OpenBEM - An open source Boundary Element Method software in Acoustics

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Abstract

OpenBEM is a collection of open source programs for solving the Helmholtz Equation using the Boundary Element Method. The collection is written in Matlab by the authors and contains codes for dealing with exterior and interior problems in two or three dimensions as well as implementation of axi-symmetric and half-space problems. It also contains a number of improvements such a dealing with thin objects and close surfaces, meshing for 2D and axisymmetrical problems, analytical solutions for verification, and a number of additional functions.

This paper gives an overview of the capabilities of the program with examples of its use. Previous research results where OpenBEM was employed will be mentioned.

Keywords: BEM, Numerical Methods.

1   Introduction

Numerical methods such as the Finite Element Method (FEM), the Boundary Element Method (BEM) and Finite Differences are regularly used in Acoustics. The BEM only requires the discretization of the domain boundary, saving computing time and storage. However, the BEM, unlike the FEM, cannot cope in principle with an inhomogeneous medium and produces fully populated coefficient matrices.

All in all, the BEM has clear advantages over the remaining methods for harmonic analysis of stationary systems, in particular when the domain is unbounded. Examples are characterization of microphones and loudspeakers, calibration systems, noise barriers, scattering by objects, etc. There are a number of commercial and non-commercial software packages implementing the BEM, and some researchers have developed their own codes for specific purposes.

In this paper a specific implementation of the Boundary Element Method for Acoustics is presented, the OpenBEM.[1] The openness of OpenBEM not only means that it is distributed free of charge to those interested, but also that its code can and is constantly modified by the users to suit their needs. The different geometrical implementations of BEM included in OpenBEM will be reviewed and some of the main features highlighted. Finally, some examples of the use of OpenBEM in research will be mentioned.

2 Theory

OpenBEM implements the BEM variant called *direct collocation method*. This way of implementing BEM deals directly with the acoustic variables (sound pressure *p* and particle velocity *v*) and boundary conditions and it is thus simpler to use and interpret. The BEM can also be implemented using other formulations such as the *indirect variational method*, where the acoustic variables are replaced by quantities representing their variation across the boundaries.[2] The indirect variational BEM has some computational advantages, like symmetric matrices and the possibility of dealing with thin objects. However, some of the improvements in OpenBEM's direct collocation implementation can effectively counterbalance its weaknesses, as is the case of the non-uniqueness and the thin shape problems mentioned later in this paper. The direct collocation method is briefly explained in the following.
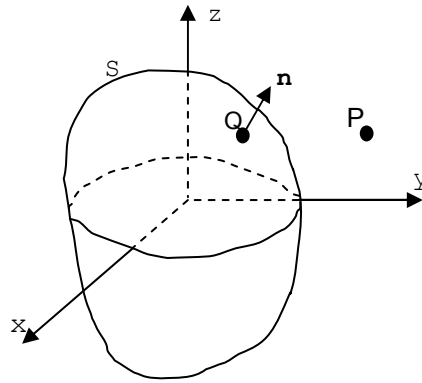

Figure 1. Generic integration domain and boundary surface.

The BEM approach to acoustic radiation and scattering problems is based on the Helmholtz Integral Equation that relates the pressure *p(Q)* and normal velocity *v(Q)* on the surface of a body of any shape (see figure 1) with the pressure at any point *p(P)* and the pressure of an incoming wave *p^I(P)*.[3,4] The harmonic time dependence $e^{i\omega t}$ is omitted, giving:

$$C(P)p(P) = \int_S \left(\frac{\partial G}{\partial n} p(Q) + ikz_0 v(Q)G \right) dS + 4\pi p^I(P) \tag{1}$$

where *S* is the surface of the body, *Q* a point on that surface and *P* any exterior or interior point. The normal vector **n** is directed into the computational domain. The factor *C(P)* is the geometrical constant and represents the exterior solid angle at *P*.
The Green's function for 3-D free space is

$$G(R) = \frac{e^{-ikR}}{R}, \qquad R = |P - Q| \tag{2}$$

In BEM, the surface S is discretized into elements, resulting into a matrix equation:

$$\mathbf{C}\,\mathbf{p} = \mathbf{A}\,\mathbf{p} + i\,k\,z_o\,\mathbf{B}\,\mathbf{v} + 4\,\pi\,\mathbf{p}^I \tag{3}$$

