Raport Kalman

Szymon

MPU9250 Settings

Gyroscope

Scale: $\pm 250 \frac{\text{rad}}{s}$

Sample rate: 3.6 kSa/s Can be set to 8.8 kSa/s

Internal DLPF off.

Accelerometer

Scale: ± 2 g. *The ecompass algorithm operates under no acceleration assumption*

Sample rate: 1.13 kSa/s

Noise density: $250 \frac{\mu g}{\text{rt Hz}}$

Magnetometer

Scale is fixed at $\pm 4800 \mu T$.

Sample rate is 100 Sa/s (MODE2).

Filtering schemes

Extended Kalman Filter

In case the magnetometer is not used, the yaw angle is estimated from the gyroscope measurements only. That is, the observation of the yaw angle is taken from the state vector. The equations remain unchanged.

State:

$$x(k) = \begin{bmatrix} \boldsymbol{q} \\ \omega_w \\ \omega_u \\ \omega_v \end{bmatrix}$$

The angular velocities refer to the local frame.

Update equation

$$\boldsymbol{x}_{k+1|k} = \boldsymbol{F}(t)\boldsymbol{x}_k = \begin{bmatrix} q \\ \omega_w \\ \omega_u \\ \omega_v \end{bmatrix} = \begin{bmatrix} 1 & \omega_w & -\omega_v & \omega_u & 0 & 0 & 0 \\ -\omega_w & 1 & \omega_u & \omega_v & 0 & 0 & 0 \\ \omega_v & -\omega_u & 1 & \omega_w & 0 & 0 & 0 \\ -\omega_u & -\omega_v & -\omega_w & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} q_w \\ q_x \\ q_y \\ q_z \\ \omega_w \\ \omega_u \\ \omega_v \end{bmatrix}$$

Dealing with different sample rates for the sensors

Since the gyroscope could be sampled much more quickly, it has a separate update equation to allow for sampling and updating independently of the accelerometer/magnetometer pair.

Additionally, since the accelerometer is also sampled much more quickly than the magnetometer, the magnetometer data is interpolated.

Interpolation scheme:

- assumption of being constant inbetween measurements
- linear interpolation (?)
- separate LKF for the magnetometer (?)

Measurements vector:

$$y = \begin{bmatrix} \omega_w \\ \omega_u \\ \omega_v \\ a_x \\ a_y \\ a_z \\ M_x \\ M_y \end{bmatrix}$$

Magnetometer measures the magnetic field strength in the global frame M. The magnetic field strength in the local frame is denoted as m.

M_Z is not estimated. It is dependent on the exact location on Earth.

Mapping the state space to the measurement space: the observation equations

$$\omega_w = \omega_w$$

$$\omega_u = \omega_u$$

$$\omega_v = \omega_v$$

Using the Euler angles:

$$\psi = \arctan\left(\frac{2q_w q_x + q_y q_z}{1 - 2\left(q_x^2 + q_y^2\right)}\right)$$

$$\theta = \frac{-\pi}{2} + 2\arctan\left(\frac{\sqrt{1 + 2\left(q_w q_y - q_x q_z\right)}}{\sqrt{1 - 2\left(q_w q_y - q_x q_z\right)}}\right)$$

$$\phi = \arctan\left(\frac{2\left(q_w q_z + q_x q_y\right)}{1 - 2\left(q_y^2 q_z^2\right)}\right)$$

Numerical problems around 0 and 1.

