6. Zmienna X ma standardowy rozkład normalny $X \sim N(0,1)$. Niech $\sigma > 0, \mu \in \mathbb{R}$. Znaleźć rozkład zmiennej $Y = \sigma X + \mu$.

def. $X \sim N(0,1)$, tzn. $f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$Y = \sigma X + \mu$

$$F_Y(y) = P(Y < y) = P(\sigma X + \mu < y) = P\left(X < \frac{y-\mu}{\sigma}\right) = \int_0^{\frac{y-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx =$$

$$= \int_0^{\frac{y-\mu}{\sigma}} f_X(x)\,dx = \Big[F_X(x)\Big]_0^{\frac{y-\mu}{\sigma}} = F_X\left(\frac{y-\mu}{\sigma}\right) - F_X(0)$$

$$f_Y(y) = F_Y'(y) = \frac{d}{dy} F_X\left(\frac{y-\mu}{\sigma}\right) - \frac{d}{dy} F_X(0) = f_X\left(\frac{y-\mu}{\sigma}\right) \cdot \frac{d}{dy}\left(\frac{y-\mu}{\sigma}\right) - f_X(0) \cdot \frac{d}{dy} 0 =$$

$$= \frac{1}{\sigma} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{y-\mu}{\sigma}\right)^2} - f_X(0) \cdot 0 = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{y-\mu}{\sigma}\right)^2}$$