

〈色の塗り方〉

$\text{colorYB}(x, n, z)$ def $\begin{cases} Y & (0 \leq z-x \leq n \text{ \&\& } \text{odd}(z-x) == \text{false}) \\ B & (0 \leq z-x \leq n \text{ \&\& } \text{odd}(z-x) == \text{true}) \\ Y & (\text{otherwise}) \end{cases}$

Lem YB1

$\forall x, n,$

$\forall i (0 \leq i \leq n \text{ \&\& } \text{odd}(i) == \text{false} \rightarrow \text{colorYB}(x, n, x+i) = Y).$

prf. $0 \leq i \leq n \text{ \&\& } \text{odd}(i) == \text{false}$ をとする x, n, i をとる.

$(x+i) - x = i \neq 0$ $\text{colorYB}(x, n, x+i) = Y. //$

Lem YB2

$\forall x, n,$

$\forall i (0 \leq i \leq n \text{ \&\& } \text{odd}(i) == \text{true} \rightarrow \text{colorYB}(x, n, x+i)).$

prf. $0 \leq i \leq n \text{ \&\& } \text{odd}(i) == \text{true}$ をとする x, n, i をとる.

$(x+i) - x = i \neq 0$ $\text{colorYB}(x, n, x+i) = B. //$

Lem YB3

$\forall x, n, \text{odd}(n) == \text{false} \rightarrow$

$(\text{colorYB}(x, n, x) = \text{colorYB}(x, n, x+n))$.

prf. $\text{odd}(n) == \text{false}$ とする x, n をとる.

$0 \leq 0, 0 \leq n \Rightarrow 0 \leq 0 \leq n$ であり $\text{odd}(0) == \text{false}$.

Lem YB1 により $\text{colorYB}(x, n, x) = \text{colorYB}(x, n, x+0) = Y$.

$0 \leq n, n \leq n \Rightarrow 0 \leq n \leq n$ であり $\text{odd}(n) == \text{false}$.

Lem YB1 により $\text{colorYB}(x, n, x+n) = Y$.

よって, $\text{colorYB}(x, n, x) = \text{colorYB}(x, n, x+n)$. //

Even A

$\forall x, y, n,$

$$\forall i (0 \leq i \leq n \rightarrow C_{\text{pos}}(x+i, y, \text{colorYB}(x, n, x+i))) \rightarrow \forall i (0 \leq i \leq n-1 \rightarrow C_{\text{pos}}(x+i, y+1, R))$$

prf. $x, y, n \in \Sigma$.

$\forall i (0 \leq i \leq n \rightarrow C_{\text{pos}}(x+i, y, \text{colorYB}(x, n, x+i)))$ を仮定する.

$0 \leq i \leq n-1$ とする i をとる.

$i \leq n-1, n-1 \leq n \Rightarrow i \leq n$ となるので $0 \leq i \leq n$.

$\vdash \exists, C_{\text{pos}}(x+i, y, \text{colorYB}(x, n, x+i))$.

また, $i \leq n-1 \Rightarrow i+1 \leq n$ となるので, $0 \leq i+1 \leq n$

$\vdash \exists, C_{\text{pos}}(x+i+1, y, \text{colorYB}(x, n, x+i+1))$.

$C\text{-exists } \exists' C_{\text{pos}}(x+i, y+1, C) \text{ とする } C \text{ の序数}.$

$C = C' \wedge \exists \exists' C_{\text{pos}}(x+i, y+1, C')$.

$C\text{-mix } \exists' C' = \text{mix}(\text{colorYB}(x, n, x+i), \text{colorYB}(x, n, x+i+1))$.

- $\text{odd}(i) == \text{false}$ のとき

$\text{odd}((x+i)-x) == \text{odd}(i) == \text{false} \Rightarrow \text{colorYB}(x, n, x+i) = Y$

$\text{odd}((x+i+1)-x) == \text{odd}(i+1) == \text{true} \Rightarrow \text{colorYB}(x, n, x+i+1) = B$

$\vdash \exists, C' = \text{mix}(Y, B) = R \vdash \exists' C_{\text{pos}}(x+i, y+1, R)$.

