HNFMS

平衡树

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- **营业额统计(turnover.cpp）**
- **【问题描述】**账本上记录了公司成立以来每天的营业额。
- 当最小波动值越大时，就说明营业情况越不稳定。而分析整个公司的从成立到现在营业情况是否稳定，只需要把每一天的最小波动值加起来就可以了。你的任务就是编写一个程序计算这一个值。第一天的最小波动值为第一天的营业额。
- 输入
- 6
- 5 1 2 5 4 6
- 输出
- 12
- 结果说明：
- 5+|1-5|+|2-1|+|5-5|+|4-5|+|6-5|=5+4+1+0+1+1=12

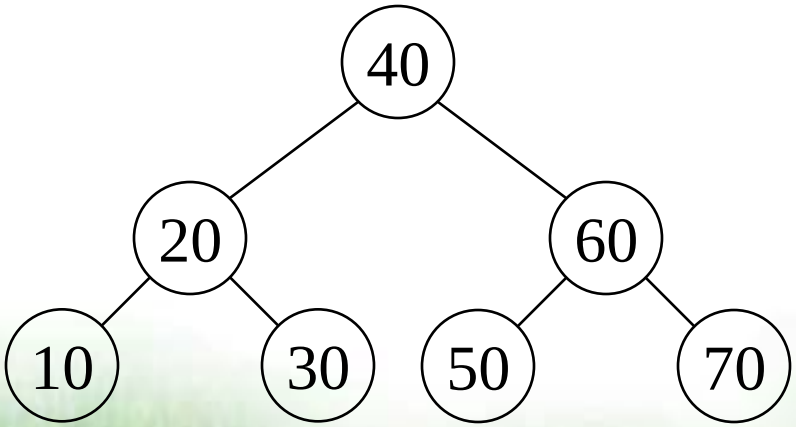
链表(反向处理,跳跃表)

线段树

平衡树

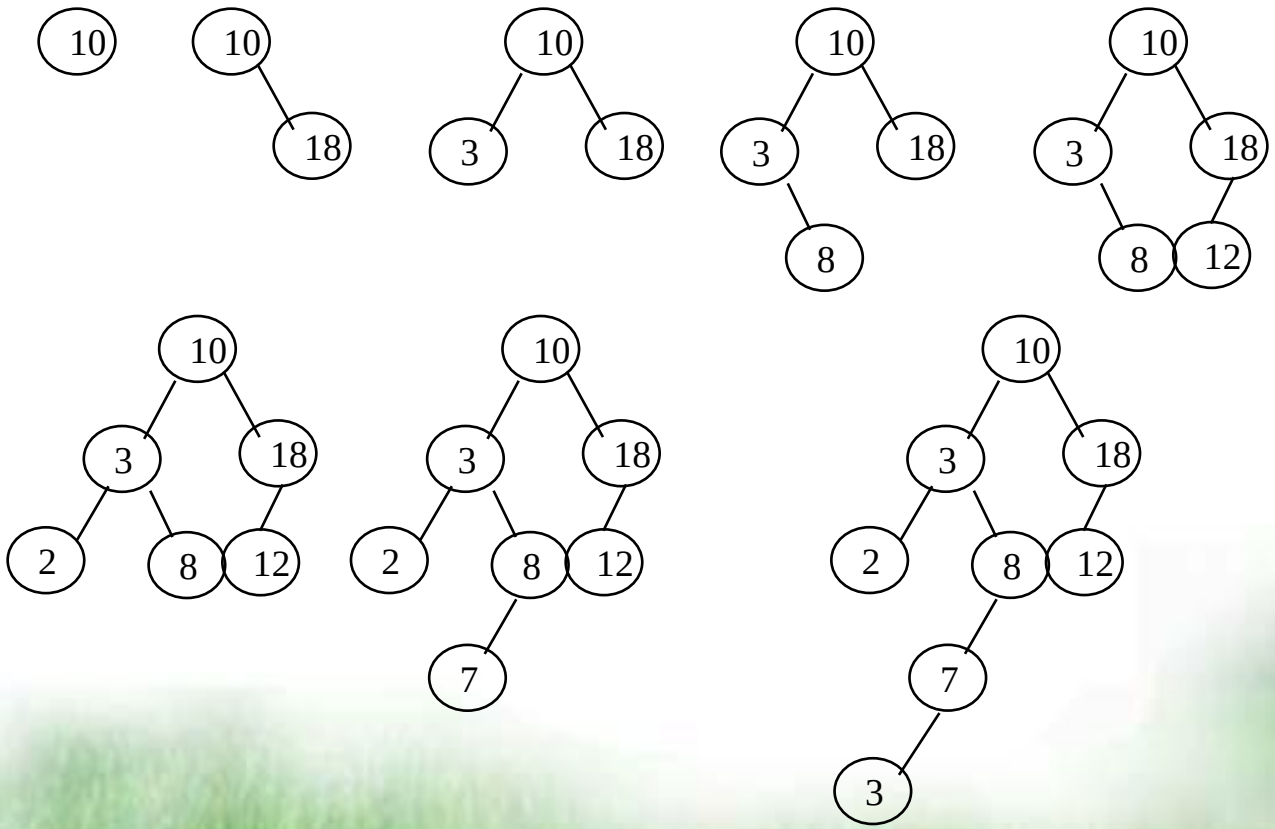
– 二叉查找树

- 定义：二叉排序树或是一棵空树，或是具有下列性质的二叉树：
  - 若它的左子树不空，则左子树上所有结点的值均小于它的根结点的值
  - 若它的右子树不空，则右子树上所有结点的值均大于或等于它的根结点的值
  - 它的左、右子树也分别为二叉排序树



– 插入算法

例 {10， 18， 3， 8， 12， 2， 7， 3}



- 二叉排序树的删除

要删除二叉排序树中的p结点，分三种情况：

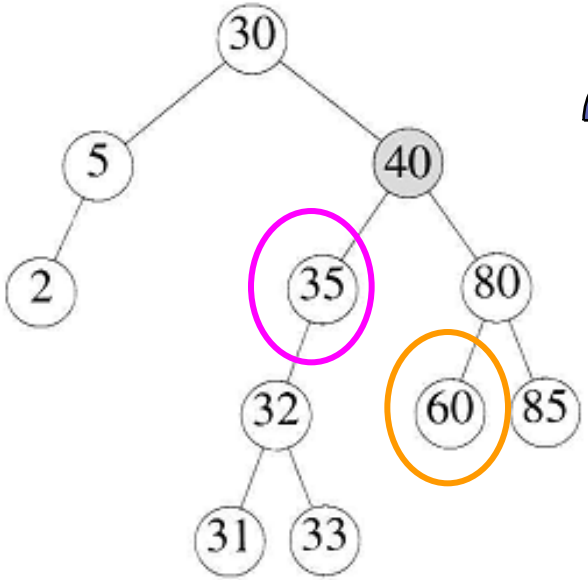
– (1) p为叶子结点

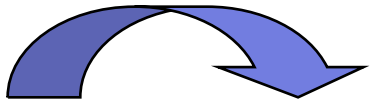
– (2) p只有左子树或右子树

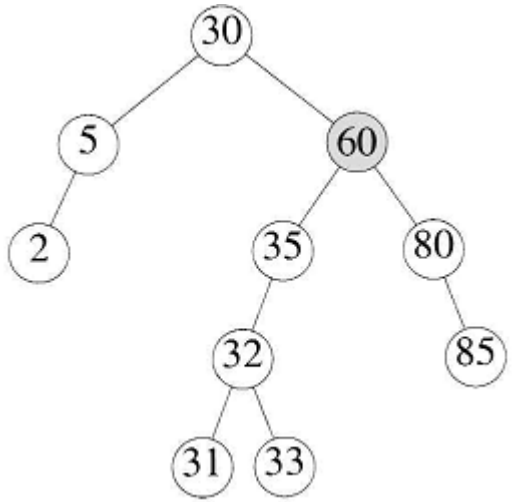
– (3)p左、右子树均非空

情况3-删除示例

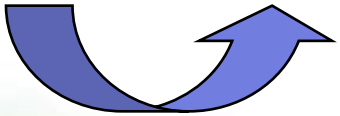
- 删除关键值为40的节点

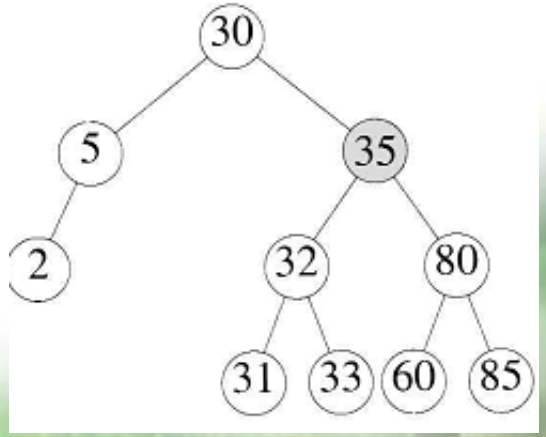






右子树中的最小元素

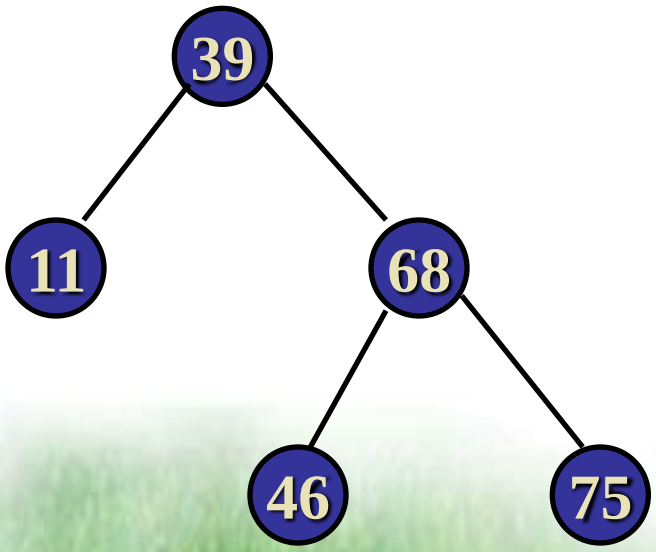


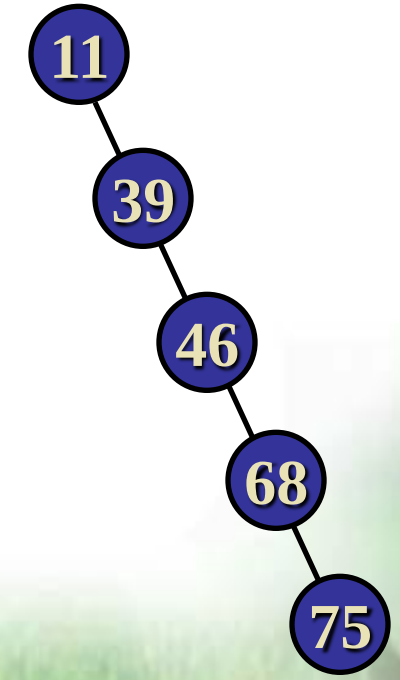


左子树中的最大元素

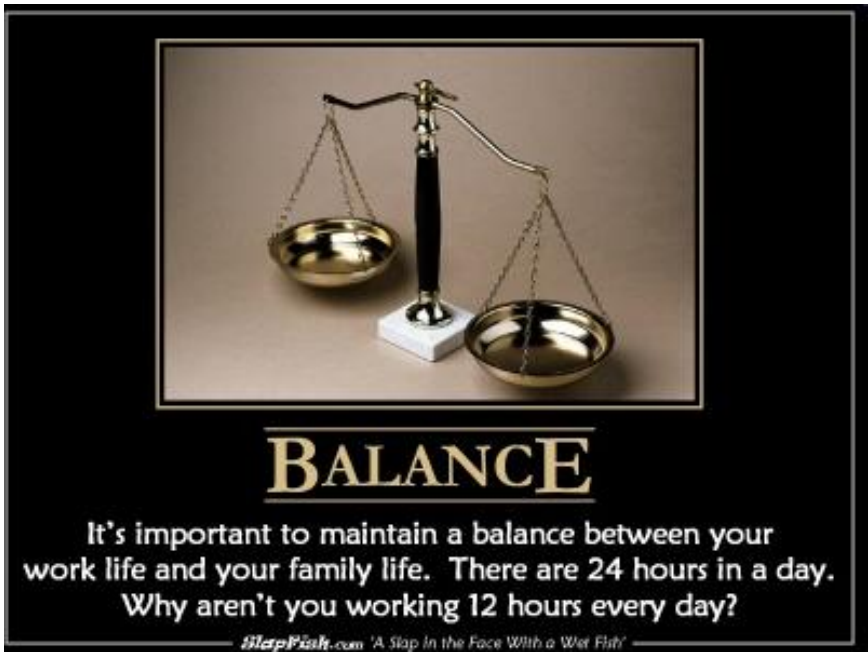
BST的高度

- 含有n个结点的二叉搜索树不是唯一的，从而树的高度就不一定相同。
- 一棵n元素的二叉搜索树的高度可以与n一样大。





平衡树



- 0、暴力平衡(替罪羊树)
- 1、高度平衡(avl)
- 2、重量平衡（treap）
- 3、自动平衡（红黑树，伸展树）

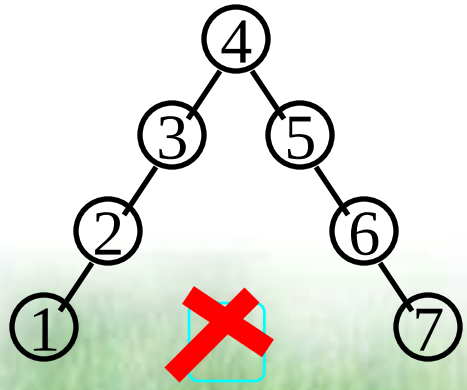
Adelson-Velskii-Landis (AVL) Trees (1962)

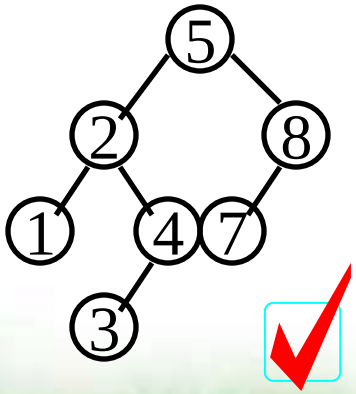
【Definition】An empty binary tree is height balanced. If T is a nonempty binary tree with T_L and T_R as its left and right subtrees, then T is height balanced iff

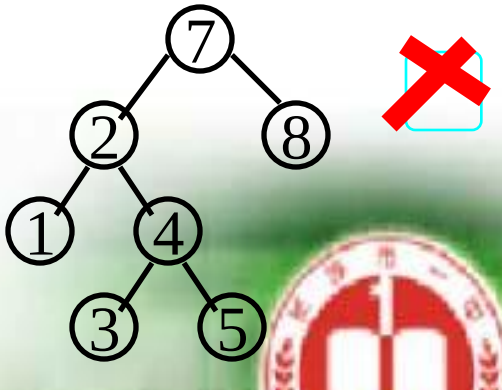
(1) T_L and T_R are height balanced, and

(2) $|h_L - h_R| \leq 1$ where h_L and h_R are the heights of T_L and T_R, respectively.

【Definition】The balance factor $BF(\text{node}) = h_L - h_R$. In an AVL tree, $BF(\text{node}) = -1, 0, \text{or } 1$.

