8. Niech $Y = X^2$ (X określona na $\mathbb{R}$). Wykazać, że

$$f_Y(y) = \frac{f_X(\sqrt{y}) + f_X(-\sqrt{y})}{2\sqrt{y}}, \quad \text{dla } y > 0.$$

$Y = X^2$

$f_Y(y) = F'_Y(y)$, więc wyznaczmy $F_Y(y)$

$$F_Y(y) = P(Y<y) = P(X^2<y) = P(-\sqrt{y} < X < \sqrt{y}) = \int_{-\sqrt{y}}^{\sqrt{y}} f_X(x)\,dx = \left[f_{cx}(x) \right]_{-\sqrt{y}}^{\sqrt{y}} =$$

$$= f_{cx}(\sqrt{y}) - f_{cx}(-\sqrt{y})$$

$$F'_Y(y) = \left(f_{cx}(\sqrt{y}) - f_{cx}(\sqrt{-y}) \right) = \frac{1}{2\sqrt{y}} f(\sqrt{y}) - \frac{1}{2\sqrt{y}} f(-\sqrt{y}) = \frac{f(\sqrt{y}) - f(-\sqrt{y})}{2\sqrt{y}}$$