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2013 | INNSBRUCK | AUSTRIA
15.-18. SEPTEMBER 2013
NOISE CONTROL FOR QUALITY OF LIFE

Verification of an Acoustic 3D BEM with Visco-Thermal Losses

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ABSTRACT

Sound waves propagating in the interior of devices such as acoustic transducers, hearing aids and mobile phones undergo a significant amount of losses due to viscous and thermal effects. In some cases like microphones, the performance of the device even relies on controlling these loss mechanisms.

A newly implemented three-dimensional Boundary Element Method with visco-thermal losses will be tested in this work using a number of cases, including idealized setups with analytical solutions and actual measurements on existing devices.

In particular, measurement microphones will be used as test cases. These devices are challenging due to the high degree of coupling between diaphragm, internal gap, back cavity and external medium. In this work they are modeled using a coupled FEM-BEM model, where the Finite Element Method is used on the diaphragm.

Keywords: BEM, Visco-thermal losses

1. INTRODUCTION

Numerical modeling of acoustic transducers has been carried out more or less routinely for a couple of decades, and modeling techniques range from lumped parameter models, typically implemented as equivalent electrical circuits to numerical models based of Finite Difference, Finite Element or Boundary Element Methods. The Boundary Element Method (BEM) has become popular for modeling the sound field exterior of the transducer – for instance the directivity of loudspeakers and microphones, since the BEM readily allows for an infinite domain. For the modeling of e.g. directivity, acoustic centre and other quantities defined in the far field, computational models that do not take losses into account are normally quite adequate. However, the complexity of the models used has progressed from the lossless models and in some cases it is necessary to be able to model the viscous and thermal losses, which take place close to the surfaces of the domain. The condenser microphone is one example of a device, in which the viscous and thermal losses are important since the damping due to these losses is actively used in order to shape the response of the device. Hence, effort in the later years has been out into developing numerical methods such as the FEM and the BEM, which are able to deal with viscous-thermal losses in axis-symmetric and full 3-D space.

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A full 3-D BEM model, that include viscous and thermal losses has recently been developed and is presented in an accompanying paper. The implementation involves several substantial changes and additions to the standard lossless BEM, and verification of the model developed becomes an essential interactive part of the implementation. Verification of numerical models is not always straight forward. On one hand analytical solutions are needed in order to provide ultimate tests of the accuracy and convergence of the numerical model, since there is a need to know the true solution within fractions of a dB. On the other hand, the numerical models are being developed because of the need of being able to model geometries and/or boundary conditions, for which analytical solutions do not exist, and in these cases models has to be verified using either alternative numerical models or measurements.

The present paper presents two verification cases for the 3-D BEM model developed: i) a simplified microphone, for which an analytical solution has been presented in the literature [1] and ii) measurements on actual condenser microphones.

2. MATRIX EQUATIONS

In the accompanying paper [2] the dicretization and assembling of the linearized visco-thermal differential equations into a matrix equation by the use of a direct collocation Boundary Element Method is described. The result is a matrix equation relating the total pressure at the boundary nodes, $\mathbf{p}_\mathrm{a}$, to the boundary velocity components $\mathbf{v}_{\mathrm{boundary},n}$ and $\vec{\mathbf{v}}_{\mathrm{boundary},t}$ (the normal and the two tangential components respectively) and an incoming pressure at boundary nodes $\mathbf{p}^\mathrm{I}$ (if present),

$$\mathbf{X}\,\mathbf{p}_a = \mathbf{Y}_1\mathbf{v}_{boundary,n} + \mathbf{Y}_2\vec{\mathbf{v}}_{boundary,t} + \mathbf{Y}_3\,\mathbf{p}^\mathrm{I}. \tag{1}$$

The matrices $\mathbf{X}$, $\mathbf{Y}_1$, $\mathbf{Y}_2$ and $\mathbf{Y}_3$ are M by M, where M is the number of nodes in the surface mesh. In the test cases studied present paper, the visco-thermal acoustic domain is the interior of a cavity, which is excited by a diaphragm. Therefore both the incoming pressure and the tangential components of the velocity are zero leading to,

$$\mathbf{v}_{\mathrm{boundary},n} = \mathbf{Y}_1^{-1}\mathbf{X}\,\mathbf{p}_a = \mathbf{VT}\mathbf{p}_a. \tag{2}$$

