Mean-Reverting Portfolio Optimization via a Surrogate Risk Measure - Conditional Desirability Value at Risk

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Abstract. The financial crisis in 2008 highlighted the demise of the most widely used risk measure, Value-at-Risk. Unlike the Basel-IV-required Conditional VaR model of Rockafellar & Uryasev, VaR ignores the possibility of abnormal returns and is not even coherent. Our novelty here is introducing an annual Desirability Value (DV) for each candidate company before creating a portfolio of their stocks. We utilize only the downside changes in DVs in CVaR-optimization instead of simply utilizing annual stock-price returns. The DV of a company is the perpendicular distance from the fundamental position of that company to the best separating hyperplane that separates profitable companies from losers during training. Thus, we introduce a novel coherent surrogate risk measure, Conditional-Desirability-Value-at-Risk (CDVaR). Our machine-learning-fundamental-analysis-based CDVaR portfolio optimization results are comparable to those of mainstream price-returns-based CVaR optimizers. We show that these two optimal portfolios and the market index, BIST100 (all three portfolios in USD terms), are cointegrated. When backtested using in-sample data, the resulting Mean-Reverting Portfolio, labelled MRP, returns an APR of 5.26 (in USD) over nine years. In contrast, the market index produces negative returns over the same period. Since the existence of cointegration points to a long-term trend, the fact that this conclusion comes from in-sample data seems to be acceptable.

Keywords: Portfolio optimization · Basel 4 · Cointegration · Mean reversion · Statistical arbitrage · Risk management · Downside risk · Conditional value at risk · Machine learning · Linear programming · Fundamental analysis

1 Introduction

Set by regulators, the minimum capital requirement for a bank acts as a cushion to protect it from insolvency. One may think of a bank's assets and liabilities as a single portfolio. The minimum capital requirement, that is, the amount of cash

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that must be added to this portfolio to make its risk acceptable to regulators, is calculated by a certain risk measure [17]. However, the mathematical properties of available risk measures are vastly different from each other. They are so far apart from each other that while one is known to be a coherent risk measure, another is proven to be incoherent. Note that the definition of a coherent risk measure was published in the year 2000 [26]. It is difficult to come to terms with the fact that one of the major causes of the 2008 global financial meltdown was the utilization of an incoherent risk measure, Value at Risk (VaR).

The revision of the Internal/Advanced Model Approach (IMA) is among the key elements of the new standards that Basel IV brings to determining market risk [15]. Probably, the most important of these revisions is the replacement of the VaR by the Expected Shortfall (ES), which is a coherent risk measure. The implementation date for this replacement is Jan 1, 2022 [19]. Note that when the returns of the constituents of a portfolio are from a continuous probability distribution function (pdf), Expected Shortfall is synonymous with Conditional Value at Risk (CVaR) [2].

Obviously, one of the reasons why people use the risk measure, CVaR, is to determine the minimum capital requirement for a bank, given the pdf of the constituents of its assets and liabilities in a maximum stress year, as required by Basel 4 [19]. In this paper, however, our goal is to use a variation of CVaR as a tool to determine an optimal portfolio of stocks that are listed in Borsa Istanbul (BIST). The variation that we shall describe is interesting in the way that it is a pseudo risk-measure. We call it a pseudo risk-measure because instead of computing the expected shortfall of some portfolio in terms of dollar units, it produces an *abstract quantity* that we call "Expected Shortfall in Desirability" as well as "Conditional Desirability Value at Risk" (CDVaR).

We employ a novel machine learning model to enhance an existing portfolio optimization tool (Rockafellar-Uryasev [27]) to come up with a well-balanced portfolio (which we call "CDVaR-optimal") that consists of long stock positions only. Via Machine Learning, we design this portfolio to beat the market index at the end of a preset portfolio duration.[1] However, since the market index in Turkey, BIST100, has been going downhill in USD terms since 2010, we needed to take an extra step.

The seminal paper [10] won Clive Granger and Robert F. Engle the Nobel Memorial Prize in Economics in 2003. The two had made contributions to time series analysis that fundamentally changed the way in which economists analyze financial and macroeconomic data. What they did was to open the door to pairs trading in which a portfolio of a linear combination of two (or more) financial instruments each of which moves as non-stationary time series could be made to move as a stationary time series by picking the correct linear combination coefficients (if they exist). They call the resulting stationary time series "cointegrated".