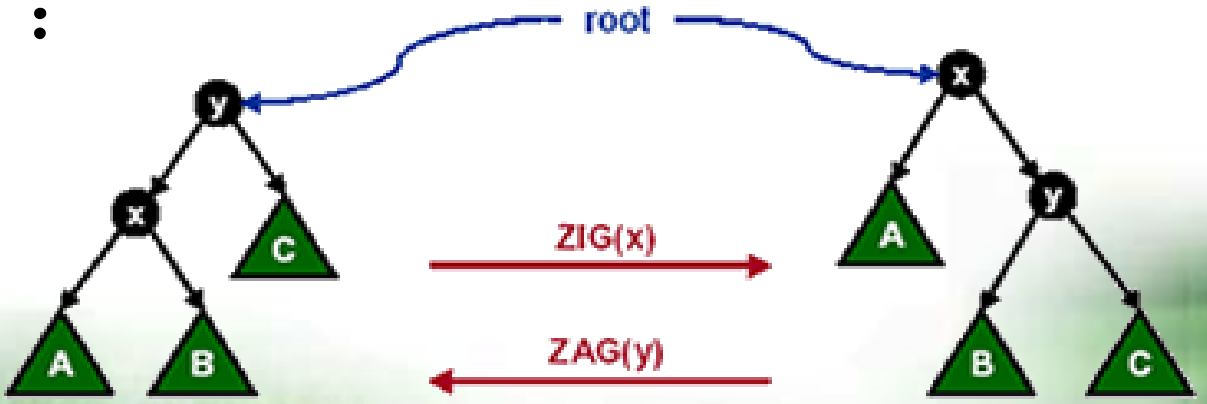




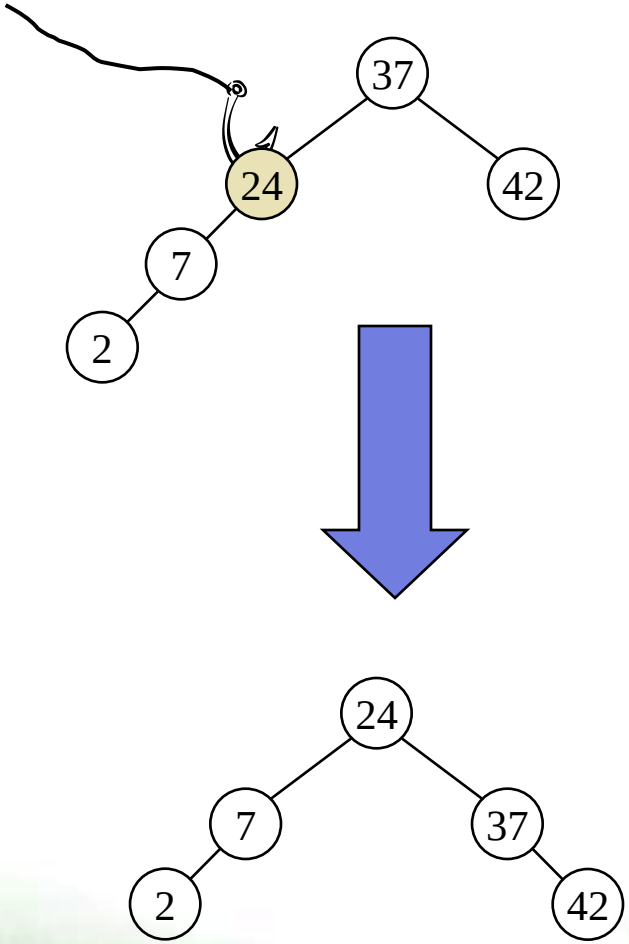


二叉排序树

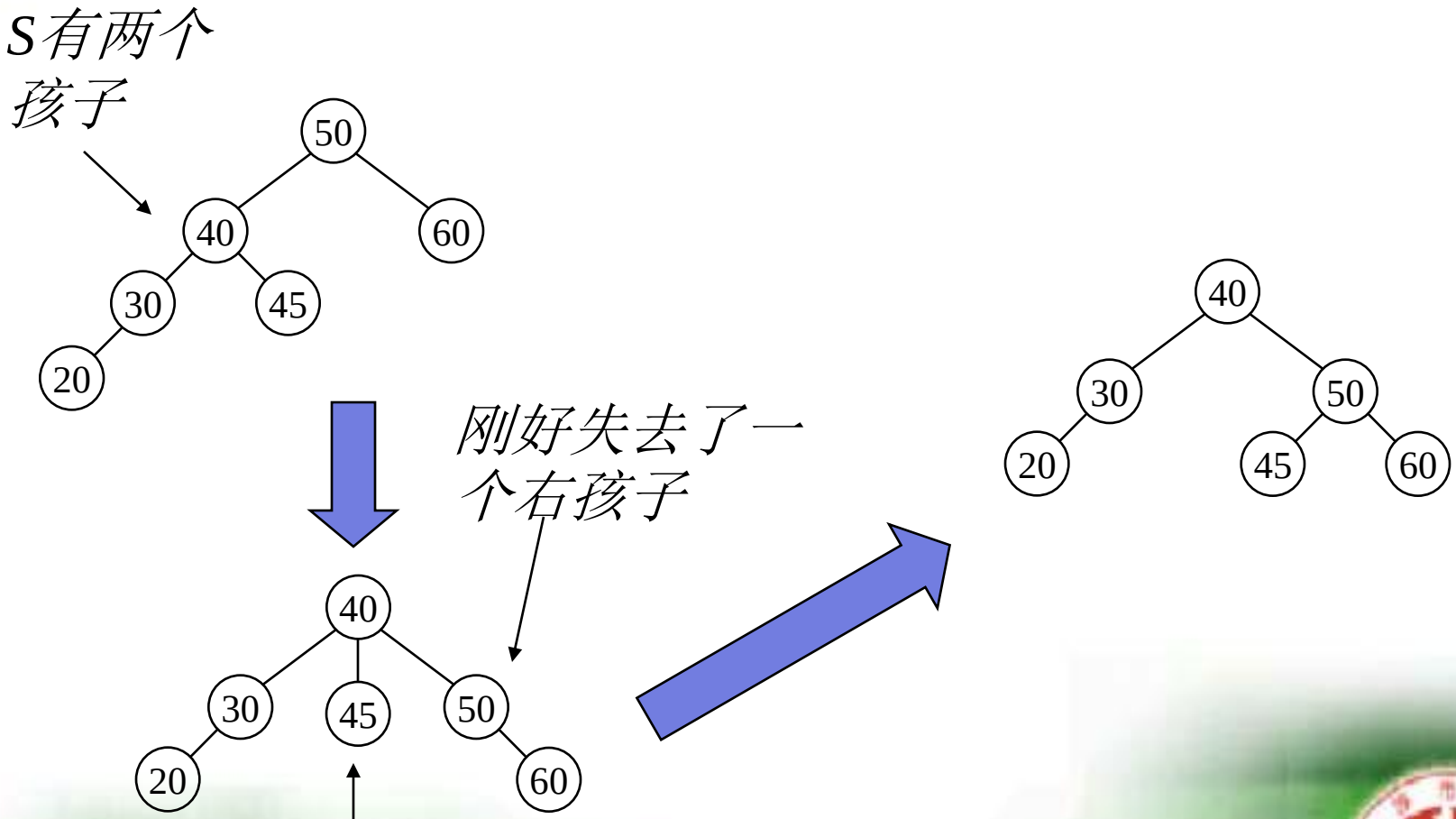
- 旋转操作是二叉排序树的众多变种的一个共同的理论基础
- 下图是右旋操作示意图(反之就是左旋)：



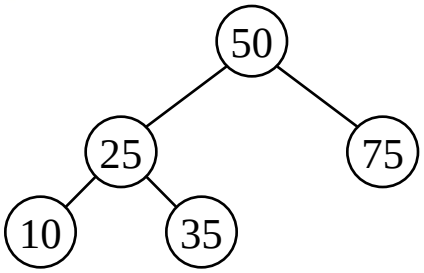
一个不平衡的BST



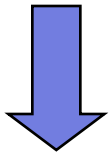
两个孩子的情况

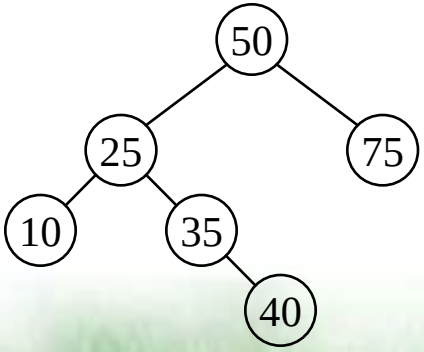

S有两个孩子
50
40
60
30
45
20
刚好失去了一个右孩子
40
30
45
50
20
60
把它放哪里?
40
30
50
20
45
60

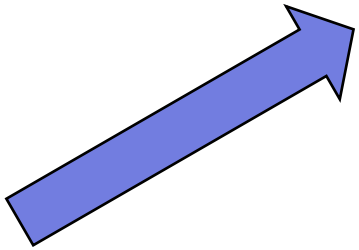
一个单旋??

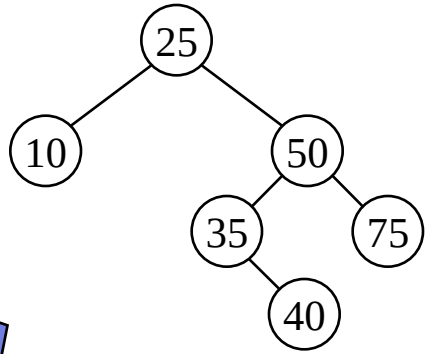


Insert 40



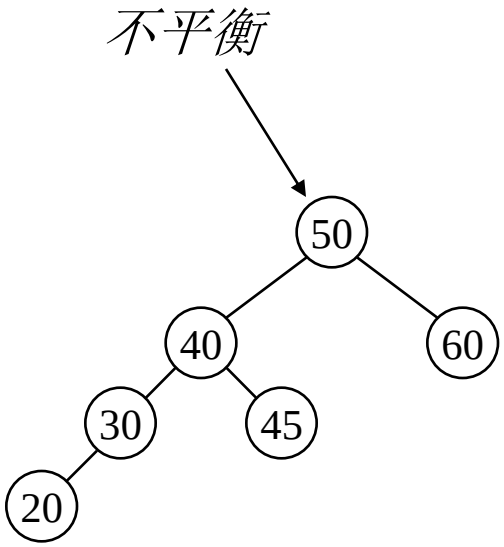


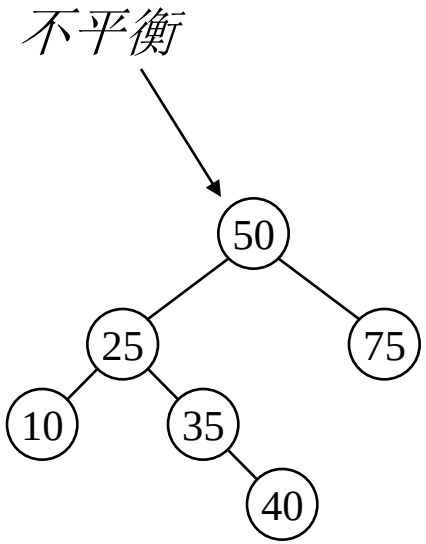




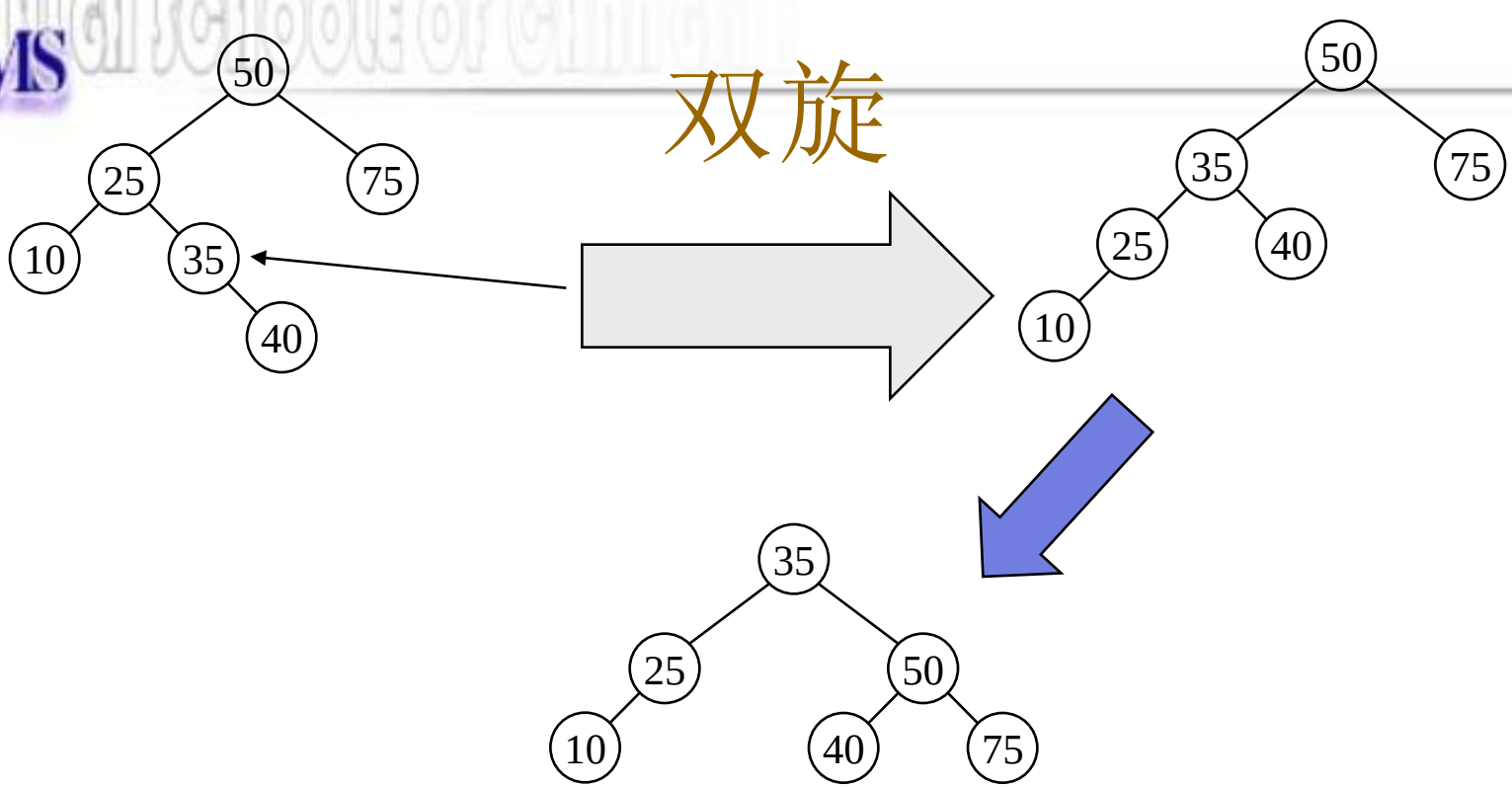
还是不平衡!!

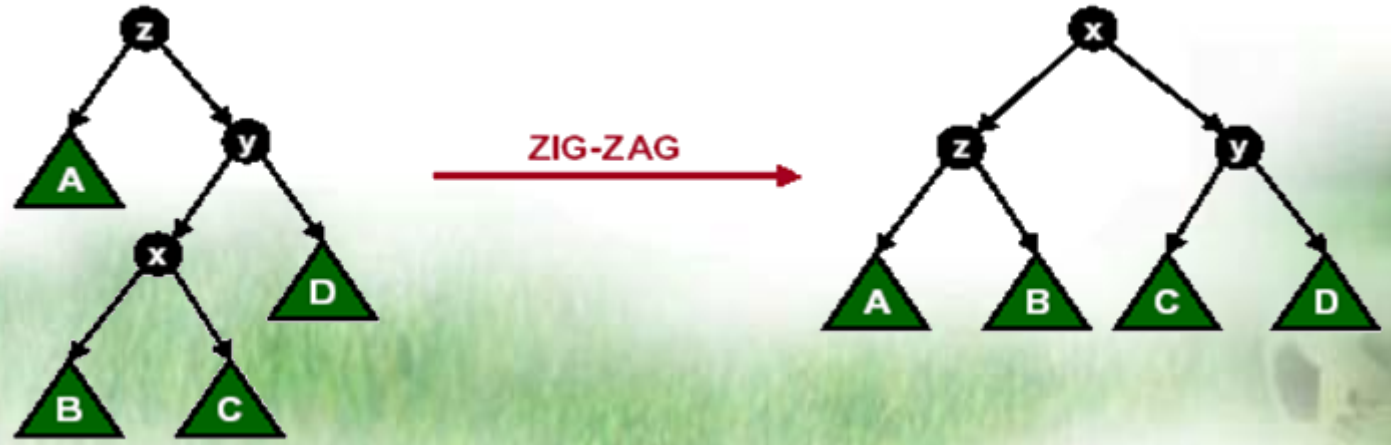
它们的差别是什么?





