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of Technology**

**FACULTY OF AUTOMATIC CONTROL, ELECTRONICS
AND COMPUTER SCIENCE**

**PROGRAMME: CONTROL, ELECTRONIC
AND INFORMATION ENGINEERING**

Master Thesis

**Improving the efficiency of lossless image compression
using extensions of Part 2 of the JPEG 2000 standard**

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O Ś W I A D C Z E N I E

Wyrażam zgodę/nie wyrażam* zgody na udostępnienie mojej pracy dyplomowej/rozprawy doktorskiej*

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Abstract

Lorem ipsum...

Keywords: loseless image compression, image processing, JPEG 2000, discrete wavelet transform, entropy estimation, multithreading, modern c++

Contents

1	Introduction	1
1.1	Preface	1
1.2	Objective of the project	2
1.3	Scope of the thesis	3
1.4	Thesis outline	4
2	Problem analysis	5
2.1	Discrete Wavelet Transform	5
2.1.1	One dimensional DWT	5
2.1.2	Two dimensional DWT	7
2.1.3	DWT features summary	9
2.2	Part 2 of the JPEG 2000	9
2.2.1	Introduction	9
2.2.2	Arbitrary Decomposition	11
2.2.3	Arbitrary Wavelet Transforms	12
2.3	Computer architecture	13
2.4	Known solutions	13
3	Subject of the thesis	15
3.1	Solution to the problem	15
3.2	Rationale of applied algorithms	15
4	Experiments	17
4.1	Methodology	17
4.2	Data sets	17
4.3	Results	17
5	Summary	19
5.1	Results	19
5.2	Conclusions	19
5.3	Future development	19
	Appendices	21

Technical documentation	23
List of abbreviations and symbols	25
Contents of attached CD	27

Chapter 1. Introduction

1.1 Preface

The usage of digital images is constantly growing across whole world. There are multiple types of application where memory usage matters to the users. Image compression is a possible solution to this problem in some of these fields. For example it is mission critical component in medical and picture archiving and communication systems (PACSs). [15] There are two major types of such compression. The one is lossy variant and the other one is lossless. Applying lossy methods to the image can result in the occurrence of compression artifacts. However, there are applications where such disadvantage is negligible, e.g. natural images and photographs processing in Internet day-to-day usage. [4] On the other hand lossless image compression does not produce such artefacts, sacrificing some performance and bitrates optimizations. It is employed in mentioned before medical systems. Images used for the sake of diagnostics can be taken as an example. In some countries there are regulations that forbid applying lossy compression to such images. [15] Moreover, the usage of lossless variant is more desired when there exists some uncertainty whether information contained in the image can be discarded. In these scenarios not using any variant of compression can be the only substitute of lossless one. [15]

Taking into account mentioned before reasons, some compression algorithms have been introduced as ISO standards. [15] Some notable examples of such papers are PNG, JPEG and JPEG 2000 (often written as JP2). The latter was originally developed from 1997 to 2000 with the desire of expanding JPEG capabilities. The main feature of this standard is usage of discrete wavelet transform (DWT) instead of discrete cosine transform (DCT) which was introduced in the predecessor. [5] The other feature of JPEG 2000 are support for lossy and lossless compression. As can be described before, such compression is needed to be performed in mission critical systems such as medicine. Therefore, the JPEG 2000 standard is utilized in PACSs and Digital Imaging and Communications in Medicine DICOM standard. [15] This standard consist of 16 ISO parts which contain wide set of features. Some notable ones are core system coding and its extensions, motion images, testing and reference software. [5]

The successor of JPEG standard improved several aspects over its predecessor. With the usage of its algorithms, e.g. DWT, it was possible to improve compression performance over JPEG. Moreover, there are other improved areas with even greater importance. The few examples of such features are scalability and editability. [5] The JPEG 2000 standard supports both very low and very high rates of the compression. It comes crucial in applications that require such flexibility. Another main advantage of this standard is the ability of effective handling large range of bit rates. It allows to reduce number of steps taken in processing certain images in comparison to JPEG. As an example, reducing the number of bits in some image below certain amount using JPEG standard compliant solution requires reducing the resolution of the input at first. Only after this procedure encoding of the image can be applied. The JPEG 2000 standard supplies adequate feature named multiresolution decomposition structure which makes such transformation transparent and one step only. [5]