For a specific example, we may cite [30]. There, applying a set of statistical tests to postwar U.S. data on the federal funds rate along with the 3- and 12-

[1] Our portfolio is rebalanced every mid-February and closed at the end of each year.

month treasury bill interest rates, Stock and Watson concluded that these three time series appear to be cointegrated.

Since a stationary time series has a habit of reverting to its mean, it is easy to place bets on it. Let us first consider the case in which there is no statistically significant trend (a.k.a. drift) in the time series. Whenever the time series is more than two standard deviations below its mean, we would open a long position and close this position when the time series hits its mean. Else, we would open a short position and close the position when the time series reaches its mean.

In the case that the stationary time series has a positive (or negative) drift in its long-term behavior, we would take a long (or short) position and expect to make money when the drift comes to fruition.

Surprisingly, we noticed that our novel CDVaR-optimal portfolio optimization tool produced an annual portfolio wealth history that appeared to be cointegrated with the competing CVaR-optimal portfolio wealth history and the market index, BIST100.

In short, we came up with a market neutral (or cointegrated) portfolio that comprises a linear combination of the CDVaR-optimal portfolio and the CVaR-optimal portfolio and BIST100, which not only beats the market but also produces and APR of 5.26 (in USD terms), over the span of 2010–2019. There is, however, some in-sample testing involved in this computation. On the other hand, since the existence of cointegration points to a long-term trend, the fact that this conclusion comes from in-sample data seems to be acceptable. Also, note that bringing in a positive APR from the Borsa Istanbul in USD terms is a big feast because BIST100 has the tendency of going down in USD terms, over the same time span.

Note that we shall leave the utilization of the cointegration concept to the end of this paper and concentrate on how to beat the market first, which we also describe in [5].

In our setting, a financial instrument, such as a stock or stock portfolio, is said to beat the market if and only if it returns more than a specific popular market index does, between Feb 16 and Dec 31. The reason for choosing a peculiar starting date for the comparison is as follows: We believe that it is the balance sheet quality that causes a stock to beat the market, and in Turkey, the year-end (third party audited) financial reports of companies are required to be made public before Feb 16 of the following year.

In portfolio optimization, beating the market is an issue mainly if no short positions are allowed in a portfolio. In this case, one is forced to (a) choose stocks that will outperform the market *and* (b) allocate optimal amounts of capital to each such stock, which requires a portfolio optimization method.

If, on the other hand, short positions were allowed, we would use a market neutral portfolio selection method, an example of which is in [4]. This paper uses one of the techniques mentioned in [4], namely, picking cointegrated pairs. Also Perlin [23] shows that pairs trading is a profitable strategy at the Brazilian market. A primer on using the cointegration concept in finance can be found in [8]. Off-the-shelf implementations of the R programming language that are related to cointegrated time series may be found in [31].

Here, we shall define what we mean by the "standard usage" of the Linear Programming (LP) model in [27]. If one employs stock-price-return histories to produce an optimal portfolio of financial instruments - optimal in the sense that it has minimum CVaR while having an expected return higher than a threshold - then one is using this model in the standard way. Moreover, no short positions are allowed in the standard usage.

In this paper, we coin the term "desirability value" (abbreviated as DV) of a company[2]. We start by stating that a company is desirable if its stock price increases faster than the most popular stock market index in the associated country, within a fixed amount of time. If we can quantify the concept of desirability of a company, we can start using mathematical optimization tools to create portfolios that are made up of stocks of multiple desirable companies each of which are expected to beat the market.

A *risk measure* that specifies minimum capital requirements is the amount of cash that must be added to a portfolio to make its risk acceptable to regulators [17]. Pflug has a proof that shows CVaR is a *coherent* risk measure while VaR is not [26]. A simplified definition of a coherent risk measure can be found in [17]. Çobandağ-Güloğlu and Weber [7] describe a method to produce a Robust CVaR risk measure, RCVaR.

