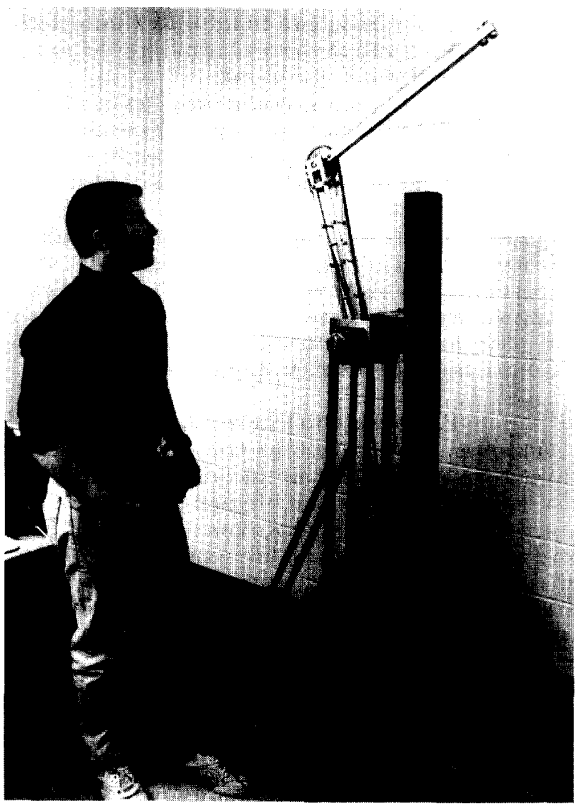

The Swing Up Control Problem For The Acrobot

Mark W. Spong

Underactuated mechanical systems are those possessing fewer actuators than degrees of freedom. Examples of such systems abound, including flexible joint and flexible link robots, space robots, mobile robots, and robot models that include actuator dynamics and rigid body dynamics together. Complex internal dynamics, nonholonomic behavior, and lack of feedback linearizability are often exhibited by such systems, making the class a rich one from a control standpoint. In this article we study a particular underactuated system known as the Acrobot: a two-degree-of-freedom planar robot with a single actuator. We consider the so-called swing up control problem using the method of partial feedback linearization. We give conditions under which the response of either degree of freedom may be globally decoupled from the response of the other and linearized. This result can be used as a starting point to design swing up control algorithms. Analysis of the resulting zero dynamics as well as analysis of the energy of the system provides an understanding of the swing up algorithms. Simulation results are presented showing the swing up motion resulting from partial feedback linearization designs.

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Introduction

In this paper we study the swing up control problem for the Acrobot, a two-link, underactuated robot that we are using to study problems in nonlinear control and robotics (refer to Fig. (1)). The Acrobot dynamics are complex enough to yield a rich source of nonlinear control problems, yet simple enough to permit a complete mathematical analysis.

The swing up control problem is to move the Acrobot from its stable downward position to its unstable inverted position and balance it about the vertical. Because of the large range of motion, the swing up problem is highly nonlinear and challenging. We derive two distinct algorithms for the swing up control. Both of our algorithms are based on the notion of partial feedback linearization [11], but also share a common design philosophy with the recent method of integrator backstepping [12]. As we shall see, our first algorithm is useful in the case that there are no limits on the rotation of the second link, while our second algorithm can be used in cases where the second link is restricted to less than a full 360° rotation.

The Acrobot model that we use is a two-link planar robot arm with an actuator at the elbow (joint 2) but no actuator at the shoulder (joint 1). The equations of motion of the system are [23]

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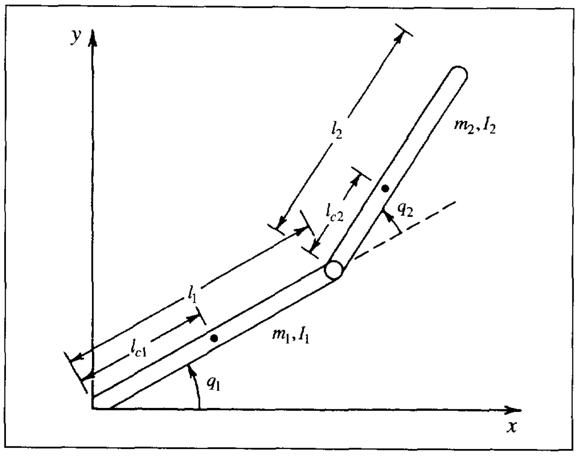

Fig. 1. The Acrobot.