The diaphragm is modeled as a membrane with no stiffness,

$$\Delta\varepsilon + k_\mathrm{m}^2\varepsilon = \frac{p_d}{T}, \tag{3}$$

where Δ is the two-dimensional Laplace operator, ε is the normal displacement, k_m is the wavenumber of the mechanical wave, T is the membrane tension and p_d is the difference of sound pressures acting on the diaphragm, internally and externally. This equation is implemented using a two-dimensional Finite Element Method implementation producing the matrix equation:

$$T\left(\mathbf{K} + k_m^2\mathbf{M}\right)\varepsilon = \mathbf{p_d} \tag{4}$$

where $\mathbf{K}$ and $\mathbf{M}$ are the stiffness and mass matrices respectively. The matrix equations (2) and (4) are coupled though the total pressure acting on the diaphragm, $\mathbf{p_d}$=$\mathbf{p_a}$-$\mathbf{p_{inc}}$ and continuity of normal velocity on the interior of the diaphragm $\mathbf{v}_{\mathrm{boundary},n} = -\mathrm{j}\omega\varepsilon$ (the time convension is $\mathrm{e}^{\mathrm{j}\omega t}$ – the minus is due to the fact that in the BEM, the velocity is along the inward normal, whereas the displacement is along the outward normal in the FEM),

$$\left(\begin{array}{c|c} T(\mathbf{K} + k_m^2\mathbf{M}) & \mathbf{D} \\ \hline -\mathrm{j}\omega & -\mathbf{VT} \end{array}\right)\begin{pmatrix} \vdots \\ \boldsymbol{\varepsilon} \\ \vdots \\ \hline \vdots \\ \mathbf{p_a} \\ \vdots \end{pmatrix} = \begin{pmatrix} \vdots \\ -\mathbf{p_{inc}} \\ \vdots \\ \hline \vdots \\ 0 \\ \vdots \end{pmatrix} \tag{5}$$

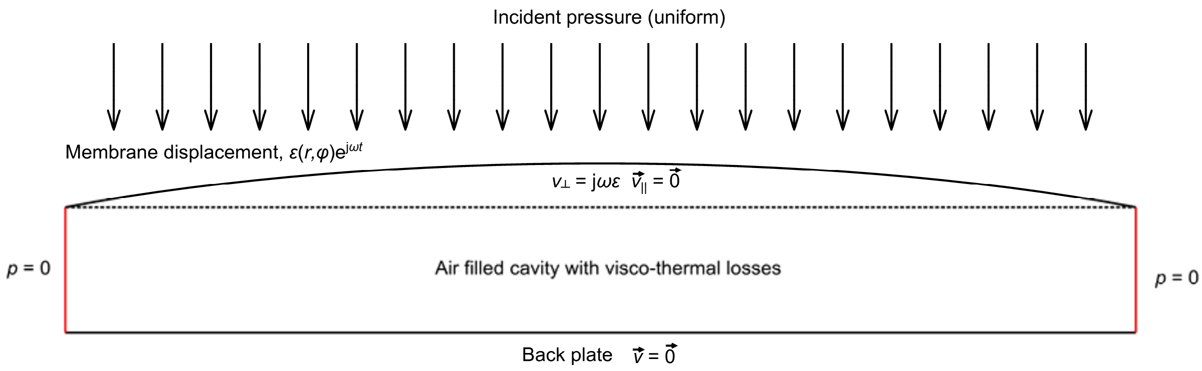

Figure 1. Geometry of the analytical test case. A circular membrane is loaded with a harmonic uniform pressure on the exterior side and forms the upper part of a cylindrical cavity on its other side