Equation (3) is the result of *collocating* point *P* on every one of the discrete *Q* points, the *nodes* of the surface mesh. Here **p** and **v** denote the pressure and normal velocities at the nodes and matrices **A** and **B** contain integrals of the kernel functions defined in (1) and (2). Given the fact that point P has been placed at the surface, the left-hand side can be subtracted from the diagonal of the first term in the right-hand side of (3). Then the pressure on the nodes can be expressed as a function of the normal velocity and/or the incident pressure on the surface, for radiation and scattering problems respectively, by solving the system of equations. The sound pressure on any point of the domain can then be obtained from the surface values of pressure and normal velocity by integrating (1) again.

In BEM, therefore, obtaining a solution over the domain is made in two phases: i) solving the BEM system of equations and getting pressure and normal velocity on the surface, and ii) choosing and calculating points in the domain using the previous results. There are, however, many cases where only phase i) is performed, e.g., to obtain the pressure on the membrane of a microphone. In phase ii), one can choose to calculate only the relevant parts of the sound field, such as normal planes, far field points, etc.

3   OpenBEM: history and description

OpenBEM is written in Matlab, a high-level interpreted programming environment. Matlab is a commercial product, but the OpenBEM code can be easily adapted to run in the non-commercial open-source Matlab clone Octave (www.octave.org). The origins of OpenBEM date back to the authors' Ph.D. work at the Technical University of Denmark.[4,5] The first implementation of the axisymmetrical formulation made in Pascal was reprogrammed then in Matlab and new features were added. Since then, the code has been continuously revised and employed in research projects, not only by the authors, but also many other researchers.

OpenBEM consists of three independent formulations: axisymmetrical (AxiBEM), three-dimensional (3DBEM), and bi-dimensional (2DBEM). They share a common structure and many of the features, but differ in the definition of the geometry and its implementation.

3.1   The axisymmetrical formulation AxiBEM

This was the first of the three formulations to be developed. It was originated by the thesis work of one of the authors.[4] Taking as a startpoint the general BEM theory outlined in section 2, the problem is reduced to an axisimmetrical geometry by using cylindrical coordinates, and expressing the rotational variation of the acoustic variables analytically using cosine expansions.

$$p = \sum_{m=0}^{\infty} p_m \cos(m\theta) \qquad v = \sum_{m=0}^{\infty} v_m \cos(m\theta) \tag{4}$$

The expressions in (4) are introduced in the Helmholtz integral equation, which is then discretized. In this way, axisymmetrical objects can be described in three dimensions and the sound field can be non-axisymmetrical.

Only the generator of the body needs to be discretized, that is, the mesh is further reduced to one-dimensional, as represented in figure 2. A special function is included in the package to generate the meshes, allowing any kind of axisymmetrical geometry, including user-defined curves. Other functions facilitate the definition of the different terms of the cosine expansion of excitation and solution.
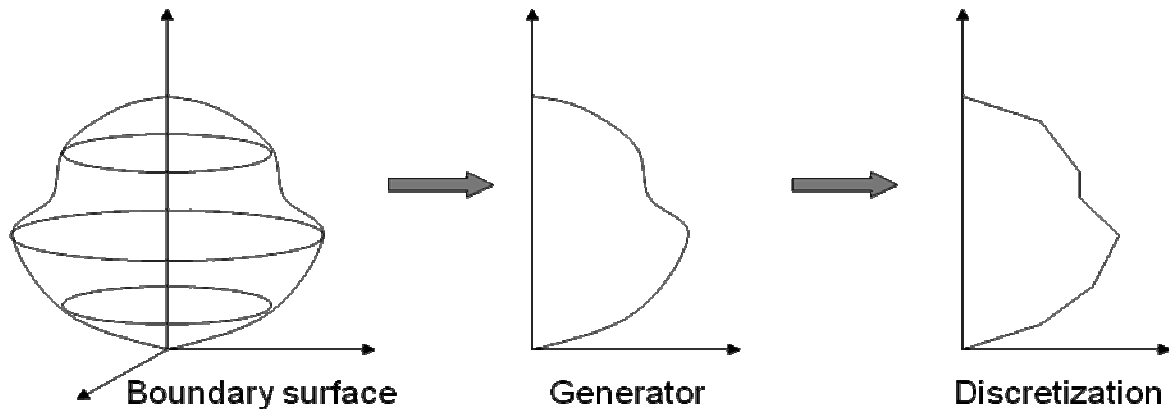

Figure 2. Schematic representation of the meshing process in AxiBEM

The cosine series needs of course to be truncated, but the experience shows that a rather small number of terms is normally enough to represent a setup. Every term should be calculated independently and the solution assembled afterwards. In many cases, a full axisymmetrical description is sufficient, so only the $m=0$ term needs to be considered.

