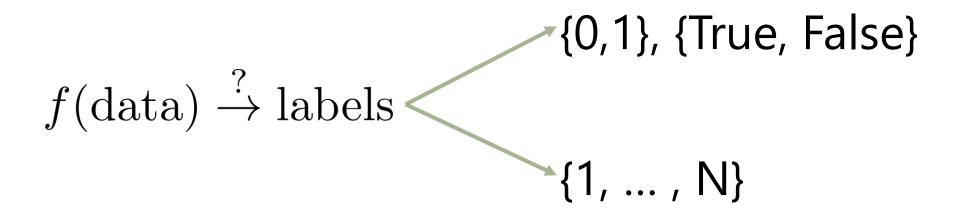
CSE 258 – Lecture 4

Web Mining and Recommender Systems

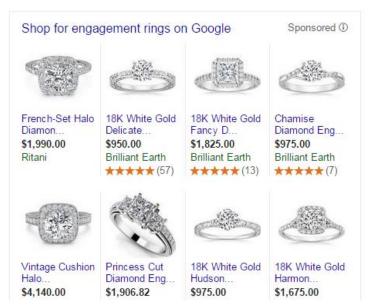
Last lecture...

How can we predict **binary** or **categorical** variables?



Last lecture...





Will I **purchase** this product?

(yes)

Will I **click on** this ad?

(no)

Last lecture...

Naïve Bayes

- Probabilistic model (fits p(label|data))
- Makes a conditional independence assumption of the form $(feature_i \perp \perp feature_j | label)$ allowing us to define the model by computing $p(feature_i | label)$ for each feature
- Simple to compute just by counting

Logistic Regression

 Fixes the "double counting" problem present in naïve Bayes

SVMs

Non-probabilistic: optimizes the classification error rather than the likelihood

1) Naïve Bayes

posterior prior likelihood
$$p(label|features) = \frac{p(label)p(features|label)}{p(features)}$$
 evidence

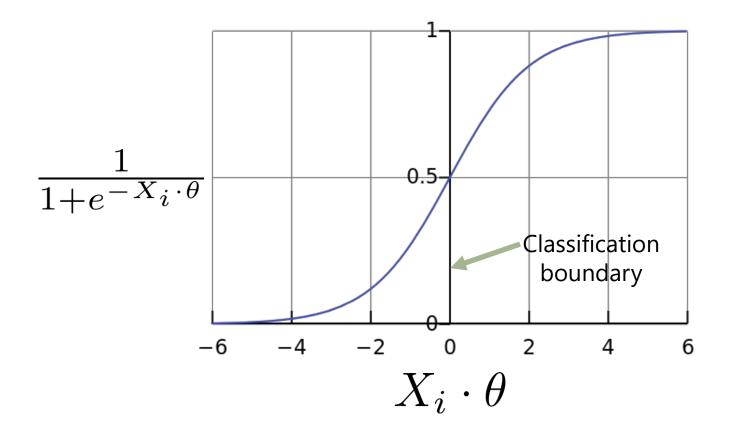
due to our conditional independence assumption:

$$p(label|features) = \frac{p(label)\prod_{i}p(feature_{i}|label)}{p(features)}$$

