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Research article

Tracking control of an underactuated ship by modified dynamic inversion

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HIGHLIGHTS

- The underactuated ship tracking control problem is solved in a clear and beautiful manner by using modified dynamic inversion.
- The proposed method inherits the advantage of dynamic inversion control which does not require a careful construction of Lyapunov functions.
- A unified framework of modified dynamic inversion is built, which removes some inherent limitations of dynamic inversion control, making it applicable to a wide variety of underactuated systems.

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ABSTRACT

The tracking control problem of an underactuated ship is investigated. We intend to use the underactuated ship as an example to extend the traditional dynamic inversion control method to underactuated systems. The difficulty lies in the fact that the system has no relative degree, which prevents the application of standard dynamic inversion. Three modified dynamic inversion methods are proposed that are applicable to this system. The first is the well-known dynamic extension-based dynamic inversion (DEDI), which treats an input as a state and takes dynamic extension to achieve a relative degree. The second is virtual input-based dynamic inversion (VIDI), which treats a state as a virtual input to achieve a relative degree. The third is output redefinition-based dynamic inversion (ORDI), which selects a particular variable as a new output to achieve a relative degree. The three methods are generalizations of dynamic inversion control and remove some of its inherent limitations, making it applicable to a wide variety of underactuated systems. The effectiveness of the proposed methods is verified by numerical simulations.

1. Introduction

Tracking control of underactuated systems is regarded as a challenging problem and has become an active research area [1]. A control system is underactuated if it has fewer independent control actuators than the degrees of freedom to be controlled. In practice, many mechanical systems are underactuated due to their dynamic nature and some others may become underactuated during actuator failure. A typical example is an underactuated ship with two propellers, which has three degrees of freedom (yaw, sway and surge) while only two controls (surge force and yaw moment) are available. During the past two decades, a lot of research has been done on the trajectory tracking of underactuated ships and has promoted the development of underactuated system control theory.

Some research focused on converting the tracking problem into a stabilization problem by assuming all reference states and inputs be generated by a virtual ship. In [2,3], a coordinate transformation was made to transform the original model into a triangular-like structure, then a backstepping controller was designed for the transformed system. In [4,5], global exponential tracking was achieved by using Lyapunov's direct method, including a passivity-based approach and a combined cascade-backstepping approach. In [6], a continuous time-varying tracking controller was designed based on Lyapunov theory. In [7], the model was transformed into a linear time-varying system, and a cascade controller was designed to stabilize it. In [8], a finite-time switching controller was developed.

Some other tracking control methods were derived directly with the output reference trajectories. In [9], a backstepping controller was designed based on several nonlinear coordinate changes. In [10], a dynamic surface controller was proposed. In [11], a two-level sliding mode controller was designed by using Lyapunov's direct method. In [12], the model was discretized and the control signals for exact tracking were obtained by solving a set of linear equations. In [13], a robust adaptive trajectory tracking algorithm based on proportional-integral sliding mode control and backstepping technique was proposed. In [14], an adaptive output feedback

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controller was developed by using neural networks to estimate the unknown system parameters and nonlinearities. In [15], an online adaptive near-optimal controller was designed considering uncertain parameters.

Although various methods have been proposed, to the authors' knowledge, nearly none of them have attempted to use dynamic inversion. The main reason may be that the underactuated ship system is not transformable into a chained system since it has no relative degree. Consequently, dynamic inversion cannot be applied directly. However, this restriction can be removed with some modifications to the standard dynamic inversion method. Due to the powerful ability of dynamic inversion in dealing with nonlinear tracking control problems, it may have some advantages over other control methods if it can be applied. This is the start point of this paper and our goal is to solve the underactuated ship tracking control problem by using modified dynamic inversion.

Dynamic inversion, which is also known as feedback linearization [16], is a powerful nonlinear tracking control method but has some inherent limitations in applicable systems, e.g., it cannot be applied to systems with no relative degree and nonminimum phase systems. However, some efforts are made to enlarge its application fields. A well-known modification is the combination with dynamic extension [16], which can be applied to systems with no relative degree. The input is treated as a state and its derivative is viewed as a new input to achieve a relative degree. This method has been applied to many practical systems, such as a quadrotor [17], a car-like robot [18], and manipulators [19], just to name a few. In this paper, we show that dynamic extension can also be applied to underactuated ships. However, we find it requires calculating the higher-order output derivative and results in a control law with a long expression. Therefore, we intend to find a better way to do this. This is challenging since there are no other reported methods available, neither for underactuated ships nor for other systems with no relative degree.

Motivated by the output redefinition technique [20] which is developed for nonminimum phase systems, as well as the virtual input concept in backstepping control [21], we find out two other ways to extend dynamic inversion control to underactuated ships, which simplify the control law design and give more options for the control of underactuated systems with no relative degree. Finally, three modified dynamic inversion methods are developed for an underactuated ship. The first is dynamic extension-based dynamic inversion (DEDI), which treats an input as a state and takes dynamic extension to achieve a relative degree. The second is virtual input-based dynamic inversion (VIDI), which treats a state as a virtual input to achieve a relative degree. The third is output redefinition-based dynamic inversion (ORDI), which selects a particular variable as output to achieve a relative degree. The advantages and disadvantages of the three methods are summarized. The main contributions of this paper are twofold. Firstly, with the proposed modified dynamic inversion methods, the underactuated ship tracking control problem is solved in a clear and beautiful manner. Compared to the existed methods, the proposed method inherits the advantage of dynamic inversion control which does not require a careful construction of Lyapunov function. Secondly, through the underactuated ship example, a unified framework of modified dynamic inversion is built, which extends the applicability of dynamic inversion to a wide variety of underactuated systems.