This formulation has been employed extensively in a number of projects. The definition of the mesh is very simple as compared to the full three-dimensional version, and there are many setups in Acoustics that can be simplified as axisymmetrical objects: micropones, loudspeakers, cavities, etc.

3.2   The three-dimensional formulation 3DBEM

This is the formulation that corresponds directly to the theory in section 2. From the programming point of view, it is simpler than the other two, but it requires more complicated meshes and generates heavier calculations in terms of memory and computer time, if we consider problems of similar geometrical size in relation to the wavelength.

OpenBEM does not provide 3D meshing functions, but it allows importing meshes generated in other programs, e.g. ANSYS. Figure 3 shows examples of 3D meshes generated in other environments and used in OpenBEM projects.
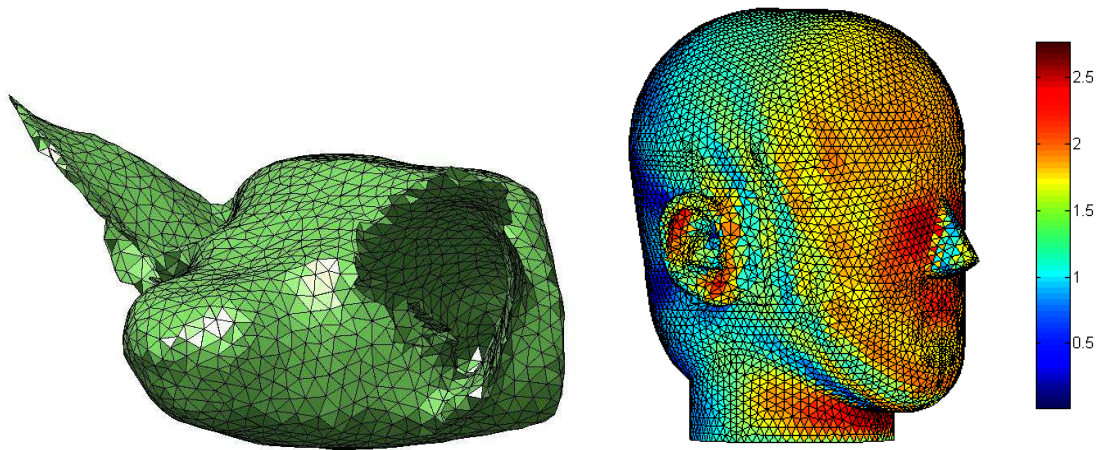
Figure 3. Examples of 3DBEM meshes: a bat head obtained from a CT-scan in the ChiRoPing project and a Head-And-Torso Simulator (Brüel&Kjær HATS).

3.3 The two-dimensional formulation 2DBEM

In the 2DBEM formulation, objects and calculations are defined in the x-y plane. The z direction is considered with no variation. This, on the one hand, simplifies the meshing very much, but, on the other hand, makes the numerical implementation somewhat more complicated. This formulation was first motivated by research on sound barriers, and therefore the possibility of including an impedance plane was implemented in it.[6] Figure 4 shows a result for a simple rectangular barrier calculated by 2DBEM, which includes an impedance plane.
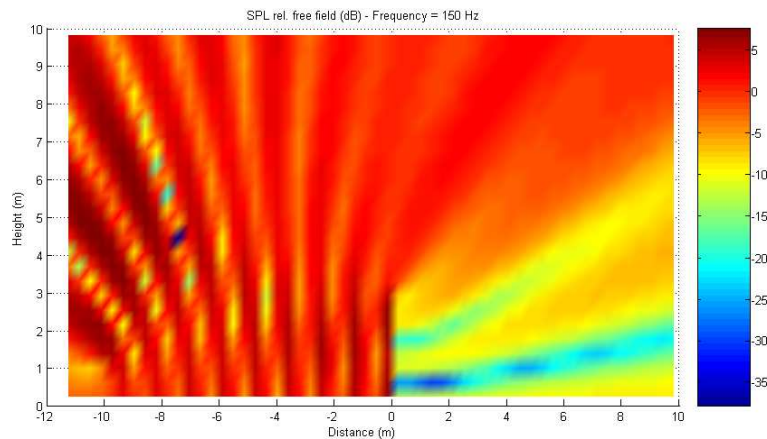

