Preliminaries

$u_{max} = 0.6$

Coloring the plane

In[1]:=
```
Imax = 0.1;
R0 = 3.64;
umax = 0.6;
Rc = (1 - umax) R0;

(*R0 = 2;
Rc = 4/3;

Imax = 0.01;*)
```

Formulas for the transition times

In[5]:=
```
μ[S_, Is_] := Is - 1/R0 Log[S] + S
ν[S_, Is_] := Is - 1/Rc Log[S] + S

Ss[v_, mu_] := Exp[R0 Rc/(R0 - Rc) (mu - v)]

Is[v_, mu_] := R0/(R0 - Rc) mu - Rc/(R0 - Rc) v - Exp[R0 Rc/(R0 - Rc) (mu - v)]

(*hatSmu[mu_]:=-ProductLog[-e^(-R0 mu) R0]/R0
   hatSnu[nu_]:=-ProductLog[-e^(-Rc nu) Rc]/Rc*)

hatSmu[mu_] := Module[{S, sols},
  sols = NSolve[mu == -1 / R0 Log[S] + S, S, Reals];
  Max[S /. sols]
 ]

hatSnu[nu_] := Module[{S, sols},
  sols = NSolve[nu == -1 / Rc Log[S] + S, S, Reals];
```

```
  Max[S /. sols]
 ]

muf = Imax + (Log[R0] + 1)/R0
vmin = -(1/Rc) Log[hatSmu[muf]] + hatSmu[muf]

mumax = -(1/R0) Log[hatSnu[vmin]] + hatSnu[vmin]
vmax = Imax + Log[R0]/Rc + 1/R0



T0[ν1_, μ1_, ν0_, μ0_] := NIntegrate[1/((R0/Rc) μ0 - v - ((R0-Rc)/Rc) Exp[(R0 Rc)/(R0-Rc) (μ0 - v)]),
  {v, ν0, ν1}, Method → "LocalAdaptive", MaxRecursion → 100]
Tc[ν1_, μ1_, ν0_, μ0_] := NIntegrate[1/(-μ + (Rc/R0) ν1 + ((R0-Rc)/R0) Exp[(R0 Rc)/(R0-Rc) (μ - ν1)]),
  {μ, μ0, μ1}, Method → "LocalAdaptive", MaxRecursion → 100]

(*T0[ν1_,μ1_,ν0_,μ0_]:=
 NIntegrate[1/((R0/Rc)μ0-v-((R0-Rc)/Rc)Exp[(R0 Rc)/(R0-Rc)(μ0-v)]),{v,ν0,ν1}, MaxRecursion→100]
   Tc[ν1_,μ1_,ν0_,μ0_]:=
  NIntegrate[1/(-μ+(Rc/R0)ν1+((R0-Rc)/R0)Exp[(R0 Rc)/(R0-Rc)(μ-ν1)]),{μ,μ0,μ1},MaxRecursion→100]*)

T[νf_?NumericQ, μf_?NumericQ, ν0_?NumericQ, μ0_?NumericQ] :=
 T0[νf, μ0, ν0, μ0] + Tc[νf, μf, νf, μ0]

(*T0[1.5,1.2,ν0,μ0]
 Tc[1.5,1.2,ν0,μ0]
 T[1.5,1.2,ν0,μ0]*)
```

Out[11]= 0.729666

Out[12]= 0.95412

Out[13]= 0.865225

Out[14]= 1.26208

In[•]:=
```
vmin
vmax

muf
mumax
```

Out[•]= 0.95412

Out[•]= 1.26208

Out[•]= 0.729666

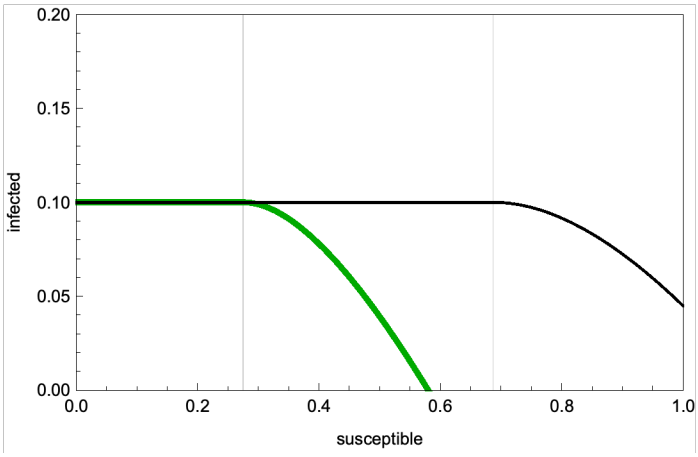
Out[•]= 0.865225

Switching surfaces

In[18]:=
```
Phi[S_, R_] := Module[{},
  If[S ≤ 1/R, Imax, Imax + 1/R (Log[R S] + 1 - R S)]
 ]
```

In[19]:=
```
fig1 = Show[
  Plot[Phi[S, R0], {S, 0, 1},
   PlotRange → {{0, 1}, {0, 2 Imax}},
   GridLines → {{1 / R0, 1 / Rc}, {}},
   FrameTicks → {{Automatic, None}, {Automatic, None}},
   PlotStyle → Directive[Darker@Green, Thickness[0.01]],
   FrameLabel → {"susceptible", "infected"},
   Frame → True
  ]
  ,
  Plot[Phi[S, Rc], {S, 0, 1},
   PlotStyle → Black,
   PlotRange → All]
 ]
```

Out[19]=



In[20]:=
```
I0 = 1*^-2 // N
S0 = 1 - I0 // N

(*v0 = v[S0, I0]*)
v0 = Mean[{vmin, vmax}]

munudata = ParallelTable[
    {μ0,
     vf /. Last@NMinimize[{Evaluate@T[vf, muf, vmin, μ0], vmin ≤ vf ≤ vmax}, vf]}
    , {μ0, muf, mumax, (mumax - muf) / 10}];

Psi = {Ss[#[[2]], #[[1]]], Is[#[[2]], #[[1]]]} & /@ munudata
```

