INDRAPRASTHA INSTITUTE *of*
INFORMATION TECHNOLOGY **DELHI**

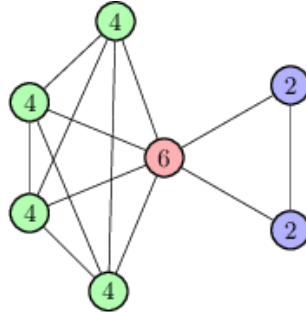# Graph Theory Problems

1. In the following, find (**½ + ½ points**)
   a. A maximum matching
   b. A vertex cover

2. Prove that every tree is bipartite and give a visual cue to identify partitions. (**1 point**)
3. Prove that a graph is bipartite if and only if every induced cycle is of even length. (**2 points**)
4. Do both parts: (both a and b = **3 points**) (one of a or b = **2 points**)
   a. Prove that given a graph G with a min degree δ (where $\delta > 1$), then there exists a path of length at least δ.
   b. In the previous question, show that there also exists a cycle of length at least $\delta + 1$.
5. Give an algorithm to find the maximum independent set on a vertex-weighted path. (**3 points**)
6. For bipartite graphs, prove that there exists a vertex which is contained in every maximum matching. (**7 points**)