Figure 4. Sound field (relative to free field) around a sound barrier excited by a line source at 150 Hz, as calculated by 2DBEM-OpenBEM.

The implementation of the Green's function is more involved in 2DBEM. The free field Green's function contains a Hankel function instead of an exponential function and it can also be modified to account for an infinite plane with impedance. Mesh generation is implemented in a similar way as in AxiBEM, so it is possible to generate the meshes, which are one-dimensional, inside OpenBEM.

5

In 2DBEM, the point sources are replaced by line sources, and objects extend in the z direction. This makes it an obvious choice for studying sound barriers, but it also has other uses, such as the study of diffusers in room acoustics.

It has also proven to be a very good tool for Acoustics teaching, since it is possible to run, in a reasonably short time, several interesting examples of, for example, scattering around objects, modes in cavities and other interesting phenomena.

4 OpenBEM features

OpenBEM is an open package of Matlab functions. This means there is not a user interface and the users are expected to adapt the functions to their needs. Each of the three formulations has the following parts, each grouped together:

- A kernel of functions implementing the BEM calculation. With these functions the user can calculate the coefficient matrices or obtain the results on field points. It also includes meshing functions, incoming field generators and other utilities.
- A number of test scripts implementing sample calculations
- Functions to generate analytical solutions for simple objects such as spheres and discs. They are included for verification purposes.
- A number of example meshes, in the 3DBEM formulation.

Some of the main features of OpenBEM are briefly explained below. The reader is referred to the relevant publications.

- Possiblity of using constant, linear or quadratic isoparametric elements.
- In 3DBEM, triangular and quadrilateral elements are possible.
- Non-uniqueness: exterior BEM calculations are subject to errors caused by resonances in the reciprocal interior problem. In OpenBEM, it is possible to include CHIEF points as described in reference 7.
- In direct collocation BEM, calculation difficulties show up when parts of the surface are very close together in relation to the overall dimensions. This is the case of narrow slits and thin bodies. The difficulties are also observed when field points are calculated at close distance of the boundary. The effect is caused by a near-singular integrand. In OpenBEM this condition is dealt with using an adapted integration scheme.[8,9]
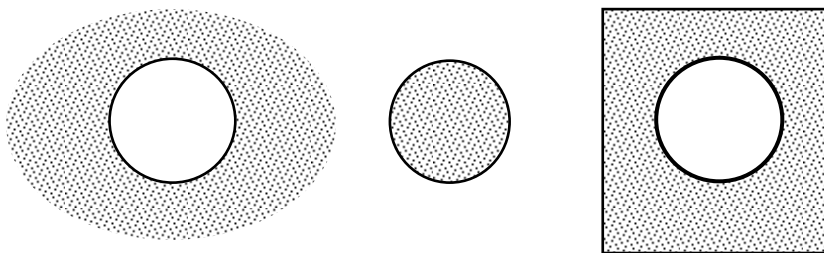

Figure 5. From left to right, sketch of an exterior problem, an interior problem and a combined interior-exterior problem. The dotted area represents the calculation domain.

- For any of the independent bodies defined in a specific calculation it is possible to consider either the exterior or the interior domain. It is also possible to mix exterior and interior domains, for example when an object is contained into another. Figure 5 illustrates these possibilities.

5 Performing a calculation in OpenBEM

A BEM simulation in OpenBEM contains a number of phases that can be written in one or more Matlab scripts:

1. Parameters of the simulation: input quantities like frequency, key geometrical dimensions, ambient conditions, etc. can be grouped at the beginning for convenience.
2. Mesh generation or retrieval from existing files: prior to the calculation, the meshes are defined and generated, except for the 3DBEM case, where they have to be imported.
3. Definition of the boundary conditions: incoming waves, prescribed velocities, pressures and impedances can be defined at this stage.
4. Calculation of the coefficient matrices using the kernel functions of OpenBEM.
5. Solution of the system of equations to obtain values on the boundary.
6. Definition of field points and calculation of the sound field on them using the appropriate kernel functions.
7. Presentation of results, either on the field points or on the surface.

