niedziela, 7 lutego 2021 14:50

$w(x) = x^3 + 3x^2$

x_i	y_i			
-1	2			
0	0	-2		
2	20	10	4	
3	54	34	8	1

$\underline{w(x)} = 2 - 2(x+1) + 4(x+1)x + \underbrace{(x+1)x(x-2)}$

Postać Czebyszewa:

$\underline{w(x)} = c_0 + c_1 T_1(x) + c_2 T_2(x) + c_3 \underbrace{T_3(x)}$

$1 \quad x \quad 2x^2-1 \quad 4x^3-3$

$(n-1)^2 + (n-2)^2 + \ldots + 1^2 \quad O(n^3)$

$A = L \cdot U \quad n^3$

$L\underline{Ux} = b \quad n^2$

n

2

1

$1 + \ldots + n \quad O(n^2)$

$Ax_1 = b_1 \quad Ax_2 = b \quad \ldots \quad Ax = b_k$

$n^3 \quad n^3 \quad n^3 \quad k \cdot n^3$

$n^2 \quad n^2 \quad n^2$

n^3 wyznaczenie LU

$LU \quad n^3 + k \cdot n^2$

$A[x_1 : x_2 : \ldots : x_k] = [Ax_1 : Ax_2 : \ldots : Ax_k] = [b_1 : b_2 : \ldots : b_k]$

X

$$AX = B$$

$Ax = b_1 \quad Ax = b_2 \quad \ldots$

Z 15.33

$c_k = 2^{(At_k^2 + 2018)^{-1}}$

$\log_2 c_k = (At_k^2 + 2018)^{-1}$

$\frac{1}{\log_2 c_k} - 2018 = \underline{A \cdot t_k^2}$

komb. liniowa

$y_k \approx A \cdot t_k^2$

Apr. średn.

$f \approx \underline{a_0 g_0(x) + \ldots + a_k g_k(x)}$

$At^2 + B \cdot \underline{2018}$

$At^2 + C \cdot 1$

$h(A) = \sum_{k=0}^{N} (y_k - At_k^2)^2 \quad \left(= A^2 \sum t_k^4 + \ldots \right)$

$h'(A) = -2 \sum_{k=0}^{N} (y_k - At_k^2) t_k^2 = \emptyset$

(1) $\sum_{k=0}^{N} y_k t_k^2 = A \cdot \sum_{k=0}^{N} t_k^4$

$\bar{A}$ już jest

$h''(A)\big|_{A=\bar{A}} > \phi.$ Wprost.

$\searrow\nearrow$

Aproks. średn.

$\langle f, g \rangle = \sum_{k=0}^{N} f_k g_k$

$\langle y, t^2 \rangle = A \langle t^2, t^2 \rangle$ ✓

Ukł. równań normalnych $y \approx \operatorname{lin}\{g_0, g_1 \ldots g_k\}$

a_i – rozwiązania układu $w_k^* = \sum_{i=0}^{k} a_i g_i$

$$\begin{bmatrix} \langle g_0, g_0 \rangle & \langle g_0, g_1 \rangle & \ldots & \langle g_0, g_k \rangle \\ \langle g_1, g_0 \rangle & \ldots & \ldots & \langle g_1, g_k \rangle \\ \vdots & & & \vdots \\ \langle g_k, g_0 \rangle & \ldots & \ldots & \langle g_k, g_k \rangle \end{bmatrix} \begin{bmatrix} a_0 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} \langle g_0, y \rangle \\ \langle g_1, y \rangle \\ \vdots \\ \langle g_k, y \rangle \end{bmatrix}$$

$y \approx A \cdot t^2$

$[\underline{\langle t^2, t^2 \rangle}]\,[A] = [\langle \underline{t^2, y} \rangle]$ ✓

$y \approx a + b \cdot x$ $\qquad y \quad \operatorname{lin}\{1, x\}$

$a \cdot 1 + b \cdot x$

$$\begin{bmatrix} \langle 1, 1 \rangle & \langle 1, x \rangle \\ \langle x, 1 \rangle & \langle x, x \rangle \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \langle 1, y \rangle \\ \langle x, y \rangle \end{bmatrix}$$

$$y = \frac{ax^2 - 3}{x^2 + 1}$$

$\underbrace{y(x^2+1) + 3}_{Z} = ax^2$

$Z = ax^2$

$h(a) = \sum_{i=0}^{N} (z_i - a x_i^2)$ analiza

$\langle x^2, x^2 \rangle \cdot a = \langle z, x^2 \rangle$ równania norm.

