

Stable non-Abelian semi-superfluid vortices in dense QCD

Chandrasekhar Chatterjee^a and Muneto Nitta^b

*Department of Physics, and Research and Education Center for Natural Sciences,
Keio University, Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan*

E-mail: chandra.chtrj@gmail.com^a, chandra@phys-h.keio.ac.jp^a, nitta(at)phys-h.keio.ac.jp^b

(Received October 5, 2016)

Color superconductivity is expected to be formed in high density quark matter where color symmetry is spontaneously broken in the presence of di-quark condensate. Stable non-Abelian vortices or color magnetic flux tubes exist in the color-flavor locked phase at asymptotically high density. CP^2 Nambu-Goldstone(NG) bosons and Majorana fermions belonging to the triplet representation are localized around a non-Abelian vortex. We discuss the zero mode analysis and the low-energy effective world sheet theory of a non-Abelian vortex. We determine the interactions of these bosonic and fermionic modes by using the nonlinear realization method. We also discuss the Aharonov-Bohm (AB) phases of charged particles, such as, electrons, muons, and color-flavor locked mesons made of tetra-quarks encircling around a non-Abelian vortex in the presence of electro-magnetic fields. This is a review based on our recent works [1–3].

KEYWORDS: Color superconductor, CFL Phase, Non-Abelian vortices, Zero mode effective action, Majorana Fermion

1. Introduction

As it is well known that quantum chromodynamics (QCD) describes dynamics of interacting quarks and gluons. However free quarks and gluons are not detected in nature: they are confined to hadrons. The asymptotic freedom of $SU(3)$ gauge theory describes QCD correctly at high energies by perturbative quantum field theory, where the coupling constant is small. At low energies the coupling constant becomes very strong and the mechanism of confinement remains unsolved. However if we increase the density sufficiently the system becomes asymptotically free and quarks condensate to color-superconductor due to existing attractive force among them. At asymptotically high densities the mass of the strange quark can be neglected and the system reaches to the most symmetric color-flavor locked (CFL) phase [4,5] (see Ref. [6] as a review). In this case, the baryon number symmetry along with the $SU(3)_c$ color $SU(3)_f$ flavor symmetries are spontaneously broken by forming di-quark condensates. This creates a color superconductor, and by an analogy with ordinary metallic superconductor, one would expect the formation of vortices here. Because the $U(1)_B$ baryon number symmetry is a global symmetry, stable vortices which can be created in this medium is superfluid vortices. These vortices carry color magnetic fluxes, so we may call them as chromo-magnetic flux tubes. The situation is dual to confining flux tubes in hadronic phase where quarks are confined by chromo-electric flux tubes. Beside these theoretical analogies, it is expected that a color superconductor can be found at the core of compact stars. The vortices in color superconductors then could effect the rotation dynamics of compact stars.

In this talk, based on our recent works [1–3], we review some developments of non-Abelian vortices in the CFL phase after the comprehensive review paper [7]. We discuss construction of vortices and their orientational zero modes using the Ginzburg-Landau (GL) formalism and Bogoliubov-de Gennes (BdG) theory for fermions. We discuss interaction of fermion modes with bosonic orientational modes and write down effective interacting action. By introducing the electromagnetic interac-

tions, we study AB phases of electrons, muons and CFL mesons around a vortex.

2. Ginzburg-Landau free energy and non-Abelian vortices in the CFL phase

2.1 Ginzburg-Landau free energy

Let us first introduce the GL description of color superconductors in the CFL phase. The GL order parameters are defined close to the critical temperature T_c by the di-quark condensates as $\Phi_{La}^A \sim \epsilon_{abc} \epsilon^{ABC} q_{Lb}^B C q_{Lc}^C$, $\Phi_{Ra}^A \sim \epsilon_{abc} \epsilon^{ABC} q_{Rb}^B C q_{Rc}^C$, where $q_{L/R}$ are left/right handed quarks carrying fundamental color indices a, b, c ($SU(3)_c$) and fundamental flavour ($SU(3)_{L/R}$) indices A, B, C . At ground state, the chiral symmetry is spontaneously broken due to $\Phi_L = -\Phi_R \equiv \Phi$. The order parameter Φ transforms as $\Phi' = e^{i\theta_B} U_c \Phi U_F^{-1}$, $e^{i\theta_B} \in U(1)_B$, $U_c \in SU(3)_c$, $U_F \in SU(3)_F$. After subtraction of the redundant discrete symmetries the actual symmetry group is given by $G = \frac{SU(3)_c \times SU(3)_F \times U(1)_B}{\mathbb{Z}_3 \times \mathbb{Z}_3}$.