Depending on the boundary conditions BEM calculations vary in the way the system of equations is solved. Matrix equation (3) can be expressed:

$$0 = \mathbf{A}\,\mathbf{p} + i\,k\,z_o\,\mathbf{B}\,\mathbf{v} + 4\,\pi\,\mathbf{p}^I \tag{5}$$

Where we consider that the left-hand side of (3) has already been subtracted from the first term of the right-hand side. For example, if we are considering a problem where an incoming wave is scattered around an acoustically hard object (infinite impedance), we have:

$$0 = \mathbf{A}\mathbf{p} + 4\,\pi\,\mathbf{p}^I$$
$$\rightarrow \quad \mathbf{p} = \mathbf{A}^{-1}\left(-4\,\pi\,\mathbf{p}^I\right) \tag{6}$$

On the other hand, a radiation problem with no external excitation would have normal velocities defined on the boundary. It would be:

$$0 = \mathbf{A}\mathbf{p} + i\,k\,z_o\,\mathbf{B}\mathbf{v}$$
$$\rightarrow \quad \mathbf{p} = \mathbf{A}^{-1}\left(-i\,k\,z_o\,\mathbf{B}\mathbf{v}\right) \tag{7}$$

It is also possible to combine a prescribed velocity $\mathbf{v}^0$ and an impedance boundary condition through the velocity term as shown in (8).

$$\mathbf{v} = \mathbf{Y}\,\mathbf{p} + \mathbf{v}^o \tag{8a}$$

$$\begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_M \end{pmatrix} = \begin{pmatrix} Y_1 & 0 & \dots & 0 \\ 0 & Y_2 & \vdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & Y_M \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{pmatrix} + \begin{pmatrix} v_1^o \\ v_2^o \\ \vdots \\ v_M^o \end{pmatrix} \tag{8b}$$

The Y_i are the admittances on the nodes, and the v^o_i are the fixed normal velocities on the nodes in radiation problems. Only one of the terms in the right-hand side will be usually present, but in some practical cases both could exist. This happens when lining material covers the surface, and therefore the normal surface velocity is different from the air normal velocity. As an example, let us consider a scattering case with finite surface impedance:

$$0 = \mathbf{A}\mathbf{p} + i\,k\,_{z_o}\,\mathbf{B}\mathbf{v} + 4\,\pi\,\mathbf{p}^I$$
$$\rightarrow \quad 0 = \mathbf{A}\mathbf{p} + i\,k\,_{z_o}\,\mathbf{B}\mathbf{Y}\mathbf{p} + 4\,\pi\,\mathbf{p}^I \tag{9}$$
$$\rightarrow \quad \mathbf{p} = \left(\mathbf{A} + i\,k\,_{z_o}\,\mathbf{B}\mathbf{Y}\right)^{-1}\left(-4\,\pi\,\mathbf{p}^I\right)$$

When pressure and velocity boundary conditions are mixed along the boundary, the system of equations must be rearranged in order to solve it. In the example (10) one node has a pressure boundary condition and its corresponding column is exchanged:

$$
\begin{pmatrix}
a_{11} & a_{12} & \cdots & -b_{1i} & \cdots & a_{1M} \\
a_{21} & a_{22} & & -b_{2i} & & a_{2M} \\
\vdots & \vdots & & \vdots & & \vdots \\
a_{M1} & a_{M2} & \cdots & -b_{Mi} & \cdots & a_{MM}
\end{pmatrix}
\begin{pmatrix}
p_1 \\ \vdots \\ v_i \\ p_M
\end{pmatrix} =
$$
$$
\begin{pmatrix}
b_{11} & b_{12} & \cdots & -a_{1i} & \cdots & b_{1M} \\
b_{21} & b_{22} & & -a_{2i} & & b_{2M} \\
\vdots & \vdots & & \vdots & & \vdots \\
b_{M1} & b_{M2} & \cdots & -a_{Mi} & \cdots & b_{MM}
\end{pmatrix}
\begin{pmatrix}
v_1 \\ \vdots \\ p_i \\ v_M
\end{pmatrix} +
\begin{pmatrix}
p_1^I \\ p_2^I \\ \vdots \\ p_M^I
\end{pmatrix} \tag{10}
$$

6 Examples

OpenBEM has been used in a large number of projects. Here are some selected examples where the authors have been involved. Again, references are provided for the interested reader.