$$a_x = \sin(\theta)$$

$$a_y = -\cos(\theta)\sin(\psi)$$

$$a_z = \cos(\theta)\cos(\psi)$$

$$m_x =$$

$$m_y =$$

Observation matrix Jacobian

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jac =

[(cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))*((imag(q_y/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) - real(q_y/(2*q_w*q_y -
2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z -
2*q_w*q_y + 1)^(1/2))) + ((imag(q_y/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) + real(q_y/
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) +
real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) -
real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))
+ real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2), -(cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*((imag(q_z/(2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)) - real(q_z/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_z/(2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)) + real(q_z/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2), (cos(pi/2 - atan2((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*((imag(q_w/(2*q_x*q_z -
2*q_w*q_y + 1)^(1/2)) - real(q_w/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y -
2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_w/(2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2)) + real(q_w/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z
- 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z
- 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2), -(cos(pi/2 -
atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*((imag(q_x/
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) - real(q_x/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/
(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))
+ ((imag(q_x/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) + real(q_x/(2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)))^2), 0, 0, 0]
[                    - (cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2),
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2*((imag(q_y/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) -
real(q_y/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_y/(2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) + real(q_y/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2) + (sin(pi/2 - atan2((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2*((imag(q_y/
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) - real(q_y/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/
(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))
+ ((imag(q_y/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) + real(q_y/(2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z +
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1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))^2),                                        (cos(pi/2 - atan2((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2*((imag(q_z/(2*q_x*q_z -
2*q_w*q_y + 1)^(1/2)) - real(q_z/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y -
2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_z/(2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2)) + real(q_z/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z
- 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z
- 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y
-  2*q_x*q_z  +  1)^(1/2))  -  real((2*q_x*q_z  -  2*q_w*q_y  +  1)^(1/2)))^2)  -
(sin(pi/2  -  atan2((2*q_w*q_y  -  2*q_x*q_z  +  1)^(1/2),  (2*q_x*q_z  -  2*q_w*q_y  +
1)^(1/2)))^2*((imag(q_z/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) - real(q_z/(2*q_w*q_y -
2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z -
2*q_w*q_y + 1)^(1/2))) + ((imag(q_z/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) + real(q_z/
(2*q_x*q_z  -  2*q_w*q_y  +  1)^(1/2)))*(imag((2*q_x*q_z  -  2*q_w*q_y  +  1)^(1/2))  +
real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) -
real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))
+ real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2),                                       -
(cos(pi/2  -  atan2((2*q_w*q_y  -  2*q_x*q_z  +  1)^(1/2),  (2*q_x*q_z  -  2*q_w*q_y  +
1)^(1/2)))^2*((imag(q_w/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) - real(q_w/(2*q_w*q_y -
2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z -
2*q_w*q_y + 1)^(1/2))) + ((imag(q_w/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) + real(q_w/
(2*q_x*q_z  -  2*q_w*q_y  +  1)^(1/2)))*(imag((2*q_x*q_z  -  2*q_w*q_y  +  1)^(1/2))  +
real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) -
real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))
+  real((2*q_w*q_y  -  2*q_x*q_z  +  1)^(1/2)))^2  +  (imag((2*q_w*q_y  -  2*q_x*q_z  +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2) + (sin(pi/2 - atan2((2*q_w*q_y
-   2*q_x*q_z   +   1)^(1/2),   (2*q_x*q_z   -   2*q_w*q_y   +   1)^(1/2)))^2*((imag(q_w/
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) - real(q_w/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/
(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))
+ ((imag(q_w/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) + real(q_w/(2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)))^2),                 (cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2),
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2*((imag(q_x/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) -
real(q_x/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) -
real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_x/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
+ real(q_x/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))
+ real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))
+ real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2) - (sin(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2*((imag(q_x/(2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)) - real(q_x/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_x/(2*q_w*q_y - 2*q_x*q_z
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+ 1)^(1/2)) + real(q_x/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)/((imag((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2), 0, 0, 0]
[- (2*cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)))*imag(q_x))/abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i + 1)
