

←  
111-000

# CSCI S607, Exam #2



Noah Hendrickson

hend0800@umn.edu : TO: 5920241

: S# malch9

Problem #1:  $r = 3$ ,  $c = (2, 2, 10)$  orig =  $(0, 0, 0)$  int-p =  $(1, 4, 8)$

$$\eta_i = 1.0 \quad \eta_t = 1.5$$

$$\text{Snell's law: } \frac{\sin \theta_i}{\sin \theta_t} = \frac{\eta_t}{\eta_i}$$

a) find angle of incidence,  $\theta_i$ :  $\theta_i = \arccos(N \cdot I)$

$N$  is normal at int-p,  $I$  is negative ray direction

$$N = \text{int-p} - c = (1, 4, 8) - (2, 2, 10) = (-1, 2, -2) = \left(-\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)$$

$$I = \text{orig} - \text{int-p} = (0, 0, 0) - (1, 4, 8) = (-1, -4, -8) = \left(-\frac{1}{9}, -\frac{4}{9}, -\frac{8}{9}\right)$$

$$\theta_i = \arccos(N \cdot I) = \arccos\left(\frac{1}{27} - \frac{8}{27} + \frac{16}{27}\right) = \arccos\left(\frac{9}{27}\right) = \arccos\left(\frac{1}{3}\right) = 70.53^\circ$$

b) find angle of reflection,  $\theta_r$ :  $\theta_i = \theta_r$ , thus  $\theta_r = 70.53^\circ$

c) find direction of reflected ray:  $\vec{R} = \vec{A} + \vec{S}$

$$A = (aN), a = N \cdot I = \frac{1}{3}, A = \frac{1}{3}N = \left(-\frac{1}{9}, \frac{2}{9}, -\frac{2}{9}\right)$$

$$S = (aN - I) = \left(-\frac{1}{9}, \frac{2}{9}, -\frac{2}{9}\right) - \left(-\frac{1}{9}, -\frac{4}{9}, -\frac{8}{9}\right) = \left(0, \frac{2}{3}, \frac{2}{3}\right)$$

$$R = \left(-\frac{1}{9}, \frac{2}{9}, -\frac{2}{9}\right) + \left(0, \frac{6}{9}, \frac{6}{9}\right) = \left(-\frac{1}{9}, \frac{8}{9}, \frac{4}{9}\right)$$

d) find angle of transmission,  $\theta_t$ :  $\theta_t = \arcsin\left(\frac{\eta_i}{\eta_t} \sin(\theta_i)\right)$

$$\theta_t = \arcsin\left(\frac{1.0}{1.5} \sin(70.53^\circ)\right) = \arcsin(0.6285491) = 38.94^\circ$$

e) find direction of transmitted ray:  $T = \cos \theta_t (-N) + \sin \theta_t \left(\frac{S}{\sin \theta_i}\right)$

$$T = \cos(38.94^\circ) \left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right) + \sin(38.94^\circ) \left(\frac{(0, \frac{2}{3}, \frac{2}{3})}{\sin(70.53)}\right)$$

$$= 0.7778 \left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right) + 0.6285 \left((0, \frac{2}{3}, \frac{2}{3}) / 0.9428\right)$$

$$= (0.2593, -0.5185, 0.5185) + (0, 0.4444, 0.4444)$$

$$= (0.2593, -0.0741, 0.9629)$$

## Problem #2:

The sequence is as such:

→ rotate +90° around y to align with x

→ apply matrix to align with view direction

→ apply transformation matrix.

$$V_f = TRV_0$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

~~$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$~~

~~R requires a bit more work~~

normalize v and d:  $c = (\frac{2}{3}, \frac{1}{3}, -\frac{2}{3})$   $a = (-\frac{2}{3}, \frac{2}{3}, -\frac{1}{3})$

w' and d' are orthogonal. Cross product to find v:

$$a = c \times b = (\frac{1}{3}, \frac{2}{3}, \frac{2}{3})$$

$$R = \begin{bmatrix} a_x, a_y, a_z, 0 \\ b_x, b_y, b_z, 0 \\ c_x, c_y, c_z, 0 \\ 0, 0, 0, 1 \end{bmatrix}^T = \begin{bmatrix} -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}, 0 \\ \frac{2}{3}, \frac{2}{3}, \frac{1}{3}, 0 \\ \frac{1}{3}, \frac{2}{3}, -\frac{2}{3}, 0 \\ 0, 0, 0, 1 \end{bmatrix}$$

