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Задание 1.   Найти частные производные

$$z = e^{-(x+2y)} \cdot \cos(x+2y)$$

$$\frac{dz}{dx} = \left(e^{-x+2y}\right)' \cos(x+2y) + e^{-x+2y} \cdot (\cos(x+2y))' =$$

$$= e^{-(x+2y)} \cdot (-1) \cdot \cos(x+2y) + e^{-(x+2y)} \cdot (-\sin(x+2y)) \cdot 1 =$$

$$= -e^{-(x+2y)} \left(\cos(x+2y) + \sin(x+2y) \right)$$

$$\frac{dz}{dy} = \left(e^{-x+2y}\right)' \cos(x+2y) + e^{-x+2y} \cdot (\cos(x+2y))' =$$

$$= e^{-(x+2y)} \cdot (-2) \cdot \cos(x+2y) + e^{-(x+2y)} \cdot -\sin(x+2y) \cdot 2$$

$$= -2 e^{-(x+2y)} \left(\cos(x+2y) + \sin(x+2y) \right)$$

Задание 2   Вычислить приближённо

$$\sqrt{(3.02)^3 - 2(1.93)^3 - 2 \cdot (1.02)^6}$$

$$f(x,y,z) = \sqrt{x^3 - 2y^3 - 2z^6}$$

$$f(x+\Delta x, \; y+\Delta y, \; z+\Delta z) \approx f(x,y,z) + df$$

$x = 3 \qquad \Delta x = 0,02$ $\qquad f(x,y,z) = \sqrt{27 + 16}$ =

$y = 2 \qquad \Delta y = -0,01$ $\qquad = \sqrt{27 - 16 - 2} = \sqrt{9} = \boxed{3}$ ⊕

$z = 1 \qquad \Delta z = 0,02$ $\qquad df = f_x'(x,y,z) \cdot \Delta x + f_y'(x,y,z) \cdot \Delta y +$

$$+ \; f_z'(x,y,z) \cdot \Delta z$$

$$f'_x(x,y,z) = \frac{3x^2}{2\sqrt{x^3-2y^3-2z^6}} = \frac{27}{2\cdot 3} = \frac{27}{6} = \boxed{4,5}$$

$$f'_y(x,y,z) = \frac{-6y^2}{2\sqrt{x^3-2y^3-2z^6}} = \frac{-6y^2}{6} = \boxed{-4}$$

$$f'_z(x,y,z) = \frac{-12z^5}{2\sqrt{x^3-2y^3-2z^6}} = \frac{-12}{6} = \boxed{-2} \quad \oplus$$

$$df = 4,5\cdot(0,02) + 4\cdot(-0,01) - 2\cdot 0,02 =$$
$$= 0,09 + 0,04 - 0,04 = \boxed{0,09} \quad \oplus$$

Ответ: $3 + 0,09 = \cancel{2,91}\ 3,09.$ $\quad \oplus$

Задание 3. Найти экстремумы функции

$$Z = x^3 + 8y^3 - 6xy + 5.$$

$$\begin{cases} z'_x = 3x^2 - 6y = 0 \\ z'_y = 24y^2 - 6x = 0 \end{cases} \quad \Big| \quad y = \frac{x^2}{2}; \quad y_1 = 0, \quad y_{2,3} = \frac{1}{2}$$
$$6x^4 - 6x = 0 /:6$$
$$x(x^3-1) = 0, \quad x_1 = 0$$
$$x_{2,3} = 1$$

$$M_1(0;0), \quad M_2\left(1;\frac{1}{2}\right) \quad \oplus$$

M_1, M_2 — крит. точки.

$$z''_{xx} = 6x \quad \Big| \quad z''_{xx}(M_1) = 0 \quad \Big| \quad z''_{xx}(M_2) = 6$$
$$z''_{yy} = 48y \quad \Big| \quad z''_{yy}(M_1) = 0 \quad \Big| \quad z''_{yy}(M_2) = 24$$
$$z''_{xy} = -6. \quad \Big| \quad z''_{xy}(M_1) = -6 \quad \Big| \quad z''_{xy}(M_2) = -6.$$

$$\psi(M_1) = \begin{pmatrix} 0 & -6 \\ -6 & 0 \end{pmatrix}, \quad \Delta_1 = 0 \qquad \cancel{\bcancel{\ }}\ M_1\ \text{---}\ \text{не экстремум}$$
$$\Delta_2 = -36 < 0$$
$$\oplus$$

$\vartheta(M_2) = \begin{pmatrix} 6 & -6 \\ -6 & 24 \end{pmatrix}$; $\Delta_1 = 6 > 0$

M_2 - т. макс. мин $\oplus$

$\Delta_2 = 108 > 0$

Ответ: $z_{min} = 1 + 1 - 3 + 5 = \boxed{4}$ $\oplus$