Numerical Acoustics - Spring

Exercise 3: Numerical integration and convergence

In this exercise numerical integration will be used in connection with the concept of convergence of numerical methods. Numerical integrals and convergence are core concepts in numerical methods such as BEM and FEM.

1. Write expressions in Matlab to calculate, numerically, an integral using the left Riemann sum, the trapezoidal rule, Simpson's rule, and Gauss integration. For the Gaussian integration, you can obtain the parameters using the Matlab function:

```
function [bp,wf]=gaussrule(n)
% n: order, bp: base points, wf: weight factors
u=(1:n-1)./sqrt((2*(1:n-1)).^2-1);
[vc,bp]=eig(diag(u,-1)+diag(u,1));
[bp,k]=sort(diag(bp));
wf=2*vc(1,k)'.^2;
```

   This function gives the *base points (bp)* and *weight factors (wf).* The (*bp,wf*) in the function *gaussrule* correspond to (x_s,w_s) on the slides, with the *bp* defined for Gauss integration over the interval (-1,1). For a generic (a,b) interval, a variable change is needed:

```
bp = bp*(b-a)/2 + (b+a)/2;
```

   and the result of the integral must be multiplied by `(b-a)/2`.

2. Use your functions to evaluate the simple test function:

$$\int_1^4 \sqrt{x}dx = 14/3$$

   This integral contains a function with infinite derivatives (C^∞) and it has an analytical value to compare with. Write a script to evaluate this function for different numbers of divisions and plot the error versus the number of divisions. If you use a double-logarithmic plot (*loglog*), you should see straight lines. This is called a *convergence plot*. What do you observe?

3. Use again your functions to evaluate the same function in another interval:

$$\int_0^1 \sqrt{x}dx = 2/3$$

   Prepare again a convergence plot. Observe the results and try to explain them.

   Again, prepare for a short explanation to your classmates.

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