4) $f(x) = \lambda \exp(-\lambda x)$, $\lambda > 0$.

(a) $\int_0^\infty \lambda e^{-\lambda x} dx = \lambda \int_0^\infty e^{-\lambda x} dx = \lambda \cdot \left[-\frac{1}{\lambda} e^{-\lambda x} \right]_0^\infty = \lambda \left[-\lim_{x \to \infty} \frac{1}{\lambda \cdot e^{\lambda x}} + \frac{1}{\lambda} e^{-\lambda \cdot 0} \right] =$

$= \lambda \left[0 + \frac{1}{\lambda} \right] = 1$

(b) $\int_0^\infty x f(x) dx = \int_0^\infty x \lambda e^{-\lambda x} dx = \lambda \int_0^\infty x e^{-\lambda x} dx = \left| \begin{matrix} f = x & g' = e^{-\lambda x} \\ f' = 1 & g = \frac{-1}{\lambda} \cdot e^{-\lambda x} \end{matrix} \right| =$

$= \lambda \left[\left[x \cdot \frac{-1}{\lambda} e^{-\lambda x} \right]_0^\infty - \int_0^\infty \frac{-1}{\lambda} \cdot e^{-\lambda x} dx \right] = -\left[x \cdot e^{-\lambda x} \right]_0^\infty + \int_0^\infty e^{-\lambda x} dx =$

$= -\lim_{x \to \infty} \frac{x}{e^{\lambda x}} + 0 \cdot e^{-\lambda \cdot 0} + \left[\frac{-1}{\lambda} e^{-\lambda x} \right]_0^\infty = 0 + 0 - \underbrace{\lim_{x \to \infty} \frac{1}{\lambda e^{\lambda x}}}_{=0} + \frac{1}{\lambda} \cdot e^{-\lambda \cdot 0} = \frac{1}{\lambda}$