2) logistic regression

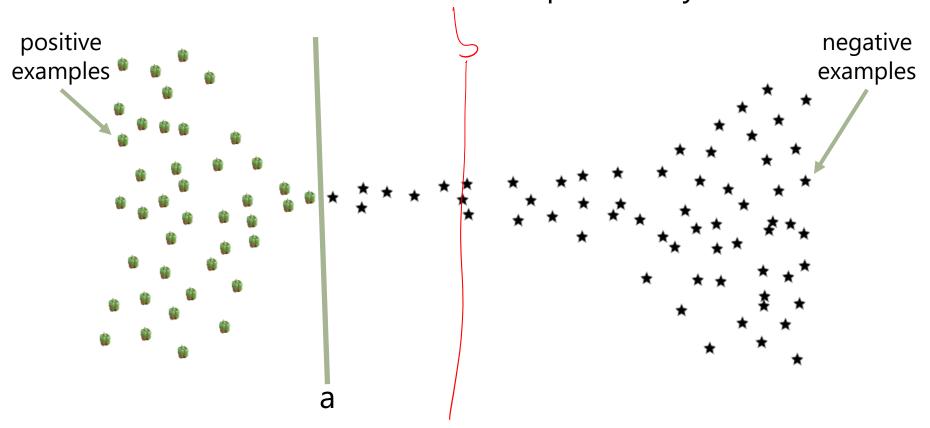
sigmoid function:
$$\sigma(t) = \frac{1}{1+e^{-t}}$$

$$\sigma(t) = \frac{1}{1+e^{-t}}$$



3) Support Vector Machines

Try to optimize the **misclassification error** rather than maximize a probability



Pros/cons

Naïve Bayes

- ++ Easiest to implement, most efficient to "train"
- ++ If we have a process that generates feature that *are* independent given the label, it's a very sensible idea
- -- Otherwise it suffers from a "double-counting" issue

Logistic Regression

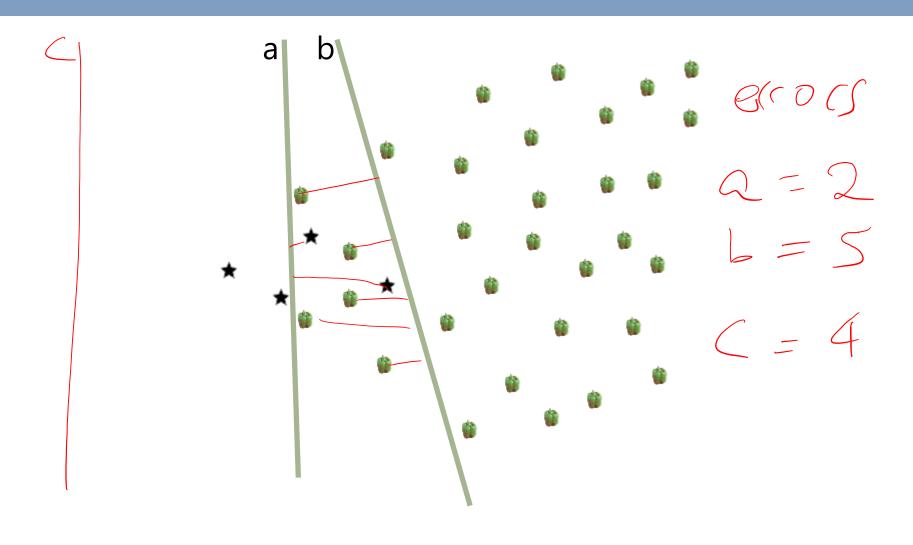
- ++ Fixes the "double counting" problem present in naïve Bayes
- -- More expensive to train

SVMs

- ++ Non-probabilistic: optimizes the classification error rather than the likelihood
- -- More expensive to train

CSE 258 – Lecture 4

Web Mining and Recommender Systems

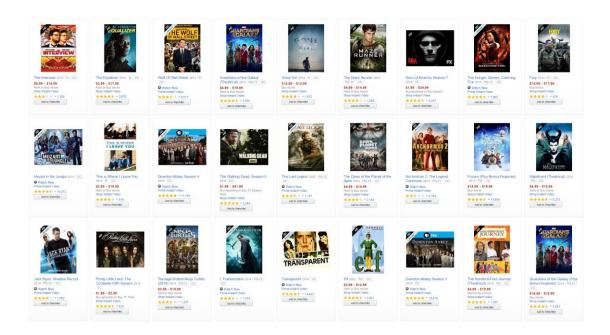


The solution which minimizes the #errors may not be the best one

1. When data are highly imbalanced

If there are far fewer positive examples than negative examples we may want to assign additional weight to negative instances (or vice versa)

e.g. will I purchase a product? If I purchase 0.00001% of products, then a classifier which just predicts "no" everywhere is 99.99999% accurate, but not very useful



