Critical torque and speed of eddy current brake with widely separated soft iron poles

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Abstract: A theoretical model is derived for eddy current disc brakes with iron pole shoes with a wide spacing. The eddy current in the disc is not a simple periodic function allowing representation by sine functions. In the high speed region, the current distribution around the pole shoe occupies a limited zone width proportional to the air gap. The theory based on this effect leads to reasonably accurate values of the critical torque and speed compared with experimental data. The predicted air gap dependency of the critical values agrees better with experimental results than Rüdenberg's theory.

List of symbols

ρ = specific resistance of disc material
d = disc thickness
D = diameter of soft iron pole, for noncircular pole shape D denotes diameter of circle with same area as pole face
x = air gap between pole faces including disc thickness or co-ordinate perpendicular to air gap
ξ = ratio of zone width, in asymptotic current distribution around poles, to air gap
c = ratio of total contour resistance to resistance of contour part under pole; $c = 0.5$ if disc has infinite diameter
v = tangential speed, measured at centre of pole
v_k = critical speed, i.e., speed at which exerted force is maximum
v_k^0 = theoretical value of v_k divided by value at $x/D = 0.2$
v^* = experimental value of critical speed divided by theoretical value at $x/D = 0.2$
F_e = force of electromagnetic origin exerted by pole on disc
$\hat{F}_e$ = maximum value of exerted force F_e as function of v
f = theoretical value of $\hat{F}_e$ divided by value at $x/D = 0.2$
f^* = experimental value of $\hat{F}_e$ divided by value at $x/D = 0.2$
B_0 = air gap induction at $v = 0$
$_{exp}$ = (index) experimental value
y = co-ordinate in direction of movement
K = constant
R = distance from centre of disc to centre of pole
A = half of disc diameter
j = volume current density in Am^{-2}
P = electrical (mechanical) power
P_∞ = dissipation per pole in disc for speed tending to infinity
m = amount by which half of disc diameter, or half of strip width, exceeds that required to just cover pole
ω = angular speed of disc brake
T_e = torque of electromagnetic origin

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1 Introduction

The problem of the eddy current brake was successfully investigated by Rüdenberg. Rüdenberg took a cylindrical machine as a point of departure to design a brake that was to be energised by a direct current. At the time Rüdenberg wrote 'Energie der Wirbelströme' [1], all electrical machines had to be designed to obtain maximum power for a machine of certain dimensions. To develop a theory for the disc brake Rüdenberg supposed the poles were situated near to each other, so that the magnetic fields and the current patterns could be described sufficiently accurately by sinusoidal functions.

Improved construction materials enabled a power density that can lead to heat problems in the disc. This necessitated better cooling and wider spacing of the poles, since an eddy current brake is a machine in which all of the mechanically absorbed power is dissipated in the (thin) disc. The designer was, despite the non-applicability of the theory, able to construct a brake. This was because the design was constrained by heat transfer problems, and not by the prediction of the correct excitation current to torque ratio.

Zimmermann [4] noted that the critical torque and speed were not proportional to the air gap, as predicted by the theory of Rüdenberg, but showed 'ein schwächeres als proportionales Anwachsen'.*

Smythe [2] studied the current distribution in the disc around the pole. That study was successful for low speeds, but for the high speed region the results were not suitable. The asymptotic behaviour shows a fall-off of the torque more rapid than ω^{-1} in the high speed region, which is in contradiction with experimental results.

Schieber [10, 5] found the same dragging forces as Smythe, but extended the theory so that it became valid for a rotating disc as well as for a linearly moving strip. Schieber did not investigate the high speed region.

Only global methods will provide a satisfactory solution, since no methods exist to explicitly find expressions for the current and magnetic field distributions to improve the physical understanding of the eddy current

* 'a slower than proportional ascent' [4], p. 146

brake. Spek [7] used the supposition that the current distribution under the pole will be the same at the critical speed, regardless of the values of D, x or d and independent of z, the co-ordinate perpendicular to y. The function $B(y)$ under the pole at the critical speed may be experimentally determined. It can be approximated by a simple power series in y/D, giving values for $(\partial B/\partial y)$ and for the current distribution under the pole. On the base of these data the critical force may be found as a function of x, D and B_0

$$\hat{F}_e = KD \times B_0^2 \tag{1}$$

in which K is a constant, the value of which is experimentally determined to be about 2.48×10^5 $\mathrm{AmV^{-1}\,s^{-1}}$ [7].