3. VERIFICATION

3.1 The idealized Microphone

An analytical model for predicting the acoustic response of a stretched circular diaphragm (membrane) that is separated from a rigid back plate by a thin air film has been presented in reference [1] and adapted in a Matlab program [3]. The axis-symmetric set-up is as sketched in Figure 1: A diaphragm is stretched above a back plate (back electrode). The exterior side of the diaphragm is excited by a uniform sound pressure and the interior side is exposed to a thin air film. A pressure release boundary condition is assumed at the rim of the small back cavity. The simplicity of the boundary conditions allows the three modes of the sound field to be expressed in an orthogonal set of cylindrical Bessel functions. The diaphragm itself is fixed at its rim and it is supposed to be lossless (an ideal membrane equation), so that all losses of the system takes place in the air film. The analytical solution is found by coupling the diaphragm to the air in the cavity and to the exterior cavity by matching (by continuity) the boundary conditions for each term in the modal expansions, making use of orthogonality [1].

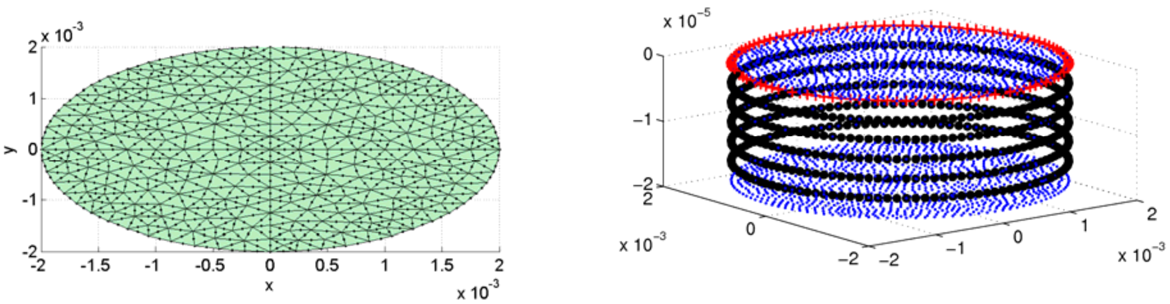

Figure 2. Left: The membrane of the simplified microphone meshed using 313 quadratic triangular elements. Right: The boundary nodes of the membrane and the cavity. Blue points: nodes with velocity boundary condition; red points: membrane nodes with fixed boundary condition; black points: nodes with pressure release boundary condition. Note the scale on the vertical axis.

Table 1. Parameters used for the simplified microphone

Uniform exterior pressure amplitude	1 Pa
Membrane radius, Back plate radius	2 mm
Gap thickness, d	0.018 mm
Membrane tension	3128 N/m
Membrane surface density	57.7 g/m^2

The numerical solution corresponding to the simplified microphone is carried out by coupling the visco-thermal 3-D BEM model with a 2-D FEM model for the membrane. Both models were implemented in Matlab. In order to compare the analytical and the numerical results in a configuration relevant for a microphone, parameters for the setup were chosen as summarized in Table 1.

Figure 3 shows the deflection of the membrane for four frequencies ranging from 200 Hz to 200 kHz. It is evident that the deflection shape is axisymmetric due to the axisymmetry of both the geometry and the boundary conditions. However, no assumption of axisymmetry is made in the numerical model – the restriction of axisymmetry is only necessary in order to compare with the analytical solution.

Figure 4 makes use of the fact, that the membrane displacement is axisymmetric. The figure shows the modulus of the displacement along a radius of the membrane. It is evident that the deflection becomes more complicated at higher frequencies. The good agreement between the numerical and the analytical solution is also evident.

Finally, Figure 5 shows the normalized sensitivity of the simplified microphone as a function of the frequency. The normalized sensitivity is found as a weighted average of the membrane displacement [4]. The series of curves correspond to different mesh resolutions as summarized in Table 2. A finer mesh leads to more accurate results – in particular at higher frequencies.

