Green’s function in a room

Preparation: Read sect. 8.2 in reference [1] carefully and think about the relation between an appropriate number of modes in the sum, the frequency range, and the dimensions of the room. Consider that between the natural frequencies of the room many modes contribute to the sound field.

The Green’s function is a normalised transfer function between the volume acceleration of a point monopole source at an arbitrary position and the sound pressure at another arbitrary position. The purpose of this exercise is to study such a Green’s function in a lightly damped rectangular room, experimentally as well as by a simulation study.

Experimental determination of the Green’s function

The Green’s function in a room can be measured using a device with two matched quarter-inch microphones (A and B) in a tube mounted on a Brüel & Kjær ‘Omnisource’ loudspeaker, and another microphone (C) that represents the observation point. The measurement involves determining the frequency responses between microphone (A) and the two other microphones (B and C), $\mathrm{H_{AB}(f)}$ and $\mathrm{H_{AC}(f)}$, and the frequency response between microphones B and C, $\mathrm{H_{BC}(f)}$. If it can be assumed that the sound field in the tube is one-dimensional it is easy to show that the volume velocity at the outlet of the tube is

$$Q = \frac{S}{\rho c} \frac{p_A \cos(kl) - p_B \cos\big(k(l + \Delta l)\big)}{j \sin(k\Delta l)},$$

where p_A and p_B are the signals from microphones A and B, k is the wavenumber, c is the speed of sound, ρ is the density of air, S is the cross-sectional area of the tube, Δl is the distance between the two microphones in the tube, and l is the distance between the opening of the tube and the nearest microphone (B). It now follows that the Green’s function is

$$G(\boldsymbol{r}, \boldsymbol{r_0}) = \frac{p_c(x,y,z)}{j\omega\rho Q} = \frac{p_c(x,y,z) Q^*}{j\omega\rho |Q|^2} =$$
$$\frac{\sin(k\Delta l)}{kS} \frac{H_{AC}(f)\cos(kl) - |H_{AB}(f)|^2 H_{BC}(f) \cos\big(k(l+\Delta l)\big)}{\cos^2(kl) - 2\mathcal{R}e\{H_{AB}(f)\}\cos(kl)\cos\big(k(l+\Delta l)\big) + |H_{AB}(f)|^2 \cos^2\big(k(l+\Delta l)\big)},$$

where $\boldsymbol{r} = (x, y, z)$ is the position of microphone C, and $\boldsymbol{r}_0 = (x_0, y_0, z_0)$ is the position of the outlet of the tube. The detailed derivation can be found in refs. [2,3].

The three frequency responses are measured using a Brüel & Kjær ‘PULSE’ analyser. It can be recommended to measure with the analyser in the ‘zoom FFT mode’, to use a frequency span of 200 Hz, a centre frequency of, say, 150 Hz, and a resolution of 62.5 mHz (corresponding to 3200 frequency lines). At higher frequencies the imperfect geometry of the room causes deviations between the theory and the measurements.

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Simulation study

The theoretical Green's function for a rectangular room is [3]

$$G(\boldsymbol{r},\boldsymbol{r}_0) = \frac{-1}{V}\sum_{m=0}^{\infty}\frac{\psi_m(\boldsymbol{r})\psi_m(\boldsymbol{r}_0)}{k^2 - k_m^2 - jk/(\tau_m c)},$$

where $\boldsymbol{r} = (x, y, z)$ is the point at which the sound pressure is determined and $\boldsymbol{r}_0 = (x_0, y_0, z_0)$ is the position of the source. The dimensions of the room are l_x, l_y and l_z, $V = l_x l_y l_z$ is the volume, and τ_m is the time constant of the *m'th* mode. The corresponding reverberation time is

$$T_{60} = 6\ln(10)\,\tau_m.$$

The quantity

$$k_m = \left(\left(\frac{n_x\pi}{l_x}\right)^2 + \left(\frac{n_y\pi}{l_y}\right)^2 + \left(\frac{n_z\pi}{l_z}\right)^2\right)^{1/2}$$

is the wavenumber that corresponds to the natural frequency of mode $m = (n_x, n_y, n_z)$,

$$f_m = \frac{c}{2}\left(\left(\frac{n_x}{l_x}\right)^2 + \left(\frac{n_y}{l_y}\right)^2 + \left(\frac{n_z}{l_z}\right)^2\right)^{1/2}$$

where $n_x = 0,1,2,\ldots$, $n_y = 0,1,2,\ldots$, and $n_z = 0,1,2,\ldots$, so the Green's function is a triple summation. The quantity

$$\psi_m(x,y,z) = \Lambda_\mathrm{m}\cos\left(\frac{n_x\pi}{l_x}x\right)\cos\left(\frac{n_y\pi}{l_y}y\right)\cos\left(\frac{n_z\pi}{l_z}z\right)$$

is the mode shape, and

$$\Lambda_m = \sqrt{\varepsilon_{n_x}\varepsilon_{n_y}\varepsilon_{n_z}}$$

is a normalisation constant in which $\varepsilon_m = 1$ for $m = 0$ and $\varepsilon_m = 2$ for $m > 0$.

Construct a MATLAB program that can plot the Green's function for a given set of positions as a function of the frequency, in decibels, with a linear frequency axis. Use the room data listed below. For simplicity all modes can be given the same time constant, although better results might be obtained if all individual time constants were determined. Evidently the infinite sum must be truncated, but more modes than those with natural frequency in the frequency range of concern should be included.

Construct another similar program for plotting the sound pressure at a given frequency as a function of the position $\boldsymbol{r} = (x, y, z)$, for example on a line across the room from one corner to another. Compare measured Green's functions for two different source/receiver configurations with predictions.

Report

Discuss the Green's functions from the simulation study and the agreement (or lack of agreement) between the experimental and theoretical Green's functions. It is important that you relate the physical phenomena that you observe in your simulation/measurement with the theory, and that you demonstrate an understanding of both the theory and the observations (please be precise and concise in your observations).

This should be submitted as a combined report with the simulation exercise of the diffuse sound field. The report should be submitted two weeks after starting the later exercise. For guidance: expected total length of the combined report is approx. 7-10 pages (Intro 0.5p; Theory 1-2 pp; Setups 1 p. Results & discussion 4-6 pp.; Conclusion 0.5 p. These lengths and sections are just for guidance on total length). It makes sense to present the results of the two exercises separately [Green's function and diffuse sound field], as two parts of the same report.

The report should be complete and concisely written.

Appendix

Dimensions of the room: l_x = 3.14 m (width); l_y = 4.38 m (length); l_z = 3.27 m (height)
Reverberation time: T_{60} = 3.5 s (this a rough average value at the frequencies of interest)
Data for the tube: S = 0.00113 m^2; l = 0.03 m; Δl = 0.02 m
(Note: sensitivity of the 1⁄4 inch microphones in this freq. range ~ Mic A: 3 mV/Pa, Mic B: 3.8 mV/Pa)

References

[1] F. Jacobsen and P. M. Juhl, Fundamentals of General Linear Acoustics, John Wiley and Sons, 2013 (Corresponding to chapter 2 in note no 31261 "The sound field in a reverberation room", 2011).
[2] S. Gade, N. Møller, J. Hald and L. Alkestrup: The use of volume velocity source in transfer measurements. Proceedings of Inter-Noise 2004, Prague, Czech Republic, 2004.
[3] Y. Luan and F. Jacobsen: A method of measuring the Green's function in an enclosure. Journal of the Acoustical Society of America 123, 4044-4046 (2008).