Numerical Acoustics – Spring 2025

Exercise 5: Non-uniqueness in 2D BEM

Non-uniqueness occurs for exterior problems in BEM at frequencies where the corresponding interior domain has an eigenfrequency (Atalla, section 7.10, p. 280). This numerical problem will be shown in this exercise and a remedy using the combined Helmholtz integral equation formulation (CHIEF) method will be implemented. The BEM in 2D from the previous exercise dealing with the scattering of a plane wave by a cylinder is used as a basis. A new version has been uploaded containing a working script, *Test_ScatCyl*.

1. Identify the frequencies where the non-uniqueness problem arises. To this end, consider that the instability of the equations will show as a high condition number of the coefficient matrices. Create a loop for k values in a range such as k=2 to k=8, with as many values as can be calculated in a reasonable time (50-100). There is no need to solve the equations; just obtain the A matrices and calculate and store their condition numbers for all k values. Use the Matlab function *cond(A)*. Represent the condition numbers in a figure as a function of k. Interpret the result and deduce the frequencies with a non-uniqueness issue. You may repeat the process in a zoomed-in k-scale to obtain more precise k values. Keep in mind that maxima can be hidden in between k values, and their amplitude is rather an indication of how close the calculated k values are from the non-uniqueness than of how "strong" the effect is.

2. Choose one of the k values found in 1 to give a very high condition number and run the full calculation of the pressure on the surface of the cylinder, comparing it with the analytical solution. Does it work? You may have to run again the loop in 1 with a deeper zoom to make the k value more precise and provoke the problem. You may need up to 10-15 significant figures, which will depend and change depending on the computer you use.

3. The package includes the script *Test_ScatCyl_Convergence.m*. Run it; it produces a convergence plot of the BEM solution for 150 Hz (k=2.74), a frequency where non-uniqueness is not a problem. Try it now for the k values found in section 1. Observe how non-uniqueness affects convergence.

4. To solve the non-uniqueness, include additional equations in the BEM system of equations, corresponding to points collocated in the interior of the setup (CHIEF points). Use the function *fieldpoints,* with the CHIEF points as an input, for obtaining the extra rows of the A matrix (`[A;Ap]`). Notice that the system of equations is now overdetermined (more rows than columns), but Matlab can still solve it using the backslash operator (`[A;Ap]\(–2*pi*[`p^I`;`p^I_{ch}`])`). You will also need the incident pressure on the CHIEF points (p^I_{ch}). Experiment with the number and positions of the points, observing the calculation results. Not many points may be needed; often even one is sufficient.

5. Test whether the convergence behavior is restored by including the CHIEF points in the convergence calculation too.

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