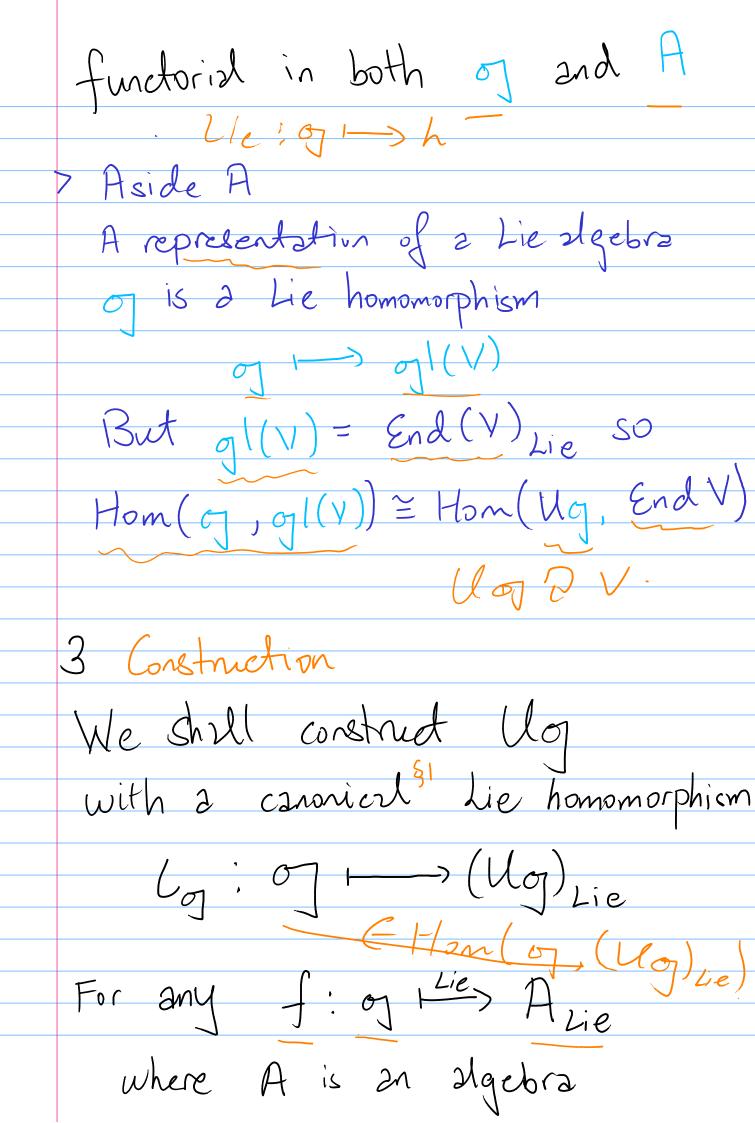
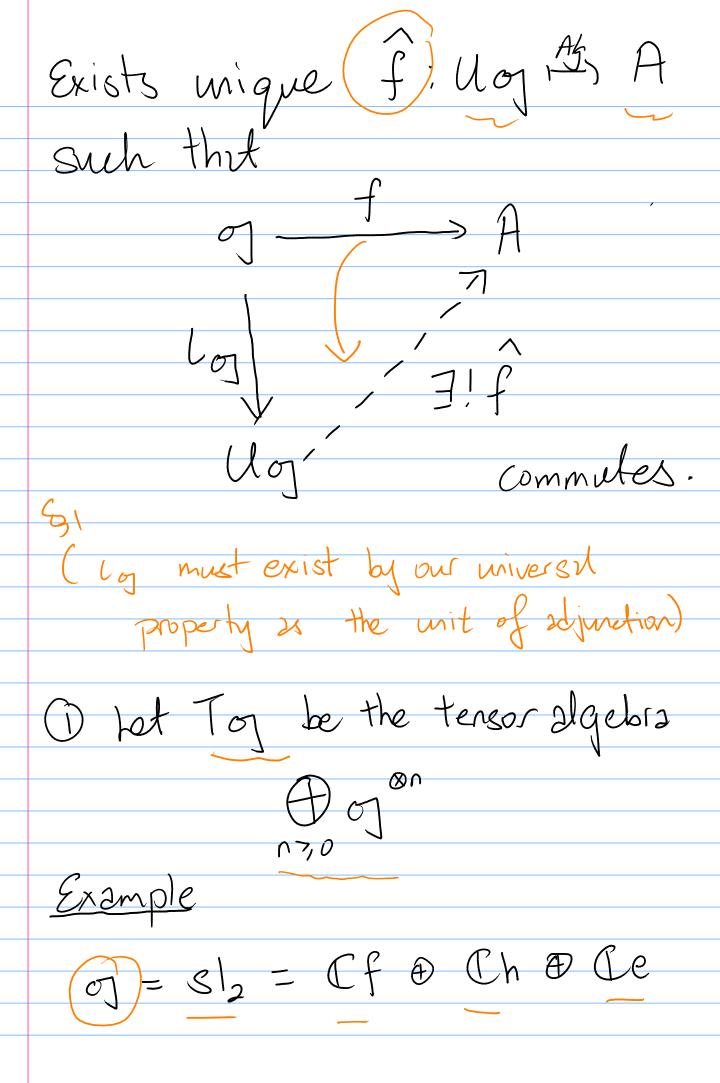
Iniversal Enveloping Algebras Syx Pex 27th October 1 Background and Representations | Modures |
of algebra >
CG 2 Group G/C Natually. Representations Modules Lie 21gebre 01/C of algebra> For today all representations