Out[20]= 0.01

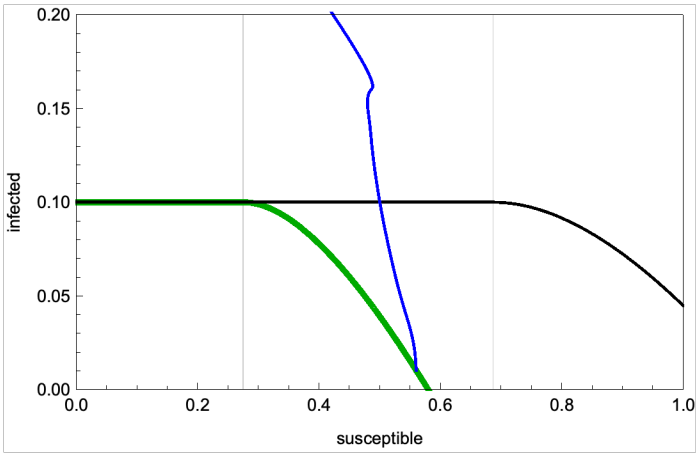
Out[21]= 0.99

Out[22]= 1.1081

Out[24]= {{0.56029, 0.0102271}, {0.554391, 0.0267745},
{0.533972, 0.0504396}, {0.517845, 0.0716974}, {0.505279, 0.0910707},
{0.495694, 0.10895}, {0.488623, 0.12563}, {0.483693, 0.14133},
{0.480599, 0.156217}, {0.479096, 0.170416}, {0.389447, 0.216705}}

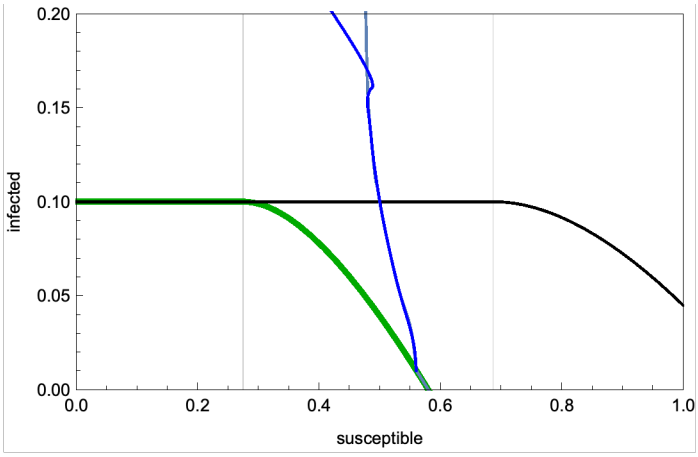
In[25]:=
```
Show[
 fig1
 ,
 ListPlot[Psi,
  Joined → True,
  PlotStyle → {Blue, Thickness[0.005]},
  InterpolationOrder → 2]
 ,
 PlotRange → {All, {0, 0.2}}
]
```

Out[25]=



In[26]:=
```
Psifun = Interpolation[Psi, InterpolationOrder → 5];

Psifun2[S_?NumericQ] := Module[{psivalue},
  Which [
   S <  Min[First@# & /@ Psi], psivalue = 1,
   S > Max[First@# & /@ Psi], psivalue = Phi[S, R0],
   True, psivalue = Psifun[S]
  ];
  psivalue
 ]

Show[
 fig1
 ,
 Plot[Psifun[S], {S, Min[First@# & /@ Psi], Max[First@# & /@ Psi]}]
 ,
 Plot[Psifun2[S], {S, 0, 1}]
 ,
 ListPlot[Psi,
  Joined → True,
  PlotStyle → {Blue, Thickness[0.005]},
  InterpolationOrder → 2]
 ,
 PlotRange → {All, {0, 0.2}}
]
```

Out[28]=



Calculate S*

In[29]:=
```
temp = Table[
{S, Norm[Psifun2[S] - Imax]},
{S, Min[First@# & /@ Psi], Max[First@# & /@ Psi],
     (Max[First@# & /@ Psi] - Min[First@# & /@ Psi]) / 1000}];
indx = Flatten@Position[temp[[All, 2]], Min@temp[[All, 2]]]
Sstar = temp[[indx, 1]][[1]]
```

Out[30]= {648}

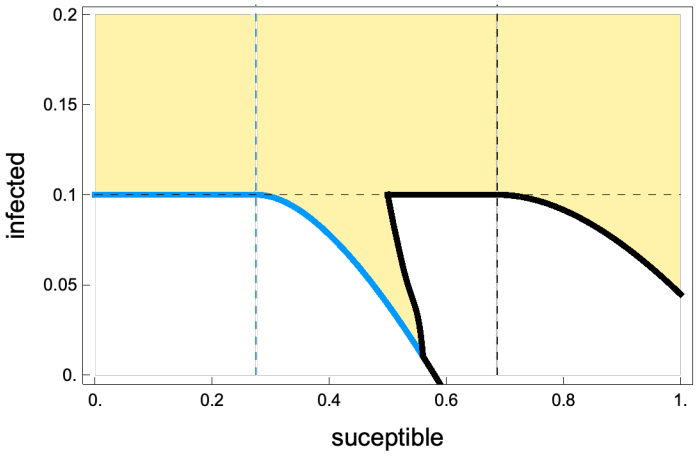
Out[31]= 0.499983

Color the (S,I) plane

In[32]:=
```
ϵ = 0.7*^-3;

optimalcontrol[S_, Is_] := Module[{uopt},
  uopt = Which[
    Is ≤ Phi[S, R0], 0,
    Is ≥  Phi[S, Rc], umax,
    Is ≥ Phi[S, R0] && Is ≤ Psifun2[S], umax,
    (*S≥ Sstar && S ≤ 1/Rc, 1-1/(β S),*)
    True, 0
   ];
  uopt
 ]

optimalcontrol2[S_, Is_] := Module[{uopt},
  uopt = Which[
    Is < Phi[S, R0] + ϵ, 0,
    Is > Phi[S, Rc] + ϵ, umax,
    Phi[S, Rc] - ϵ ≤ Is ≤  Phi[S, Rc] + ϵ && S ≥ Sstar, (1 / ϵ) (Is - Phi[S, Rc]),
    (*Is<Phi[S,Rc]-ϵ,0,*)
    (*Is == Phi[S, Rc], If[S≥ Sstar && S ≤ 1/Rc, 1-1/(β S), umax],*)
    Is ≥ Phi[S, R0] + ϵ && Is ≤ Psifun2[S] && S > 1 / R0, umax,
    (*S≥ Sstar && S ≤ 1/Rc, 1-1/(β S),*)
    True, 0
   ];
  uopt
 ]


figplane = Show[
  DensityPlot[optimalcontrol[S, Is], {S, 0, 1}, {Is, 0, 0.2},
   (*ColorFunction→(Hue[0,2/3,1-#]&),*)
   ColorFunction → (Blend[{White, □}, #] &),
```