The Rockafellar-Uryasev CVaR portfolio optimizer is also important because, just as VaR does, it reduces only downside risk and does not touch "upside risk", a term that does not make much sense in portfolio optimization [33]. Our method goes one step further and reduces downside risk *selectively* using a surrogate risk measure. That is, we first define a direction along which to reduce a fundamental risk and then make an attempt to reduce it along that direction, using our surrogate risk measure. The concept of surrogate risk measures appear in Vos [32] and Johnson and Maxwell [22], among others. While Vos uses surrogate risk measures derived from balance sheets in an attempt to price small unlisted businesses for sale, Johnson and Maxwell describe how to cluster Australian companies into homogeneous risk groups using surrogate risk measures. Neither employs a mathematical portfolio optimization tool. To our knowledge, [5] *is the first paper* that utilizes a surrogate risk measure in portfolio optimization. The current paper that you are reading now is a continuation of [5].

Note that a portfolio that is formed by optimizing our surrogate risk measure allows itself to assessment by classical monetary risk measures VaR and CVaR. So, there is nothing wrong with using a surrogate risk measure to form a well-balanced portfolio.

Literature in the area of applying machine learning to portfolio selection: Pau describes "technical analysis for portfolio trading by syntactic pattern recognition" in [25]. Oh et al. use a genetic algorithm to do portfolio optimization for index fund management [24]. Ince and Trafalis use Kernel methods for short-term portfolio management in [20]. The book Computational Finance 1999 [1], cov-

[2] The DV is not related to the desirability function of [16].

ers machine learning in portfolio optimization among other topics. Freitas et al. describe a prediction-based portfolio optimization model using Neural Networks [13]. Karaçor and Erkan compare the quantitative predictabilities of different financial instruments in the context of machine learning [18]. Furhter portfolio optimization examples can be seen in [9] on pages 150–152.

2 Machine Learning Model

Companies in Turkey report their balance sheets using International Financial Reporting Standards [21]. End-of-year balance-sheet data were downloaded from [12]. All stock prices as well as the index values were converted to USD using the appropriate exchange rate USDTRY for the day. The ML model that we employed using these data is described in detail in [5].

2.1 How to Compute Desirability Values Using the Separating Hyperplane

We describe how to compute Desirability Values for companies at the end of each year in [5]. Note that the only aspect of the standard usage of [27] that we changed in [5] is the input data, not the constraint structure. Therefore, we did not put extra constraints in the portfolio optimizer that would allow it to *only pick stocks with high scores* because it was beyond the scope of [5]. We suspect that this would enhance our optimization because while the nominal values of prices that are established enough to be a part of BIST 50 have no economic meaning due to splitting stocks (among other reasons), the score, S_1, does have an economic meaning. The higher its value, the more likely that the company's stock would beat the market.

To compare our method with the standard usage of [27], we will have three different experiments whose portfolio performances we shall compare with the performance of BIST 100:

- Using $s_{DV}(i, j)$, the annual differences of Desirability Values in the LP model (7). This is what we call the CDVaR-optimized portfolio.
- Using annual relative returns $s_{i,j}$ derived from prices in (7). This will be the CVaR-optimized portfolio, as in the standard usage of [27].
- An equally weighted portfolio (EWP) that accepts all 22 candidate stocks. This is the trivial solution since neither machine learning nor portfolio optimization is at work here.[3]

We found out that neither s_{DV} nor s were normally distributed in our experiments. However, since the CVaR-optimization model described in [27] does not specify the distribution of returns that can be used, we are fine.

[3] Even though the EWP consists of stocks from BIST 50, we make the comparison to the more popular sister index BIST 100.

3 CVaR and CDVaR Portfolio Optimization Models Based on LP

The general purpose in forming a CVaR portfolio optimization model is to minimize the coherent risk measure, CVaR, while keeping the portfolio's expected return greater than a threshold level [27]. The specific reason that we use this model in this paper is to minimize a coherent surrogate risk measure Conditional Value at Risk, CDVaR, while keeping our portfolio's expected surrogate return above a threshold. We do this by simply replacing $s_{i,j}$ with $s_{DV}(i, j)$, for all i and j.

3.1 Definitions of VaR and CVaR

In [5], we listed the well-established mathematical definitions of Value at Risk (VaR) and Conditional Value at Risk (CVaR). Some terminology needs to be addressed here: There are two other terms that are widely used in the context of risk measures: Tail Conditional Expectation (TCE) and Expected Shortfall (ES). TCE is identical to CVaR. On the other hand, ES is the generalized version of CVaR since ES is coherent under both continuous and discrete return distributions. This is because the definition of ES includes a correction term for discrete distributions [2]. Since our paper only considers continuous return distributions, we base our CDVaR model on CVaR.