2. When mistakes are more costly in one direction

False positives are nuisances but false negatives are disastrous (or vice versa)





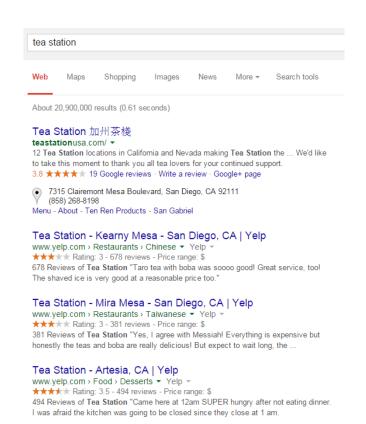


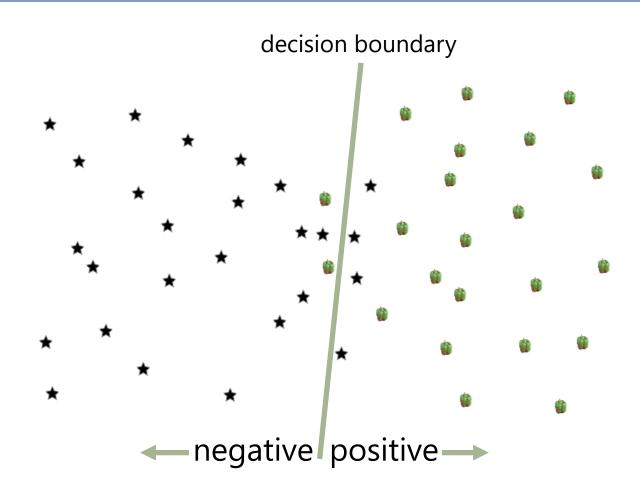


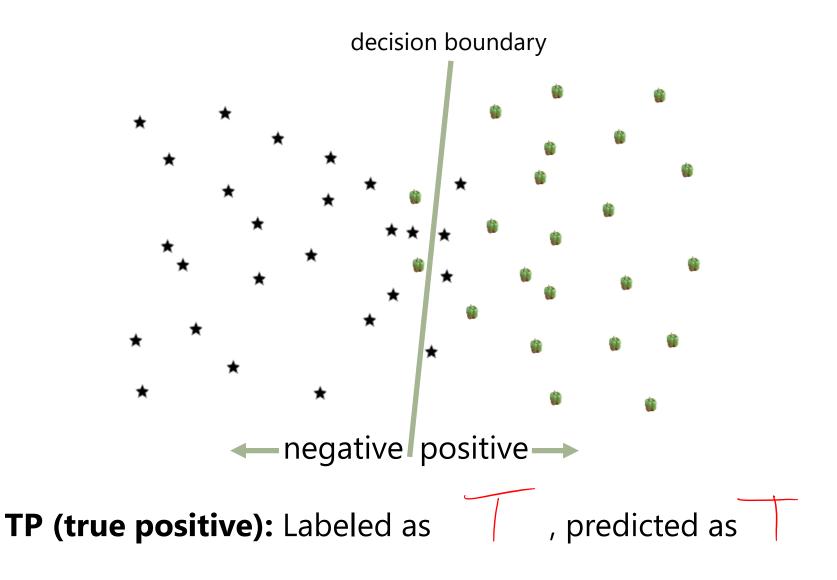
e.g. which of these bags contains a weapon?