This sequence will rotate the vertices into proper location

### Problem #3:

a) rotate around z axis:  $\theta = 23.5^\circ$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0.917 & -0.399 & 0 & 0 \\ 0.399 & 0.917 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

b) given an angle  $\theta$ , the series is;

$$M' = R_2 R_1 M \quad \text{where}$$

$$R_1 = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad R_2 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c) extra credit: orbiting

the series of transformations would be

$$M' = R_y T R_z \quad \text{where } R_y = R_2 \text{ from part b,}$$

$R_z = R_1$  from part b, and

$$T = \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3, 2, 4  
4, 1, 3

5, 0, 2

6, -1, 1

-1 1 1 : -1 1 1  
1 0 1 : 1 0 1

### Problem #7: Camera/View

$$\text{eye} = (0, 0, 0) \quad -n = (0, 0, -1) \quad \text{up} = (0, 1, 0)$$

a) when eye = (2, 3, 5), dir = (1, -1, -1), and up = (0, 1, 0)

$$n' = (-1, 1, 1) \quad u' = \text{up} \times n' = (1, 0, -1) \quad v = n' \times u' = (1, 2, -1)$$
$$n = \left( -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3} \right) \quad u = \left( \frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2} \right) \quad v = \left( \frac{\sqrt{6}}{6}, \frac{2\sqrt{6}}{6}, -\frac{\sqrt{6}}{6} \right)$$

$$d = (\text{dot}(\text{eye}, u), \text{dot}(\text{eye}, v), \text{dot}(\text{eye}, n)) \\ = \left( \frac{7\sqrt{2}}{2}, \frac{\sqrt{6}}{2}, 2\sqrt{3} \right)$$

$$V = \begin{bmatrix} u_x & u_y & u_z & d_x \\ v_x & v_y & v_z & d_y \\ n_x & n_y & n_z & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & \frac{7\sqrt{2}}{2} \\ \frac{\sqrt{6}}{6} & \frac{2\sqrt{6}}{6} & -\frac{\sqrt{6}}{6} & \frac{\sqrt{6}}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 2\sqrt{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) the vectors  $u$ ,  $v$ , and  $n$  will stay the same and so will their corresponding components in  $V$ .

The vector  $d$  and its components in  $V$  will change.

$d_x$  will stay the same,  $d_y$  will also stay the same.

$d_z$  will gradually decrease.

c)  $n$  will change to match the negative of the view dir,

thus,  $u$  will also change because it's dependent on

both  $u$  and  $n$ . ~~it will change because it is dependent on  $u$  and  $n$ .~~

~~and  $u$  will change because it is dependent on  $n$ .~~  $d$  will also change because it is

dependent on all three.  $n$  will spin clockwise

with the view dir so will  $u$  as it is always  $90^\circ$

to  $n$ .  $V$  won't change as the plane spanned by  $u$  and  $n$

doesn't change (still should though to reorthogonalize)

d)  $n$  and  $v$  will change and thus so will  $d$ .

$u$  will stay same because plane spanned by  $n$  and  $v$

does not change.  $n$  and  $v$  both rotate the same

way as view dir with  $n$   $180^\circ$  of it and

$N$   $90^\circ$  more

$$\begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{matrix}$$

$$\begin{matrix} 1 & 0 & 0 \\ 0 & -1 & 0 \end{matrix}$$

$$(0, 0, -1)$$

$$(0, 1, 0)$$

### Problem #5:

a) blue :

b) need to rotate -90 degrees ( $x$  to  $z$ ) around  $y$

$$M = \begin{bmatrix} \cos(-90) & 0 & \sin(-90) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(-90) & 0 & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

c) eye =  $(5, 0, -5)$  view =  $(-1, 0, 0)$  up =  $(0, 1, 0)$

$n = (1, 0, 0)$   $u = \text{up} \times n = (0, 0, -1)$   $v = n \times u = (0, 1, 0)$

$d = (\text{dot}(\text{eye}, u), \text{dot}(\text{eye}, v), \text{dot}(\text{eye}, n)) = (5, 0, 5)$

$$V = \begin{bmatrix} u_x & u_y & u_z & dx \\ v_x & v_y & v_z & dy \\ n_x & n_y & n_z & dz \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 & 0 & -1 & 5 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