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + h_1 + \phi_1 = 0 \tag{1}$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + h_2 + \phi_2 = \tau, \tag{2}$$

where

$$d_{11} = m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos(q_2)) + I_1 + I_2$$
$$d_{22} = m_2 l_{c2}^2 + I_2$$
$$d_{12} = m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2$$
$$d_{21} = m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2$$
$$h_1 = -m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2^2 - 2m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2\dot{q}_1$$
$$h_2 = m_2 l_1 l_{c2}\sin(q_2)\dot{q}_1^2$$
$$\phi_1 = (m_1 l_{c1} + m_2 l_1)g\cos(q_1) + m_2 l_{c2} g\cos(q_1 + q_2)$$
$$\phi_2 = m_2 l_{c2} g\cos(q_1 + q_2).$$

The difference between the system (1)-(2) and the standard model of a two-link planar robot [23] is, of course, the absence of an input torque to the first equation (1).

There have been a number of previous studies of underactuated mechanical systems; only a few will be mentioned here. The term "Acrobot" was coined at Berkeley, where the first studies of its controllability properties were performed by Murray and Hauser [14]. More recently, Berkemeier and Fearing [3] have investigated the application of nonlinear control to achieve sliding and hopping gaits of an Acrobot that has its first link free, as opposed to this paper in which the first link is pinned.

The first experimental results for the Acrobot were produced by Bortoff [5] in his Ph.D. thesis. The technique of pseudolinearization was used to design both observers and controllers to balance the Acrobot along its (unstable) equilibrium manifold of balancing configurations. The so-called Rolling Acrobot, which is similar to the mechanism of Berkemeier and Fearing, was also studied in this thesis (see also [4]).

In [17] a similar mechanism was designed and built to investigate so-called brachiation motions. Excellent experimental results were achieved using control algorithms quite different from the type considered here. The control of other gymnast-type robots has been considered in [24, 25] and [18, 20]. The control of manipulators with passive joints has been considered in [1] and [2]. These mechanisms used brakes on the passive joints, which introduces a reduced amount of actuation to the passive joints that is unavailable for the Acrobot.

The area of space robotics contains many opportunities for the study of underactuated systems. The papers by Papadopoulos and Dubowsky [8,15,16], for example, have shown the existence of so-called dynamic singularities in the task space control which greatly complicates the control problems.

A number of other related studies can be mentioned, such as the control of the more classical inverted pendulum [9]. Most previous works have used open loop strategies, sinusoidal excitation, etc., for swing up control. A notable exception is the paper [26], which discusses controlling the energy of the system; an approach related to the one of the algorithms in this paper.

Partial Feedback Linearization

It has been shown [14] that the Acrobot dynamics are not feedback linearizable with static state feedback and nonlinear coordinate transformation. This is typical of a large class of underactuated mechanical systems. However, as we will show, we may achieve a linear response from either degree of freedom by suitable nonlinear feedback. In this section, we derive and analyze two distinct nonlinear controllers to achieve two distinct systems, which we call Σ_1 and Σ_2, and which represent the linearization of the response of link 1 and link 2, respectively. We will use these two systems to generate two distinct approaches for the swing up control problem.

The easiest way to see how the partial feedback linearization is accomplished is as follows. In equation (1) suppose that we solve for either $\ddot{q}_2$ or $\ddot{q}_1$ and use the resulting expression in the second equation (2). In this way the second equation will be a feedback linearizable equation involving only $\ddot{q}_1$ in the first case or only $\ddot{q}_2$ in the second case. Upon choosing τ to linearize the resulting equation (2), we achieve either the system Σ_1

$$d_{12}\ddot{q}_2 + h_1 + \phi_1 = -d_{11}v_1 \tag{3}$$

$$\ddot{q}_1 = v_1, \tag{4}$$

or the system Σ_2

$$d_{11}\ddot{q}_1 + h_1 + \phi_1 = -d_{12}v_2 \tag{5}$$

$$\ddot{q}_2 = v_2, \tag{6}$$

where the terms v_1 and v_2 are additional (outer loop) control inputs to be designed. (This will be clarified below.) We use the term *non-collocated linearization* to describe the system Σ_1 since the unactuated joint response is linearized, and we use the term *collocated linearization* to describe the system Σ_2 in which the actuated joint response is linearized. (See [20] for further details.)

