
Charges, Energies and Potentials

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1 PROBLEM ONE

$$Q_A = -1[C] \qquad W_A = 60[J] \qquad (1.1)$$

$$Q_B = -2[C] \qquad W_B = 10[J] \qquad (1.2)$$

Two points A and B exist in a vacuum. They have energies and charges as described above.

c) How much work is necessary to get 2.5 trillion electrons from infinitely far away to Point A?

We start our investigation by calculating the electrostatic charge of $2.5 \cdot 10^{18}$ electrons.

$$e^- = -1.6 \cdot 10^{-19}[C] \qquad (1.3)$$

Using 1.3 for the charge of a single electron we let Q_C be the total charge of the 2.5 trillion electrons and calculate that

$$Q_C = 2.5 \cdot 10^{18} \cdot e^- \qquad (1.4)$$

$$= -0.4[C] \qquad (1.5)$$

In order to calculate how much work it takes to move Q_C from infinitely far away to point A, we need to know how strong the electric field is at any point on the journey. Let r be the distance from point A to any point in 3D space and ϵ_0 ¹ the permittivity in empty space (vacuum), then the electric field E at any distance r from point A is

$$E = \frac{Q_A}{4\pi\epsilon_0 r^2} \qquad (1.6)$$

¹ $\epsilon_0 = 8.854 \cdot 10^{-12}[m/F]$

And the force F at any point is

$$F = E \cdot Q_C \quad (1.7)$$

$$= \frac{Q_A}{4\pi\epsilon_0 r^2} \cdot Q_C \quad (1.8)$$

To get the work it takes to move Q_C towards point A from distance ∞ to a distance 0 from point A, we then take the integral of the force from ∞ to 0. Note how we introduce the concept of the mathematical limit to be able to deal with infinities.

$$W = -\lim_{x \rightarrow \infty} \int_x^0 \frac{Q_A Q_C}{4\pi\epsilon_0 r^2} \cdot dr \quad (1.9)$$

$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} \lim_{x \rightarrow \infty} \int_x^0 \frac{1}{r^2} \cdot dr \quad (1.10)$$

$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{1}{0} \right) \quad (1.11)$$

$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} \left(0 - \frac{1}{0} \right) \quad (1.12)$$

Now we find ourselves confronted with the infamous divide by 0 problem and it dawns on us, that bringing a charge Q_C on top on another charge Q_A , where the force between the two charges is known to be

$$F = \frac{Q_A Q_C}{4\pi\epsilon_0 r^2} \quad (1.13)$$

will lead to a infinitely large force pushing Q_C away from Q_A . After checking that a integral of the form of

$$\int_{\infty}^0 \frac{1}{r^2} \cdot dr \quad (1.14)$$

$$(1.15)$$

indeed is divergant² we give in to the fact that electrons can not exist on top of each other and reinterpret "How much work is necessary to get 2.5 trillion electrons from infinitely far away to point A" as "How much work is necessary to get 2.5 trillion electrons from infinitely far away to very close to point A".

²See example 8 here: <http://tutorial.math.lamar.edu/Classes/CalcII/ImproperIntegrals.aspx>
and result here: <https://www.wolframalpha.com/input/?i=integral+of+1%2Fx%5E2+from+infinity+to+0>

$$W = -\lim_{x \rightarrow \infty} \int_x^{0.000000001[m]} \frac{Q_A Q_C}{4\pi\epsilon_0 r^2} \cdot dr \quad (1.16)$$

$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} \lim_{x \rightarrow \infty} \int_x^{0.000000001[m]} \frac{1}{r^2} \cdot dr \quad (1.17)$$

$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} \lim_{x \rightarrow \infty} \left(\frac{1}{x} - \frac{1}{0.000000001[m]} \right) \quad (1.18)$$

$$= -\frac{Q_A Q_C}{4\pi\epsilon_0} \left(0 - \frac{1}{0.000000001[m]} \right) \quad (1.19)$$

$$= -\frac{-1[C] \cdot -0.4[C]}{4\pi\epsilon_0} \left(-\frac{1}{0.000000001[m]} \right) \quad (1.20)$$

$$= -\frac{-1[C] \cdot -0.4[C]}{4\pi \cdot 8.854 \cdot 10^{-12}[F/m]} \left(-\frac{1}{0.000000001[m]} \right) \quad (1.21)$$

$$= \frac{0.4 \cdot 10^{12}[C^2]}{4\pi \cdot 8.854[F]} \left(\frac{1}{0.000000001} \right) \quad (1.22)$$

$$= 3.595 \cdot 10^{18} \frac{[C^2]}{[F]} \quad (1.23)$$

$$= 3.595 \cdot 10^{18} [C \cdot V] \quad (1.24)$$

$$= 3.595 \cdot 10^{18} \left[A \cdot s \cdot \frac{N \cdot m}{A \cdot s} \right] \quad (1.25)$$

$$= 3.595 \cdot 10^{18} [N \cdot m] \quad (1.26)$$

$$= 3.595 \cdot 10^{18} [J] \quad (1.27)$$

We come to the conclusion that it takes $3.5950 \cdot 10^{18} [J]$ to get 2.5 trillion electrons from infinitely far away to 1 [nm] near point A.

Note that we only considered the field from Q_A even though we know that there is also a charge Q_B emitting an electric field E_B . We could take field E_b into account if we knew the position in 3D space relative to point A. But we don't. So we can't. Sounds fishy? Yeah, let's give this another try.

2 PROBLEM ONE, SECOND ITERATION

Everything gets better with the second iteration. This time the author is confident that he is on point with how the problem is meant to be solved; This problem is setup in such a way that there is no electric field being caused by Q_A or Q_B . Which is not to say that there is no electric field at all. There is one – because without, the charges wouldn't have potential energies – but it is implicit. There is no need to calculate it.

The trick to solving problem c) is to use the conservation of energy. Charge Q_A has a potential energy of 60[J] because 60[J] of work was done to get it there (pushing it against the implicit static electric field).

We observe that it took 60[J] to get a charge of -1[C] to point A. since the work necessary to move a charge in a electric field is proportional to the amount of charge, we come to the conclusion that the work necessary to get a charge of -0.4[C] to point A must be 24[J] (ignoring the field from the charge Q_A which is already at point A).