- (cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))*(q_x*(conj(q_w)*conj(q_x)*2i + conj(q_y)*conj(q_z)*1i + 2*conj(q_x)^2 +
2*conj(q_y)^2 - 1)*2i + conj(q_x)*(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i +
1)*2i)*(2*real(q_x^2) + 2*real(q_y^2) + imag(q_y*q_z) - real(q_w*q_x*2i) - 1))/(2*abs(-
2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i + 1)^2*(-(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2
+ q_z*q_y*1i + 1)*(conj(q_w)*conj(q_x)*2i + conj(q_y)*conj(q_z)*1i + 2*conj(q_x)^2 +
2*conj(q_y)^2 - 1))^(1/2)) - (sin(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2),
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*((imag(q_y/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) -
real(q_y/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_y/(2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) + real(q_y/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2*(2*real(q_x^2) + 2*real(q_y^2)
+ imag(q_y*q_z) - real(q_w*q_x*2i) - 1))/(abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 +
q_z*q_y*1i + 1)*((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)))^2)), - (cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z -
2*q_w*q_y + 1)^(1/2)))*(2*imag(q_w) + 4*real(q_x)))/abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2
+ q_z*q_y*1i + 1) - (cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z -
2*q_w*q_y + 1)^(1/2)))*((conj(q_w)*2i + 4*conj(q_x))*(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2
+ q_z*q_y*1i + 1) + (q_w*2i - 4*q_x)*(conj(q_w)*conj(q_x)*2i + conj(q_y)*conj(q_z)*1i
+ 2*conj(q_x)^2 + 2*conj(q_y)^2 - 1))*(2*real(q_x^2) + 2*real(q_y^2) + imag(q_y*q_z)
- real(q_w*q_x*2i) - 1))/(2*abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i +
1)^2*(-(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i + 1)*(conj(q_w)*conj(q_x)*2i
+ conj(q_y)*conj(q_z)*1i + 2*conj(q_x)^2 + 2*conj(q_y)^2 - 1))^(1/2)) + (sin(pi/2 -
atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*((imag(q_z/
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) - real(q_z/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/
(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))
+ ((imag(q_z/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) + real(q_z/(2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)))^2*(2*real(q_x^2) + 2*real(q_y^2) + imag(q_y*q_z) - real(q_w*q_x*2i) -
1))/(abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i + 1)*((imag((2*q_x*q_z -
2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))^2 + (imag((2*q_w*q_y
- 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)), - (cos(pi/2 -
atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag(q_z)
+ 4*real(q_y)))/abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i + 1) -
(cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)))*((4*conj(q_y) + conj(q_z)*1i)*(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i
+ 1) - (4*q_y - q_z*1i)*(conj(q_w)*conj(q_x)*2i + conj(q_y)*conj(q_z)*1i + 2*conj(q_x)^2
+ 2*conj(q_y)^2 - 1))*(2*real(q_x^2) + 2*real(q_y^2) + imag(q_y*q_z) - real(q_w*q_x*2i)
- 1))/(2*abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i + 1)^2*(-(- 2*q_x^2 +
q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i + 1)*(conj(q_w)*conj(q_x)*2i + conj(q_y)*conj(q_z)*1i
+ 2*conj(q_x)^2 + 2*conj(q_y)^2 - 1))^(1/2)) - (sin(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*((imag(q_w/(2*q_x*q_z - 2*q_w*q_y +
1)^(1/2)) - real(q_w/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z +
```

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1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_w/(2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)) + real(q_w/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2*(2*real(q_x^2) + 2*real(q_y^2)
+ imag(q_y*q_z) - real(q_w*q_x*2i) - 1))/(abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 +
q_z*q_y*1i + 1)*((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)))^2)), - (cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z
- 2*q_w*q_y + 1)^(1/2)))*imag(q_y))/abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i
+ 1) - (cos(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2), (2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)))*(q_y*(conj(q_w)*conj(q_x)*2i + conj(q_y)*conj(q_z)*1i + 2*conj(q_x)^2 +
2*conj(q_y)^2 - 1)*1i + conj(q_y)*(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i +
1)*1i)*(2*real(q_x^2) + 2*real(q_y^2) + imag(q_y*q_z) - real(q_w*q_x*2i) - 1))/(2*abs(-
2*q_x^2 + q_w*q_x*2i - 2*q_y^2 + q_z*q_y*1i + 1)^2*(-(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2
+ q_z*q_y*1i + 1)*(conj(q_w)*conj(q_x)*2i + conj(q_y)*conj(q_z)*1i + 2*conj(q_x)^2 +
2*conj(q_y)^2 - 1))^(1/2)) + (sin(pi/2 - atan2((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2),
(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*((imag(q_x/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) -
real(q_x/(2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)))/(imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))
- real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2))) + ((imag(q_x/(2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) + real(q_x/(2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))*(imag((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2))))/(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2)*(imag((2*q_w*q_y - 2*q_x*q_z +
1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)))^2*(2*real(q_x^2) + 2*real(q_y^2)
+ imag(q_y*q_z) - real(q_w*q_x*2i) - 1))/(abs(- 2*q_x^2 + q_w*q_x*2i - 2*q_y^2 +
q_z*q_y*1i + 1)*((imag((2*q_x*q_z - 2*q_w*q_y + 1)^(1/2)) + real((2*q_w*q_y - 2*q_x*q_z
+ 1)^(1/2)))^2 + (imag((2*q_w*q_y - 2*q_x*q_z + 1)^(1/2)) - real((2*q_x*q_z - 2*q_w*q_y
+ 1)^(1/2)))^2)), 0, 0, 0]
```

Error-State Extended Kalman Filter

Dealing with gyroscope bias

Commonly, there is a thermal model that is used to estimate the gyroscope bias. However, Machony et al. proved, that since the bias is a DC component of the measurements, it can be estimated via integration:

$$b_i = \int_0^t \omega_i \, \mathrm{d}t$$

To allow the sensor to change environments rapidly, I (plan to) use the so-called leaky integrator:

$$b_i = \int_{t-T}^t \omega_i \, \mathrm{d}t$$

The time constant T can be used to tune the filter. Additionally, the leaky integrator is trivial to implement.

Magdwick filter