Probability and Statistics for JMC
Exercises 6 — Estimation

1. If $(X_1, \ldots, X_n)$ are a random sample from an exponential distribution with rate parameter λ, find the maximum likelihood estimate for λ.

2. Derive the maximum likelihood estimate for λ for n independent samples from Poisson(λ).

3. In a study of traffic congestion, data were collected on the number of occupants in private cars on a certain road. These data, collected for 1469 cars, are given below

Count	1	2	3	4	5	≥ 6
Frequency	902	403	106	38	16	4

One theory suggests that these data may have arisen from a modified geometric distribution, in which the probability that there are x occupants in a car is

$$p(x) = p(1-p)^{x-1}, \ x = 1, 2, \ldots$$

(a) Find the maximum likelihood estimate of the parameter p of the geometric distribution for these data. (Note that $\mathrm{P}(X \geq x) = (1-p)^{x-1}$.)

(b) *[To be attempted after the lectures on hypothesis testing]* Describe how a hypothesis test could be carried out, at the 1% level, to see if these data do come from a geometric distribution.

4. (a) For a random sample of size n from a normal distribution with unknown mean μ and known variance σ^2, what is the confidence level for for each of the following confidence intervals for μ?

i. $\bar{x} \pm 1.96\dfrac{\sigma}{\sqrt{n}}$

ii. $\bar{x} \pm 1.645\dfrac{\sigma}{\sqrt{n}}$

iii. $\bar{x} \pm 2.575\dfrac{\sigma}{\sqrt{n}}$

iv. $\bar{x} \pm 0.99\dfrac{\sigma}{\sqrt{n}}$

(b) A random sample of 64 observations from a population produced the following summary statistics:

$$\sum_i x_i = 700 \qquad \sum_i (x_i - \bar{x})^2 = 4238.$$

Find a 95% confidence interval for μ, and interpret this interval.

5. Compute confidence intervals at the 95% level for the means of the distributions from which the following sample values were obtained:

   (a) $n = 100, \quad \sum_i x_i = 250, \quad \sum_i x_i^2 = 725000$

   (b) $n = 100, \quad \bar{x} = 83.2, \quad s_{n-1} = 6.4$

6. The following random sample was selected from a normal distribution:

   $$7.53, \quad 4.35, \quad 7.66, \quad 7.54, \quad 5.83, \quad 1.92, \quad 3.14, \quad 4.41$$

   (a) Construct a 90% confidence interval for the population mean.

   (b) Construct a 99% confidence interval for the population mean.

Partial answers:

1. $1/\bar{x}$
2. $\bar{x}$
3. (a) 0.643
4. (a) i. 95%; ii. 90%; iii. 99%; iv. 68%; (b) 10.9 ± 2
5. (a) 2.5 ± 16.7656; (b) 83.2 ± 1.254
6. (a) $[3.83, 6.77]$; (b) $[2.59, 8.01]$