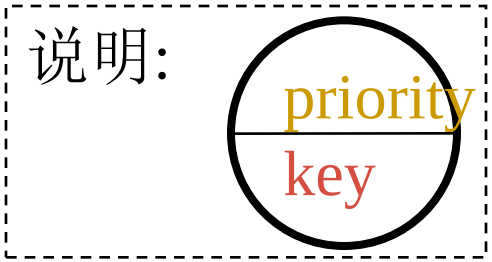
双旋

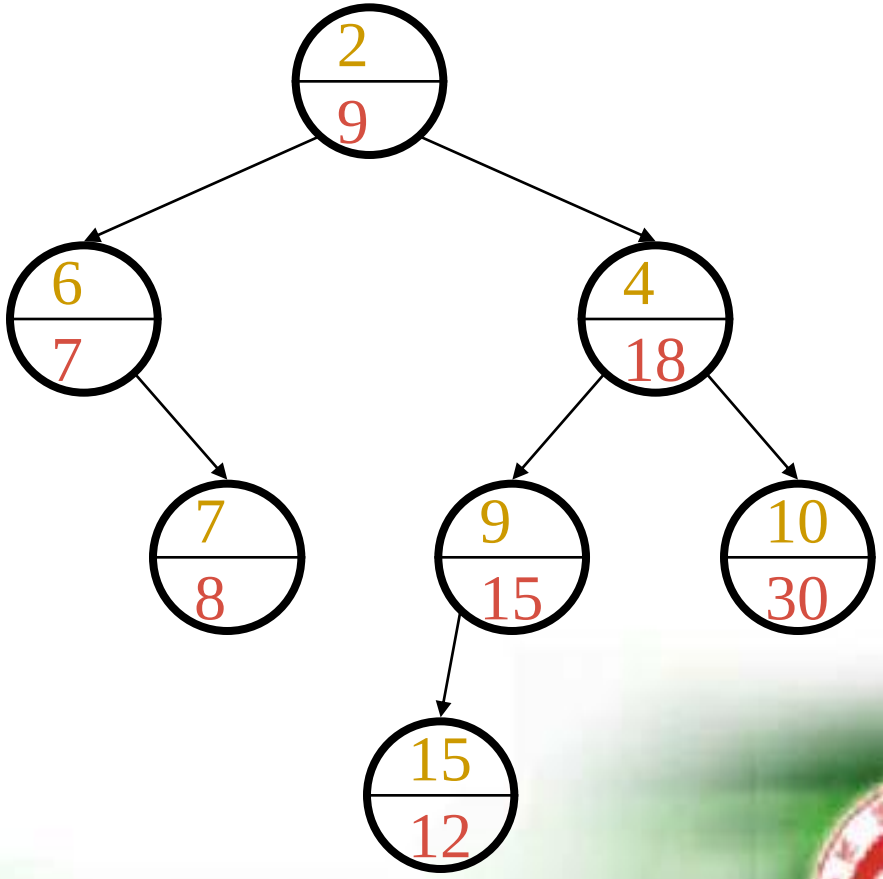

50
25
75
10
35
40
50
35
75
25
40
10
35
25
50
10
40
75


z
A
y
x
D
B
C
ZIG-ZAG
x
z
y
A
B
C
D

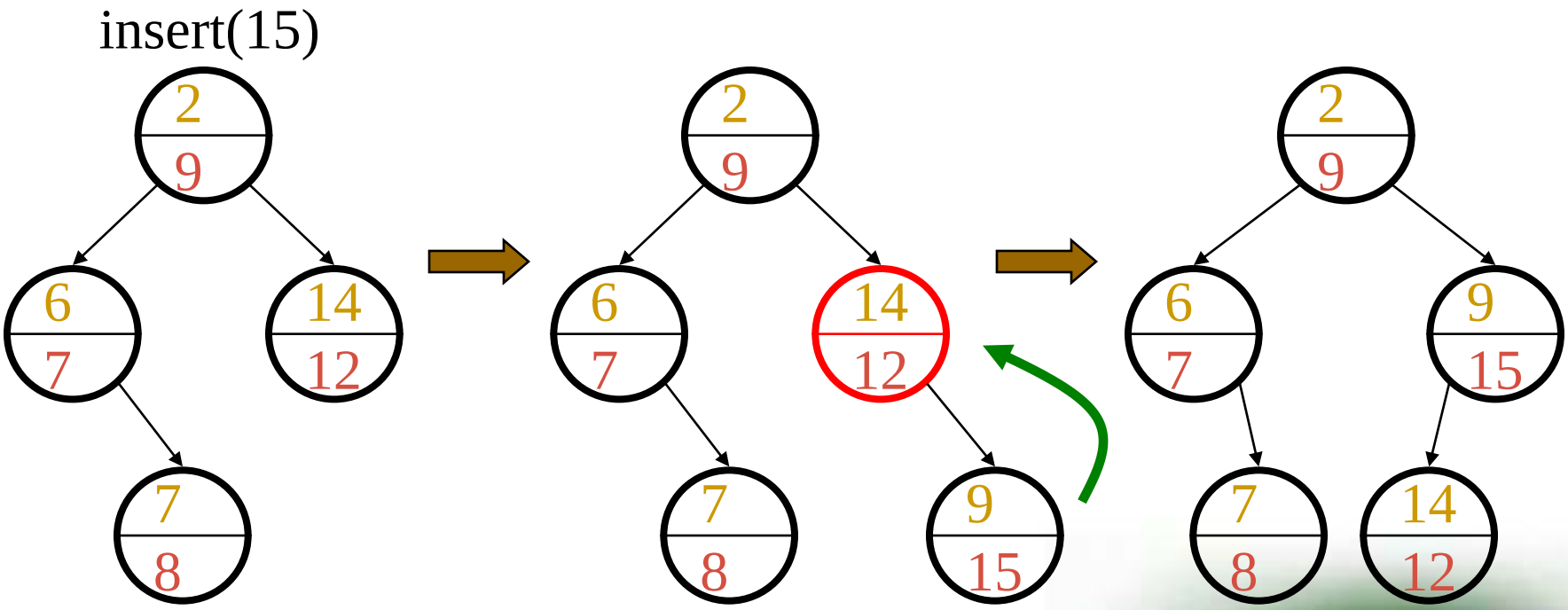

```
void rotate(int x)
{    int y=fa[x],g=fa[y],c=child[y][1]==x;
   child [y][c]=child [x][c^1];fa[child[y][c]]=y;
   child [x][c^1]=g;fa[y]=x;
   fa[x]=g;
   if(g)
        child[g][child[g][1]==y]=x ;
}
```

Treap



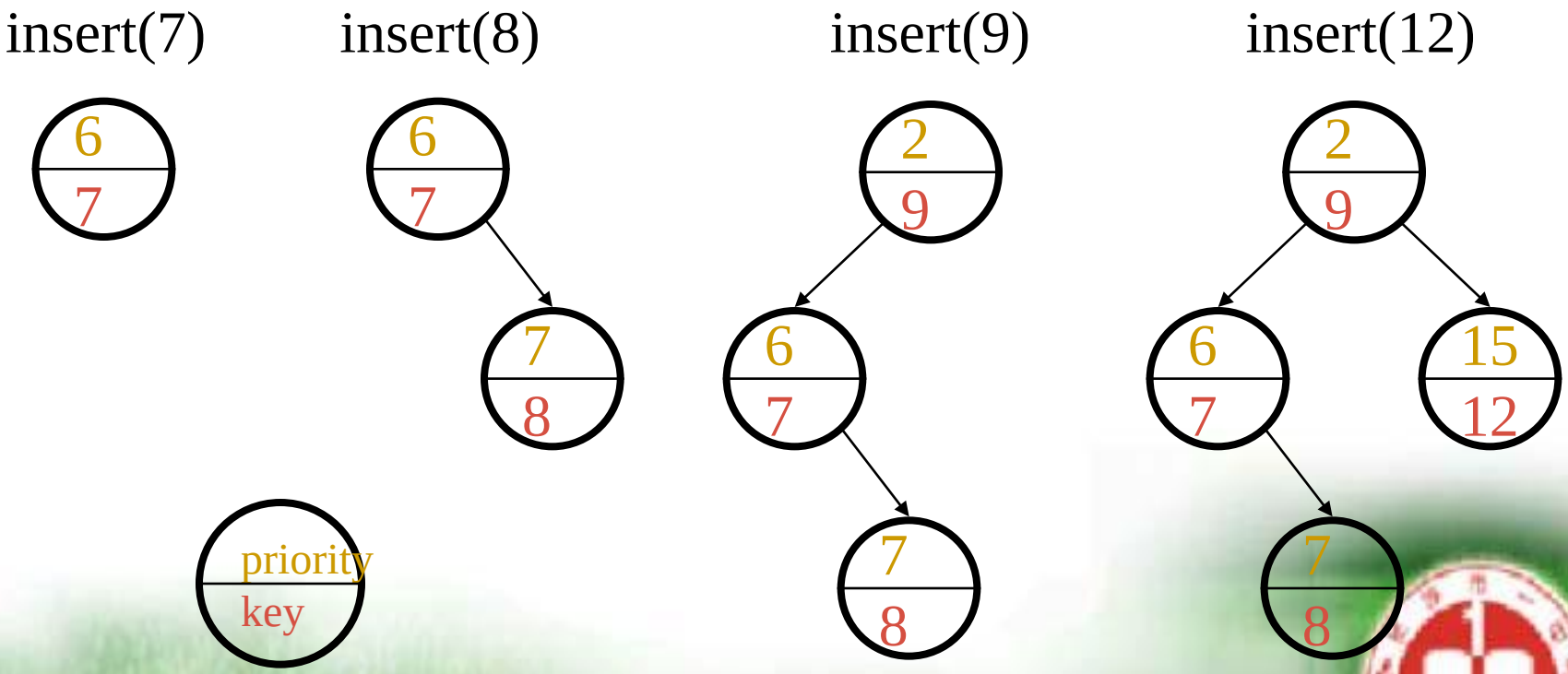


Treap Insert



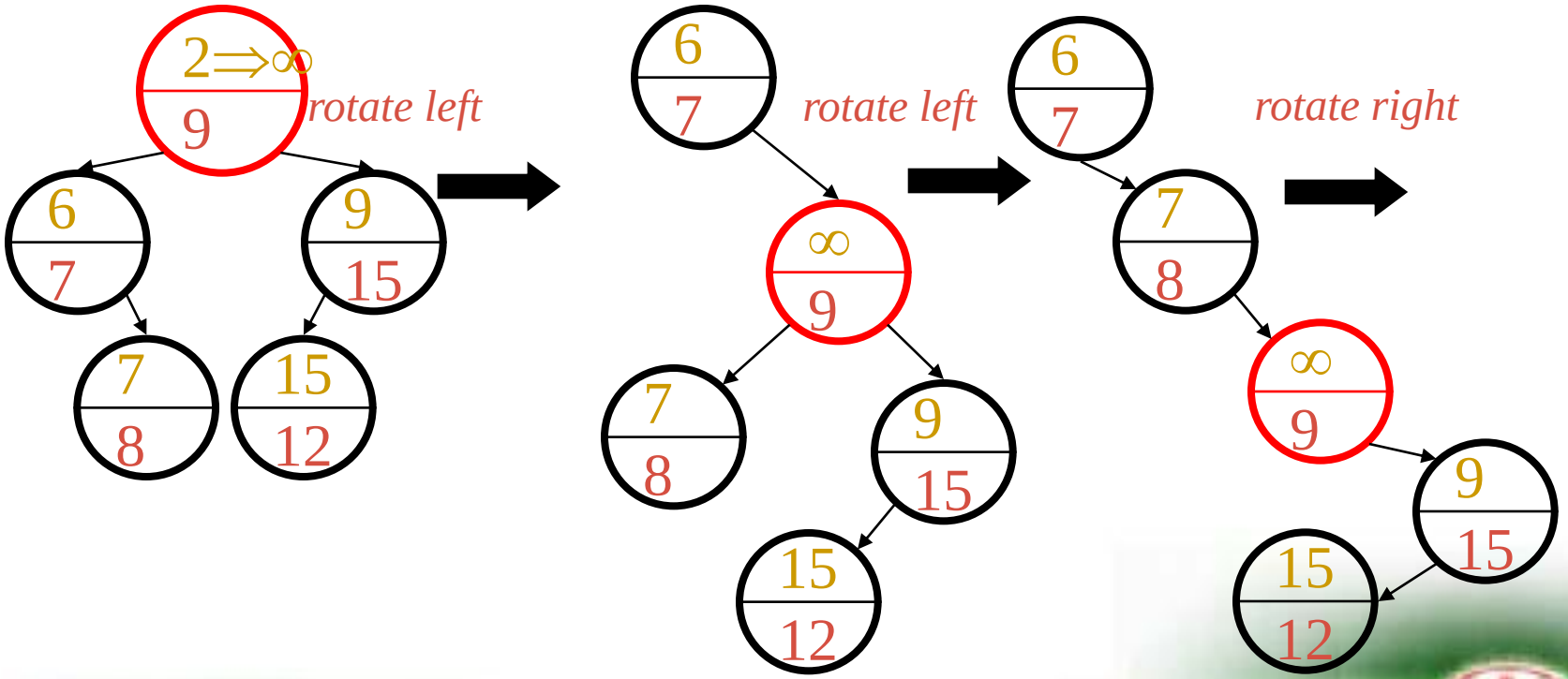
Tree + Heap

按顺序插入,结果会是什么样子?

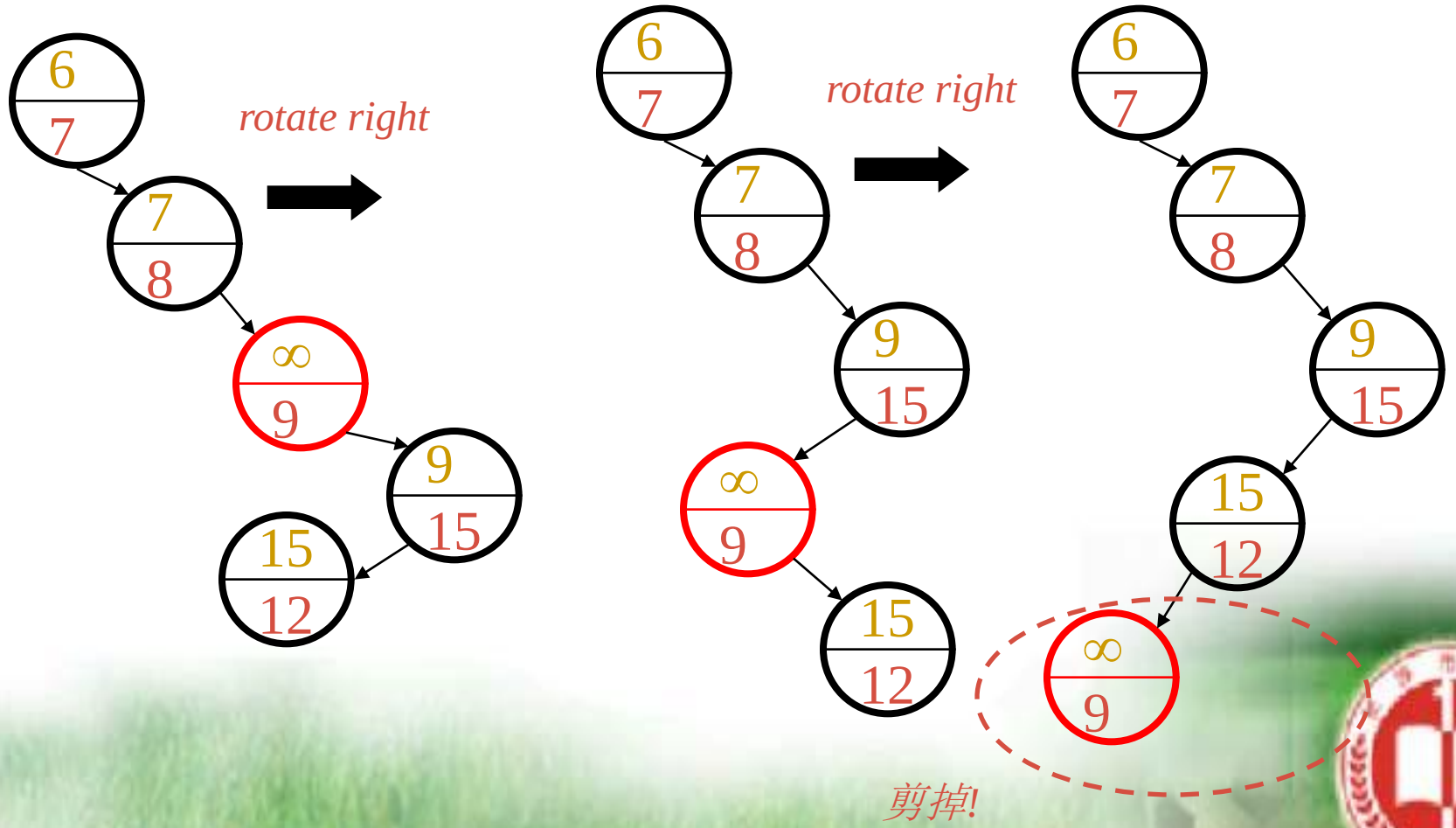


Treap Delete

delete(9)

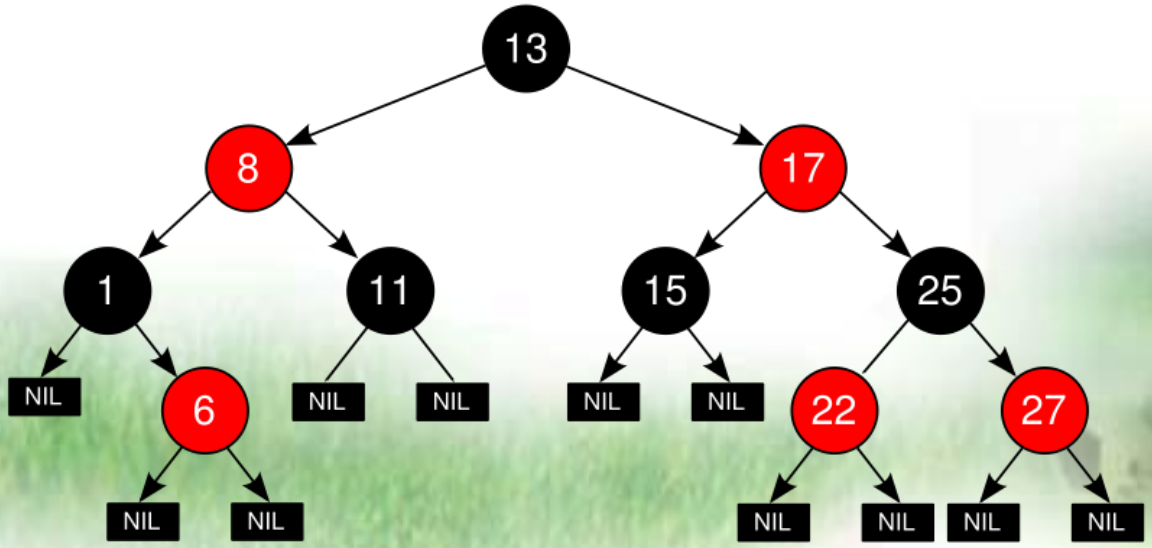


Treap Delete, cont.