1.2 Objective of the project

The standard way of performing discrete wavelet transform (DWT) in the JPEG 2000 compliant with Part 1 is to decompose the image into subbands using a pair of low and high-pass filters. This decomposition is applied multiple times using higher DWT orders. The standard order which is used across whole industry is five. [11] [8] The Part 2 of the standard contains several types of extensions which can be applied to modify the encoding algorithm. For instance DWT can be modified in a way that makes decomposition of the image into subbands of different shapes possible. Moreover, the strict selection of the pair of filters imposed by Part 1 of the standard can be broken. However, the same pair has to be used for all subbands of the image. [11] The other type of applicable modification is skipping some steps of discrete wavelet transform (SS-DWT). It is usually beneficial for processing non-photographic and screen content images. Another way of achieving improvement in terms of compression ratio is applying the reversible histogram packing. This type of extension significantly improves the ratio of compression when the histogram of the image is sparse. It means that unused levels appear between frequently used brightness levels. With the help of described Part 2 compliant extensions to the JPEG 2000 standard it is possible to adaptively adjust the transform for a specified image to improve the compression ratio. The result of this operation can still be correctly decoded by every decompressor which is compatible with the Part 2 of the JPEG 2000 standard.

The objective of the thesis is to develop, implement and test several form of heuristics which can determine the optimal transform in terms of compression ratio of the given image. Transform shall be compliant with the Part 2 of JPEG 2000. The heuristics shall be rather fast and use entropy as an estimation of the JPEG 2000 encoding. Moreover, they can be greedy and use trial and error approach to some extent. The implementation of the program shall be done in modern C++ to utilize such language capabilities as cross-platform threads. The main target of the application are multi-core CPU architectures. The result of the project work is a tool that quickly determines the transform for the specified image and invokes the JPEG 2000 encoder with selected transform. However, it is acceptable to achieve small time overhead in terms of the entire compression process. The resulting image shall come with the improvement of the lossless JPEG 2000 compression ratio.

1.3 Scope of the thesis

- Initial research in fields of image processing and compression, analysis of JPEG 2000 algorithm.
- More advanced research of algorithms such as DWT, SS-DWT, HP and JPEG 2000 implementation - Kakadu.
- Development of basic DWT implementation.
- Development of advanced 2D DWT implementation with possibility of skipping transformation of columns or rows.
- Setup of Continuous Integration system and implementation of DWT testing component
- Development of initial heuristics allowing to study the effects of DWT modifications compliant with the JPEG 2000 standard such as decompositions into subbands and usage of different filters
- Support of loading and storing both grayscale and color images.
- Initial implementation of multi-threaded heuristics.
- Conducting preliminary tests and selecting modifications or their variants to be included in the final heuristics.
- Development of multi-threaded optimized implementation of final heuristics.

- Research on final heuristics - comparison in terms of obtained compression ratio and time with: unmodified JPEG 2000, SS-DWT transformation and the transformation determined by an exhaustive search.

1.4 Thesis outline

At the beginning of this paper there is introduction to the domain problem of image processing and compression. Some methods of applying this kind of compression are described in Introduction. Moreover, objective and scope of the thesis are described there.

The last chapter is Summary which wraps up all results and makes some valuable conclusions. At the end there are appendices available such as technical documentation and list of used tables, listings, etc.

Chapter 2. Problem analysis

2.1 Discrete Wavelet Transform

2.1.1 One dimensional DWT

The linear convolution (filtering) of sequences $x(n)$ and $h(n)$ is defined as in equation 2.1:

$$y(n) = \sum_{m=-\infty}^{\infty} x(m)h(n-m) \quad (2.1)$$

The one dimensional discrete wavelet transform can be depicted as successive applications (convolutions) of one selected pair of high and low-pass filters. The output of such application is then followed by downsampling by the factor of two. For example, it can be achieved by discarding samples with odd indices after each of filtering operation. It is better visualized in the Figure 2.1. [11] The pair of low and high-pass filters is known as analysis filter bank in the encoding process. In the signal decoding process it is featured as a synthesis filter bank. The decoding step requires using the inverse of discrete wavelet transform.