The remainder of this paper is organized as follows. In Section 2, the model and problem formulation are given. Section 3 introduces the controller design of DEDI, VIDI, and ORDI one by one. The numerical simulation results are given in Section 4 and conclusions are given in Section 5.

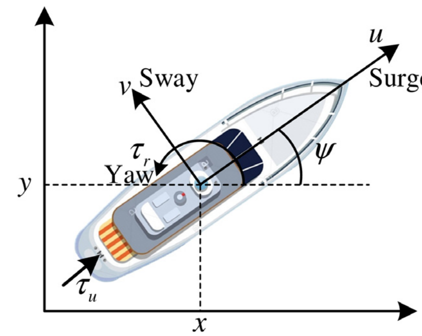

Fig. 1. Diagram of an underactuated ship.

2. Model and problem formulation

The model considered in this paper is a 3 degrees of freedom model of an underactuated surface ship as shown in Fig. 1, which can move in surge, sway, and yaw. The ship is underactuated since it has only two propellers, with one providing the surge force τ_u and the other generating the yaw moment τ_r, while no independent actuator is available in the sway axis.

Following [5], the dynamic equations of the underactuated surface ship are written as follows

$$\begin{cases} \dot{x} = u\cos\psi - v\sin\psi \\ \dot{y} = u\sin\psi + v\cos\psi \\ \dot{\psi} = r \\ \dot{u} = \left(m_{22}vr - d_{11}u + \tau_u\right)/m_{11} \\ \dot{v} = \left(-m_{11}ur - d_{22}v\right)/m_{22} \\ \dot{r} = \left(\left(m_{11} - m_{22}\right)uv - d_{33}r + \tau_r\right)/m_{33} \end{cases} \tag{1}$$

where x, y denote the position of the ship in the earth fixed frame, ψ is the heading angle of the ship, and u, v and r represent the velocity in surge, sway and yaw, respectively. The control inputs are the surge force τ_u and the yaw moment τ_r. The parameters $m_{ii}, d_{ii}, i = 1, 2, 3$ are positive constants which denote the ship inertia and damping.

From the model, it can be found that only the surge velocity and yaw velocity are directly controlled by the control inputs. Rewrite their dynamics as follows

$$\begin{aligned} \dot{u} &= f_u + g_u\tau_u \\ \dot{r} &= f_r + g_r\tau_r \end{aligned} \tag{2}$$

with

$$\begin{aligned} f_u &= \left(m_{22}vr - d_{11}u\right)/m_{11}, \quad g_u = 1/m_{11} \\ f_r &= \left(\left(m_{11} - m_{22}\right)uv - d_{33}r\right)/m_{33}, \quad g_r = 1/m_{33} \end{aligned} \tag{3}$$

The tracking control objective is to let the position x, y track the given reference trajectories $x_d(t), y_d(t)$. Define the tracking errors as $e_x = x - x_d, e_y = y - y_d$. To obtain the input–output dynamics, take the second-order derivative of e_x, e_y

$$\begin{aligned} \ddot{e}_x &= \dot{u}\cos\psi - u\dot{\psi}\sin\psi - \dot{v}\sin\psi - v\dot{\psi}\cos\psi - \ddot{x}_d \\ \ddot{e}_y &= \dot{u}\sin\psi + u\dot{\psi}\cos\psi + \dot{v}\cos\psi - v\dot{\psi}\sin\psi - \ddot{y}_d \end{aligned} \tag{4}$$

Since the input τ_u appears in $\dot{u}$, the input–output dynamics (4) can be rewritten in an affine form as follows

$$\begin{bmatrix} \ddot{e}_x \\ \ddot{e}_y \end{bmatrix} = F + G\begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix} \tag{5}$$

where

$$\begin{aligned} F &= \begin{bmatrix} f_u\cos\psi - ur\sin\psi - \dot{v}\sin\psi - vr\cos\psi - \ddot{x}_d \\ f_u\sin\psi + ur\cos\psi + \dot{v}\cos\psi - vr\sin\psi - \ddot{y}_d \end{bmatrix} \\ G &= \begin{bmatrix} \cos\psi/m_{11} & 0 \\ \sin\psi/m_{11} & 0 \end{bmatrix} \end{aligned} \tag{6}$$

It can be observed that the control matrix G is singular for any ψ, thus the system has no relative degree and dynamic inversion cannot be directly applied to this system.

3. Tracking controller design by modified dynamic inversion

In this section, three modified dynamic inversion controllers are developed for the underactuated ship. The main idea is to modify the input or output such that the new input–output dynamics has a nonsingular control matrix. The first adopts the traditional dynamic extension technique which views an input as a state to achieve a relative degree, while the second views a state as a virtual input to achieve a relative degree. And the third redefines a new output to achieve a relative degree. A comparison is made for these three methods in the end.

