Mathematical Model for School Assignment Problem

Parameters

- S: Total number of schools
- G: Total number of student groups
- N: Total number of neighborhoods
- $Capacity_{s,g}$: Capacity of school s for student group g, for $s = 1, \ldots, S$ and $g = 1, \ldots, G$
- $Population_{n,g}$: Population of student group g in neighborhood n, for $n = 1, \ldots, N$ and $g = 1, \ldots, G$
- $Distance_{n,s}$: Distance between neighborhood n and school s, for $n = 1, \ldots, N$ and $s = 1, \ldots, S$

Decision Variables

- $x_{n,g,s}$: Number of students from group g in neighborhood n assigned to school s

Objective Function

Minimize the total distance traveled by all students:

$$\min \sum_{n=1}^{N} \sum_{g=1}^{G} \sum_{s=1}^{S} Distance_{n,s} \cdot x_{n,g,s}$$

Constraints

1. Total number of students from each student group g assigned from neighborhood n to schools does not exceed the population of group g in neighborhood n:

$$\sum_{s=1}^{S} x_{n,g,s} \leq Population_{n,g} \quad \forall n = 1, \ldots, N, \forall g = 1, \ldots, G$$

2. Total number of students from each student group g assigned to school s does not exceed the capacity of school s for student group g:

$$\sum_{n=1}^{N} x_{n,g,s} \leq Capacity_{s,g} \quad \forall s = 1, \ldots, S, \forall g = 1, \ldots, G$$

3. Each student is assigned to exactly one school:

$$\sum_{s=1}^{S} x_{n,g,s} = Population_{n,g} \quad \forall n = 1, \ldots, N, \forall g = 1, \ldots, G$$

4. All populations, capacities, and distances are non-negative:

$$x_{n,g,s} \geq 0 \quad \forall n = 1, \ldots, N, \forall g = 1, \ldots, G, \forall s = 1, \ldots, S$$