?? 8 vertices?? not 6?? ??

$$V_6 = (\sqrt{3}, -\sqrt{3}, -4\sqrt{3}) \quad V_7 = (-\sqrt{3}, -\sqrt{3}, -4\sqrt{3})$$

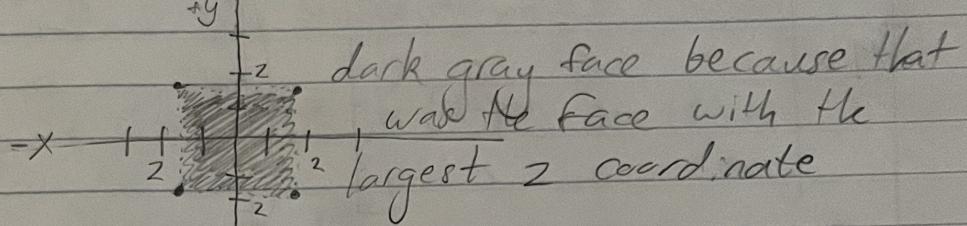
Problem #6:  $V_0 = (\sqrt{3}, \sqrt{3}, -2\sqrt{3}) \quad V_1 = (-\sqrt{3}, \sqrt{3}, -2\sqrt{3}) \quad V_2 = (\sqrt{3}, -\sqrt{3}, -2\sqrt{3})$

a)  $V_3 = (-\sqrt{3}, -\sqrt{3}, -2\sqrt{3}) \quad V_4 = (\sqrt{3}, \sqrt{3}, -4\sqrt{3}) \quad V_5 = (-\sqrt{3}, \sqrt{3}, -4\sqrt{3})$

i)  $V_0 = (\sqrt{3}, \sqrt{3}, 0) \quad V_1 = (-\sqrt{3}, \sqrt{3}, 0) \quad V_2 = (\sqrt{3}, -\sqrt{3}, 0) \quad V_3 = (-\sqrt{3}, -\sqrt{3}, 0)$

$V_4 = (\sqrt{3}, \sqrt{3}, -4\sqrt{3}) \quad V_5 = (-\sqrt{3}, \sqrt{3}, 0) \quad V_6 = (\sqrt{3}, -\sqrt{3}, 0) \quad V_7 = (-\sqrt{3}, -\sqrt{3}, 0)$

ii)



iii) only matrix needed for this is

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b)  $d = (1, 0, \sqrt{3})$

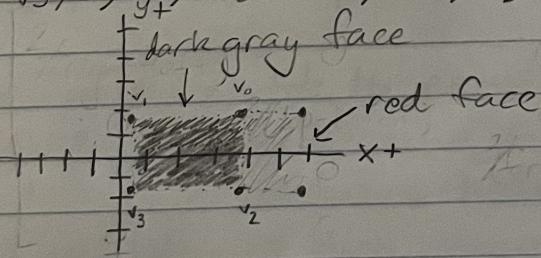
i)  $x_n = x + (-z)\frac{1}{\sqrt{3}} \quad y_n = y + (-z)\frac{0}{\sqrt{3}} = y \quad z_n = 0$

$V_0 = (\sqrt{3}+2, \sqrt{3}, 0) \quad V_1 = (-\sqrt{3}+2, \sqrt{3}, 0) \quad V_2 = (\sqrt{3}+2, -\sqrt{3}, 0)$

$V_3 = (\sqrt{3}+2, -\sqrt{3}, 0) \quad V_4 = (\sqrt{3}+4, \sqrt{3}, 0) \quad V_5 = (\sqrt{3}+4, -\sqrt{3}, 0)$

$V_6 = (\sqrt{3}+4, -\sqrt{3}, 0) \quad V_7 = (-\sqrt{3}+4, -\sqrt{3}, 0)$

ii)



iii) two matrices are needed:

$$ObI = MS \text{ where }$$

$$S = \begin{bmatrix} 1 & 0 & \sqrt{3} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

S shears in the direction d and M is basic orthographic projection.