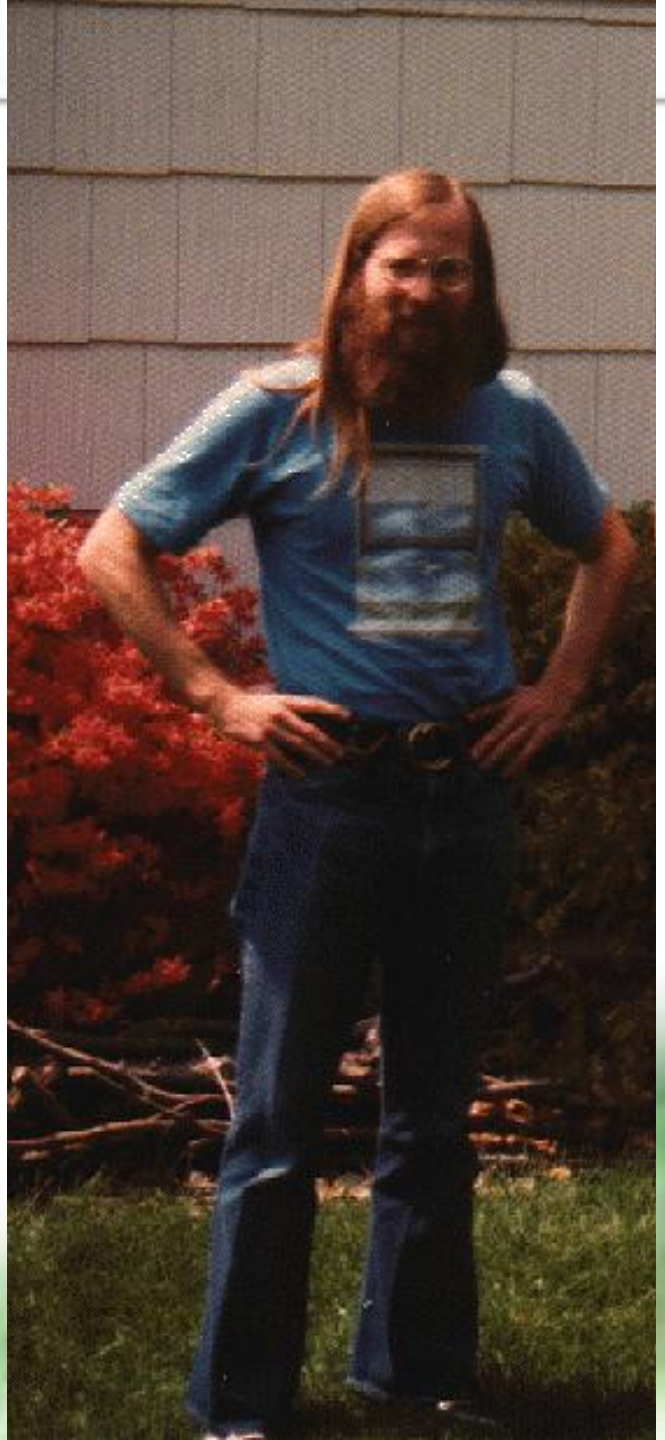


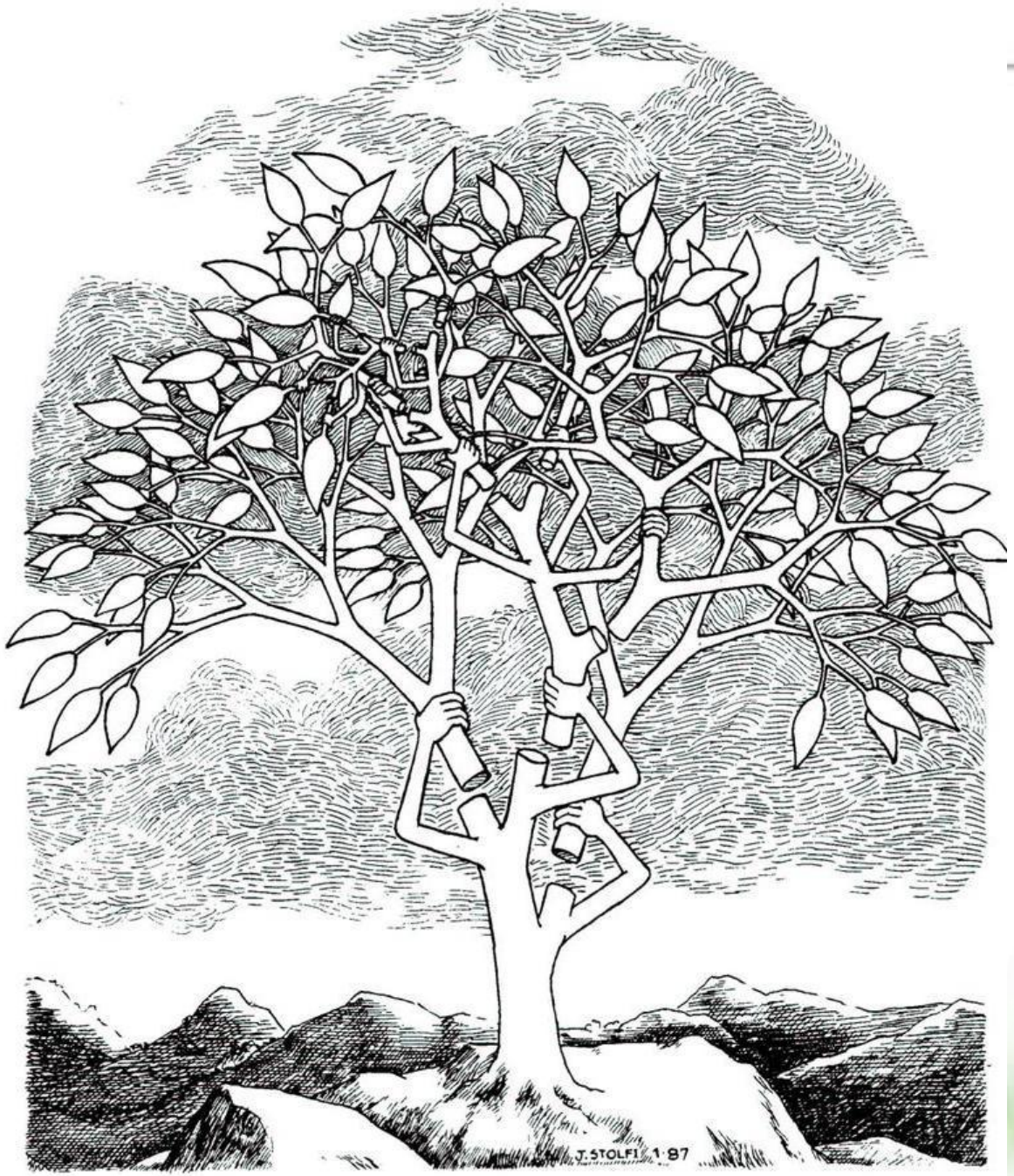
红 黑 树

性质1. 节点是红色或黑色。
性质2. 根是黑色。
性质3. 所有叶子都是黑色（包括NIL）。
性质4. 每个红色节点的两个子节点都是黑色。
性质5. 从任一节点到其每个叶子的所有路径都包含相同数目的黑色节点。



Sleator and *Tarjan* (1985)



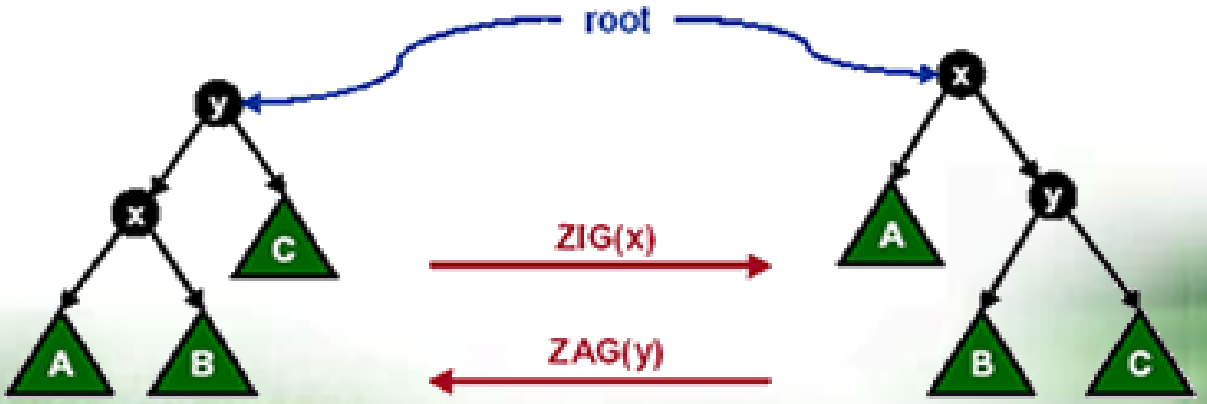

J. STOLFI 1·87

Self-Organization in Biological Systems

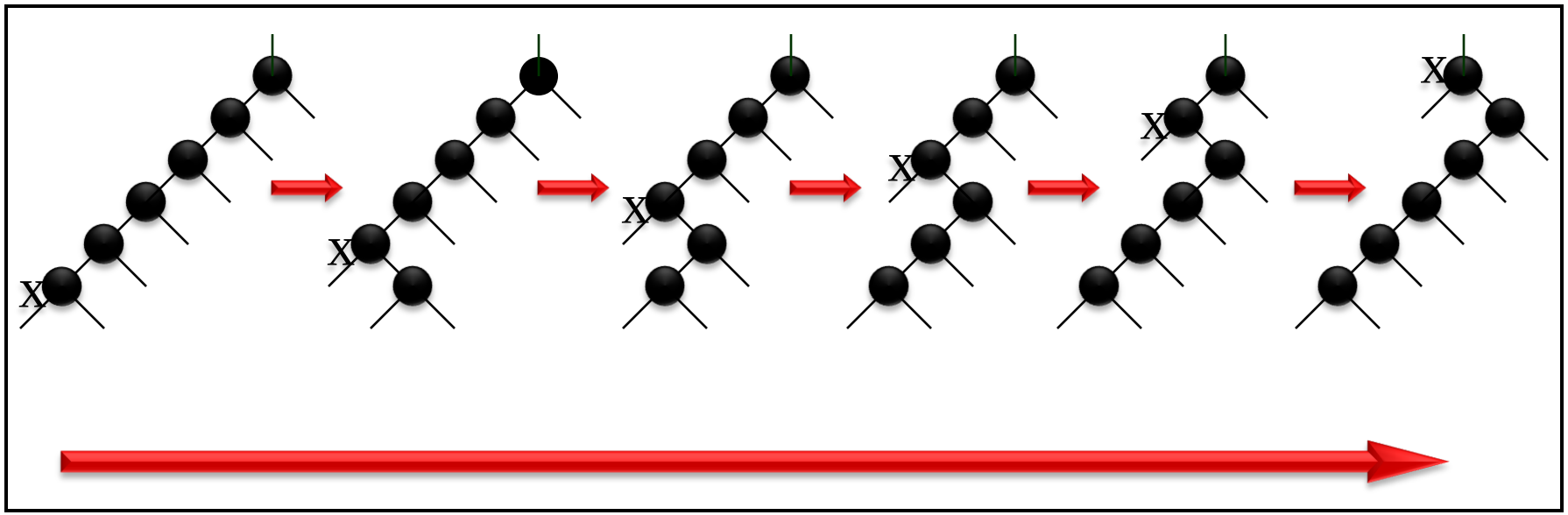
Scott Camazine Jean-Louis Deneubourg Nigel R. Franks
James Sneyd Guy Theraulaz Eric Bonabeau

PRINCETON STUDIES IN COMPLEXITY

单旋操作

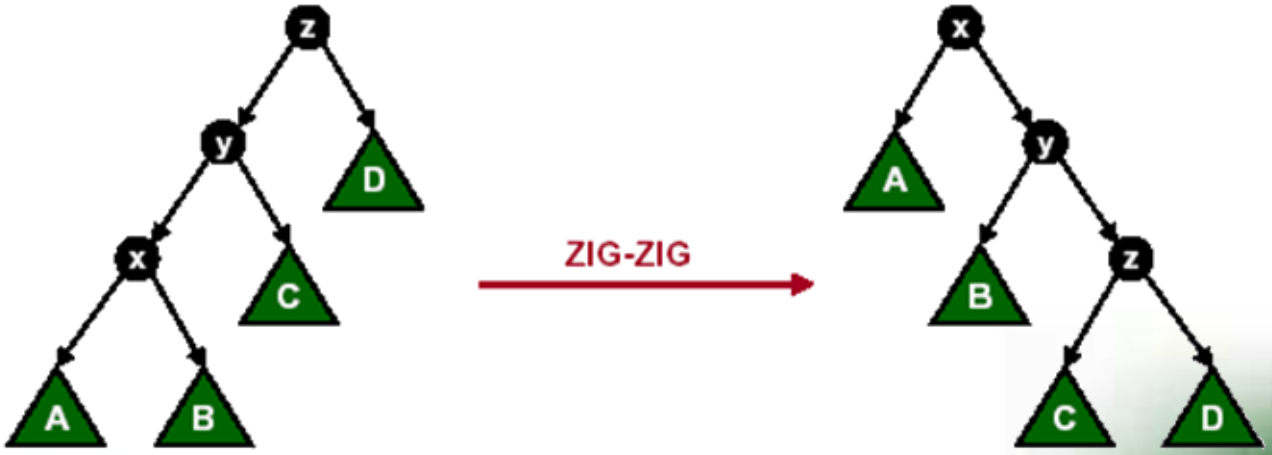


naivesplay

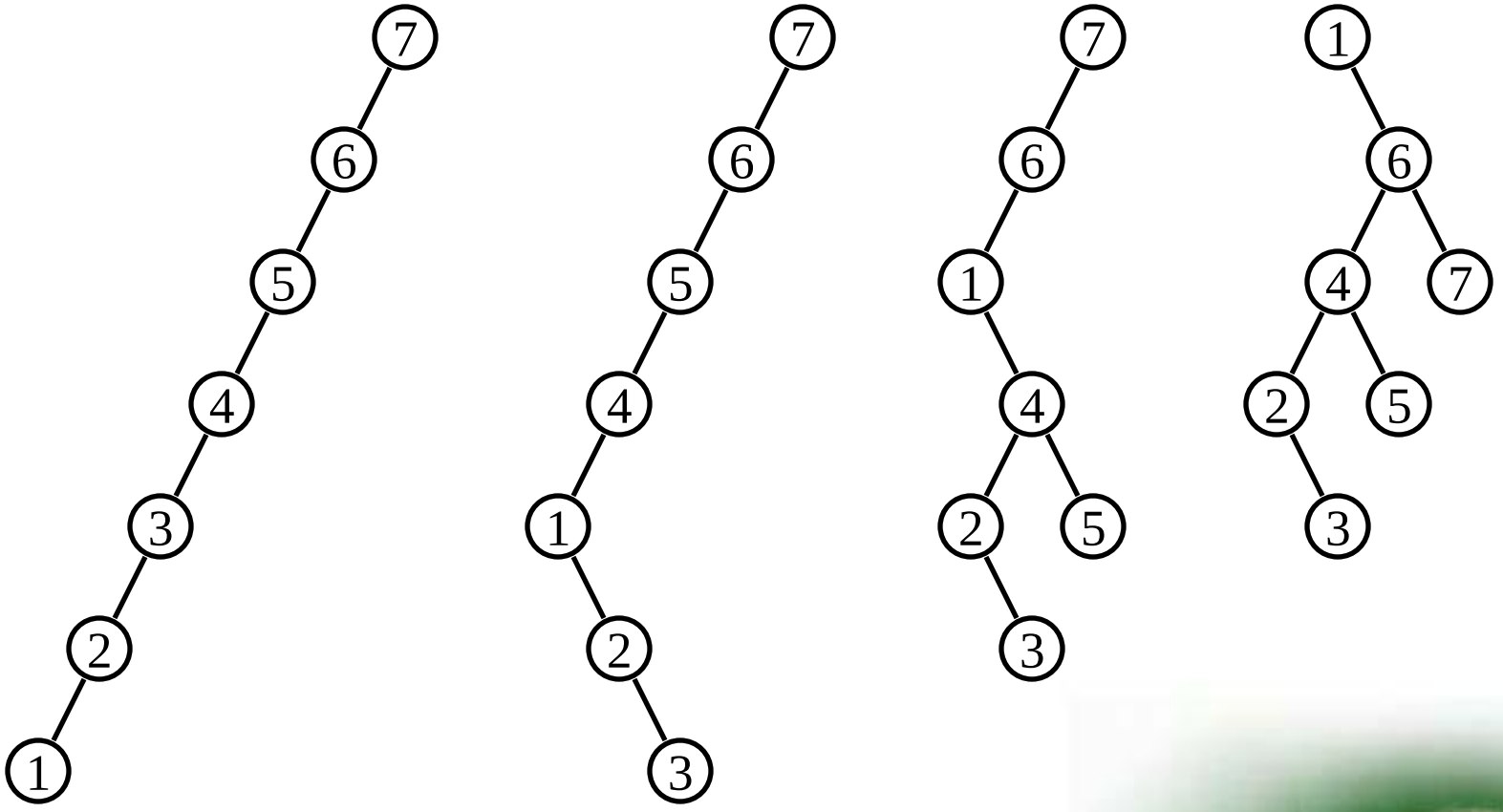



```
void naivesplay(int x)
{    for(int y;y=fa[x];rotate(x));
      root=x;
}
```

