Name: Andy Favez

ID: 242FC242H6

Lecture Group: TC1L

$\frac{\quad}{14}$ marks

a_1 a_2 a_3

1. Given.... $y-6$, $2y+3$, $4y-1$.... are three consecutive terms of an arithmetic sequence. Find the value of y and the common difference of the sequence. [2 marks]

Value of y:

$2y+3-y+6 = 4y-1-2y-3$

$2y-y+3+6 = 4y-2y-1-3$

$y+9 = 2y-4$

$y-2y = -4-9$

$\cancel{-}y = \cancel{-}13$

$y = 13$

a_1: $13-6 = 7$

a_2: $2(13)+3 = 29$

a_3: $4(13)-1 = 51$

d:

$a_3 - a_2$

$51-29 = 22$

$a_2 - a_1$

$29-7 = 22$

$\therefore y = 13$

$d = 22$

2. Given two terms of a geometric sequence, $a_5 = 96$ and $a_8 = 768$. Find the first term and the common ratio. [2 marks]

$a_n = ar^{n-1}$

$a_5 = 96$

$ar^{n-1} = a_n$

$a = \frac{a_n}{r^{n-1}}$

$a = \frac{96}{r^{5-1}}$

$a = \frac{96}{r^4}$

a_8

$a_8 = ar^{8-1}$

$768 = ar^7$

$768 = \left(\frac{96}{\cancel{r^4}}\right)\cancel{r^7}^{\,3}$

$\frac{768}{96} = r^3$

$8 = r^3$

$r^3 = 8$

$r = \sqrt[3]{8}$

$r = 2$

$96 = a(2)^{5-1}$

$96 = a(2)^4$

$a(2)^4 = 96$

$16a = 96$

$a = \frac{96}{16}$

$a = 6$

$\therefore$ First term $= 6$

Common ratio $= 2$

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3. Express $1.2\overline{48}$ as a fraction. Show all steps.

$1.2\overline{48}$

$= 1.2 + 0.048$

$= 1.2 + S_\infty$

$= 1.2 + \frac{8}{165}$

$= \frac{206}{165}$ //

$a = 0.048$

$a_2 = 0.00048$

$r = \frac{0.00048}{0.048} = 0.01$

$S_\infty = \frac{0.048}{0.01} = \frac{8}{165}$

4. Find the term that contains x^9 in the expression of $\left(x^2 + \frac{2}{x}\right)^{12}$

$= {}^{12}C_r (x^2)^{12-r} \left(\frac{2}{x}\right)^r$

$= {}^{12}C_r \, x^{2(12-r)} \, 2x^{-r}$

$= {}^{12}C_r \, 2x^{24-2r-r}$

let $24 - 2r - r = 9$

$24 - 2r - r = 9$

$24 - 3r = 9$

$-3r = -15$

$r = 5$

Term contains x^9

$= {}^{12}C_5$

$= 792$

$\therefore (792 \times 2)\, x^9$

$1584x^9$ //

5. Given the following system of linear equations :

$$3x + 2y + z = 34$$

$$2x + 5y + 6z = 57$$

$$3x + 4y + 5z = 56$$

i) write the system in the form of $AX = B$ where $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$.

$$\begin{array}{ccc} a_{11} & a_{12} & a_{13} \end{array}$$

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 5 & 6 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 34 \\ 57 \\ 56 \end{pmatrix}$$

ii) Find the cofactor, adjoint, determinant and A^{-1} of matrix A. Hence, find the values of x, y, z. [6 marks]

Show all necessary intermediate steps.

$$C = \begin{pmatrix} +\begin{vmatrix} 5 & 6 \\ 4 & 5 \end{vmatrix} & -\begin{vmatrix} 2 & 6 \\ 3 & 5 \end{vmatrix} & +\begin{vmatrix} 2 & 5 \\ 3 & 4 \end{vmatrix} \\ -\begin{vmatrix} 2 & 1 \\ 4 & 5 \end{vmatrix} & +\begin{vmatrix} 3 & 1 \\ 3 & 5 \end{vmatrix} & -\begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} \\ +\begin{vmatrix} 2 & 1 \\ 5 & 6 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 2 & 6 \end{vmatrix} & +\begin{vmatrix} 3 & 2 \\ 2 & 5 \end{vmatrix} \end{pmatrix}$$

$$C = \begin{pmatrix} +|1| & -|-8| & +|-7| \\ -|6| & +|12| & -|6| \\ +|7| & -|12| & +|11| \end{pmatrix}$$

$$C = \begin{array}{c} \begin{array}{ccc} c_{11} & c_{12} & c_{13} \end{array} \\ \begin{pmatrix} 1 & 8 & -7 \\ -6 & 12 & -6 \\ 7 & -16 & 11 \end{pmatrix} \end{array}$$

$$|A| = 3(1) + 2(8) + 1(-7)$$
$$= 3 + 16 - 7$$
$$= 12$$

$$C^T = \begin{pmatrix} 1 & -6 & 7 \\ 8 & 12 & -16 \\ -7 & -6 & 11 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} C^T$$

$$= \frac{1}{12} \begin{pmatrix} 1 & -6 & 7 \\ 8 & 12 & -16 \\ -7 & -6 & 11 \end{pmatrix} \begin{pmatrix} 34 \\ 57 \\ 56 \end{pmatrix}$$

$$= \frac{1}{12} \begin{pmatrix} 84 \\ 60 \\ 36 \end{pmatrix}$$

$$= \begin{pmatrix} 7 \\ 5 \\ 3 \end{pmatrix}$$

$\therefore x = 7$

$y = 5$

$z = 3$