characteristic

	Why Algebas?
	Key problem of representation theory Given an algebra A find
	Troy problem of representation meng
	Given an algebra H find
	In A Def S (Equivalence classes of) irreducible representations?
	of A
	But we have the A-(left)
	module A. For any SE IrrA,
	we have a homomorphism
	$\int_{\Lambda} A \longrightarrow S$
	$\alpha \longrightarrow \alpha s$
	Sirreducible so fis epi
	So $S \cong A/I$ where I is 2
	maximal ideal of A
4	& Apply techniques of algebra

2 Definition. Recall we have a functor · ie: C-Alg -> C-Lie Take V2 C-Alg, [u,v] = uv - vu, u,v ∈ V Restated goal multiplication as an algebra Find a fundor U: (I-Lie > C-Alq left adjoint to Lie Explicitly for any Lie algebra of get the satisfying the universal property: for any algebra A Hom (Uo, A) = Hom (of, Azie)
as algebras Lie algebras





= span (1) = C 0 = span (s/2) = span (e,f,h)=8/2.

= span (e,f,h)=8/2.

(e+f) & (h) = e & h+f & l

= sl_2 & sl_2 & m & & B | x,B

= sl_2 & sl_2 & m & & B | x,B

= span fee e8 f e8 h

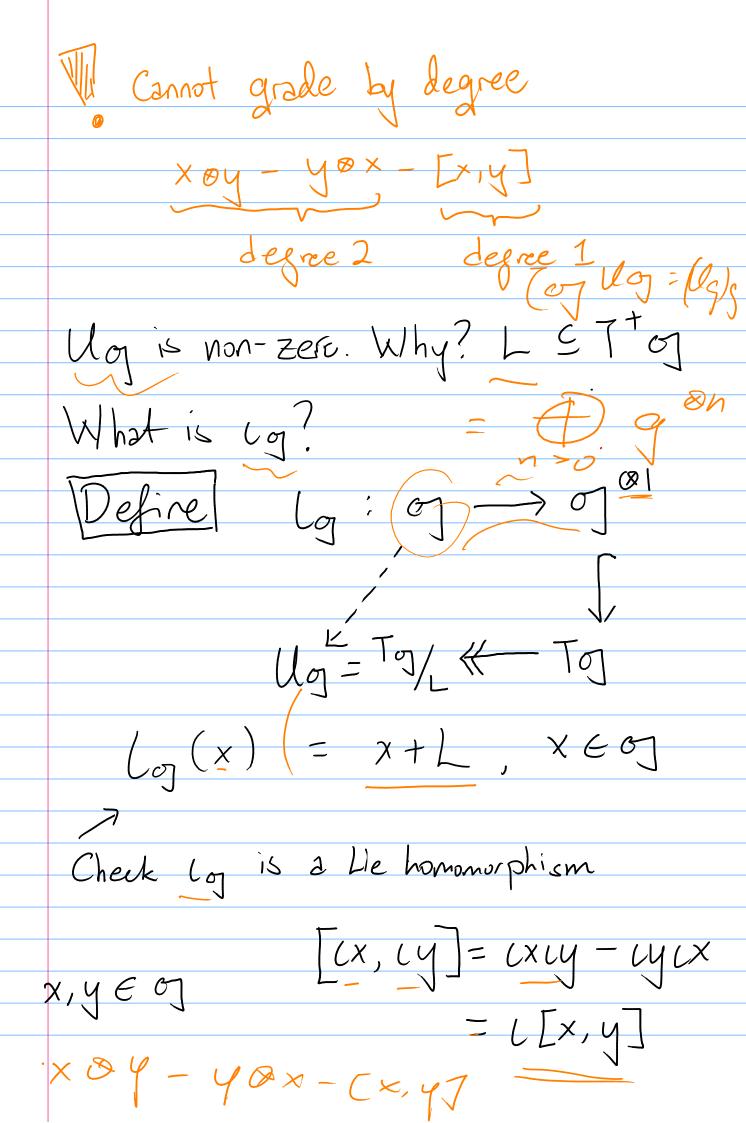
= span fee fof feh

fee hoe hof hah

Toj = hon-commutative

polynomists of e,f & h 2 Define Ug = Tg/L where $L = (x \otimes y - y \otimes x - [x, y] | x, y \in g)$

This has Lie bracket [f,e]=h, [h,e]=[h,f]=0 So $\frac{TH}{he-eh=0}$ hf-fh=0Consider the algebra of operators on Ik[x] generated by In and It This is called the Weyl Algebra W. But $\left(\frac{d}{dx}x - x\right)\frac{d}{dx}g = \frac{d}{dx}(xg) - xg = g$ $e + \frac{1}{5hows} + \frac{2}{4} = \frac{1}{2}$ $e + \frac{1}{5hows} + \frac{1}{4} = \frac{1}{2}$ This turns out to be an infinite dimensional irreducible representation of M