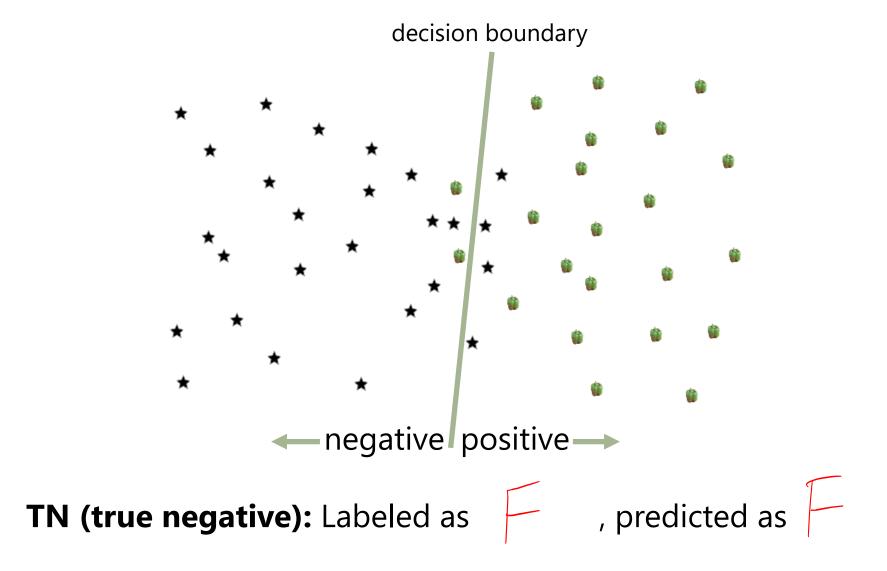
3. When we only care about the "most confident" predictions

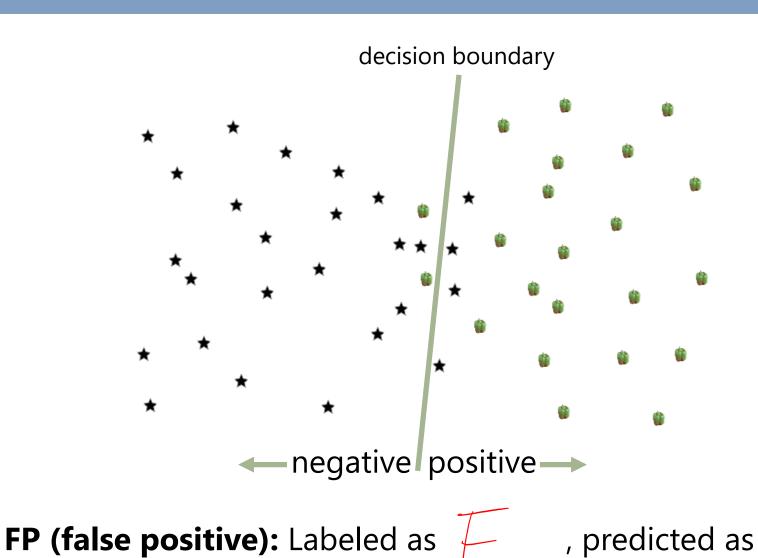
e.g. does a relevant result appear among the first page of results?

