ДЗ№1. Элементарные функции и пределы

Задача 1.

1. $\lim\limits_{n\to\infty} \dfrac{(4-n)^3+(4+n)^3}{(3+n)^3+(3-n)^3}$.
2. $\lim\limits_{n\to\infty} \dfrac{(5-n)^3+(5+n)^3}{(4-n)^3+(4+n)^3}$.
3. $\lim\limits_{n\to\infty} \dfrac{(2-n)^3+(2+n)^3}{(4-n)^3+(4+n)^3}$.
4. $\lim\limits_{n\to\infty} \dfrac{(4-3n)^3+(4+3n)^3}{(4-n)^3+(4+n)^3}$.
5. $\lim\limits_{n\to\infty} \dfrac{(3-2n)^3+(3+2n)^3}{(4-3n)^3+(4+3n)^3}$.
6. $\lim\limits_{n\to\infty} \dfrac{(4-5n)^3+(4+5n)^3}{(3-2n)^3+(3+2n)^3}$.
7. $\lim\limits_{n\to\infty} \dfrac{(n-3)^3-(n+3)^3}{(4-n)^2+(4+n)^2}$.
8. $\lim\limits_{n\to\infty} \dfrac{(n-5)^3-(5+n)^3}{(4-n)^2+(4+n)^2}$.
9. $\lim\limits_{n\to\infty} \dfrac{(n-3)^4-(n+3)^4}{(4-n)^3-(4+n)^3}$.
10. $\lim\limits_{n\to\infty} \dfrac{(n-2)^4-(n+2)^4}{(3-n)^3-(3+n)^3}$.
11. $\lim\limits_{n\to\infty} \dfrac{(n-6)^4-(n+6)^4}{(n-2)^4-(n+2)^4}$.
12. $\lim\limits_{n\to\infty} \dfrac{(n-6)^4-(n+6)^4}{(n+2)^4-(n-2)^4}$.
13. $\lim\limits_{n\to\infty} \dfrac{(4-2n)^3+(4+2n)^3}{(4+n)^3+(4-n)^3}$.
14. $\lim\limits_{n\to\infty} \dfrac{(3-2n)^3+(3+2n)^3}{(2-n)^3+(2+n)^3}$.
15. $\lim\limits_{n\to\infty} \dfrac{(2-3n)^3+(2+3n)^3}{(6-n)^3+(6+n)^3}$.
16. $\lim\limits_{n\to\infty} \dfrac{(4-3n)^3+(4+3n)^3}{(4-n)^3+(4+n)^3}$.
17. $\lim\limits_{n\to\infty} \dfrac{(3-2n)^3+(3+2n)^3}{(4-3n)^3+(4+3n)^3}$.
18. $\lim\limits_{n\to\infty} \dfrac{(4-5n)^3+(4+5n)^3}{(3-2n)^3+(3+2n)^3}$.
19. $\lim\limits_{n\to\infty} \dfrac{(2n-3)^3-(2n+3)^3}{(4-3n)^2+(4+3n)^2}$.
20. $\lim\limits_{n\to\infty} \dfrac{(2n-5)^3-(5+2n)^3}{(4-n)^2+(4+n)^2}$.
21. $\lim\limits_{n\to\infty} \dfrac{(2n-3)^4-(2n+3)^4}{(4-n)^3-(4+n)^3}$.
22. $\lim\limits_{n\to\infty} \dfrac{(3n-2)^4-(3n+2)^4}{(3-n)^3-(3+n)^3}$.
23. $\lim\limits_{n\to\infty} \dfrac{(3n-6)^4-(3n+6)^4}{(n-2)^4-(n+2)^4}$.
24. $\lim\limits_{n\to\infty} \dfrac{(2n-6)^4-(2n+6)^4}{(3n+2)^4-(3n-2)^4}$.
25. $\lim\limits_{n\to\infty} \dfrac{(n-3)^3-(n+3)^3}{(4-3n)^2+(4+3n)^2}$.
26. $\lim\limits_{n\to\infty} \dfrac{(2n-5)^3-(5+2n)^3}{(4-3n)^2+(4+3n)^2}$.
27. $\lim\limits_{n\to\infty} \dfrac{(2n-5)^4-(2n+5)^4}{(4-2n)^3-(4+2n)^3}$.
28. $\lim\limits_{n\to\infty} \dfrac{(3n-1)^4-(3n+1)^4}{(5-n)^3-(5+n)^3}$.
29. $\lim\limits_{n\to\infty} \dfrac{(3n-2)^4-(3n+2)^4}{(2n-1)^4-(2n+1)^4}$.
30. $\lim\limits_{n\to\infty} \dfrac{(2n-5)^4-(2n+5)^4}{(3n+1)^4-(3n-1)^4}$.
31. $\lim\limits_{n\to\infty} \dfrac{(4-5n)^3+(4+5n)^3}{(3+n)^3+(3-n)^3}$.
32. $\lim\limits_{n\to\infty} \dfrac{(7-2n)^3+(7+2n)^3}{(4-n)^3+(4+n)^3}$.
33. $\lim\limits_{n\to\infty} \dfrac{(2-5n)^3+(2+5n)^3}{(4-3n)^3+(4+3n)^3}$.

Задача 2.