- $\text{odd}(i) == \text{true}$ のとき

$\text{odd}((x+i)-x) == \text{odd}(i) == \text{true} \Rightarrow \text{colorYB}(x, n, x+i) = B$.

$\text{odd}((x+i+1)-x) == \text{odd}(i+1) == \text{false} \Rightarrow \text{colorYB}(x, n, x+i+1) = Y$.

$\vdash \exists, C' = \text{mix}(B, Y) = R \vdash \exists' C_{\text{pos}}(x+i, y+1, R)$.

//

All Red (証明済み)

$\forall x, y, n$

$$(\forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, R)) \rightarrow \text{Cpos}(x, y+n, R).$$

Even B

$\forall x, y, n \in \mathbb{N}$

$$\forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, \text{colorYB}(x, n, x+i))) \rightarrow \text{Cpos}(x, (y+n), R).$$

prf. $x, y, n \in \mathbb{N}$.

$\forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, \text{colorYB}(x, n, x+i)))$ を仮定する。

EvenA $\models' \forall i (0 \leq i \leq n-1 \rightarrow \text{Cpos}(x+i, y+1, R))$

AllRed $\models' \text{Cpos}(x, (y+1)+(n-1), R)$.

$\models \tau, \text{Cpos}(x, y+n, R) : //$

false Color (証明済み)

$\forall x, y, \forall c_0, c_1$

$$((c_0 \neq c_1) \wedge \text{Cpos}(x, y, c_0) \wedge \text{Cpos}(x, y, c_1)) \rightarrow \text{false}.$$

EvenC

$\forall x, y, n$

$$\left(\text{odd}(n) == \text{false} \right) \rightarrow \neg \left(\begin{array}{l} \forall C: \text{color}, \forall f: \text{nat} \rightarrow \text{Color} \\ \text{Triangle}(x, y, n, f(x), f(x+n), C) \end{array} \right)$$

prf. x, y をとる.

$\text{odd}(n) == \text{false}$ となる n をとる.

$\left(\begin{array}{l} \forall C: \text{color}, \forall f: \text{nat} \rightarrow \text{Color} \\ \text{Triangle}(x, y, n, f(x), f(x+n), C) \end{array} \right)$ を仮定して矛盾を示す.

C-paint $\vdash \forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, \text{colorYB}(x+i)))$

o EvenB より $\text{Cpos}(x, y+n, R)$.

o $0 \leq 0 \leq n$, $\text{odd}(0) == \text{false}$ より $\text{Cpos}(x+0, y, \text{colorYB}(x, n, x))$

$0 \leq n \leq n$, $\text{odd}(n) == \text{false}$ より $\text{Cpos}(x+n, y, \text{colorYB}(x, n, x+n))$.

C-exists より $\text{Cpos}(x, y+n, C)$ となる C が存在する.

$C = C'$ とすると $\text{Cpos}(x, y+n, C')$.

仮定 $\vdash \text{Triangle}(x, y, n, \text{colorYB}(x, n, x), \text{colorYB}(x, n, x+n), C')$

よって, $C' = \text{mix}(\text{colorYB}(x, n, x), \text{colorYB}(x, n, x+n))$

さらに, LemYB 3 より $\text{colorYB}(x, n, x) = \text{colorYB}(x, n; x+n)$ より

$C' = \text{mix}(\text{colorYB}(x, n, x), \text{colorYB}(x, n, x))$.