Table 2. Mesh densities for the calculations on the simplified microphone

Mesh name	BEM nodes	BEM elements	FEM elements
9	2946	1472	313
12	4930	2464	545
15	7842	3920	889
18	11266	5632	1297

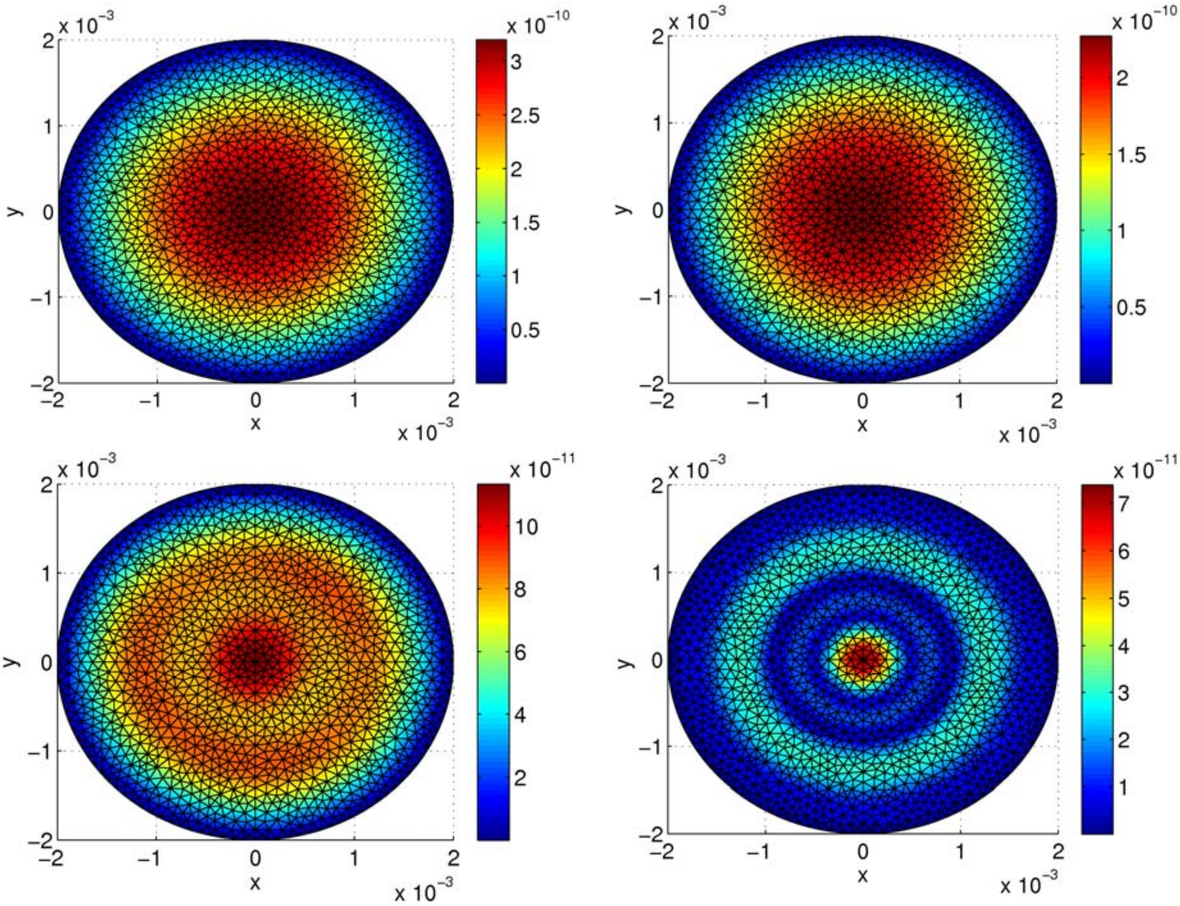

Figure 3 Amplitude of the membrane displacement for the simplified microphone for a number of frequencies; upper left: 0.2 kHz; upper right: 35 kHz; lower left: 110 kHz; lower right: 200 kHz