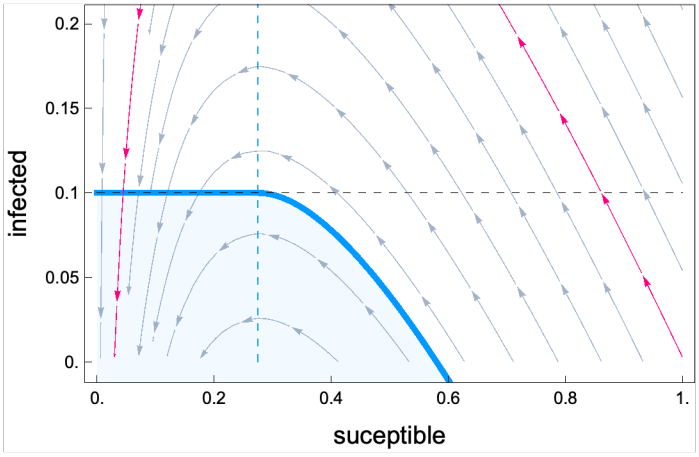
```
  PlotPoints → 100]
 ,
 Plot[Phi[S, R0], {S, 0, 1},
  PlotStyle → Directive[■, Thickness[0.01]],
  (*Filling→Bottom,*)
  FillingStyle → Directive[■, Opacity[0.05]]
 ]
 ,
 Plot[Phi[S, Rc], {S, Sstar, 1}, PlotStyle → Directive[Black, Thickness[0.01]]]
 ,
 Plot[Psifun2[S], {S, Sstar, 1}, PlotStyle → Directive[Black, Thickness[0.01]]
 ]
 ,
 Graphics[{■, Dashed, Line[{{1 / R0, 0}, {1 / R0, 1}}]}]
 ,
 Graphics[{Dashed, Line[{{1 / Rc, 0}, {1 / Rc, 1}}]}]
 ,
 Graphics[{Dashed, Line[{{0, Imax}, {1, Imax}}]}]
 ,
 PlotRange → {Automatic, {-0.001, 0.2}},
 FrameLabel → {Style["suceptible", 13], Style["infected", 13]},
 FrameTicks → {{Range[0, 1, 0.05], None}, {Range[0, 1, 0.2], None}},
 AspectRatio → 1 / GoldenRatio
]
```

Out[35]=



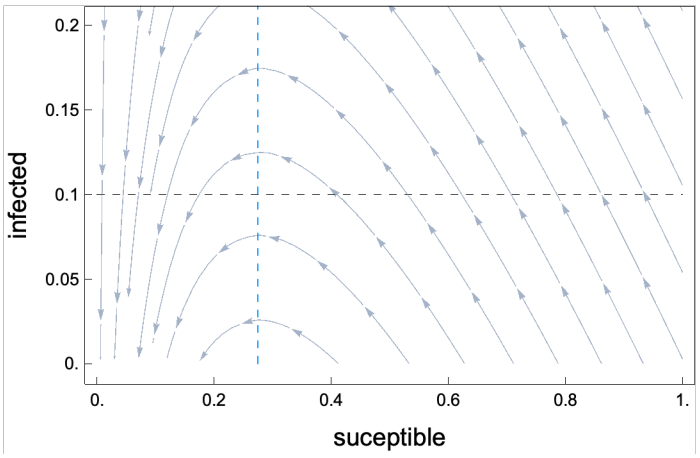
In[122]:=
```
population = 8.855*^6;

Is0 = 1 / population;
S0 = 1 - Is0;

Show[
 Plot[Phi[S, R0], {S, 0, 1},
  PlotStyle → Directive[■, Thickness[0.01]],
  Filling → Bottom,
  FillingStyle → Directive[■, Opacity[0.05]]
 ]
 ,
 Graphics[{■, Dashed, Line[{{1 / R0, 0}, {1 / R0, 1}}]}]
 ,
 Graphics[{Dashed, Line[{{0, Imax}, {1, Imax}}]}]
 ,
 StreamPlot[{-R0 * Svar * Ivar, R0 * Svar * Ivar - Ivar}, {Svar, 0, 1}, {Ivar, 0, 1},
  StreamPoints → {{{{0.99, 0.01}, ■}, Automatic}},
  StreamStyle → Directive[Opacity[0.5]]
 ]
 ,
 PlotRange → {Automatic, {-0.001, 0.2}},
 Frame → True,
 FrameLabel → {Style["suceptible", 13], Style["infected", 13]},
 FrameTicks → {{Range[0, 1, 0.05], None}, {Range[0, 1, 0.2], None}},
 AspectRatio → 1 / GoldenRatio
]
```

Out[125]=



In[132]:=
```
Show[
 Plot[Phi[S, R0], {S, 0, 1},
  PlotStyle → Directive[■, Thickness[0.01], Opacity[0]],
  Filling → Bottom,
  FillingStyle → Directive[■, Opacity[0.05], Opacity[0]]
 ]
 ,
 Graphics[{■, Dashed, Line[{{1 / R0, 0}, {1 / R0, 1}}]}]
 ,
 Graphics[{Dashed, Line[{{0, Imax}, {1, Imax}}]}]
 ,
 StreamPlot[{-R0 * Svar * Ivar, R0 * Svar * Ivar - Ivar}, {Svar, 0, 1}, {Ivar, 0, 1},
  StreamPoints → {{{{0.99, 0.01}, Automatic}, Automatic}},
  StreamStyle → Directive[Opacity[0.5]]
 ]
 ,
 PlotRange → {Automatic, {-0.001, 0.2}},
 Frame → True,
 FrameLabel → {Style["suceptible", 13], Style["infected", 13]},
 FrameTicks → {{Range[0, 1, 0.05], None}, {Range[0, 1, 0.2], None}},
 AspectRatio → 1 / GoldenRatio
]
```

