CSE 258 – Lecture 7

Web Mining and Recommender Systems

Recommender Systems

Announcements

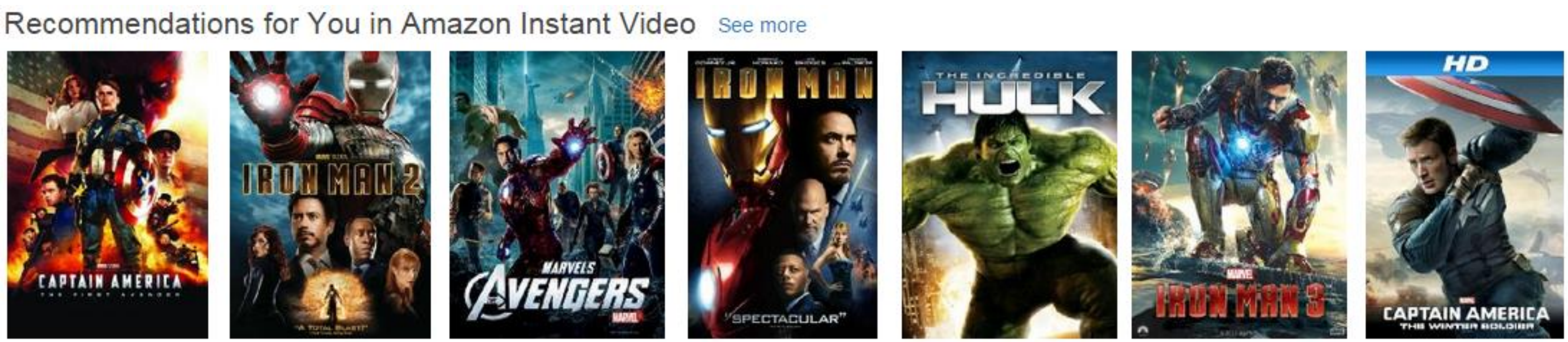
- Assignment 1 is out
- It will be due in week 8 **on Monday before class**
- HW3 will help you set up an initial solution

- HW1 solutions have been posted to Piazza

Why recommendation?

The goal of recommender systems is…

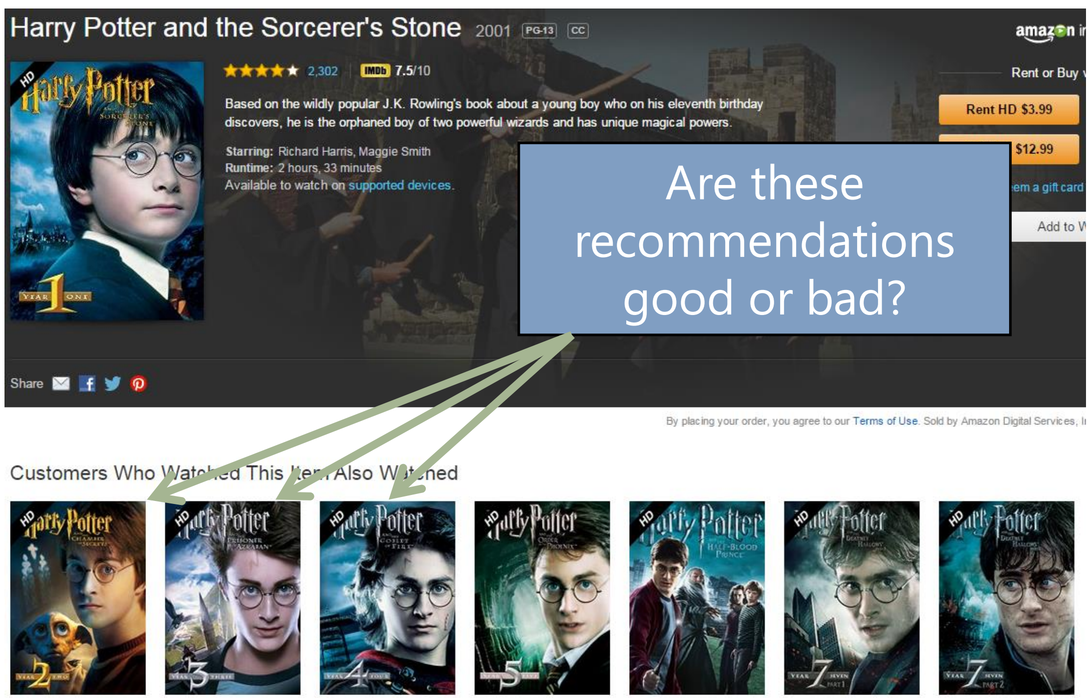
- To help people discover new content



Why recommendation?

The goal of recommender systems is…

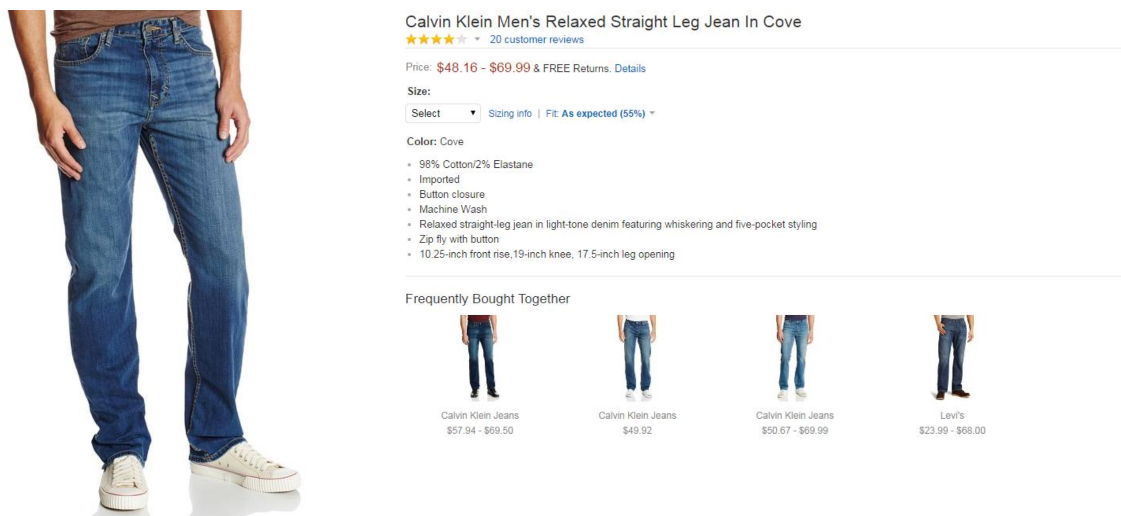
- To help us find the content we were already looking for



Why recommendation?