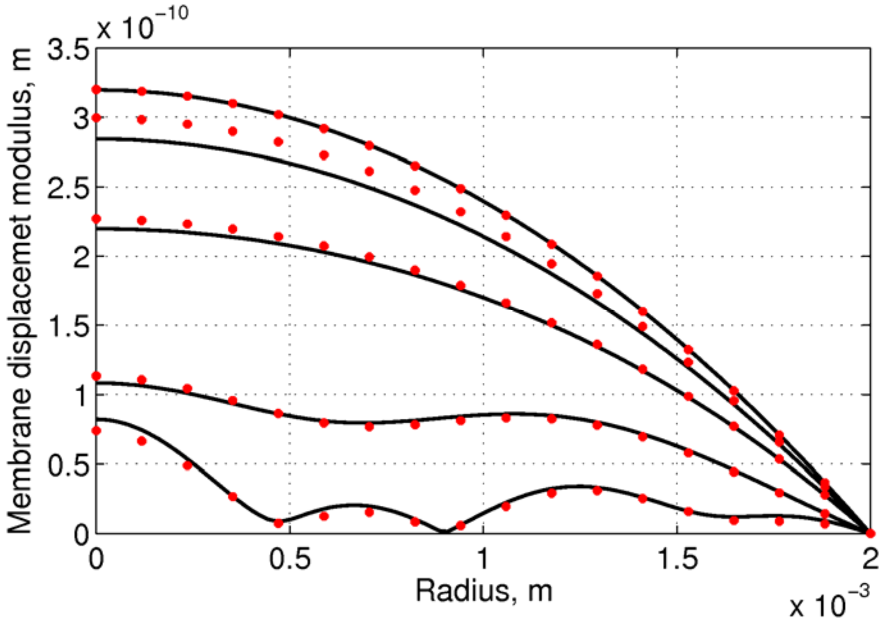

Figure 4 Amplitude of the membrane displacement of the simplified microphone along a radius. Solid line: analytical solution; red dots: visco-thermal BEM solution at computational nodes. The pair of curves corresponds from upper (larges amplitudes) to lower (smaller amplitudes) to the frequencies: 0.2 kHz; 13 kHz; 35 kHz; 110 kHz and 200 kHz.

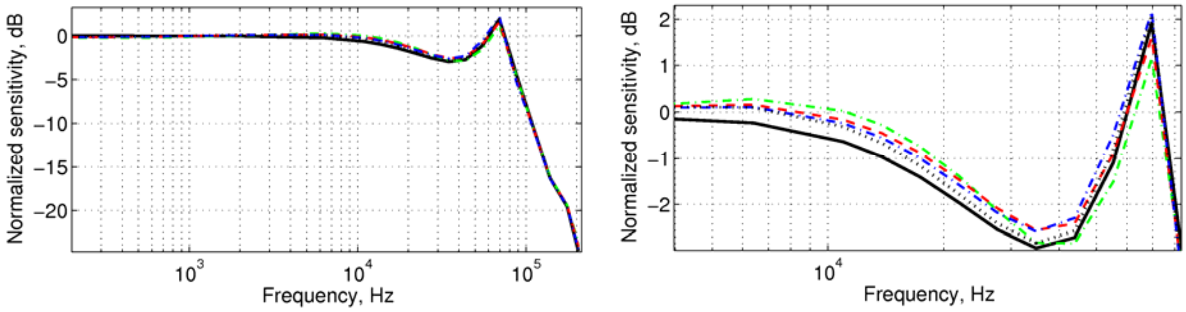

Figure 5 Normalized sensitivity of the simplified microphone for a number of computational mesh sizes; left: full frequency range; right: zoom near the resonance of the system. Solid line: analytical solution; green line: mesh 9; red line: mesh 12; blue line: mesh 15; dotted black line: mesh 18 (refer to Table 2).

3.2 B&K 4938 microphone

In order to compare the implementation to measurements a microphone has been modeled. The Brüel and Kjær quarter inch type 4938 (see Figure 6) has been chosen for this comparison since measurements of the sensitivity of a number of microphones could be supplied by the manufacturer. The manufacturer also kindly supplied the nominal parameters of the microphone – see Table 3.