1. $\lim\limits_{n\to\infty} n^2\left(\sqrt{n^4+7}-\sqrt{n^4-3}\right)$. 2. $\lim\limits_{n\to\infty} n^3\left(\sqrt{n^6+9}-\sqrt{n^6-3}\right)$.

3. $\lim\limits_{n\to\infty} (n+1)^2\left(\sqrt{n^4+7}-\sqrt{n^4-3}\right)$. 4. $\lim\limits_{n\to\infty} (n-1)^2\left(\sqrt{n^4+11}-\sqrt{n^4-2}\right)$.

5. $\lim\limits_{n\to\infty} (n-1)^3\left(\sqrt{n^6+9}-\sqrt{n^6-1}\right)$. 6. $\lim\limits_{n\to\infty} (n-3)^2\left(\sqrt{n^4+5}-\sqrt{n^4-1}\right)$.

7. $\lim\limits_{n\to\infty} n\left(\sqrt{n^4+7n}-\sqrt{n^4-3}\right)$. 8. $\lim\limits_{n\to\infty} n^2\left(\sqrt{n^6+9n}-\sqrt{n^6-3n}\right)$.

9. $\lim\limits_{n\to\infty} (n-1)^2\left(\sqrt{n^6+5n}-\sqrt{n^6-n}\right)$. 10. $\lim\limits_{n\to\infty} n^2\left(\sqrt{n^6+7n-1}-\sqrt{n^6-2n+3}\right)$.

11. $\lim\limits_{n\to\infty} (n-2)^2\left(\sqrt{n^6+8n-2}-\sqrt{n^6-3n}\right)$. 12. $\lim\limits_{n\to\infty} n^3\left(\sqrt{n^8+9n}-\sqrt{n^8-5}\right)$.

13. $\lim\limits_{n\to\infty} \sqrt[3]{(n-2)^2}\left(\sqrt[3]{n+4}-2\sqrt[3]{n+1}+\sqrt[3]{n}\right)$. 14. $\lim\limits_{n\to\infty} \sqrt[3]{(n+2)^2}\left(\sqrt[3]{n+5}-2\sqrt[3]{n+1}+\sqrt[3]{n}\right)$.

15. $\lim\limits_{n\to\infty} \sqrt[3]{(n+2)^2}\left(\sqrt[3]{n+6}-2\sqrt[3]{n+1}+\sqrt[3]{n}\right)$. 16. $\lim\limits_{n\to\infty} \sqrt[3]{(n+4)^2}\left(\sqrt[3]{n+7}-2\sqrt[3]{n+5}+\sqrt[3]{n+1}\right)$.

17. $\lim\limits_{n\to\infty} \sqrt[3]{(n-2)^2}\left(\sqrt[3]{n+9}-2\sqrt[3]{n+3}+\sqrt[3]{n+1}\right)$. 18. $\lim\limits_{n\to\infty} \sqrt[3]{(n+2)^2}\left(\sqrt[3]{n+7}-2\sqrt[3]{n+3}+\sqrt[3]{n}\right)$.

19. $\lim\limits_{n\to\infty} \sqrt[3]{(n+4)^2}\left(\sqrt[3]{n+6}-2\sqrt[3]{n+5}+\sqrt[3]{n}\right)$. 20. $\lim\limits_{n\to\infty} \sqrt[3]{(n+5)^2}\left(\sqrt[3]{n+8}-2\sqrt[3]{n+3}+\sqrt[3]{n}\right)$.

21. $\lim\limits_{n\to\infty} \sqrt{n-2}\left(\sqrt{n+4}-2\sqrt{n+1}+\sqrt{n}\right)$. 22. $\lim\limits_{n\to\infty} \sqrt{n+2}\left(\sqrt{n+5}-2\sqrt{n+1}+\sqrt{n}\right)$.

23. $\lim\limits_{n\to\infty} \sqrt{n+2}\left(\sqrt{n+6}-2\sqrt{n+1}+\sqrt{n}\right)$. 24. $\lim\limits_{n\to\infty} \sqrt{n+4}\left(\sqrt{n+7}-2\sqrt{n+5}+\sqrt{n+1}\right)$.

25. $\lim\limits_{n\to\infty} \sqrt{n-2}\left(\sqrt{n+9}-2\sqrt{n+3}+\sqrt{n+1}\right)$. 26. $\lim\limits_{n\to\infty} \sqrt{n+2}\left(\sqrt{n+7}-2\sqrt{n+3}+\sqrt{n}\right)$.

27. $\lim\limits_{n\to\infty} \sqrt{n+4}\left(\sqrt{n+6}-2\sqrt{n+5}+\sqrt{n}\right)$. 28. $\lim\limits_{n\to\infty} \sqrt{n+5}\left(\sqrt{n+8}-2\sqrt{n+3}+\sqrt{(n+1)}\right)$.

29. $\lim\limits_{n\to\infty} n\left(\sqrt{n^4+6n}-\sqrt{n^4-3n}\right)$. 30. $\lim\limits_{n\to\infty} n^2\left(\sqrt{n^6+9n}-\sqrt{n^6-4n}\right)$.

31. $\lim\limits_{n\to\infty} (n+1)\left(\sqrt{n^4+6n}-\sqrt{n^4-2n}\right)$. 32. $\lim\limits_{n\to\infty} (n-1)^2\left(\sqrt{n^6+8n}-\sqrt{n^6-3n}\right)$.