3.1. Dynamic extension-based dynamic inversion

Recall the input–output dynamics (4), it has no relative degree since that only one input τ_u appears in $\ddot{e}_x$ and $\ddot{e}_y$. However, the other input τ_r will appear if one more differentiation is taken on $\ddot{e}_x, \ddot{e}_y$, i.e.,

$$\begin{aligned} \dddot{e}_x &= \left(\ddot{u} - \dot{v}\dot{\psi} - v\ddot{\psi}\right)\cos\psi - \left(\dot{u} - v\dot{\psi}\right)\dot{\psi}\sin\psi \\ &- \left(\ddot{v} + \dot{u}\dot{\psi} + u\ddot{\psi}\right)\sin\psi - \left(\dot{v} + u\dot{\psi}\right)\dot{\psi}\cos\psi - \dddot{x}_d \\ \dddot{e}_y &= \left(\ddot{u} - \dot{v}\dot{\psi} - v\ddot{\psi}\right)\sin\psi + \left(\dot{u} - v\dot{\psi}\right)\dot{\psi}\cos\psi \\ &+ \left(\ddot{v} + \dot{u}\dot{\psi} + u\ddot{\psi}\right)\cos\psi - \left(\dot{v} + u\dot{\psi}\right)\dot{\psi}\sin\psi - \dddot{y}_d \end{aligned} \tag{7}$$

Now τ_r appears on the right-hand side since $\ddot{\psi}$ contains τ_r. Meanwhile, the early-occurred input τ_u is differentiated in $\ddot{u}$. By treating τ_u as a state with the following dynamic extension:

$$\dot{\tau}_u = \xi_u \tag{8}$$

where ξ_u is treated as a new input. Then it follows that

$$\begin{aligned} \ddot{u} &= \left(m_{22}\dot{v}r + m_{22}v\dot{r} - d_{11}\dot{u} + \xi_u\right)/m_{11} \\ \ddot{v} &= \left(-m_{11}\dot{u}r - m_{11}u\dot{r} - d_{22}\dot{v}\right)/m_{22} \end{aligned} \tag{9}$$

Rewrite (9) as follows

$$\begin{aligned} \ddot{u} &= a_u + \xi_u/m_{11} + b_u\tau_r \\ \ddot{v} &= a_v + b_v\tau_r \end{aligned} \tag{10}$$

with

$$\begin{aligned} a_u &= \left(m_{22}\dot{v}r + m_{22}vf_r - d_{11}\dot{u}\right)/m_{11}, \qquad b_u = m_{22}v/m_{11}m_{33} \\ a_v &= \left(-m_{11}\dot{u}r - m_{11}uf_r - d_{22}\dot{v}\right)/m_{22}, \; b_v = -m_{11}u/m_{22}m_{33} \end{aligned} \tag{11}$$

Substitute (10) into (7), then the new input–output dynamics turns into

$$\begin{bmatrix} \dddot{e}_x \\ \dddot{e}_y \end{bmatrix} = F_1 + G_1 \begin{bmatrix} \xi_u \\ \tau_r \end{bmatrix} \tag{12}$$

where F_1, G_1 as in Eq. (13) given in Box I. It is assumed that $m_{11} \neq m_{22}$ and the velocity of the ship is nonzero so that the control matrix G_1 is nonsingular. Therefore, the system now has a well-defined relative degree and dynamic inversion controller can be designed as follows

$$\begin{bmatrix} \xi_u \\ \tau_r \end{bmatrix} = G_1^{-1}\left(-F_1 + \begin{bmatrix} -k_{11}\ddot{e}_x - k_{12}\dot{e}_x - k_{13}e_x \\ -k_{14}\ddot{e}_y - k_{15}\dot{e}_y - k_{16}e_y \end{bmatrix}\right) \tag{14}$$

where $k_{11}, k_{12}, k_{13}, k_{14}, k_{15}, k_{16}$ are positive parameters to be designed. This control law is actually a very long expression when expanded since it contains $\ddot{e}_x, \ddot{e}_y$ and the complicated F_1, G_1. The real input τ_u is obtained by integrating ξ_u with zero initial value. It leads to the following closed-loop system

$$\begin{cases} \dddot{e}_x = -k_{11}\ddot{e}_x - k_{12}\dot{e}_x - k_{13}e_x \\ \dddot{e}_y = -k_{14}\ddot{e}_y - k_{15}\dot{e}_y - k_{16}e_y \end{cases} \tag{15}$$

According to Routh–Hurwitz criterion, the parameters can be selected as $k_{11}, k_{13}, k_{14}, k_{16} > 0$, $k_{12} > k_{13}/k_{11}$, $k_{15} > k_{16}/k_{14}$ so that the tracking errors will converge to zero asymptotically.

3.2. Virtual input-based dynamic inversion

Although dynamic extension is an effective way to achieve a relative degree, it requires calculating the higher-order output derivative which involves complex calculation. Motivated by the virtual input concept in backstepping control [21], a new way is proposed to achieve a relative degree and enable the application of dynamic inversion to the underactuated ship.

Still recall the input–output dynamics (4), although the input τ_r does not appear in $\ddot{e}_x, \ddot{e}_y$, the state r which is controlled by τ_r does appear. Therefore, the state r can be treated as a virtual input and then the input–output dynamics (4) is rewritten as

$$\begin{bmatrix} \ddot{e}_x \\ \ddot{e}_y \end{bmatrix} = F_2 + G_2 \begin{bmatrix} \tau_u \\ r \end{bmatrix} \tag{16}$$

where F_2, G_2 as in Eq. (17) given in Box II. Now the control matrix becomes G_2. Still assume that $m_{11} \neq m_{22}$ and the velocity of the ship is nonzero so that G_2 is nonsingular. Therefore, the system has a relative degree and the dynamic inversion controller can be designed as

