1)Prove that if we subtract 1 from a positive odd square number the answer is always divisible by 8.

2) Prove that there are no integers a, b which satisfy the following equation :-
a^2 - 8b=7

3) Given a, b are positive odd integers , with a>b . Show that if a+b is a multiple of 4 then a-b can't be a multiple of 4.

4) Prove $\log_{10}(3)$ is an irrational number.

5) Prove that the square of a positive integer can never be of the form 3k+2 where k is a natural number/whole number.

6) Prove that V(A+B) = V(A) + V(B), given A and B are independent. (V is variance)

7) Prove that for all positive real numbers a and b; $a^3 + 2b^3 >= 3ab^2$

8) Suppose A_1,A_2,...A_n are sets in some universal set U, and n≥2. Prove that
$(A_1 \cup A_2 \cup ... \cup A_n)'$ =
A_1' intersection A_2' intersection….A_n'

9) Suppose that A,B and C are sets and C is not equal to the empty set. Prove that if AxC=BxC then A=B.

10) A={n:n^2 ends with 6 and n belongs to N}
For y belonging to A prove that y^2-36 is always divisible by 10.

11) For all n>2, where n belongs to Naturals,N. Prove that the successor and predecessor of n^3 are not prime.

-

12) Prove that the number of real numbers in the intervals (0,1] and [1,∞) are the same

13) Prove that for an interior point O of triangle ABC, p/2<AO+BO+CO<p, where p is the perimeter of ABC

14) Prove that all points (x,y) satisfying the equation $x\sqrt{1 - y^2} + y\sqrt{1 - x^2} = 1$ lie on the unit circle.

15) Prove that for positive real numbers a,b and c, $a^3 + b^3 + c^3 \geq a^2b + b^2c + c^2a$

16) Prove that the infinite series $\sum_{n=0}^{\infty} n/(n^4 + n^2 + 1) = ½$

17) Prove that the following identity holds $\sum_{r=1}^{n} (C_r^n)^2 = C_n^{2n}$

18) Prove that for any 9 randomly chosen natural numbers less than 16, there exists at least one pair of numbers that sum up to 16.

19) Prove that there cannot exist a perfect square with sum of digits 2022

20) Prove that if a,b and c are positive real numbers,
$1/(a + b) + 1/(b + c) + 1/(c + a) \leq 1/2(1/a + 1/b + 1/c)$

21) Prove that every point in R^2 satisfies the equation $2\sqrt{(x^2 + y^2)} \geq |x - y|$

22) For a, b and c belonging to non-zero real numbers, such that a+1/b=b+1/c=c+1/a ,prove that |abc|=1

-

23) For a function f(x) defined on [0,1] such that f(0)=f(1)=1 and |f(a)-f(b)|<|a-b| for all a ≠ B in the interval [0,1].

Prove that |f(a)-f(b)|<⅔

24) Prove that the length of the longest diagonal in an odd-sided regular polygon of is L/2sin(π/2n), where L is the length of each side and n is the number of sides

25) Prove that $20 < \sum_{n=1}^{120} 1/\sqrt{n} < 21$

26) Prove that there are infinitely many prime numbers

27) If gcd(a,b)=1 and a and b are positive integers, x^a-1 and x^b-1 have a unique common root

28) If we take a solution to the inequality x+y+z ≥ 1, prove that
$x^2/(y + z) + y^2/(z + x) + z^2/(x + y) \geq 1$

29) The number n^4+4 is always a composite number

30) The nth term of the fibonacci sequence is given by $F_n = \frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$

31) $1.2+2.3+3.4+...n.(n+1)=n(n+1)(n+2)/3$

32) We cannot have an equilateral triangle in cartesian plane with all integral vertices

33) let $p>7$ be a prime, then prove that the number 111….1($p-1$ times) is divisible by p.