Analysis 2, Complex Analysis

Mid-Term test 2021

9:00, Wednesday, 24th of February, 2021

1. [5p]

Find out the geometric meaning of $\Omega \subset \mathbb{C}$ defined by

$$|z - 1| \geq 2\,|z - i|.$$

2. [5p]

Let f be continuous on $\gamma = \{z \in \mathbb{C} : |z| = 1\}$. Prove

$$\overline{\oint_\gamma f(z)\,dz} = -\oint_\gamma \frac{\overline{f(z)}}{z^2}\,dz.$$

3. [5p]

Compute

$$f(w) = \frac{1}{2\pi i} \oint_\gamma \frac{dz}{z(z - w)}$$

for all $w : |w| \neq 1$, where

$$\gamma = \{z \in \mathbb{C} : |z| = 1\}.$$

4. [5p]

Let $A = \{z : r \leq |z| \leq R\}$, where $0 < r < R < \infty$. Show that there is a positive number ε such that for an arbitrary polynomial p

$$\max_{z \in A} |p(z) - z^{-1}| > \varepsilon.$$