$$\begin{bmatrix} \tau_u \\ r_d \end{bmatrix} = G_2^{-1}\left(-F_2 + \begin{bmatrix} -k_{21}\dot{e}_x - k_{22}e_x \\ -k_{23}\dot{e}_y - k_{24}e_y \end{bmatrix}\right) \tag{18}$$

where $k_{21}, k_{22}, k_{23}, k_{24}$ are positive parameters to be designed. Note that r is actually a state which cannot be set directly, so the notation r_d is used in (18) which represents a desired value for the virtual input r. If $r = r_d$, then it leads to the following closed-loop system

$$\begin{cases} \ddot{e}_x = -k_{21}\dot{e}_x - k_{22}e_x \\ \ddot{e}_y = -k_{23}\dot{e}_y - k_{24}e_y \end{cases} \tag{19}$$

which can be made asymptotically stable. However, the real input τ_r needs to be designed to drive r to the desired value r_d. Define the virtual input error $e_r = r - r_d$. Then the actual closed-loop system is

$$\begin{cases} \ddot{e}_x = -k_{21}\dot{e}_x - k_{22}e_x + \\ \left(m_{22}v\cos\psi/m_{11} + m_{11}u\sin\psi/m_{22} - u\sin\psi - v\cos\psi\right)e_r \\ \ddot{e}_y = -k_{23}\dot{e}_y - k_{24}e_y + \\ \left(m_{22}v\sin\psi/m_{11} - m_{11}u\cos\psi/m_{22} + u\cos\psi - v\sin\psi\right)e_r \end{cases} \tag{20}$$

Now consider the virtual input error dynamics

$$\dot{e}_r = f_r + g_r\tau_r - \dot{r}_d \tag{21}$$

Design the real input as

$$\tau_r = g_r^{-1}\left(\dot{r}_d - f_r - k_{25}e_r\right) \tag{22}$$

where $k_{25} > 0$. It follows that

$$\dot{e}_r = -k_{25}e_r \tag{23}$$

Inspect (20) and (23), they form a cascade system. Since (20) is asymptotically stable with $e_r = 0$ and (23) is also asymptotically stable, the overall system is stable around zero according to [22].

Remark 1. To avoid the "explosion of terms" problem [23] in calculating the analytical expressions of the virtual input derivative $\dot{r}_d$, a first-order differentiator is adopted to make an approximation

$$\dot{r}_d \approx \dot{r}_c = \lambda\left(r_d - r_c\right) \tag{24}$$

where $\lambda > 0$. This is a regular strategy used in dynamic surface control [23].

$$F_1 = \begin{bmatrix} \left(a_u - 2\dot{v}r - vf_r - ur^2\right)\cos\psi - \left(a_v + 2\dot{u}r + uf_r - vr^2\right)\sin\psi - \dddot{x}_d \\ \left(a_u - 2\dot{v}r - vf_r - ur^2\right)\sin\psi + \left(a_v + 2\dot{u}r + uf_r - vr^2\right)\cos\psi - \dddot{y}_d \end{bmatrix}$$
$$G_1 = \begin{bmatrix} \cos\psi/m_{11} & \left(\left(m_{22}/m_{11} - 1\right)v\cos\psi + \left(m_{11}/m_{22} - 1\right)u\sin\psi\right)/m_{33} \\ \sin\psi/m_{11} & \left(\left(m_{22}/m_{11} - 1\right)v\sin\psi + \left(1 - m_{11}/m_{22}\right)u\cos\psi\right)/m_{33} \end{bmatrix} \tag{13}$$

Box I.

$$F_2 = \begin{bmatrix} -d_{11}u\cos\psi/m_{11} + d_{22}v\sin\psi/m_{22} - \ddot{x}_d \\ -d_{11}u\sin\psi/m_{11} - d_{22}v\cos\psi/m_{22} - \ddot{y}_d \end{bmatrix}$$
$$G_2 = \begin{bmatrix} \cos\psi/m_{11} & m_{22}v\cos\psi/m_{11} + m_{11}u\sin\psi/m_{22} - u\sin\psi - v\cos\psi \\ \sin\psi/m_{11} & m_{22}v\sin\psi/m_{11} - m_{11}u\cos\psi/m_{22} + u\cos\psi - v\sin\psi \end{bmatrix} \tag{17}$$

Box II.

3.3. Output redefinition-based dynamic inversion

Unlike DEDI and VIDI, which make modifications on the input side to achieve a relative degree, modifications are made on the output side to achieve a relative degree in ORDI.

To achieve a relative degree, we need the control matrix to be nonsingular, that is, we need both inputs to appear in the input–output dynamics. As shown in Section 2, for the original output x, y, only one input τ_u appears in the input–output dynamics, while the other input τ_r appears in $\dddot{\psi}$. Therefore, it is naturally to think about constructing a new output by combining the original output and ψ to achieve a relative degree. For the output redefinition method, it is a traditional way to select the position of a fixed point in the vehicle body as a new output [24]. Inspired by [24], the new output x_1, y_1 is defined as follows

$$\begin{aligned} x_1 &= x + l\cos\psi \\ y_1 &= y + l\sin\psi \end{aligned} \tag{25}$$

which represents the position of a fixed point on the ship's centerline with a distance of l to the mass center. Since ψ enters the new output, the input τ_r will appear on the second order derivative of the new output. Denote the tracking errors as $e_{x_1} = x_1 - x_d$, $e_{y_1} = y_1 - y_d$. The new input–output dynamics are now derived as

$$\begin{bmatrix} \ddot{e}_{x_1} \\ \ddot{e}_{y_1} \end{bmatrix} = F_3 + G_3 \begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix} \tag{26}$$

where

$$\begin{aligned} F_3 &= F + \begin{bmatrix} -lf_r\sin\psi - lr^2\cos\psi \\ lf_r\cos\psi - lr^2\sin\psi \end{bmatrix} \\ G_3 &= \begin{bmatrix} \cos\psi/m_{11} & -l\sin\psi/m_{33} \\ \sin\psi/m_{11} & l\cos\psi/m_{33} \end{bmatrix} \end{aligned} \tag{27}$$

