

Probability and Statistics for JMC

Exercises 2b — More probability

1. (a) In one spin of a European roulette wheel (which has pockets numbered 0, 1, 2, up to and including 36) what is the probability that the outcome is odd?

(b) An urn contains x red balls and y green ones (both x and y are larger than 2). You remove them one at a time until the urn is empty.
 - i. What is the chance that the first is red?
 - ii. What is the chance that the second is red?
 - iii. What is the chance that the first two are red?
 - iv. What is the chance that the second to last one is red?
2. (a) An experiment consists of tossing a fair coin and rolling a fair die. What is the probability of the joint event “heads with an odd number of dots”?

(b) In a particular class, 30% of students were female, and 90% of the males and 80% of the females passed the examination. What percentage of the class passed the examination altogether?
3. On any day the chance of rain is 25%. The chance of rain on two consecutive days is 10%.
 - (a) Does this mean that the events of rain on two consecutive days are independent or dependent events?
 - (b) Given that it is raining today, what is the chance of rain tomorrow?
 - (c) Given that it will rain tomorrow, what is the chance of rain today?
4. You forget your umbrella with probability $\frac{1}{4}$ every time you visit a shop (and, once you leave it behind, you do not collect it again).
 - (a) You set out with your umbrella to visit four different shops. What is the probability that you will leave it in the fourth shop?
 - (b) If you arrive home without your umbrella, what is the probability that you left it in the fourth shop?
 - (c) If you arrive home without it, but were seen carrying it after leaving the first shop, what is the probability that you left it in the fourth shop?

5. A warehouse contains packs of electronic components. Forty percent of the packs contain components of low quality for which the probability that any given component will prove satisfactory is 0.8; forty percent contain components of medium quality for which this probability is 0.9; and the remaining twenty percent contain high quality components which are certain to be satisfactory.
 - (a) If a pack is chosen at random and one component from it is tested, what is the probability that this component is satisfactory?
 - (b) If a pack is chosen at random and two components from it are tested, what is the probability that exactly one of the components tested is satisfactory?
 - (c) If it was found that just one of the components tested was satisfactory, what is the probability that the selected pack contained medium quality components?
 - (d) If both components were found to be satisfactory, what is the probability that the selected pack contained high quality components?
6. Prove that if $P(A) > P(B)$ then $P(A | B) > P(B | A)$.
7. Show that if three events A , B , and C are independent, then A and $B \cup C$ are independent.
8. Two factories produce similar parts. Factory 1 produces 1000 parts, 100 of which are defective. Factory 2 produces 2000 parts, 150 of which are defective. A part is selected at random and found to be defective. What is the probability that it came from factory 1?
9. In an experiment in which two fair dice are thrown, let A be the event that the first die is odd, let B be the event that the second die is odd, and let C be the event that the sum is odd. Show that events A , B , and C are pairwise independent, but A , B , and C are not jointly independent.
10. In a gambling game called craps, a pair of dice is rolled and the outcome is the sum of the dice. The player wins on the first roll if the sum is 7 or 11 and loses if the sum is 2, 3, or 12. If the sum is 4, 5, 6, 8, 9, or 10, that number is called the players "point". Once the point is established, the rule is: If the player rolls a 7 before the point, the player loses; but if the point is rolled before a 7, the player wins. Compute the probability of winning in the game of craps.

Partial answers:

1. (a) $18/37$; (b.i) $x/(x+y)$; (b.ii) $x/(x+y)$; (b.iii) $\frac{x-1}{x+y-1} \frac{x}{x+y}$; (b.iv) $x/(x+y)$
2. (a) $1/4$; (b) 87%
3. (a) dependent; (b) 0.4
4. (a) $27/256$; (b) 0.15; (c) 0.24

5. (a) 0.88; (b) 0.2; (c) 0.36; (d) 0.26
8. $1/12$
10. $0.492929\dots$