# Analysis 2, Complex Analysis
## Assessed Coursework 1

Deadline the 19th of February, 2021.

**Q1. [5]**

a) Compute

$$D_n = \frac{1}{\pi}\left(\frac{1}{2} + \cos x + \cos 2x + \cdots + \cos nx\right)$$

**Q2. [5]**

Let $a \in \mathbb{C}$ and let $f$ be holomorphic in $D = \{z : |z| \leq 1\}$. Compute

$$\oint_{|z|=1} \frac{\overline{f(z)}}{z-a}\,dz$$

where the integration is counterclockwise

   a) if $|a| < 1$
   b) if $|a| > 1$.

**Q3. [5]**

Use Morera's theorem to show that

$$f(z) = \int_0^1 \frac{dt}{1-zt}$$

is holomorphic in the unit disc.

**Q 4.[5]**

Let $f$ be holomorphic in open connected set $\Omega$. Prove that if $|f(z)| \equiv C = \text{const}$ in $\Omega$, then $f$ is a constant.