3.6. $a_n = 1 + 2 + \dots + 2^n + (-\sqrt{2})^n$

$$a_n \cancel{b_n} = \sum_{n=0}^{\infty} \left(\sum_{i=0}^{n} 2^i x^i + (-\sqrt{2})^n \right) x^n$$

$$\cancel{b_n\ b(x) = \sum_{n=0}^{\infty} c_n + (-\sqrt{2})^n x^n}$$

$$\cancel{c_n = \sum_{i=0}^{n} 2^i x^i}$$

$$a_n = (2^{n+1} - 1) + (-\sqrt{2})^n$$

$$a_n = 2 \cdot 2^n - 1 + (-\sqrt{2})^n$$

$$\alpha_n = 2 \cdot \frac{1}{1-2x}$$

$$\beta = \frac{1}{1+x}$$

$$\gamma = \frac{1}{1+\sqrt{2}}$$

$$a_n = \frac{2}{1-2x} + \frac{1}{1+x} + \frac{1}{1+\sqrt{2}}$$

$$a_n = \sum_{n=0}^{\infty} \underbrace{(x^0 + 2x + 2^2 x^2 + \dots + 2^n x^n) x^n}_{\alpha(x)} + \underbrace{(-\sqrt{2})^n x^n}_{\beta(x)}$$

$$\alpha(x) = 1x^0 + (1+2)x^1 + (1+2+2^2)x^2 + \dots = 1(x^0 + x^1 + x^2 + \dots) + 2(x^1 + x^2 + \dots) + 2^2(x^2 + x^3 + \dots) + \dots = 1(x^0 + x^1 + \dots) + 2x^1(x^0 + x^1 + \dots) + 2^2 x^2 (x^0 + x^1 + \dots) + \dots =$$

$$= \sum_{n=0}^{\infty} 2^n x^n \sum_{i=0}^{\infty} x^i = \sum_{n=0}^{\infty} 2^n x^n \frac{1}{1-x} = \frac{1}{1-x} \sum_{n=0}^{\infty} 2^n x^n = \frac{1}{1-x} \cdot \frac{1}{1-2x}$$

$$\beta(x) = \sum_{n=0}^{\infty} (-\sqrt{2})^n x^n = \frac{1}{1+\sqrt{2}x}$$

$$A(x) = \frac{1}{(1-x)(1-2x)} + \frac{1}{1+\sqrt{2}x}$$