Analysis 2, Complex Analysis
Assessed Coursework 2
Deadline the 26th of March, 2021.

Q1. [5] Find the Laurent series for the function
$$\frac{z^2 - 2z + 3}{z - 2}$$
in the region $|z - 1| > 1$.

Q2. [5]

a) [2] Find $\Omega \subset \mathbb{R}^2$ where the function
$$u(x, y) = \frac{x^2 + y^2 + x}{x^2 + y^2}$$
is harmonic.

b) [2] Find all harmonic conjugates $v(x, y)$.
Hint: use C-R equations in polar coordinates.

c) [1] If $f(z) = u(x, y) + i v(x, y)$, find $f(z)$ in terms of z.

Q3. [5] Use complex integration to compute
$$\int_0^\infty \frac{1 - \cos x}{x^2}\, dx.$$

Q4. [5]

a) [3] Let ψ be holomorphic in $D = \{z : |z| \leq 1\}$. Prove that there exists $0 < \rho$ such that for any $w \in \{z : |z| < \rho\}$ the equation $z = w\,\psi(z)$ has exactly one root.

b) [2] Find the Möbius transform $w = f(z)$ that maps points $z_1 = 1$, $z_2 = i$ and $z_3 = -i$ onto the given points $w_1 = -i$, $w_2 = i$ and $w_3 = \infty$, respectively.