

6.4,

$$a_{n+2} = 2a_{n+1} - a_n + 5^{2n}, \quad a_0 = a_1 = 0$$

$$a_{n+2} - 2a_{n+1} + a_n - 5^{2n} = 0$$

$$\langle a_n \rangle (E^2 - 2E + 1) (E - 5^2) = 0$$

$$\langle a_n \rangle (\alpha n^2 + \beta n + \gamma) (n + \dots)$$

$$(E - 1)^2 (E - 5^2) \text{ annihilator}$$

$$\alpha n + \beta + 25^n \gamma = 0$$

$$a_2 = 2 \cdot 0 - 0 + 5^0 = 1, \quad a_0 = 0, a_1 = 0$$

$$\begin{cases} 0 = 2\alpha + \beta + 25\gamma \\ 0 = \alpha + \beta + 25^2\gamma \\ 1 = 2\alpha + \beta + 25^2\gamma \end{cases}$$

$$\begin{aligned} (E - a) \langle a^{2i} \rangle &= \langle (i+1) a^{2i+2} - a i a^{2i} \rangle \\ &= \langle i a^{2i+2} + a^{2i+2} - i a^{2i+2} \rangle \\ &= \langle a^{2i+2} \rangle \end{aligned}$$