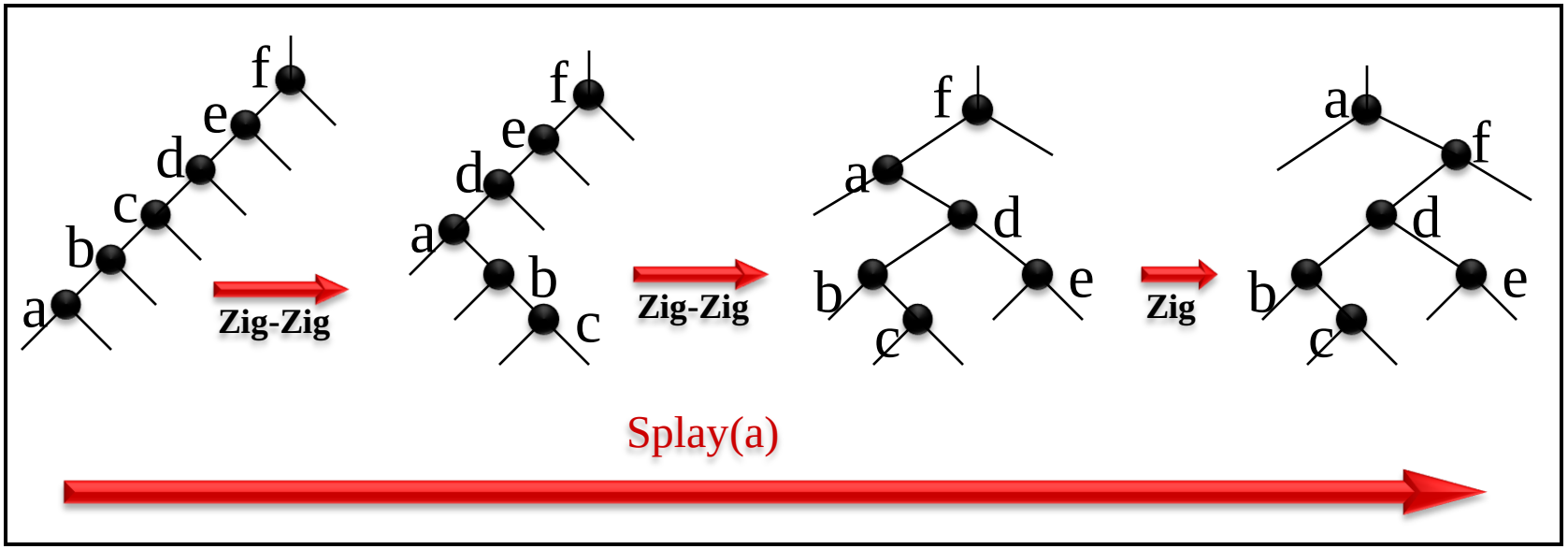
双旋操作



Insert: 1, 2, 3, 4, 5, 6, 7 Find: 1



splay

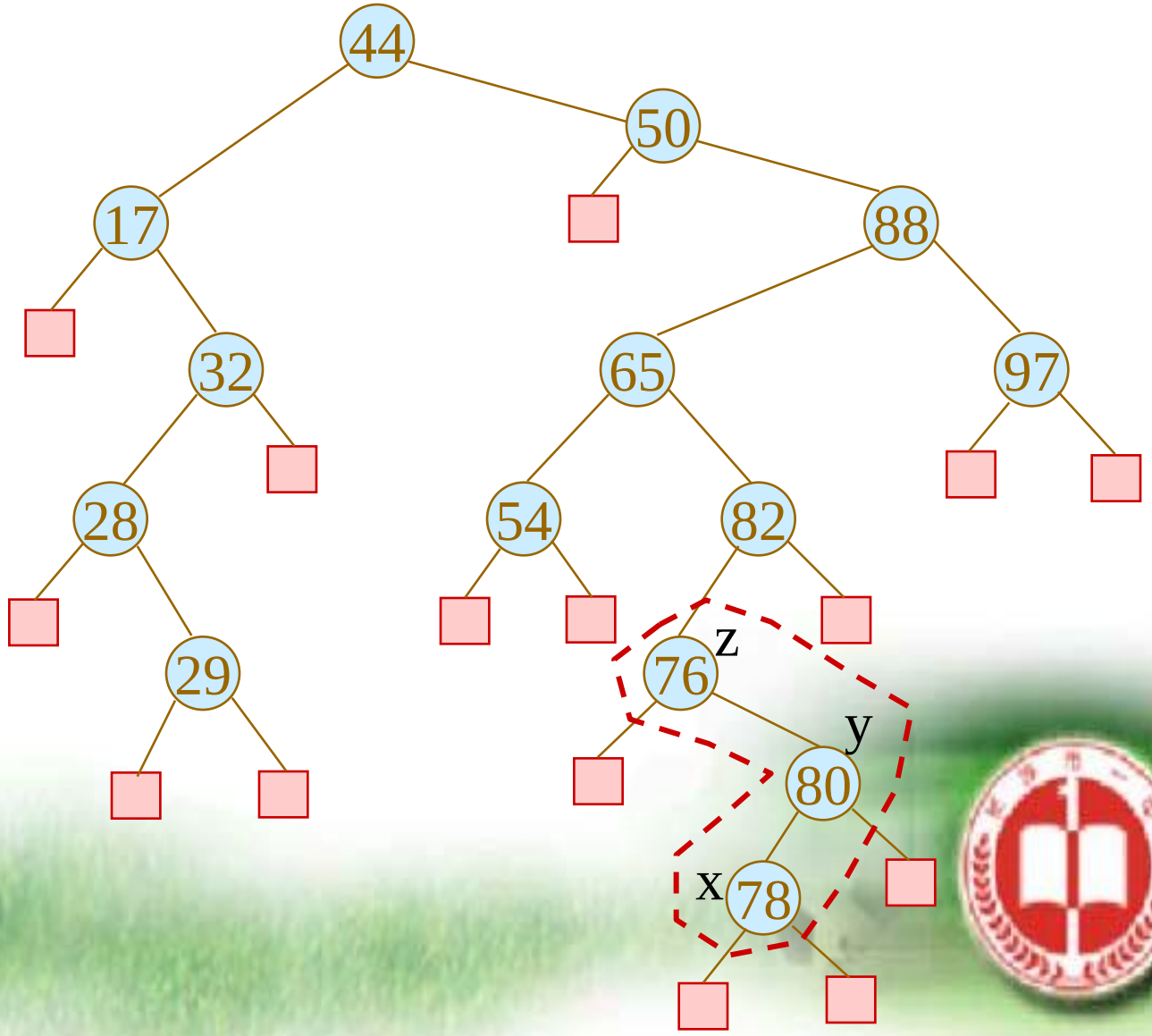



```
void splay(int x)
{  for(int y;y=fa[x];rotate(x))
       if(fa[y])
          rotate((x==child[y][1]))==(y==child[fa[y]][1]))?y:x);
   root=x;
}
```

Complete Example

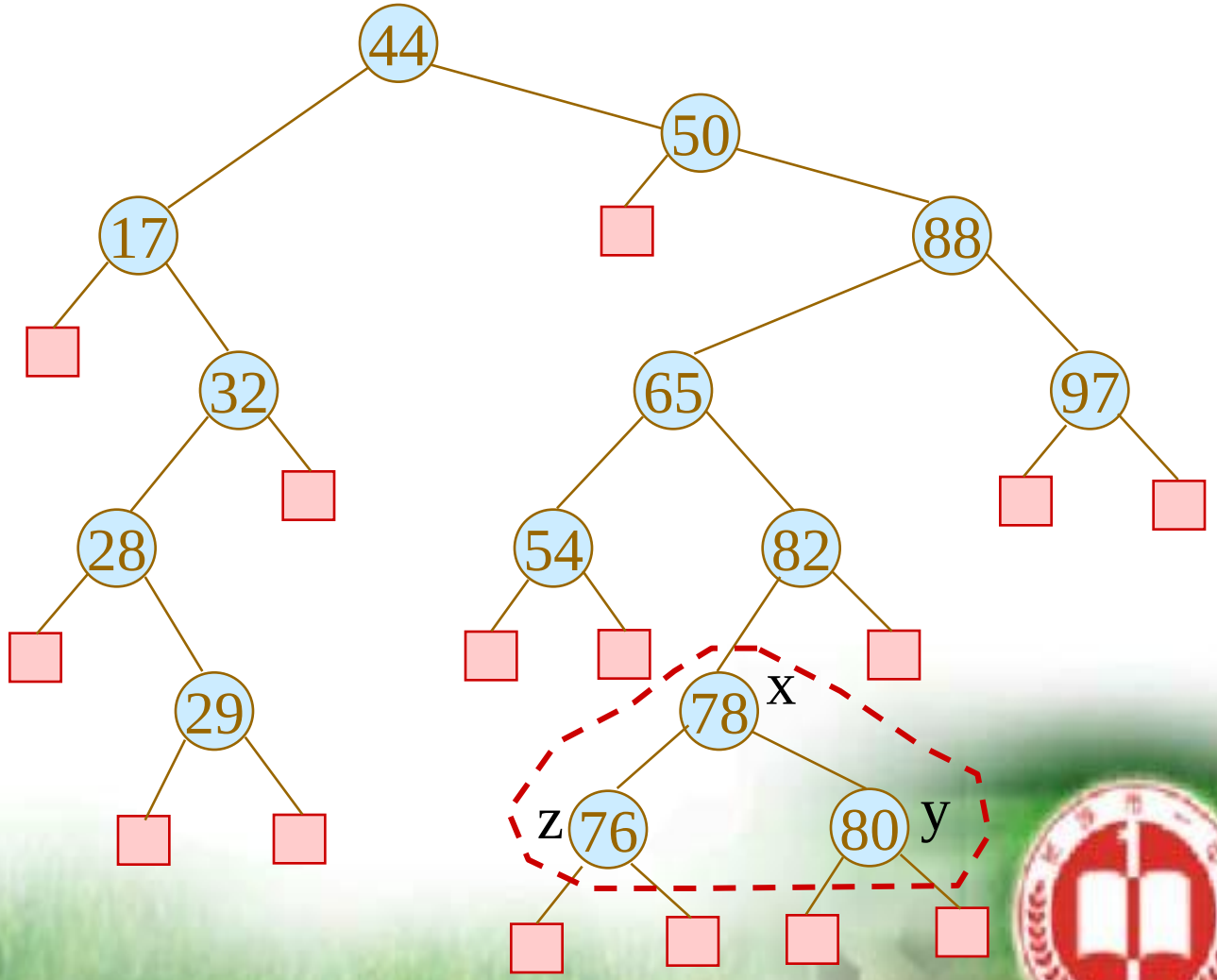
Splay(78)

zig-zag



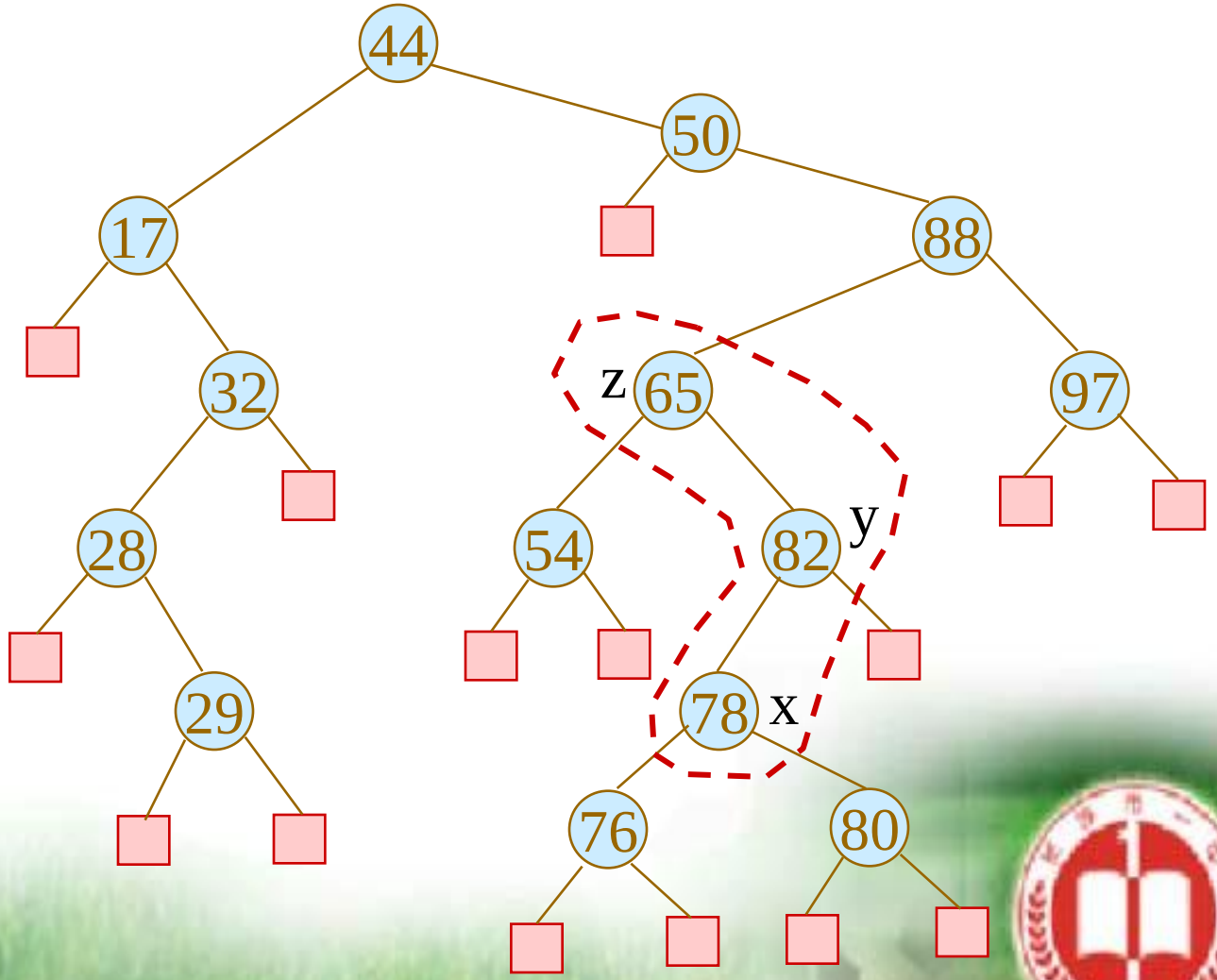
Splay(78)

zig-zag



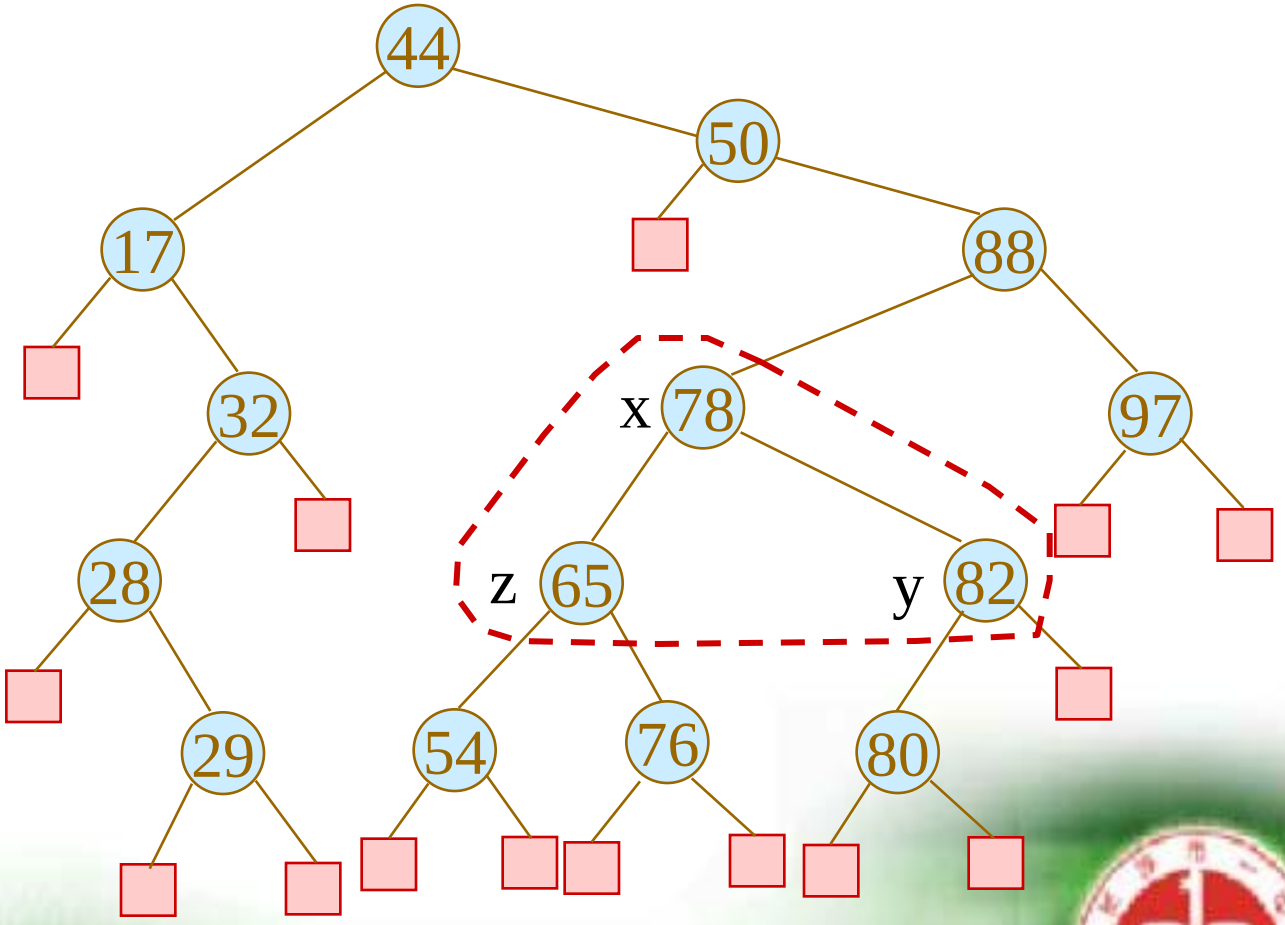
Splay(78)

