RPiS Lista 1 Krystian Jasionek

1. (2p)

a) $\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = 1$

Zauważmy, że ze wzoru dwumianowego Newtona $(a+b)^n = \sum_{k=0}^{n} a^k b^{n-k}$, zatem jeśli $a=p$, $b=p-1$,

wtedy

$$\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = (p+1-p)^n = 1^n = 1$$

b) $\sum_{k=0}^{n} k\binom{n}{k} p^k (1-p)^{n-k} = np$

$$\sum_{k=0}^{n} k\binom{n}{k} p^k (1-p)^{n-k} = \sum_{k=0}^{n} k\cdot\frac{n!}{(n-k)!\,k!} p^k(1-p)^{n-k} = \sum_{k=0}^{n} \frac{n!}{(n-k)!\,(k-1)!} p^k(1-p)^{n-k} =$$

$$= \sum_{k=1}^{n} \frac{n!}{(n-k)!\,k!} p^k(1-p)^{n-k} = \sum_{k=0}^{n-1} \frac{n!}{(n-k-1)!\cdot k!} p^{k+1}(1-p)^{n-1-k} = np\sum_{k=0}^{n-1} \frac{(n-1)!}{k!\,(n-k-1)!} p^k(1-p)^{n-1-k} =$$

$$= np\sum_{k=0}^{n-1} \binom{n-1}{k} p^k(1-p)^{n-1-k} = np\cdot(p+1-p)^{n-1} = np$$