The goal of recommender systems is…

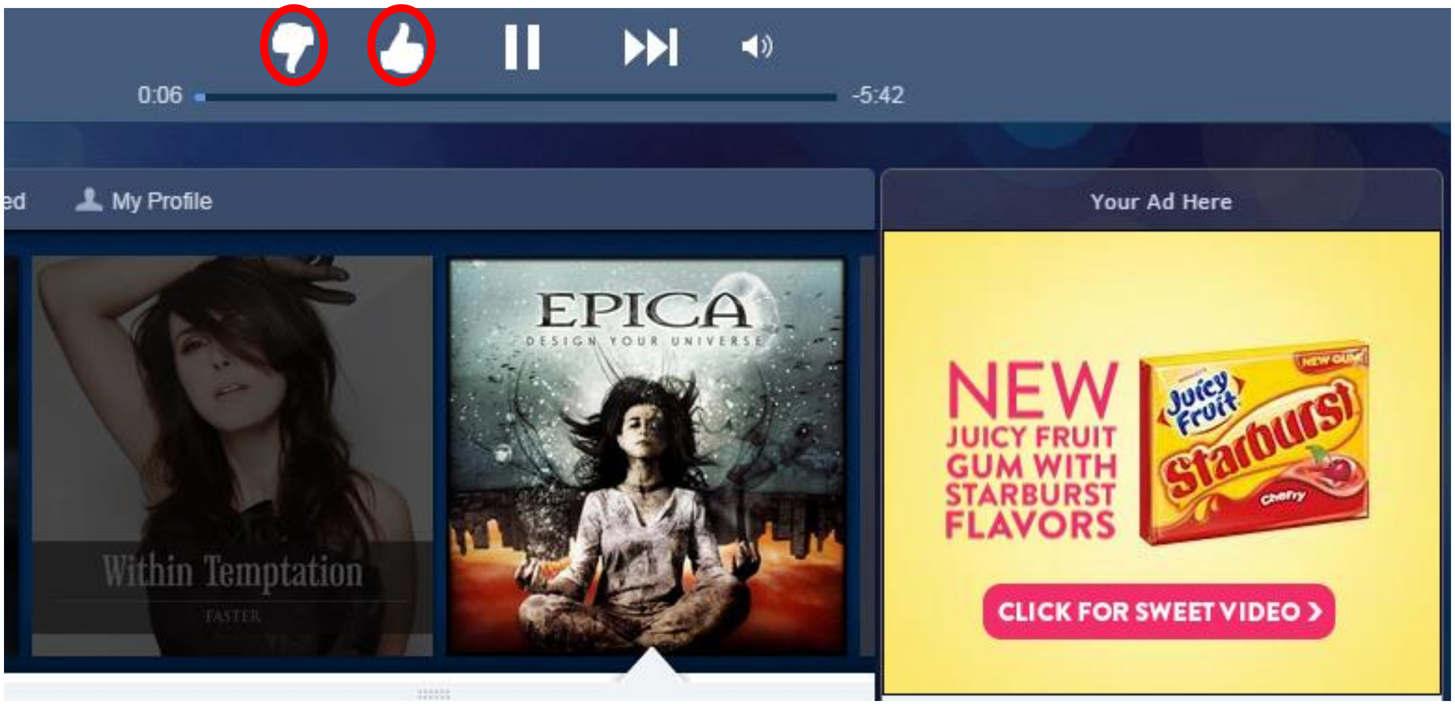
- To discover which things go together



Why recommendation?

The goal of recommender systems is…

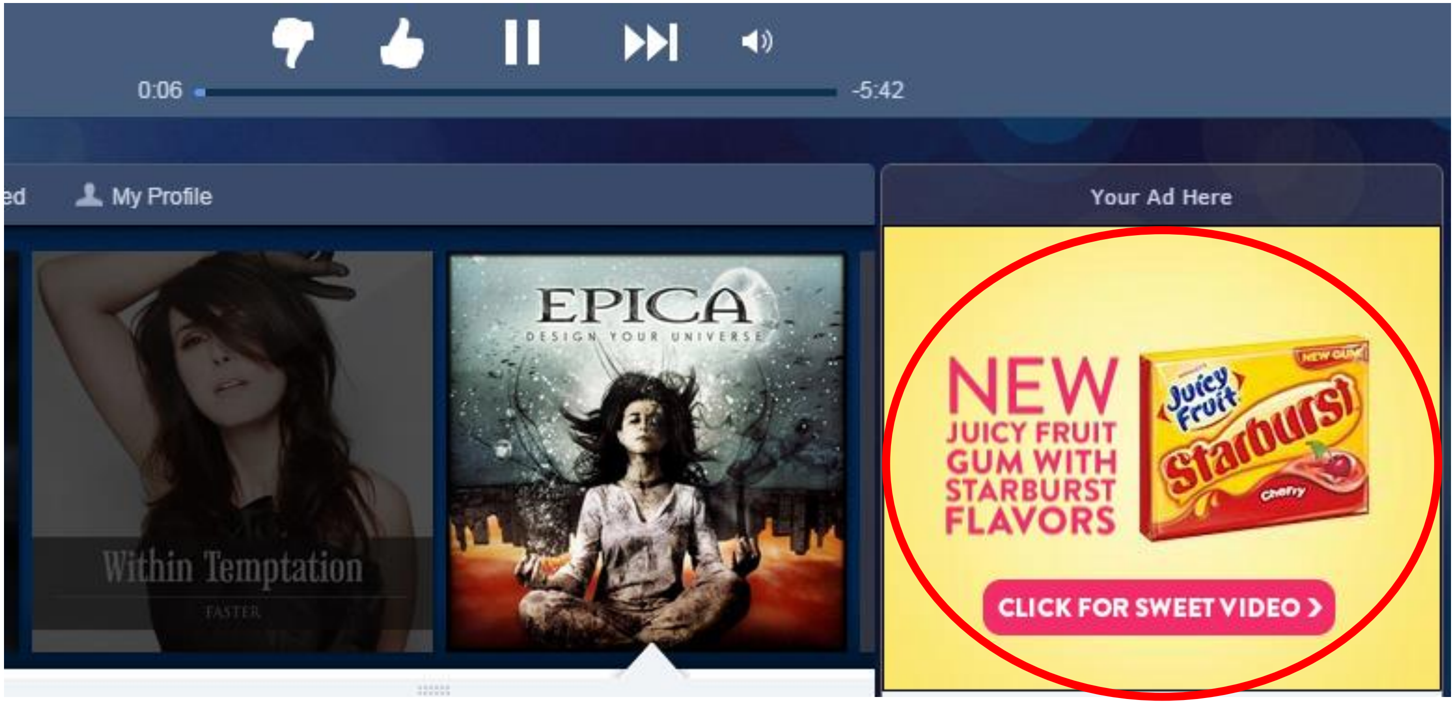
- To personalize user experiences in response to user feedback



Why recommendation?

The goal of recommender systems is…

- To recommend incredible products that are relevant to our interests



Why recommendation?

The goal of recommender systems is…

- To identify things that we **like**

Why recommendation?

The goal of recommender systems is...

- To help people discover new content
- To he[...]e were
- To dis[...]ogether
- To p[...]ces in
  r[...]ck
- To identify things that we **like**

To **model** people's preferences, opinions, and behavior

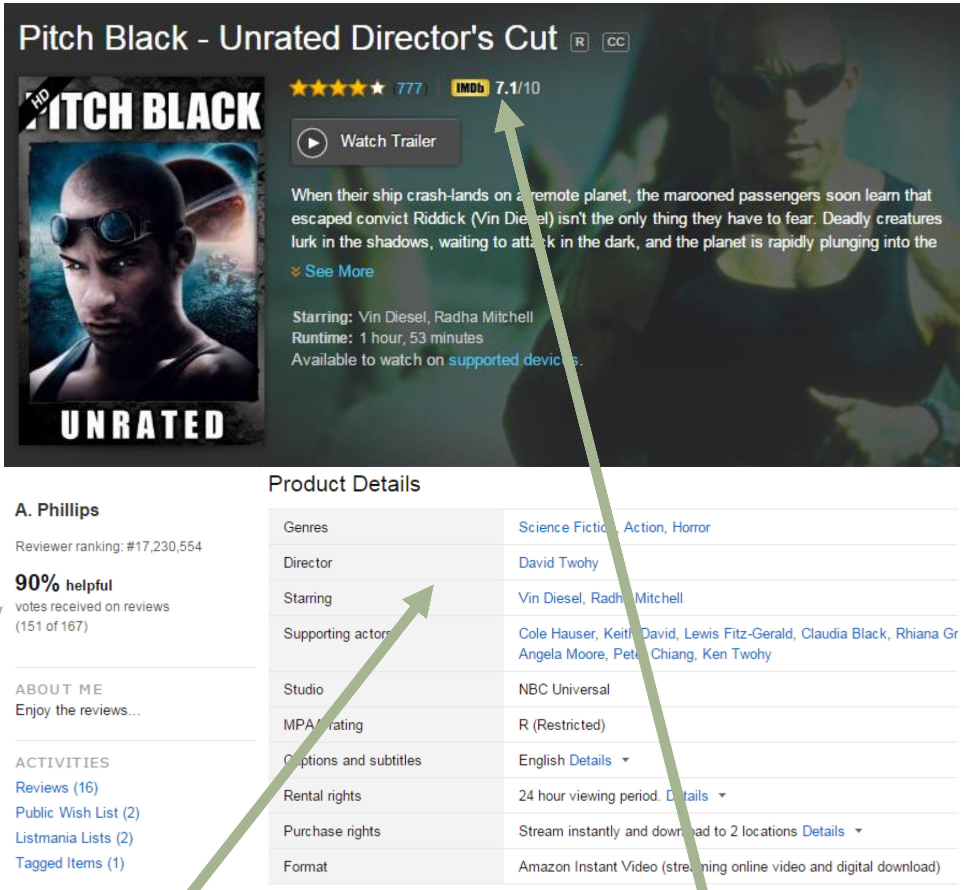
Recommending things to people

Suppose we want to build a movie recommender

e.g. which of these films will I rate highest?