zig-zag



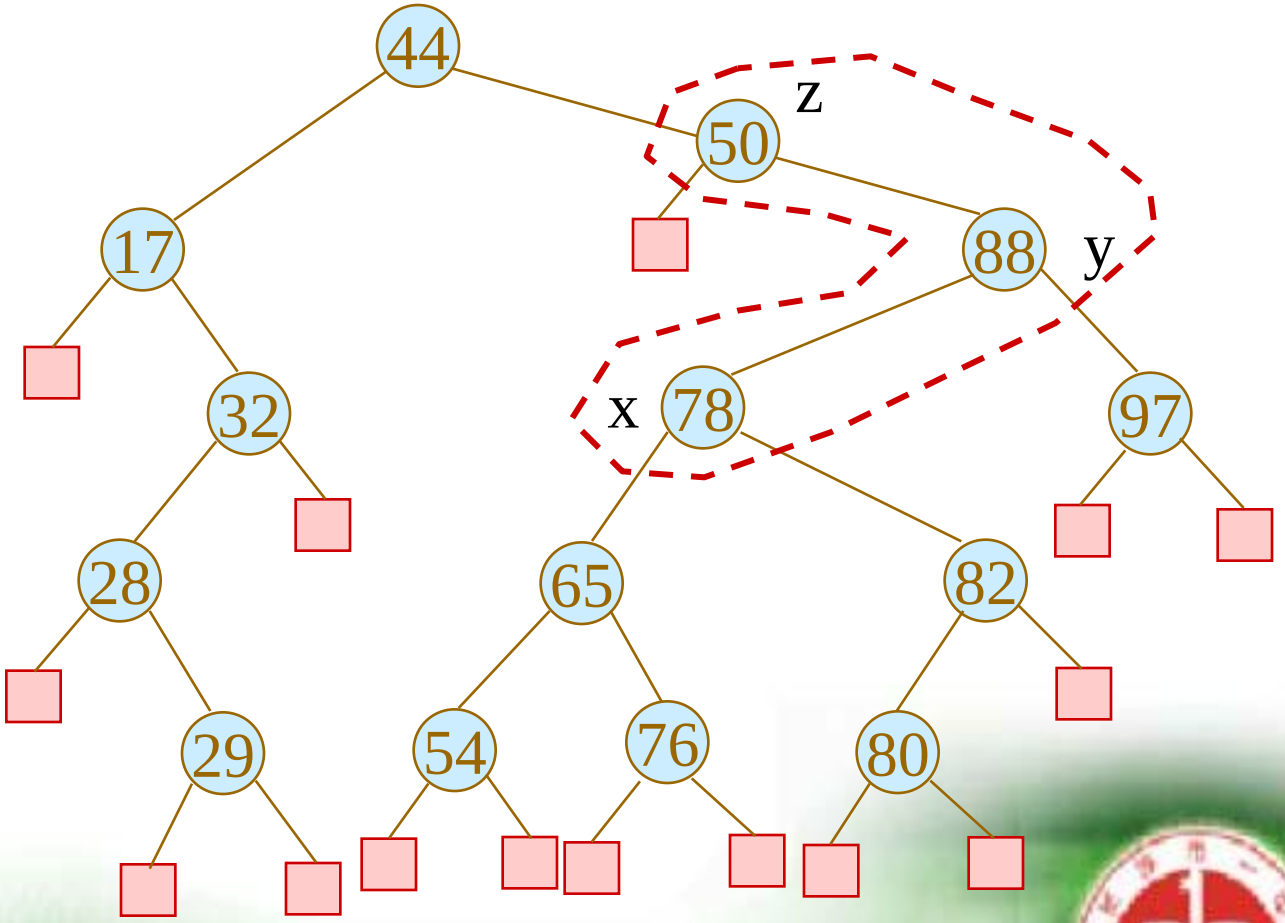
Splay(78)

zig-zag



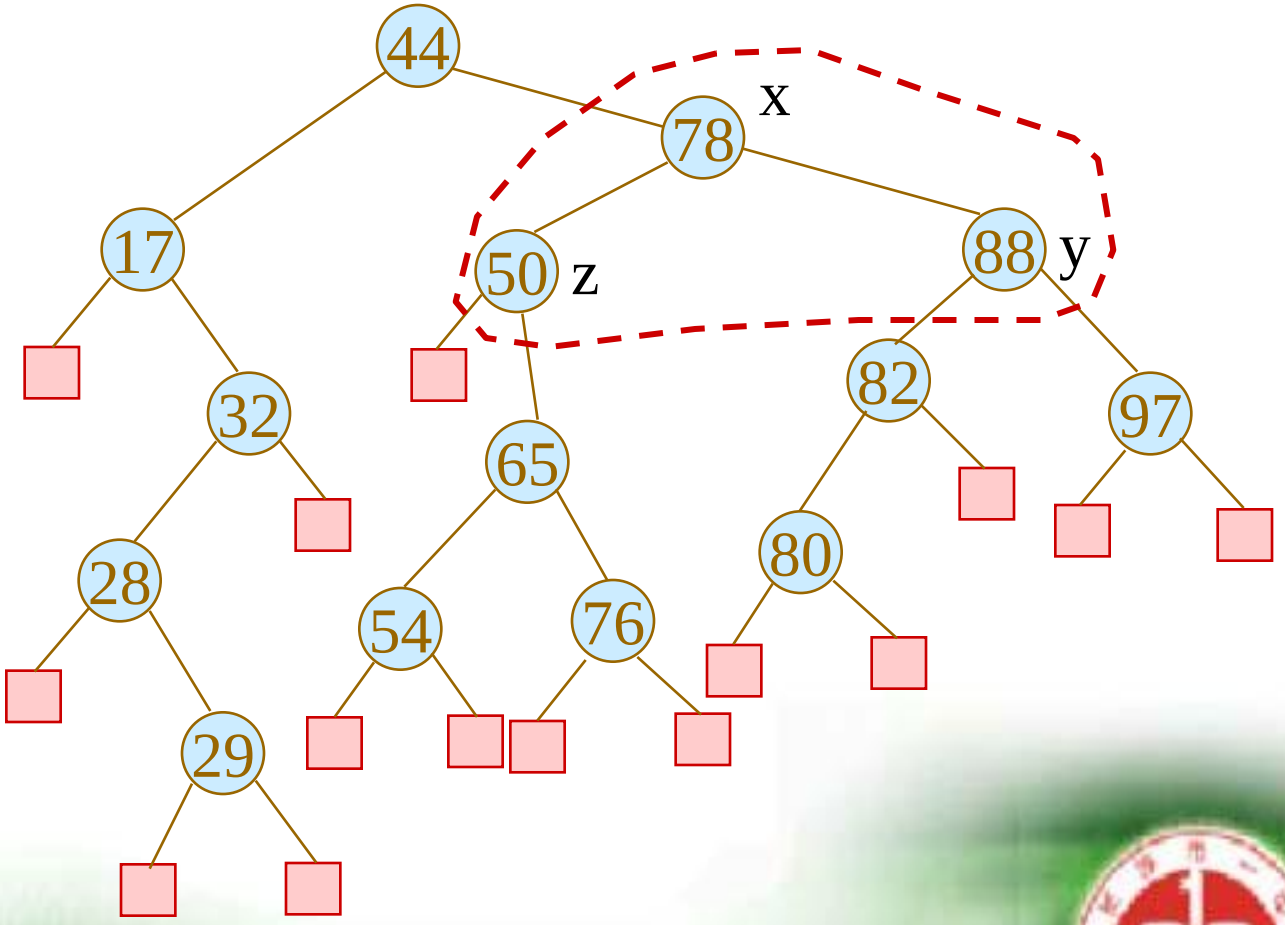
Splay(78)

zig-zag



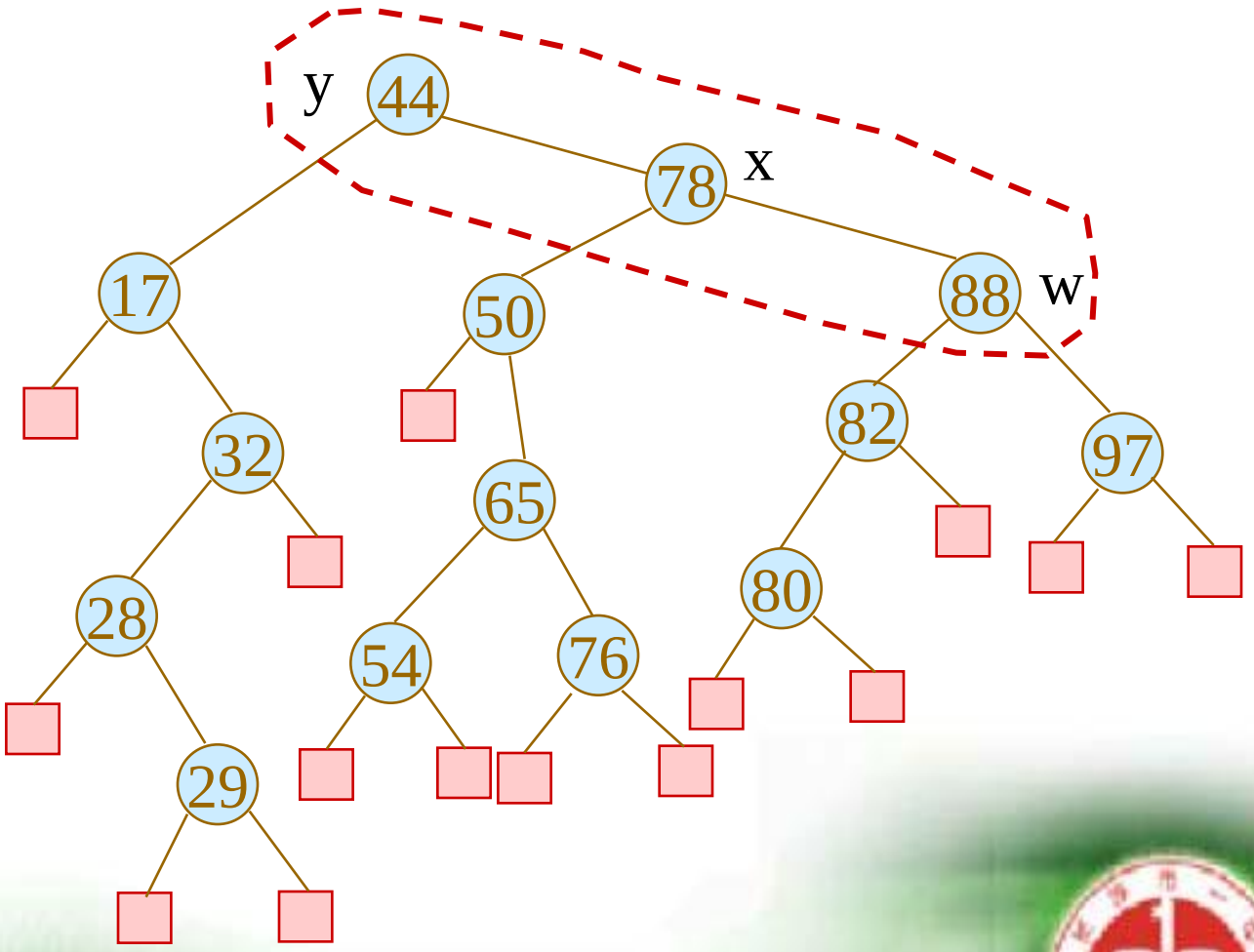
Splay(78)

zig-zag



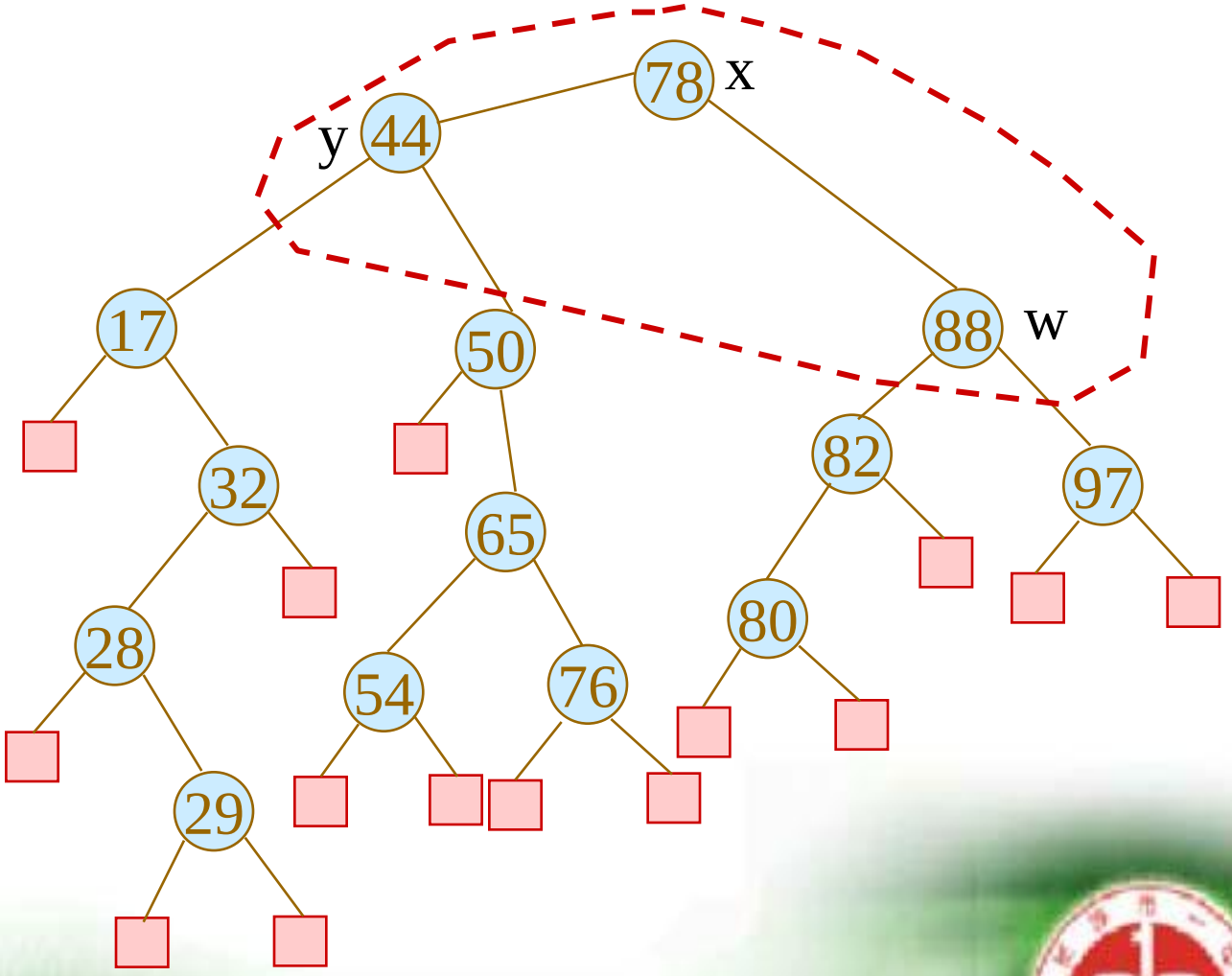
Splay(78)

zig



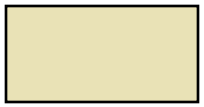
Splay(78)