Take into consideration a one dimensional signal $x(n) = \{55, 234, 70, 21, 88, 37\}$. It can be better understood as values of pixels in a part of the grayscale image row. It is followed with a pair of low and highpass filters designated by $h_0(n)$ and $h_1(n)$ respectively. An example of such pair is a lowpass filter $h_0(n) = \{-1, 2, 6, 2, -1\}/8$ and a highpass filter $h_1(n) = \{-1, 2, -1\}/2$. They are both symmetric and consist of

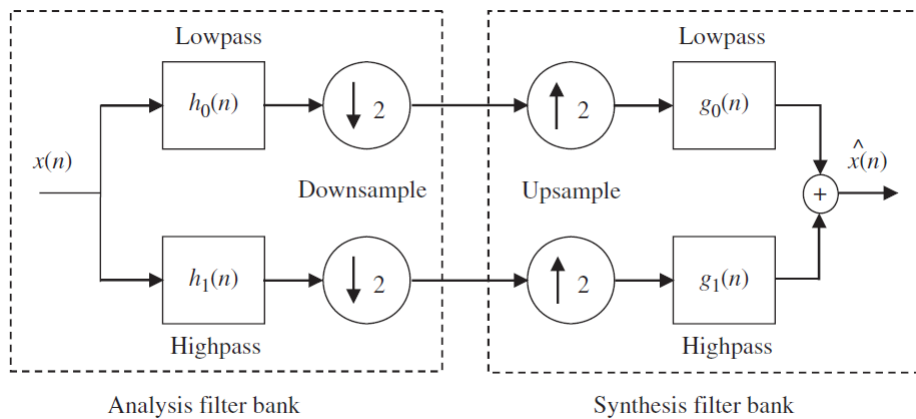


Figure 2.1: 1-D DWT, two-band wavelet analysis and synthesis filter banks [11]

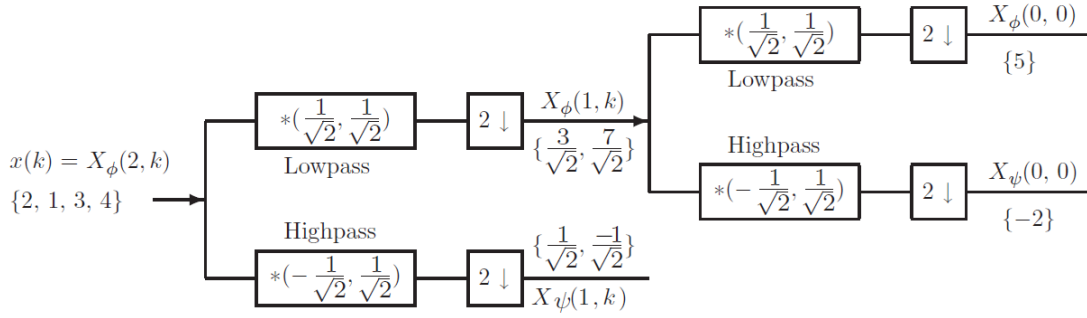


Figure 2.2: Computation of a 2-level 4-point DWT using a two-stage two-channel Haar analysis filter bank [18]

only integer taps. Such pair can be presented in the notation of (5, 3) filter bank. This convention indicates that the length of lowpass filter is five and the length of highpass filter is three. In fact the analysis filter bank presented here was firstly proposed by LeGall and Tabatabai in 1988 and is used in the JPEG 2000 standard for lossless compression of images. The filtering operation has to be defined at the signal boundaries. Therefore, the one dimensional signal is extended in both directions. The Part 1 of the JPEG 2000 standard requires symmetrical extension to be performed in such case. [11] After applying the required symmetrical padding the signal is extended to $x(n) = \{21, 70, 234, 55, 55, 234, 70, 21, 88, 37, 37, 88, 21, 70\}$. Then, the lowpass filter is applied resulting in $x'_0(n) = \{197.25, 75.5, 98.375, 67.125, 45.375\}$ and the highpass one which results in $x'_1(n) = \{44.75, -85.75, 29, 12.75, -29.5\}$.