The GL free energy can be expressed as [8–10]:

$$\Omega = \text{Tr} \left[\frac{1}{4\lambda_3} F_{ij}^2 + \frac{\epsilon_3}{2} F_{0i}^2 + K_3 \mathcal{D}_i \Phi^\dagger \mathcal{D}_i \Phi \right] + \alpha \text{Tr} (\Phi^\dagger \Phi) + \beta_1 [\text{Tr} (\Phi^\dagger \Phi)]^2 + \beta_2 \text{Tr} [(\Phi^\dagger \Phi)^2] \quad (1)$$

where $i, j = 1, 2, 3$ are space coordinates indices, λ_3 and ϵ_3 are the magnetic permeability and the dielectric constant for gluons, respectively. Here, $\mathcal{D}_\mu \Phi = \partial_\mu \Phi - ig_s A_\mu^a T^a \Phi$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig_s [A_\mu, A_\nu]$ are the covariant derivative and the field strength of gluons, respectively, where μ, ν indices are the spacetime coordinates and g_s is defined as the $SU(3)_c$ coupling constant. The microscopic calculations of the GL parameters $\alpha, \beta_{1,2}, K_3, \mu$ can be found in Refs. [8, 9].

The ground state is found to be $\langle \Phi \rangle = \Delta_{\text{CFL}} \mathbf{1}_3$ where $\Delta_{\text{CFL}} \equiv \sqrt{-\frac{\alpha}{8\beta}}$, by which the full symmetry group G is spontaneously broken down to $H \simeq \frac{SU(3)_{c+F}}{\mathbb{Z}_3}$. The order parameter space is found to be $G/H \simeq \frac{SU(3) \times U(1)}{\mathbb{Z}_3} = U(3)$.

2.2 Non-Abelian vortices, Nambu-Goldstone modes and Effective action

We now review non-Abelian vortices in the CFL phase. The existence of vortices are supported by non-zero fundamental group $\pi_1(G/H) = \mathbb{Z}$. Here the vortices are global (superfluid) vortices since the broken $U(1)_B$ is a global symmetry and broken color symmetry generates chromo-magnetic fields inside the vortex [11]. We can write down a particular vortex ansatz as

$$\Phi^0(r, \theta) = \Delta_{\text{CFL}} \begin{pmatrix} e^{i\theta} f(r) & 0 & 0 \\ 0 & g(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}, \quad A_i^0(r) = -\frac{1}{3g_s} \frac{\epsilon_{ij} x_j}{r^2} [1 - h(r)] \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (2)$$

The form of the profile functions $f(r)$ and $h(r)$ can be computed numerically with boundary condition, $f(0) = 0$, $\partial_r g(r)|_0 = 0$, $h(0) = 1$, $f(\infty) = g(\infty) = \Delta_{\text{CFL}}$, $h(\infty) = 0$.

As it can be shown from the boundary condition of the profile functions that the vortex configuration in Eq. (2) breaks spontaneously the unbroken ground state symmetry group $SU(3)_{c+F}$ into a subgroup $SU(2) \times U(1)$ inside the vortex core. This symmetry breaking generates continuous degeneracies in vortex solutions and the corresponding NG modes are parametrized by the coset space $\frac{SU(3)}{SU(2) \times U(1)} \simeq \mathbb{C}P^2$ [12]. Applying a global transformation by a reducing matrix $U = \frac{1}{\sqrt{X}} \begin{pmatrix} 1 & -B^\dagger \\ B & X^{\frac{1}{2}} Y^{-\frac{1}{2}} \end{pmatrix}$, $X = 1 + B^\dagger B$, $Y = 1_3 + B B^\dagger$, we can generate the generic solutions on the $\mathbb{C}P^2$ space, where $B = \{B_1, B_2\}$ are inhomogeneous coordinates of $\mathbb{C}P^2$:

$$\Phi(r, \theta) = U \Phi^0(r, \theta) U^\dagger, \quad A_i(r) = U A_i^0(r) U^\dagger. \quad (3)$$