3.2 How to Compute VaR and CVaR of Any Portfolio

The VaR of *any* portfolio $\hat{\mathbf{x}}$ can easily be computed using the classical method: We would sort, in decreasing order, *all* available historical portfolio returns. We would then skip the first $100\beta\%$ elements of this list and pick the next one as VaR. On the other hand, CVaR is the average of all those entries that are worse than or equal to the VaR.

3.3 The LP Model for CVaR and How to Tweak It into CDVaR

The Linear Programming model that we use in this paper minimizes the coherent risk measure CVaR while keeping the expected portfolio return $E[R_{j+1}]$ above a certain threshold. This model is from Rockafellar and Uryasev [27]. Also see [5] for our implementation of the model. Since the optimization deals with only the bad tail of the risk distribution, it does not have to be nonlinear. Historically, the nonlinearity in risk management has been based on the dogma that the standard deviation was the culprit, thus both tails had to be dealt with. With the introduction of Downside Risk Sharpe Ratio [33] and of CVaR, researchers and some practitioners started dealing with only the bad tail. This is good news for portfolio managers, because they are no more advised to curtail the *good* tail, which was an obscure reality in practice until recently [33].

There are a number of useful properties of $(\gamma^*, \alpha^*, \mathbf{x}^*, \mathbf{z}^*)$, the optimal solution of the LP model given in [5]. First, α^* gives $\text{VaR}_\beta(\mathbf{x}^*)$, which is VaR_β of the optimally resource-allocated portfolio, $\mathbf{x}^*$. Moreover, CVaR_β of the optimally allocated portfolio equals the optimal objective function value, γ^*.

We use the following values for the input parameters: $\beta = 0.95$, $L = 0$, $\mathbf{u} = 0.25$, $\mu = \min(0.01, \rho)$ where ρ is the mean of h_i among all i. Note that there were times when $\rho < 0$ even though, ideally, the targeted expected return $\mu \geq L \geq 0$. However, since the computation of the expected value is nowhere near being precise, we simply ignored this rare exceptional situation. Moreover, $q = 3$ for our CVaR optimization, and $q = 1$ for our CDVaR optimization. The q values for both optimizations were determined using the training data.

We assume zero transaction costs. Our three portfolios and BIST 100 are cash-only in USD, or flat, between Dec 31 and Feb 15 of each year awaiting the publication of annual balance sheets. We assume that no interest accrues during these 46 days.

The optimal solution of LP model [5] gives a CVaR-optimal portfolio when annual relative returns from price histories are substituted for $s_{i,j}$. On the other hand, it gives a CDVaR-optimal portfolio if s_{DV}, the annual difference in DV, is substituted into $s_{i,j}$. *This is the first of the two main contributions* in this paper.[4]

All the three experimental portfolios listed in Subsect. 2.1 were first composed on Feb 16, 2010. This is because the earliest $E(m)$ and consequently the earliest $S_1(m)$ were computed for $j = 7$. Thus, the earliest s_{DV} was computed for $j = 8$. If $(q+1)$, the look-back parameter for CDVaR, were as low as the lowest possible value of 2, then we could start the first CDVaR portfolio in mid-February 2009. However, since the best optimization look-back parameter for CVaR (discovered running the CVaR algorithm over the training stock-price data) was $q + 1 = 4$ years, we could start our very first CVaR portfolio in 2010 (after having received balance sheet results from the end of 2009, i.e. $j = 9$). Since the optimization look-back parameter of the CDVaR portfolio was $q + 1 = 2$ the first CDVaR portfolio was easily composed on Feb 16, 2010. For purposes of comparison, the first equally weighted portfolio, EWP, was also put together on Feb 16, 2010.

While a computer code for the CVaR-LP model (written in NuOpt, a high level programming language) can be found in [29], we did our own coding in GAMS [14,28]. Additional coding for I/O and bookkeeping was done in Python.

3.4 How to Define and Compute DVaR and CDVaR of Any Portfolio?

We describe these in [5].

4 Numerical Results Before Cointegration Is Introduced

The optimal portfolio weights $\mathbf{x}^*$, described in Sect. 3 were never changed within a portfolio cycle, which starts in mid-February and ends at the end of the same

[4] The second main contribution is the observation that the CDVaR-optimal portfolio, the CVaR-optimal portfolio and the market index BIST100 (all three portfolios in USD terms) are cointegrated.

year. First, we shall look at the portfolio optimization results with training and testing data combined because we have too few testing years. We compared the cumulative portfolio wealth of the portfolios that are (a) CDVaR-optimal (b) CVaR-optimal with that of the equally weighted portfolio (EWP) and with the cumulative return of the BIST 100 index. See [5] for more details.