The next example shows how to compute the two levels of discrete wavelet transform. To speed up the process no padding option is chosen this time which makes it non-compliant with the JPEG 2000 standard. The filter used here is the most basic one, i.e. Haar analysis filter bank. It is the first wavelet from the Daubechies wavelet family. The calculation process is visualized in the Figure 2.2. [18]

The input is chosen as 4-point signal $X_\phi(2, k) = \{2, 1, 3, 4\}$. This notation emphasizes the fact that it is approximation of the input at scale 2. The so called scaling coefficients (or in other term approximation at scale 1) $X_\phi(1, k)$ are computed by convolving the input $x(k)$ with the low-pass Haar filter impulse response $l(k) = \{1/\sqrt{2}, 1/\sqrt{2}\}$. In the next step there is downsampling by a factor of 2 applied. The output of convolution has five values. The middle three from these five correspond to cases where both the given input values overlap with the impulse response. As it was described earlier, the odd values are preserved in the downsampling process. In a result first and third value of these three middle ones are the approximation output $X_\phi(1, k)$. In the similar way, the detail coefficients at scale 1 $X_\psi(1, k)$ are computed. The input $x(k)$ is convolved with the high-pass filter impulse response $h(k) = \{-1/\sqrt{2}, 1/\sqrt{2}\}$. Then the downsampling by factor of 2 is

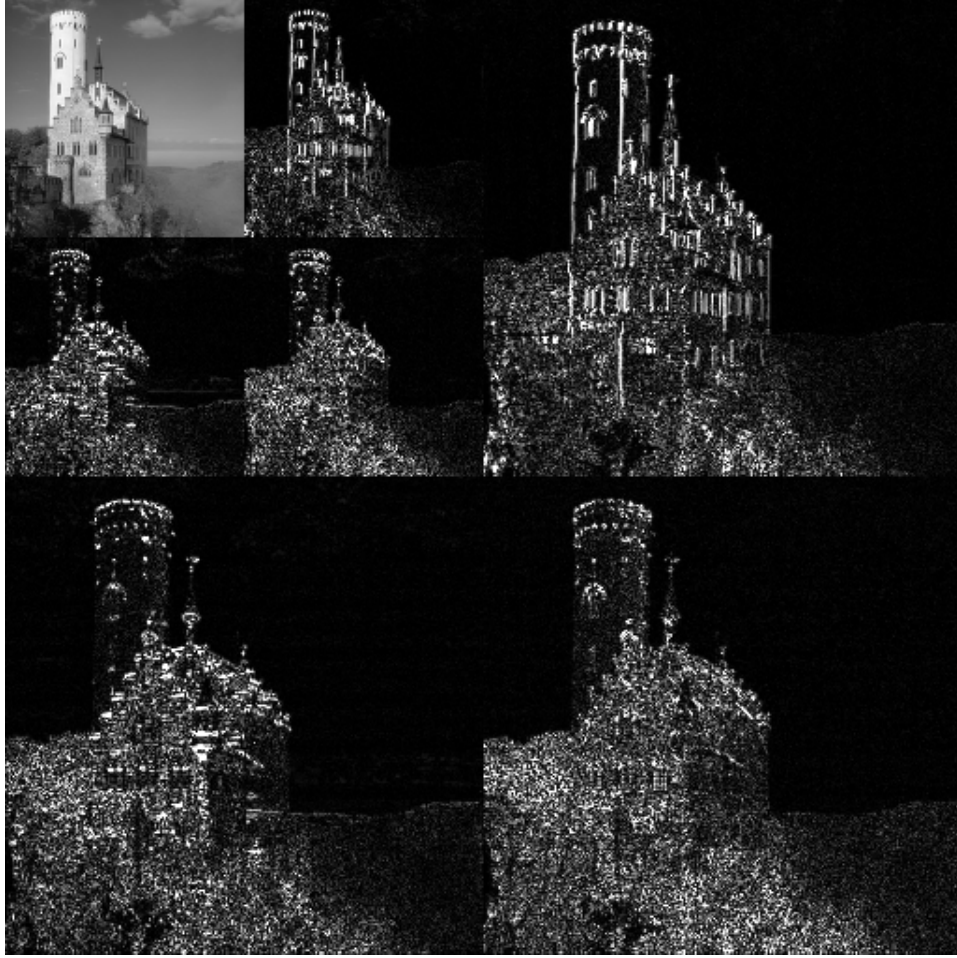


Figure 2.3: 2D DWT applied 2 times to an exemplary image [1]

performed. Note that only the approximation output $X_\phi(1, k)$ of the first stage goes to the second one. The $X_\phi(0, 0)$ and $X_\psi(0, 0)$ are calculated accordingly at the end of the second stage. [18]