6.1 Improvement of sound intensity probes

In this investigation a sound intensity probe measurement was simulated using AxiBEM-OpenBEM. This made possible to define a new procedure for extending its frequency range.[10] This was one of the first applications of OpenBEM, which had been used previously to model the free-field correction of condenser microphones.

6.2 Design of a sound intensity calibrator

One of Brüel & Kjær products, the sound intensity calibrator type 4297, was designed using AxiBEM-OpenBEM.[11]
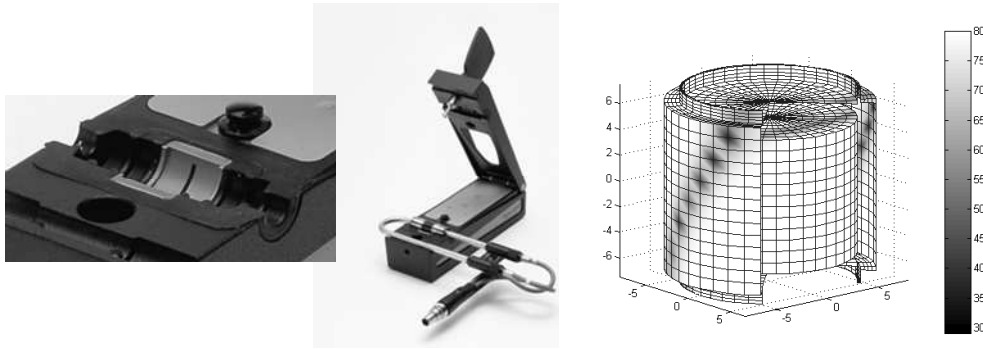
Figure 6. B&K sound intensity calibrator type 4297 and one of the OpenBEM results.

This design was in fact only possible using the BEM numerical model, since the design by prototypes and lumped parameter models had not been successful.

6.3   Acoustic levitation

OpenBEM has been applied in the recent years to the study of acoustic levitation.[12,13] This is a phenomenon where forces produced by strong sound fields, generally in the ultrasonic range, are used to maintain small objects suspended in the air. Figure 7 presents some examples of calculations in an acoustic levitator.
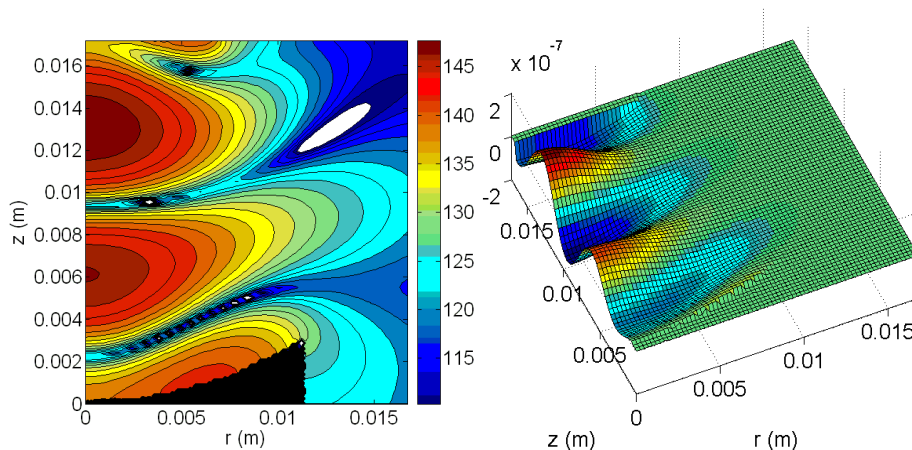

Figure 7. Pressure sound filed inside an acoustic levitator (left) and its associated force-generating Gorkov potential (right).

7   Conclusions

The features and capabilities of the Boundary Element Method software OpenBEM have been presented in this paper. A number of examples show the usefulness of this free package in research. It is expected that more researchers will be interested in using and contributing to it in the future.

Acknowledgments

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