このとき, $0 \leq 0 \leq n$, $\text{odd}(x-x) == \text{odd}(0) == \text{false}$ であるから

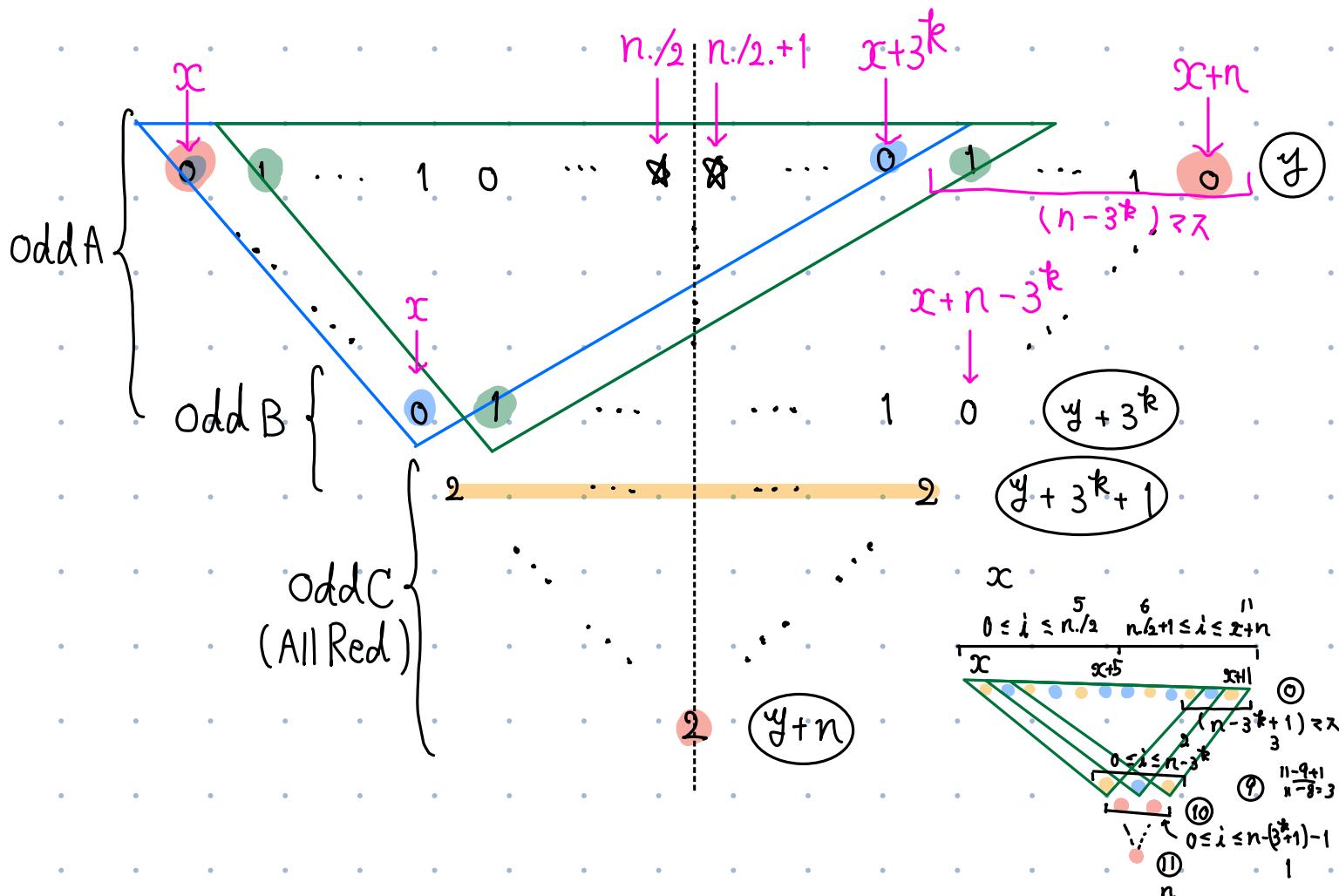
LemYB 1 より $\text{colorYB}(x, n, x) = Y$ となり $C' = \text{mix}(Y, Y) = Y$.