Thus, under conditions that we will state below, the systems Σ_1 and Σ_2 are both feedback equivalents of the Acrobot dynamics. Either of these systems, Σ_1 or Σ_2, may be used to generate a swing up control strategy, as we will show, after first giving the details of the derivations of Σ_1 and Σ_2.

Derivation of the System Σ1: The Non-Collocated Case

Consider the first equation (1)

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + h_1 + \phi_1 = 0 \tag{7}$$

and assume that the term

$$d_{12} = m_2(l_{c2}^2 + l_1 l_{c2}\cos(q_2)) + I_2$$

is nonzero for all values of $q2$. This condition is termed *strong inertial coupling* in [18] and generalizes to the multi-degree-of-freedom case where d_{12} is a matrix function of q_2. Note that the strong inertial coupling condition imposes some restrictions on the inertia parameters of the robot, namely that $I_2 > m_2 l_{c2}(l_1 - l_{c2})$. Under this assumption we can solve for $\ddot{q}_2$ from (7) as

$$\ddot{q}_2 = -\frac{1}{d_{12}}(d_{11}\ddot{q}_1 + h_1 + \phi_1) \tag{8}$$

and substitute the resulting expression (8) into (2) to obtain

$$\bar{d}_1\ddot{q}_1 + \bar{h}_1 + \bar{\phi}_1 = \tau, \tag{9}$$

where the terms $\bar{d}_1, \bar{h}_1, \bar{\phi}_1$ are given by

$$\bar{d}_1 = d_{21} - d_{22}d_{11}/d_{12}$$

$$\bar{h}_1 = h_2 - d_{22}h_1/d_{12}$$

$$\bar{\phi}_1 = \phi_2 - d_{22}\phi_1/d_{12}.$$

The term $\bar{d}_1$ can easily be shown to be strictly positive as a consequence of the positive definiteness of the robot inertia matrix and strong inertial coupling. A feedback linearizing controller can therefore be defined for equation (9) according to

$$\tau = \bar{d}_1 v_1 + \bar{h}_1 + \bar{\phi}_1, \tag{10}$$

where v_1 is an additional outer loop control term that will be used to complete the generation of the swing up control law. The complete system Σ_1, to this point, is given by

$$d_{12}\ddot{q}_2 + h_1 + \phi_1 = -d_{11}v_1 \tag{11}$$

$$\ddot{q}_1 = v_1. \tag{12}$$

If $q_1^d(t)$ is a given reference trajectory for q_1 we may choose the input term v_1 as

$$v_1 = \ddot{q}_1^d + k_d(\dot{q}_1^d - \dot{q}_1) + k_p(q_1^d - q_1), \tag{13}$$

where k_p and k_d are positive gains. With state variables

$$z_1 = q_1 - q_1^d$$

$$z_2 = \dot{q}_1 - \dot{q}_1^d$$

$$\eta_1 = q_2$$

$$\eta_2 = \dot{q}_2, \tag{14}$$

the closed loop system may be written as

$$\dot{z}_1 = z_2 \tag{15}$$

$$\dot{z}_2 = -k_p z_1 - k_d z_2 \tag{16}$$

$$\dot{\eta}_1 = \eta_2 \tag{17}$$

$$\dot{\eta}_2 = -\frac{1}{d_{12}}(h_1 + \phi_1) - \frac{d_{11}}{d_{12}}v_1. \tag{18}$$

It is interesting to note that the same result can be obtained by choosing an output equation

$$y = q_1 - q_1^d = z_1 \tag{19}$$

for the original system (1)-(2), differentiating the output y until the input appears, and then choosing the control input to linearize the resulting equation. The system therefore has relative degree 2 with respect to the output y. The manner in which we have arrived at the system Σ_1 has the advantage that the computation and analysis of the resulting zero dynamics is simple.

It is, at first glance, surprising that we can achieve a linear response from the first degree of freedom even though it is not directly actuated but is instead driven only by the coupling forces arising from motion of the second link. The motion of link 2 necessary to achieve this may be complex and precisely defines the zero dynamics of the system. For this reason the analysis of the zero dynamics [11] is crucial to the understanding of the behavior of the complete system. The zero dynamics, with respect to the output $y = z_1$ are computed by specifying that the q_1 identically track the reference trajectory q_1^d. We will analyze the zero dynamics for the case of a constant reference command in the next section.

