MULTIMEDIA UNIVERSITY

STUDENT ID NO

Tutorial:

MULTIMEDIA UNIVERSITY

TEST 1

TRIMESTER 2, 2022/2023

PMT0301 – MATHEMATICS III

(All sections/groups)

25 MAY 2023
8.00 p.m – 10.00 p.m
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 6 pages with 4 questions only.

2. Answer **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the designated space of each question.

QUESTION 1 (10 MARKS)

a) Given a sequence 10, 24, 38, ... Find the first term that exceeds 400. [3 marks]

$a = 10, \quad d = 24 - 10 = 14$

$a_n > 400$
$10 + (n-1)(14) > 400$ [1]
$14n > 404$
$n > 28.86$ [0.5]
$n = 29$ [0.5]

$a_{29} = 10 + 28(14)$ [0.5]
$a_{29} = 402$ [0.5]

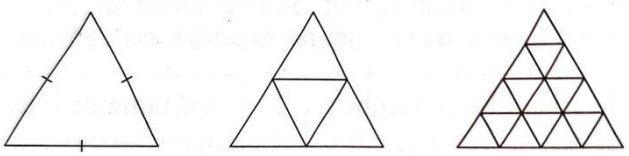
b) Express the recurring decimals 0.272727...as a fraction. Show all steps. [2 marks]

$0.272727\ldots$
$= 0.27 + 0.0027 + 0.000027 + \ldots$

$a = 0.27, \quad r = \frac{0.0027}{0.27} = 0.01$ [0.5 + 0.5]

$0.272727\ldots$
$= \frac{0.27}{1 - 0.01}$ [0.5]
$= \frac{29}{99}$
$= \frac{3}{11}$ [0.5]

c) The diagram shows the arrangements of equilateral triangles made from wire and welded together when more equilateral triangles are added. The triangles are of sides 16 cm, 8 cm, 4 cm, ... [3 marks]



i. Find the length of side of equilateral triangles of the 10^{th} diagram.

ii. Find the number of equilateral triangles of the 10^{th} diagram.

i) Length of sides of the equilateral triangles:

$16, \quad 8, \quad 4, \quad \ldots \Rightarrow \; a = 16, \quad r = \frac{1}{2}$ [0.5]

The length of side of equilateral triangles of the 10th diagram

$a_{10} = 16\left(\frac{1}{2}\right)^{10-1}$ [0.5]

$a_{10} = \frac{1}{32}$ cm. [0.5]

ii) Number of equilateral triangle: $1, \quad 4, \quad 16, \quad \ldots \Rightarrow \quad a = 1, \quad r = 4$ [0.5]

$a_{10} = 1(4^{10-1})$ [0.5]

$a_{10} = 262144$ triangles. [0.5]

d) Find the term that contains y^3 in the expansion of $(3x+2y)^7$. [2 marks]

$^7C_3(3x)^4(2y)^3$ [1]

$= 22680x^4y^3$ [1]

QUESTION 2 (10 MARKS)

a) Using Gauss-Jordan Elimination, solve the following system of linear equations. **[Note: The system may have unique solution, infinitely many solutions, or no solution]**

$$\begin{aligned} 3x + y + 2z &= 7 \\ 2x + 2y + 2z &= 6 \\ 2x + 4y + 3z &= 8 \end{aligned}$$

[10 marks]

$$\begin{bmatrix} 3 & 1 & 2 & 7 \\ 2 & 2 & 2 & 6 \\ 2 & 4 & 3 & 8 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 3 & 1 & 2 & 7 \\ 1 & 1 & 1 & 3 \\ 2 & 4 & 3 & 8 \end{bmatrix}$$ [1]

$$\xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 3 & 1 & 2 & 7 \\ 2 & 4 & 3 & 8 \end{bmatrix}$$ [1]

$$\xrightarrow[R_3 - 2R_1 \to R_3]{R_2 - 3R_1 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & -2 & -1 & -2 \\ 0 & 2 & 1 & 2 \end{bmatrix}$$ [2]

$$\xrightarrow{-\frac{1}{2}R_2 \to R_2} \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & \frac{1}{2} & 1 \\ 0 & 2 & 1 & 2 \end{bmatrix}$$ [1]

$$\xrightarrow[R_3 - 2R_2 \to R_3]{R_1 - R_2 \to R_1} \begin{bmatrix} 1 & 0 & 1/2 & 2 \\ 0 & 1 & 1/2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ [2]

Now, we have

$$x = 2 - \tfrac{1}{2}z \quad \text{and} \quad y = 1 - \tfrac{1}{2}z$$

Let $z = t$, hence the system has **infinitely many solutions** such that:

$$x = 2 - \frac{1}{2}t$$ [1]

$$y = 1 - \frac{1}{2}t$$ [1]

$$z = t$$ [1]