The measured torque proved to be about twice the value predicted by eqn. 1. This may be caused by the currents and magnetic field outside the pole area. Another major limitation of this method is the assumption that the critical current distribution is independent of x, which is essentially false. The same x-proportionality is predicted by Rüdenberg and no value is predicted for the critical speed.

A second method is proposed. It makes use of two known phenomena of the high speed region. The drag force F_e becomes proportional to v^{-1}, i.e. the developed power asymptotically tends to a constant. The original magnetic induction under the pole tends to be cancelled by the current induced around it in the disc [7, 4]. This, together with the already known asymptotic behaviour around $v = 0$ [2, 5, 10], enables a global theory. This theory is useful for the designer and the physical understanding of the eddy current brake.

2 Current distribution in the low speed region

When the disc of an eddy current brake is moved, an electrical field $\boldsymbol{E} = \boldsymbol{v} \times \boldsymbol{B}$ is induced perpendicular to both the speed of movement $\boldsymbol{v}$ and the magnetic induction $\boldsymbol{B}$. If the speed is low with respect to the critical speed, i.e., and magnetic induction caused by the current pattern is negligible compared with the original induction B_0 at zero velocity, then the magnetic induction perpendicular to the plane of the disc may be assumed to be equal to B_0.

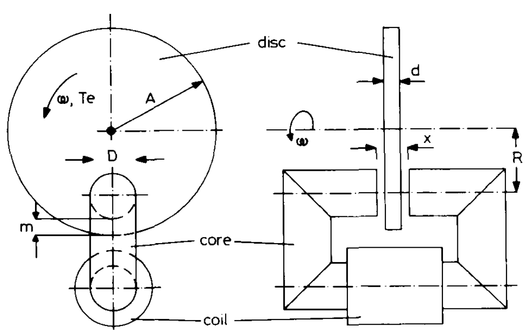

Fig. 1 *Eddy current brake*

The question of the current pattern in the disc, solved in detail by Smythe [2] and Schieber [5, 10], may be elucidated by the following thought experiment. Suppose the pole is surrounded by a (super) conducting ring slipping over the disc, forming a resistanceless return path for the currents under the pole.

This 'ideal' eddy current brake generates the current pattern of Fig. 2.

The current density $\boldsymbol{j}$ under the pole in the hypothetical brake is $1/\rho(\boldsymbol{v} \times \boldsymbol{B})$ because of the simple boundary

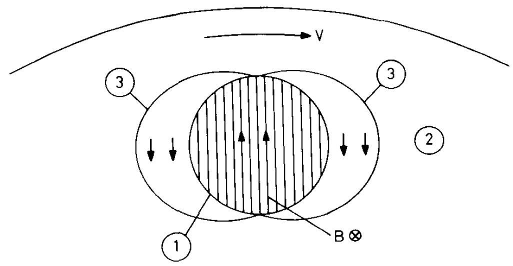

Fig. 2 *'Ideal' eddy current brake at low speed*

$c = 1$
1: hypothetical superconducting ring around pole slipping over disc
2: disc
3: tangential current in ring (sinusoidal around circumference for circular pole profile)

condition embodied by the short circuit ring.† The total dissipation may be found simply by integrating ρj^2 over the cylindrical volume $\frac{1}{4}\pi D^2 d$ resulting in

$$P_{diss} = \frac{1}{4}\frac{\pi}{\rho} D^2\, dB_0^2 v^2 \tag{2}$$

Dividing by v on both sides, eqn. 2 changes into

$$F_e = \frac{\pi}{4\rho} D^2\, dB_0^2\, v \tag{3}$$

which means that, for low speeds, the dragging force is proportional to the speed.

In real systems, the return path of the induced currents is not of zero resistance, three factors now have to be considered

(i) Resistance increases
(ii) Total dissipation decreases
(iii) Finite diameter of the disc or the finite width of the strip has an (minor) influence on the dragging force

If the result of Schieber [5] is compared with eqn. 3 then a form like

$$F_e = \frac{1}{4}\frac{\pi}{\rho} D^2\, dB_0^2\, cv \tag{4}$$

is found in which

$$c = \frac{1}{2}\left[1 - \frac{1}{4}\frac{1}{\left(1 + \frac{R}{A}\right)^2 \left(\frac{A - R}{D}\right)^2}\right] \tag{5}$$

eqns. 4 and 5 completely agree with Smythe's [2] initial result.