Analysis of the Zero Dynamics: The Autonomous Case

If the reference input q_1^d is a constant, then the system is autonomous and we may write (15)-(18) as

$$\dot{z} = Az \tag{20}$$

$$\dot{\eta} = w(z, \eta), \tag{21}$$

with suitable definitions of the matrix A and the function $w(z, \eta)$ (see [18]). We see from the above that the surface $z = 0$ in state space defines an invariant manifold for the system. Since A is Hurwitz for positive values of k_p and k_d this invariant manifold is globally attractive. The dynamics on this manifold are given by

$$\dot{\eta} = w(0, \eta) \tag{22}$$

and are referred to as the "zero dynamics" with respect to the the output y defined above [11]. Since we are interested in the swing up control problem, we consider the case $q_1^d = \pi/2$. Substituting $q_1^d = \pi/2$, $\dot{q}_1^d = 0 = \ddot{q}_1^d$ into the equation (18) and using the original description of the system (1), we arrive at the following expression for the zero dynamics of the system:

$$(m_2 l_{c2}^2 + m_2 l_1 l_{c2}\cos(q_2) + I_2)\ddot{q}_2 - m_2 l_1 l_{c2}\sin(q_2)\dot{q}_2^2 - m_2 l_{c2} g \sin(q_2) = 0 \tag{23}$$

The system (23), considered as a dynamical system on the cylinder, has two equilibrium points, $p_1 = (0, 0)^T$, which is a saddle, and $p_2 = (\pi, 0)^T$, which is a center. A typical phase portrait of this system (23) is shown in Fig. 2.

It follows (locally) that, for initial conditions, $z(0) = z_0$, $\eta(0) = \eta_0$, the state $z(t)$ converges exponentially to zero, while the state $\eta(t)$ converges to a trajectory of the system (23). The proof of this fact relies on the Center Manifold Theorem [6] and can be found in [11].

It is interesting to note that the expression for the zero dynamics, Equation (23), is independent of the gains k_p and k_d used in the outer loop control (13). These gains, however, together with the intial conditions, completely determine the particular trajectory of the zero dynamics to which the response of the complete system converges. We will see then that the tuning of these gains is crucial to the achievement of a successful swingup.

Since almost all trajectories of the system (23) are periodic, the typical steady state behavior is for the first link to converge exponentially to $q_1 = \pi/2$ and for the second link to oscillate, either about the center point equilibrium $(\pi, 0)$ of (23), or "outside" the homoclinic orbit of the saddle point equilibrium. The strategy for the swing up control is then to determine an appropriate set of gains k_p, k_d for the outer loop control (13) that swings the second link close to its saddle point equilibrium and then to switch from the above partial feedback linearization controller to a linear, quadratic regulator designed to balance the Acrobot about this equilibrium, whenever the trajectory enters the basin of attraction defined by the LQR controller. This will be illustrated in the next section by simulation results.

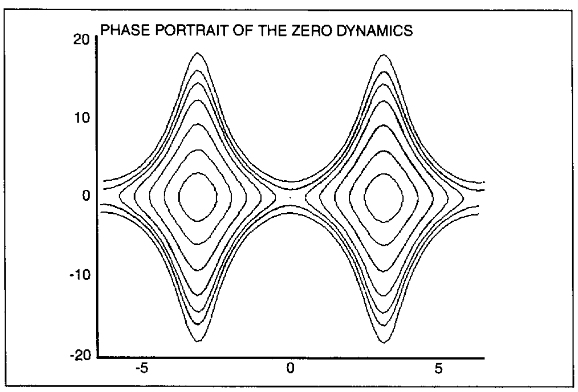

Fig. 2. Phase portrait of the zero dynamics.

Simulation Results

We have simulated the Acrobot in Simnon [7], using the parameters in Table 1. The links are modeled as uniform thin rods and so the moments of inertia are given by the formula $I = \frac{1}{12}ml^2$. It can easily be checked that the Strong Inertial Coupling condition holds for this set of parameters.

Fig. 3 shows the response of the partial feedback linearization controller with gains $k_p = 16$, $k_d = 8$. The angle q_2 is plotted modulo 2π, which is the reason for any apparent jumps in the joint angle during the transient response.