Note that one of the main parameters, $\mathbf{u}$, was held at 0.25 before the concept of cointegration was introduced into our model. Also, our choices for the look-back period $q+1$ were 2 and 4 for the CDVaR- and the CVaR-optimized portfolios, respectively. In the next section, where we investigate the effects of cointegration in our model, we shall switch to $\mathbf{u} = 0.20$ and $q+1 = 5$ for both the CDVaR- and the CVaR-optimized portfolio. Thus, we shall be able to present a larger subset of system parameters.

5 The Concepts of Stationarity and Cointegration

The concept of cointegration will be utilized at this point in order to make the results in the previous section better (in the sense that they are more robust). If we can prove that a portfolio (actually, the time series that represents the portfolio wealth) that we found is stationary with a long-term linear or quadratic deterministic trend (or drift), we can conclude that the time series will wrap around this deterministic trend – for at least a limited amount of time in the future. See the bottom of Fig. 1, where we succeeded to show that our final portfolio, which we label MRP, is indeed a stationary, i.e., a mean-reverting portfolio that has a (convex) quadratic deterministic trend. Consequently, we can hope to make money solely by opening a long position in MRP and waiting for the upward quadratic deterministic trend to continue to materialize. Below, we show that if a number of time series are cointegrated, then they form a single aggregate time series, which is stationary, a.k.a. a mean-reverting time series with a finite and constant variance.

5.1 Stationarity

Nonstationarity of time series leads to several econometric problems. For example, it opens the door to a deceitful (spurious) relationship among the levels of economic variables because the parameter estimates from a regression of one such variable upon others may be inconsistent unless the variables are cointegrated [8]. *The most common mistake that professional traders do is to bet on the outcome of a nonstationary time series and expect to make money.*

5.2 Cointegration

If we have two time series $A(t)$ and $B(t)$, and we find a (nonzero) constant β such that the newly formed time series $C(t) = A(t) + \beta B(t)$ is $I(0)$, then the time series $A(t)$ and $B(t)$ are said to be cointegrated with cointegration parameter β.

The most important property of the stationary time series $C(t)$ is its tendency to revert to its mean in a finite amount of time since its variance is finite.

Now, let us take a look at the case when there is actually a positive (or negative) drift in the realization of $C(t)$ which is a stationary time series that is formed by a cointegrated pair of time series $A(t)$ and $B(t)$. Here, we would just take a long (or short) position in $C(t)$ and expect to benefit from the deterministic drift. *We shall utilize this concept as the second of our two main contributions in this paper.* That is, from this point on, we shall describe a method to create a statistical arbitrage position. In other words, we describe a method to combine three portfolios, one of which is the market index BIST100, and the other two are our own CDVaR and CVaR portfolios, to come up with a mean-reverting portfolio, which we shall call MRP. MRP is expected to climb a deterministic upward drift or trend, which in our case, is a quadratic function of time. Furthermore, this trend is a long-term trend which would not be destroyed easily or quickly.

There is one caveat to exploiting cointegrated time series in this fashion. The *longevity* of a cointegrated time series is a serious issue. One has to check every now and then if stationarity is lost or not. For example, one would use the statistical test in [3], which determines if a time series has its cointegration property broken down.

Whether there is a drift, or not, traders call such a situation "opening a statistical arbitrage position" because, *under the assumption that the cointegration property of $A(t)$ and $B(t)$ will not break down before the end of the trade*, they know with, say, 95% confidence, that their bet will make money.

5.3 Description of the Statistical Tests Used

Here, we need to mention two statistical tests, each of which may be easily run using EViews [11] or a similar econometrics software. The Johansen Cointegration Test is more versatile than the Augmented Dickey-Fuller (ADF) Test because the latter accepts as input only a single time series and checks if it is stationary, whereas the former accepts as input a combination of any number of distinct time series and checks if there exists a linear combination of them which is stationary. The former also gives the coefficients for the linear combination(s) that end(s) up with no unit roots, i.e., stationary.