There is a connection with another publication of Schieber [10] regarding the dragging force exerted on a linearly moving strip under the same condition of negligible magnetic field generated by the induced eddy currents. Referring to Fig. 3 for the physical dimensions, Schieber's result may be written as

$$c' = \frac{1}{2}\left[1 - \frac{\pi^2}{24}\left(\frac{D}{2h}\right)^2\right] \tag{6}$$

with a similar expression as eqn. 4 for F_e with c' instead of c.

† Though this is not exact, the R to D ratio is supposed to be so large that the situation is the same as with a strip moving linearly.

At low speed the construct of the eddy current brake acts as a linear damper, i.e., the dragging force exerted is

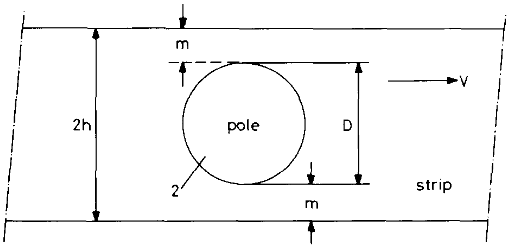

Fig. 3 *Dimensional reference for Schiebers' expression*

proportional to the speed. If the extension of the disc or the strip is infinite, then the proportionality factor c is exactly half that of the hypothetical brake with the superconducting slipping ring around the pole.

An excess copper of the strip, or disc, outside the pole exceeding a quarter of the pole diameter D is of a limited help only in enlarging the proportionality factor c or c'. Table 1 illustrates this. With a strip, or disc, only two times $\frac{1}{4}D$ wider than that required to just cover the pole, more than 81% of the potential dragging force is obtained.

Table 1: Copper excess against proportionality factor *c* for low speed dragging force

Schrieber for moving strip			Schrieber and Smythe for disc brake		
c'	$\frac{2h}{D}$	m(2-sided)	c	A	m
0.295	1	0	0.205	0.150	0
0.409	1.5	0.25D	0.418	0.1625	0.25D
0.449	2.0	0.50D	0.458	0.175	0.50D
0.477	3.0	1.00D	0.485	0.200	1.00D
0.500	∞	∞	0.500	∞	∞

3 Asymptotical behaviour for large speed

A similarity exists between an eddy current disc brake and a DC current fed induction machine. The original flux remains uninfluenced by the rotor current generated magnetic field for low speeds and a speed proportional torque is generated. The behaviour is dominated by the current source character of the rotor circuit for very high speed, pushing away the original main flux into the leakage ways, perpendicular to the teeth. This causes the rotor dissipation and the power $T_e \omega$ to tend to a constant. If the same physical mechanism is found for the disc brake, a theoretical base for the function $F_e(v)$ will be found without the need to worry about what happens in the transition regions.

When the air gap magnetic field is studied at different speeds, three remarkable phenomena are observed

(*a*) At very low speeds the field differs only slightly from the field at zero speed. At the side at which the copper disc or strip enters the pole air gap, the magnetic induction is slightly less than B_0 and slightly higher at the exit side.*

(*b*) At the speed at which the maximum dragging force is exerted, the mean induction under the pole is already significantly les tha B_0, though in a small region at the exit side B_0 is still exceeded.

* Assume the strip or disc moves from left to right in every diagram.

(*c*) At higher speeds the magnetic induction tends to further decrease. Fig. 4 shows that the mean induction under the pole is already reduced to about one quarter of B_0 at a speed of about three times the critical value.