Cube, vertices:  $c_0 = (0.5, 0.5, -4)$   $c_1 = (-0.5, 0.5, -4)$   
 $c_2 = (0.5, -0.5, -4)$   $c_3 = (-0.5, -0.5, -4)$

Cube<sub>2</sub> vertices:  $b_0 = (1, 1.5, -7)$   $b_1 = (-1, 1.5, -7)$   
 $b_2 = (1, -0.5, -7)$   $b_3 = (-1, -0.5, -7)$

### Problem #7:

a)  $\theta_v = 90^\circ$ , near = 1 far = 10 aspect = 1

$$\text{ProjM} = \begin{bmatrix} \cot(\frac{\theta_v}{2}) & 0 & 0 & 0 \\ 0 & \cot(\frac{\theta_v}{2}) & 0 & 0 \\ 0 & 0 & -\frac{\text{far+near}}{\text{far-near}} & -\frac{2 \cdot \text{far} \cdot \text{near}}{\text{far}-\text{near}} \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{9} & -\frac{20}{9} \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Camera matrix is identity

$$c'_0 = (0.5, 0.5, 2.6666, 1) = (0.125, 0.125, 0.6666, 1)$$

$$c'_1 = (-0.5, 0.5, 2.6666, 1) = (-0.125, 0.125, 0.6666, 1)$$

$$c'_2 = (0.5, -0.5, 2.6666, 1) = (0.125, -0.125, 0.6666, 1)$$

$$c'_3 = (-0.5, -0.5, 2.6666, 1) = (-0.125, -0.125, 0.6666, 1)$$

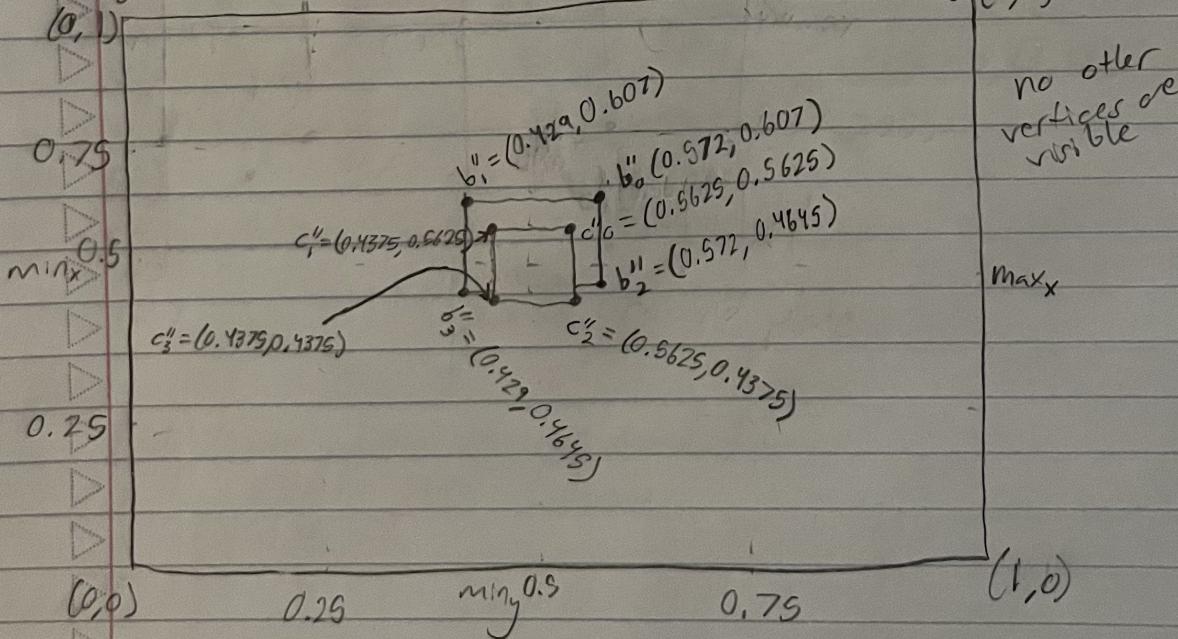
$$b'_0 = (1, 1.5, 6.3333, 1) = (0.143, 0.214, 0.905, 1)$$

$$b'_1 = (-1, 1.5, 6.3333, 1) = (-0.143, 0.214, 0.905, 1)$$

$$b'_2 = (1, -0.5, 6.3333, 1) = (0.143, -0.071, 0.905, 1)$$

$$b'_3 = (-1, -0.5, 6.3333, 1) = (-0.143, -0.071, 0.905, 1)$$

max<sub>y</sub> (1, 1)



camera eye:  $(0, 0, -2)$

$$V = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{Proj } M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{1}{9} & -\frac{20}{9} \\ 0 & -1 & 0 \end{bmatrix}$$