Bijection (fx)(fy) - (fy)(fx) Hom (Ug, A) = {f & Hom(Tg, A) = f [x,y] Alg $\begin{cases} \forall x,y \in 0 \end{cases}$ $\begin{cases} \forall x,y$ is the universal property of tensor algebras. (Tensor funder is the left adjoint of the forgetful fundor) Functoriality! Is Us functor

Left check U respects identity
and composition Hom (Ug, A) = Hom (of, Azie)

as algebras

functorial in both of and A Omitted. 3 Poincaré - Birkhoff - With On: Is in injective? On What is a basis of U07?

Then
$$Toj = C(X)$$

Then $Toj = C(X)$

Ind

Free algebra

 $(X_i, X_i - X_j, X_j - [X_i, X_j])$

Asi de B

Old Problem on mathematical flow.

Square matrices satisfying certain relations must have dimension divisible by 3

Asked 7 years, 8 months ago Active 7 years, 7 months ago Viewed 1k times



I saw this tucked away in a MathOverflow comment and am asking this question to preserve (and advertise?) it. It's a nice problem!

35

Problem: Suppose A and B are real $n \times n$ matrices with $A^2 + B^2 = AB$. If AB - BA is invertible, prove n is a multiple of 3.

()

linear-algebra matrices contest-math

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Step 1 Consider the algebra $\frac{1}{a^2+b^2-ab}$ free 21 gebra ideal generated. Then we have a representation of T sending a > A What is the basis? Give a lexicographical order axb, then abbaba try and reduce words a(al-a)ala using $b^2 = ab - a^2$

Formalizing, we ask if the set of basis elements

without b² is linearly independent Technique (Bergman's Diamond Lemma) Consider 2 reduction system of an algebra with presentation $A = \frac{C(x)}{(b^2 + ab^{-2})}$ $A = \frac{(w_k - f_k) k \in S}{where}$ where of X from C(X) The reduction system is the set of (wx, fx).

