SAPPORO, DEPARTMENT OF UNDEFINED STUDIES

Charges, Energies and Potentials

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1 PROBLEM ONE

$$Q_A = -1[C] \qquad W_A = 60[J] \tag{1.1}$$
$$Q_B = -2[C] \qquad W_B = 10[J] \tag{1.2}$$

Two points A and B exist in a vacuum. They have energies and charges as describes above.
c) How much work is necessary to get 2.5 trillion electrons from infinitely far away to Point A?
We start our investigation by calculating the electrostatic charge of $2.5 \cdot 10^{18}$ electrons.

$$e^- = -1.6 \cdot 10^{-19}[C] \tag{1.3}$$

Using 1.3 for the charge of a single electron we let Q_C be the total charge of the 2.5 trillion electrons and calculate that

$$Q_C = 2.5 \cdot 10^{18} \cdot e^- \tag{1.4}$$
$$= -0.4[C] \tag{1.5}$$

In order to calculate how much work it takes to move Q_C from infinitely far away to point A, we need to know how strong the electric field is at any point on the journey. Let r be the distance from point A to any point in 3D space and ϵ_0[1] the permitivity in empty space (vacuum), then the electric field E at any distance r from point A is

$$E = \frac{Q_A}{4\pi\epsilon_0 r^2} \tag{1.6}$$

[1] $\epsilon_0 = 8.854 \cdot 10^{-12}[m/F]$

And the force F at any point is

$$F = E \cdot Q_C \tag{1.7}$$
$$= \frac{Q_A}{4\pi\epsilon_0 r^2} \cdot Q_C \tag{1.8}$$

To get the work it takes to move Q_C towards point A from distance ∞ to a distance 0 from point A, we then take the integral of the force from ∞ to 0. Note how we introduce the concept of the mathematical limit to be able to deal with infinities.

$$W = -lim_{x\to\infty} \int_x^0 \frac{Q_A Q_C}{4\pi\epsilon_0 r^2} \cdot dr \tag{1.9}$$
$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} lim_{x\to\infty} \int_x^0 \frac{1}{r^2} \cdot dr \tag{1.10}$$
$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} lim_{x\to\infty} \left(\frac{1}{x} - \frac{1}{0}\right) \tag{1.11}$$
$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} \left(0 - \frac{1}{0}\right) \tag{1.12}$$

Now we find ourselves confronted with the infamous divide by 0 problem and it dawns on us, that bringing a charge Q_C on top on another charge Q_A, where the force between the two charges is known to be

$$F = \frac{Q_A Q_C}{4\pi\epsilon_0 r^2} \tag{1.13}$$

will lead to a infinitely large force pushing Q_C away from Q_A. After checking that a integral of the form of

$$\int_\infty^0 \frac{1}{r^2} \cdot dr \tag{1.14}$$
$$\tag{1.15}$$

indeed is divergant[2] we give in to the fact that electrons can not exist on top of each other and reinterpret "How much work is necessary to get 2.5 trillion electrons from infinitely far away to point A" as "How much work is necessary to get 2.5 trillion electrons from infinitely far away to very close to point A".

[2] See example 8 here: http://tutorial.math.lamar.edu/Classes/CalcII/ImproperIntegrals.aspx
and result here: https://www.wolframalpha.com/input/?i=integral+of+1

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\begin{aligned}
W &= -lim_{x\to\infty}\int_x^{0.000000001[m]} \frac{Q_A Q_C}{4\pi\epsilon_0 r^2}\cdot dr && (1.16)\\
&= -\frac{Q_A Q_C}{4\pi\epsilon_0} lim_{x\to\infty}\int_x^{0.000000001[m]} \frac{1}{r^2}\cdot dr && (1.17)\\
&= -\frac{Q_A Q_C}{4\pi\epsilon_0} lim_{x\to\infty}\left(\frac{1}{x}-\frac{1}{0.000000001[m]}\right) && (1.18)\\
&= -\frac{Q_A Q_C}{4\pi\epsilon_0}\left(0-\frac{1}{0.000000001[m]}\right) && (1.19)\\
&= -\frac{-1[C]\cdot -0.4[C]}{4\pi\epsilon_0}\left(-\frac{1}{0.000000001[m]}\right) && (1.20)\\
&= -\frac{-1[C]\cdot -0.4[C]}{4\pi 8.854\cdot 10^{-12}[F/m]}\left(-\frac{1}{0.000000001[m]}\right) && (1.21)\\
&= \frac{0.4\cdot 10^{12}[C^2]}{4\pi\cdot 8.854[F]}\left(\frac{1}{0.000000001}\right) && (1.22)\\
&= 3.595\cdot 10^{18}\frac{[C^2]}{[F]} && (1.23)\\
&= 3.595\cdot 10^{18}[C\cdot V] && (1.24)\\
&= 3.595\cdot 10^{18}\left[A\cdot s\cdot\frac{N\cdot m}{A\cdot s}\right] && (1.25)\\
&= 3.595\cdot 10^{18}[N\cdot m] && (1.26)\\
&= 3.595\cdot 10^{18}[J] && (1.27)
\end{aligned}
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We come to the conclusion that it takes $3.5950\cdot 10^{18}[J]$ to get 2.5 trillion electrons from infinitely far away to 1 [nm] near point A.