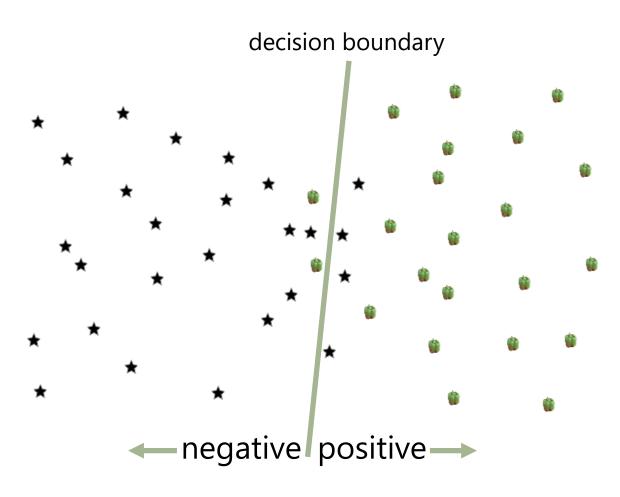












FN (false negative): Labeled as

, predicted as

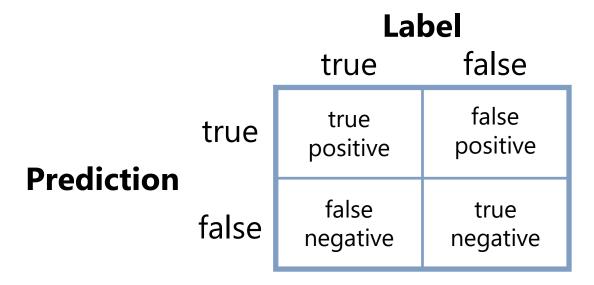
Label false true false true true positive positive **Prediction** false true false negative negative

Classification accuracy

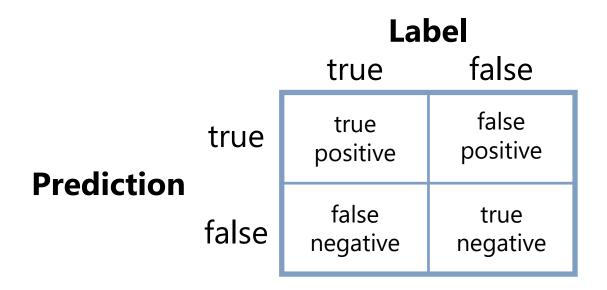
= correct predictions / #predictions = (TP+TN) (TP+TV+FN)

Error rate 1/2 according

= incorrect predictions / #predictions = (FP+FN) / (TP+TN+FN)



True negative rate (**TNR**) = true negatives / #labeled negative = $\tau \mathcal{N}$ / ($\tau \mathcal{N} + \mathcal{F}_{\mathcal{P}}$)



= ½ for a random/naïve classifier, 0 for a perfect classifier

e.g. $\mathbf{y} = [1, -1, 1, 1, 1, -1, 1, 1, -1, 1]$ $\mathbf{Confidence} = [1.3, -0.2, -0.1, -0.4, 1.4, 0.1, 0.8, 0.6, -0.8, 1.0]$ $\mathsf{TP} \quad \mathsf{TN} \quad \mathsf{FN} \quad \mathsf{FN} \quad \mathsf{TP} \quad \mathsf{$

$$TR = \frac{5}{7}$$
 $TR = \frac{3}{4}$
 $DER = 1 - \frac{1}{2} \left(\frac{5}{7} + \frac{2}{4} \right)$

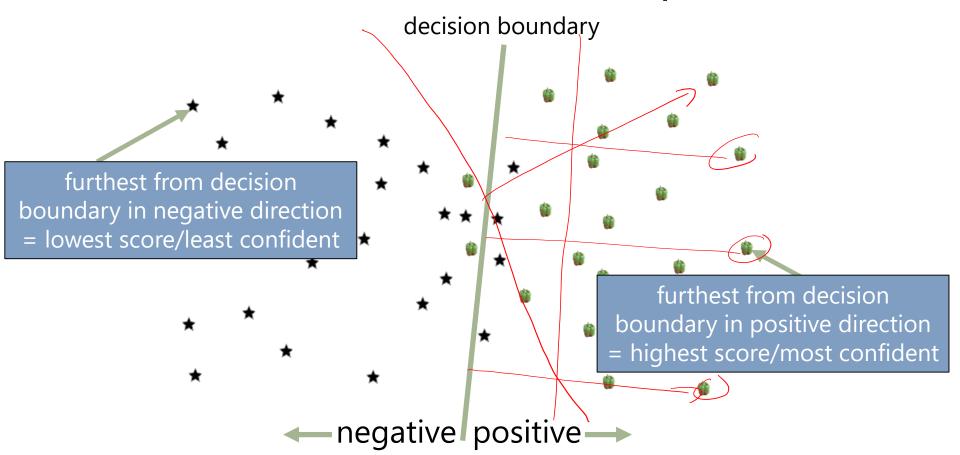
How to optimize a balanced error measure:

$$L_{\theta}(y|X) = \prod_{y_i=1} p_{\theta}(y_i|X_i) \prod_{y_i=0} (1 - p_{\theta}(y_i|X_i))$$

$$\mathcal{L}_{\theta}(y|X) = \underbrace{\sum_{y_i=1} y_{\theta}(X_i, y_i)}_{y_i=0} + \underbrace{\sum_{y_i=0} l_{\theta}g\left(-\sigma(X_i, y_i)\right)}_{y_i=0}$$

$$= \frac{1}{|y| = 1} \sum_{i=1}^{\log r(x_i, o)} + \frac{1}{|y_i - o|} \sum_{j=0}^{\log (1 - o(x_i, o))}$$

The classifiers we've seen can associate **scores** with each prediction



The classifiers we've seen can associate **scores** with each prediction

- In ranking settings, the actual labels assigned to the points (i.e., which side of the decision boundary they lie on) don't matter
- All that matters is that positively labeled points tend to be at higher ranks than negative ones

The classifiers we've seen can associate **scores** with each prediction

- For naïve Bayes, the "score" is the ratio between an item having a positive or negative class
 - For logistic regression, the "score" is just the probability associated with the label being 1
 - For Support Vector Machines, the score is the distance of the item from the decision boundary (together with the sign indicating what side it's on)

The classifiers we've seen can associate **scores** with each prediction

e.g. y = [1, -1, 1, 1, 1, -1, 1, 1, -1, 1] Confidence = [1.3, -0.2, -0.1, -0.4, 1.4, 0.1, 0.8, 0.6, -0.8, 1.0]

Sort **both** according to confidence:

The classifiers we've seen can associate **scores** with each prediction

Suppose we have a fixed budget (say, six) of items that we can return (e.g. we have space for six results in an interface)

- Total number of **relevant** items =
- Number of items we returned =
- Number of **relevant items** we returned = 5

The classifiers we've seen can associate **scores** with each prediction

```
precision = \frac{|\{relevant\ documents\} \cap \{retrieved\ documents\}|}{|\{retrieved\ documents\}|}
```

"fraction of retrieved documents that are relevant"

$$recall = \frac{|\{relevant\ documents\} \cap \{retrieved\ documents\}|}{|\{relevant\ documents\}|}$$

"fraction of relevant documents that were retrieved"

The classifiers we've seen can associate **scores** with each prediction

precision@k = precision when we have a budget of k retrieved documents

e.g.

- Total number of **relevant** items = 7
- Number of items we returned = 6
- Number of relevant items we returned = 5

The classifiers we've seen can associate **scores** with each prediction

$$F_1 = 2 \cdot \frac{\text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}$$

(harmonic mean of precision and recall)

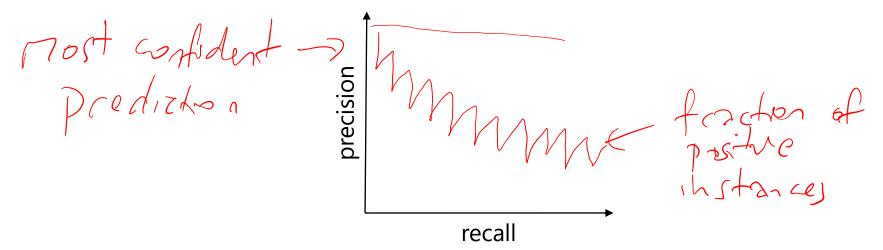
$$F_{\beta} = (1 + \beta^2) \cdot \frac{\text{precision} \cdot \text{recall}}{\beta^2 \text{precision} + \text{recall}}$$

(weighted, in case precision is more important (low beta), or recall is more important (high beta))

Precision/recall curves

How does our classifier behave as we "increase the budget" of the number retrieved items?