Out[132]=



In[121]:=
```
Pink
```

■

Simulation with optimal control

In[251]:=
```
SolveSIRoptControl[I0_, S0_, γ_, β_, u_, tf_] := Module[{s},
   s = Flatten@NDSolve[{
        S'[t] == -(1 - u[S[t], Is[t]]) β S[t] * Is[t],
        Is'[t] == (1 - u[S[t], Is[t]]) β S[t] * Is[t] - γ Is[t],
        S[0] == S0, Is[0] == I0},
       {S, Is}, {t, tf}, WorkingPrecision → 10, MaxSteps → 1*^8]
  ];

u0[S_, Is_] := 0

uopt[S_?NumericQ, Is_?NumericQ] := Evaluate[optimalcontrol2[S, Is]]
```

In[ ]:=
```
Clear[SolveSIRoptControl]
```

In[254]:=
```
tf = 120;

γ = 1 / 7;
 β = 0.52;

population = 8.855*^6;

Is0 = 1 / population;
S0 = 1 - Is0;


s0opt = SolveSIRoptControl[Is0, S0, γ, β, uopt, tf ];
```

NDSolve: The precision of the differential equation

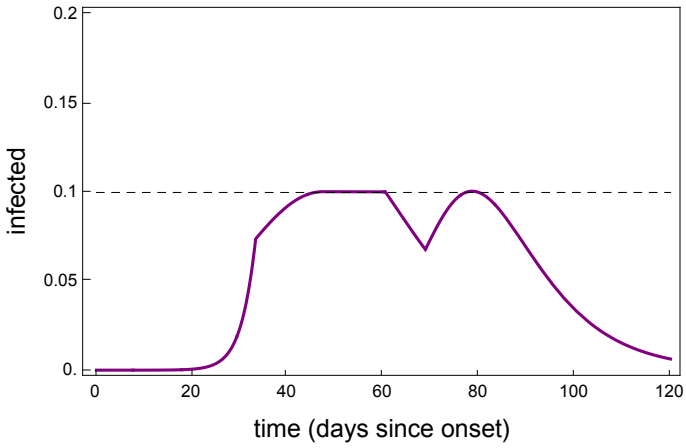
$$\left(\left\{S'[t] == 0.52\, Is[t]\, S[t]\, (-1 + uopt[S[t], Is[t]]),\ Is'[t] == -\frac{Is[t]}{7} + 0.52\, Is[t]\, S[t]\, (1 - uopt[\ll 2 \gg]),\ S[0] == 1.,\ Is[0] == 1.12931 \times 10^{-7}\right\}, \{\}, \{\}, \{\}, \{\}\right)$$ is less than WorkingPrecision (10.`).

In[262]:=
```
s0opt
```

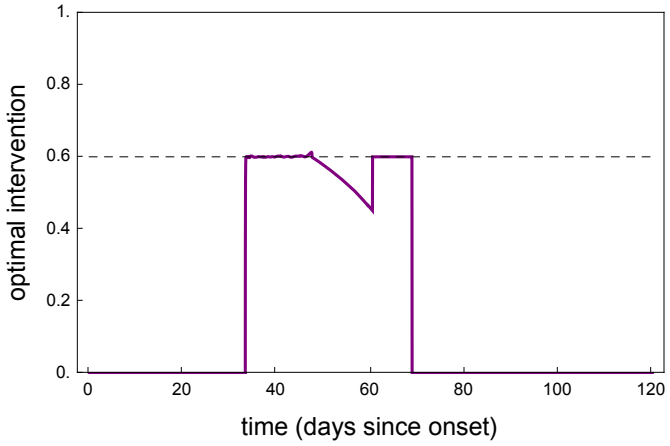
Out[262]=
```
{S → InterpolatingFunction[ 

],

 Is → InterpolatingFunction[ 

]}
```

In[143]:=
```
Show[
 Plot[{
   Evaluate[Is[t] /. s0opt]
  }, {t, 0, tf},
  Frame → True,
  PlotStyle → {
    Directive[■],
    Directive[Darker@Red],
    Directive[Purple]
   },
  PlotRange → {Automatic, {-0.004, 0.204}},
  FrameLabel → {Style["time (days since onset)", 13], Style["infected", 13]},
  FrameTicks → {{Range[0, 10, 0.05], None}, {Range[0, tf, 20], None}},
  AspectRatio → 1 / GoldenRatio,
  ImageSize → {Automatic, 230}
 ]
 ,
 Graphics[{Dashed, Line[{{0, Imax}, {tf, Imax}}]}]
]

Show[
 Plot[{
   Evaluate[uopt[S[t], Is[t]] /. s0opt]
  }, {t, 0, tf},
  Frame → True,
  PlotStyle → {
    Directive[■],
    Directive[Darker@Red],
    Directive[Purple]
   },
  PlotRange → {Automatic, {-0.004, 1}},
  FrameLabel →
   {Style["time (days since onset)", 13], Style["optimal intervention", 13]},
  FrameTicks → {{Range[0, 10, 0.2], None}, {Range[0, tf, 20], None}},
  AspectRatio → 1 / GoldenRatio,
  ImageSize → {Automatic, 230}
 ]
 ,
 Graphics[{Dashed, Line[{{0, umax}, {tf, umax}}]}]
]
```

Out[143]=



Out[144]=



In[145]:=

```
tf = 120;

γ = 1 / 7;
 β = 0.52;

population = 8.855*^6;

Is0 = 1 / population;
S0 = 0.65;


s1opt = SolveSIRoptControl[Is0, S0, γ, β, uopt, tf ];
```

NDSolve: The precision of the differential equation

$$\left(\left\{S'[t] == 0.52\, Is[t]\, S[t]\, (-1 + uopt[S[t], Is[t]]),\ Is'[t] == -\frac{Is[t]}{7} + 0.52\, Is[t]\, S[t]\, (1 - uopt[\ll 2 \gg]),\ S[0] == 0.65,\ Is[0] == 1.12931 \times 10^{-7}\right\}, \{\}, \{\}, \{\}, \{\}\right)$$ is less than WorkingPrecision (10.`).