2.1.2 Two dimensional DWT

The idea of using lowpass filter is the preservation of low frequencies of a signal while trying to eliminate or at least attenuate the high frequencies. In a result the output signal is the blurred version of the original one. Therefore, the operating principle of the highpass filter is completely opposite. As a result of applying such filter, the high frequencies of the signal are preserved and the low ones are discarded or at least diminished. The output is a signal consisting of edges, textures and other details. [11]

There is presented an example of the effects of the two dimensional discrete wavelet transform on the Figure 2.3. The DWT used here is compliant with the Part 1 of the JPEG2000 standard. The number of DWT stages presented in this example

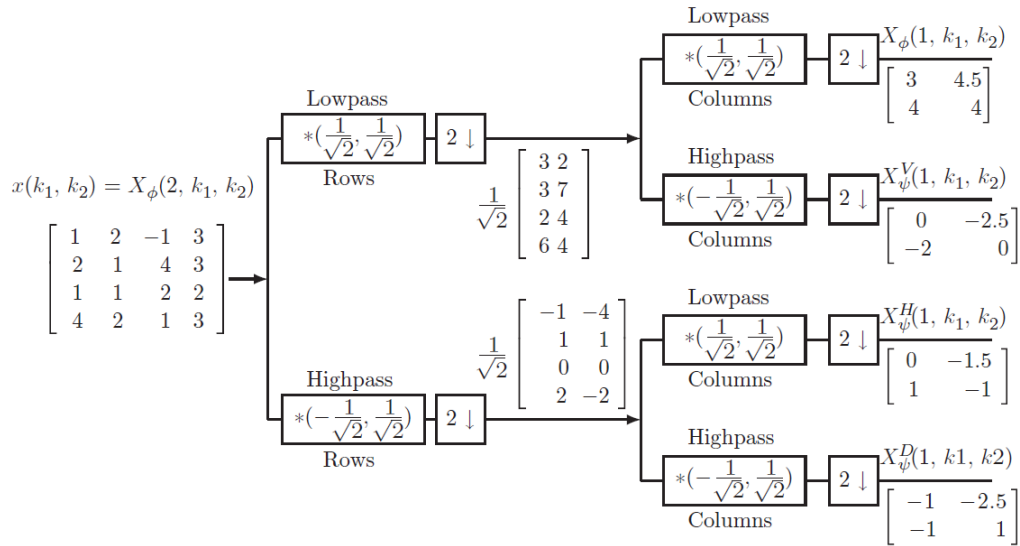


Figure 2.4: Computation of a 1-level 4 × 4 2-D Haar DWT using a two-stage filter bank [18]

is equal to two. Two dimensional discrete wavelet transform applied first time to the original image yields four same sized subimages. The LL layer (upper left subimage) is an approximation of the image and contains the low frequencies. This layer is once more transformed in the next stage. The LH layer (upper right subimage) preserves high frequencies from the rows of the image. As a result vertical lines and details (brightness) can be seen in the produced subimage. On the other hand, the HL layer (bottom left) contains high frequencies from the columns of the image. The horizontal details and lines can be noticed there. Lastly, the HH layer (bottom right) preserves the diagonal lines. [1]

The process of computing a 1 level two dimensional discrete wavelet transform with usage of two-stage analysis Haar filter bank is shown in Figure 2.4. Coefficients X_ϕ are calculated as a result of lowpass filtering and downsampling to each row of the two dimensional data. Next, similar process process, i.e. lowpass convolution and downsampling is applied to each column of resulting data. The rest of coefficients is obtained in very similar fashion to the previous ones. Coefficients X_ψ^H are calculated by applying high-pass filtering and downsampling to each row of the 2-D data x and then followed by applying sequence of low-pass filtering and downsampling to each column of the resulting data. Coefficients X_ψ^V are obtained by applying low-pass filtering and then downsampling to each column of the resulting data. Lastly, coefficients X_ψ^D are obtained by applying high-pass filtering and downsampling to each row of the 2-D data x followed by applying highpass filtering and downsampling to each column of the resulting data. In the next stage of more com-

plex dwt calculating process only the coefficients X_ϕ are taken into consideration. [18]