- [20] M. Kobayashi, E. Nakano and M. Nitta, JHEP **1406**, 130 (2014) [arXiv:1311.2399 [hep-ph]].
- [21] M. A. Zubkov, arXiv:1610.08041 [hep-th].
- [22] M. Eto, M. Nitta and N. Yamamoto, Phys. Rev. D **83**, 085005 (2011) [arXiv:1101.2574 [hep-ph]].
- [23] M. G. Alford and A. Sedrakian, J. Phys. G **37**, 075202 (2010) [arXiv:1001.3346 [astro-ph.SR]].

The low-energy excitation and interaction of these NG zero modes are described by the effective action as the $\mathbb{C}P^2$ nonlinear sigma model on the world-sheet coordinate (t, z) as [3, 13, 14]:

$$\mathcal{L}_{\mathbb{C}P^2} = C_\alpha \left[|\partial_\alpha \hat{n}|^2 + (\hat{n}^\dagger \partial_\alpha \hat{n})^2 \right], \quad \hat{n}^T = \frac{1}{\sqrt{1 + |B_i|^2}} (1, B_1, B_2) \quad (4)$$

where C_α are constants written as the integrations of profile functions. This result was obtained first by taking a singular gauge [13, 14] and has been recently confirmed also in a regular gauge [3].

3. Fermion zero modes and effective action

3.1 Fermion zero modes in a particular vortex

To compute fermion zero modes in a particular vortex background we start with the BdG Hamiltonian

$$\mathcal{H} = \bar{\Psi}_a^A \left(\hat{\mathcal{H}}_0 \delta_{ab} \delta^{AB} + \tilde{\Phi}_{ab}^{AB} \right) \Psi_b^B, \quad \Psi = \begin{pmatrix} \psi \\ \psi_c \end{pmatrix}, \quad \psi_a^A = (\psi)_{Aa} = \begin{pmatrix} s_b & s_g & s_r \\ d_b & d_g & d_r \\ u_b & u_g & u_r \end{pmatrix} \quad (5)$$

where $a, b = \{b, g, r\}$, $\{A, B\} = \{s, d, u\}$. Ψ is the quark quasi-particle field written in the Nambu-Gor'kov basis and $\psi_c = e^{i\eta_c} i\gamma^2 \psi^*$ where η_c is an arbitrary phase. Here ψ is written as a 3×3 matrix whose entries are quarks with color and flavor indices. $\hat{\mathcal{H}}_0$ and $\tilde{\Phi}_{ab}^{AB}$ can be expressed as

$$\hat{\mathcal{H}}_0 = \begin{pmatrix} -i\vec{\gamma} \cdot \vec{\nabla} - \gamma^0 \mu & 0 \\ 0 & -i\vec{\gamma} \cdot \vec{\nabla} + \gamma^0 \mu \end{pmatrix}, \quad \tilde{\Phi}_{ab}^{AB} = \begin{pmatrix} 0 & \Phi_{ab}^{AB} \gamma^5 \\ -\Phi_{ab}^{*AB} \gamma^5 & 0 \end{pmatrix}, \quad (6)$$

where Φ_{ab}^{AB} is the generic gap function of a particular vortex defined as $\Phi_{ab}^{AB} \sim \epsilon_{abc} \epsilon^{ABC} \Phi_c^C$ with Φ defined in Eq. (2). The BdG eigenvalue equation is found to be, $\mathcal{H}\Psi = \mathcal{E}\Psi$. A normalizable triplet zero mode can be shown to exist by solving BdG equation and the equations for the triplet states are found to be