zig

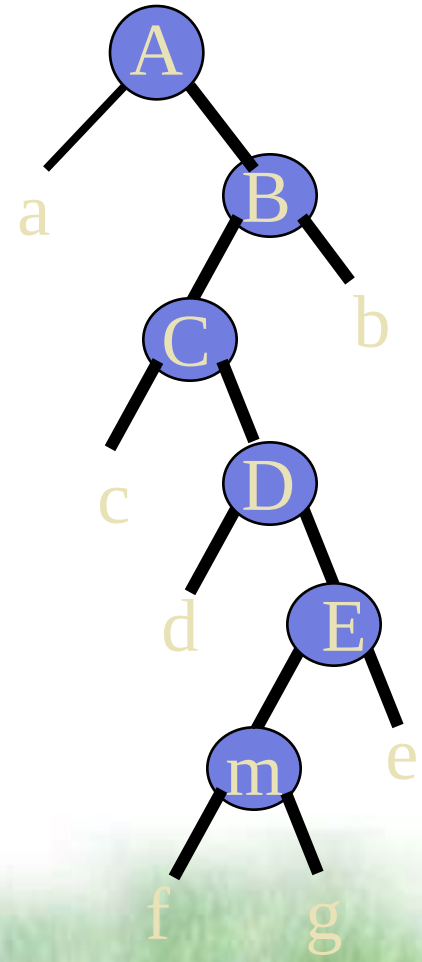


Split A Binary Search Tree

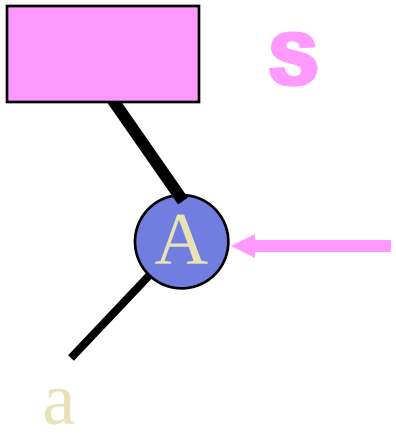
S

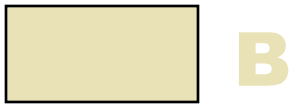


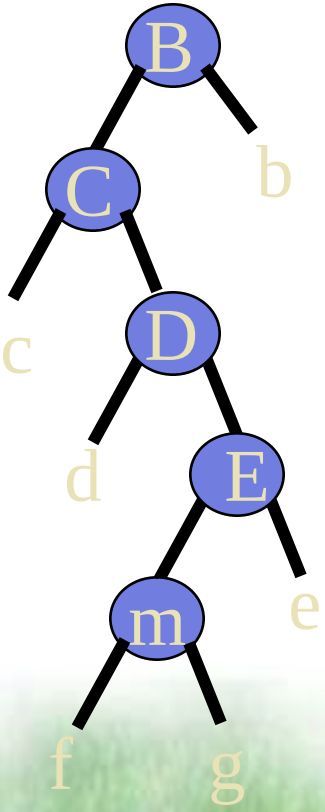
B



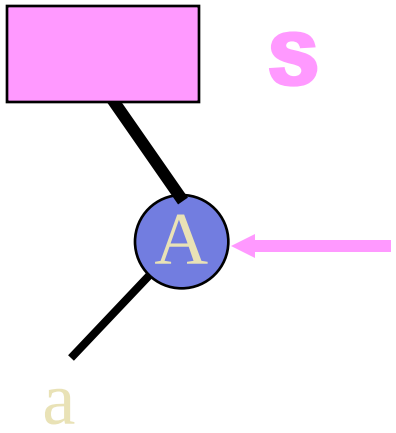
Split A Binary Search Tree

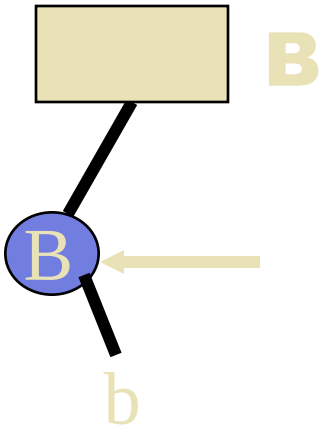


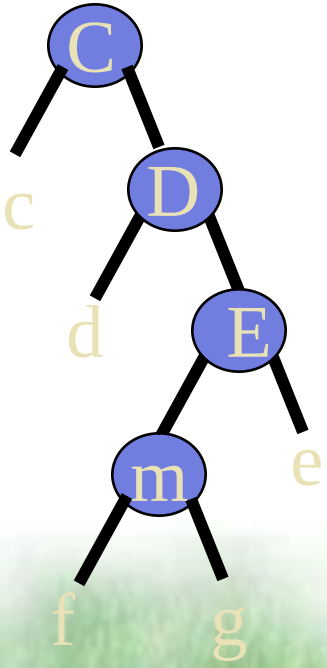




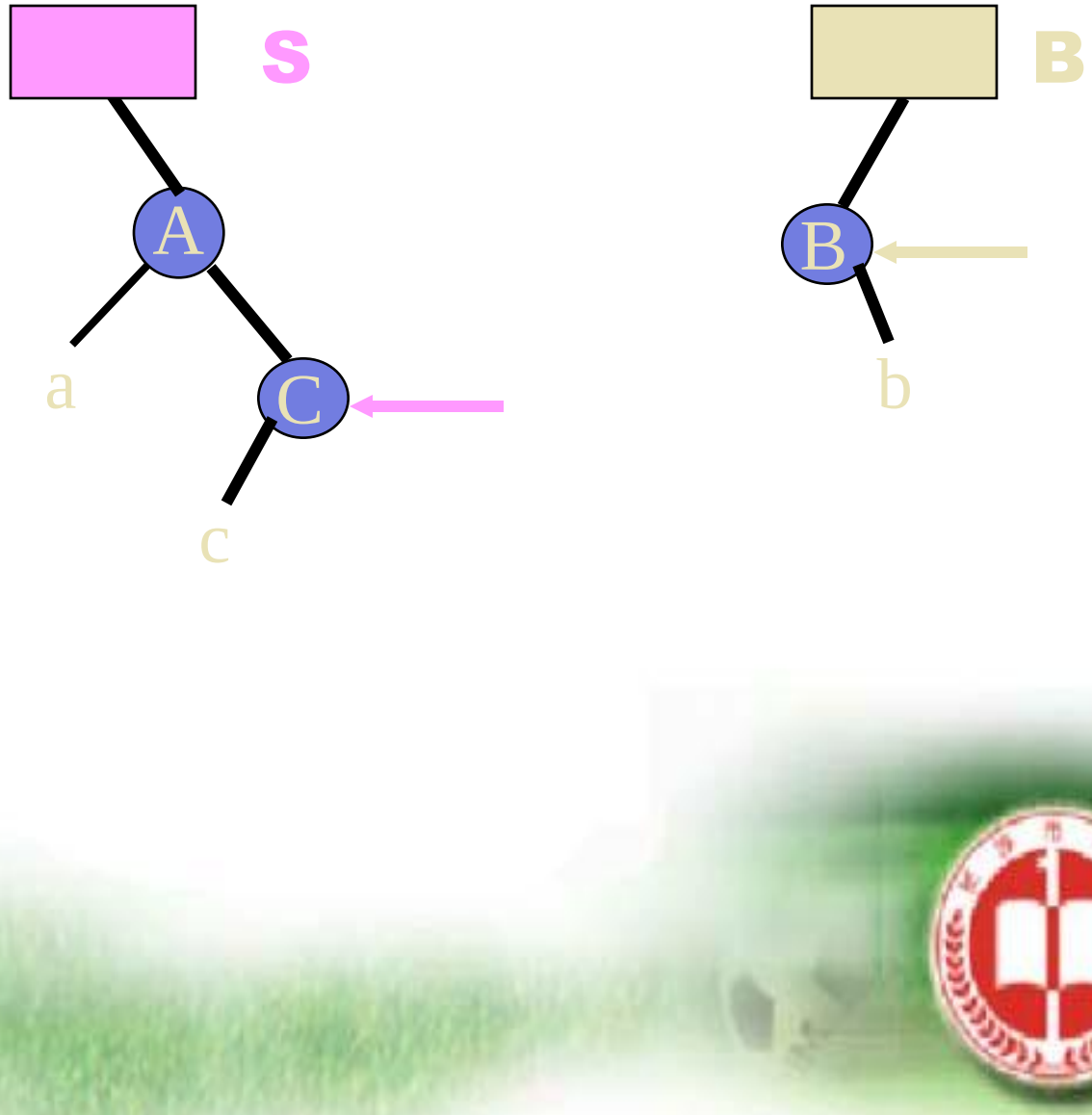
Split A Binary Search Tree



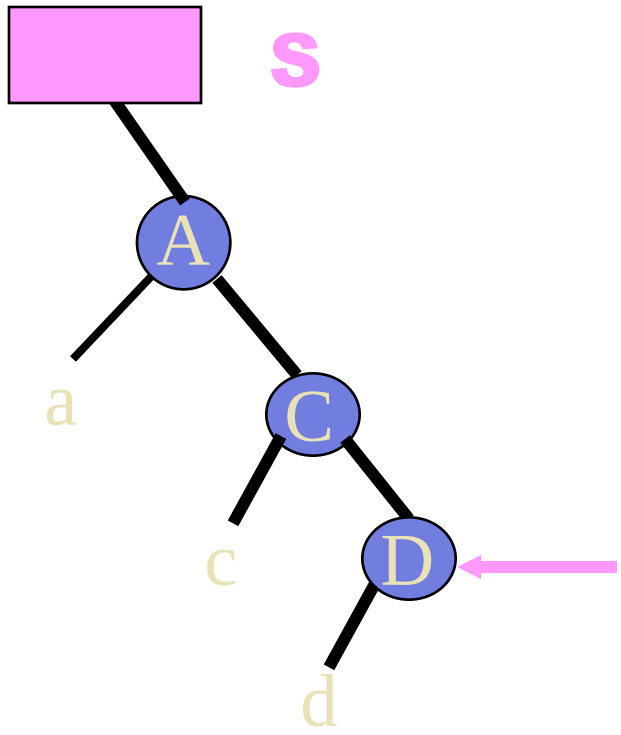


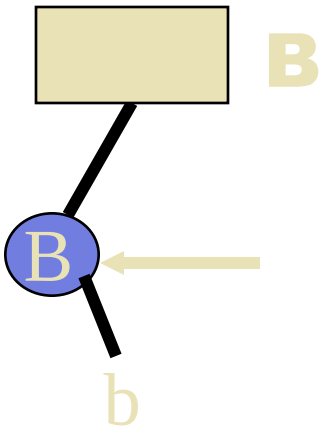


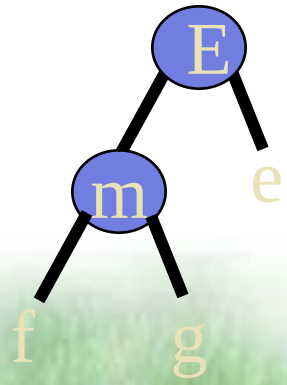
Split A Binary Search Tree



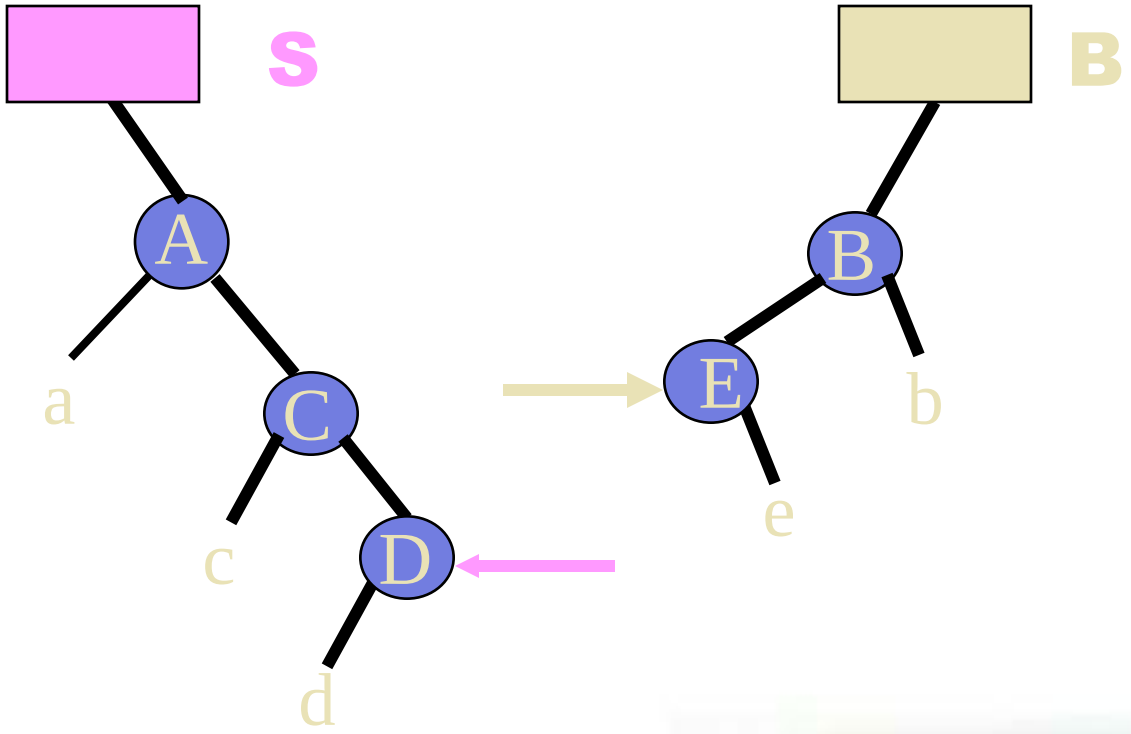
Split A Binary Search Tree

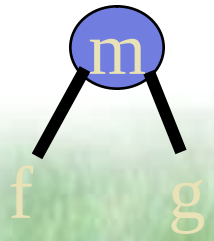




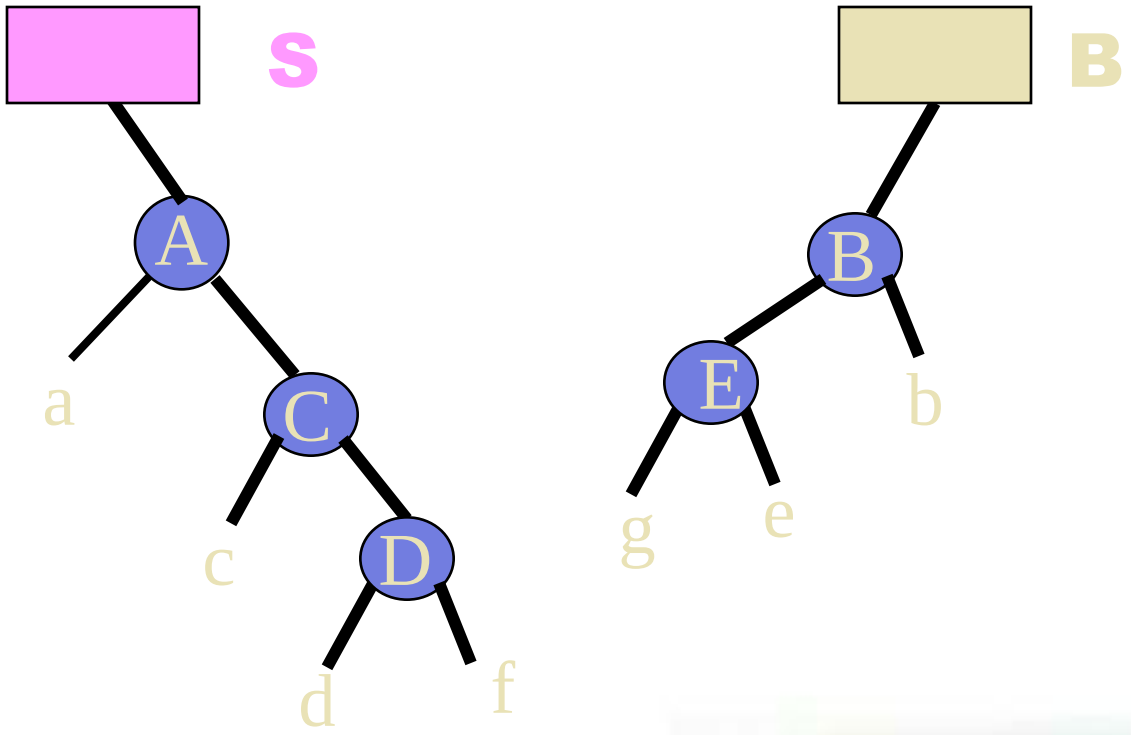


Split A Binary Search Tree





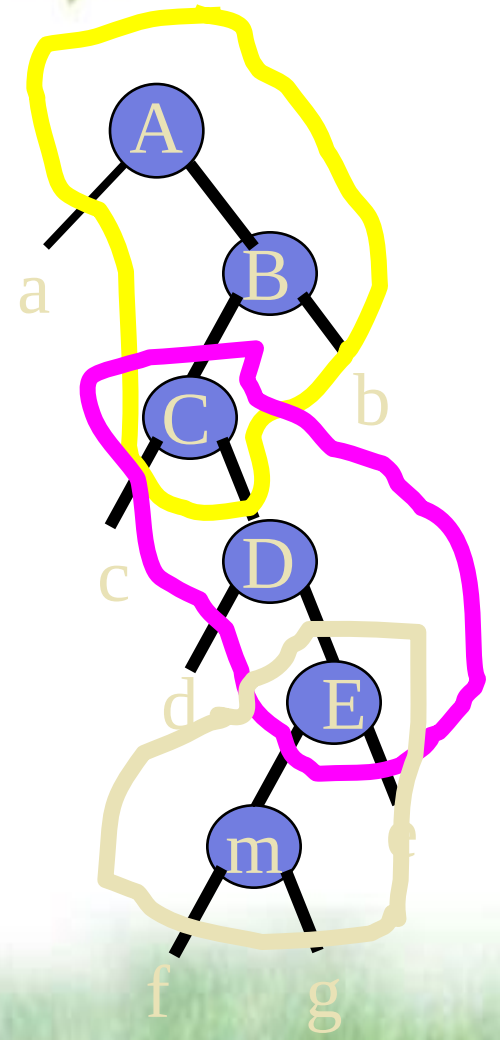
Split A Binary Search Tree



m

- Let m be the splay node.

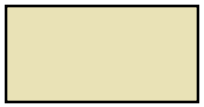
Two-Level Moves



- Let m be the splay node.
  - RL move from A to C.
  - RR move from C to E.
  - L move from E to m.