QUESTION 3 (10 MARKS)

a) Find the vector equation for the line which passes through the point $(-7, 3, 0)$ and perpendicular to the plane $-x - 3y + 8z = 12$. [2.5 marks]

Given a point: $(-7,3,0)$ and direction$\langle -1, -3, 8\rangle$
Vector Equation
$\bar{r} = \bar{r}_o + t\bar{v}$
$\langle x, y, x\rangle = \langle -7,3,0\rangle + t\langle -1, -3, 8\rangle$ **0.5m+0.5m**
$= \langle -7 - t, 3 - 3t, 8t\rangle$ **0.5m+0.5m+0.5m**

b) Where does the line intersect the plane? [2.5 marks]

$-(-7 - t) - 3(3 - 3t) + 8(8t) = 12$ **0.5m**
$7 + t - 9 + 9t + 64t = 12$
$74t = 14$
$t = \frac{7}{37}$ **0.5m**
$\therefore x = -7 - \frac{7}{37} = -\frac{266}{37}$ **0.5m**
$y = 3 - 3\left(\frac{7}{37}\right) = \frac{90}{37}$ **0.5m**
$z = 8\left(\frac{7}{37}\right) = \frac{56}{37}$ **0.5m**
$\therefore \left(-\frac{266}{37}, \frac{90}{37}, \frac{56}{37}\right)$

c) Find an equation of the plane through the points $(2, -5, 3)$ and contains the line $x = -2 + t$, $y = -4 - 3t$ and $z = 5t$. [5 marks]

Given a known point on the plane, **A**: $(2, -5, 3)$
Known point on the line, **B**: $(-2, -4, 0)$
The direction of the line is $\vec{v} = \langle 1, -3,5\rangle$

$\therefore \overrightarrow{AB} = \langle -2 - 2, -4 - (-5), 0 - 3\rangle = \langle -4, 1, -3\rangle$ **0.5m+0.5m**

The cross product of $\overrightarrow{AB}$ and $\vec{v}$
$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -4 & 1 & -3 \\ 1 & -3 & 5 \end{vmatrix}$ **0.5m**
$= \vec{i}(5 - 9) - \vec{j}(-20 - (-3)) + \vec{k}(12 - 1)$ **1m**
$= -4\vec{i} + 17\vec{j} + 11\vec{k}$ or $\langle -4,17,11\rangle$ **0.5m**

The plane equation
$\vec{n}.(\vec{r}-\vec{r_o})=0$
$\langle -4,17,11\rangle.(\langle x,y,z\rangle-\langle 2,-5,3\rangle)$ **0.5m**
$-4(x-2)+17(y+5)+11(z-3)=0$ **0.5m**
$-4x+8+17y+85+11z-33=0$ **0.5m**
$-4x+17y+11z+60=0$ or $4x-17y-11z-60=0$ **0.5m**

QUESTION 4 (10 MARKS)

a) Abu collected the ages of 15 colleagues and recorded his results.

32, 19, 36, 28, 27, 45, 32, 19, 31, 40, 45, 32, 55, 24, 20

i. Find the **mean**, **median**, **mode**, **range**, **variance**, and **standard deviation** value.
[6.5 marks]

Mean:
$\bar{x}=\frac{\Sigma x}{n}$
$\bar{x}=\frac{32+19+36+28+27+45+32+19+31+40+45+32+55+24+20}{15}$ **0.5**
$\bar{x}=32.33$ **0.5**

Median:
1st: Sort in ascending order
19, 19, 20, 24, 27, 28, 31, 32, 32, 32, 36, 40, 45, 45, 55 **0.5**
$\tilde{x}=\left(\frac{n+1}{2}\right)th$
$\tilde{x}=\left(\frac{15+1}{2}\right)th$ **0.5**
$\tilde{x}=8th=32$ **0.5**

Mode: 32 **0.5**

Range:
Largest Value – Smallest Value
$55-19=36$ **1**

Variance:
$s^2=\frac{\Sigma x^2-\frac{(\Sigma x)^2}{n}}{n-1}$
$s^2=\frac{17215-\frac{(485)^2}{15}}{15-1}$ **1**
$s^2=109.5238$ **0.5**

Standard deviation:

$s = \sqrt{variance}$

$s = \sqrt{109.5238}$ **0.5**

$s = 10.4654$ **0.5**

ii. Interpret the meaning of mean and median value. [2 marks]

Mean: On average, the ages of Abu's colleague is 32.333 = 32 years old **1**

Median: 50% of the age of Abu's colleagues are more than 32 years old, and another 50% of his colleague are less than 32 years old. **1**

iii. Determine the shape of the distribution based on the value obtained. [1.5 marks]

Mean – Mode = Positive

32.333 – 32 = 0.333 (Positive) **0.5**

Therefore, the shape of distribution is skewed to the right/positively skewed. **1**