8. $\vec{\mu}, X \in \mathbb{R}^n$, $\Sigma \in \mathbb{R}^{n\times n}$. Niech $S = (X-\vec{\mu})^T \Sigma^{-1} (X-\vec{\mu})$ oraz $Y = AX$, gdzie A jest odwracalna. Pokażmy, że $S = (Y - A\vec{\mu})^T (A\Sigma A^T)^{-1} (Y - A\vec{\mu})$.

$$(Y-A\vec{\mu})^T (A\Sigma A^T)^{-1} (Y-A\vec{\mu}) = (AX-A\vec{\mu})^T (A\Sigma A^T)^{-1} (AX-A\vec{\mu}) =$$

$$= \left((AX)^T - (A\mu)^T\right) \left(A(A\Sigma)^T\right)^{-1} A(X-\vec{\mu}) =$$

$$= (X^T A^T - \mu^T A^T)\left((A\Sigma^T)^T\right)^{-1} A^{-1} A(X-\vec{\mu}) = (X^T-\mu^T) A^T \left((A\Sigma^T)^{-1}\right)^T I (X-\vec{\mu}) =$$

$$= (X-\mu)^T A^T \left((\Sigma^T)^{-1} A^{-1}\right)^T (X-\vec{\mu}) = (X-\mu)^T A^T (A^{-1})^T \left((\Sigma^{-1})^T\right)^T (X-\vec{\mu}) =$$

$$= (X-\mu)^T (A^{-1}A)^T \Sigma^{-1} (X-\vec{\mu}) = (X-\mu)^T \cdot I^T \Sigma^{-1} (X-\vec{\mu}) = (X-\mu^T)\Sigma^{-1}(X-\vec{\mu})$$