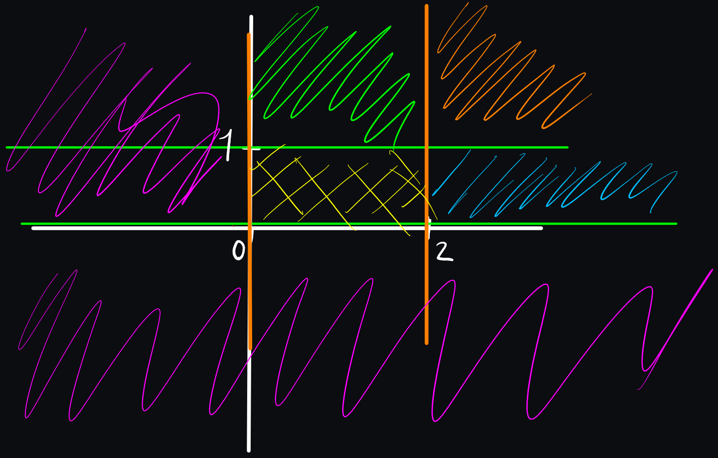
7. **2p.** Zmienna (X,Y) ma rozkład o gęstości $f(x,y)=xy$, na obszarze $[0,2]\times[0,1]$. Wyznaczyć dystrybuantę tej zmiennej, czyli obliczyć $F_{XY}(s,t)=\int_{-\infty}^{s}\int_{-\infty}^{t} xy\,dy\,dx$.

Sprawdzmy $\int_0^2\int_0^1 xy\,dy\,dx = 1.$

$$\int_0^2\int_0^1 xy\,dy\,dx = \int_0^2 \left[\frac{xy^2}{2}\right]_0^1 dx = \int_0^2 \frac{x}{2}\,dx = \left[\frac{x^2}{4}\right]_0^2 = \frac{4}{4} = 1 \quad \checkmark$$



1° Jeśli $s\in(-\infty,0)$ lub $t\in(-\infty,0)$, wtedy $F_{XY}(s,t)=0$.

2° Jeśli $s\in[2,+\infty)$ i $t\in[1,+\infty)$, wtedy $F_{XY}(s,t)=1$

3° $s\in[0,2]$ i $t\in[1,+\infty)$, wtedy $F_{XY}(s,t)=\int_0^1\int_0^s xy\,dx\,dy = \int_0^1\left[\frac{x^2y}{2}\right]_0^s dy =$

$$= \int_0^1 \frac{s^2y}{2} = \left[\frac{s^2y^2}{4}\right]_0^1 = \frac{s^2}{4}$$

4° $s\in[2,+\infty)$ i $t\in[0,1]$, wtedy $F_{XY}(s,t)=\int_0^t\int_0^2 xy\,dx\,dy = \int_0^t\left[\frac{x^2y}{2}\right]_0^2 dy = \int_0^t 2y\,dy =$

$$= \left[y^2\right]_0^t = t^2$$

5° $s\in[0,2]$ i $t\in[0,1]$, wtedy $F_{XY}(s,t)=\int_0^t\int_0^s xy\,dx\,dy = \int_0^t\left[\frac{x^2y}{2}\right]_0^s dy = \int_0^t \frac{s^2y}{2}\,dy =$

$$= \left[\frac{s^2y^2}{4}\right]_0^t = \frac{s^2t^2}{4}$$