Суриков Н.С. - ИУК4-21Б, В-17.

1. Найти e^{At}

$$A = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}$$

Приведём к диагональному виду.

$$D = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

$$\begin{vmatrix} 3-\lambda & 4 \\ 2 & 1-\lambda \end{vmatrix} = 0 \quad , \quad (3-\lambda)(1-\lambda) - 8 = 0$$

$$3 - \lambda - 3\lambda + \lambda^2 - 8 = 0$$

$$\lambda^2 - 4\lambda - 5 = 0.$$

$$| \lambda_1 = -1 \quad \lambda_2 = +5 |$$

$$D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & +5 \end{pmatrix} \quad +$$

Защита: 6 баллов

Посещаемость: 3 балла

Всего: 9 баллов

$$\begin{cases} (3-\lambda)x + 4y = 0 \\ 2x + (1-\lambda)y = 0 \end{cases}$$

~~При $\lambda = 1$.~~

$$\cancel{\begin{cases} 2x + 4y = 0 \\ 2x = 0 \end{cases}} \quad \cancel{x = 0;\ y = 0} \quad \cancel{\bar{S}_1 = \{0, 0\}}$$

~~При $\lambda = -5$~~

$$\cancel{\begin{cases} 8x + 4y = 0 \\ 2x + 6y = 0 \end{cases}}$$

При $\lambda_1 = -1$

$$\begin{cases} 4x + 4y = 0 \\ 2x + 2y = 0 \end{cases} \quad x = -y \quad \bar{S}_1 = \{1, -1\} \quad +$$

При $\lambda = 5$

$$\begin{cases} -2x + 4y = 0 \\ 2x - 4y = 0 \end{cases} \Bigg| \quad \begin{matrix} x - 2y = 0 \\ x = 2y \end{matrix} \qquad \bar{S}_2 = \{2, 1\}$$

(+)

$$U = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}, \quad |U| = 3, \quad U^* = \begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$

$$U^{-1} = \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix}$$ (+)

$$A = U B U^{-1}$$

$$f(At) = U \begin{pmatrix} f(\lambda_1 t) & 0 \\ 0 & f(\lambda_2 t) \end{pmatrix} U^{-1}$$

$$e^{At} = U \begin{pmatrix} e^{-t} & 0 \\ 0 & e^{5t} \end{pmatrix} U^{-1} = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} e^{-t} & 0 \\ 0 & e^{5t} \end{pmatrix} \frac{1}{3}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} =$$

$$= \frac{1}{3}\begin{pmatrix} e^{-t} & 2e^{5t} \\ -e^{-t} & e^{5t} \end{pmatrix}\begin{pmatrix} 1 & -2 \\ 1 & 1 \end{pmatrix} = \frac{1}{3}\begin{pmatrix} e^{-t} + 2e^{5t} & -2e^{-t} + 2e^{5t} \\ -e^{-t} + e^{5t} & 2e^{-t} + e^{5t} \end{pmatrix}$$

Ответ: $$e^{At} = \begin{pmatrix} \frac{e^{-t} + 2e^{5t}}{3} & \frac{-2e^{-t} + 2e^{5t}}{3} \\ \frac{-e^{-t} + e^{5t}}{3} & \frac{2e^{-t} + e^{5t}}{3} \end{pmatrix}.$$ (+)

2. $\underbrace{xy}_{f(x,y)} + \underbrace{2x + y}_{f(x,y)} + \frac{5}{2} = 0$

$f(x,y) = xy.$

$A = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}$; $\begin{vmatrix} -\lambda & \frac{1}{2} \\ \frac{1}{2} & -\lambda \end{vmatrix} = 0 =$

$= \lambda^2 - \frac{1}{4} = 0$

$\lambda^2 = \frac{1}{4}$

$\lambda = \pm\frac{1}{2}$

При $\lambda = \frac{1}{2}$

$\cancel{\frac{1}{2}}$ $\begin{cases} -\frac{1}{2}x + \frac{1}{2}y = 0 \\ \frac{1}{2}x + \frac{1}{2}y = 0 \end{cases}$ $x = y$ $\bar{S}_1 = \{1, 1\}$

$\bar{S}_1^0 = \{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\}$

При $\lambda = -\frac{1}{2}$

$\frac{1}{2}x + \frac{1}{2}y = 0$

$\frac{1}{2}x + \frac{1}{2}y = 0$ $x = -y$ $\bar{S}_2 = \{-1, \cancel{1}\}$

$\bar{S}_2^0 = \{-\frac{1}{\sqrt{2}}; \frac{1}{\sqrt{2}}\}$

$U = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} x' \\ y' \end{pmatrix} =$ $\cancel{xz}$ $\left.\begin{matrix} \frac{1}{\sqrt{2}}x' - \frac{1}{\sqrt{2}}y' = x \\ \frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y' = y \end{matrix}\right\}$

$\begin{cases} x = \frac{1}{\sqrt{2}}x' - \frac{1}{\sqrt{2}}y' \\ y = \frac{1}{\sqrt{2}}x' + \frac{1}{\sqrt{2}}y' \end{cases}$ +

$f(x,y) = 2x+y = 2\left(\frac{1}{\sqrt2}x' - \frac{1}{\sqrt2}y'\right) + \frac{1}{\sqrt2}x' + \frac{1}{\sqrt2}y' = \frac{3}{\sqrt2}x' - \frac{1}{\sqrt2}y'$

В системе координат $x'oy'$:

$$\frac12(x')^2 - \frac12(y')^2 + \frac{3}{\sqrt2}x' - \frac{1}{\sqrt2}y' + \frac52 = 0.$$

$$\frac12\left((x')^2 + \frac{6}{\sqrt2}x' + \frac92\right) - \frac94 - \frac12\left((y')^2 + \frac{2}{\sqrt2}y' + \frac12\right) + \frac{1}{\cancel{8}\,4} + \frac52 = 0$$

$$\frac12\left(x' + \frac{3}{\sqrt2}\right)^2 - \frac12\left(y' + \frac{1}{\sqrt2}\right)^2 - \frac94 + \frac{1}{\cancel{8}\,4} + \frac52 = 0$$

$$\frac12\left(x' + \frac{3}{\sqrt2}\right)^2 - \frac12\left(y' + \frac{1}{\sqrt2}\right) = -\cancel{\frac{3}{8}}\ \frac14$$

$$\left(x' + \frac{3}{\sqrt2}\right)^2 - \left(y' + \frac{1}{\sqrt2}\right) = \cancel{-\frac{3}{4}}$$

$$\begin{cases} x' + \frac{3}{\sqrt2} = x'' \\ y' + \frac{1}{\sqrt2} = y'' \end{cases} \qquad \begin{cases} x' = x'' - \frac{3}{\sqrt2} \\ y' = y'' - \frac{1}{\sqrt2} \end{cases}$$

$$(x'')^2 - (y'')^2 = -\frac34 \quad \Big| : -\frac34$$

$O'\left(-\frac{3}{\sqrt2}, -\frac{1}{\sqrt2}\right)$

$$\frac{(y'')^2}{\cancel{3/4}\ 1} - \frac{(x'')^2}{\cancel{3/4}\ 1} = 1 \ \text{— гипербола.}$$

$a = \sqrt{\frac34} = \frac{\sqrt3}{2}$

$b = \frac{\sqrt3}{2}$.

$\frac{\sqrt3}{2} \approx 0{,}85$