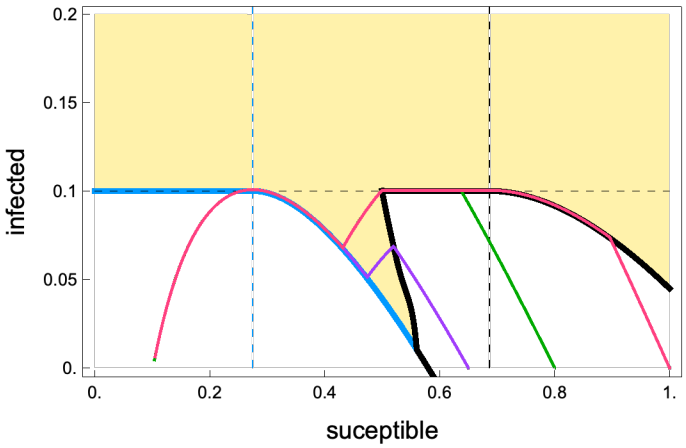
In[152]:=
```
tf = 120;

γ = 1 / 7;
 β = 0.52;

population = 8.855*^6;

Is0 = 1 / population;
S0 = 0.8;


s2opt = SolveSIRoptControl[Is0, S0, γ, β, uopt, tf ];
```

NDSolve: The precision of the differential equation

$$\left(\left\{\left\{S'[t] == 0.52\, Is[t]\, S[t]\, (-1 + uopt[S[t], Is[t]]),\ Is'[t] == -\frac{Is[t]}{7} + 0.52\, Is[t]\, S[t]\, (1 - uopt[\ll 2 \gg]),\ S[0] == 0.8,\ Is[0] == 1.12931 \times 10^{-7}\right\}, \{\}, \{\}, \{\}, \{\}\right\}\right)$$ is less than WorkingPrecision (10.`).

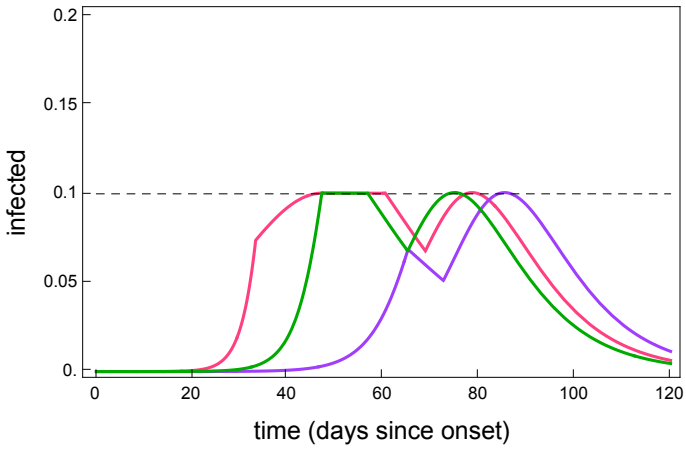
In[159]:=
```
fig1 = Show[
  figplane
  ,
  ParametricPlot[
   Evaluate[{S[t], Is[t]} /. s1opt], {t, 0, tf},
   PlotStyle → Directive[■]
  ]
  ,
  ParametricPlot[
   Evaluate[{S[t], Is[t]} /. s2opt], {t, 0, tf},
   PlotStyle → Directive[Darker@Green]
  ]
  ,
  ParametricPlot[
   Evaluate[{S[t], Is[t]} /. s0opt], {t, 0, tf},
   PlotStyle → Directive[■]
  ]
  ,
  ImageSize → {Automatic, 230}
 ]
```

Out[159]=



In[160]:=
```
fig2 =
 Show[
  Plot[{
    Evaluate[Is[t] /. s0opt],
    Evaluate[Is[t] /. s1opt],
    Evaluate[Is[t] /. s2opt]
   }, {t, 0, tf},
   Frame → True,
   PlotStyle → {
     Directive[■],
     Directive[■],
     Directive[Darker@Green]
    },
   PlotRange → {Automatic, {-0.004, 0.204}},
   FrameLabel → {Style["time (days since onset)", 13], Style["infected", 13]},
   FrameTicks → {{Range[0, 10, 0.05], None}, {Range[0, tf, 20], None}},
   AspectRatio → 1 / GoldenRatio,
   ImageSize → {Automatic, 230}
  ]
  ,
  Graphics[{Dashed, Line[{{0, Imax}, {tf, Imax}}]}]
 ]
```