b) Camera matrix no longer just negates. 82

$\text{Proj}_M$  stays the same

$$\text{Order is } V_F = \text{Proj } M \cdot V \cdot V_0 = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{3} & -\frac{14}{3} \\ 0 & 0 & -1 & -2 \end{vmatrix} V_0$$

$$c'_0 = (0.5, 0.5, 0.2222, 2) = (0.25, 0.25, 0.111111, 1) = (0.625, 0.625)$$

$$C_1' = (-0.5, 0.5, 0.2272, 2) = (-0.25, 0.25, 0.111111, 1) = (0.375, 0.625)$$

$$C_2' = (0.5, -0.5, 0.222, 2) = (0.25, -0.25, 0.1111, 1) = (0.625, 0.375)$$

$$C_3 = (-0.5, -0.5, 0.222, 2) = (-0.25, -0.25, 0.1111, 1) = (0.375, 0.375)$$

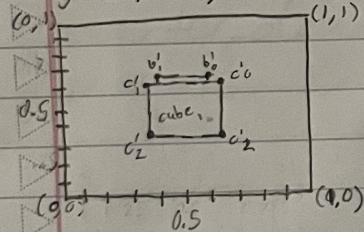
$$b_0' = (1, 1.9, 3.8888, 5) = (0.2, 0.3, 0.7777, 1) = (0.6, 0.65)$$

$$b_1' = (-1, 1.5, 3.8888, 5) = (-0.2, 0.3, 0.7777, 1) = (0.4, 0.65)$$

$$b_2' = (1, -0.5, 3.8888, 5) = (0.2, -0.1, 0.7777, 1) = (0.6, 0.45,$$

$$b'_1 = (-1, -0.5, 3.8888, 5) = (-0.2, -0.1, 0, 7777, 1) = (0.4, 0.45)$$

$(0,1)$   $\xrightarrow{+}$   $(1,1)$



no other vertices are visible

c) projection matrix  
changes in rot values

$$P = \begin{bmatrix} 2.41 & 0 & 0 \\ 0 & 2.41 & 0 \\ 0 & 0 & -\frac{1}{9} - \frac{20}{9}i \end{bmatrix}$$

because  $\theta_v$  is changing.  
Will bring cubes closer

$$C' = (1.205, 1.205, 0.2222, 2) = (0.6025, 0.6025, 0.1111, 1) = (0.80, 0.80)$$

$$C' = (-1.205, 1.205, 0.2222, 2) = (-0.6025, 0.6025, 0.1111, 1) = (0.2, 0.8)$$

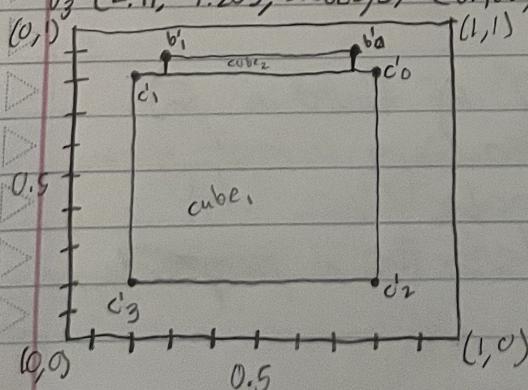
$$\triangle C'_6 = (0.8, 0.2) \quad C'_3 = (0.2, 0.2)$$

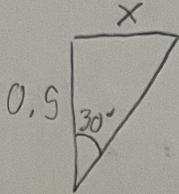
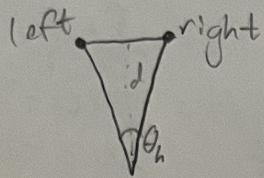
$$b' = (0.482, 0.723, 0.777, 1) = (0.741, 0.862)$$

$$b_1 = (-2.41, 3.615, 3.88885) = (-0.482, 0.723, 0.7777, 1) = (0.259, 0.862)$$

$$k' = (2.41, -1.205, 3.888885) = (0.482, -0.241, 0.7222, 1) = (0.791, 0.3795)$$

$$b_2' = (-2.41, -0.705, 3.8888, 5) = (-0.482, -0.241, 0.7222, 1) = (0.289, 0.3715)$$





