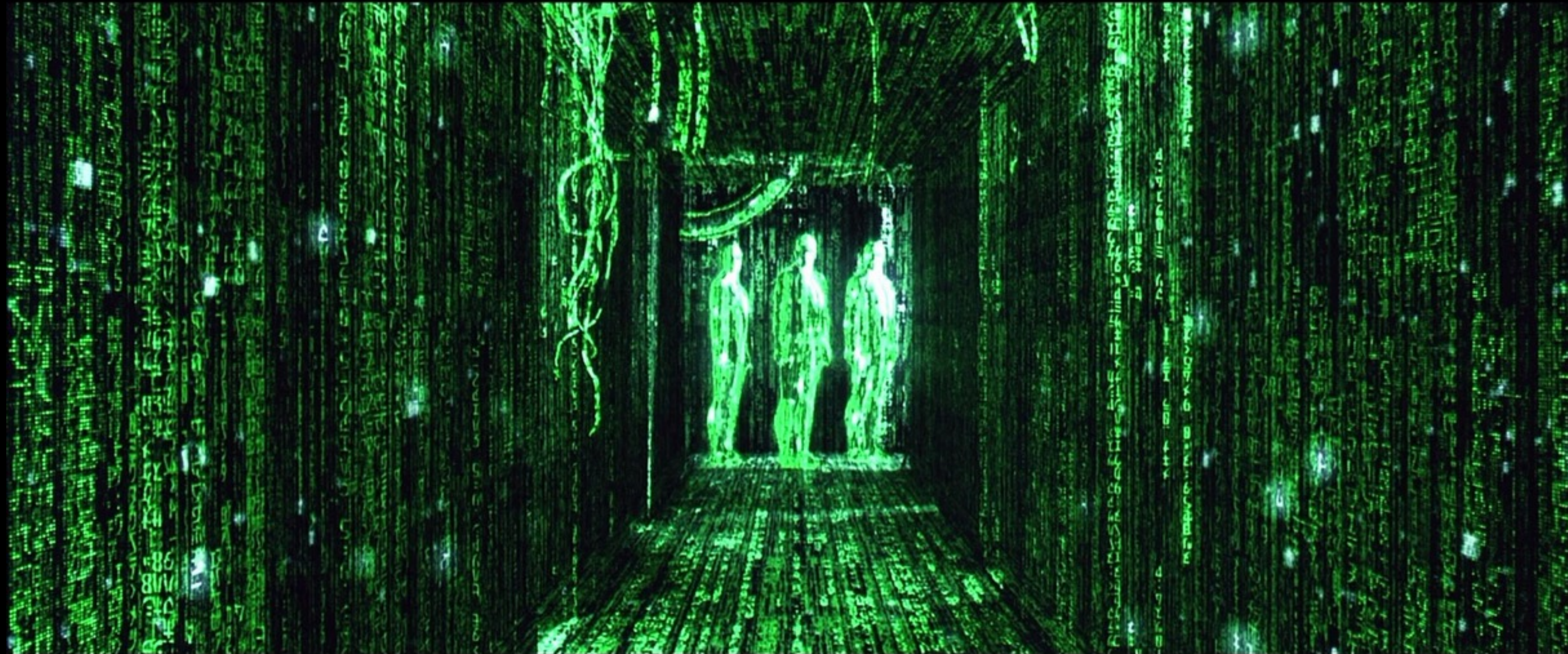


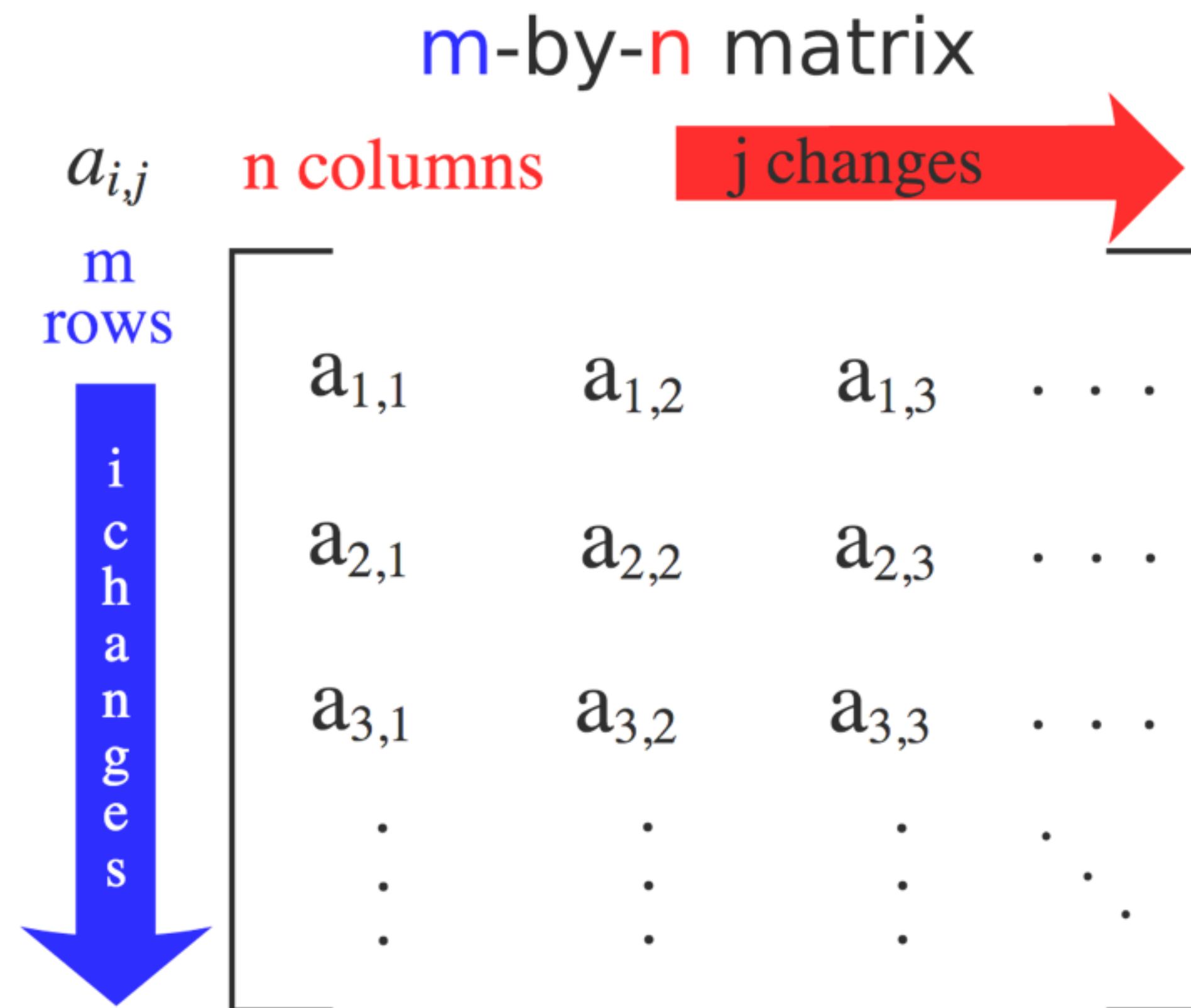
Matrix transformations.

Part 1



Matrix math.

A matrix.



A 2x3 matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \end{bmatrix}$$

A 3x3 matrix.

$$\begin{bmatrix} 1 & 2 & 0 \\ 4 & 3 & 2 \\ 3 & 4 & 2 \end{bmatrix}$$

Matrix operations.

Matrix addition.

To add two matrices, add their corresponding entries.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} + \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} = \begin{bmatrix} A+J & B+K & C+L \\ D+M & E+N & F+O \\ G+P & H+Q & I+R \end{bmatrix}$$

Matrix subtraction.

To **subtract** two matrices, **subtract** their **corresponding** entries.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} - \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} = \begin{bmatrix} A-J & B-K & C-L \\ D-M & E-N & F-O \\ G-P & H-Q & I-R \end{bmatrix}$$

Matrix addition and subtraction can only
happen
with **matrices that are the same size!**

Transpose of a matrix.

Transpose of a matrix is a matrix whose columns are the rows of the original matrix (and its rows are the columns).

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}$$

M

$$\begin{bmatrix} A & D \\ B & E \\ C & F \end{bmatrix}$$

M^T

Matrix/scalar multiplication.

Multiply each entry of the matrix by the scalar.

$$S \times \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} = \begin{bmatrix} S \times A & S \times B & S \times C \\ S \times D & S \times E & S \times F \\ S \times G & S \times H & S \times I \end{bmatrix}$$

Matrix/matrix multiplication.

You can only multiply **two matrices**
if the **number of columns of the first matrix** equals the
number of rows of the second.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = ?$$

It results in a matrix that is **number of rows of first matrix** by **number of columns of second matrix**.

