

# Grand Finale Finals

Integral Cup

IIT Bombay

September 8, 2025



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# The finale

Five minutes per integral



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## Question 1

$$\int_{-1}^1 \sqrt{5x^2 + 33 + 2\sqrt{6x^4 + 91x^2 + 200}} \, dx$$



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## Question 1

$$\int_{-1}^1 \sqrt{5x^2 + 33 + 2\sqrt{6x^4 + 91x^2 + 200}} \, dx$$
$$= \left[ \sqrt{11} + 3\sqrt{3} + \frac{8\sqrt{3}}{3} \ln \left( \frac{\sqrt{3} + \sqrt{11}}{2\sqrt{2}} \right) + \frac{25\sqrt{2}}{2} \ln \left( \frac{3\sqrt{3} + \sqrt{2}}{5} \right) \right]$$



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## Question 2

$$\int_{-1}^1 \frac{1}{1 - \frac{x}{1 + x - \frac{x^2}{1 + x^2 - \frac{x^3}{1 + x^3 - \dots \frac{x^{2025}}{1 + x^{2025}}}}} dx$$



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## Question 2

$$\int_{-1}^1 \cdots dx$$

$$= 2 \sum_{k=0}^{2025} \frac{1}{1 + 2k(4k + 1)} + \frac{1}{1 + (2k + 2)(4k + 3)}$$



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**Question 3**



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$$\int_{\pi/2}^{3\pi/2} \arcsin \left( \sqrt{\ln(x)} \right) dx$$

we ignore this question right now, the limits are wrong! but the main idea was using integration by parts, i'll give a corrected solution in a pdf i will make and send soon



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## Question 3



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$$\int_{\pi/2}^{3\pi/2} \arcsin \left( \sqrt{\ln(x)} \right) dx$$

we ignore this question right now, the limits are wrong! but the main idea was using integration by parts, i'll give a corrected solution in a pdf i will make and send soon

$$= \left[ \frac{3\pi}{2} \arcsin \sqrt{\ln \frac{3\pi}{2}} - \frac{\pi}{2} \arcsin \sqrt{\ln \frac{\pi}{2}} - \sum_{k \geq 1} \frac{\binom{2k-1}{k}}{k! 2^{k-1}} \pi \right]$$





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## Question 4

$$\int_{\pi}^{\infty} \frac{\log_{\pi}(e)}{\pi x^2} \frac{x\pi^{\pi/x} - \pi^2}{1 - \log_{\pi} x} dx$$



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## Question 4

$$\int_{\pi}^{\infty} \frac{\log_{\pi}(e)}{\pi x^2} \frac{x\pi^{\pi/x} - \pi^2}{1 - \log_{\pi} x} dx$$
$$= \boxed{\sum_{k \geq 1} \frac{\ln(k) \ln^k \pi}{\pi k!} - \ln 2}$$



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## Question 5

$$\int_{-1}^1 \sum_{a=1}^{2025} \sum_{b=1}^{2025} \left[ \frac{x}{a+b} - \frac{1}{2} - \left\lfloor \frac{x}{a+b} \right\rfloor \right] \sin(a\pi x) \sin(b\pi x) dx$$



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## Question 5

$$\int_{-1}^1 \sum_{a=1}^{2025} \sum_{b=1}^{2025} \left[ \frac{x}{a+b} - \frac{1}{2} - \left\lfloor \frac{x}{a+b} \right\rfloor \right] \sin(a\pi x) \sin(b\pi x) dx$$
$$= \boxed{-\frac{2025}{2}}$$



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## Question 6

$$\int_0^{\pi} \arctan(\sin^2 x) \, dx$$



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## Question 6

$$\int_0^{\pi} \arctan(\sin^2 x) \, dx$$
$$= \boxed{\pi \arctan\left(\sqrt{\frac{1}{\sqrt{2}}} - \frac{1}{2}\right)}$$



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## Question 7

$$\int_0^{5!} \sum_{n=0}^{\infty} \left\{ \frac{x}{n!} \right\} dx$$



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## Question 7

$$\int_0^{5!} \sum_{n=0}^{\infty} \left\{ \frac{x}{n!} \right\} dx = \boxed{5!(60e - 100)}$$





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## Tiebreaker 1

$$\int_0^1 \log^2(1-x) \log^2(1+x) dx$$



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## Tiebreaker 1

$$\int_0^1 \log^2(1-x) \log^2(1+x) dx$$

$$= \begin{aligned} &24 - 8\zeta(2) - 8\zeta(3) - \zeta(4) + 8\log(2)\zeta(2) \\ &\quad - 4\log^2(2)\zeta(2) + 8\log(2)\zeta(3) - 24\log 2 \\ &\quad + 12\log^2(2) - 4\log^3(2) + \log^4(2) \end{aligned}$$



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## Tiebreaker 2

$$\int_0^1 \frac{1}{(1+yx)\sqrt{1-x^2}} dx$$



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## Tiebreaker 2

$$\int_0^1 \frac{1}{(1+yx)\sqrt{1-x^2}} dx$$
$$= \boxed{\frac{\arccos(y)}{\sqrt{1-y^2}}}$$



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## Tiebreaker 3

$$\int_0^{\pi/2} \frac{x^3}{\sin^2(x)} dx$$



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## Tiebreaker 3

$$\int_0^{\pi/2} \frac{x^3}{\sin^2(x)} dx$$
$$= \boxed{\frac{3}{8} \left( \pi^2 \ln 4 - 7\zeta(3) \right)}$$