Fig. 4 shows the response of the partial feedback linearization controller for the gains $k_p = 20$ and $k_d = 8$. In this case link 2 rotates 360° in the steady state.

The "tuning problem" is then to choose a set of gains to move the Acrobot as close as possible to the saddle point equilibrium and then switch to a "balancing" controller to capture and balance the Acrobot about this equilibrium. We illustrate this below using a linear, quadratic regulator to balance the Acrobot about the vertical.

The Balancing Controller

Linearizing the Acrobot dynamics about the vertical equilibrium $q_1 = \pi/2$, $q_2 = 0$, using the parameters in Table 1 results in the controllable linear system

$$\dot{x} = Ax + Bu, \tag{24}$$

where the state vector $x = (q_1 - \pi/2, q_2, \dot{q}_1, \dot{q}_2)$, the control input $u = \tau$, and the matrices A and B are given by

Table 1
Parameters of the Simulated Acrobot

m_1	m_2	l_1	l_2	l_{c1}	l_{c2}	I_1	I_2	g
1	1	1	2	0.5	1	0.083	0.33	9.8

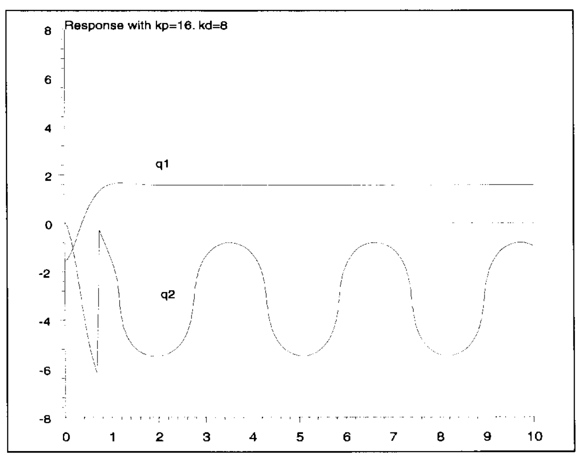

Fig. 3. Partial feedback linearization response with gains $k_p = 16$, $k_d = 8$.

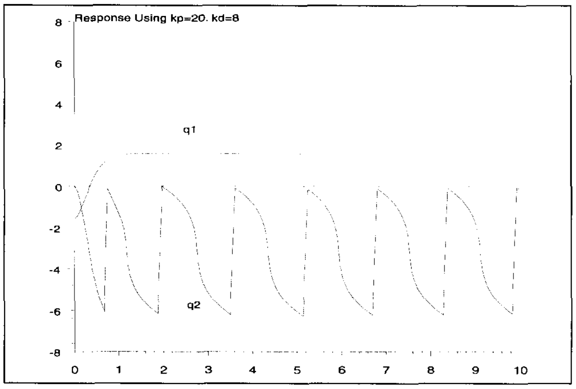

Fig. 4. Partial feedback linearization response with gains k_p = 20, k_d = 8.

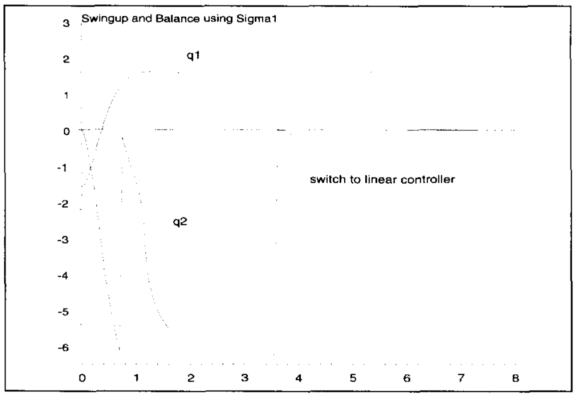

Fig. 5. Swing up motion of the Acrobot using Σ_1.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 12.49 & -12.54 & 0 & 0 \\ -14.49 & 29.36 & 0 & 0 \end{bmatrix} \tag{25}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ -2.98 \\ 5.98 \end{bmatrix}. \tag{26}$$

Using Matlab, an LQR controller was designed with weighting matrices

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

and $R = 1$, yielding the state feedback controller $u = -Kx$, where

$$K = [-242.52,\ -96.33,\ -104.59,\ -49.05]. \tag{28}$$

The linear control law is switched on whenever the Acrobot reaches the near vertical configuration. Fig. 5 shows a plot of a successful swing up and balance using the partial feedback linearization followed by the linear, quadratic regulator.

