8. def. $B(p,q) = \int_0^1 t^{p-1}(1-t)^{q-1}\,dt$, $p,q>0$

(a) $B(p,q+1) = B(p,q)\frac{q}{p+q}$

$$B(p,q+1) = \int_0^1 t^{p-1}(1-t)^{q}\,dt = \int_0^1 t^{p-1}(1-t)^{q-1}(1-t)\,dt = \int_0^1 t^{p-1}(1-t)^{q-1} - t^{p}(1-t)^{q-1}\,dt =$$

$$= \int_0^1 t^{p-1}(1-t)^{q-1}\,dt - \int_0^1 t^{p}(1-t)^{q-1}\,dt = B(p,q) - B(p+1,q)$$

$$B(p,q+1) = B(p,q) - B(p+1,q)$$

$$\boxed{B(p,q) = B(p,q+1) + B(p+1,q)}$$

$$B(p,q+1) = \int_0^1 t^{p-1}(1-t)^{q}\,dt = \left| \begin{matrix} f = (1-t)^q & g' = t^{p-1} \\ f' = -q(1-t)^{q-1} & g = \frac{t^p}{p} \end{matrix} \right| = \left[\frac{t^p}{p}(1-t)^q\right]_0^1 + \int_0^1 \frac{q}{p} t^p (1-t)^{q-1}\,dt =$$

$$= [0-0] + \frac{q}{p}\int_0^1 t^p(1-t)^{q-1}\,dt = \frac{q}{p}B(p+1,q) = \frac{q}{p}\left(B(p,q) - B(p,q+1)\right) =$$

$$= \frac{q}{p}B(p,q) - \frac{q}{p}B(p,q+1)$$

$$B(p,q+1) + \frac{q}{p}B(p,q+1) = \frac{q}{p}B(p,q)$$

$$\frac{p+q}{p}B(p,q+1) = \frac{q}{p}B(p,q) \quad / \cdot \frac{p}{p+q}$$

$$B(p,q+1) = \frac{q}{p}\cdot\frac{p}{p+q}B(p,q)$$

$$B(p,q+1) = \frac{q}{p+q}B(p,q)$$

(b) $B(p,q) = B(p,q+1) + B(p+1,q)$ wykazałem w podpunkcie (a) (nie zrobiono).