33. $\lim\limits_{n\to\infty} (n+3)\left(\sqrt{n^4+9n}-\sqrt{n^4+2n}\right)$. 34. $\lim\limits_{n\to\infty} (n-2)^2\left(\sqrt{n^6+n}-\sqrt{n^6+3n}\right)$.

Задача 3.

1. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2+4}{3x^2-2}\right)^{2x^2+x}$. 2. $\lim\limits_{x\to\infty}\left(\dfrac{5x^2+4}{5x^2-4}\right)^{3x^2+x}$. 3. $\lim\limits_{x\to\infty}\left(\dfrac{6x^2+5}{6x^2+1}\right)^{3x^2-x+1}$.

4. $\lim\limits_{x\to\infty}\left(\dfrac{4x^2-3}{4x^2+6}\right)^{5x^2-3}$. 5. $\lim\limits_{x\to\infty}\left(\dfrac{5x^2+3}{5x^2+6}\right)^{2x^2-3}$. 6. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2-4}{3x^2+5}\right)^{4x^2-x}$.

7. $\lim\limits_{x\to\infty}\left(\dfrac{2x^2+3}{2x^2-1}\right)^{3x^2+x-3}$. 8. $\lim\limits_{x\to\infty}\left(\dfrac{6x^2+5}{6x^2+1}\right)^{3x^2-x+1}$. 9. $\lim\limits_{x\to\infty}\left(\dfrac{1+2x}{5+2x}\right)^{x-3}$

10. $\lim\limits_{x\to\infty}\left(\dfrac{5x^2+1}{5x^2-1}\right)^{2x^2+x-1}$. 11. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2+5}{3x^2+2}\right)^{x^2-2x+1}$ 12. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2+4}{3x^2-1}\right)^{x^2+6x-7}$.

13. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2+2x+2}{3x^2+2x-6}\right)^{2x^2+x}$. 14. $\lim\limits_{x\to\infty}\left(\dfrac{5x^2+x+4}{5x^2+x-4}\right)^{6x^2+3x}$. 15. $\lim\limits_{x\to\infty}\left(\dfrac{6x^2+5x+2}{6x^2+5x+1}\right)^{3x^2-4x+1}$.

16. $\lim\limits_{x\to\infty}\left(\dfrac{4x^3-3}{4x^3+6}\right)^{5x^3-3x}$. 17. $\lim\limits_{x\to\infty}\left(\dfrac{5x^2+3x+5}{5x^2+3x+4}\right)^{2x^2-3}$. 18. $\lim\limits_{x\to\infty}\left(\dfrac{3x^2-4x-4}{3x^2-4x+5}\right)^{4x^2-4x+2}$.

19. $\lim\limits_{x\to\infty}\left(\dfrac{2x^2+3x+2}{2x^2+3x-1}\right)^{3x^2+x-3}$. 20. $\lim\limits_{x\to\infty}\left(\dfrac{6x^3+5}{6x^3+1}\right)^{3x^3-x+1}$. 21. $\lim\limits_{x\to\infty}\left(\dfrac{x^3+2x+5}{x^3+4x+4}\right)^{x^2-3}$

22. $\lim\limits_{x\to\infty}\left(\dfrac{5x^3-5x+2+1}{5x^3-x+3}\right)^{2x^2+x-1}$. 23. $\lim\limits_{x\to\infty}\left(\dfrac{3x^3+5x-4}{3x^3+2x+3}\right)^{x^2-2x+1}$.

24. $\lim\limits_{x\to\infty}\left(\dfrac{3x^3+4x+2}{3x^3-x+7}\right)^{x^2+6x-7}$. 25. $\lim\limits_{x\to\infty}\left(\dfrac{2x^2-3x+2}{2x^2-3x-5}\right)^{3x^2+2x-3}$.

26. $\lim\limits_{x\to\infty}\left(\dfrac{7x^3+5}{7x^3+3}\right)^{3x^3-4x+1}$. 27. $\lim\limits_{x\to\infty}\left(\dfrac{x^3+5x+7}{x^3+3x+4}\right)^{x^2-5}$

28. $\lim\limits_{x\to\infty}\left(\dfrac{5x^3-6x+2+1}{5x^3-2x+3}\right)^{2x^2+3x-1}$. 29. $\lim\limits_{x\to\infty}\left(\dfrac{3x^3+7x-2}{3x^3+2x+3}\right)^{x^2-2x+5}$

30. $\lim\limits_{x\to\infty}\left(\dfrac{3x^3+3x+4}{3x^3-2x+7}\right)^{x^2+6x-2}$. 31. $\lim\limits_{x\to\infty}\left(\dfrac{7x^2+3x+2}{7x^2+3x-5}\right)^{3x^2+4x-3}$.

32. $\lim\limits_{x\to\infty}\left(\dfrac{6x^3+5}{6x^3+1}\right)^{2x^3-5x+1}$. 33. $\lim\limits_{x\to\infty}\left(\dfrac{x^3-5x+5}{x^3-4x+4}\right)^{x^2-6}$

Задача 4.

