

ZADANIA 5-8. Zmienna $Y = \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ ma rozkład $N(\mu, \Sigma)$, gdzie $\mu = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ oraz $\Sigma = \begin{bmatrix} 4 & 1 \\ 1 & 9 \end{bmatrix}$. Niech $U = \frac{1}{2\sqrt{15}} (-3Y_1 + 2Y_2)$ oraz $V = \frac{1}{2\sqrt{21}} (3Y_1 + 2Y_2 - 12)$.

5. (2 p.) Wykazać, że $Z_1 = U^2 + V^2$. Z_1 jak w zadaniu 8.

8. Jaki jest rozkład zmiennej $Z_1 = (Y - \mu)^T \Sigma^{-1} (Y - \mu)$?

$$Z_1 = U^2 + V^2$$

$$Z_1 = (Y - \mu)^T \Sigma^{-1} (Y - \mu)$$

$$Z_1 = \left(\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right)^T \begin{bmatrix} 4 & 1 \\ 1 & 9 \end{bmatrix}^{-1} \left(\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right)$$

$$Z_1 = \begin{bmatrix} Y_1 - 2 & Y_2 - 3 \end{bmatrix} \begin{bmatrix} \frac{9}{35} & -\frac{1}{35} \\ -\frac{1}{35} & \frac{4}{35} \end{bmatrix} \begin{bmatrix} Y_1 - 2 \\ Y_2 - 3 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} \frac{9Y_1 - 18 - Y_2 + 3}{35} & \frac{-Y_1 + 2 - 4Y_2 - 12}{35} \end{bmatrix} \begin{bmatrix} Y_1 - 2 \\ Y_2 - 3 \end{bmatrix}$$

$$Z_1 = \left(\frac{9Y_1 - Y_2 - 15}{35} \right) (Y_1 - 2) + \frac{(4Y_2 - Y_1 - 10)(Y_2 - 3)}{35}$$

$$Z_1 = \frac{9Y_1^2 - 18Y_1 - Y_2Y_1 + 2Y_2 - 15Y_1 + 30 + 4Y_2^2 - Y_1Y_2 - 10Y_2 - 12Y_2 + 3Y_1 + 30}{35}$$

$$Z_1 = \frac{9Y_1^2 - 30Y_1 - 2Y_1Y_2 - 20Y_2 + 4Y_2^2 + 60}{35}$$

$$U^2 + V^2 = \left[\frac{1}{2\sqrt{15}} (-3Y_1 + 2Y_2) \right]^2 + \left[\frac{1}{2\sqrt{21}} (3Y_1 + 2Y_2 - 12) \right]^2$$

$$= \left[\frac{9Y_1^2 - 12Y_1Y_2 + 4Y_2^2}{60} \right] + \left[\frac{9Y_1^2 + 12Y_1Y_2 + 4Y_2^2 - 72Y_1 - 48Y_2 + 144}{84} \right]$$

$$= \frac{7[\dots]}{420} + \frac{5[\dots]}{420}$$

$$= \frac{63Y_1^2 - 84Y_1Y_2 + 28Y_2^2 + 45Y_1^2 + 60Y_1Y_2 + 20Y_2^2 - 360Y_1 - 240Y_2 + 720}{420}$$

$$= \frac{108Y_1^2 - 360Y_1 - 24Y_1Y_2 - 240Y_2 + 48Y_2^2 + 720}{35 \cdot 12}$$

$$u^2 + v^2 = \frac{8y_1^2 - 30y_1 - 27y_2 - 20y_2 + 4y_2^2 + 60}{35}$$

$$\text{Zudem } u^2 + v^2 = Z_1$$