2.1.3 DWT features summary

- In a nutshell, the discrete wavelet transform is a set of bandpass filters. Usually it is implemented with the usage of low and high-pass filters recursively.
- The computational complexity of computing the DWT in the best case is linear, i.e. $O(N)$.
- The first approach to implement the DWT efficiently is evaluation of the required convolutions with the usage of the polyphase filter structure.
- The second approach is factorization of the polyphase matrix into a product of a set of sparse matrices.
- The two dimensional discrete wavelet transform (with separable filters) is usually computed by the row-column method. One dimensional DWT of all the columns is computed at first. Then the 1-D DWT of all the resulting rows is calculated. The order of the computation does not matter in terms of achieving the same result.
- Additional memory of approximate half the size of the given data is required in the implementation of the DWT.
- Data reordering is required for an in-place computation of the DWT.
- Data expansion problem can occur due to the finite length of the data in the implementation of the asymmetric filters.
- Symmetric filters provide linear phase response and an effective solution to the border problem. [18]

2.2 Part 2 of the JPEG 2000

2.2.1 Introduction

Many ideas have been emerging as the JPEG 2000 was developed. These concept were full of value-added capabilities. However, they were not that important to be gone through the time-consuming ISO standardization process. The Part 1 (ISO/IEC, 2004a) of the standard, i.e. Core coding system, was originally published

in 2000. There was a need to create additional parts to include missing features. The Part 2 of the standard, published as ISO/IEC 15444-2 or ITU Recommendation T.801 (ISO/IEC, 2004b), contains multiple such extensions. There is present group of rather small additions that could not merit entire documents of their own. In the Part 1 Core of JPEG 2000 standard decoders are supposed to handle all of the code-stream functionality. The Part 2 is different from first one in this aspect. It is a collection of options that can be implemented on demand to meet very specific requirements of the given market. Moreover, sections within an extension annex can be implemented separately. For example, subsets of extended file format JPX can be used on their own. Therefore, some features of the Part 2 may be present in the wide spectrum of JPEG 2000 applications while the other ones can be less common in the decoders. [11]

As it was shown in the previous paragraph, the extensions present in the Part 2 consist of very different set of topics that can modify or add some features to the Part 1 JPEG 2000 compliant processing chain. Some tools can result in the compression efficiency improvement. Others can ameliorate the visual appearance of compressed images. Another group of extensions can modify or extend some functionalities in the other ways. The list of the major topics is presented below. [11]

Compression efficiency:

- Variable DC offset (VDCO) - Annex B
- Variable scalar quantization (VSQ) - Annex C
- Trellis coded quantization (TCQ) - Annex D
- Extended visual masking - Annex E
- Arbitrary wavelet decomposition - Annex F
- Arbitrary wavelet transform kernel - Annexes G and H
- Multiple component transform - Annex J
- Nonlinear point transform - Annex K [11]

Functionalities:

- Geometric manipulation - Annex I
- Single-sample overlap (SSO/TSSO) - Annex I

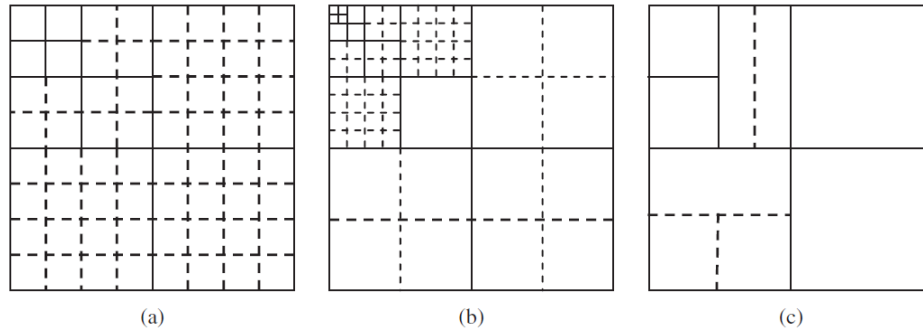


Figure 2.5: Some examples of decomposition compliant with the Part 2 [11]