$\|h\| = \max_{[a,b]} |h(x)|$

Błąd interpolacji

23. $\|f - L_n\|^{\infty}_{[-1,1]} \leq 10^{-8}$

24. $\|f - L_n\|^{\infty}_{[0,1]} \leq 10^{-15}$

(3) $\|f - L_n\|^{\infty}_{[\;]} \leq \dfrac{\|f^{(n+1)}\|}{(n+1)!} \|p_{n+1}(x)\|$

oszacow. dla

1° $\|f^{(n+1)}\|$

2° $\|p_{n+1}\|$

Z 15.23

$f(x) = \cos\left(\frac{x}{2}\right)$

Ad 1° $f^{(n+1)}(x) = (-1)^s\, s(x) \cdot \left(\frac{1}{2}\right)^{n+1}$ $\qquad s \in \{\sin, \cos\}$

$\|f^{(n+1)}\|_{[-1,1]} \leq \dfrac{1}{2^{n+1}}$

$$\| f^{(n+1)} \|_{[-1,1]} \le \frac{1}{2^{n+1}}$$

Ad 2° $\quad p_{n+1}(x) = \frac{1}{2^n} T_{n+1}(x) \qquad \| p_{n+1} \|_{[-1,1]} = \frac{1}{2^n}$

$$(3) \qquad \| f - L_n \|_{[-1,1]} \le \frac{1}{2^{n+1}} \frac{1}{(n+1)!} \frac{1}{2^n} < 10^{-8}$$

Z 15.24 $\qquad T_{n+1}(x) = \cos((n+1) \arccos x)$

$$\arccos x_k = \frac{\frac{\pi}{2} + k\pi}{n+1}$$

$$x = \frac{1}{2} t + \frac{1}{2} \qquad t \in [-1, 1]$$

$$\Downarrow$$

$$x \in [0, 1]$$

$$x_k = \cos \frac{2k+1}{2n+2} \pi \qquad \checkmark$$

Początek: $\quad x \in [0,1] \qquad f(x) = \sin \frac{x}{2}$

przechodzimy do zmiennej t $\qquad \underline{\underline{t = 2x - 1}}$

Jak się zmienia $f(x)$

$$g(t) = f(2x-1) = \sin \frac{2x-1}{2} = \sin\left(x - \frac{1}{2}\right)$$

$$\| g(t) - L_n(t) \| \le \frac{\| g^{(n+1)} \|}{(n+1)!} \| p_{n+1} \| \le \frac{1}{(n+1)!} \frac{1}{2^n}$$

$$\boxed{B_i^n}(t) = \frac{n+1-i}{n+1} B_i^{n+1}(t) + \frac{i+1}{n+1} B_{i+1}^{n+1}(t) \qquad (4)$$

$$\frac{n-i+1}{n+1}$$

$$p(t) = \sum_{k=0}^{n} a_k B_k^n = \sum_{k=0}^{n+1} a_k^{(1)} B_k^{n+1} \quad \Leftarrow$$

Skąd (z jakich wzorów powstaje B_k^{n+1}

$$a_k B_k^n = \underline{\underline{\frac{n-k+1}{n+1} B_k^{n+1}}} + \underline{\quad} B_{k+1}^{n+1}$$

$$a_{k-1} B_{k-1}^n = \underline{\quad} B_{k-1}^{n+1} \quad \underline{\underline{\frac{k}{n+1} B_k^{n+1}}}$$

$$a_k^{(1)}$$

$$\begin{array}{lllll}
T_1 \equiv T_{0,0} & & & & \\
T_2 \equiv T_{1,0} & T_{1,1} & & & \\
\vdots & T_{2,1} & T_{2,2} & & \\
T_{2^k} \equiv T_{k,0} & T_{k,1} & T_{k,2} & \cdots & T_{k,k} \\
\mathcal{O}(h^2) & \mathcal{O}(h^4) & \mathcal{O}(h^6) & & \mathcal{O}(h^{2k+2})
\end{array}$$

$$T_{i,j} = \frac{4^j T_{i,j-1} - T_{i-1,j-1}}{4^j - 1}$$

$$T_{i,0}(f) = I(f) + \mathcal{O}(h^2)$$

$$T_{i,1}(f) = I(f) + \mathcal{O}(h^4)$$

$2^k + 1$ punktów w których obliczamy f.