よって, $\text{Cpos}(x, y+n, Y)$

以上より $R \neq Y$ であるから falsecolor より 矛盾 //

〈色の塗り方〉

$$\text{colorYBBY}(x, n, z) \stackrel{\text{def}}{=} \begin{cases} Y & \left(0 \leq z-x \leq +n/2 \& \& \text{odd}(z-x) == \text{false} \right) \\ Y & \left(n/2+1 \leq z-x \leq n \& \& \text{odd}(z-x) == \text{true} \right) \\ B & \left(0 \leq z-x \leq +n/2 \& \& \text{odd}(z-x) == \text{true} \right) \\ B & \left(n/2+1 \leq z-x \leq n \& \& \text{odd}(z-x) == \text{false} \right) \\ Y & (\text{otherwise}) \end{cases}$$



Lem YBBY1

$\forall x, n,$

$\forall i ((0 \leq i \leq n/2 \ \&\& \text{odd}(i) == \text{false}) \rightarrow \text{colorYBBY}(x, n, x+i) = Y).$

prf. $0 \leq i \leq n/2 \ \&\& \text{odd}(i) == \text{false}$ とすると x, n, i をとる.

$(x+i) - x = i \neq 0 \quad \text{colorYBBY}(x, n, x+i) = Y //$

Lem YBBY2

$\forall x, n,$

$\forall i ((n/2+1 \leq i \leq n \ \&\& \text{odd}(i) == \text{true}) \rightarrow \text{colorYBBY}(x, n, x+i) = Y).$

prf. $n/2+1 \leq i \leq n \ \&\& \text{odd}(i) == \text{true}$ とすると x, n, i をとる.

$(x+i) - x = i \neq 0 \quad \text{colorYBBY}(x, n, x+i) = Y //$

Lem YBBY3

$\forall x, n,$

$\forall i ((0 \leq i \leq n/2 \ \&\& \text{odd}(i) == \text{true}) \rightarrow \text{colorYBBY}(x, n, x+i) = B).$

prf. $0 \leq i \leq n/2 \ \&\& \text{odd}(i) == \text{true}$ とすると x, n, i をとる.

$(x+i) - x = i \neq 0 \quad \text{colorYBBY}(x, n, x+i) = B //$

Lem YBBY4

$\forall x, n,$

$\forall i ((n/2+1 \leq i \leq n \ \&\& \text{odd}(i) == \text{false}) \rightarrow \text{colorYBBY}(x, n, x+i) = B).$

prf. $n/2+1 \leq i \leq n \ \&\& \text{odd}(i) == \text{false}$ とする x, n, i をとる.

$(x+i) - x = i \neq 0 \quad \text{colorYBBY}(x, n, x+i) = B //$

Lem YBBY5

$\forall x, n, \text{odd}(n) == \text{true} \rightarrow$

$(\text{colorYBBY}(x, n, x) = \text{colorYBBY}(x, n, x+n)).$

prf. $\text{odd}(n) == \text{true}$ とする x, n をとる.

$0 \leq 0, 0 \leq n \ \&\& \ 0 \leq 0 \leq n \text{ で } \text{odd}(0) == \text{false}.$

Lem YBBY1 より $\text{colorYBBY}(x, n, x) = \text{colorYBBY}(x, n, x+0) = Y.$

$n/2+1 \leq n, n \leq n \ \&\& \ n/2+1 \leq n \leq n \text{ で } \text{odd}(n) == \text{false}.$

Lem YBBY2 より $\text{colorYBBY}(x, n, x+n) = Y.$

よって, $\text{colorYBBY}(x, n, x) = \text{colorYBBY}(x, n, x+n) //$

OddA

$\forall x, y, n, k,$

- $3^k \leq n \leq 3^k * 2 \& \& \text{odd}(i) == \text{true} \rightarrow$
 - $\forall x', y', c_0, c_1, c_2. \text{Triangle}(x', y', 3^k, c_0, c_1, c_2) \rightarrow$
 - $\forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i))) \rightarrow$
- $\forall i (0 \leq i \leq n - 3^k \rightarrow \text{Cpos}(x+i, y+3^k, \text{colorYB}(x, n - 3^k, x+i)))$

prf. $x, y \in \mathbb{Z}$.