1. $\lim\limits_{x\to 3} \dfrac{\operatorname{arctg}(9-x^2)}{x^2-5x+6}$. 2. $\lim\limits_{x\to 3} \dfrac{\arcsin(9-x^2)}{x^2-8x+15}$. 3. $\lim\limits_{x\to 2} \dfrac{2^{(4-x^2)}-1}{x^2-5x+6}$. 4. $\lim\limits_{x\to 4} \dfrac{\sin(\pi(x^2-16))}{x^2-5x+4}$.

5. $\lim\limits_{x\to 5} \dfrac{\sin(2\pi(x-5))}{x^2-7x+10}$. 6. $\lim\limits_{x\to 3} \dfrac{\operatorname{tg}(3\pi(9-x^2))}{x^2-7x+12}$. 7. $\lim\limits_{x\to 3} \dfrac{\operatorname{arctg}(9-x^2)}{x^3-27}$. 8. $\lim\limits_{x\to 2} \dfrac{3^{(4-x^2)}-1}{x^3-8}$.

9. $\lim\limits_{x\to 5} \dfrac{\sin(2\pi(x^2-25))}{x^2-8x+15}$. 10. $\lim\limits_{x\to 2} \dfrac{\operatorname{tg}(5\pi(4-x^2))}{x^3-2x^2+4x-8}$. 11. $\lim\limits_{x\to 2} \dfrac{\operatorname{arctg}(8-x^3)}{x^3-2x^2+6x-12}$.

12. $\lim\limits_{x\to 3} \dfrac{4^{(27-x^3)}-1}{x^3-3x^2+5x-15}$. 13. $\lim\limits_{x\to 1} \dfrac{5^x-5x}{x^2+4x-5}$. 14. $\lim\limits_{x\to 5} \dfrac{\ln(x^2-10x+26)}{x^2-8x+15}$.

15. $\lim\limits_{x\to 3} \dfrac{\ln(x^2-6x+10)}{x^2-7x+12}$. 16. $\lim\limits_{x\to 2} \dfrac{\ln(x^2-4x+5)}{x^2-8x+12}$.

17. $\lim\limits_{x\to 2} \dfrac{\sin(2\pi(4-x^2))}{x^3-2x-4}$. 18. $\lim\limits_{x\to 3} \dfrac{\operatorname{tg}(4\pi(9-x^2))}{x^3-x^2-4x-6}$.

19. $\lim\limits_{x\to 1} \dfrac{4^x-4x}{x^2-7x+6}$. 20. $\lim\limits_{x\to -2} \dfrac{\sin(4\pi(4-x^2))}{x^3-2x+4}$. 21. $\lim\limits_{x\to -3} \dfrac{\operatorname{tg}(6\pi(9-x^2))}{x^3+x^2-4x+6}$.

22. $\lim\limits_{x\to 1} \dfrac{3^x-3x}{x^2-4x+3}$. 23. $\lim\limits_{x\to 1} \dfrac{2^x-2x}{x^2-3x+2}$. 24. $\lim\limits_{x\to 1} \dfrac{7^x-7x}{x^2-8x+7}$.

25. $\lim\limits_{x\to \frac{1}{2}} \dfrac{8x^3-1}{2^{1-2x}-1}$. 26. $\lim\limits_{x\to \frac{1}{3}} \dfrac{27x^3-1}{3^{1-3x}-1}$. 27. $\lim\limits_{x\to \frac{1}{4}} \dfrac{64x^3-1}{5^{1-4x}-1}$.

28. $\lim\limits_{x\to -\frac{1}{2}} \dfrac{8x^3+1}{2^{1+2x}-1}$. 29. $\lim\limits_{x\to -\frac{1}{3}} \dfrac{27x^3+1}{6^{1+3x}-1}$. 30. $\lim\limits_{x\to -\frac{1}{4}} \dfrac{64x^3+1}{6^{1+4x}-1}$.

31. $\lim\limits_{x\to 2} \dfrac{2^x-4}{\ln(x^2-x-1)}$. 32. $\lim\limits_{x\to 2} \dfrac{3^x-9}{\ln(x^2+x-5)}$. 33. $\lim\limits_{x\to 2} \dfrac{4^x-16}{\ln(x^2+2x-7)}$.

Задача 5.

$$1.\ \lim_{x\to 0}\frac{\sqrt{9+x^2}-3}{\sqrt{16+x^2}-4}.\quad 2.\ \lim_{x\to 0}\frac{\sqrt{9+x^2}-3}{\sqrt{36+x^2}-6}.\quad 3.\ \lim_{x\to 2}\frac{\sqrt{5+x^2}-3}{\sqrt{21+x^2}-5}.$$

$$4.\ \lim_{x\to 3}\frac{\sqrt{7+x^2}-4}{\sqrt{27+x^2}-6}.\quad 5.\ \lim_{x\to 4}\frac{\sqrt{9+x^2}-5}{\sqrt{20+x^2}-6}.\quad 6.\ \lim_{x\to 5}\frac{\sqrt{11+x^2}-6}{\sqrt{24+x^2}-7}.$$

$$7.\ \lim_{x\to 2}\frac{\sqrt[3]{25+x}-\sqrt[3]{29-x}}{x-2}.\quad 8.\ \lim_{x\to 3}\frac{\sqrt[3]{61+x}-\sqrt[3]{67-x}}{x-3}.\quad 9.\ \lim_{x\to 1}\frac{\sqrt[3]{7+x}-\sqrt[3]{9-x}}{x-1}.$$