$$(\hat{\mathcal{H}}_0 - \tilde{\Delta}_1) \Psi^1 = 0, \quad (\hat{\mathcal{H}}_0 + \tilde{\Delta}_1) \Psi^2 = 0, \quad (\hat{\mathcal{H}}_0 - \tilde{\Delta}_1) \Psi^3 = 0, \quad (7)$$

where $\psi^1 = \frac{dr+ug}{\sqrt{2}}$, $\psi^2 = \frac{dr-ug}{i\sqrt{2}}$, $\psi^3 = \frac{ur-dg}{\sqrt{2}}$ and $\tilde{\Delta}_1 = \Delta_{\text{crl}} \begin{pmatrix} 0 & f(r)e^{i\theta} \gamma^5 \\ -f(r)e^{-i\theta} \gamma^5 & 0 \end{pmatrix}$. The triplet zero modes were found in Refs. [15, 16]. To write down the effective action we introduce the t, z dependence in a factorized way as: $\Psi_L^\rho(t, z, x, y) = \chi_L^\rho(t, z) \Psi_{0L}^\rho(x, y)$, $\Psi_R^\rho(t, z, x, y) = \chi_R^\rho(t, z) \Psi_{0R}^\rho(x, y)$ where $\rho = \{1, 2, 3\}$ (no summation over ρ). Inserting this into the original action we may write down the effective action as

$$\mathcal{L}_{\text{eff}} = -i\text{Tr} \chi^\dagger \left(\dot{\chi} - v_{\text{fermi}} \sigma^3 \partial_z \chi \right), \quad \chi = \chi^\rho \tau^\rho, \quad \tau^\rho = \frac{1}{2} \sigma^\rho \quad (8)$$

where v_{fermi} is the velocity and the two-dimensional spinors $\chi^\rho(t, z)$ are defined by $\chi^\rho(t, z) = \begin{pmatrix} \chi_L^\rho(t, z) \\ \chi_R^\rho(t, z) \end{pmatrix}$.

3.2 The interaction between NG mode and fermion zero mode: nonlinear realization

So far we discussed NG modes and fermion zero modes in the background of a particular vortex solution along the vortex configuration at $B_i = 0$ separately. However they interact once we excite the NG modes (B_i) in the presence of fermion zero modes. This can be understood as follows. The fermion modes are computed in the particular vortex background along the vortex configuration at $B_i = 0$. At that point on $\mathbb{C}P^2$ moduli space the fermion zero modes transform under unbroken $SU(2)$ as triplet linearly. However when we excite the NG modes along a path on the $\mathbb{C}P^2$ moduli space

then the transformation matrices of the fermion zero mode changes since the background vortex configuration changes along the path on moduli space. So the transformation becomes a nonlinear action of the full group $SU(3)$ on the fermion modes. Since NG modes($B_i(t, z)$) are functions of the t, z coordinates, the transformation becomes local on the $t - z$ plane and it generates gauge fields which is basically projection of pure gauge along the $SU(2)$ subspace.

$$A_\alpha = -i \left[U^\dagger(\vec{B}(t, z)) \partial_\alpha U(\vec{B}(t, z)) \right]_\perp, \quad \alpha = \{0, 3\} \quad (9)$$

here \perp shows the projection along unbroken $SU(2)$. The effective action can be written as,

$$\mathcal{L}_{\text{fermi}} = -i \text{Tr} \left[\chi^\dagger(t, z) \left\{ \mathcal{D}_0 \chi(t, z) - v_{\text{fermi}} \sigma^3 \mathcal{D}_z \chi(t, z) \right\} \right], \quad (10)$$

where v is the same for all three triplet zero modes [1].