Table 3. Nominal parameters of the B&K microphone type 3938

Membrane radius	2 mm
Back plate radius	1.75 mm
Gap thickness, *d*	0.019 mm
Membrane tension	3128 N/m
Membrane surface density	57.7 g/m^2

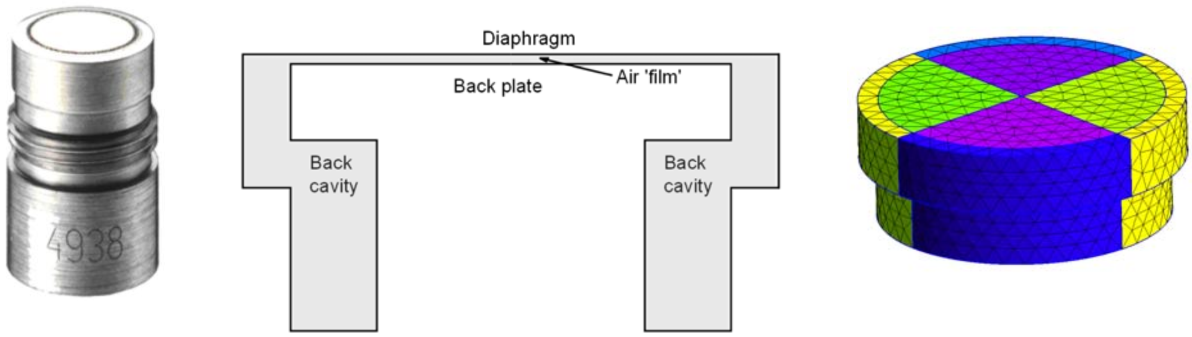

Figure 6 Left: photograph of the Brüel & Kjær quarter inch microphone type 4938; middle: sketch of the cross section of the microphone; right: computational mesh

Since no measurement of the deflection of the diaphragm is available the comparison concerns the normalized sensitivity only. Figure 7 shows the normalized sensitivity of the microphone. The 'measurement' presented is the average of 183 measurements using an electrostatic actuator on different microphones and the calculations presented makes use of the nominal parameters (Table 3). The computational mesh consists of 2342 quadratic triangular elements (4686 nodes). In order to access the importance of losses, a calculation without considering losses is also presented. It is evident that the viscous-thermal losses are important for the response of the microphone for frequencies above 10 kHz, and that the viscous-thermal BEM calculations agree well with the measurements, even though some difference between calculations and measurements can be observed.

The details of the response of the microphone depend on the actual parameters. The actual parameters of a given microphone may differ from the nominal value due to production tolerances, but also due to the fact that when producing the microphone, each microphone is adjusted to a certain sensitivity at a low frequency (within a tolerance) rather than to the nominal tension of the diaphragm, for instance. Figure 8 shows the normalized sensitivity of an average of 183 microphones as well as the maximum and minimum sensitivity measured. Also shown in Figure 8 is a calculation corresponding to changing the tension of the diaphragm to 3666 N/m, the surface density of the diaphragm to 61.1 g/m^2. It is evident that the difference from a particular microphone to a 'nominal' microphone is of the same order than the difference of calculations using perturbed parameters. The difference between calculation results and measurement average is probably both due to numerical errors (the finite number of elements and other approximations) as well as lack of knowledge of the precise parameters of the 'average' microphone.

