

5. $D_n = n$

$$D_n = \begin{vmatrix} 1 & -1 & -1 & \dots & -1 & -1 \\ & 1 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & 1 & \\ & & & & & 1 \end{vmatrix}$$

$D_1 = |1| = 1$

$$D_n = \sum_{i=1}^n (-1)^{i+j} a_{ij} \det A_{ij}$$

$$a_{00} A_{00} = \begin{vmatrix} 1 & -1 & -1 & \dots & -1 & -1 \\ & 1 & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & 1 & \\ & & & & & 1 \end{vmatrix} \cdot 1 = 1 \cdot \begin{vmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \ddots & \\ & & & & 1 \end{vmatrix} = 1 \cdot 1 = 1$$

da schrittweise

$$\begin{aligned} R_2 &= R_2 - R_1 \\ R_3 &= R_3 - R_1 \\ &\vdots \\ R_n &= R_n - R_1 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & -1 & \dots & -1 \\ 0 & 2 & 1 & 1 & \dots & 1 \\ 0 & 1 & 2 & 1 & \dots & 1 \\ 0 & 1 & 1 & 2 & 1 & \dots & 1 \\ \vdots & & & & \ddots & \\ 0 & & & & & 2 \end{bmatrix}$$

$$\begin{aligned} R_3 &= R_3 - \frac{1}{2} R_2 \\ R_4 &= R_4 - \frac{1}{2} R_2 \\ &\vdots \\ R_n &= R_n - \frac{1}{2} R_2 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & -1 & \dots & -1 \\ 0 & 2 & 1 & 1 & \dots & 1 \\ 0 & 0 & \frac{3}{2} & \frac{1}{2} & \dots & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{3}{2} & \dots & \frac{1}{2} \\ \vdots & \vdots & & & \ddots & \\ 0 & 0 & & & & \frac{3}{2} \end{bmatrix}$$

$$\begin{aligned} R_4 &= R_4 - \frac{1}{3} R_3 \\ R_5 &= R_5 - \frac{1}{3} R_3 \\ &\vdots \\ R_n &= R_n - \frac{1}{3} R_3 \end{aligned}$$

$$\begin{bmatrix} 1 & -1 & -1 & \dots & -1 \\ 0 & 2 & 1 & 1 & \dots & 1 \\ 0 & 0 & \frac{3}{2} & \frac{1}{2} & \dots & \frac{1}{2} \\ 0 & 0 & 0 & \frac{4}{3} & \dots & \frac{1}{3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \frac{1}{3} & \dots & \frac{4}{3} \end{bmatrix} \rightarrow \dots \rightarrow$$

$$\begin{aligned} R_k &= R_k - \frac{1}{k-1} R_{k-1} \\ &\vdots \\ R_n &= R_n - \frac{1}{n-1} R_{n-1} \end{aligned}$$

$$\rightarrow \begin{bmatrix} 1 & -1 & -1 & \dots & -1 \\ 0 & 2 & 1 & 1 & \dots & 1 \\ 0 & 0 & \frac{3}{2} & \frac{1}{2} & \dots & \frac{1}{2} \\ 0 & 0 & 0 & \frac{4}{3} & \frac{1}{3} & \dots & \frac{1}{3} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \dots & \dots & \frac{n}{n-1} & \frac{1}{n-1} \end{bmatrix} = A$$

$$\det A = 1 \cdot 2 \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdot \dots \cdot \frac{n-1}{n-2} \cdot \frac{1}{n-1}$$

$$\det A = 1 \cdot n = n$$