$3^k \leq n \leq 3^k * 2 \& \& \text{odd}(i) == \text{true}$ となる $n, k \in \mathbb{Z}$ とする。

$\left(\begin{array}{l} \forall x', y', c_0, c_1, c_2. \text{Triangle}(x', y', 3^k, c_0, c_1, c_2) \\ \forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i))) \end{array} \right)$ を仮定する。

$0 \leq i \leq n - 3^k$ となる $i \in \mathbb{Z}$ とする。

$i \leq n - 3^k, n - 3^k \leq n \Rightarrow i \leq n$ となるから $0 \leq i \leq n$.

$\exists, \exists, \text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i)))$.

$i \leq n - 3^k \Rightarrow i + 3^k \leq n$ となるから $0 \leq i + 3^k \leq n$.

$\exists, \exists, \text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i+3^k)))$.

C-exists すり $\text{Cpos}(x+i, y+3^k, c)$ となる c の存在する。 $(c = c' \in \mathbb{Z})$

仮定すり $\text{Triangle}(x, y, n, \text{colorYBBY}(x, n, x+i), \text{colorYBBY}(x, n, x+i+3^k), c')$

よし, $c' = \text{mix}(\text{colorYBBY}(x, n, x+i), \text{colorYBBY}(x, n, x+i+3^k))$.

- $\text{odd}(i) == \text{false}$ のとき

$\text{odd}((x+i) - x) == \text{odd}(i) == \text{false}$ となるから LemYBBY1 すり $\text{colorYBBY}(x, n, x+i) = Y$

$\text{odd}((x+i+3^k) - x) == \text{odd}(i+3^k) == \text{true}$ となるから LemYBBY2 すり $\text{colorYBBY}(x, n, x+i+3^k) = Y$

よし, $c' = \text{mix}(Y, Y) = Y = \text{colorYB}(x, n - 3^k, x+i)$ となる, (\odot LemYB1)

$\text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i))$

- $\text{odd}(i) == \text{true}$ のとき

$\text{odd}((x+i)-x) == \text{odd}(i) == \text{true}$ ならば LemYBBY3 が $\text{colorYBBY}(x, n, x+i) = B$

$\text{odd}((x+i+3^k)-x) == \text{odd}(i+3^k) == \text{false}$ ならば LemYBBY4 が $\text{colorYBBY}(x, n, x+i+3^k) = B$.

$f, t, c' = \text{mix}(B, B) = B = \text{colorYB}(x, n-3^k, x+i)$ ならば, (\odot LemYB2)

$\text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i))$

Odd B

$\forall x, y, n, k,$

- $3^k \leq n \leq 3^k * 2 \& \& \text{odd}(i) == \text{true} \rightarrow$
 - $\forall x', y', C_0, C_1, C_2. \text{Triangle}(x', y', 3^k, C_0, C_1, C_2) \rightarrow$
 - $\forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i))) \rightarrow$
- $\forall i (0 \leq i \leq (n-3^k)-1 \rightarrow \text{Cpos}(x+i, y+(3^k+1), R))$

prf. $x, y, n, k \in \mathbb{Z}$.