Recommending things to people

We already have a few tools in our "supervised learning" toolbox that may help us



$$\boxed{f(\text{user features}, \text{movie features}) \overset{?}{\rightarrow} \text{star rating}}$$

Recommending things to people

$$f(\text{user features}, \text{movie features}) \stackrel{?}{\rightarrow} \text{star rating}$$

Movie features: genre, actors, rating, length, etc.

Product Details

Genres	Science Fiction, Action, Horror
Director	David Twohy
Starring	Vin Diesel, Radha Mitchell
Supporting actors	Cole Hauser, Keith David, Lewis Fitz-Gerald, Claudia Black, Rhiana Gr Angela Moore, Peter Chiang, Ken Twohy
Studio	NBC Universal
MPAA rating	R (Restricted)
Captions and subtitles	English Details
Rental rights	24 hour viewing period. Details
Purchase rights	Stream instantly and download to 2 locations Details
Format	Amazon Instant Video (streaming online video and digital download)

User features: age, gender, location, etc.

A. Phillips

Reviewer ranking: #17,230,554

90% helpful

votes received on reviews
(151 of 167)

ABOUT ME

Enjoy the reviews...

ACTIVITIES

Reviews (16)
Public Wish List (2)
Listmania Lists (2)
Tagged Items (1)

Recommending things to people

$$f(\text{user features}, \text{movie features}) \xrightarrow{?} \text{star rating}$$

With the models we've seen so far, we can build predictors that account for…

- Do women give higher ratings than men?
- Do Americans give higher ratings than Australians?
- Do people give higher ratings to action movies?
- Are ratings higher in the summer or winter?
- Do people give high ratings to movies with Vin Diesel?

So what **can't** we do yet?

Recommending things to people

$$f(\text{user features}, \text{movie features}) \xrightarrow{?} \text{star rating}$$

Consider the following linear predictor (e.g. from week 1):

$$f(\text{user features}, \text{movie features}) =$$
$$\langle \phi(\text{user features}); \phi(\text{movie features}), \theta \rangle$$

$$\langle \phi(\text{user}), \theta^{\text{user}} \rangle + \langle \phi(\text{movie}), \theta^{\text{item}} \rangle$$

Recommending things to people

But this is essentially just two separate predictors!

$$f(\text{user features}, \text{movie features}) =$$

$$= \underbrace{\langle \phi(\text{user features}), \theta_{\text{user}} \rangle}_{\text{user predictor}} + \underbrace{\langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle}_{\text{movie predictor}}$$

That is, we're treating user and movie features as though they're **independent!**

Recommending things to people

But these predictors should (obviously?) **not** be independent

$$f(\text{user features}, \text{movie features}) = f(\text{user}) + f(\text{movie})$$

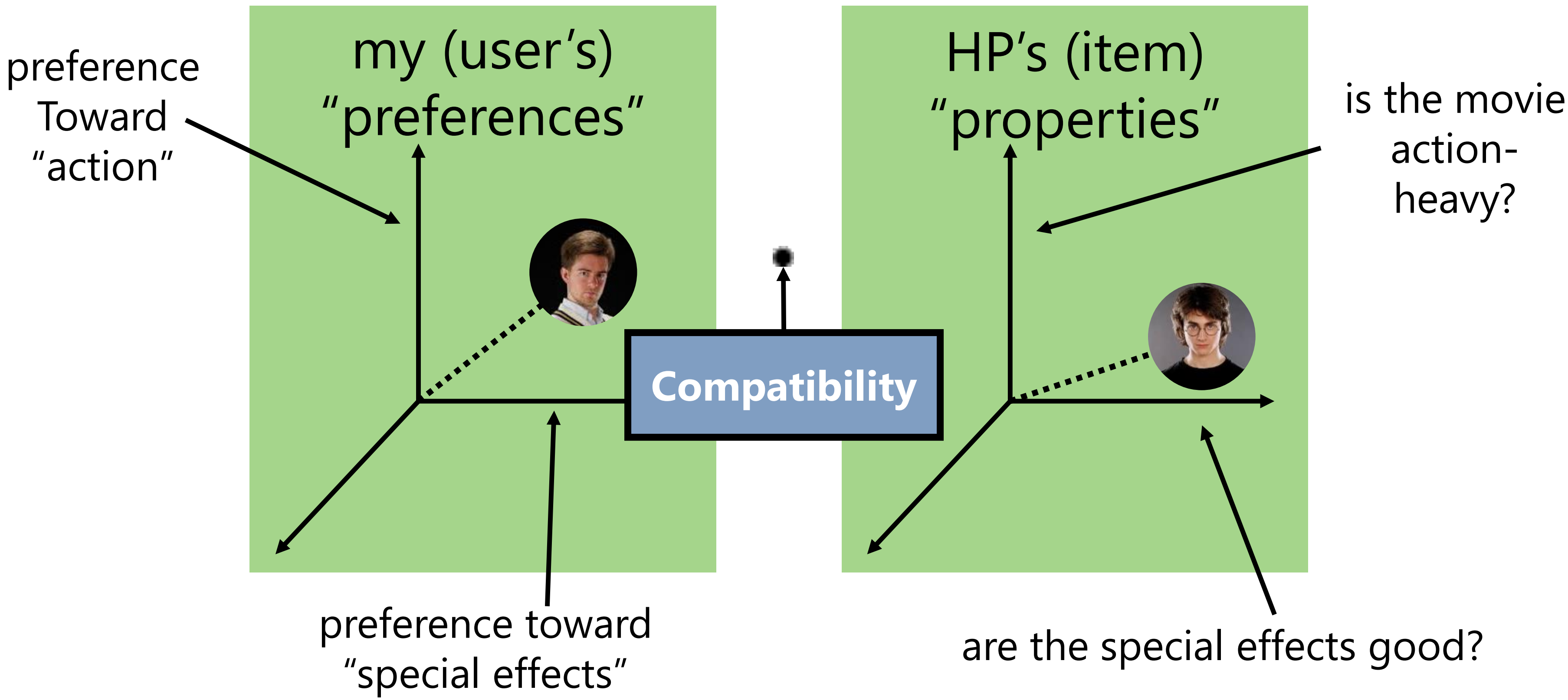
do I tend to give high ratings?

does the population tend to give high ratings to this genre of movie?

But what about a feature like "do **I** give high ratings to **this genre** of movie"?

Recommending things to people

Recommender Systems go beyond the methods we've seen so far by trying to model the **relationships** between people and the items they're evaluating



Today

Recommender Systems

1. Collaborative filtering

(performs recommendation in terms of user/user and item/item similarity)

2. Assignment 1
3. (next lecture) Latent-factor models

(performs recommendation by projecting users and items into some low-dimensional space)

4. (next lecture) The Netflix Prize

Defining similarity between users & items

Q: How can we measure the **similarity** between two **users?**
A: In terms of the **items** they purchased!

Q: How can we measure the similarity between two **items?**
A: In terms of the users who purchased them!

Defining similarity between users & items

e.g.: Amazon

Calvin Klein Men's Relaxed Straight Leg Jean In Cove

20 customer reviews

Price: $48.16 - $69.99 & FREE Returns. Details

Size:

Select | Sizing info | Fit: **As expected (55%)**

Color: Cove

- 98% Cotton/2% Elastane
- Imported
- Button closure
- Machine Wash
- Relaxed straight-leg jean in light-tone denim featuring whiskering and five-pocket styling
- Zip fly with button
- 10.25-inch front rise,19-inch knee, 17.5-inch leg opening

Frequently Bought Together

Customers Who Viewed This Item Also Viewed

Customers Who Bought This Item Also Bought

Page 3

Definitions

I_u = set of items purchased by user u

U_i = set of users who purchased item i

Definitions

Or equivalently…

$$R = \overbrace{\begin{pmatrix} 1 & 0 & \cdots & 1 \\ 0 & 0 & & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{pmatrix}}^{\text{items}} \left.\vphantom{\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 1 \end{pmatrix}}\right\} \text{users}$$

R_u = binary representation of items purchased by *u*

$R_{\cdot,i}$ = binary representation of users who purchased *i*

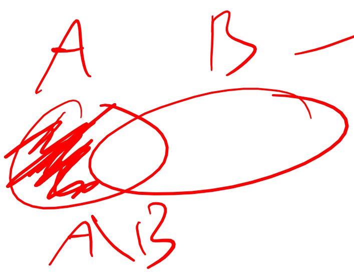
$$I_u = \{ i \mid R_{ui} = 1 \} \qquad U_i = \{ u \mid R_{ui} = 1 \}$$