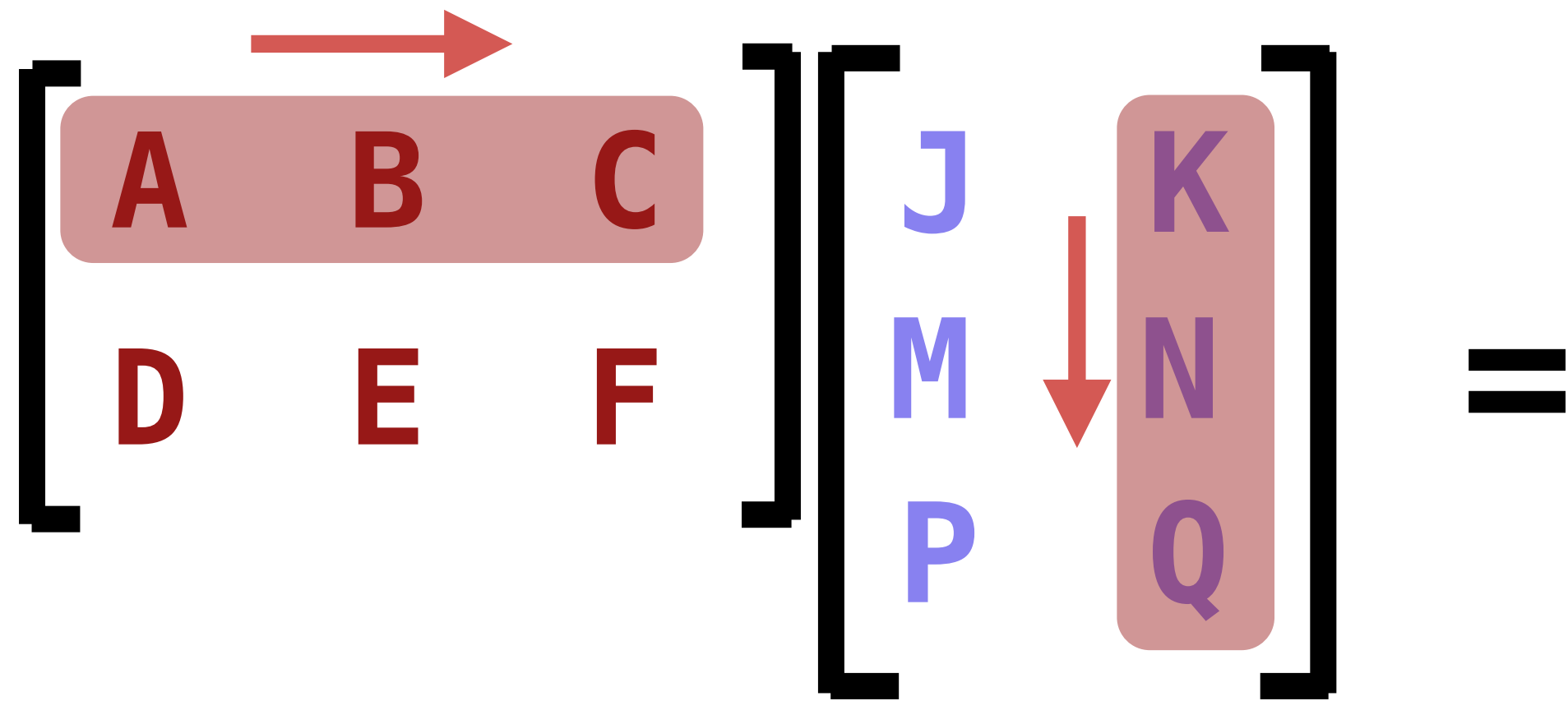
$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

For each row, find dot product with each column.

The diagram illustrates the dot product of a row from a matrix and a column from another matrix. The first matrix is $\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix}$. A red arrow points to the first row $[A \ B \ C]$, which is highlighted with a red background. The second matrix is $\begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix}$. A red arrow points to the first column $\begin{bmatrix} J \\ M \\ P \end{bmatrix}$, which is highlighted with a red background. An equals sign follows the matrices.