$$x = 0.2887$$

$$\tan(30) = \frac{x}{0.5}$$

$$0.5 \tan(30) = x$$

### Problem #8:

a)  $d_{near} = 0.5$   $h_{fov} = 60^\circ$   $v_{fov} = 60^\circ$

$\text{left} = \text{bottom} = -0.2887$   $\text{right} = \text{top} = 0.2887$

$$P = \begin{bmatrix} 1.73 & 0 & 0 & 0 \\ 0 & 1.73 & 0 & 0 \\ 0 & 0 & -1.05 & -1.03 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

b)  $\text{top} = 2 \cdot \text{top} = 0.5774$   $\text{bottom} = -0.5774$

$$P = \begin{bmatrix} 1.73 & 0 & 0 & 0 \\ 0 & 0.866 & 0 & 0 \\ 0 & 0 & -1.05 & -1.03 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

c)  $\theta_h = 60^\circ$  (didn't change)

$$\theta_v = 2 \tan^{-1}(0.5774 / 0.5) = 2 \cdot 49.1 = 98.2^\circ$$

$$d) \begin{bmatrix} 1.73 & 0 & 0 & 0 \\ 0 & 1.73 & 0 & 0 \\ 0 & 0 & -1.05 & -1.03 \\ 0 & 0 & -1 & 0 \end{bmatrix} = P_{alt}$$

e)  $\begin{bmatrix} 0.866 & 0 & 0 & 0 \\ 0 & 1.73 & 0 & 0 \\ 0 & 0 & -1.05 & -1.03 \\ 0 & 0 & -1 & 0 \end{bmatrix}$  aspect ratio is  $\frac{1}{2}$   $= P_{alt}$

f)  $\theta_v = 60^\circ$ , stays same.  $\theta_h = 98.2^\circ$  as found in c)

The contents change by expanding what you can see horizontally but it looks like it's zoomed out? I dk how to describe it properly, like it's distorted.

$$g) P_{att} = \begin{bmatrix} 2.46 & 0 & 0 & 0 \\ 0 & 1.73 & 0 & 0 \\ 0 & 0 & -1.05 & -1.03 \\ 0 & 0 & -1 & 0 \end{bmatrix} \text{ aspect ratio is } 2$$

h)  $\theta_v$  stays at  $60^\circ$  content in the screen zooms in because of the decrease in horizontal fov to find  $\theta_h$ :

$$\text{height}_{1/2} = \tan\left(\frac{\theta_v}{2}\right) \cdot 0.5 = 0.2886 \quad \text{and because}$$

$$\text{aspect ratio} = \frac{h}{w} = \frac{2}{1} \rightarrow \text{width} = \frac{h}{2} = 0.1993$$

$$\theta_h = 2 \arctan\left(\frac{1.993}{0.5}\right) = 32.1^\circ$$

i) you would need to adjust the field of view along with the aspect ratio. i.e. if the window is made taller, increase  $\theta_v$  and if its made shorter, decrease  $\theta_v$ . or alternatively just dont use  $P_{att}$

Problem #9:  $p_0 = (3, 3)$   $p_1 = (9, 5)$   $p_2 = (11, 11)$

a)  $e_1 = p_1 - p_0$   $e_2 = p_2 - p_1$   $e_3 = p_0 - p_2$

$n_1 = (-(p_{1y} - p_{0y}), (p_{1x} - p_{0x}))$

$n_2 = (-(p_{2y} - p_{1y}), (p_{2x} - p_{1x}))$

$n_3 = (-(p_{0y} - p_{2y}), (p_{0x} - p_{2x}))$

$$e_{(p_1-p_0)}(x, y) = (-(p_{ny} - p_{my}), (p_{nx} - p_{mx})) \cdot ((x, y) - (p_{mx}, p_{my})) \\ = -(p_{ny} - p_{my}) \cdot x + (p_{nx} - p_{mx})y + [(p_{ny} - p_{my}) \cdot p_{mx} - (p_{nx} - p_{mx}) \cdot p_{my}]$$

a point  $(x, y)$  is in triangle defined by  $p_0, p_1, p_2$   
if,  $\forall e$  defined by those vertices,  $e(x, y) \geq 0$ :