0. Euclidean distance

Euclidean distance:

e.g. between two items i,j (similarly defined between two users)

$$|U_i \setminus U_j| + |U_j \setminus U_{\cancel{j}}| = \|R_i - R_j\|$$



i

Euclidean distance:

e.g.: U_1 = {1,4,8,9,11,23,25,34}
U_2 = {1,4,6,8,9,11,23,25,34,35,38}
U_3 = {4}
U_4 = {5}

$$|U_1 \setminus U_2| + |U_2 \setminus U_1| = 3$$

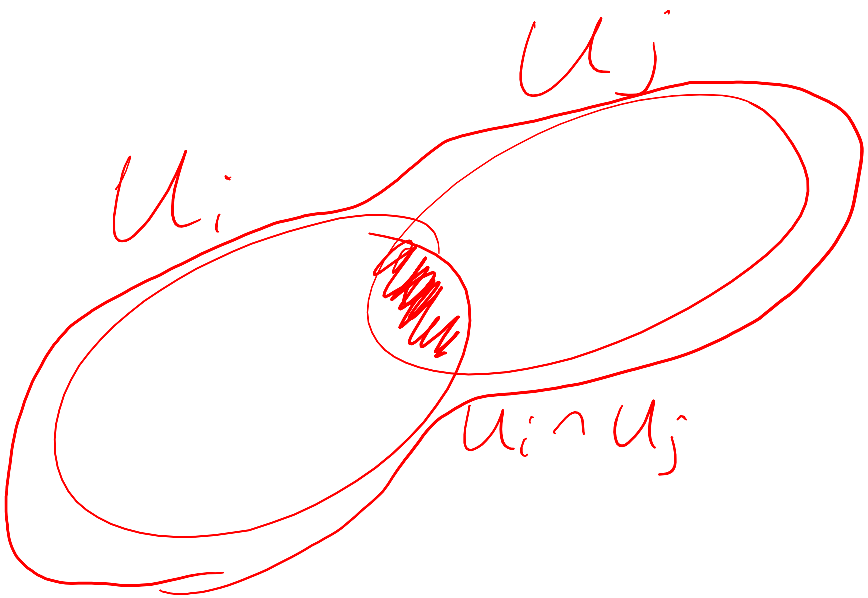
$$|U_3 \setminus U_4| + |U_3 \setminus U_4| = 2$$

Problem: favors small sets, even if they have few elements in common

1. Jaccard similarity

$$\text{Jaccard}(A, B) = \frac{|A \cap B|}{|A \cup B|}$$

$$\text{Jaccard}(U_i, U_j) = \frac{|U_i \cap U_j|}{|U_i \cup U_j|}$$



→ Maximum of 1 if the two users purchased **exactly the same** set of items
(or if two items were purchased by the same set of users)

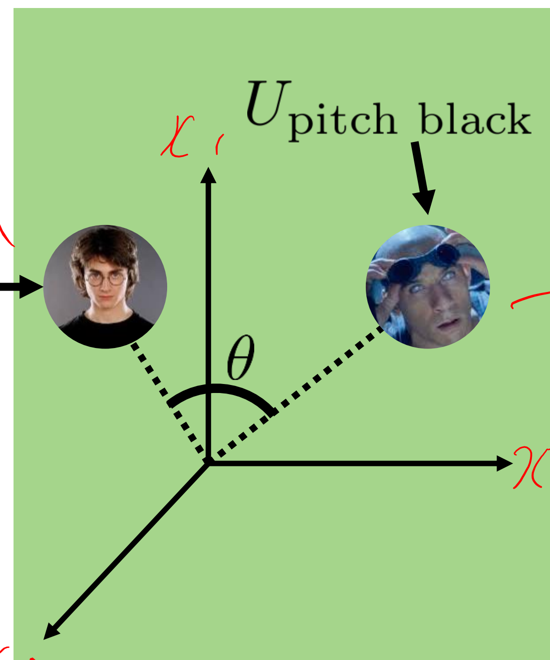
→ Minimum of 0 if the two users purchased **completely disjoint** sets of items
(or if the two items were purchased by completely disjoint sets of users)

2. Cosine similarity

$U_{\text{harry potter}}$

(vector representation of users who purchased harry potter)

(0,1,1)



(1,1,0)

$$\cos(\theta) = 1$$

(theta = 0) → *A* and *B* point in exactly the same direction

$$\cos(\theta) = -1$$

(theta = 180) → *A* and *B* point in opposite directions (won't actually happen for 0/1 vectors)

$$\cos(\theta) = 0$$

(theta = 90) → *A* and *B* are orthogonal

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{\|A\| \, \|B\|}\right)$$

$$\cos\theta = \frac{u_i \cdot u_j}{\|u_i\| \, \|u_j\|} \sim |u_i \cap u_j| \quad \text{(for binary)}$$

2. Cosine similarity

Why cosine?

- Unlike Jaccard, works for arbitrary vectors
- E.g. what if we have **opinions** in addition to purchases?

$$R = \begin{pmatrix} 1 & 0 & \cdots & 1 \\ 0 & 0 & & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{pmatrix} \rightarrow \begin{pmatrix} -1 & 0 & \cdots & 1 \\ 0 & 0 & & -1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & -1 \end{pmatrix}$$

bought and **liked**

didn't buy

bought and **hated**

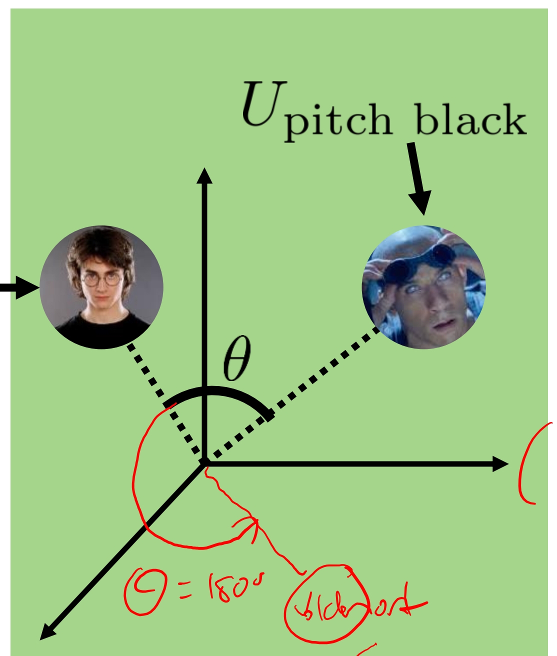
2. Cosine similarity

E.g. our previous example, now with "thumbs-up/thumbs-down" ratings

(0, 1, 1)

$U_{\text{harry potter}}$

(vector representation of users' **ratings of** Harry Potter)



(1, 1, 0)

(0, -1, -1)

$$\cos(\theta) = 1$$

(theta = 0) → Rated by the same users, and they all agree

$$\cos(\theta) = -1$$

(theta = 180) → Rated by the same users, but they **completely disagree** about it

$$\cos(\theta) = 0$$

(theta = 90) → Rated by different sets of users

4. Pearson correlation

What if we have numerical ratings (rather than just thumbs-up/down)?

$$R = \begin{pmatrix} -1 & 0 & \cdots & 1 \\ 0 & 0 & & -1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 4 & 0 & \cdots & 2 \\ 0 & 0 & & 3 \\ \vdots & & \ddots & \vdots \\ 5 & 0 & \cdots & 1 \end{pmatrix}$$