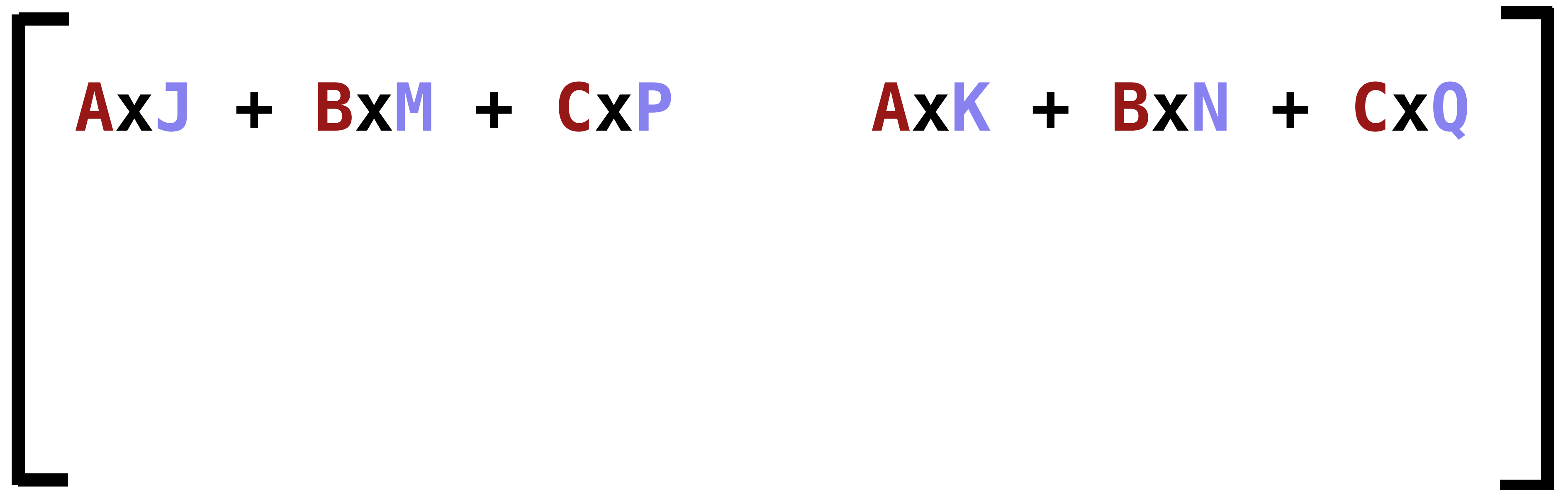
The diagram shows the calculation of the dot product for the first row and column. The expression $A \times J + B \times M + C \times P$ is enclosed in large square brackets. The terms are color-coded: A and B are red, C is dark red, J , M , and P are blue, and K , N , and Q are light blue.

For each row, find dot product with each column.



The diagram illustrates the process of finding the dot product for a specific row and column in a matrix multiplication. It shows two matrices separated by an equals sign. The first matrix is a 2x3 matrix with elements A, B, C in the first row and D, E, F in the second row. A red arrow points to the first row, and a red box highlights the elements A, B, and C. The second matrix is a 3x2 matrix with elements J, M, P in the first column and K, N, Q in the second column. A red arrow points to the second column, and a red box highlights the elements K, N, and Q.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$



The diagram shows the resulting dot products for the first row of the first matrix. It is a large bracketed expression containing two dot products. The first dot product is A times J plus B times M plus C times P. The second dot product is A times K plus B times N plus C times Q.

$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \end{bmatrix}$$

For each row, find dot product with each column.

The diagram illustrates the dot product of a row from matrix A with a column from matrix B. Matrix A is represented as a 2x3 grid with elements A, B, C in the first row and D, E, F in the second row. A red arrow points from the first row of A towards the right. Matrix B is represented as a 3x2 grid with elements J, K in the first column and M, N in the second column, and P, Q in the third column. A red arrow points from the first column of B downwards. The two matrices are separated by an equals sign, indicating the result of the dot product operation.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} =$$

The diagram shows the resulting dot products for each row of matrix A. The first row of the result is calculated as $A \times J + B \times M + C \times P$ and $A \times K + B \times N + C \times Q$. The second row of the result is calculated as $D \times J + E \times M + F \times P$. The entire result is enclosed in large square brackets.