Here, we shall mention a technicality: If two distinct time series $A(t)$ and $B(t)$ are cointegrated, typically both of them must be non-stationary. This is because the nonstationarity in one of them is cancelled out with the nonstationarity in the other, with the contribution of the cointegration parameter β. Also, two nonstationary time series and a stationary one could just as well be cointegrated.

5.4 Numerical Results

We found out that Mean Reverting Portfolio (designated as MRP) is a linear combination of BIST100, CDVaR-optimized, and CVaR-optimized portfolios, where

$$\mathrm{MRP} = \mathrm{CDVaR}(5) - 2.525 * \mathrm{CVaR}(5) - 2.705 * \mathrm{BIST100} \tag{1}$$

Note that the arguments of CDVaR and CVaR both equal to the look-back period $q + 1 = 5$. That is, we use $q + 1$ consecutive year-end balance sheets to optimize the CDVaR and CVaR portfolios. First, we ran the Johansen Cointegration Test using the time series data of MRP (from Feb 16 2010 to Dec 31, 2018, inclusive) as input. We chose the option "Quadratic Deterministic Trend". We received a p-value of 0.0004 from both cointegrations tests[5], so we were able to conclude that C6 a.k.a. MRP is cointegrated, that is, it does not have a unit root, by definition. So, MRP is a stationary time series. Second, we ran the Augmented Dickey-Fuller Test with lag = 2 and found a p-value of 0.0156. Since this value is less than 5%, we have found yet another proof that the MRP is stationary in the long-run.

Figure 1 shows the USD denominated levels of all the portfolios that we put together.

5.5 How to Exploit the Cointegration Property

Note that the cointegrated time series MRP does have a convex quadratic drift in it, the dotted line (a second-order polynomial) in the bottom graph in Fig. 1. We formulated this long-term drift equation as $y = 0.0027x^2 + 0.0371x - 1.1067$. Where $x = 0...9$ denotes the years. Note that $y(0) = -1.1067$ and $y(9) = -0.458$. So, the portfolio makes about 58.6% in USD terms, in 9 years. So, the Average Percentage Rate is 5.26. In order to take advantage of the long-term positive drift, we just take a long position in MRP in mid February 2019 and wait for the positive drift to materialize. Unless the cointegration property is destroyed before the end of the trade on December 31, 2019, we would expect to make an APR of around 5.26. While an APR of this magnitude may not sound glamorous, since the market index BIST100 goes downhill in USD terms between the years 2010 and 2019 (as seen in the third line from the top in Fig. 1), we believe that it is a significant improvement.

6 Conclusions

The fact that we have reduced the balance sheet reading problem into reading (or rather, computing) only two fundamental ratios is also important in and of itself. We assign a Desirability Value to each company at end of each year by checking their third party audited year-end balance sheets.

$$DV(m) = 0.247 * E(m, 27) - 0.011 * V(m, 32) \tag{2}$$

Note that in the above equation m is the company-year index, whereas $E(m, 27)$ is the synthetic variable that is based on the 2-year-long route that the company's

[5] The Johansen Cointegration Test actually applies two different cointegration tests under the hood.

Earnings Per Share has taken with regards to other competitors, as defined in Sect. 2. Moreover, $V(m, 32)$ is the ratio of *Short-Term Liabilities* to Net Sales (%). The second term on the right hand side of Eq. (2) is pretty similar to the second term of the equation that defines the Turkish Economic Stability Index (TESI) introduced by Boduroğlu and Erenay [6]. The first terms of each equation are also similar in effect. Since there are no other terms except for a constant in TESI, these two analyses seem to support each other. That is, it seems as if what makes a company desirable for investors is very similar to what makes an emerging market country desirable for them. This is because having a low short-term debt to net sales ratio in the corporate world and having a low short-term external debt to international reserves ratio as an EM country are fundamentally good for both actors. Likewise, having a high EPS for a company is similar to having a strong banking sector in an EM country. We suspect that this parallelism between promising companies and promising EM countries would also be seen in EM countries other than Turkey.

The CDVaR-optimized portfolio beats (in terms of both the total return and the Sharpe Ratio) the CVaR-optimized portfolio, the Equally Weighted Portfolio (formed using 22 stocks in BIST 50), and the market index BIST 100 when only the testing data was used. During testing, portfolio cycles started from mid-February 2015 and ended at the end of 2017. The reason EWP performed better than the BIST 100 may be because the former's components had been selected from BIST 50, the larger cap index than BIST 100. Moreover, no financial or holding companies were allowed in EWP, which might have also effected this outcome.