Out[160]=

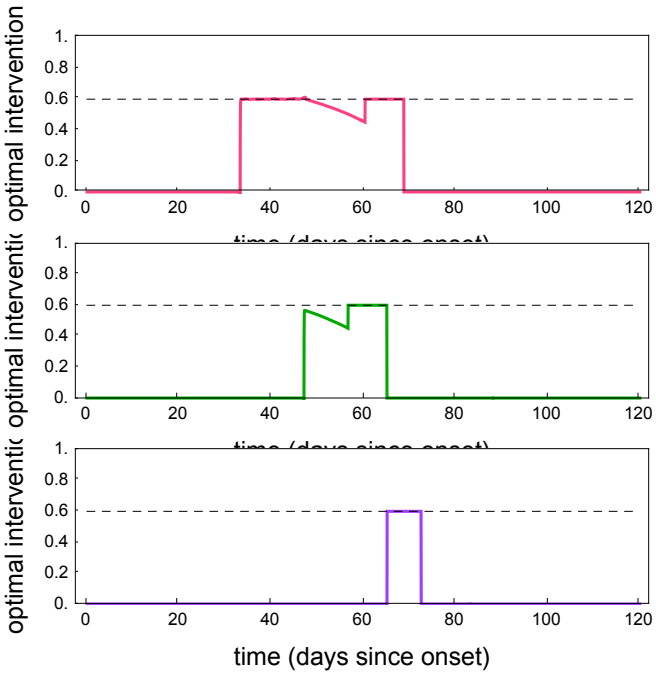


In[161]:=
```
figu1 = Show[
   Plot[{
     Evaluate[uopt[S[t], Is[t]] /. s0opt]
    }, {t, 0, tf},
    Frame → True,
    PlotStyle → {
      Directive[■]
     },
```

```
    PlotRange → {Automatic, {-0.004, 1}},
    FrameLabel →
     {Style["time (days since onset)", 13], Style["optimal intervention", 13]},
    FrameTicks → {{Range[0, 10, 0.2], None}, {Range[0, tf, 20], None}},
    AspectRatio → 1 / 6 GoldenRatio,
    ImageSize → {340, Automatic}
   ]
   ,
   Graphics[{Dashed, Line[{{0, umax}, {tf, umax}}]}]
  ];

figu2 = Show[
   Plot[{
     Evaluate[uopt[S[t], Is[t]] /. s2opt]
    }, {t, 0, tf},
    Frame → True,
    PlotStyle → {
      Directive[Darker@Green]
     },
    PlotRange → {Automatic, {-0.004, 1}},
    FrameLabel →
     {Style["time (days since onset)", 13], Style["optimal intervention", 13]},
    FrameTicks → {{Range[0, 10, 0.2], None}, {Range[0, tf, 20], None}},
    AspectRatio → 1 / 6 GoldenRatio,
    ImageSize → {340, Automatic}
   ]
   ,
   Graphics[{Dashed, Line[{{0, umax}, {tf, umax}}]}]
  ];

figu3 = Show[
   Plot[{
     Evaluate[uopt[S[t], Is[t]] /. s1opt]
    }, {t, 0, tf},
    Frame → True,
    PlotStyle → {
      Directive[■]
     },
    PlotRange → {Automatic, {-0.004, 1}},
    FrameLabel →
     {Style["time (days since onset)", 13], Style["optimal intervention", 13]},
    FrameTicks → {{Range[0, 10, 0.2], None}, {Range[0, tf, 20], None}},
    AspectRatio → 1 / 6 GoldenRatio,
    ImageSize → {340, Automatic}
```

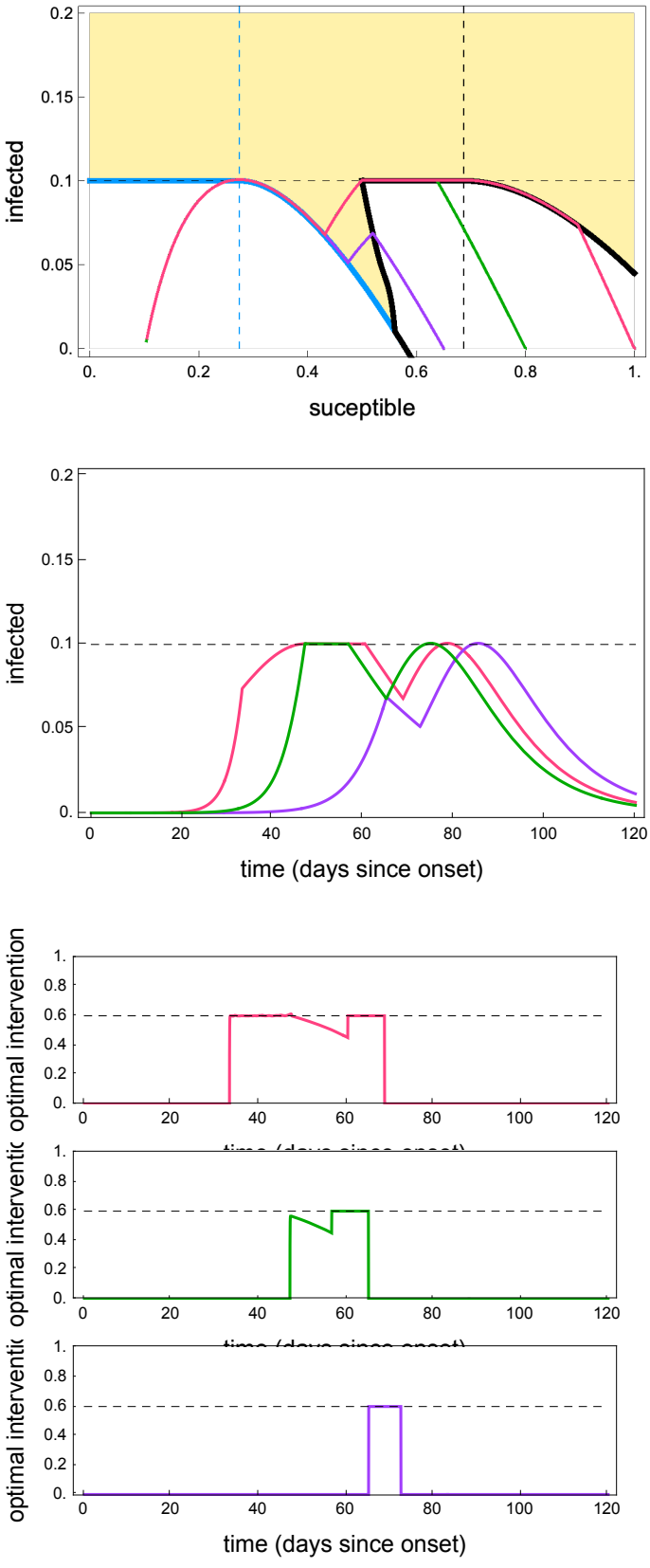
```
    ]
    ,
    Graphics[{Dashed, Line[{{0, umax}, {tf, umax}}]}]
   ];

figu = Column[{figu1, figu2, figu3}, Spacings → -2]
```