Derivation of the System Σ_2: Collocated Linearization

In this section we derive an alternative swing up control algorithm which can be used in the case that the second link is constrained to rotate less than a full revolution, as for example, with the experimental Acrobot considered in [5]. The alternative algorithm derived here is based on linearizing the system with respect to $q2$ instead of q_1. Consider the Equation (2),

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + h_2 + \phi_2 = \tau. \tag{29}$$

This time we solve for $\ddot{q}_1$ from Equation (1) and substitute the resulting expression into (29) to obtain

$$\bar{d}_2\ddot{q}_2 + \bar{h}_2 + \bar{\phi}_2 = \tau, \tag{30}$$

where the terms $\bar{d}_2$, $\bar{h}_2$, $\bar{\phi}_2$ are given by

$$\bar{d}_2 = d_{22} - d_{21}d_{12}/d_{11}$$

$$\bar{h}_2 = h_2 - d_{21}h_1/d_{11}$$

$$\bar{\phi}_2 = \phi_2 - d_{21}\phi_1/d_{11}.$$

Note that this requires that the term d_{11} be nonzero over the configuration manifold of the robot. This, however, involves no restrictions on the inertia parameters since d_{11} is always bounded away from zero as a consequence of the uniform positive definiteness of the robot inertia matrix. A feedback linearizing controller can be defined for equation (30) according to

$$\tau = \bar{d}_{22}v_2 + \bar{h}_2 + \bar{\phi}_2. \tag{31}$$

Substituting the control (31) into (30) yields the system Σ_2

$$d_{11}\ddot{q}_1 + h_1 + \phi_1 = -d_{12}v_2 \tag{32}$$

$$\ddot{q}_2 = v_2 \tag{33}$$

The input term v_2 can now be chosen so that q_2 tracks any given reference trajectory q_2^d. The important problem now is to choose the reference signal q_2^d to execute the swing up maneuver. In [19] an energy pumping strategy was used to solving the swing up control problem. The result in [19] contains an analysis of the resulting zero dynamics for Σ_2 similar to that contained here for Σ_1. We will not repeat the analysis of the zero dynamics in this paper. Instead we will discuss the original energy pumping interpretation of our algorithm that was the original motivation for its derivation.

Energy Based Swing Up Algorithm

If the second link angle $q2$ is constrained to lie in an interval $q_2 \in [-\beta, \beta]$ then we choose an α less than β and swing the second link as follows: Let the reference q_2^d for link 2 be given as

$$q_2^d = \alpha \arctan(\dot{q}_1) \tag{34}$$

and choose the outer loop control term v_2 as

$$v_2 = k_p(q_2^d - q_2) - k_d \dot{q}_2 \tag{35}$$

with k_p and k_d positive gains. The idea behind this choice of reference position for q_2 is to "pump energy" into the system by swinging link 2 "in phase" with the motion of link 1 so that energy is transferred from link 2 to link 1. In this way, the amplitude of link 1 may be increased with each swing.

To see how this might be expected to work, consider the motion of a single link with a force F acting at the end of the link. Assume that the force F is directed perpendicular to the link for simplicity. Then the torque acting at the joint is equal to lF and the equation of motion is

$$I\ddot{q}_1 + mgl_c \sin(q_1) = lF. \tag{36}$$

The total energy of the system is given by

$$V = \frac{1}{2} I \dot{q}_1^2 + mgl_c(1 - \cos(q_1)) \tag{37}$$

and the derivative of V along trajectories of the system is given by

$$\dot{V} = lF\dot{q}_1. \tag{38}$$

Therefore, the change in total energy over a time interval [T-1, T] is

$$V(T) - V(T-1) = l\int_{T-1}^{T} F(t)\dot{q}_1(t)dt. \tag{39}$$

Suppose that the force $F(t)$ is any so-called 1st and 3rd quadrant function of $\dot{q}_1$, i.e., suppose that,

$$F = |F| \, sgn\,(\dot{q}_1(t)). \tag{40}$$

Then we see from (39) that

$$V(T) - V(T-1) = l\int_{T-1}^{T} |F| \cdot |\dot{q}_1| \, dt \geq 0, \tag{41}$$

i.e., the change in energy during the time interval [T-1, T] is nonnegative. Our strategy for swinging link 2 rapidly in the direction of motion of $\dot{q}_1$ is designed to produce a net force during the time [T-1, T] of each swing with the "correct sign" as above.