It is obvious that the new control matrix G_3 is nonsingular for any ψ, so the dynamic inversion controller can be designed as

$$\begin{bmatrix} \tau_u \\ \tau_r \end{bmatrix} = G_3^{-1}\left(-F_3 + \begin{bmatrix} -k_{31}\dot{e}_{x_1} - k_{32}e_{x_1} \\ -k_{33}\dot{e}_{y_1} - k_{34}e_{y_1} \end{bmatrix}\right) \tag{28}$$

where $k_{31}, k_{32}, k_{33}, k_{34}$ are positive parameters to be designed. The closed-loop system becomes

$$\begin{cases} \ddot{e}_{x_1} = -k_{31}\dot{e}_{x_1} - k_{32}e_{x_1} \\ \ddot{e}_{y_1} = -k_{33}\dot{e}_{y_1} - k_{34}e_{y_1} \end{cases} \tag{29}$$

If l is chosen as small enough, then $x_1 \approx x, y_1 \approx y$. As e_{x_1}, e_{y_1} converge to zero, the actual tracking error e_x, e_y will also converge to a small region around zero.

3.4. Summarization of the three modified dynamic inversion methods

In the underactuated ship example, dynamic inversion is extended to systems with no relative degree such that the underactuated systems can also enjoy the benefit from the powerful nonlinear control method. The main features of the three modified dynamic inversion methods are summarized as follows.

Dynamic extension-based dynamic inversion (DEDI): This is a systematic and universal way to deal with systems with no relative degree. It achieves a relative degree by taking dynamic extension on the input, i.e., viewing the input derivative (or higher-order derivative) as a new input to get a higher-order input–output dynamics with a well-defined relative degree. The advantage is that it provides a systematic solution to the no-relative-degree problem and is most widely applicable.

Virtual input-based dynamic inversion (VIDI): This is inspired from the backstepping control theory. The original input–output dynamics are kept, while a relative degree is achieved by viewing a state as a virtual input. And the real input is obtained to drive the virtual input to the desired value. The resulted controller is generally simpler than DEDI since it is obtained by taking inversion of the original input–output dynamics. However, it requires the input–output dynamics be affine to the virtual input.

Output redefinition-based dynamic inversion (ORDI): This method which is originally proposed for nonminimum phase systems is shown also suitable for systems with no relative degree. It selects a particular variable (usually a combination of the original output and other state) as a new output, leading to new input–output dynamics with a well-defined relative degree. However, the tracking task is not changed though the output is changed. The new output should be carefully selected as a good approximation to the original output so that the tracking error of the original output remains small when the new output tracks the reference trajectory.

To make it clear, a comparison of the three modified dynamic inversion methods is listed in Table 1. Generally speaking, the disadvantage of VIDI and ORDI lies in the restriction of applicable systems. But as long as they are available (such as the underactuated ship example in this paper), VIDI and ORDI will be more effective since they do not require calculating the higher-order output derivative and can yield a simpler control law.

From the underactuated ship example, modified dynamic inversion shows great potential to control underactuated systems, which greatly enlarge the application fields of dynamic inversion method. As shown in this paper, underactuated systems with no

Table 1
Comparison of three modified dynamic inversion methods.

	Way to achieve a relative degree	Advantage	Disadvantage
DEDI	Taking dynamic extension on the input	Systematic; most widely applicable	Require calculating the higher-order output derivative
VIDI	Treating a state as a virtual input	Simpler control law	Virtual input should be affine
ORDI	Redefining a new output	Also applicable to nonminimum phase systems	New output should be a good approximation to the original output

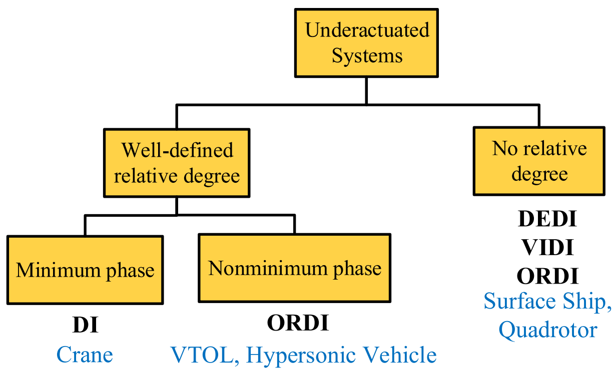

Fig. 2. Classification of underactuated systems and applicable dynamic inversion methods.

relative degree can be controlled by DEDI, VIDI, or ORDI. Typical examples include underactuated surface ships and quadrotors [17]. Besides, some other underactuated systems although has a well-defined relative degree, dynamic inversion cannot be directly used due to the nonminimum phase behavior caused by unstable zero dynamics. Examples are vertical take-off and landing aircrafts (VTOL) [1] and hypersonic vehicles [20,25]. The modified dynamic inversion method, ORDI, is shown to be applicable to this kind of systems [20]. As for standard dynamic inversion, it can be used only when the system has a well-defined relative degree and is minimum phase such as a crane system [1]. A classification of underactuated systems and the applicable dynamic inversion methods are shown in Fig. 2. With the modified dynamic inversion methods introduced in this paper, more options become available when facing the control problem of underactuated systems.

