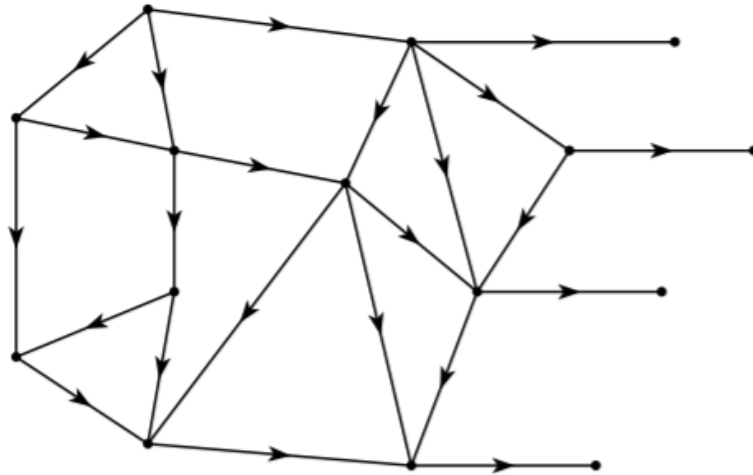


# ZPC 7 : Introduction to Combinatorial Games on Graphs

## Challenge Questions

Total : 25 marks

Q1). Report as an integer the sum of SG-values of all the vertices in the given graph. [2 marks]



Q2). Find the SG function of the subtraction game with the subtraction set  $S = \{1, 3, 4\}$ . [3 marks]

Q3). Consider the one-pile game with the rule that you may remove  $c$  chips from a pile of  $n$  chips if and only if  $c$  is a divisor of  $n$ , including 1 and  $n$ . For example, from a pile of 12 chips, you may remove 1, 2, 3, 4, 6, or 12 chips. The only terminal position is 0. This game is called **Dim<sup>+</sup>** in Winning Ways.

Find the Sprague-Grundy function. [5 marks]

Q4). Show that subtraction games with finite subtraction sets have Sprague-Grundy functions that are eventually periodic. [6 marks]

Q5). The following directed graphs have cycles and so are not finite. Find the Sprague-Grundy values of all nodes if the function exists, otherwise report does not exist. [1 + 2 + 6 = 9 marks]

(a)



(b)



(c)