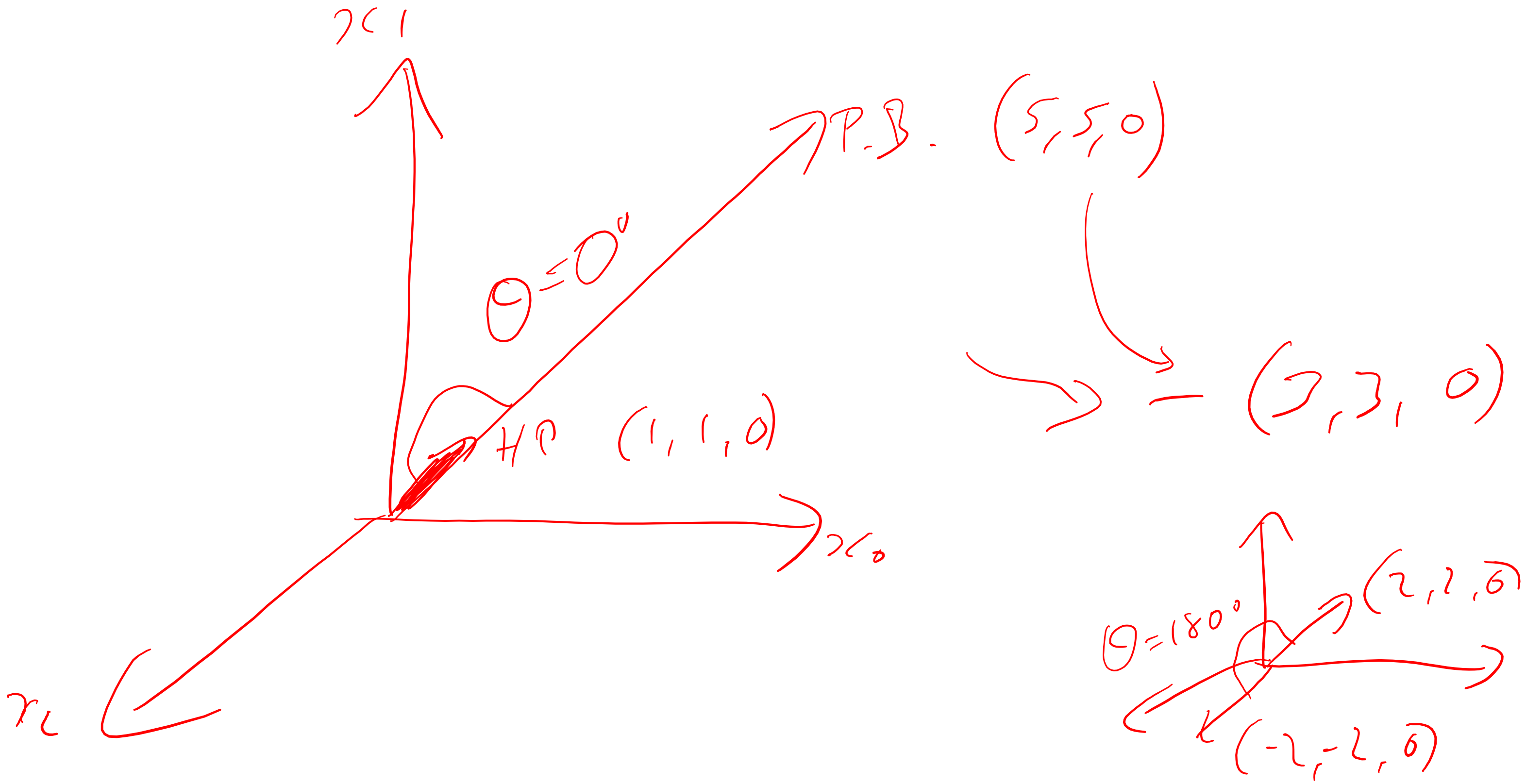
bought and **liked**

didn't buy

bought and **hated**

4. Pearson correlation

What if we have numerical ratings (rather than just thumbs-up/down)?



4. Pearson correlation

What if we have numerical ratings (rather than just thumbs-up/down)?

- We wouldn't want 1-star ratings to be parallel to 5-star ratings
- So we can subtract the average – values are then **negative** for below-average ratings and **positive** for above-average ratings

items rated by both users

average rating by user *v*

$$\mathrm{Sim}(u,v) = \frac{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R_u})(R_{v,i} - \bar{R_v})}{\sqrt{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R_u})^2 \sum_{i \in I_u \cap I_v} (R_{v,i} - \bar{R_v})^2}}$$

4. Pearson correlation

Compare to the cosine similarity:

Pearson similarity (between users):

items rated by both users average rating by user *v*

$$\text{Sim}(u,v) = \frac{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R_u})(R_{v,i} - \bar{R_v})}{\sqrt{\sum_{i \in I_u \cap I_v} (R_{u,i} - \bar{R_u})^2 \sum_{i \in I_u \cap I_v} (R_{v,i} - \bar{R_v})^2}}$$

Cosine similarity (between users):

$$\text{Sim}(u,v) = \frac{\sum_{i \in I_u \cap I_v} R_{u,i} R_{v,i}}{\sqrt{\sum_{i \in I_u \cap I_v} R_{u,i}^2 \sum_{i \in I_u \cap I_v} R_{v,i}^2}} = \frac{I_u \cdot I_v}{\|I_u\| \|I_v\|}$$

Collaborative filtering in practice

How does amazon generate their recommendations?

$\underset{j}{\arg\max}\ \mathrm{sim}(i,j)$

Given a product: $i =$ 

Let U_i be the set of users who viewed it

Rank products according to: $\dfrac{|U_i \cap U_j|}{|U_i \cup U_j|}$ (or cosine/pearson)

.86 .84 .82 .79 …

Linden, Smith, & York (2003)

Collaborative filtering in practice

Note: (surprisingly) that we built something pretty useful out of **nothing but rating data** – we didn't look at any features of the products whatsoever

Collaborative filtering in practice

But: we still have
a few problems left to address…

1. This is actually kind of slow given a huge enough dataset – if one user purchases one item, this will change the rankings of **every other item that was purchased by at least one user in common**
2. Of no use for **new users** and **new items** ("cold-start" problems
3. Won't necessarily encourage diverse results

Questions

CSE 258 – Lecture 7

Web Mining and Recommender Systems

Latent-factor models

Latent factor models

So far we've looked at approaches that try to define some definition of user/user and item/item **similarity**

Recommendation then consists of

- Finding an item *i* that a user likes (gives a high rating)
- Recommending items that are similar to it (i.e., items *j* with a similar rating profile to *i*)

Latent factor models

What we've seen so far are **unsupervised** approaches and whether the work depends highly on whether we chose a "good" notion of similarity

So, can we perform recommendations via **supervised** learning?

Latent factor models

e.g. if we can model

$$f(\text{user features}, \text{movie features}) \rightarrow \text{star rating}$$

Then recommendation will consist of identifying

$$recommendation(u) = \arg\max_{i \in \text{unseen items}} f(u, i)$$

The Netflix prize

In 2006, Netflix created a dataset of **100,000,000** movie ratings
Data looked like:

$$(\text{userID}, \text{itemID}, \text{time}, \text{rating})$$

The goal was to reduce the (R)MSE at predicting ratings:

$$\text{RMSE}(f) = \sqrt{\frac{1}{N} \sum_{u,i,t \in \text{test set}} (f(u,i,t) - r_{u,i,t})^2}$$

model's prediction

ground-truth

Whoever first manages to reduce the RMSE by **10%** versus Netflix's solution wins **$1,000,000**

The Netflix prize

This led to **a lot** of research on rating prediction by minimizing the Mean-Squared Error

(it also led to a lawsuit against Netflix, once somebody managed to de-anonymize their data)

We'll look at a few of the main approaches

Rating prediction

Let's start with the
simplest possible model:

$$f(u, i) = \alpha$$

user item

$$\alpha = \frac{1}{N} \sum_{u,i \in R} R_{u,i}$$

$$RMSE = s.d.(R)$$

Rating prediction

What about the **2nd** simplest model?

$$f(u, i) = \alpha + \beta_u + \beta_i$$

user item

how much does this user tend to rate things above the mean?

does this item tend to receive higher ratings than others

e.g.

$$\alpha = 4.2$$

$$\beta_{\text{pitch black}} = -0.1$$

$$\beta_{\text{julian}} = -0.2$$

Rating prediction

$$f(u,i) = \alpha + \beta_u + \beta_i$$

This is a linear model!