Odd A が $\forall i (0 \leq i \leq n-3^k \rightarrow \text{Cpos}(x+i, y+3^k, \text{colorYB}(x, n-3^k, x+i)))$

Even A が $\forall i (0 \leq i \leq (n-3^k)-1 \rightarrow \text{Cpos}(x+i, (y+3^k)+1, R)) //$

All Red (証明済み)

$\forall x, y, n$

$$(\forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, R)) \rightarrow \text{Cpos}(x, y+n, R).$$

Odd C

$\forall x, y, n, k,$

$$\left\{ \begin{array}{l} \circ 3^k \leq n \leq 3^k * 2 \&& \text{odd}(i) == \text{true} \rightarrow \\ \circ \forall x', y', c_0, c_1, c_2. \text{Triangle}(x', y', 3^k, c_0, c_1, c_2) \rightarrow \\ \circ \forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i))) \rightarrow \\ \text{Cpos}(x, y+n, R) \end{array} \right.$$

prf. x, y, n, k をとる。

$$\text{OddB } \vdash \forall i (0 \leq i \leq (n - 3^k) - 1 \rightarrow \text{Cpos}(x+i, y+(3^k+1), R))$$

$$\text{AllRed } \vdash \text{Cpos}(x, (y + 3^k + 1) + ((n - 3^k) - 1), R)$$

$$d, \Gamma, \text{Cpos}(x, y+n, R) //$$

false Color (証明済み)

$\forall x, y, \forall c_0, c_1$

$$(\text{c}_0 \neq \text{c}_1 \wedge \text{Cpos}(x, y, c_0) \wedge \text{Cpos}(x, y, c_1)) \rightarrow \text{false}.$$

Odd D

$\forall x, y, n, R,$

$$\left(\begin{array}{l} 3^k \leq n \leq 3^k * 2 \\ \& \& \\ \text{odd}(n) == \text{true} \end{array} \right) \rightarrow \exists \left(\begin{array}{l} \forall C: \text{color}, \forall f: \text{nat} \rightarrow \text{Color} \\ \text{Triangle}(x, y, n, f(x), f(x+n), C) \end{array} \right)$$

prf. x, y をとる。

$\text{odd}(n), 3^k \leq n \leq 3^k * 2$ となる n, R をとする。

$\left(\begin{array}{l} \forall C: \text{color}, \forall f: \text{nat} \rightarrow \text{Color} \\ \text{Triangle}(x, y, n, f(x), f(x+n), C) \end{array} \right)$ を仮定して 矛盾を示す。

十分性は示されてるので $\forall x', y', c_0, c_1, c_2. \text{Triangle}(x', y', 3^k, c_0, c_1, c_2)$

C-paint たり $\forall i (0 \leq i \leq n \rightarrow \text{Cpos}(x+i, y, \text{colorYBBY}(x, n, x+i)))$

- Odd C たり $\text{Cpos}(x, y+n, R)$.

- $0 \leq o \leq n \rightarrow \text{Cpos}(x+o, y, \text{colorYBBY}(x, n, x+o))$.

- $0 \leq n \leq n \rightarrow \text{Cpos}(x+n, y, \text{colorYBBY}(x, n, x+n))$.

C-exists たり $\text{Cpos}(x, y+n, c)$ となる c が存在する。 $(c = c' \text{ とする})$

仮定より $\text{Triangle}(x, y, n, \text{colorYBBY}(x, n, x+o), \text{colorYBBY}(x, n, x+n), c')$

よって, $c' = \text{mix}(\text{colorYBBY}(x, n, x), \text{colorYBBY}(x, n, x+n))$.

さらに, Lem YBBY 5 より $\text{colorYBBY}(x, n, x) = \text{colorYBBY}(x, n, x+n)$ だから

$c' = \text{mix}(\text{colorYBBY}(x, n, x), \text{colorYBBY}(x, n, x+n))$.

$\text{Cpos}(x, y+n, \text{mix}(\text{colorYBBY}(x, n, x), \text{colorYBBY}(x, n, x+n)))$.

このとき, $0 \leq o \leq n$, $\text{odd}(x-o) == \text{odd}(o) == \text{false}$ であるから

LemYBBY1 より $\text{colorYB}(x, n, x) = Y$ となり $c' = \text{mix}(Y, Y) = Y$.

よって, $\text{Cpos}(x, y+n, Y)$

以上より $R \neq Y$ であるから falsecolor より 矛盾 //