$$10.\ \lim_{x\to 2}\frac{2x-\sqrt{16x-16}}{\sqrt{x^2+4}-\sqrt{4x}}.\quad 11.\ \lim_{x\to 3}\frac{\sqrt{3}x-\sqrt{18x-27}}{\sqrt{x^2+9}-\sqrt{6x}}.\quad 12.\ \lim_{x\to 4}\frac{\sqrt{6}x-\sqrt{32x-32}}{\sqrt{x^2+16}-\sqrt{8x}}$$

$$13.\ \lim_{x\to 0}\frac{\sqrt[3]{5+x}-\sqrt[3]{5-x}}{\sqrt{4-x}-\sqrt{4+x}}.\quad 14.\ \lim_{x\to 0}\frac{\sqrt[3]{6+x}-\sqrt[3]{6-x}}{\sqrt{3-x}-\sqrt{3+x}}.\quad 15.\ \lim_{x\to 0}\frac{\sqrt[3]{7+x}-\sqrt[3]{7-x}}{\sqrt{5-x}-\sqrt{5+x}}.$$

$$16.\ \lim_{x\to 0}\frac{\sqrt[3]{8+x}-\sqrt[3]{8-x}}{\sqrt{2-x}-\sqrt{2+x}}.\quad 17.\ \lim_{x\to 0}\frac{\sqrt[3]{9+x}-\sqrt[3]{9-x}}{\sqrt{3-x}-\sqrt{3+x}}.\quad 18.\ \lim_{x\to 0}\frac{\sqrt[3]{5+x}-\sqrt[3]{5-x}}{\sqrt{5-x}-\sqrt{5+x}}.$$

$$19.\ \lim_{x\to 1}\frac{\sqrt[3]{7+x}-\sqrt[3]{9-x}}{x-1}.\quad 20.\ \lim_{x\to 3}\frac{\sqrt[3]{24+x}-\sqrt[3]{30-x}}{x-3}.\quad 21.\ \lim_{x\to 4}\frac{\sqrt[3]{23+x}-\sqrt[3]{31-x}}{x-4}.$$

$$22.\ \lim_{x\to\infty}\frac{\sqrt{x^4+x^3}-x^2}{x}\quad 23.\ \lim_{x\to\infty}\left(\sqrt{x^4+x^3}-x^2-\frac{1}{2}x\right).\quad 24.\ \lim_{x\to+\infty}\frac{x^2+\sin x}{x^2+x\sqrt{x^2-1}}.$$

$$25.\ \lim_{x\to-\infty}\frac{x^2+\sin x}{x^2-x\sqrt{x^2-1}}\quad 26.\ \lim_{x\to+\infty}\left(\frac{x^2+\sin\left(\frac{1}{x}\right)}{x+\sqrt{x^2-1}}-\frac{1}{2}x\right).\quad 27.\ \lim_{x\to-\infty}\left(\frac{x^2+\sin\left(\frac{1}{x}\right)}{x-\sqrt{x^2-1}}-\frac{1}{2}x\right)$$

$$28.\ \lim_{x\to+\infty}\frac{x^2+\cos x}{x^2+x\sqrt{x^2-1}}.\quad 29.\ \lim_{x\to-\infty}\frac{x^2+\cos x}{x^2-x\sqrt{x^2-1}}.\quad 30.\ \lim_{x\to+\infty}\left(\frac{x^2+\cos\left(\frac{1}{x}\right)}{x+\sqrt{x^2-1}}-\frac{1}{2}x\right).$$

$$31.\ \lim_{x\to-\infty}\left(\frac{x^2+\cos\left(\frac{1}{x}\right)}{x-\sqrt{x^2-1}}-\frac{1}{2}x\right).32.\ \lim_{x\to 0}\frac{\sqrt[3]{11+x}-\sqrt[3]{11-x}}{\sqrt{3-x}-\sqrt{3+x}}.\quad 33.\ \lim_{x\to 0}\frac{\sqrt[3]{6+x}-\sqrt[3]{6-x}}{\sqrt{6-x}-\sqrt{6+x}}.$$

Задача 6.

$$1.\ \lim_{x\to 4}\frac{\operatorname{tg}^3\left(\frac{\pi}{4}x\right)}{\left(2^x-2^4\right)^3}.\quad 2.\ \lim_{x\to 0}\frac{\left(e^{\alpha x}-e^{\beta x}\right)^2}{(\sin\alpha x-\sin\beta x)^2}.\quad 3.\ \lim_{x\to 3}\frac{\operatorname{tg}^3\left(\frac{\pi}{3}x\right)}{\left(2^x-2^3\right)^3}.$$

$$4.\ \lim_{x\to 1}\frac{\ln\cos(2\pi x)}{\ln\cos 2(x-1)}.\quad 5.\ \lim_{x\to 2}(2-x)\operatorname{tg}\frac{\pi x}{4}.\quad 6.\ \lim_{x\to 0}\frac{e^{\cos 2x}-e^{\cos x}}{4^{x^2}-1}.$$