$$f(u,i) = \langle \phi(u,i), \theta \rangle$$

position u position i

$$\phi(u,i) = [1 \mid 0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0 \mid 0\ 0\ 0 \mid 0\ 0\ 0\ 0\ 0]$$

$$\theta = [\alpha \mid \beta_u \mid \beta_i]$$

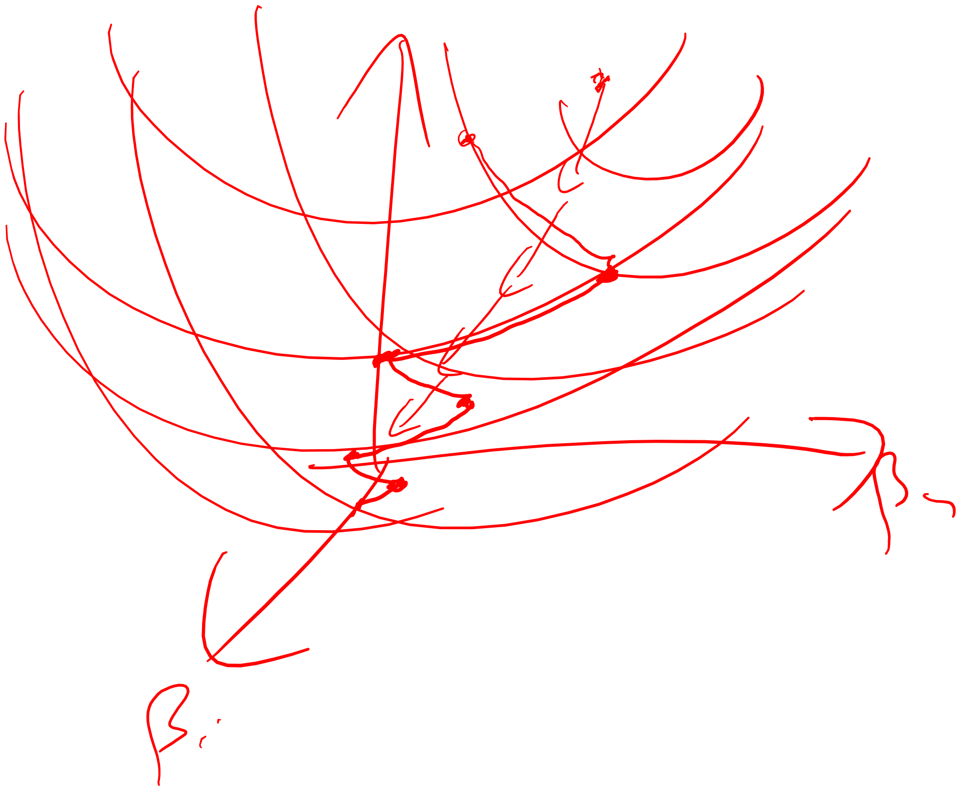
Rating prediction

The optimization problem becomes:

$$\arg\min_{\alpha,\beta} \underbrace{\textstyle\sum_{u,i}(\alpha + \beta_u + \beta_i - R_{u,i})^2}_{\text{error}} + \underbrace{\lambda \left[\textstyle\sum_u \beta_u^2 + \sum_i \beta_i^2\right]}_{\text{regularizer}}$$

Jointly convex in \beta_i, \beta_u. Can be solved by iteratively removing the mean and solving for beta

Jointly convex?



Rating prediction

Differentiate:

obj

$$\arg\min_{\alpha,\beta} \sum_{u,i} (\alpha + \beta_u + \beta_i - R_{u,i})^2 + \lambda \left[\sum_u \beta_u^2 + \sum_i \beta_i^2\right]$$

$$\frac{\partial\, obj}{\partial \beta_u} \qquad \sum_{i \in I_u} 2(\alpha + \beta_u + \beta_i - R_{u,i}) + 2\lambda\beta_u$$

solve for β_u

$$-\lambda\beta_u - |I_u|\beta_u = \sum_{i \in I_u} (\alpha + \cancel{\beta_u} + \beta_i - R_{u,i})$$

$$\beta_u = \frac{-\sum_{i \in I_u} (\alpha + \beta_i - R_{u,i})}{\lambda + |I_u|}$$

Rating prediction

Iterative procedure – repeat the following updates until convergence:

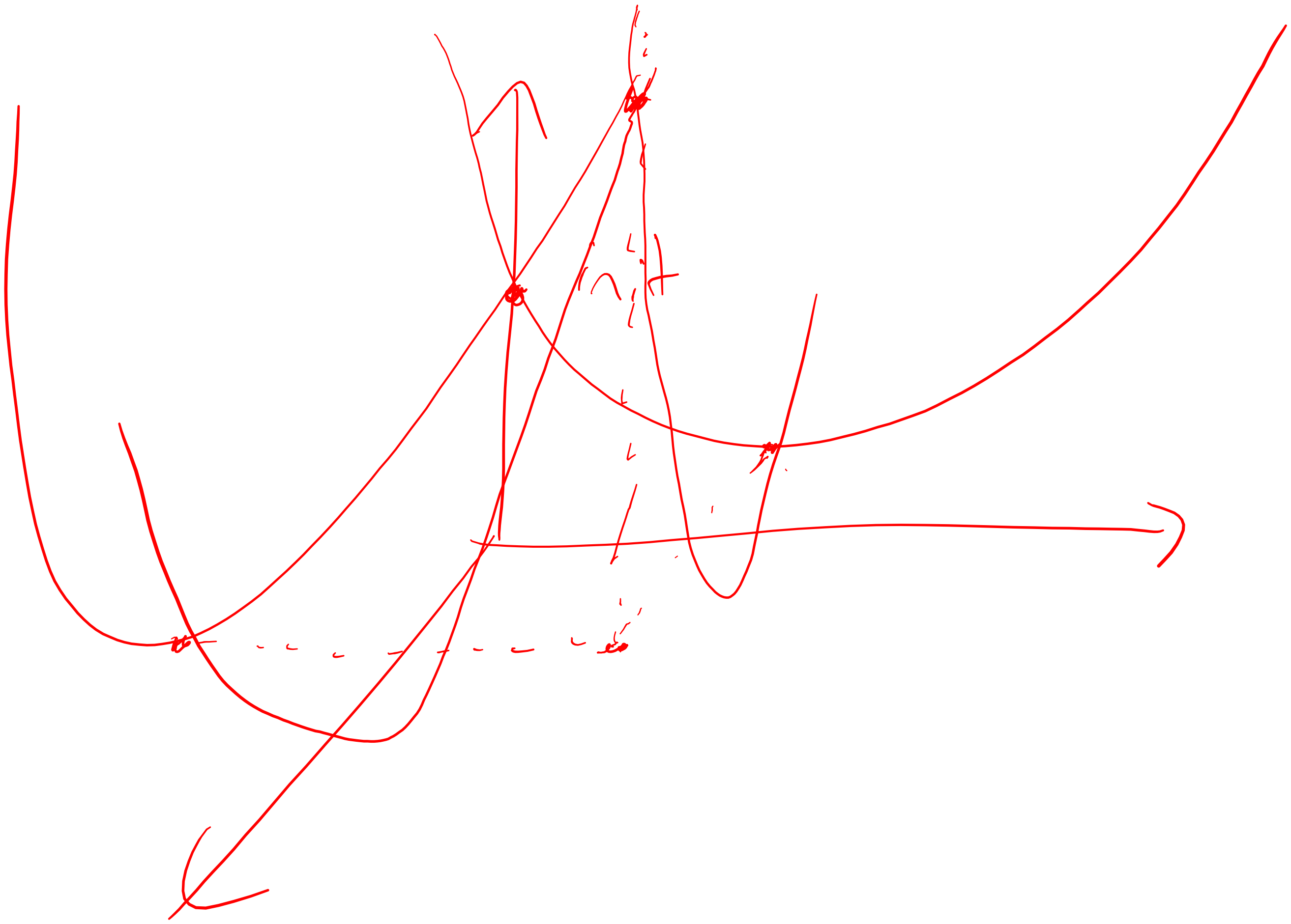
$$\alpha^{(t)} = \frac{\sum_{u,i \in \text{train}} (R_{u,i} - (\beta_u + \beta_i))}{N_{\text{train}}}$$

$$\beta_u^{(t+1)} = \frac{\sum_{i \in I_u} R_{u,i} - (\alpha + \beta_i)}{\lambda + |I_u|}$$

$$\beta_i^{(t+2)} = \frac{\sum_{u \in U_i} R_{u,i} - (\alpha + \beta_u)}{\lambda + |U_i|}$$

(exercise: write down derivatives and convince yourself of these update equations!)

One variable at a time or all at once?