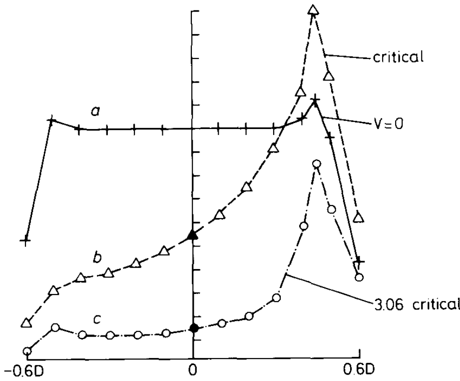

Fig. 4 *Magnetic induction measured under round pole* [7]

$B_0 = 0.1$ Vs/m^2; $D = 50$ mm; $x = 12$ mm; $d = 8.25$ mm
a: standstill
b: experimentally determined critical speed
c: 3.06 times critical speed

The magnetic induction shown in Fig. 4 was measured with a device made of polymethylmethacrylate with 13 sleeves, exactly large enough to enclose the Hall probe, used for the measurements. Fig. 5 shows the device.

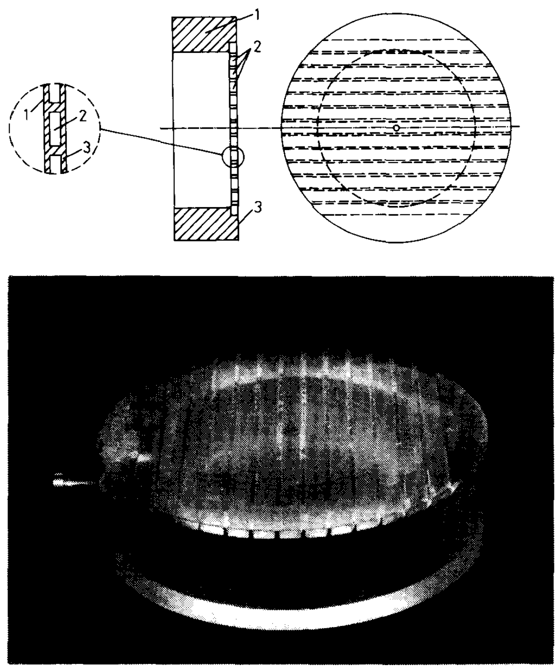

Fig. 5 *Pole covered with sleeves for induction measurement*

1: cylindrical body fitting over pole
2: sleeves
3: safety cover
4: photograph

A similar result was obtained by Zimmermann [4]. A figure on page 152 of Reference 4 shows what happens when the speed of movement is increased beyond the critical value.

Fig. 6 shows that at about 10 times the critical speed the mean original field is reduced to less than 20%. The hypothesis, that for infinite speed the original magnetic field will be completely cancelled by the induced eddy

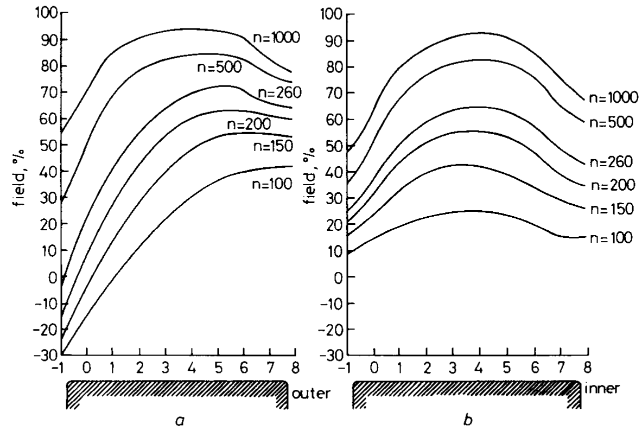

Fig. 6 *Magnetic field induced by eddy currents [4]*
a Distribution in direction of movement
b Distribution in transverse direction

currents, is evident on the basis of Zimmermann's experimental results.

This hypothesis becomes a matter of fact by the following thought experiment.†‡

Suppose a particle is on the copper strip, moving with a very large speed and passing the air gap zone of a construction like the brake of Fig. 1. Far from the pole an arbitrary contour like C in Fig. 7 links no magnetic flux.