$$e_{p_1-p_0}(x, y) = -2x + 6y + (6 - 18) = -2x + 6y - 12 \geq 0 \text{ and}$$

$$e_{p_2-p_1}(x, y) = -6x + 2y + (8y - 10) = -6x + 2y + 4y \geq 0 \text{ and}$$

$$e_{p_0-p_2}(x, y) = 8x - 8y + (-8y + 88) = 8x - 8y \geq 0$$

b)  $e_{p_1-p_0}(6, 4) = -2(6) + 6(4) - 12 = 0 \geq 0$  True

$$e_{p_2-p_0}(6, 4) = -6(6) + 2(4) + 4y = 16 \geq 0$$
 True

$$e_{p_0-p_2}(6, 4) = 8(6) - 8(4) = 16 \geq 0$$
 True

point  $(6, 4)$  lies inside

$$e_{p_1-p_0}(7, 7) = -2(7) + 6(7) - 12 = 16 \geq 0$$
 True

$$e_{p_2-p_0}(7, 7) = -6(7) + 2(7) + 4y = 16 \geq 0$$
 True

$$e_{p_0-p_2}(7, 7) = 8(7) - 8(7) = 0 \geq 0$$
 True

point  $(7, 7)$  lies inside

$$e_{p_1-p_0}(10, 8) = -2(10) + 6(8) - 12 = 16 \geq 0$$
 True

$$e_{p_2-p_0}(10, 8) = -6(10) + 2(8) + 4y = 0 \geq 0$$
 True

$$e_{p_0-p_2}(10, 8) = 8(10) - 8(8) = 16 \geq 0$$
 True

point  $(10, 8)$  lies inside

## Problem #10:

for help with this, bounding box bottom left is (0.5, 0.5) and top right is (7.5, 7.5) (each box is 1 unit)  
 $v_0 = (1.5, 8.5)$   $v_1 = (1.5, 1.5)$   $v_2 = (3.5, 6.5)$   $v_3 = (3.5, -0.5)$   
 $v_4 = (6.5, 7.5)$  and  $v_5 = (6.5, 2.5)$

because  $v_0$  and  $v_3$  lie outside clip such that

$v_0$  is split into  $v_{00} = (1.5, 7.5)$  and  $v_{01} = (2.5, 7.5)$

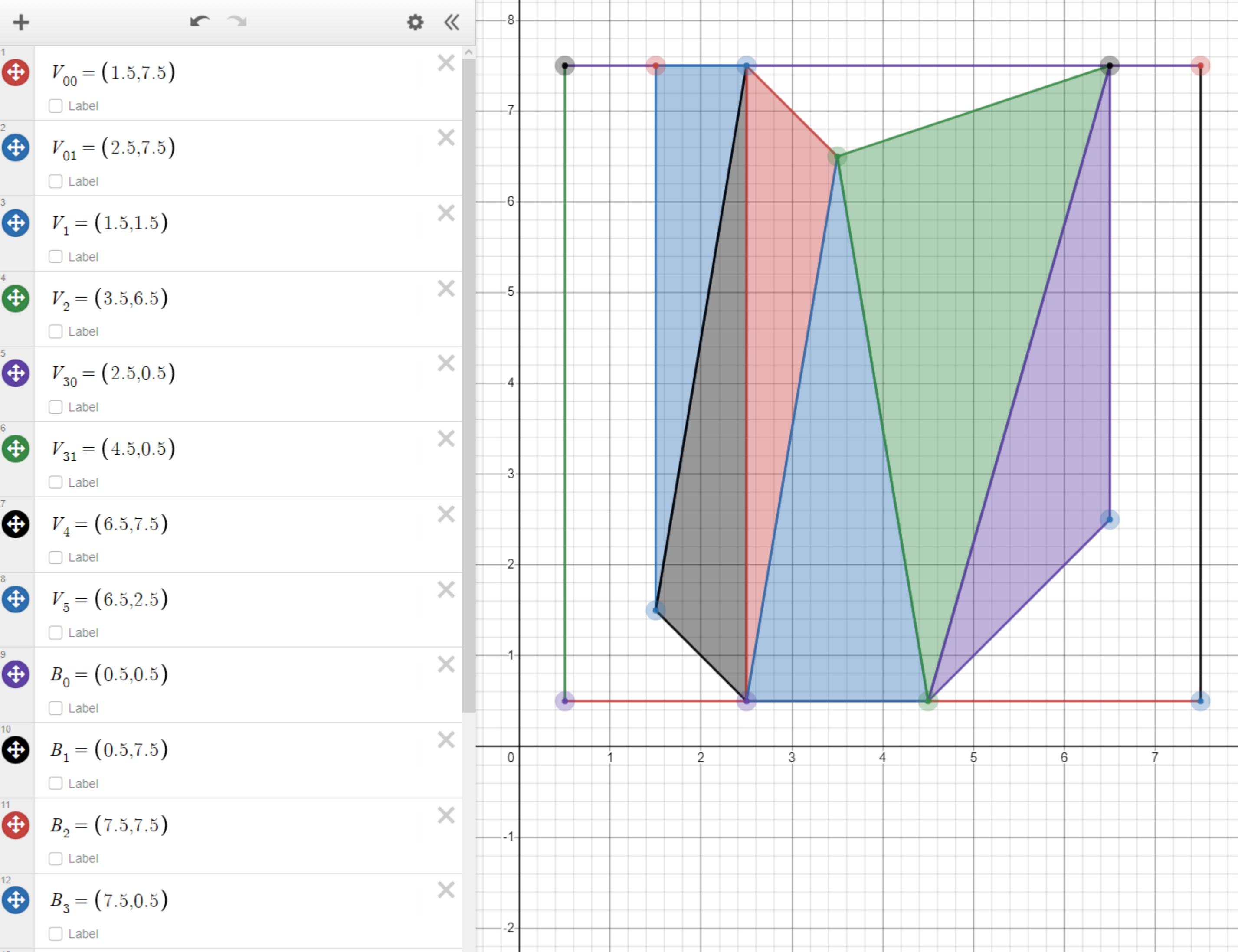
$v_3$  is split into  $v_{30} = (2.5, 0.5)$  and  $v_{31} = (4.5, 0.5)$

