9. Dana jest $n$-wymiarowa zmienna losowa $\mathbf{X} = (X_1, \ldots, X_n)^T$. Zmienną $\mathbf{Y} = (Y_1, \ldots, Y_n)^T$ określamy następująco:

$$Y_1 = \bar{\mathbf{X}}, \quad Y_k = X_k - \bar{\mathbf{X}} \ \text{ dla } k = 2, \ldots, n.$$

Znaleźć wartość Jacobianu

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} & \cdots & \frac{\partial x_1}{\partial y_n} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} & \cdots & \frac{\partial x_2}{\partial y_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial x_n}{\partial y_1} & \frac{\partial x_n}{\partial y_2} & \cdots & \frac{\partial x_n}{\partial y_n} \end{vmatrix}.$$

$y_1 = \bar{X}$

$y_k = X_k - \bar{X} \longrightarrow X_k = y_k + \bar{X} = y_k + y_1$

$X_2 = y_2 + y_1$

$X_3 = y_3 + y_1$

$\vdots$

$X_1 = ?$

$y_1 = \bar{X} \Rightarrow y_1 = \frac{x_1 + x_2 + \ldots + x_n}{n} \quad / \cdot n$

$y_1 n = x_1 + x_2 + \ldots + x_n$

$x_1 = y_1 n - x_2 - x_3 - \ldots - x_n$

$x_1 = y_1 n - (y_2 + y_1) - (y_3 + y_1) - \ldots - (y_n + y_1)$

$x_1 = y_1 n - y_2 - y_3 - \ldots - y_n - (n-1) y_1$

$x_1 = y_1 - y_2 - y_3 - \ldots - y_n$

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} & \cdots & \frac{\partial x_1}{\partial y_n} \\ \vdots & & \ddots & \\ \frac{\partial x_n}{\partial y_1} & \frac{\partial x_n}{\partial y_2} & \cdots & \frac{\partial x_n}{\partial y_n} \end{vmatrix} = \begin{vmatrix} \frac{\partial y_1 - y_2 - \ldots - y_n}{\partial y_1} & \frac{\partial y_1 - y_2 - \ldots - y_n}{\partial y_2} & \cdots & \frac{\partial y_1 - y_2 - \ldots - y_n}{\partial y_n} \\ \vdots & & \ddots & \vdots \\ \frac{\partial y_n + y_1}{\partial y_1} & \cdots & \cdots & \frac{\partial y_n + y_1}{\partial y_n} \end{vmatrix} = \begin{vmatrix} 1 & -1 & -1 & \cdots & -1 \\ 1 & 1 & & & \\ 1 & 0 & 1 & & 0 \\ \vdots & & & \ddots & \\ 1 & & & & 1 \end{vmatrix}$$

$J = \left| \text{z zad 5 z listy 1} \right| = n$

5. Wykazać, że $D_n = n$, gdzie

$$D_n = \begin{vmatrix} 1 & -1 & -1 & \ldots & -1 \\ 1 & 1 & & & \\ 1 & & 1 & & \\ \vdots & & & \ddots & \\ 1 & & & & 1 \end{vmatrix}.$$