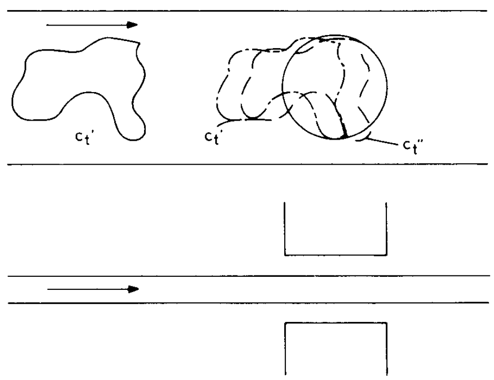

Fig. 7 *Thought experiment showing asymptotic behaviour of eddy current brake at $v \to \infty$*

The flux linkages of C do not change with time. Let the contour reach the pole region. If any magnetic flux has remained from the air gap field, then a change in the flux linkages of C may occur, because C is not required to have any specific form. Follow the contour C on from t' to t'' in Fig. 7, Since the change in the flux linkages mentioned is supposed to proceed in a time approximating zero, it requires a voltage $\oint \boldsymbol{E}\, ds$ rising to infinite, and an unlimited intensity of the induced eddy currents.

This unlimited growth of the associated eddy currents, and the nonzero resistivity of the disc material, is incompatible with the observed value of the product of the drag force F_e and the speed v.

Another approach may be the supposition of C in Fig. 7 having the form of an eddy current tube ($t' - t''$ assumed infinitesimally small). It must carry a current just effecting zero total flux to enable the effect of a limited dissipation at an unlimited speed.

The eddy current distribution at $v \to \infty$ is the same as that which would occur in a superconducting disc after it has been shifted into the air gap.

The last task is to determine the eddy current pattern that just annihilates the original magnetic field, to be able to calculate the asymptotic power P_∞.

4 Asymptotical power

Fig. 8*a* shows the magnetic field at standstill inclusive of the field at the contour of the pole. Fig. 8*b* the disc is supposed to move with a very large speed so that the net

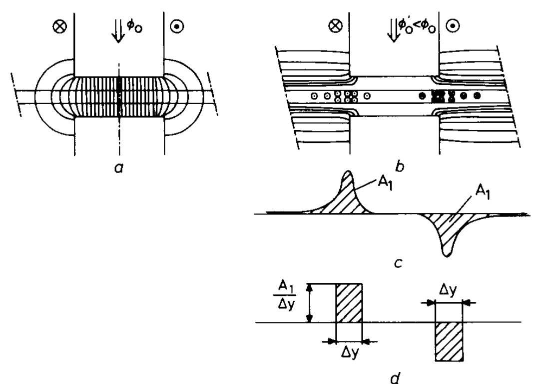

Fig. 8 *Magnetic field and eddy current*
a Magnetic field at standstill
b Magnetic field at infinite speed
c Eddy current at standstill
d Eddy current at infinite speed

magnetic induction in vertical direction tends to zero everywhere in the disc.

The current distribution is given in Fig. 8*c*. If Ampère's law is applied to a contour through the centre of the core, where the magnetic field strength H is zero (in the air gap

† J.A. Schot is the auctor intellectualis of this and many other 'experiments of thought' like this.
‡ SCHOT, J.A.: 'Private communication'

because of the annihilation effect and in the iron because of its assumed ideal properties) then the total circulating eddy current exactly balances the coil Ampere turns $(B_0 x/\mu_0)$. The current contents of A_1 in Fig. 8*c* must also be $B_0 x/\mu_0$.

This conclusion does not imply that in Fig. 8*b*, ϕ'_0 must also be zero. The flux will be forced to follow a horizontal path, and a leakage path past the boundary of the disc. In Fig. 8*d* a current distribution is given with the same total current, but a rectangular shape. To determine the effective sone width Δy is chosen such that the dissipation is the same as that obtained by experiment.

The assumption that Δy is proportional to the air gap width x, $\Delta y = \xi x$, is strongly supported, but it must remain unproven since no explicit expressions are known for either the distribution of the magnetic field at standstill or the current distribution in Fig. 8*c*.

For a restricted range of ratios $x : D$, P_∞ can be determined as follows: The effective resistance $(\pi D/d\xi x)\rho$ carries a current $(B_0 x/\mu_0)$. This results in

$$P_\infty = \pi\rho \frac{B_0^2}{\mu_0^2} \frac{xD}{\xi d} \tag{7}$$

If eqn. 7, which states the product of F_e and v to tend to P_∞, is combined with eqn. 4, giving the derivative of F_e with respect to v at $v = 0$, a function $F_e(v)$ with the same two properties of its asymptotical behaviour can be derived.