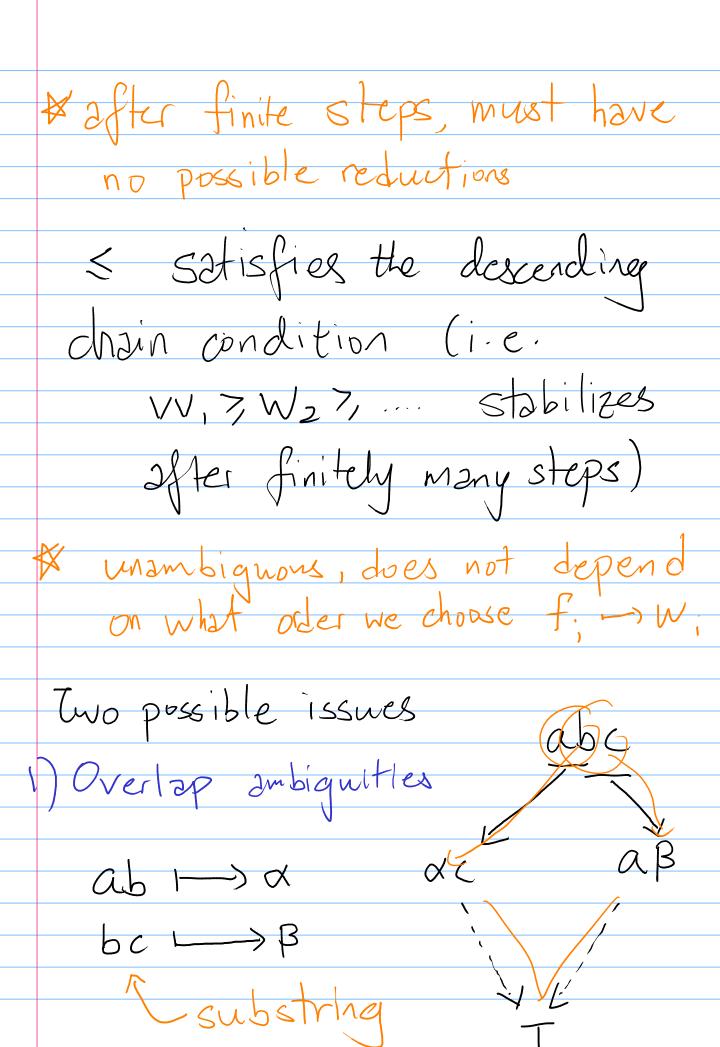
Examples has reduction system $\begin{cases} (yx, xy+1) \end{cases}$ $\begin{cases} (yx, xy+1) \end{cases}$ $\begin{cases} (yx, xy+1) \end{cases}$ $\begin{cases} ef-fe=h \\ he-eh=2e \\ hf-fh=-2f \end{cases}$ has reduction system {(ef, fe+h), (eh, he-2e), (hf, fh-2f)} Good Reduction System * af: b must be betler aw; b # after finite steps, must have no possible reductions & unambiguous, does not depend on what order we choose f; -> w;

Formally, let W be the set of monomials and \hat{W} be the set of irreducible monomials. Define a partial order \leq on W. For $t \in A$ we denote $t \leq s$ if $t_r \leq s$ for all the monomials t_r of t.

For all (wx, fx) of S,

fx > wx

For all a, b \in W



2) Indusion ambiguitles $abc \mapsto \chi$ $b \mapsto R$ $\alpha b c$ abc abc abcTheorem (Damond Lemma) T For any Good Reduction System, W (set of irreducible monomials) form a basis. $\frac{(xample)}{(x,y)}$ has reduction system
{(yx, xy+1)}

Using x < y and degree Lexicographical order shows {xxyx} abasis Similarly for C(e,f,h) ef-fe=h he-eh=2e hf-fh=-2fhas reduction system {(ef, fe+h), (eh, he-2e), (hf, fh-2f)} f<h<e and degree

Lexicographic

W = { f x h e } } But overlap ambignity

(he-eh) = hef e(fh-2f)=efh-2efh(fe+h)-2(fe+h) (feth)h-2(feth) = hfe + h2 - 2fe - 2h = feh + h2 - 2fe - 2h f (he-2e) - fhe-2fe (fh-2f)e +h2-2fe-2h - +h2-2fe-2h = +h2-2fe-2h Theorem (Poincare-Birkhoff-Witt) Let (Xi)iEI be 2 basis of of 2nd < be a total order on I, then llog has basis Xx, Xx2 - Xxx images where $d_1 \leqslant d_2 \leqslant d_k$.

Cavest with Proof.
Use & the missidering index.
If degree is the same the break
by score XIX2 Xx has
score { (i, j) x ; > x ; }
12,77
Corollary
Lg is injective
Corollary
Let h_> of be an
injective Lie homomorphism.
Then Uh > log is injective.
IfSfid
On: Is Ug noetherian? Tested.
Or: He of what of
a 1 1 1 2
On: How does the structure of on effect Uon?