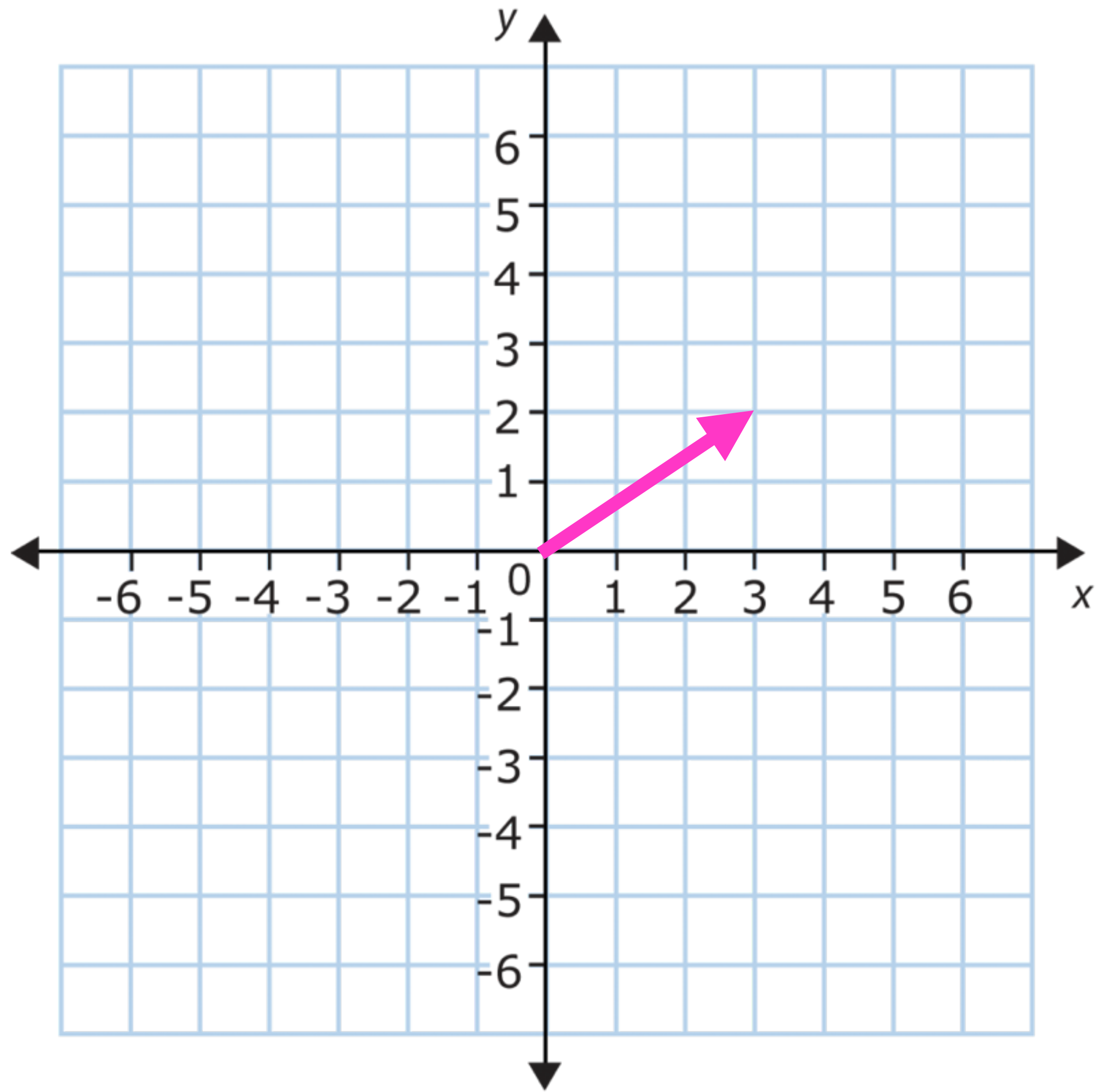
$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \\ D \times J + E \times M + F \times P \end{bmatrix}$$

For each row, find dot product with each column.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K & L \\ M & N & O \\ P & Q & R \end{bmatrix} =$$

$$\begin{bmatrix} A \times J + B \times M + C \times P & A \times K + B \times N + C \times Q \\ D \times J + E \times M + F \times P & D \times K + E \times N + F \times Q \end{bmatrix}$$

Vectors.



A **2 dimensional** vector can be represented
as a **2x1 matrix**.

$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

A **3 dimensional** vector can be represented
as a **3x1 matrix**.

$$\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$$

Matrix vector multiplication.

Multiplying a **matrix** and a **vector** is basically just
multiplying two matrices.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

Row by row, multiply **each column value** with the **each row of the vector** and **add them together**.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} AX + BY + CZ \end{bmatrix}$$

Row by row, multiply **each column value** with the **each row of the vector** and **add them together**.