This function is

$$F_e(v) = \hat{F}_e \frac{2}{\dfrac{v_k}{v} + \dfrac{v}{v_k}} \tag{8}$$

with

$$\hat{F}_e = \frac{1}{\mu_0}\sqrt{\left(\frac{c}{\xi}\right)} \frac{\pi}{4} D^2 B_0^2 \sqrt{\left(\frac{x}{D}\right)} \tag{9}$$

and

$$v_k = \frac{2}{\mu_0}\sqrt{\left(\frac{1}{c\xi}\right)} \frac{\rho}{d} \sqrt{\left(\frac{x}{D}\right)} \tag{10}$$

The only quantity in eqns. 9 and 10 that is not exactly known is the proportionality factor ξ. It is estimated to have a value of about unity. An estimation error of 20% only causes a 10% deviation in eqns. 9 and 10, so the value of ξ is not of prime importance. There are many factors causing disturbing deviations from the theory:

(i) The yoke iron is not always far from the air gap so that the field outside the gap does not decay to zero

(ii) The x to D ratio is not always small

(iii) In determining v_k at full load, the temperature influence is equivalent to about doubling the specific resistance of the disc copper. The v_k value will be inaccurate if no accurate temperature measurement is obtained.

If ξ is determined exactly for a certain case, e.g., using a finite elements method, eqns. 9 and 10 may not predict values more accurate than 10 or 20%.

5 Experimental results

Fig. 9 shows experimental results from different sources. The construction parameter x/D is plotted on a logarithmic scale as is $\hat{F}_e$ and v_k. $\hat{F}_e$ and V_k are displayed per unit with their theoretical values at $x/D = 0.2$ as a reduction base. The derivation from the theoretical value, a straight line with an arctan 0.5 ascent, can easily be read off the Figure.

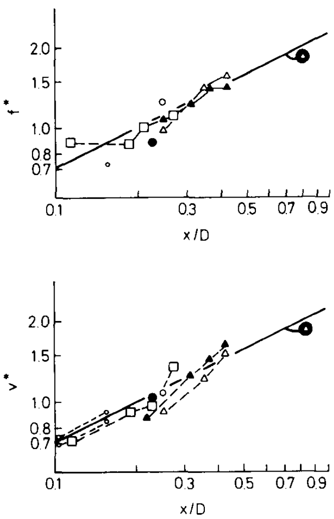

Fig. 9 *Experimental values composed with theory*

- ∘ Lentz [3]; rectangular pole
- ● Zimmermann [4]; round pole
- ○ Dumoulin [6]; round pole
- □ Berkens [8]; square pole
- ▲ Van Sterkenberg [9]; round pole
- △ Van Sterkenberg [9]; round pole
- ⬤ Theoretical value

a f^*

$$f\left(\frac{x}{D}\right) = \frac{\hat{F}_{e\,\text{theor}}\left(\frac{x}{D}\right)}{\hat{F}_{e\,\text{theor}}(0.2)}$$

$$f^* = \frac{F_{e\,\text{exp}}}{F_{e\,\text{theor}}(0.2)}$$

b v^*

$$v_k^0\left(\frac{x}{D}\right) = \frac{V_{k\,\text{theor}}\left(\frac{x}{D}\right)}{V_{k\,\text{theor}}(0.2)}$$

$$v^* = \frac{V_{k\,\text{exp}}}{V_{k\,\text{theor}}(0.2)}$$

The ξ estimate is unity for all cases. The c factor is calculated from eqn. 5 according to Smythe [2] and Schieber [5]. For the rectangular and square poles, a D-value is substituted in place of a circle with the same area as that of the real pole.

One example from Zimmermann is left out of the comparison because of its exceptional construction form: The 'equivalent' circular poles would have so large a diameter D that they would not be covered by the disc, at the outer side or the inner side. ('Siemens-Bremse' [4]; the disc had a large central hole.)

Lentz [3] was aware that the temperature had a major influence on v_k. Though not stated explicitly, it is probable that Lentz took care to avoid large temperature differences when determining the critical speed v_k. The four experimental values of v_k lie surprisingly close to the theoretical line.

6 Conclusions

Eqns. 9 and 10 predict the critical force and speed better than the theory of Rüdenberg, which gives a critical force and speed both proportional to the air gap x.

Though not completely accurate, the formulas derived give the designer a good first approximation and the reader may find it a good tool for the physical understanding of the eddy current brake.

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