$$7.\ \lim_{x\to 0}\frac{e^{\sin 2x}-e^{2\sin x}}{\ln^3(1+x2^x)}\quad 8.\ \lim_{x\to 0}\frac{\ln^3(\cos bx)}{\ln^3(\cos ax)}.\quad 9.\ \lim_{x\to 0}\frac{\ln(4-x^3)-\ln 4}{\sin 2x-2\sin x}.$$

$$10.\ \lim_{x\to 0}\frac{\cos x-\cos 2x}{\ln^2(1+4x2^x)}.\quad 11.\ \lim_{x\to 4}(4-x)\operatorname{tg}\frac{\pi x}{8}.\quad 12.\ \lim_{x\to 0}\frac{e^{\cos 4x}-e^{\cos 2x}}{5^{x^2}-1}.$$

$$13.\ \lim_{x\to 3}(3-x)\operatorname{tg}\frac{\pi x}{6}.\quad 14.\ \lim_{x\to 5}(5-x)\operatorname{tg}\frac{\pi x}{10}.\quad 15.\ \lim_{x\to -4}(4+x)\operatorname{tg}\frac{\pi x}{8}.\quad 16.\ \lim_{x\to -2}(2+x)\operatorname{tg}\frac{\pi x}{4}.$$

$$17.\ \lim_{x\to -3}(3+x)\operatorname{tg}\frac{\pi x}{6}.\quad 18.\ \lim_{x\to -5}(5+x)\operatorname{tg}\frac{\pi x}{10}.\quad 19.\ \lim_{x\to 0}\frac{\cos 4x-\cos 6x}{\ln^2(1+x3^x)}.\quad 20.\ \lim_{x\to 0}\frac{\ln(6-x^2)-\ln 6}{\ln^2(1+x5^x)}.$$

$$21.\ \lim_{x\to 0}\frac{\ln(5-x^3)-\ln 5}{\sin 4x-2\sin 2x}.\quad 22.\ \lim_{x\to 0}\frac{\ln(7-x^3)-\ln 7}{\sin 6x-2\sin 3x}.\quad 23.\ \lim_{x\to 0}\frac{\cos 4x-\cos 6x}{\ln(\cos 2\pi x)}.$$

$$24.\ \lim_{x\to 0}\frac{\cos 8x-\cos 4x}{\ln(\cos 4\pi x)}.\quad 25.\ \lim_{x\to 1}\frac{\ln(\cos(4\pi x))}{\ln(\cos 2\pi(x-1))}.\quad 26.\ \lim_{x\to 1}\frac{\ln(\cos(6\pi x))}{\ln(\cos 4\pi(x-1))}.$$

$$27.\ \lim_{x\to 2}\frac{\ln(\cos(6\pi x))}{\sqrt{x^2+4}-\sqrt{4x}}.\quad 28.\ \lim_{x\to 4}\frac{\ln(\cos(3\pi x))}{\sqrt{x^2+16}-\sqrt{8x}}.\quad 29.\ \lim_{x\to 3}\frac{\ln(\cos(6\pi x))}{\sqrt{x^2+9}-\sqrt{6x}}.$$

$$30.\ \lim_{x\to 5}\frac{\ln(\cos(8\pi x))}{\sqrt{x^2+25}-\sqrt{10x}}.\quad 31.\ \lim_{x\to 6}\frac{\ln(\cos(8\pi x))}{\sqrt{x^2+36}-\sqrt{12x}}.\quad 32.\ \lim_{x\to 6}(6-x)\operatorname{tg}\frac{\pi x}{12}.$$

$$33.\ \lim_{x\to -6}(6+x)\operatorname{tg}\frac{\pi x}{12}.$$

Задача 7.