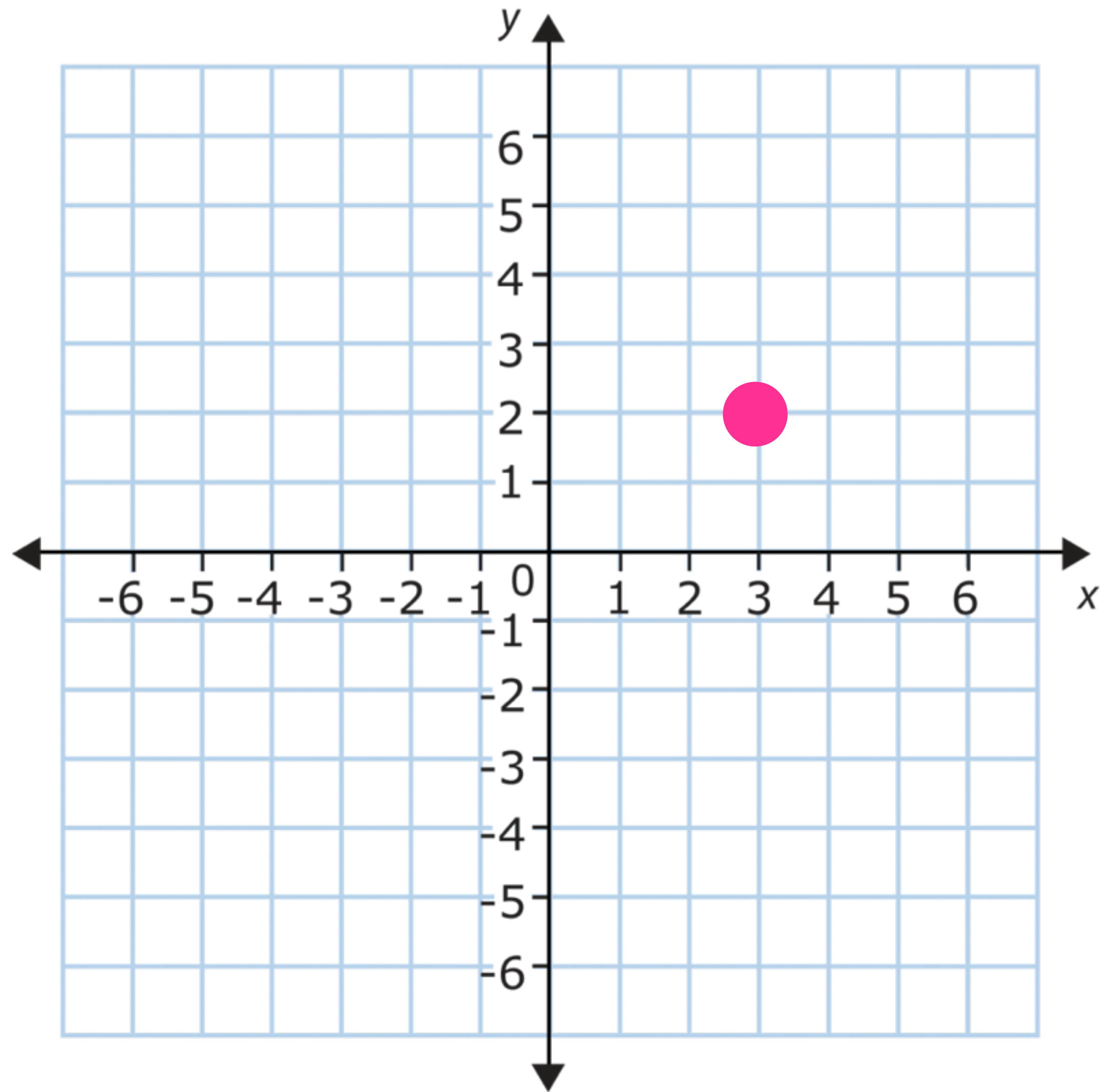
$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} AX + BY + CZ \\ DX + EY + FZ \\ \end{bmatrix}$$

Row by row, multiply **each column value** with the **each row of the vector** and **add them together**.

$$\begin{bmatrix} A & B & C \\ D & E & F \\ G & H & I \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} AX + BY + CZ \\ DX + EY + FZ \\ GX + HY + IZ \end{bmatrix}$$