4. Electromagnetic Interaction and Aharonov-Bohm(AB) Effect

So far we have neglected the electromagnetic (EM) interaction. Here we introduce the EM interaction ($U(1)_{\text{em}}$) as a part of the flavour symmetry group $SU(3)_F$ and the generator of $U(1)_{\text{em}}$ is defined as:

$$Q = \frac{1}{3} \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (11)$$

The introduction of the EM interaction brings two effects. One is that it makes some of \mathbb{CP}^2 modes massive [17] and the other is the effective \mathbb{CP}^2 model is gauged by which the localized \mathbb{CP}^2 modes interact with the bulk EM fields [18]. In this case the massive and massless diagonal gauge fields in the bulk can be expressed as

$$A_\mu^M = \frac{g_s}{g_M} A_\mu^8 - \frac{\eta e}{g_M} A_\mu^{\text{em}}, \quad A_\mu^q = \frac{\eta e}{g_M} A_\mu^8 + \frac{g_s}{g_M} A_\mu^{\text{em}}, \quad (12)$$

respectively, where $\eta = \frac{2}{\sqrt{3}}$ and $g_M^2 = g_s^2 + \eta^2 e^2$. So the EM interaction is effectively generated by A^q gauge field since all fields living in the bulk interact only with A^q effectively. Here we may define the effective EM group as $\tilde{U}(1)^{\text{em}}$. The original EM gauge potential can be written as $A_\mu^{\text{em}} = \frac{g_s}{g_M} A_\mu^q - \frac{\eta e}{g_M} A_\mu^M$, and when in the bulk the massive part vanishes, the effective EM coupling of a particle with charge q becomes $\frac{q g_s}{\sqrt{g_s^2 + \eta^2 e^2}}$. In the presence of the EM interaction, the \mathbb{CP}^2 moduli space is reduced to \mathbb{CP}^1 with one extra point on moduli space. The point is labeled as a Balachandran-Digal-Matsuura(BDM) vortex [11], which is given by Eq. (2). The ansatz for one of the \mathbb{CP}^1 vortex can be expressed as [14]

$$\begin{aligned} \Phi(r, \theta) &= \Delta_{\text{CFL}} \begin{pmatrix} g(r) & 0 & 0 \\ 0 & e^{i\theta} f(r) & 0 \\ 0 & 0 & g(r) \end{pmatrix}, \quad A_i^M(r) T^8 = \frac{1}{6 g_M} \frac{\epsilon_{ij} x_j}{r^2} [1 - h(r)] \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \\ A_i^3(r) T^3 &= -\frac{1}{2 g_s} \frac{\epsilon_{ij} x_j}{r^2} [1 - h(r)] \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \end{aligned} \quad (13)$$

As it is discussed above that the effective charge of a charged particle becomes fractional in the bulk. So when a charged particle encircles any vortex in bulk, it will pick up a non-trivial AB phase. AB phase of any particle with charge e can be written in this case as $\varphi_{\text{AB}} = \frac{\eta e^2}{2\pi g_M} \left| \oint A^M \cdot dl \right|$. The AB phases of electrons, muons and charged CFL mesons due to BDM and \mathbb{CP}^1 vortices are computed as [2]

$$\varphi_{\text{AB}}^{\text{BDM}} = \frac{\eta e^2}{2\pi g_M} \left| \oint A^M \cdot dl \right| = \frac{2e^2}{3g_s^2 + 2e^2}, \quad \varphi_{\text{AB}}^{\mathbb{CP}^1} = \frac{\eta e^2}{2\pi g_M} \left| \oint A^M \cdot dl \right| = \frac{e^2}{3g_s^2 + 2e^2}. \quad (14)$$

5. Summary and Discussion

In this talk, we have discussed non-Abelian semi-superfluid vortices in the CFL phase of dense quark matter. After introducing bosonic and fermionic zero modes around a single vortex, we have written down the interacting effective action of these modes around the vortex. After introducing the EM interaction we have written down AB phases of bulk light particles such as electrons, muons, CFL mesons.

Recent topics after the review [7] which this review cannot cover includes decay of an Abelian vortex into three non-Abelian vortices [19], a non-Abelian vortex lattice and a color magnetism [20], topological superconductivity [21].

The hadron-quark duality between the confinement and CFL phases was studied in the framework of the GL theory [22]. Introducing fermion zero modes as discussed may change the situation. The effect of AB phases in the transport of bulk particles in the CFL phase can be computed as was done in the 2SC phase [23].