- Precinct-dependent quantization - Amendment 1
- Extended region of interest - Annex L
- Extended file format/metadata (JPX) - Annexes M and N
- Extended capabilities signaling - Amendment 2 [11]

2.2.2 Arbitrary Decomposition

In the Part 1 of the JPEG 2000 standard there is only one wavelet decomposition structure allowed. This wavelet is called Mallat dyadic decomposition. Such decomposition is a good first choice to be applied across a wide spectrum of images. However, other ones can improve the quality of the image over specialized classes of the applications. The other effect of applying such decompositions are unequal reductions in the horizontal and vertical dimensional of reduced resolution extracts. [11]

Other decomposition styles can be found in the wavelet literature. They include the full packet tree processing and some of its derivatives. The applied packet decomposition derivatives can outperform the solution from Part 1 of the JPEG 2000 standard in some applications. For instance, they come crucial at maintaining regular fine-grain texture. Moreover, the applications that require processing synthetic aperture radar images can benefit from using this extension. The US Federal Bureau of Investigation actively uses a 500 ppi fingerprint compression standard, i.e. WSQ (CJIS, 1997). The decomposition is specialized for the characteristics of fingerprint imagery at 500 dpi. [11]

Some of these decomposition can be seen of the Figure 2.5. Resolution decomposition is depicted as solid lines. Dashed lines represent extra sublevel decomposition. On the first example, i.e. image (a), there is available full packet decomposition with such parameters: $NL = 3$: $Ddfs = 111$, $Doads = 321$, $Dsads = all$

Table 2.1: Analysis and synthesis filter taps for the floating-point Daubechies (9, 7) filter bank

n	Lowpass, $h_0(n)$	Lowpass, $g_0(n)$
0	+0.602949018236360	+1.115087052457000
± 1	+0.266864118442875	+0.591271763114250
± 2	-0.078223266528990	-0.057543526228500
± 3	-0.016864118442875	-0.091271763114250
± 4	+0.026748757410810	

n	Highpass, $h_1(n)$	n	Highpass, $g_1(n)$
-1	+1.115087052457000	1	+0.602949018236360
-2, 0	-0.591271763114250	0, 2	-0.266864118442875
-3, 1	-0.057543526228500	-1, 3	-0.078223266528990
-4, 2	+0.091271763114250	-2, 4	+0.016864118442875
		-3, 5	+0.026748757410810

1s. The next picture illustrates FBI decomposition wit specified parameters: NL = 5: Ddfs = 11111, Doads = 2321, Dsads = 1110111111111111. The last image is juts an arbitrary example. [11]

The prespecified decomposition structures are not the only feature of this extension. Wavelet packet analysis can be also used to design custom decompositions for specific images or some types of images. It was implemented in Coifman and Wickerhauser, 1992; Ramchandana and Vetterli, 1993; Meyer, Averbuch, and Stromberg, 2000. Such applications often start with a large decomposition tree. Then, they tend to locate a good decomposition based upon specified optimization metric. [11]

2.2.3 Arbitrary Wavelet Transforms

The Part 1 of the JPEG 2000 standard specifies only two possible wavelet transforms. The reversible one (5-3R, Table 2.2) and the irreversible one (9-7I, Table 2.1). As it was stated before, both are required to perform periodic symmetric signal extension at the boundaries. It is similar case to the Mallat dyadic decomposition in terms of generic implementation. These filters can compress quite well a wide set of image types. However, certain image classes can be compressed more efficiently with other types of wavelets. Such a flexibility is allowed in the Part 2 compliant applications. The range of wavelet transforms is broadened to include not only the wider range of whole-sample symmetric ones but also half-sample and generic non-symmetric ones. Such ability to handle generic filters makes JPEG 2000 standard

Table 2.2: Analysis and synthesis filter taps for the integer (5, 3) filter bank

n	Lowpass, $h_0(n)$	Lowpass, $g_0(n)$
0	+0.75	+1
± 1	+0.25	+0.5
± 2	-0.125	

n	Highpass, $h_1(n)$	n	Highpass, $g_1(n)$
-1	+1	1	+0.75
-2, 0	-0.5	0, 2	-0.25
		-1, 3	-0.125

a powerful research tool, together with supporting more than niche compression applications. [11]