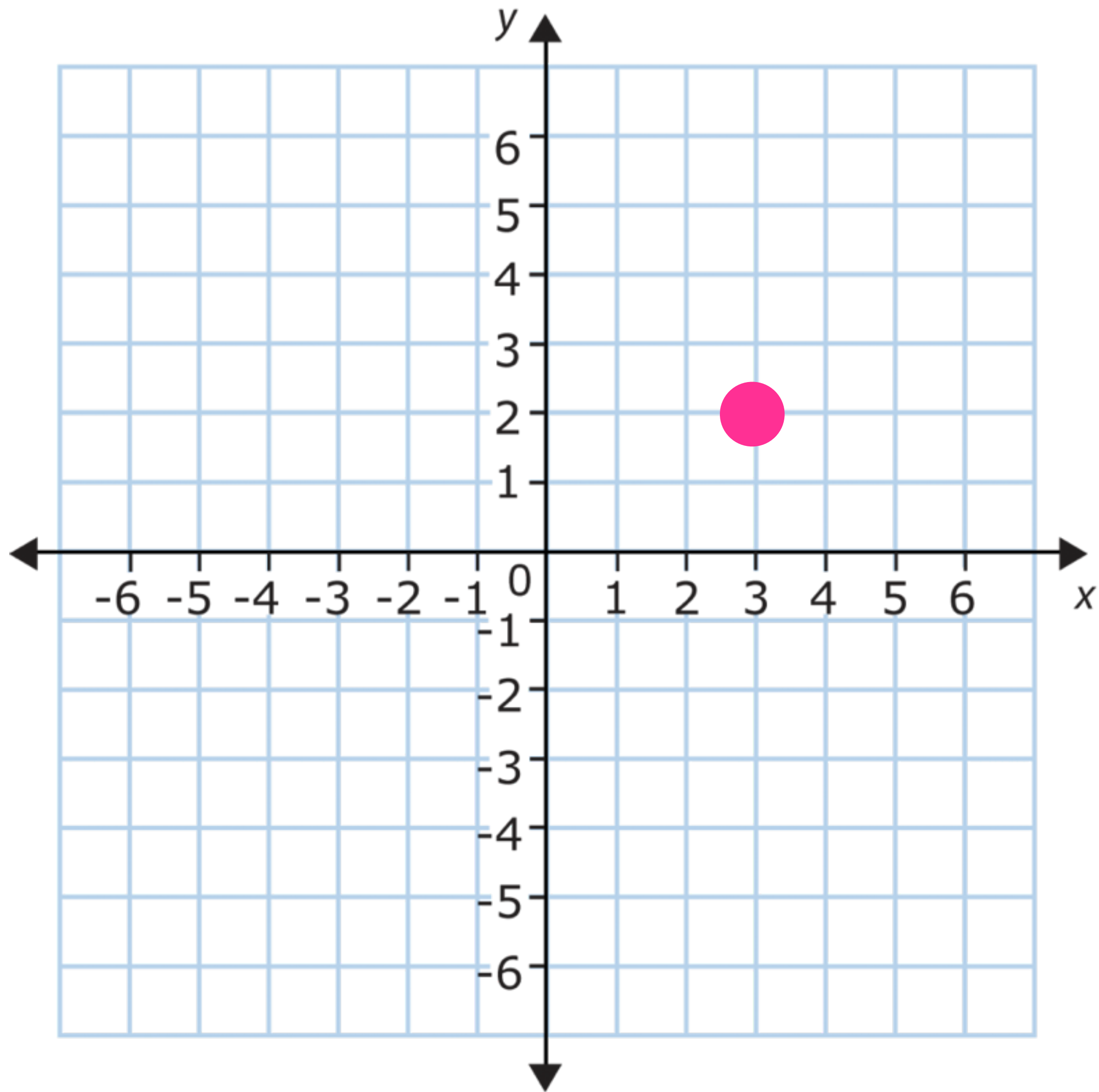
Transformation matrices.

Linear transformations stored
as matrices.



$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Example: **scale**



$$\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Scale

$$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

Scale

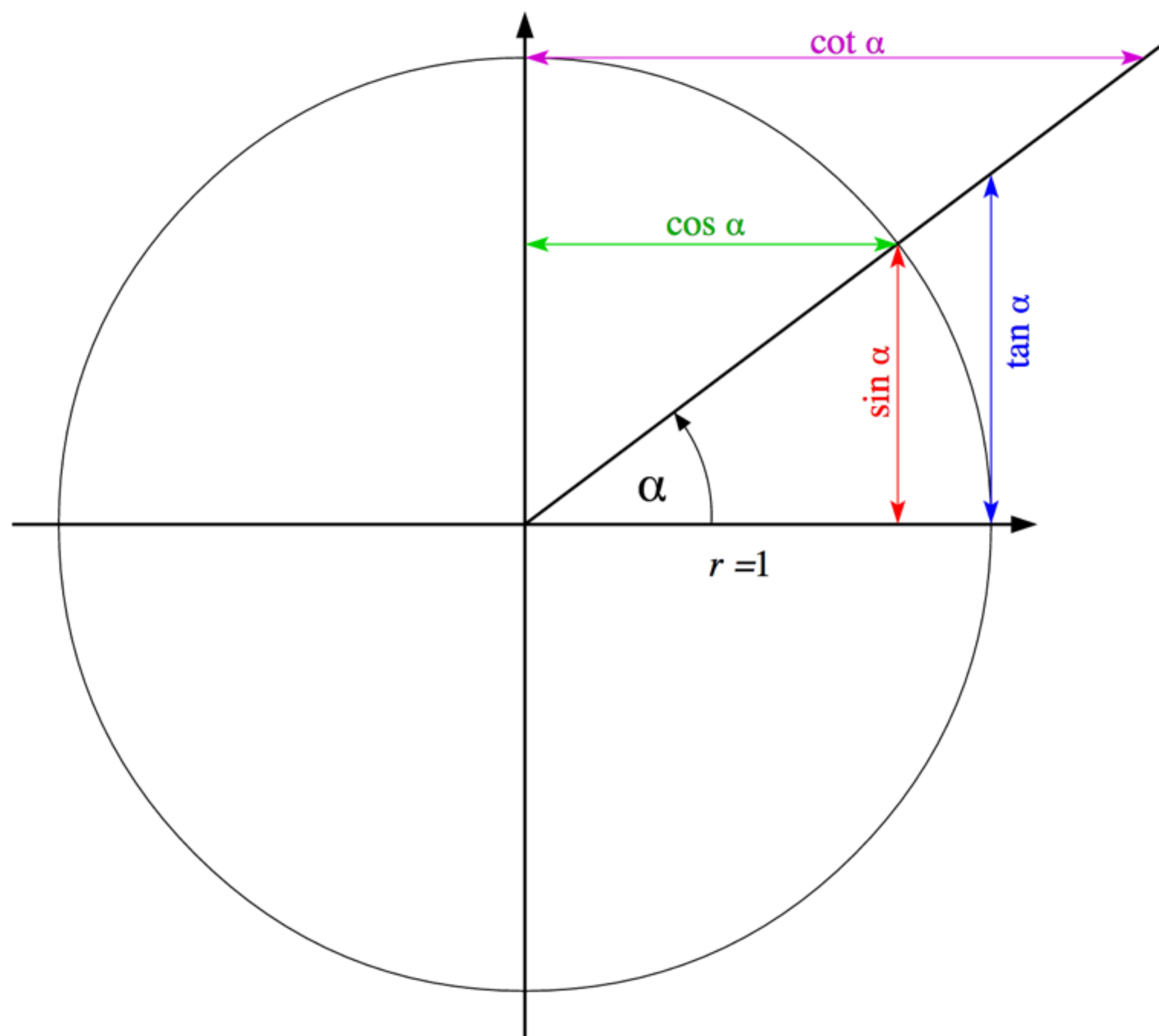
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} AX + BY \\ CX + DY \end{bmatrix}$$

