Green's function in a duct

Preparation: Read sections 7.5.1 7.5.3 and 7.6 in reference [1] carefully. In your simulation, consider carefully the cross-sectional dimensions of your duct, frequency range of interest and number of modes present in the duct (truncation order of the summation).

The Green's function is a normalised transfer function between the volume acceleration of a point monopole source at an arbitrary position and the sound pressure at another arbitrary position. The purpose of this exercise is to study the Green's function between a monopole placed at the rigid bottom of a semi-infinite duct of rectangular cross section and the sound pressure at a point in the duct in a simulation study.

Simulation study

The theoretical Green's function in a semi-infinite duct is

$$G(\mathbf{r},\mathbf{r}_0) = -\frac{\mathrm{j}}{Sk}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\psi_{mn}(x,y)\psi_{mn}(x_0,y_0)}{\sqrt{1-(k_{mn}/k)^2}}\mathrm{e}^{-\mathrm{j}k_{zmn}z} = -\frac{\mathrm{j}}{S}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{\psi_{mn}(x,y)\psi_{mn}(x_0,y_0)}{k_{zmn}}\mathrm{e}^{-\mathrm{j}k_{zmn}z},$$

where $r_0 = (x_0, y_0, 0)$ is the position of the source. The cross-sectional dimensions of the duct are a (height) and b (width), S = ab is the cross-sectional area, and k is the wavenumber. The quantity

$$k_{zmn} = \left(k^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2\right)^{1/2} = \sqrt{k^2 - k_{mn}^2}$$

is the axial wavenumber of mode (m, n), m = 0, 1, 2, ..., and n = 0, 1, 2, ... For all modes of higher order (m > 0 and/or n > 0) the axial wavenumber is seen to be purely imaginary below the cut-off frequency of the mode,

$$f_{mn} = \frac{k_{mn}c}{2\pi} = \frac{c}{2}\left(\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right)^{1/2}.$$

The quantity

$$\psi_{mn}(x,y) = \sqrt{\varepsilon_m \varepsilon_n}\cos\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)$$

is the mode shape; and ε_m = 1 for m = 0 and ε_m = 2 for m > 0.

Use MATLAB or Python to plot the Green's function for a given set of positions as a function of the frequency, in decibels, with a linear frequency axis. It is important that to choose *the right sign* of the square root in the expression for k_{zmn} when the argument of the square root is negative; the sign should correspond to a sound wave that decays exponentially with z. By default, MATLAB / Python chooses the positive imaginary square root, which corresponds to the non-plausible case of a wave that grows exponentially with the distance. Naturally, the infinite sum must be truncated, but all modes that are significant in the frequency range of concern should be included. You can verify your criterion with a convergence study, focusing on the convergence at the upper limit of that range.

Similarly, plot the amplitude of the sound pressure at a given frequency as a function of z. Several frequencies can be examined. It is particularly interesting to consider driving the source just below the

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cutoff frequency of the first higher order mode, and a somewhat higher frequency where at least two modes can propagate. You can also examine how the response changes when the source (or receiver) is placed at different cross-sectional location (x_0, y_0). Discuss the results that you observe and relate these to the theory.

Report

Present and discuss the results of the simulations. Think carefully how to best explain the phenomena that you observe, and the relevant aspects involved in the propagation of sound inside a duct (natural frequencies, mode-shapes, source and receiver position, propagating-standing-evanescent waves, phase velocities, etc.). Please consider how to examine these aspects and present them clearly and concisely.

The report should be submitted two weeks after the day of the exercise. Recommended **length 5-7 pages** (as a suggestion: Intro 0.3p; Theory 1 pp; Setup, results and discussion 3-5 pp.; Conclusion 0.5 p.). The report should be concise (though complete!).

References

[1] F. Jacobsen and P. M. Juhl, *Fundamentals of General Linear Acoustics*, John Wiley and Sons, 2013 (Corresponding to sections 6.1 and 6.3 in note no 31260 “Propagation of sound waves in ducts”. *Note no* 31260, 2011).