Rating prediction

Looks good (and actually works surprisingly well), but doesn't solve the basic issue that we started with

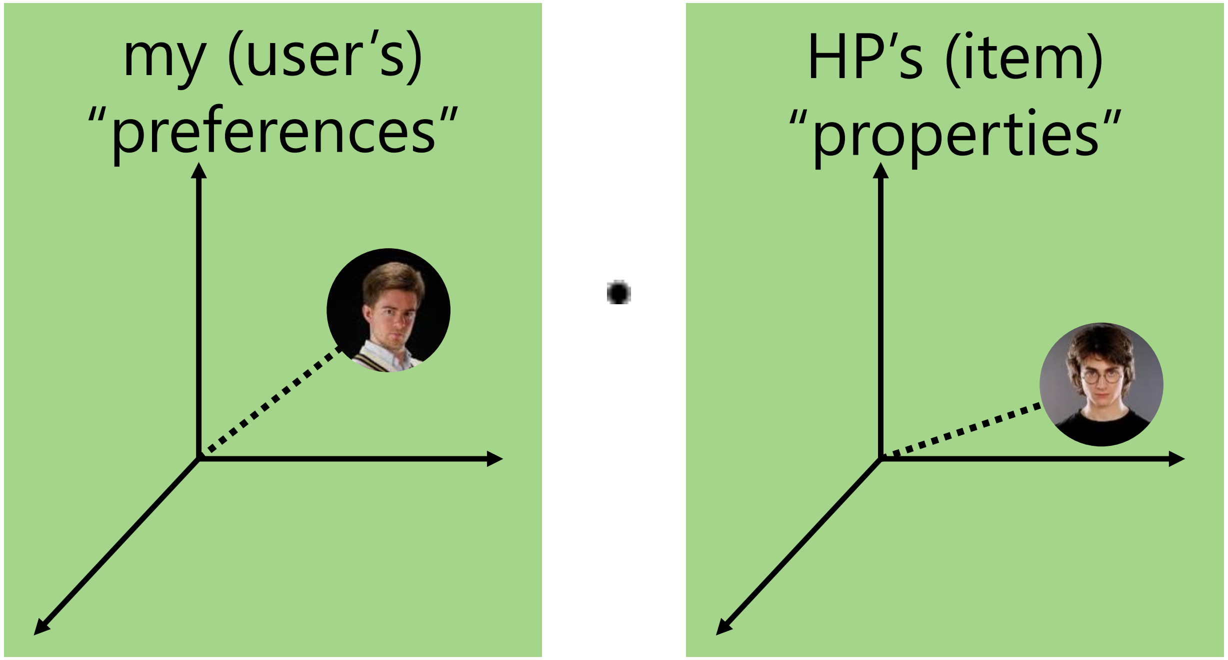
$$f(\text{user features}, \text{movie features}) =$$

$$= \underbrace{\langle \phi(\text{user features}), \theta_{\text{user}} \rangle}_{\text{user predictor}} + \underbrace{\langle \phi(\text{movie features}), \theta_{\text{movie}} \rangle}_{\text{movie predictor}}$$

That is, we're **still** fitting a function that treats users and items independently

Recommending things to people

How about an approach based on
dimensionality reduction?



i.e., let's come up with low-dimensional representations of the users and the items so as to best explain the data

Dimensionality reduction

We already have some tools that ought to help us, e.g. from week 3:

$$R = \begin{pmatrix} 5 & 3 & \cdots & 1 \\ 4 & 2 & & 1 \\ 3 & 1 & & 3 \\ 2 & 2 & & 4 \\ 1 & 5 & & 2 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 1 \end{pmatrix}$$

What is the best low-rank approximation of *R* in terms of the mean-squared error?

Dimensionality reduction

We already have some tools that ought to help us, e.g. from week 3:

$$R = \begin{pmatrix} 5 & 3 & \cdots & 1 \\ 4 & 2 & & 1 \\ 3 & 1 & & 3 \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & 1 \end{pmatrix}$$

Singular Value Decomposition

(square roots of) eigenvalues of RR^T

$$R = U\Sigma V^T$$

eigenvectors of RR^T

eigenvectors of R^TR

The "best" rank-K approximation (in terms of the MSE) consists of taking the eigenvectors with the highest eigenvalues

Dimensionality reduction

But! Our matrix of ratings is only partially observed; and it's **really big!**

$$R = \begin{pmatrix} 5 & 3 & \cdots & \cdot \\ 4 & 2 & & 1 \\ 3 & \cdot & & 3 \\ \cdot & 2 & & 4 \\ 1 & 5 & & \cdot \\ \vdots & & \ddots & \vdots \\ 1 & 2 & \cdots & \cdot \end{pmatrix}$$

Missing ratings

SVD is **not defined** for partially observed matrices, and it is **not practical** for matrices with 1Mx1M+ dimensions

Latent-factor models

Instead, let's solve approximately using gradient descent

$$
R = \underbrace{\begin{pmatrix}
5 & 3 & \cdots & \cdot \\
4 & 2 & & 1 \\
3 & \cdot & & 3 \\
\cdot & 2 & & 4 \\
1 & 5 & & \cdot \\
\vdots & & \ddots & \vdots \\
1 & 2 & \cdots & \cdot
\end{pmatrix}}_{\text{items}}
\left.\vphantom{\begin{pmatrix} 5\\4\\3\\ \cdot \\1\\ \vdots \\1\end{pmatrix}}\right\}\text{users}
$$

K-dimensional representation of each item

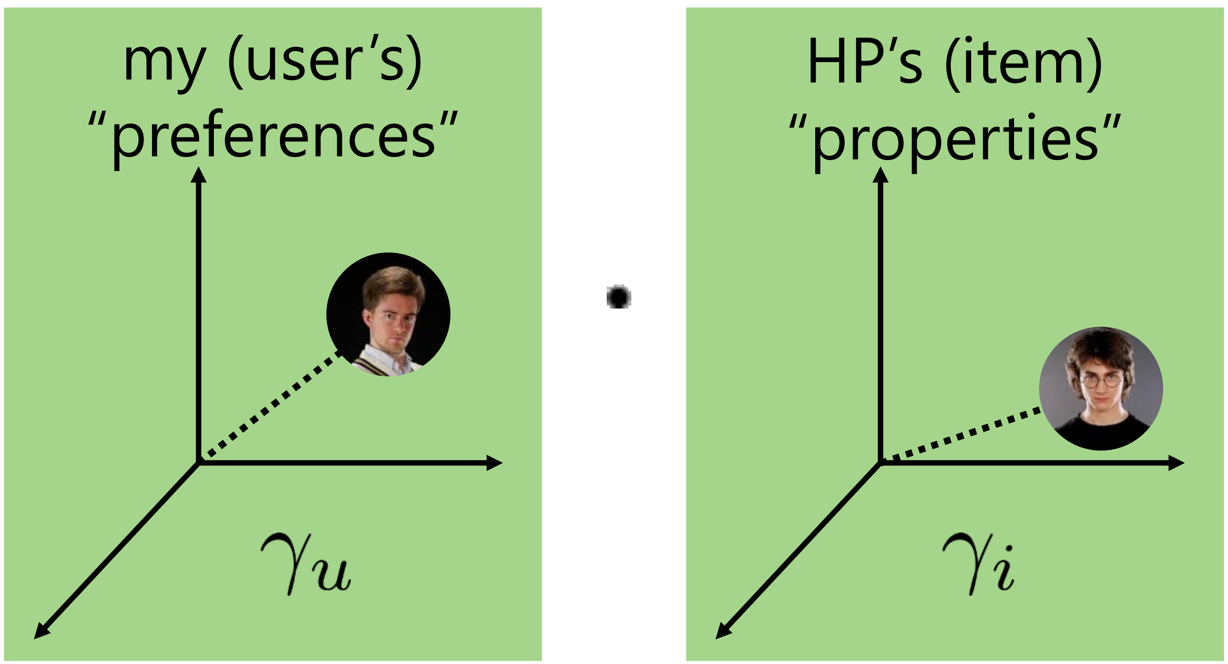
$$R \simeq UV^T$$

K-dimensional representation of each user

Latent-factor models

Let's write this as:

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$



Latent-factor models

Let's write this as:

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$

Our optimization problem is then

$$\arg\min_{\alpha,\beta,\gamma} \underbrace{\sum_{u,i}(\alpha+\beta_u+\beta_i+\gamma_u\cdot\gamma_i-R_{u,i})^2}_{\text{error}} + \underbrace{\lambda\left[\sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \|\gamma_i\|_2^2 + \sum_u \|\gamma_u\|_2^2\right]}_{\text{regularizer}}$$