Acknowledgement

This work is supported by the Ministry of Education, Culture, Sports, Science (MEXT)-Supported Program for the Strategic Research Foundation at Private Universities “Topological Science” (Grant No. S1511006). C. C. acknowledges support as an International Research Fellow of the Japan Society for the Promotion of Science (JSPS). The work of M. N. is supported in part by a JSPS Grant-in-Aid for Scientific Research (KAKENHI Grant No. 16H03984) and by a Grant-in-Aid for Scientific Research on Innovative Areas “Topological Materials Science” (KAKENHI Grant No. 15H05855) and “Nuclear Matter in Neutron Stars Investigated by Experiments and Astronomical Observations” (KAKENHI Grant No. 15H00841) from the the Ministry of Education, Culture, Sports, Science (MEXT) of Japan.

References

- [1] C. Chatterjee, M. Cipriani and M. Nitta, Phys. Rev. D **93**, no. 6, 065046 (2016) [arXiv:1602.01677 [hep-ph]].
- [2] C. Chatterjee and M. Nitta, Phys. Rev. D **93**, no. 6, 065050 (2016) [arXiv:1512.06603 [hep-ph]].
- [3] C. Chatterjee and M. Nitta, Phys. Rev. D **95**, no. 8, 085013 (2017) [arXiv:1612.09419 [hep-ph]].
- [4] M. G. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B **422**, 247 (1998) [hep-ph/9711395].
- [5] M. G. Alford, K. Rajagopal and F. Wilczek, Nucl. Phys. B **537**, 443 (1999) [hep-ph/9804403].
- [6] M. G. Alford, A. Schmitt, K. Rajagopal and T. Schafer, Rev. Mod. Phys. **80**, 1455 (2008) [arXiv:0709.4635 [hep-ph]].
- [7] M. Eto, Y. Hirono, M. Nitta and S. Yasui, PTEP **2014**, no. 1, 012D01 (2014) [arXiv:1308.1535 [hep-ph]].
- [8] I. Giannakis and H. C. Ren, Phys. Rev. D **65**, 054017 (2002) [hep-ph/0108256].
- [9] K. Iida and G. Baym, Phys. Rev. D **63**, 074018 (2001) [Phys. Rev. D **66**, 059903 (2002)] [hep-ph/0011229].
- [10] K. Iida and G. Baym, Phys. Rev. D **65**, 014022 (2002) [hep-ph/0108149].
- [11] A. P. Balachandran, S. Digal and T. Matsuura, Phys. Rev. D **73**, 074009 (2006) [hep-ph/0509276].
- [12] E. Nakano, M. Nitta and T. Matsuura, Phys. Rev. D **78**, 045002 (2008) [arXiv:0708.4096 [hep-ph]]; Prog. Theor. Phys. Suppl. **174**, 254 (2008) [arXiv:0805.4539 [hep-ph]].
- [13] M. Eto, E. Nakano and M. Nitta, Phys. Rev. D **80**, 125011 (2009) [arXiv:0908.4470 [hep-ph]].
- [14] M. Eto, M. Nitta and N. Yamamoto, Phys. Rev. Lett. **104**, 161601 (2010) [arXiv:0912.1352 [hep-ph]].
- [15] S. Yasui, K. Itakura and M. Nitta, Phys. Rev. D **81**, 105003 (2010) [arXiv:1001.3730 [hep-ph]].
- [16] T. Fujiwara, T. Fukui, M. Nitta and S. Yasui, Phys. Rev. D **84**, 076002 (2011) [arXiv:1105.2115 [hep-ph]].
- [17] W. Vinci, M. Cipriani and M. Nitta, Phys. Rev. D **86**, 085018 (2012) [arXiv:1206.3535 [hep-ph]].
- [18] Y. Hirono and M. Nitta, Phys. Rev. Lett. **109** (2012) 062501 [arXiv:1203.5059 [hep-ph]].
- [19] M. G. Alford, S. K. Mallavarapu, T. Vachaspati and A. Windisch, Phys. Rev. C **93**, no. 4, 045801 (2016) [arXiv:1601.04656 [nucl-th]].