4 Structure of Cloy Let 9, 07 be Lie algebra $1) U(\sigma_1 \times \sigma_2) \cong U\sigma_1 \otimes U\sigma_2$ 2) $U(\sigma_{1}^{\circ p}) \cong (U\sigma_{1})^{\circ p}$ $1) \qquad \begin{array}{c} x \longrightarrow (x,0) \\ 0 / \longrightarrow 0 / \times 0 / 2 \end{array} \qquad \begin{array}{c} y \\ 0 / 2 \end{array}$ ug, -> U(g, xg2) <- Ug2 Ug, & Ug, (4) exists as [(x,0),(0,y)]=0

Surjective follows from PBW

fis Lie Hom 2s

$$[x^{op}, y^{op}] = [y, x]^{op}$$

Count

9 1-> 0 is 2 lie homomorphism

 $x \mapsto 0 \text{ if } x \in \mathcal{I}$

We all & the counit

Comultiplication

of momorphism

 $\triangle: Ug \mapsto U(g \times g) = Ug \otimes Ug$

VXEOJ: X 1 — X X X + 1 & X

We call \(\Delta \) the comultiplication.

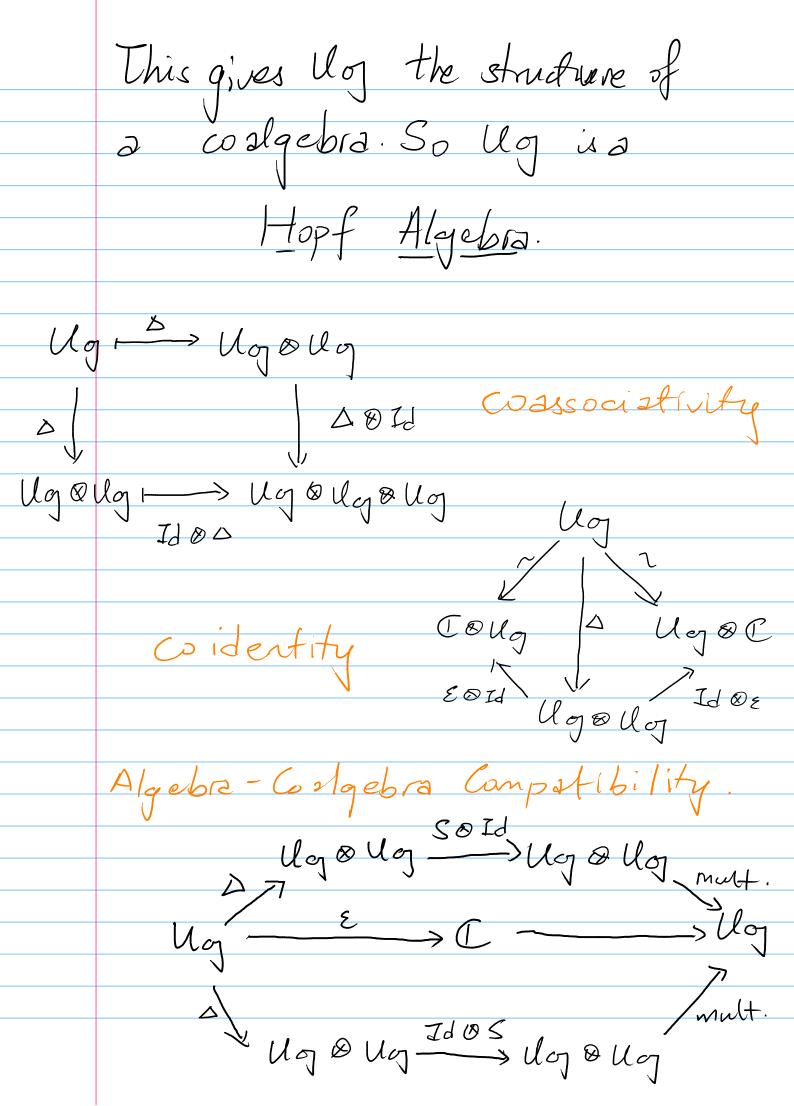
Moreover one can check that

J -> of is 2 Lle Hororo/phlsm x -> - x op

So $S: Ug \xrightarrow{\sim} (Ug)^{op}$

Vx E J: x 1---> - x op

S is the antipode and S2 = Id.



Chank You.

Any Questions!