1. $\lim\limits_{x\to 0}\dfrac{7^{2x^3}-5^{3x^3}}{\ln(3+x^2\sin 2x)-\ln 3}$.
2. $\lim\limits_{x\to 0}\dfrac{9^{2x^2}-4^{3x^2}}{\ln(3+x\operatorname{tg}2x)-\ln 3}$.
3. $\lim\limits_{x\to 0}\dfrac{5^{2x^3}-9^{3x^3}}{\ln(3+x^2\arcsin 2x)-\ln 3}$.
4. $\lim\limits_{x\to 0}\dfrac{\sqrt{5+x^2}-\sqrt{5}}{\ln(9+x\operatorname{tg}5x)-\ln 9}$.
5. $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x^2}-\sqrt{4}}{\ln(8+x\operatorname{tg}2x)-\ln 8}$.
6. $\lim\limits_{x\to 0}\dfrac{\sqrt{7+x^2}-\sqrt{7}}{\ln(2+x\arcsin 2x)-\ln 2}$.
7. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{5+x^2}-\sqrt[3]{5}}{\ln(9+x\operatorname{tg}9x)-\ln 9}$.
8. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{4+x^2}-\sqrt[3]{4}}{\ln(8+x\sin 5x)-\ln 8}$.
9. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{7+x^2}-\sqrt[3]{7}}{\ln(2+x\arcsin 5x)-\ln 2}$.
10. $\lim\limits_{x\to 0}\dfrac{\sqrt[4]{2+x^2}-\sqrt[4]{2}}{\ln(9-x\operatorname{tg}9x)-\ln 9}$.
11. $\lim\limits_{x\to 0}\dfrac{\sqrt[4]{6+x^2}-\sqrt[4]{6}}{\ln(5-x\operatorname{tg}8x)-\ln 5}$.
12. $\lim\limits_{x\to 0}\dfrac{\sqrt[4]{9+x^2}-\sqrt[4]{9}}{\ln(2-x\sin 7x)-\ln 2}$.
13. $\lim\limits_{x\to 0}\dfrac{7^{2x^3}-5^{3x^3}}{\ln(3-x^2\sin 3x)-\ln 3}$.
14. $\lim\limits_{x\to 0}\dfrac{9^{2x^2}-4^{3x^2}}{\ln(3-x\operatorname{tg}4x)-\ln 3}$.
15. $\lim\limits_{x\to 0}\dfrac{5^{2x^3}-9^{3x^3}}{\ln(3-x^2\arcsin 4x)-\ln 3}$.
16. $\lim\limits_{x\to 0}\dfrac{\sqrt{5+x^2}-\sqrt{5}}{\ln(9-x\operatorname{tg}7x)-\ln 9}$.
17. $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x^2}-\sqrt{4}}{\ln(8-x\operatorname{tg}3x)-\ln 8}$.
18. $\lim\limits_{x\to 0}\dfrac{\sqrt{7+x^2}-\sqrt{7}}{\ln(2-x\arcsin 5x)-\ln 2}$.
19. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{5+x^2}-\sqrt[3]{5}}{\ln(9-x\operatorname{tg}8x)-\ln 9}$.
20. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{4+x^2}-\sqrt[3]{4}}{\ln(8-x\sin 9x)-\ln 8}$.
21. $\lim\limits_{x\to 0}\dfrac{\sqrt[3]{7+x^2}-\sqrt[3]{7}}{\ln(2-x\arcsin 6x)-\ln 2}$.
22. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+6x^2}-\sqrt{1+2x^2}}{\ln^2(1+xe^x)}$.
23. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+8x^2}-\sqrt{1+2x^2}}{\ln^2(1+x2^x)}$.
24. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+10x^2}-\sqrt{1+4x^2}}{\ln^2(1+x5^x)}$.
25. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-6x^2}-\sqrt{1-3x^2}}{\ln^2(1+xe^x)}$.
26. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-8x^2}-\sqrt{1-2x^2}}{\ln^2(1+x2^x)}$.
27. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-10x^2}-\sqrt{1-4x^2}}{\ln^2(1+x5^x)}$.
28. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+7x^2}-\sqrt{1+3x^2}}{\ln(1+x\sin x)}$.
29. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+9x^2}-\sqrt{1+3x^2}}{\ln(1+x\operatorname{tg}x)}$.
30. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+5x^2}-\sqrt{1+2x^2}}{\ln(1+x\arcsin x)}$.
31. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-6x^2}-\sqrt{1-4x^2}}{\ln(1+x\operatorname{arctg}x)}$.
32. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-12x^2}-\sqrt{1-2x^2}}{\ln(1-x\operatorname{tg}x)}$.
33. $\lim\limits_{x\to 0}\dfrac{\sqrt{1-11x^2}-\sqrt{1-5x^2}}{\ln(1-x\sin x)}$.

Задача 8.

$$1.\lim_{x\to 0}\left(x+5^{x}\right)^{\frac{2}{x}}. \quad 2.\lim_{x\to 0}\left(x+3^{2x}\right)^{\frac{5}{x}}. \quad 3.\lim_{x\to 0}\left(5^{x}-x\right)^{\frac{4}{x}}. \quad 4.\lim_{x\to 0}\left(6^{2x}-x\right)^{\frac{5}{x}}.$$

$$5.\lim_{x\to 0}\left(1+\operatorname{tg}^{2}(3x)\right)^{\frac{1}{\ln(cos4x)}}. \quad 6.\lim_{x\to 0}\left(1-\operatorname{tg}^{2}(4x)\right)^{\frac{1}{\ln(cos8x)}}. \quad 7.\lim_{x\to 0}\left(1+\sin^{2}(5x)\right)^{\frac{4}{\ln(cos10x)}}.$$

$$8.\lim_{x\to 0}\left(\cos 2\pi x\right)^{\frac{4}{x\operatorname{tg}(\pi x)}}. \quad .9.\lim_{x\to+\infty}\left(7^{x}-x\right)^{\frac{5}{x}}. \quad 10.\lim_{x\to+\infty}\left(4^{2x}-x\right)^{\frac{8}{x}}.$$

$$11.\lim_{x\to 0}\left(\cos 4\pi x\right)^{\frac{4}{x\sin(\pi x)}}. \quad 12.\lim_{x\to+\infty}\left(9^{2x}+x\right)^{\frac{5}{x}}. \quad 13.\lim_{x\to 0}\left(1+\sin^{2}(\pi x)\right)^{\frac{4}{x\operatorname{tg}(\pi x)}}.$$

$$14.\lim_{x\to 0}\left(\cos 6\pi x\right)^{\frac{6}{\ln(6+x^{2})-\ln 6}}. \quad 15.\lim_{x\to 0}\left(1-x^{2}2^{x}\right)^{\frac{1}{1-\cos(2\pi x)}}. \quad 16.\lim_{x\to 0}\left(1+x^{2}3^{x}\right)^{\frac{1}{1-\cos(4\pi x)}}.$$