$$\begin{bmatrix} sx & 0 \\ 0 & sy \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} sxX + 0Y \\ 0X + syY \end{bmatrix}$$

Rotation

Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\theta X + -\sin\theta Y \\ \sin\theta X + \cos\theta Y \end{bmatrix}$$



$$\begin{bmatrix} \cos \theta X + -\sin \theta Y \\ \sin \theta X + \cos \theta Y \end{bmatrix}$$

Affine transformations.

Homogeneous coordinates.

ABCD is the linear part of the affine transformation matrix.

$$\begin{bmatrix} A & B & ? \\ C & D & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

Translate

Translate

$$\begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1X + 0Y + 1T_x \\ 0X + 1Y + 1T_y \\ 0X + 0Y + 1 \times 1 \end{bmatrix}$$

Identity

Identity

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} 1X + 0Y + 1 \times 0 \\ 0X + 1Y + 1 \times 0 \\ 0X + 0Y + 1 \times 1 \end{bmatrix}$$

Multiplying affine transformation **matrices**.

You can only multiply **two matrices**
if the **number of columns of the first matrix** equals the
number of rows of the second.

$$\begin{bmatrix} A & B & C \\ D & E & F \end{bmatrix} \begin{bmatrix} J & K \\ M & N \\ P & Q \end{bmatrix} = ?$$

```
matrix.identity();  
matrix.Translate(5.0, 4.0);  
matrix.Scale(2.0, 4.0);
```

```
// draw vertex at 3,2
```

```
matrix.identity();
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

```
matrix.identity(); matrix.Translate(5.0, 4.0);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ 1 \end{bmatrix}$$

`matrix.identity();`

`matrix.Translate(5.0, 4.0);`

`matrix.Scale(2.0, 4.0);`

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 5 \\ 0 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 11 \\ 12 \\ 1 \end{bmatrix}$$


```
matrix.identity();  
matrix.Scale(2.0, 4.0);  
matrix.Translate(5.0, 4.0);
```

```
// draw vertex at 3,2
```

```
matrix.identity();
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

```
matrix.identity();    matrix.Scale(2.0, 4.0);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 1 \end{bmatrix}$$

```
matrix.identity();
```

```
matrix.Scale(2.0f, 4.0f);
```

```
matrix.Translate(5.0f, 4.0f);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 10 \\ 0 & 4 & 16 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 16 \\ 24 \\ 1 \end{bmatrix}$$

Moving into **3D**

3D identity matrix and 3D position in homogeneous coordinates.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

All transformations in 3D

<p>X-Rotation in 3D</p> $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi & 0 \\ 0 & \sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	<p>Z-Rotation in 3D</p> $\begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	<p>Scale in 3D</p> $\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$
<p>Y-Rotation in 3D</p> $\begin{bmatrix} \cos\phi & 0 & \sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\phi & 0 & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$	<p>Translation in 3D</p> $\begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$	

Projection matrices are the same.

matrix.setOrthoProjection(l, r, b, t, n, f);

$$\begin{pmatrix} \frac{2}{r-l} & 0 & 0 & -\frac{(r+l)}{r-l} \\ 0 & \frac{2}{t-b} & 0 & -\frac{(t+b)}{t-b} \\ 0 & 0 & -\frac{2}{f-n} & -\frac{(f+n)}{f-n} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$