Out[164]=



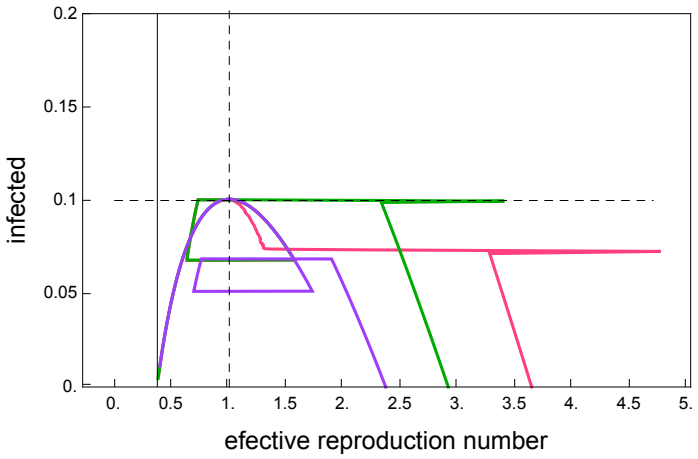
In[•]:=
```
Column[{fig1, "", fig2, "", figu}]
```

Out[•]=



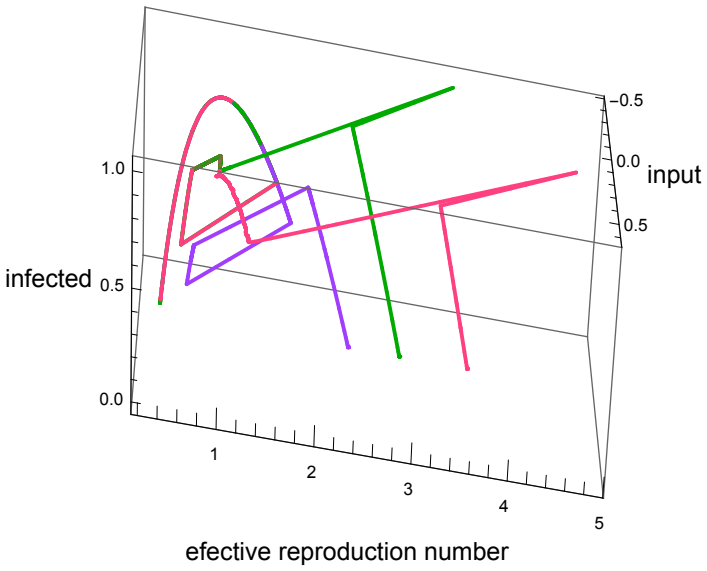
In[167]:=
```
Show[
 ParametricPlot[{
   Evaluate[{R0 (1 - uopt[S[t], Is[t]]) S[t], Is[t]} /. s0opt],
   Evaluate[{R0 (1 - uopt[S[t], Is[t]]) S[t], Is[t]} /. s2opt],
   Evaluate[{R0 (1 - uopt[S[t], Is[t]]) S[t], Is[t]} /. s1opt]
  },
  {t, 0, tf},
  PlotStyle → {
    Directive[■],
    Directive[Darker@Green],
    Directive[■]
   },
  GridLines → None,
  Frame → True,
  PlotRange → {All, {-0.001, 0.2}},
  FrameLabel →
   {Style["efective reproduction number", 13], Style["infected", 13]},
  FrameTicks → {{Range[0, 1, 0.05], None}, {Range[0, 5, 0.5], None}},
  AspectRatio → 1 / GoldenRatio
 ]
 ,
 Graphics[{Dashed, Line[{{1, 0}, {1, 1}}]}]
 ,
 Graphics[{Dashed, Line[{{0, Imax}, {4.7, Imax}}]}]
]
```

Out[167]=



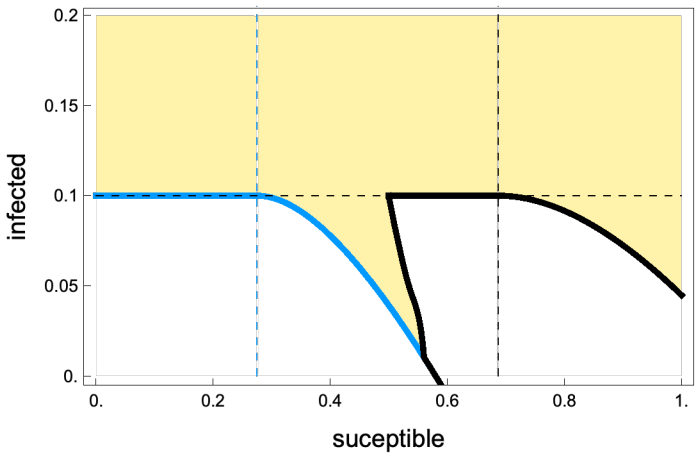
In[179]:=
```
ParametricPlot3D[{
  Evaluate[{R0 (1 - uopt[S[t], Is[t]]) S[t], 10 * Is[t], uopt[S[t], Is[t]]} /. s0opt],
  Evaluate[{R0 (1 - uopt[S[t], Is[t]]) S[t], 10 * Is[t], uopt[S[t], Is[t]]} /. s2opt],
  Evaluate[{R0 (1 - uopt[S[t], Is[t]]) S[t], 10 * Is[t], uopt[S[t], Is[t]]} /. s1opt]
 }, {t, 0, tf},
 PlotRange → All,
 AxesLabel → {Style["efective reproduction number", 13],
   Style["infected", 13], Style["input", 13]},
 PlotStyle → {
   Directive[■],
   Directive[Darker@Green],
   Directive[■]
  },
 AspectRatio → 1
]
```

Out[179]=



In[180]:=
```
Show[
 DensityPlot[optimalcontrol[S, Is], {S, 0, 1}, {Is, 0, 0.2},
  (*ColorFunction→(Hue[0,2/3,1-#]&),*)
  ColorFunction → (Blend[{White, ■}, #] &),
  PlotPoints → 100]
 ,
 Plot[Phi[S, R0], {S, 0, 1},
  PlotStyle → Directive[■, Thickness[0.01]],
  (*Filling→Bottom,*)
  FillingStyle → Directive[■, Opacity[0.05]]
 ]
 ,
 Plot[Phi[S, Rc], {S, Sstar, 1}, PlotStyle → Directive[Black, Thickness[0.01]]]
 ,
 Plot[Psifun2[S], {S, Sstar, 1}, PlotStyle → Directive[Black, Thickness[0.01]]
 ]
 ,
 Graphics[{■, Dashed, Line[{{1 / R0, 0}, {1 / R0, 1}}]}]
 ,
 Graphics[{Dashed, Line[{{1 / Rc, 0}, {1 / Rc, 1}}]}]
 ,
 Graphics[{Dashed, Line[{{0, Imax}, {1, Imax}}]}]
 ,
 PlotRange → {Automatic, {-0.001, 0.2}},
 FrameLabel → {Style["suceptible", 13], Style["infected", 13]},
 FrameTicks → {{Range[0, 1, 0.05], None}, {Range[0, 1, 0.2], None}},
 AspectRatio → 1 / GoldenRatio
]
```

