Solving the differential equation of a falling raindrop with air resistance

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1 The Problem

We want to spend some time thinking about raindrops and air resistance. In particular, we want to find a function $v_y(t)$ which maps time t to the velocity of the raindrop.

We assume that our mathematical raindrop has both a constant mass and a fixed shape. The constant mass is important because a change in mass – a real raindrop might lose some of its water molecules while falling through the air – would lead to a change in terminal velocity [1].

Furthermore, we assume that there is no wind and the only forces governing the raindrops movement are F_g (gravitational pull) and F_r (air resistance)

The air resistance in our model is proportional to $v_y(t)^2$. We introduce constant k to ...

$$F_r = k \cdot v_y(t)^2 \tag{1.1}$$

We expect the velocity to converge to some terminal value V_t. Thus, we expect the unknown function $v_y(t)$ to rise steep (start of the fall) and then flatten out (converging to terminal velocity).

2 The Math

3 The Solution

[1] https://journals.ametsoc.org/doi/pdf/10.1175/1520-0450%281969%29008%3C0249%3ATVORA%3E2.0.CO%3B2