In the new strip, 6 triangles will be encoded:

$T_1 = (v_{00}, v_1, v_{01})$   $T_2 = (v_{01}, v_1, v_{30})$   $T_3 = (v_{01}, v_{30}, v_2)$

$T_4 = (v_2, v_{30}, v_{31})$   $T_5 = (v_2, v_{31}, v_4)$   $T_6 = (v_4, v_{31}, v_5)$

image shown on next page of pdf



actual first is  
vertex shader application

## Problem #11: Scan Conversion Steps

→ first is vertex operations:

Apply the model transformation matrix, then the camera matrix, then perspective matrix to get the vertices into normalized clipping coordinates

Once all vertices have been transformed,

apply clipping algorithm and create any new necessary vertices to make the clipped polygons.

Vertices then undergo the viewport transformation which maps  $(x, y)$  values to frame buffer coordinates and  $z$  values to depth buffer values.

→ after is Rasterization

Rasterization rasterizes triangles into fragments such that each fragment maps to exactly one of the pixels enclosed by triangle. This is done by determining which pixels lie inside the triangle with the edge equations.

Anti aliasing can then be done by splitting pixels into sub pixels and coloring based on how much of the pixel is in/out of the triangle.

Information about the triangle is then interpolated across it, such as color, normals, etc...

Other operations such as hidden surface removal and depth testing are also done.

Additionally, if there is a fragment shader attached, that is applied after the fragments are determined.

→ To the Screen

Once all is applied, the frame buffer is shown to the screen

Hidden surface removal uses the  $z$  buffer to get rid of blocked triangles.

## Problem #12:

- ▶ Ray Tracing and Scan conversion are different in that raytracing is built on rays and scan conversion is built on vertices. To achieve perspective, raytracing shoots rays in different directions whereas scan conversion uses a perspective transformation matrix and a view matrix to transform vertex positions. Scan conversion requires triangles or quads whereas raytracing can render anything naturally as long as it has a parametric equation. Scan conversion is much faster than raytracing. When it comes to what each can do, raytracing is far ahead of scan conversion, being able to render incredibly realistic lighting, reflections, and refractions. This all comes at a cost that scan conversion doesn't have to deal with though.
- ▶ Ray tracing can do reflections because you can follow the rays through the scene easily and see what they interact with. With scan conversion, you only have the depth information. The same reasoning for refraction and lighting. However, scan conversion has some nice things like shaders that can be easily applied and clipping of vertices. In order to do something like clipping when ray tracing, you need to make use of hierarchical spatial data structures.
- ▶ When it comes to the shader thing, I don't think you could do that as efficiently? For example, you wouldn't be able to do something like a particle simulation nearly as fast when ray tracing as you can with scan conversion. Both have their upsides and downsides, but personally I think raytracing is far, far cooler 😊