2.3 Computer architecture

2.4 Known solutions

- state of the art, literature research (all sources in the thesis have to be referenced)
- description of known solutions, algorithms

Chapter 3. Subject of the thesis

3.1 Solution to the problem

- solution to the problem proposed by the author of the thesis
- theoretical analysis of proposed solutions

3.2 Rationale of applied algorithms

- rationale of applied methods, algorithms, and tools

Chapter 4. Experiments

This chapter presents the experiments. It is a crucial part of the thesis and has to dominate in the thesis. The experiments and their analysis should be done in the way commonly accepted in the scientific community (eg. benchmark datasets, cross validation of elaborated results, reproducibility and replicability of tests etc).

4.1 Methodology

- description of methodology of experiments
- description of experimental framework (description of user interface of research applications – move to an appendix)

4.2 Data sets

- description of data sets

4.3 Results

- presentation of results, analysis and wide discussion of elaborated results, conclusions

Table 4.1: A caption of a table is **above** it.

ζ	method						
	alg. 1	alg. 2	alg. 3			alg. 4, $\gamma = 2$	
			$\alpha = 1.5$	$\alpha = 2$	$\alpha = 3$	$\beta = 0.1$	$\beta = -0.1$
0	8.3250	1.45305	7.5791	14.8517	20.0028	1.16396	1.1365
5	0.6111	2.27126	6.9952	13.8560	18.6064	1.18659	1.1630
10	11.6126	2.69218	6.2520	12.5202	16.8278	1.23180	1.2045
15	0.5665	2.95046	5.7753	11.4588	15.4837	1.25131	1.2614
20	15.8728	3.07225	5.3071	10.3935	13.8738	1.25307	1.2217
25	0.9791	3.19034	5.4575	9.9533	13.0721	1.27104	1.2640
30	2.0228	3.27474	5.7461	9.7164	12.2637	1.33404	1.3209
35	13.4210	3.36086	6.6735	10.0442	12.0270	1.35385	1.3059
40	13.2226	3.36420	7.7248	10.4495	12.0379	1.34919	1.2768
45	12.8445	3.47436	8.5539	10.8552	12.2773	1.42303	1.4362
50	12.9245	3.58228	9.2702	11.2183	12.3990	1.40922	1.3724

Chapter 5. Summary

5.1 Results

- synthetic description of performed work

5.2 Conclusions

- conclusions
- Has the objective been reached?

5.3 Future development

- Future development, potential future research

Appendices

Technical documentation

List of abbreviations and symbols

JPEG Joint Photographic Experts Group

PNG Portable Network Graphics

PACSs Picture Archiving and Communication Systems

DICOM Digital Imaging and Communications in Medicine

ISO International Organization for Standardization

DCT Discrete Cosine Transform

DWT Discrete Wavelet Transform

SS-DWT Skipped Steps Discrete Wavelet Transform

HP Histogram Packing

LL Low and then low-pass filtered image

LH Low and then high-pass filtered image

HL High and then low-pass filtered image

HH High and then high-pass filtered image

ppi pixels per inch

N cardinality of data set

μ membership function of a fuzzy set

\mathbb{E} set of edges of a graph

\mathcal{L} Laplace transformation

Contents of attached CD

The thesis is accompanied by a CD containing:

- thesis (pdf file),
- source code of applications,
- data sets used in experiments.

List of Figures

2.1	1-D DWT, two-band wavelet analysis and synthesis filter banks [11] . .	5
2.2	Computation of a 2-level 4-point DWT using a two-stage two-channel Haar analysis filter bank [18]	6
2.3	2D DWT applied 2 times to an exemplary image [1]	7
2.4	Computation of a 1-level 4×4 2-D Haar DWT using a two-stage filter bank [18]	8
2.5	Some examples of decomposition compliant with the Part 2 [11] . . .	11
4.1	Some caption.	17

List of Tables

- 2.1 Analysis and synthesis filter taps for the floating-point Daubechies (9, 7) filter bank 12
- 2.2 Analysis and synthesis filter taps for the integer (5, 3) filter bank 13
- 4.1 A caption of a table is **above** it. 18

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