4. Numerical simulations

In this section, simulations are made in MATLAB environment to verify the effectiveness of the proposed method.

The model parameters are $m_{11} = 1.956$, $m_{22} = 2.405$, $m_{33} = 0.043$, $d_{11} = 2.436$, $d_{22} = 12.992$, $d_{33} = 0.0564$. The reference trajectories are given as a circle with radius 1 m, i.e., $x_d(t) = \sin t$, $y_d(t) = \cos t$. The initial conditions are set to $x(0) = -0.01$, $y(0) = 1.01$, $\psi(0) = 0.01$, $u(0) = 0.1$, $v(0) = 0.1$, and $r(0) = 0.1$. The control parameters for DEDI are selected as $k_{11} = 6$, $k_{12} = 12$, $k_{13} = 8$, $k_{14} = 6$, $k_{15} = 12$, $k_{16} = 8$. The control parameters for VIDI are selected as $k_{21} = 4$, $k_{22} = 4$, $k_{23} = 4$, $k_{24} = 4$, $k_{25} = 2$, $\lambda = 10$. The control parameters for ORDI are selected as $k_{31} = 4$, $k_{32} = 4$, $k_{33} = 4$, $k_{34} = 4$, $l = 0.01$.

The simulation results for DEDI are shown in Figs. 3 and 4. The simulation results for VIDI are shown in Figs. 5 and 6. The simulation results for ORDI are shown in Figs. 7 and 8.

From the figures above, it can be observed that the simulation results are very similar for the three methods. As shown in Figs. 3, 5, and 7, for the control outputs, the position x exhibits a small initial tracking error and converges to the reference x_r in about 3 s, and the position y nearly coincides with the reference y_r from the beginning. As shown in Figs. 4, 6, and 8, for the control inputs, although they are obtained by different controllers, they converge to the same steady state (τ_u at about 2.8 and τ_r at 0) after an initial adjustment. This demonstrates the effectiveness of the proposed modified dynamic inversion methods. They inherit the exact tracking ability of the dynamic inversion method.

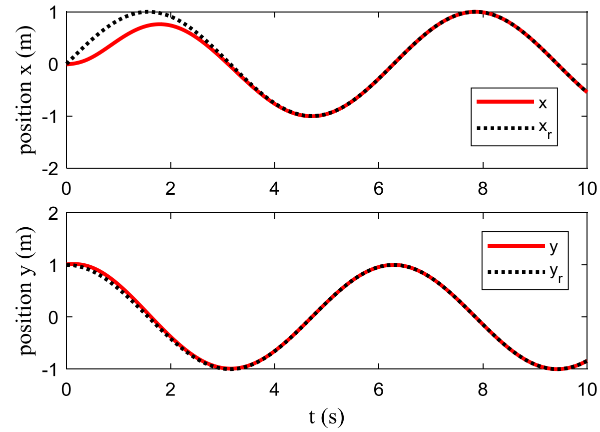

Fig. 3. Output curve for DEDI.

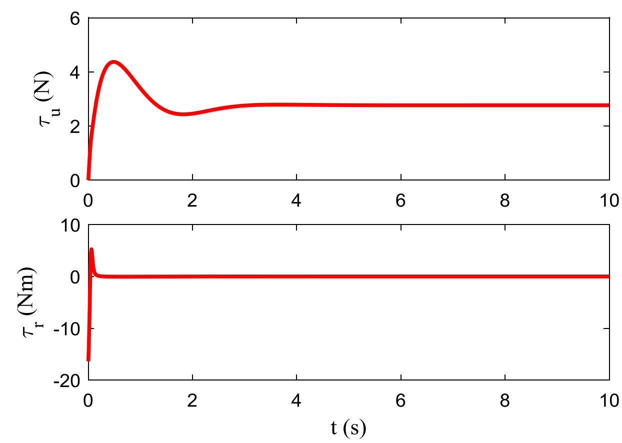

Fig. 4. Input curve for DEDI.

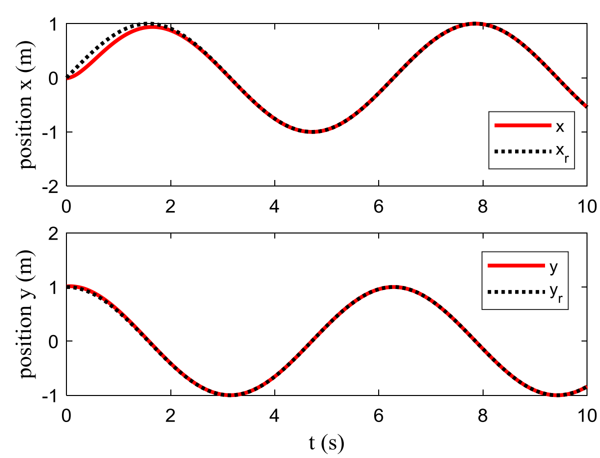

Fig. 5. Output curve for VIDI.