Latent-factor models

Problem: this is certainly not convex

Latent-factor models

Oh well. We'll just solve it approximately

Observation: if we know either the user or the item parameters, the problem becomes easy

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$

e.g. fix gamma_i – pretend we're fitting parameters for features

Latent-factor models

$$\arg\min_{\alpha,\beta,\gamma} \sum_{u,i} (\alpha+\beta_u+\beta_i+\gamma_u\cdot\gamma_i-R_{u,i})^2+\lambda \left[\sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \|\gamma_i\|_2^2 + \sum_u \|\gamma_u\|_2^2\right]$$

Latent-factor models

This gives rise to a simple (though approximate) solution

objective:

$$\underbrace{\arg\min_{\alpha,\beta,\gamma} \sum_{u,i}(\alpha+\beta_u+\beta_i+\gamma_u\cdot\gamma_i-R_{u,i})^2+\lambda\left[\sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \|\gamma_i\|_2^2 + \sum_u \|\gamma_u\|_2^2\right]}$$

$$= \arg\min_{\alpha,\beta,\gamma} objective(\alpha,\beta,\gamma)$$

1) fix γ_i. Solve $\arg\min_{\alpha,\beta,\gamma_u} objective(\alpha,\beta,\gamma)$

2) fix γ_u. Solve $\arg\min_{\alpha,\beta,\gamma_i} objective(\alpha,\beta,\gamma)$

3,4,5…) repeat until convergence

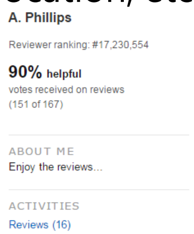
Each of these subproblems is “easy” – just regularized least-squares, like we’ve been doing since week 1. This procedure is called **alternating least squares.**

Latent-factor models

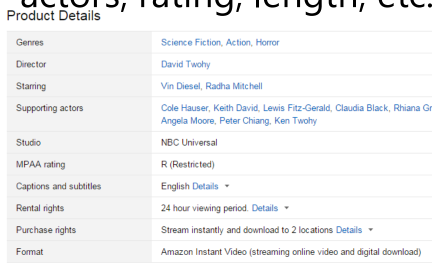
Observation: we went from a method which uses **only** features:

$$f(\text{user features}, \text{movie features}) \rightarrow \text{star rating}$$

User features: age, gender, location, etc.



Movie features: genre, actors, rating, length, etc.


Product Details

Genres	Science Fiction, Action, Horror
Director	David Twohy
Starring	Vin Diesel, Radha Mitchell
Supporting actors	Cole Hauser, Keith David, Lewis Fitz-Gerald, Claudia Black, Rhiana Gr Angela Moore, Peter Chiang, Ken Twohy
Studio	NBC Universal
MPAA rating	R (Restricted)
Captions and subtitles	English Details
Rental rights	24 hour viewing period. Details
Purchase rights	Stream instantly and download to 2 locations Details
Format	Amazon Instant Video (streaming online video and digital download)

to one which **completely ignores** them:

$$\arg\min_{\alpha,\beta,\gamma} \sum_{u,i} (\alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i - R_{u,i})^2 + \lambda \left[\sum_u \beta_u^2 + \sum_i \beta_i^2 + \sum_i \|\gamma_i\|_2^2 + \sum_u \|\gamma_u\|_2^2 \right]$$

Latent-factor models

Should we use features or not?
1) Argument **against** features:

Imagine incorporating features into the model like:

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i + \langle \phi(u), \theta_u \rangle + \langle \phi(i), \theta_i \rangle$$

which is equivalent to:

$$f(u,i) = \alpha + \beta_u + \beta_i + (\underbrace{\phi(u); \phi(i)}_{\text{knowns}}; \gamma_u) \cdot (\underbrace{\theta_u; \theta_i; \gamma_i}_{\text{unknowns}})$$

but this has fewer degrees of freedom than a model which replaces the knowns by unknowns:

$$f(u,i) = \alpha + \beta_u + \beta_i + (\gamma'_i; \gamma'_u; \gamma_u) \cdot (\theta_u; \theta_i; \gamma_i)$$

Latent-factor models

Should we use features or not? 1) Argument **against** features:

So, the addition of features adds **no expressive power** to the model. We **could** have a feature like "is this an action movie?", but if this feature were useful, the model would "discover" a latent dimension corresponding to action movies, and we wouldn't need the feature anyway

In the limit, this argument is valid: as we add more ratings per user, and more ratings per item, the latent-factor model should automatically discover any useful dimensions of variation, so the influence of observed features will disappear

Latent-factor models

Should we use features or not?
2) Argument **for** features:

But! Sometimes we **don't** have many ratings per user/item

Latent-factor models are next-to-useless if **either** the user or the item was never observed before

$$f(u,i) = \alpha + \beta_u + \beta_i + \gamma_u \cdot \gamma_i$$

reverts to zero if we've never seen the user before (because of the regularizer)

Should we use features or not?
2) Argument **for** features:

This is known as the **cold-start** problem in recommender systems. Features are not useful if we have many observations about users/items, but are useful for **new** users and items.

We also need some way to handle users who are **active**, but don't necessarily rate anything, e.g. through **implicit feedback**

Overview & recap

Tonight we've followed the programme below:

1. Measuring similarity between users/items for **binary** prediction (e.g. Jaccard similarity)
2. Measuring similarity between users/items for **real-valued** prediction (e.g. cosine/Pearson similarity)
3. Dimensionality reduction for **real-valued** prediction (latent-factor models)
4. **Finally** – dimensionality reduction for **binary** prediction

One-class recommendation

How can we use **dimensionality reduction** to predict **binary** outcomes?

- In weeks 1&2 we saw **regression** and **logistic regression.** These two approaches use the same type of linear function to predict real-valued and binary outputs
- We can apply an analogous approach to binary recommendation tasks

One-class recommendation

This is referred to as “**one-class**” recommendation

- In weeks 1&2 we saw **regression** and **logistic regression.** These two approaches use the same type of linear function to predict real-valued and binary outputs
- We can apply an analogous approach to binary recommendation tasks

One-class recommendation

Suppose we have binary (0/1) observations (e.g. purchases) or positive/negative feedback (thumbs-up/down)

$$R = \begin{pmatrix} 1 & 0 & \cdots & 1 \\ 0 & 0 & & 1 \\ \vdots & & \ddots & \vdots \\ 1 & 0 & \cdots & 1 \end{pmatrix} \text{ or } \begin{pmatrix} -1 & ? & \cdots & 1 \\ ? & ? & & -1 \\ \vdots & & \ddots & \vdots \\ 1 & ? & \cdots & -1 \end{pmatrix}$$

purchased didn't purchase liked didn't evaluate didn't like

One-class recommendation

So far, we've been fitting functions of the form

$$R \simeq UV^T$$

- Let's change this so that we maximize the **difference** in predictions between positive and negative items
- E.g. for a user who likes an item *i* and dislikes an item *j* we want to maximize:

$$\max \ln \sigma(\gamma_u \cdot \gamma_i - \gamma_u \cdot \gamma_j)$$

One-class recommendation

We can think of this as maximizing the probability of correctly predicting pairwise preferences, i.e.,

$$p(i \text{ is preferred over } j) = \sigma(\gamma_u \cdot \gamma_i - \gamma_u \cdot \gamma_j)$$

- As with logistic regression, we can now maximize the likelihood associated with such a model by gradient ascent
- In practice it isn't feasible to consider all pairs of positive/negative items, so we proceed by stochastic gradient ascent – i.e., randomly sample a (positive, negative) pair and update the model according to the gradient w.r.t. that pair

Summary

Recap

1. Measuring similarity between users/items for **binary** prediction
Jaccard similarity
2. Measuring similarity between users/items for **real-valued** prediction
cosine/Pearson similarity
3. Dimensionality reduction for **real-valued** prediction
latent-factor models
4. Dimensionality reduction for **binary** prediction
one-class recommender systems

Questions?

Further reading:

One-class recommendation:
http://goo.gl/08Rh59
Amazon's solution to collaborative filtering at scale:
http://www.cs.umd.edu/~samir/498/Amazon-Recommendations.pdf
An (expensive) textbook about recommender systems:
http://www.springer.com/computer/ai/book/978-0-387-85819-7
Cold-start recommendation (e.g.):
http://wanlab.poly.edu/recsys12/recsys/p115.pdf