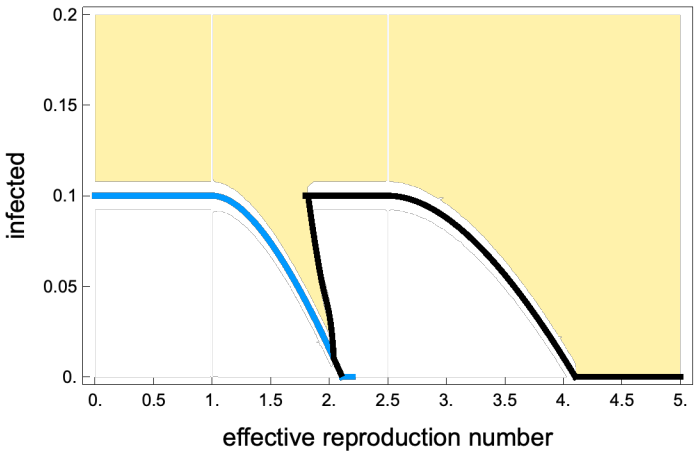
Out[180]=



In[188]:=
```
SfromRe[Re_, u_] := Re / (R0 * (1 - u))
```

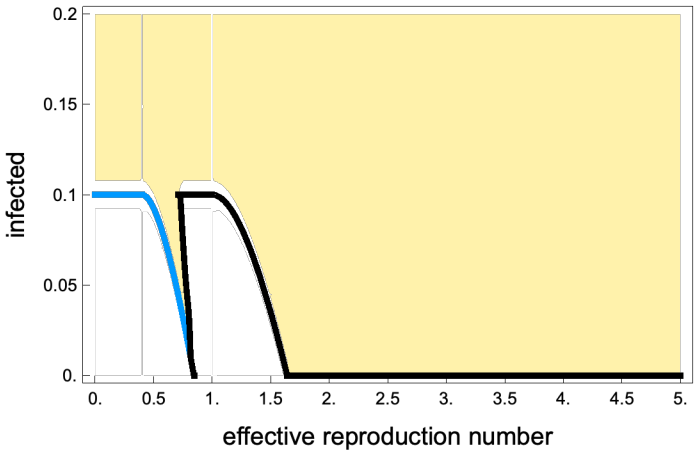
In[235]:=
```
Show[
 DensityPlot[optimalcontrol[SfromRe[Re, 0], Is], {Re, 0, 5}, {Is, 0, 0.2},
  (*ColorFunction→(Hue[0,2/3,1-#]&),*)
  ColorFunction → (Blend[{White, }, #] &),
  PlotPoints → 100]
 ,
 Plot[Max[Phi[SfromRe[Re, 0], R0], 0], {Re, 0, 2.2},
  PlotStyle → Directive[, Thickness[0.01]],
  (*Filling→Bottom,*)
  FillingStyle → Directive[, Opacity[0.05]]
 ]
 ,
 Plot[Max[0, Phi[SfromRe[Re, 0], Rc]],
  {Re, 1.8, 5}, PlotStyle → Directive[Black, Thickness[0.01]]]
 ,
 Plot[Max[0, Psifun2[SfromRe[Re, 0]]],
  {Re, 1.82, 2.1}, PlotStyle → Directive[Black, Thickness[0.01]]]
 ,
 PlotRange → All,
 FrameLabel →
  {Style["effective reproduction number", 13], Style["infected", 13]},
 FrameTicks → {{Range[0, 1, 0.05], None}, {Range[0, 5, 0.5], None}},
 AspectRatio → 1 / GoldenRatio
]
```

Out[235]=



In[236]:=
```
Show[
 DensityPlot[optimalcontrol[SfromRe[Re, umax], Is], {Re, 0, 5}, {Is, 0, 0.2},
  (*ColorFunction→(Hue[0,2/3,1-#]&),*)
  ColorFunction → (Blend[{White, }, #] &),
  PlotPoints → 100]
 ,
 Plot[Max[Phi[SfromRe[Re, umax], R0], 0], {Re, 0, 0.85},
  PlotStyle → Directive[, Thickness[0.01]],
  (*Filling→Bottom,*)
  FillingStyle → Directive[, Opacity[0.05]]
 ]
 ,
 Plot[Max[0, Phi[SfromRe[Re, umax], Rc]],
  {Re, 0.71, 5}, PlotStyle → Directive[Black, Thickness[0.01]]]
 ,
 Plot[Max[0, Psifun2[SfromRe[Re, umax]]],
  {Re, 0.73, 0.85}, PlotStyle → Directive[Black, Thickness[0.01]]]
 ,
 PlotRange → All,
 FrameLabel → {Style["effective reproduction number", 13], Style["infected", 13]},
 FrameTicks → {{Range[0, 1, 0.05], None}, {Range[0, 5, 0.5], None}},
 AspectRatio → 1 / GoldenRatio
]
```

Out[236]=



```
time = Table[t, {t, 0, tf, 1}]
controlsignal = Table[uopt[S[t], Is[t]] /. s1opt, {t, time}]
Flatten@Position[controlsignal, 0 ]
Ssignal =
```

```
Out[248]= {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23,
 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44,
 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65,
 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85,
 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104,
 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120}

Out[249]= {0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0.6, 0.6, 0.6, 0.6, 0.6, 0.6, 0.6, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

Out[250]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22,
 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
 62, 63, 64, 65, 66, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88,
 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106,
 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121}
```

$u_{max} = 0.8$