The same conclusion can also be made for the analysis over full data, training and testing, with portfolio cycles extending from mid-February 2010 to end of 2017. Since both the total return in USD terms and the Sharpe Ratio for CDVaR is higher than those of CVaR, EWP, and BIST 100, incorporating Machine Learning into portfolio optimization via CDVaR optimization seems to be a viable method even though we analyzed only a limited number of cases.

It is important to note that once a CDVaR-optimal portfolio is found using our surrogate risk measure CDVaR, one can easily compute the actual monetary risk involved using classical risk measures such as CVaR or VaR using their classical methods of calculation. Therefore, the fact that we are using a *surrogate* risk measure during the money allocation stage is insignificant.

In the second half of the paper, we show how to incorporate the concept of cointegration into our portfolio optimization model. Having constructed two distinct optimal portfolios, namely, CDVaR- and CVaR-optimized portfolios, we were able to prove that a linear combination of these two portfolios and the market index, was indeed a mean-reverting portfolio. We call this new portfolio MRP.

$$\mathrm{MRP} = \mathrm{CDVaR}(5) - 2.525 * \mathrm{CVaR}(5) - 2.705 * \mathrm{BIST100} \tag{3}$$

Note that the arguments of CDVaR and CVaR both equal to the look-back period $q+1$. That is we use $q+1=5$ consecutive year-end balance sheets to optimize the CDVaR and CVaR portfolios (See Fig. 1.) The long-term drift equation that MRP possesses is $y = 0.0027x^2 + 0.0371x - 1.1067$ where $x = 0...9$ denotes the years. So, the portfolio makes about 58.6% in 9 years. In order to take advantage of the long-term positive drift, we would just take a long position in MRP and wait for the positive drift to take place.

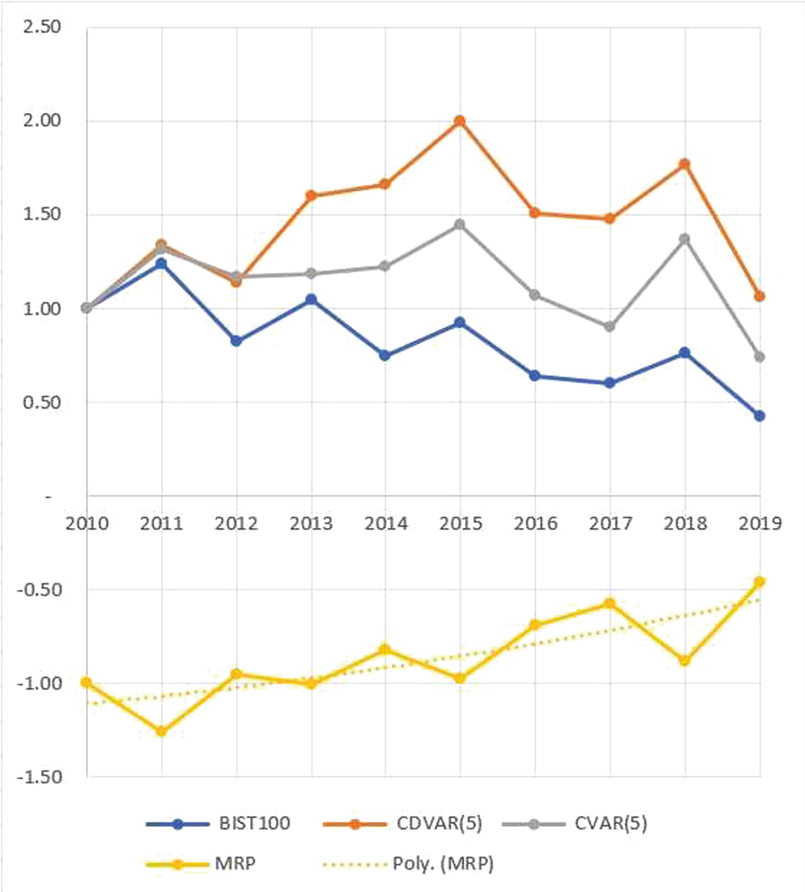

Fig. 1. Comparison of Portfolio Values in USD. Top: Portfolio closing values in decreasing order: CDVaR(5), CVaR(5), BIST100. Bottom: MRP (Mean Reverting Portfolio)

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