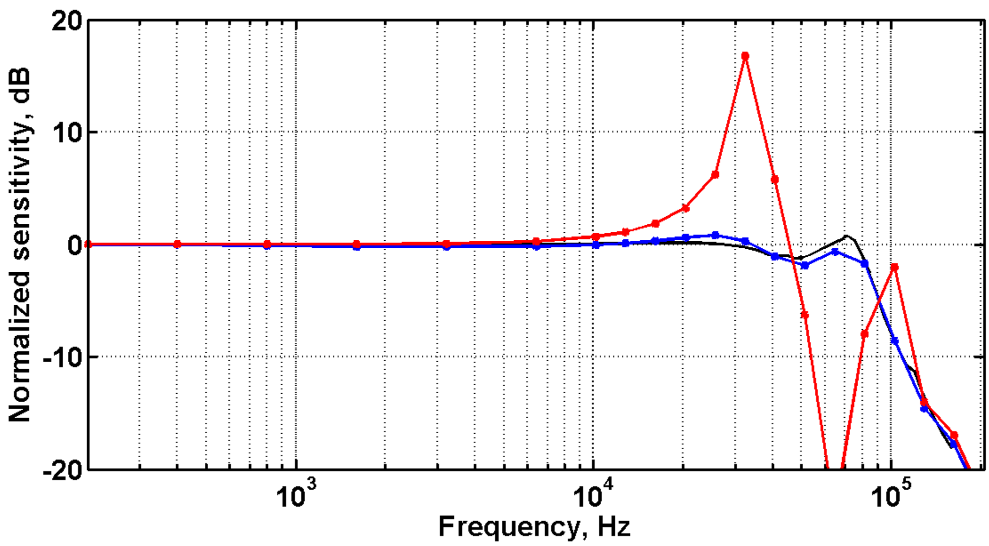

Figure 7 Normalized sensitivity of the B&K type 4938 microphone. Black curve: average of 183 microphones; blue curve: viscous-thermal BEM calculation; red curve: lossless BEM calculation. The dots indicate the calculation frequencies.

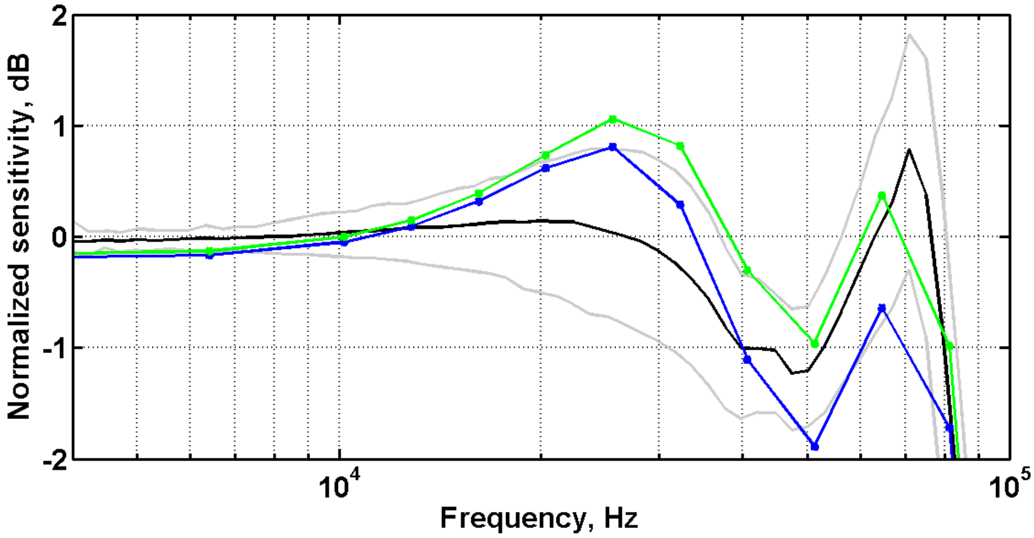

Figure 8 Normalized sensitivity of the B&K type 4938 microphone near the resonance. Black curve: average of 183 measurements; grey curves: minimum and maximum measured; blue curve: viscous-thermal BEM, nominal parameters; green curve: viscous-thermal BEM, modified parameters

4. DISCUSSION AND CONCLUSION

A full three-dimensional Boundary Element Method for sound propagation in air, which includes viscous and thermal losses, has been coupled to a two-dimensional Finite Element Method for an ideal membrane, in order to facilitate comparison of the numerical model to an analytical test case and to measurements.

The numerical model agrees very well with the analytical model, and the agreement improves when refining the computational mesh.

The agreement of the numerical model with measurements is good, although not quite as good as with the analytical test case. The reasons for discrepancy is believed to be the lack of knowledge of the actual parameters of a real microphone and to a lesser extend computational errors. The other simplifications made (ideal membrane equation for the diaphragm, the effect of polarization voltage and the non-inclusion of the radiation impedance for the microphone in the actuator setup) are believed to be of minor importance for this particular microphone.

ACKNOWLEDGEMENTS

The authors would like to thank Johan Gramtorp, Brüel and Kjær A/S for valuable discussions during the course of this study as well as making parameters and measurement results for the Brüel and Kjær type 4938 microphone available.

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