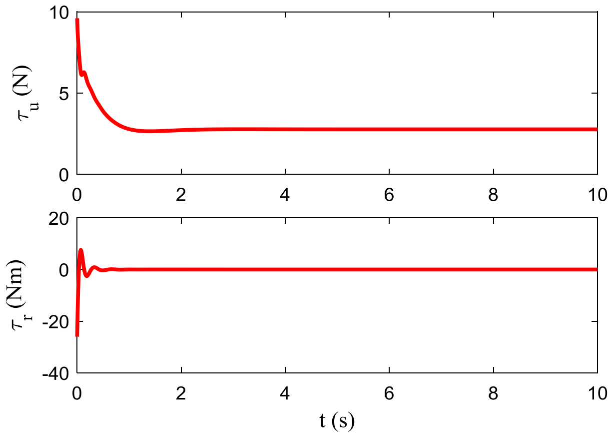

Fig. 6. Input curve for VIDI.

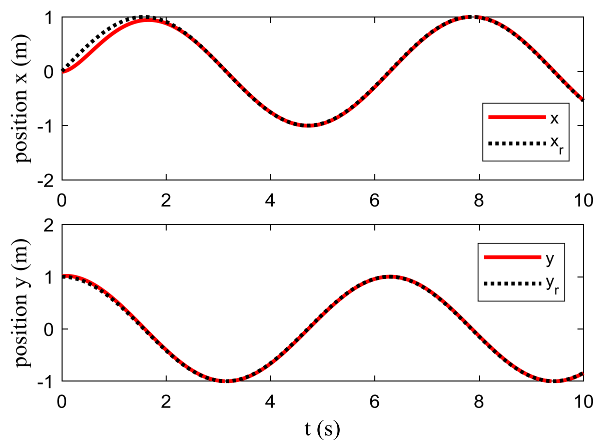

Fig. 7. Output curve for ORDI.

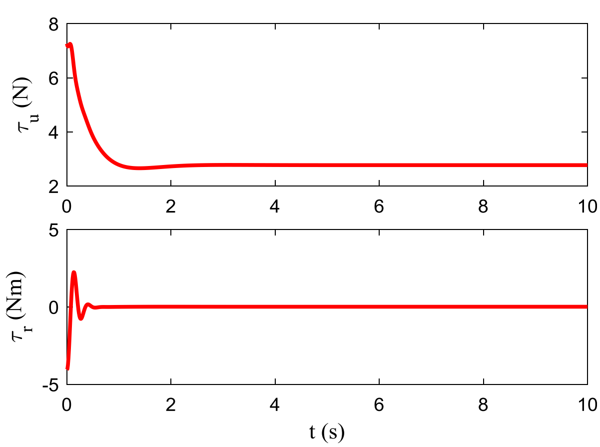

Fig. 8. Input curve for ORDI.

Furthermore, to demonstrate the robustness of the proposed methods. An additional simulation is taken with the uncertain model parameters: $m_{11} = 1.956 \times 1.5$, $m_{22} = 2.405 \times 0.6$, $m_{33} = 0.043 \times 1.2$, $d_{11} = 2.436 \times 0.5$, $d_{22} = 12.992 \times 1.6$, $d_{33} = 0.0564 \times 0.8$. The results are shown in Fig. 9. It can be seen that the tracking performance is still good. The output by ORDI only deviates a little from the reference trajectory, while DEDI deviates the most but still not bad. This demonstrates the robustness of the modified dynamic inversion methods.

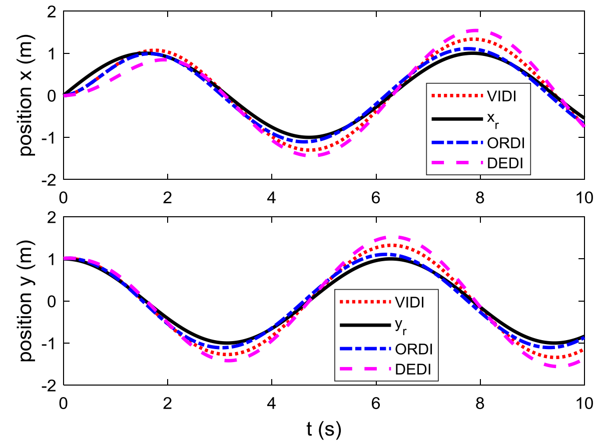

Fig. 9. Output curves under model uncertainties.

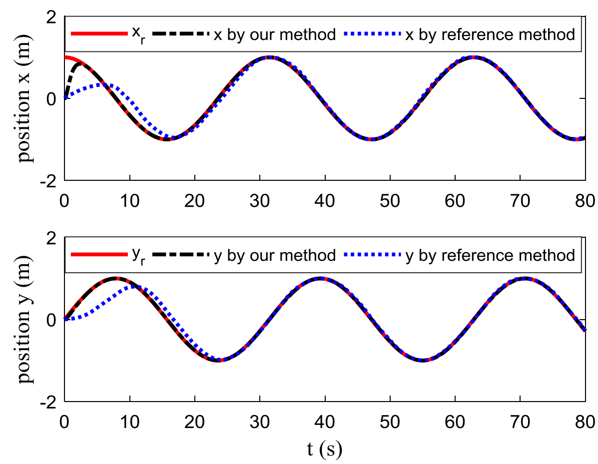

Fig. 10. Comparison of our method with reference method.