$$17.\lim_{x\to 0}\left(\cos 4\pi x\right)^{\frac{4}{\ln(4+x^{2})-\ln 4}}. \quad 18.\lim_{x\to 0}\left(1-x^{2}e^{x}\right)^{\frac{1}{1-\cos(4\pi x)}}. \quad 19.\lim_{x\to 0}\left(1+x^{2}3^{x}\right)^{\frac{1}{1-\cos(6\pi x)}}.$$

$$20.\lim_{x\to 0}\left(1+\sin^{2}6\pi x\right)^{\frac{8}{\ln(8+x^{2})-\ln 8}}. \quad 21.\lim_{x\to 0}\left(1-x^{2}2^{x}\right)^{\frac{1}{\ln(\cos(2\pi x)}}. \quad 22.\lim_{x\to 0}\left(1+x^{2}3^{x}\right)^{\frac{1}{\ln(\cos(4\pi x)}}.$$

$$23.\lim_{x\to 0}\left(x+5^{-3x}\right)^{\frac{4}{x}}. \quad 24.\lim_{x\to 0}\left(x+3^{-3x}\right)^{\frac{3}{x}}. \quad 25.\lim_{x\to 0}\left(5^{2x}-x\right)^{\frac{6}{x}}. \quad 26.\lim_{x\to 0}\left(6^{4x}-x\right)^{\frac{3}{x}}.$$

$$27.\lim_{x\to 0}\left(1+\operatorname{arctg}^{2}(5x)\right)^{\frac{1}{\ln(cos4x)}}. \quad 28.\lim_{x\to 0}\left(1-\operatorname{arctg}^{2}(5x)\right)^{\frac{1}{\ln(cos8x)}}. \quad 29.\lim_{x\to 0}\left(1+\arcsin^{2}(3x)\right)^{\frac{4}{\ln(cos10x)}}.$$

$$30.\lim_{x\to 0}\left(x+5^{-6x}\right)^{\frac{-5}{x}}. \quad 31.\lim_{x\to 0}\left(x+4^{-3x}\right)^{\frac{-2}{x}}. \quad 32.\lim_{x\to 0}\left(7^{2x}-x\right)^{\frac{-5}{x}}. \quad 33.\lim_{x\to 0}\left(2^{4x}-x\right)^{\frac{-3}{x}}.$$

Задача 9.

1. $\lim\limits_{x\to 0} \dfrac{\sqrt[25]{1+\sin^2 25x}-1+x\operatorname{arctg}\left(\frac{2}{3}x\right)}{\ln(1+\operatorname{tg}^2 2x)+\arcsin^2 2x}$.

2. $\lim\limits_{x\to 0} \dfrac{\sqrt[21]{1+\operatorname{tg}^2 21x}-1+\arcsin 4x^2}{\cos x-\cos 2x+\ln(1+3x^2)}$.

3. $\lim\limits_{x\to 0} \dfrac{\sqrt[9]{1+\operatorname{tg}^2 5x}-1+\ln^2(1+x\cdot 2^x)}{\arcsin 3x^2+\cos x-\cos 4x}$.

4. $\lim\limits_{x\to 0} \dfrac{\sqrt[7]{1+\sin^2 3x}-1+\arcsin 2x^2}{3\ln^2(1+x3^x)+e^{x^2}-1}$.

5. $\lim\limits_{x\to 0} \dfrac{\sqrt[5]{1+x\operatorname{tg} 25x}-1+x\cdot\ln(2^x+x)}{\ln^2(1+4x)+(e^x-1)^2}$.

6. $\lim\limits_{x\to 0} \dfrac{\sqrt[25]{1+\sin^2 50x}-1+x\ln(1+x2^x)}{\ln(1+\operatorname{tg}^2 4x)+\arcsin^2 4x}$.

7. $\lim\limits_{x\to 0} \dfrac{\sqrt[9]{1+\operatorname{tg}^2 9x}-1+\ln^2(1+x\cdot 4^x)}{\arcsin 3x^2+\cos 4x-\cos 6x}$.

8. $\lim\limits_{x\to 0} \dfrac{\sqrt[21]{1+\operatorname{tg}^2 21x}-1+\arcsin 6x^2}{\cos 8x-\cos 2x+\ln(1+3x^2)}$.

9. $\lim\limits_{x\to 0} \dfrac{4^{x^2}-1+\ln(1+2\sin^3 x)}{\sqrt[3]{1+x\operatorname{arctg} 3x}-1+x\sin 5x}$.

10. $\lim\limits_{x\to 0} \dfrac{5^{x^2}-1+x^2\operatorname{tg} 6x}{\sqrt[5]{1+x\arcsin 5x}-1+(e^x-1)^3}$.

11. $\lim\limits_{x\to 0} \dfrac{\sqrt[5]{1+\sin^2 25x}-1+x\operatorname{arctg}\left(\frac{4}{3}x\right)}{\ln(4+\sin^2 2x)-\ln 4}$.

12. $\lim\limits_{x\to 0} \dfrac{\sqrt[21]{1+\operatorname{tg}^2 3x}-1+\arcsin 5x^2}{\ln(6+3\operatorname{tg} x^2)-\ln 6}$.