Mathematical Model for the Traveling Salesman Problem

Parameters

- N: Number of towns to visit.
- $StartCity$: Index of the start city.
- $Distances_{ij}$: Distance from city i to city j, for $i, j \in \{1, 2, \ldots, N\}$.

Decision Variables

- x_{ij}: Binary variable that is 1 if the route goes directly from city i to city j, and 0 otherwise.
- u_i: Auxiliary variable to eliminate subtours, for $i \in \{1, 2, \ldots, N\}$.

Objective Function

Minimize the total travel distance:

$$\min \sum_{i=1}^{N} \sum_{j=1}^{N} Distances_{ij} \cdot x_{ij}$$

Constraints

1. Each town must be visited exactly once:

$$\sum_{j=1, j\neq i}^{N} x_{ij} = 1 \quad \forall i \in \{1, 2, \ldots, N\}$$

$$\sum_{i=1, i\neq j}^{N} x_{ij} = 1 \quad \forall j \in \{1, 2, \ldots, N\}$$

2. The traveler must return to the StartCity after visiting all other towns:

$$\sum_{j=1, j\neq StartCity}^{N} x_{StartCity,j} = 1$$

$$\sum_{i=1, i\neq StartCity}^{N} x_{i,StartCity} = 1$$

3. Eliminate subtours (Miller-Tucker-Zemlin formulation):

$$u_i - u_j + N \cdot x_{ij} \leq N - 1 \quad \forall i, j \in \{2, 3, \ldots, N\}, i \neq j$$

$$1 \leq u_i \leq N - 1 \quad \forall i \in \{2, 3, \ldots, N\}$$

4. Binary constraints:

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in \{1, 2, \ldots, N\}$$