Specifically, ORDI seems to be the best option for underactuated ships among the three methods. First, for tracking performance, it can be seen that the tracking performances of VIDI and ORDI are a little better than DEDI from Figs. 3, 5, and 7. Second, for control inputs, ORDI generates smaller inputs at the beginning than DEDI and VIDI from Figs. 4, 6, and 8. Last, for robustness, ORDI exhibits the smallest tracking errors under model uncertainties as shown in Fig. 9.

Finally, a comparison is made between our best method, ORDI and the method in [11]. The model parameters, output references, and initial conditions are all set the same as [11]. The results are shown in Fig. 10. It can be seen that the position converges quickly to the references within 3 s for our method, while the reference method takes about 20 s. This demonstrates the superiority of the proposed method.

5. Conclusions

Three modified dynamic inversion methods are proposed for an underactuated ship. The proposed methods aim at achieving a relative degree by modifying the input or output, making it possible to apply dynamic inversion to underactuated systems with no relative degree. The first achieves a relative degree by dynamic extension, the second by using virtual input, and the third by redefining a new output. As a result, they form three modified dynamic inversion methods: dynamic extension-based dynamic inversion (DEDI), virtual input-based dynamic inversion

(VIDI), and output redefinition-based dynamic inversion (ORDI). The proposed methods release the inherent limitations of standard dynamic inversion and provide a good option for the control of underactuated systems.

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References

[1] Liu Y, Yu H. A survey of underactuated mechanical systems. IET Control Theory Appl 2013;7:921–35.
[2] Pettersen KY, Nijmeijer H. Underactuated ship tracking control: theory and experiments. Internat J Control 2001;74:1435–46.
[3] Huang J, Wen C, Wang W, Song Y. Global stable tracking control of underactuated ships with input saturation. Systems Control Lett 2015;85:1–7.
[4] Jiang Z. Global tracking control of underactuated ships by Lyapunov's direct method. Automatica 2002;38:301–9.
[5] Do KD, Jiang Z, Pan J. Underactuated ship global tracking under relaxed conditions. IEEE Trans Automat Control 2002;47:1529–36.
[6] Behal A, Dawson DM, Dixon WE, Fang Y. Tracking and regulation control of an underactuated surface vessel with nonintegrable dynamics. IEEE Trans Automat Control 2002;47:495–500.
[7] Lefeber E, Pettersen KY, Nijmeijer H. Tracking control of an underactuated ship. IEEE Trans Control Syst Technol 2003;11:52–61.
[8] Wu Y, Zhang Z, Xiao N. Global tracking controller for underactuated ship via switching design. J Dyn Syst Meas Control 2014;136. 054506.
[9] Do KD. Practical control of underactuated ships. Ocean Eng 2010;37:1111–9.
[10] Chwa D. Global tracking control of underactuated ships with input and velocity constraints using dynamic surface control method. IEEE Trans Control Syst Technol 2011;19:1357–70.
[11] Yu R, Zhu Q, Xia G, Liu Z. Sliding mode tracking control of an underactuated surface vessel. IET Control Theory Appl 2012;6:461–6.
[12] Serrano ME, Scaglia GJ, Godoy SA, Mut V, Ortiz OA. Trajectory tracking of underactuated surface vessels: A linear algebra approach. IEEE Trans Control Syst Technol 2014;22:1103–11.
[13] Sun Z, Zhang G, Qiao L, Zhang W. Robust adaptive trajectory tracking control of underactuated surface vessel in fields of marine practice. J Mar Sci Tech-Japan 2018;1–8.
[14] Park BS, Kwon J, Kim H. Neural network-based output feedback control for reference tracking of underactuated surface vessels. Automatica 2017;77:353–9.
[15] Zhang Y, Li S, Liu X. Adaptive near-optimal control of uncertain systems with application to underactuated surface vessels. IEEE Trans Control Syst Technol 2017.
[16] Isidori A. Nonlinear Control Systems. New York, NY, USA: Springer; 2013.
[17] Akhtar A, Waslander SL, Nielsen C. Path following for a quadrotor using dynamic extension and transverse feedback linearization. In: Decision and control (CDC). 2012 IEEE 51st annual conference on; 2012. p. 3551–6.
[18] Akhtar A, Nielsen C. Path following for a car-like robot using transverse feedback linearization and tangential dynamic extension. In: Decision and control and European control conference (CDC-ECC). 2011 50th IEEE conference on; 2011. p. 7974–9.
[19] Shimizu T, Sasaki M, Okada T. Tip position control of a two link flexible manipulator based on the dynamic extension technique. Trans Soc Instrum Control Eng 2008;44:389–95.
[20] Ye L, Zong Q, Crassidis JL, Tian B. Output-redefinition-based dynamic inversion control for a nonminimum phase hypersonic vehicle. IEEE Trans Ind Electron 2018;65:3447–57.
[21] Krstic M, Kanellakopoulos I, Kokotovic PV. Nonlinear and Adaptive Control Design. Wiley; 1995.
[22] Khalil HK. Noninear Systems. New Jersey: Prentice-Hall; 1996.
[23] Swaroop D, Hedrick JK, Yip PP, Gerdes JC. Dynamic surface control for a class of nonlinear systems. IEEE Trans Automat Control 2000;45:1893–9.
[24] Martin P, Devasia S, Paden B. A different look at output tracking: control of a VTOL aircraft. In: Decision and control, 1994. Proceedings of the 33rd IEEE conference on; 1994. p. 2376-81.
[25] Wang Z, Bao W, Li H. Second-order dynamic sliding-mode control for nonminimum phase underactuated hypersonic vehicles. IEEE Trans Ind Electron 2017;64:3105–12.