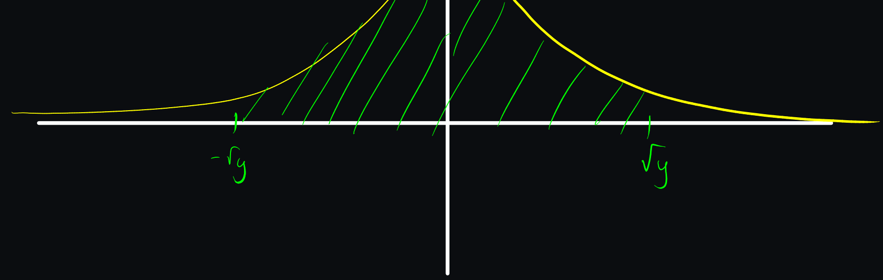
3. Zmienna losowa podlega standardowemu rozkładowi normalnemu, tzn. $f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$, gdzie $x \in \mathbb{R}$. (Skrótowo: $X \sim N(0,1)$). Znaleźć rozkład (gęstość $f_Y(y) \equiv g(y)$) zmiennej $Y = X^2$.

$$f_X(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



$$F_Y(y) = P(Y<y) = P(X^2<y) = P(-\sqrt{y} < X < \sqrt{y}) = \int_{-\sqrt{y}}^{\sqrt{y}} f_X(x)\,dx = \left[F_X(x)\right]_{-\sqrt{y}}^{\sqrt{y}} = \left[F_X(\sqrt{y}) - F_X(-\sqrt{y})\right]$$

$$f_Y(y) = F_Y'(y) = F_X'(\sqrt{y}) - F_X'(-\sqrt{y}) = (\sqrt{y})' \cdot f_X(\sqrt{y}) - (-\sqrt{y})' \cdot f_X(-\sqrt{y}) = \frac{1}{\sqrt{y}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{(\sqrt{y})^2}{2}} - \left(-\frac{1}{\sqrt{y}}\right) \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{(-\sqrt{y})^2}{2}} =$$

$$= \frac{1}{2\sqrt{y}} \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}} + \frac{1}{2\sqrt{y}} \frac{1}{\sqrt{2\pi}} e^{-\frac{y}{2}} = \underline{\frac{1}{\sqrt{y}} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{y}{2}}}$$