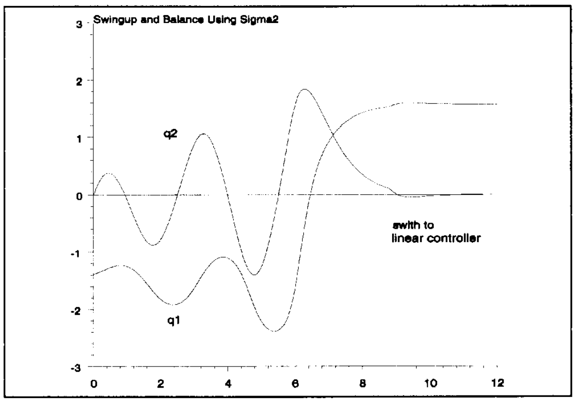

Fig. 6. Swing up motion of the Acrobot using Σ_2.

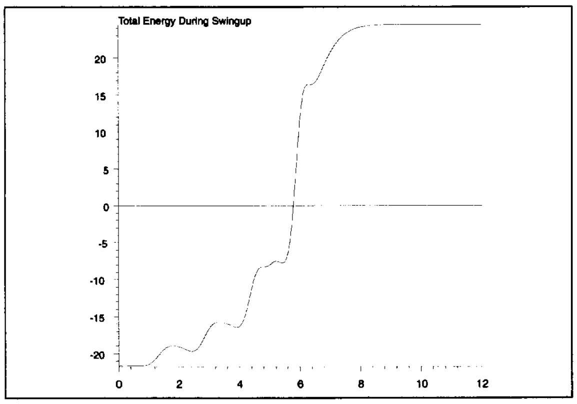

Fig. 7. Total energy during the swing up motion.

Although the above simplified analysis only approximately describes the true Acrobot, we will see below that the total energy is indeed increased with each swing as we might expect from the above considerations.

Our choice of reference position for q_2 also has the effect of straightening out the Acrobot at the top of each swing, which facilitates the capturing of the Acrobot at the vertical position. Other choices of reference q_2^d are, of course, possible such as $q_2^d = \alpha \, sgn\,(\dot{q}_1)$ or $q_2^d = \alpha \, sat\,(\dot{q}_1)$. The essential feature is that the reference function be a so-called "first and third quadrant" function of $\dot{q}_1$. See [20] for additional details and an analysis of the resulting zero dynamics.

Fig. 6 shows a swing up motion using the reference for q_2 given by (34). Again the LQR controller is switched on at the top of the swing. Fig. 7 shows a plot of the total energy during the swing up motion.

Conclusions

In this paper we have discussed two distinctly different swing up control strategies for the Acrobot, both based on the concept of partial feedback linearization. It is quite interesting that the complex swing up motions are realized in the closed-loop as the "natural responses" of autonomous nonlinear differential equations. It is also interesting that, in both cases, unstable behavior of the zero dynamics is exploited to realize the swing up motion.

The general principles discussed in this paper are applicable to a broader class of control problems. For fully actuated (and therefore feedback linearizable) systems, the nonlinear control problem is considered essentially solved once the system is linearized. We have seen in the case of the Acrobot that the second stage (or outer loop) design remains a non-trivial and nonlinear task. Interesting control problems remaining for this class of systems include the robust and adaptive control. We note that the partial feedback linearization approach leads to a system in which the inertia parameters appear nonlinearly. Thus standard adaptive techniques that have been developed for fully actuated rigid robots cannot be applied in a direct adaptive control scheme.

The simulation indicate that the response of the system is very sensitive to the values of the outer loop gains and to the switching times. Thus, the "tuning issues" in these types of problems are important, and, moreover, naturally lend themselves to methods of repetitive learning control. The reader is referred to [22] for an application of machine learning methods to this problem.

Another interesting problem is the further investigation of robust control to the balancing control. The basin of attraction of the LQR controller used here is very small, making the capture and balance phase of the swing up motion difficult. The application of more robust designs in order to increase the basin of attraction of the balancing controller is thus important and would ameliorate the difficulties of tuning the gains in the swing up phase. For example, the work of Bortoff [5] has shown that techniques such as pseudolinearization can greatly enlarge the basin of attraction of balancing controllers for these systems.

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