RL Move

S



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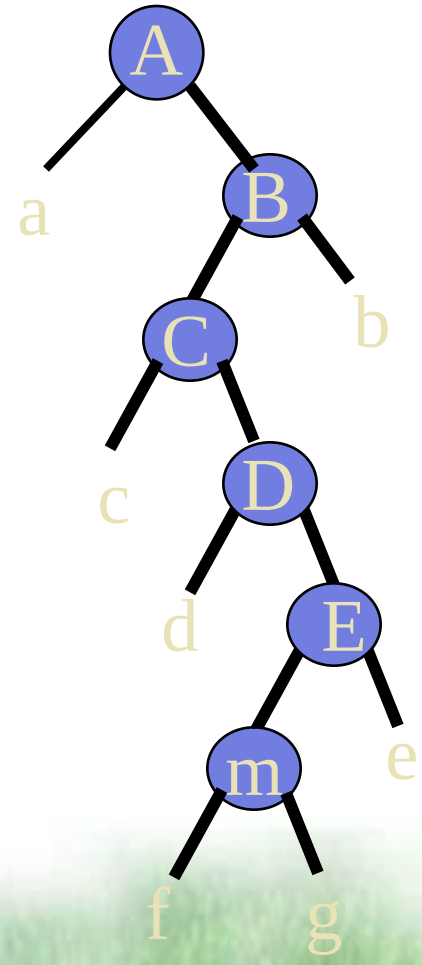
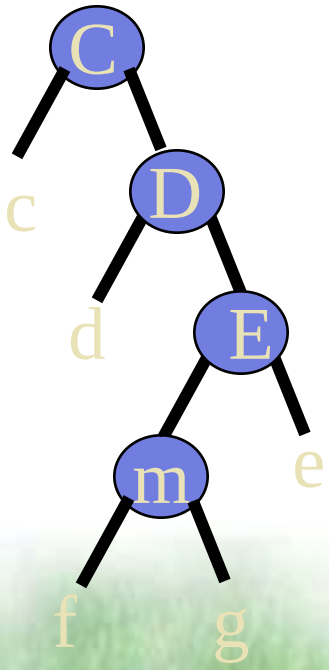


RL Move



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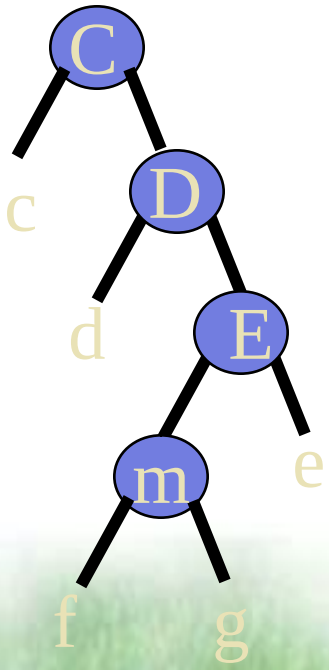
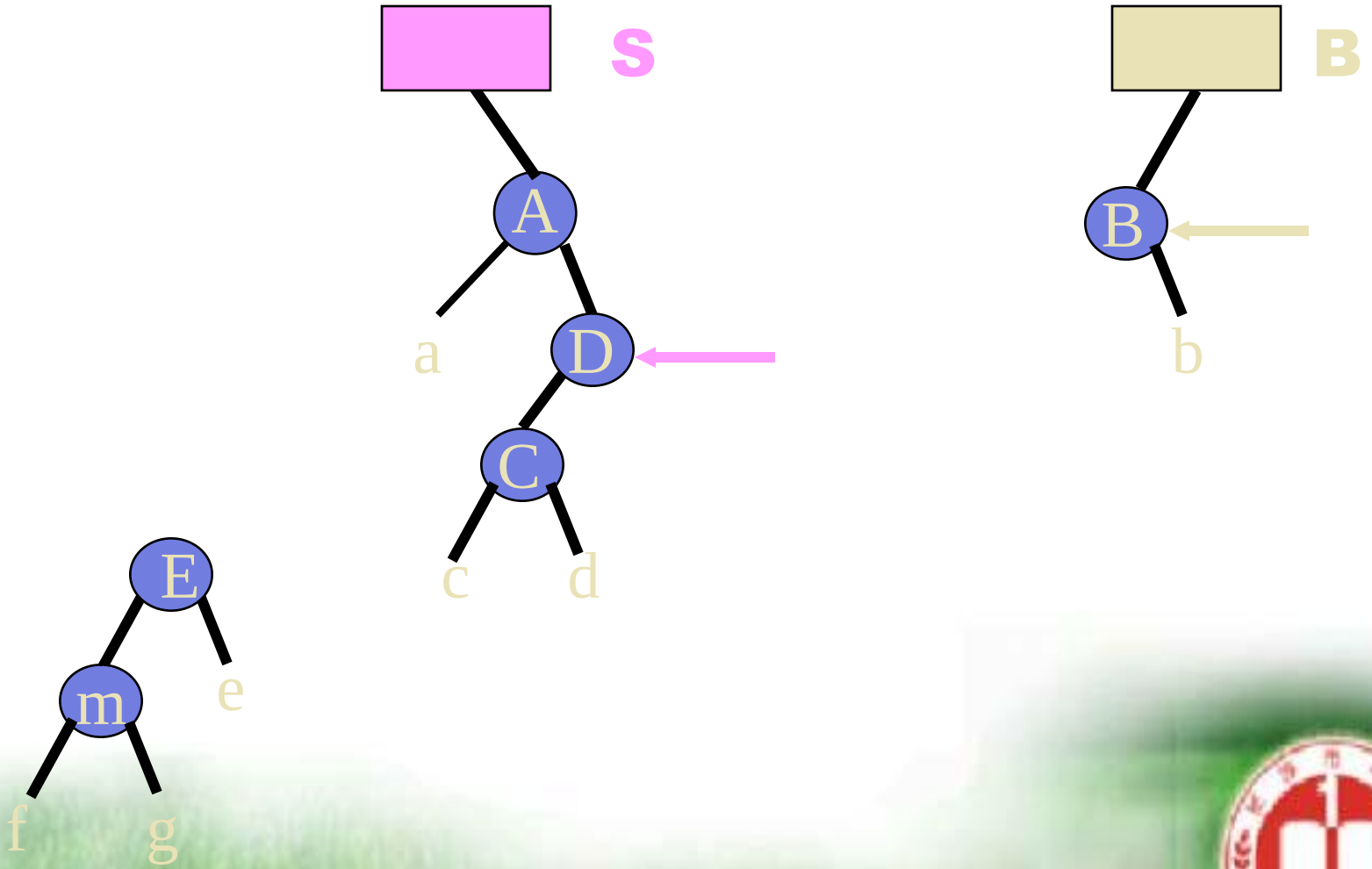
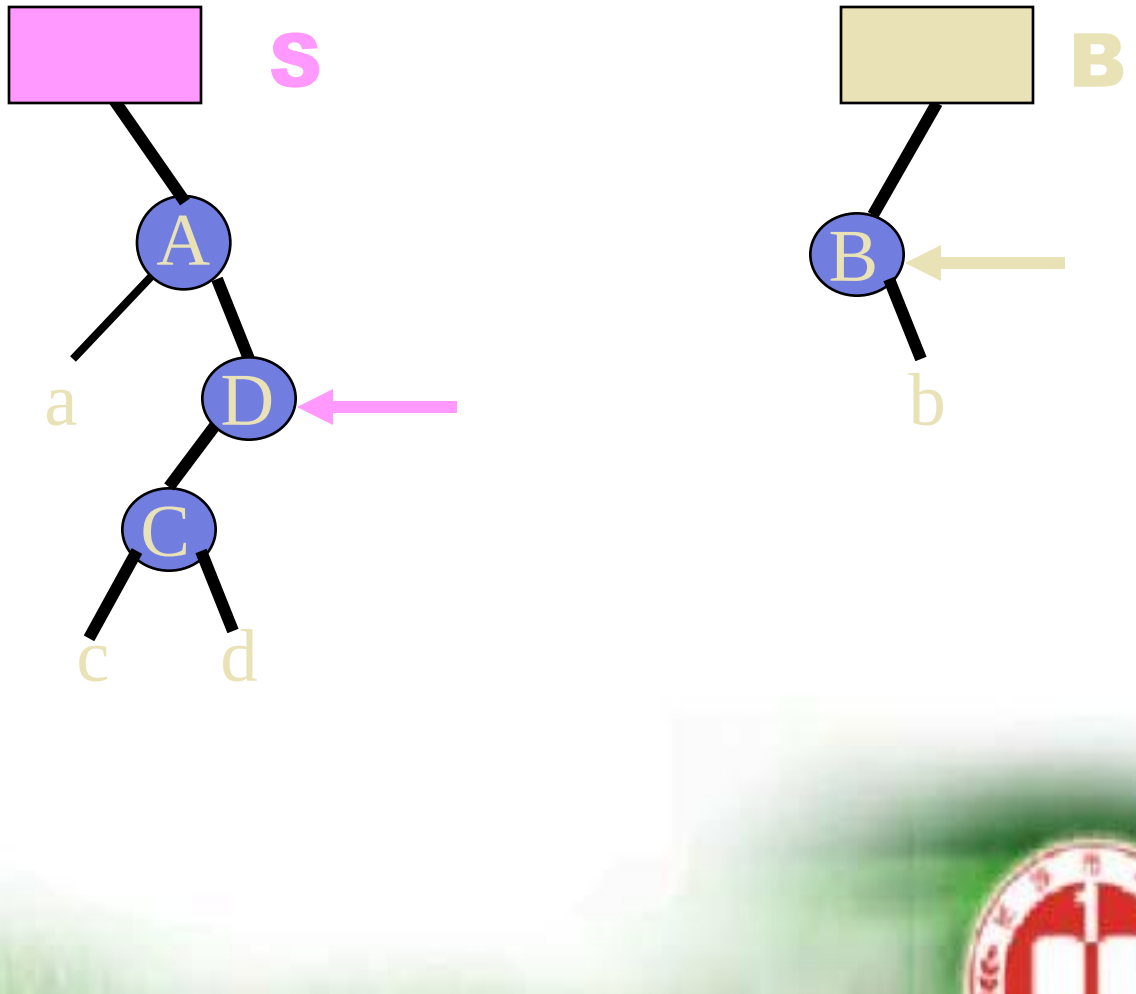


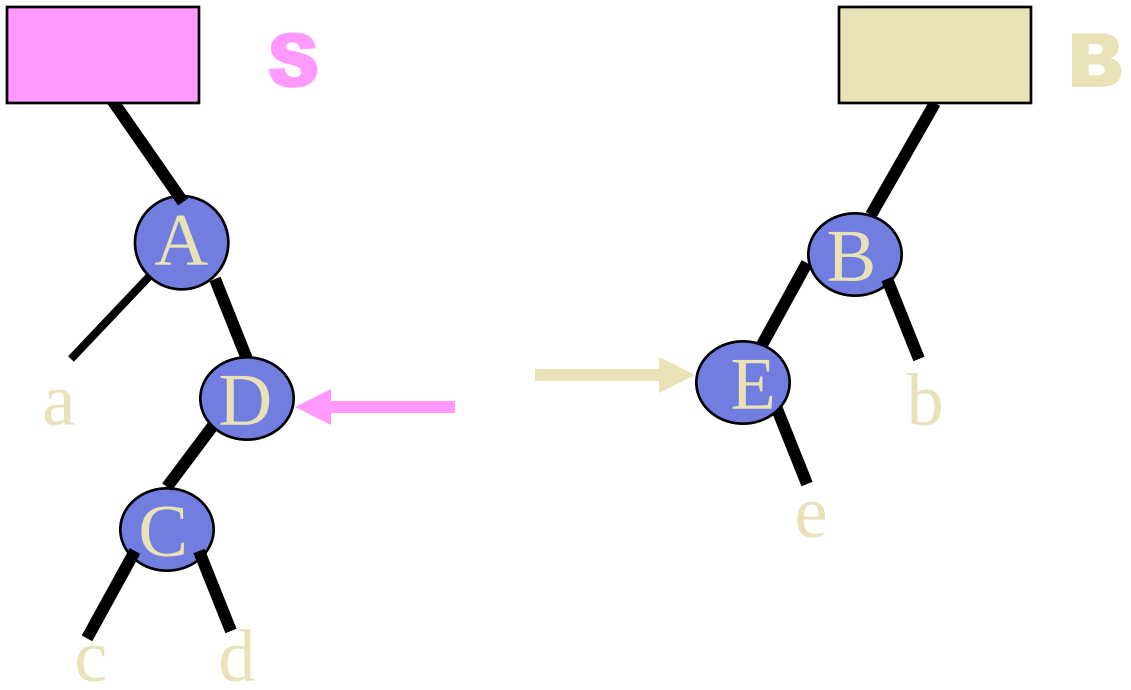
RR Move

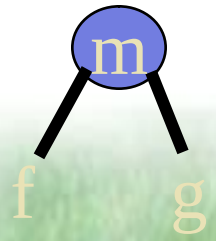


L Move

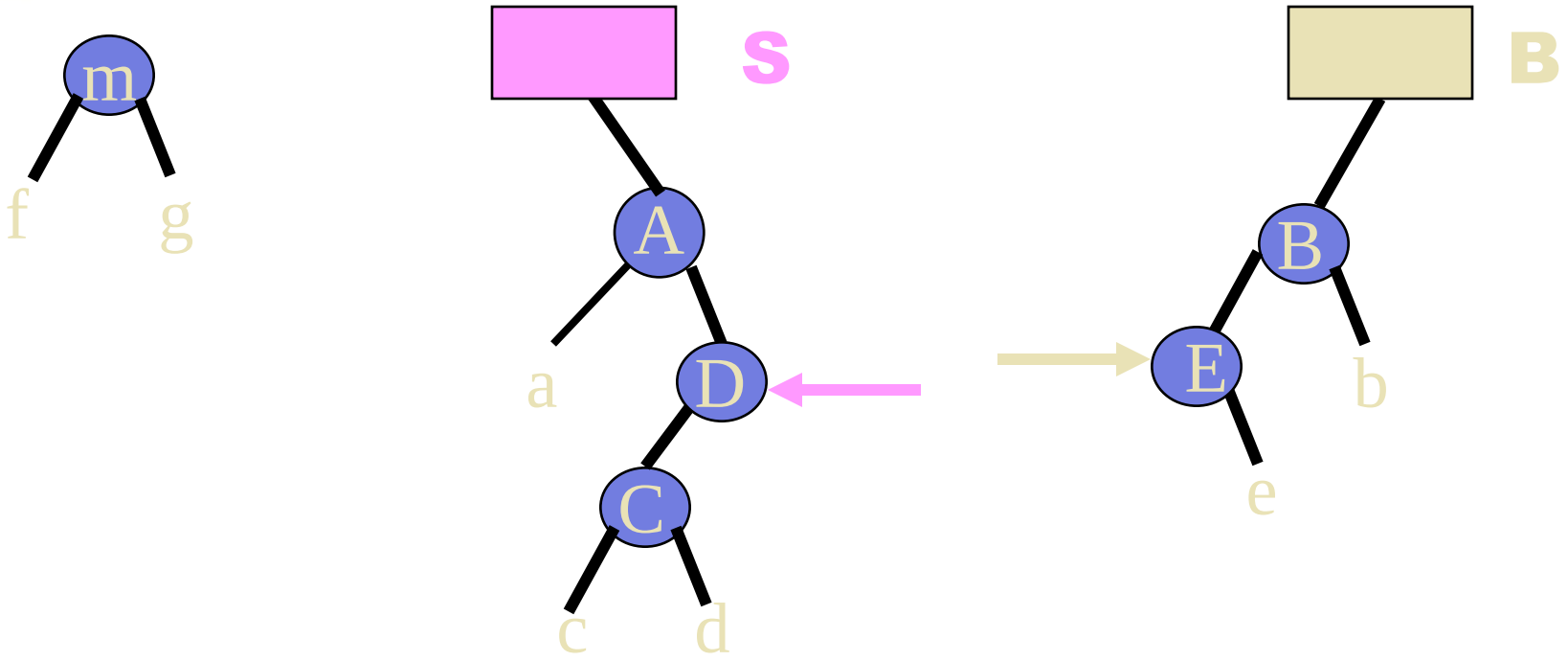


L Move

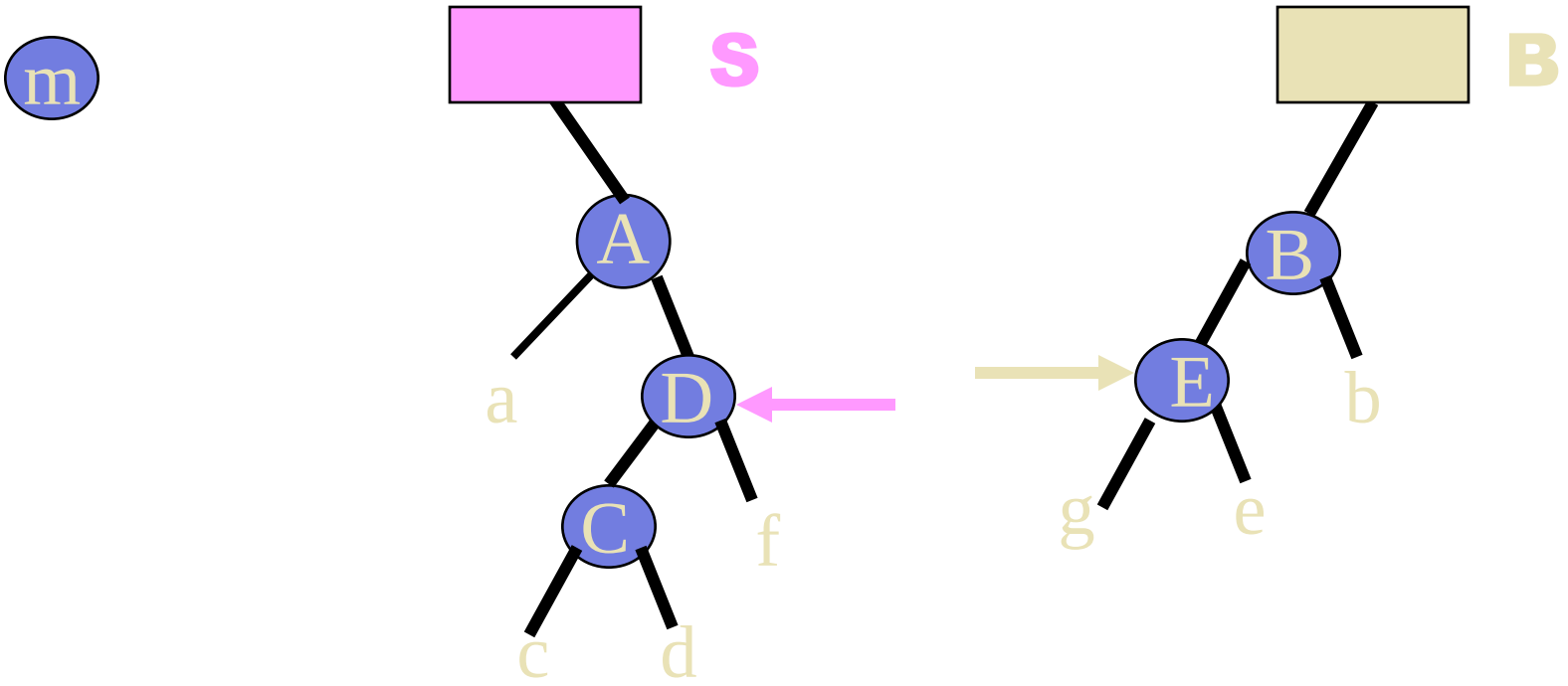




合并



合并



合并