Simulating the Movement of a Double Pendulum with Euler's Method

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1 Important note

This is a work in progress. It is not finished at this moment in time.

2 What are we trying to do here?

We want to spend some time thinking about double pendulums and numerical simulations. Our goal is, of course, to create a working simulation of a double pendulum. What has to be done to get there? What kind of math is necessary? Let's start our journey by breaking down the problem into smaller pieces.

- Defining and labeling of the mathematical double pendulum model
- Deriving the Differential Equation
- Setting up the simulation using the Differential Equation

That's better. The problem doesn't look all that daunting anymore.

3 Defining the double pendulum

4 Constructing the differential equation

In order to get the simulation up and running, we need equations (14) and (19) from scienceworld's Double Pendulum page[1]. Here they are.

[1] http://scienceworld.wolfram.com/physics/DoublePendulum.html

$$(m_1+m_2)l_1\ddot{\theta}_1 + m_2 l_2\ddot{\theta}_2\cos(\theta_1-\theta_2) + m_2 l_2\dot{\theta}_2^2\sin(\theta_1-\theta_2) + g(m_1+m_2)\sin\theta_1 = 0 \tag{4.1}$$

$$m_2 l_2\ddot{\theta}_2 + m_2 l_1\ddot{\theta}_1 cos(\theta_1-\theta_2) - m_2 l_1\dot{\theta}_1^2 sin(\theta_1-\theta_2) + m_2 g\sin\theta_2 = 0 \tag{4.2}$$

Those two equations are both second order. In order to apply euler's method to them, we need to split each of them into first order differential equations. But how does one split them? We do this by introducting λ_1 and λ_2 which are defined as follows.

$$\lambda_1 = \dot{\theta}_1 \tag{4.3}$$
$$\dot{\lambda}_1 = \ddot{\theta}_1 \tag{4.4}$$
$$\lambda_2 = \dot{\theta}_2 \tag{4.5}$$
$$\dot{\lambda}_2 = \ddot{\theta}_2 \tag{4.6}$$

Putting these lambdas into (4.1) yields

$$(m_1+m_2)l_1\dot{\lambda}_1 + m_2 l_2\dot{\lambda}_2\cos(\theta_1-\theta_2) + m_2 l_2\lambda_2^2\sin(\theta_1-\theta_2) + g(m_1+m_2)\sin\theta_1 = 0. \tag{4.7}$$

And doing the same thing to (4.2) yields

$$m_2 l_2\dot{\lambda}_2 + m_2 l_1\dot{\lambda}_1 cos(\theta_1-\theta_2) - m_2 l_1\lambda_1^2 sin(\theta_1-\theta_2) + m_2 g\sin\theta_2 = 0. \tag{4.8}$$

We are in the process of preparing our differential equations for euler's method and the next crucial step is to rewrite (4.7) and (4.8) in such a way that both have a single $\dot{\lambda}$ on the left side of the equal sign. For (4.7) we get

$$\dot{\lambda}_1 = \frac{-m_2 l_2\dot{\lambda}_2\cos(\theta_1-\theta_2) - m_2 l_2\lambda_2^2\sin(\theta_1-\theta_2)g(m_1+m_2)\sin\theta_1}{(m_1+m_2)l_1}. \tag{4.9}$$