- For budgets of size 1 to N, compute the precision and recall
- Plot the precision against the recall



Summary

1. When data are highly imbalanced

If there are far fewer positive examples than negative examples we may want to assign additional weight to negative instances (or vice versa)

e.g. will I purchase product? If I purchase 0.000019 of products, then a classifier which jus predicts "no" everywhere is 99.99999% accurate, but not very useful

Compute the true positive rate and true negative rate, and the F_1 score

F_1 score

F_1 score

F_2 is in in the last state and the last state and

Summary

2. When mistakes are more costly in one direction

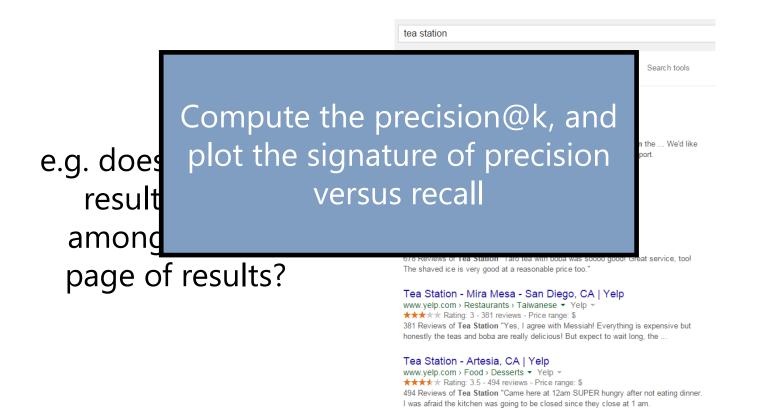
False positives are nuisances but false negatives are disastrous (or vice versa)

Compute "weighted" error measures that trade-off the precision and the recall, like the F_\beta score



Summary

3. When we only care about the "most confident" predictions



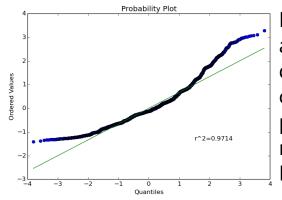
So far: Regression



How can we use **features** such as product properties and user demographics to make predictions about **real-valued** outcomes (e.g. star ratings)?

How can we prevent our models from **overfitting** by favouring simpler models over more complex ones?





How can we assess our decision to optimize a particular error measure, like the MSE?

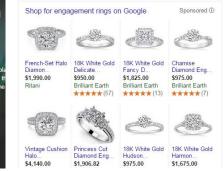
So far: Classification

Next we adapted these ideas to **binary** or **multiclass** outputs



What animal is in this image?





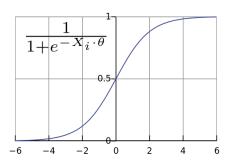
Will I purchase Will I click on this product? this ad?



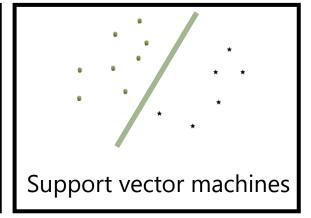




Combining features using naïve Bayes models



Logistic regression



So far: supervised learning

Given labeled training data of the form

$$\{(\mathrm{data}_1, \mathrm{label}_1), \ldots, (\mathrm{data}_n, \mathrm{label}_n)\}$$

Infer the function

$$f(\text{data}) \stackrel{?}{\rightarrow} \text{labels}$$

So far: supervised learning

We've looked at two types of prediction algorithms:

Regression
$$y_i = X_i \cdot \theta$$
 Classification
$$y_i = \begin{cases} 1 & \text{if } X_i \cdot \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

Questions?

Further reading:

- "Cheat sheet" of performance evaluation measures: http://www.damienfrancois.be/blog/files/modelperfcheatsheet.pdf
 - Andrew Zisserman's SVM slides, focused on computer vision:

http://www.robots.ox.ac.uk/~az/lectures/ml/lect2.pdf