

def russianPeasantMultiplication(a, b):

result = 0

while (a > 0):

if a % 2 == 1:

result = result + b

a = a / 2

b = b * 2

return result

Зіставимо обчислення:

Обчислимо згорі:

$$\begin{aligned}
 & \sum_{k=1}^{\log a} \log \frac{a}{2^k} + \sum_{k=1}^{\log a} \log \frac{a}{2^k} + \sum_{k=1}^{\log a} \log(b \cdot 2^k) + \log(b \cdot 2^{\log a}) + \sum_{k=1}^{\log a} \log \frac{a}{2^k} + \log b \cdot 2^k = \\
 & = 3 \sum_{k=1}^{\log a} \log \frac{a}{2^k} + 2 \sum_{k=1}^{\log a} \log(b \cdot 2^k) + \sum_{k=1}^{\log a} \log b \cdot 2^k \leq 3 \sum_{k=1}^{\log a} \log a - \log 2^k + 3 \sum_{k=1}^{\log a} \log b + \log 2^k = \\
 & = 3 \sum_{k=1}^{\log a} \log a - 3 \sum_{k=1}^{\log a} k + 3 \sum_{k=1}^{\log a} \log b + 3 \sum_{k=1}^{\log a} k = 3 \sum_{k=1}^{\log a} \log a + \log b = 3 \left[\frac{\log a + 1}{2} \log a + \log a \log b \right] \\
 & \approx 3 \frac{\log^2 a}{2} + 3 \log a \log b \in O(\log a (\log a + \log b)) = O(\log a \log ab)
 \end{aligned}$$

Обчислимо з доту

Розглянемо згорі result = result + b (if odd) and just splitting

$$\begin{aligned}
 & 3 \sum_{k=1}^{\log a} \log \frac{a}{2^k} + \sum_{k=1}^{\log a} \log b \cdot 2^k = 3 \sum_{k=1}^{\log a} \log a - 3 \sum_{k=1}^{\log a} k + \sum_{k=1}^{\log a} \log b + \sum_{k=1}^{\log a} k = \\
 & = 3 \log^2 a + \log a \log b - 2 \frac{(\log a + 1) \log a}{2} \approx 3 \log^2 a - \log^2 a + \log a \log b = \\
 & = 2 \log^2 a + \log a \log b = \log a (2 \log a + \log b) \in O(\log a (\log a + \log b)) = \\
 & = O(\log a \log ab)
 \end{aligned}$$

Зіставимо алгоритм $\in O(\log a \log ab)$