# Charges, Energies and Potentials

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## 1 Problem One

$$Q_A = -1[C] \qquad W_A = 60[J] \tag{1.1}$$
$$Q_B = -2[C] \qquad W_B = 10[J] \tag{1.2}$$

Two points A and B exist in a vacuum. They have energies and charges as described above.

### 1.1 Subproblem c

**Problem: How much work in [J] has to be done to get 2.5 trillion electrons from very far away to point A?**

We start our investigation by calculating the electrostatic charge of $2.5 \cdot 10^{18}$ electrons.

$$e^- = -1.6 \cdot 10^{-19}[C] \tag{1.3}$$

Using 1.3 for the charge of a single electron we let $Q_C$ be the total charge of the 2.5 trillion electrons and calculate that

$$Q_C = 2.5 \cdot 10^{18} \cdot e^- \tag{1.4}$$
$$= -0.4[C] \tag{1.5}$$

In order to calculate how much work it takes to move $Q_C$ from infinitely far away to point A, we need to know how strong the electric field is at any point on the journey. Let $r$ be the distance from point A to any point in 3D space and $\epsilon_0$[1] the permitivity in empty space (vacuum), then the electric field $E$ at any distance r from point A is

[1] $\epsilon_0 = 8.854 \cdot 10^{-12}[m/F]$

$$E = \frac{Q_A}{4\pi\epsilon_0 r^2} \tag{1.6}$$

And the force $F$ at any point is

$$\begin{aligned} F &= E \cdot Q_C && (1.7) \\ &= \frac{Q_A}{4\pi\epsilon_0 r^2} \cdot Q_C && (1.8) \end{aligned}$$

To get the work it takes to move $Q_C$ towards point A from distance $\infty$ to a distance 0 from point A, we then take the integral of the force from $\infty$ to 0. Note how we introduce the concept of the mathematical limit to be able to deal with infinities.

$$\begin{aligned} W &= -lim_{x\to\infty} \int_x^0 \frac{Q_A Q_C}{4\pi\epsilon_0 r^2} \cdot dr && (1.9) \\ &= -\frac{Q_A Q_C}{4\pi\epsilon_0} lim_{x\to\infty} \int_x^0 \frac{1}{r^2} \cdot dr && (1.10) \\ &= -\frac{Q_A Q_C}{4\pi\epsilon_0} lim_{x\to\infty} \left(\frac{1}{x} - \frac{1}{0}\right) && (1.11) \\ &= -\frac{Q_A Q_C}{4\pi\epsilon_0} \left(0 - \frac{1}{0}\right) && (1.12) \end{aligned}$$

Now we find ourselves confronted with the infamous divide by 0 problem and it dawns on us, that bringing a charge $Q_C$ on top on another charge $Q_A$, where the force between the two charges is known to be

$$F = \frac{Q_A Q_C}{4\pi\epsilon_0 r^2} \tag{1.13}$$

will lead to a infinitely large force pushing $Q_C$ away from $Q_A$. After checking that a integral of the form of

$$\begin{aligned} &\int_\infty^0 \frac{1}{r^2} \cdot dr && (1.14) \\ & && (1.15) \end{aligned}$$

indeed is divergant[2] we give in to the fact that electrons can not exist on top of each other and reinterpret "How much work is necessary to get 2.5 trillion electrons from infinitely far

[2] See example 8 here: http://tutorial.math.lamar.edu/Classes/CalcII/ImproperIntegrals.aspx
and result here: https://www.wolframalpha.com/input/?i=integral+of+1%2Fx%5E2+from+infinity+to+0

away to point A" as "How much work is necessary to get 2.5 trillion electrons from infinitely far away to very close to point A".

$$
\begin{aligned}
W &= -lim_{x\to\infty}\int_{x}^{0.000000001[m]} \frac{Q_A Q_C}{4\pi\epsilon_0 r^2}\cdot dr && (1.16)\\
&= -\frac{Q_A Q_C}{4\pi\epsilon_0} lim_{x\to\infty}\int_{x}^{0.000000001[m]} \frac{1}{r^2}\cdot dr && (1.17)\\
&= -\frac{Q_A Q_C}{4\pi\epsilon_0} lim_{x\to\infty}\left(\frac{1}{x}-\frac{1}{0.000000001[m]}\right) && (1.18)\\
&= -\frac{Q_A Q_C}{4\pi\epsilon_0}\left(0-\frac{1}{0.000000001[m]}\right) && (1.19)\\
&= -\frac{-1[C]\cdot -0.4[C]}{4\pi\epsilon_0}\left(-\frac{1}{0.000000001[m]}\right) && (1.20)\\
&= -\frac{-1[C]\cdot -0.4[C]}{4\pi 8.854\cdot 10^{-12}[F/m]}\left(-\frac{1}{0.000000001[m]}\right) && (1.21)\\
&= \frac{0.4\cdot 10^{12}[C^2]}{4\pi\cdot 8.854[F]}\left(\frac{1}{0.000000001}\right) && (1.22)\\
&= 3.595\cdot 10^{18}\frac{[C^2]}{[F]} && (1.23)\\
&= 3.595\cdot 10^{18}[C\cdot V] && (1.24)\\
&= 3.595\cdot 10^{18}\left[A\cdot s\cdot\frac{N\cdot m}{A\cdot s}\right] && (1.25)\\
&= 3.595\cdot 10^{18}[N\cdot m] && (1.26)\\
&= 3.595\cdot 10^{18}[J] && (1.27)
\end{aligned}
$$

We come to the conclusion that it takes $3.5950\cdot 10^{18}[J]$ to get 2.5 trillion electrons from infinitely far away to 1 [nm] near point A.

Note that we only considered the field from $Q_A$ even though we know that there is also a charge $Q_B$ emitting an electric field $E_B$. We could take field $E_b$ into account if we knew the position in 3D space relative to point A. But we don't. So we can't. Sounds fishty? Yeah, let's give this another try.

## 2 Problem One, Second Iteration

This time the author is confident that he is on point with how the problem is meant to be solved; Ignoring the electric fields from charges $Q_A$ and $Q_B$. Which is not to say that there is no electric field at all. There is one – because without, the charges wouldn't have potential energies – but it is implicit. There is no need to calculate it.

## 2.1 Subproblem A

**Problem: Calculate the electric voltage between the points.**

> According to classical electrostatics, electric potential is a scalar quantity denoted by V or occasionally $\phi$ equal to the electric potential energy of any charged particle at any location (measured in joules) divided by the charge of that particle (measured in coulombs).

Above qoute is from the english Wikipedia entry on the electric potential. To solve this problem we leverage the power of classical electrostatics and proclaim that the following equation, which connects the electric potential at a point in a static electric field to the Charge and potential energy at that point, holds true.

$$\phi = \frac{W}{Q} \tag{2.1}$$

Where $\phi$ denotes the electric potential, W the potential energy and Q the Charge. Using given values from 1.1 and 1.2:

$$\begin{aligned} \phi_A &= \frac{W_A}{Q_A} && (2.2) \\ &= -\frac{60}{1}\left[\frac{J}{C}\right] && (2.3) \\ &= -60[V] && (2.4) \end{aligned}$$

$$\begin{aligned} \phi_B &= \frac{W_B}{Q_B} && (2.5) \\ &= -\frac{10}{2}\left[\frac{J}{C}\right] && (2.6) \\ &= -5[V] && (2.7) \end{aligned}$$

Finally, the electric Voltage between point A and B is the difference between the electric potentials, so

$$\begin{aligned} \phi_{AB} &= \phi_B - \phi_A && (2.8) \\ &= 55[V] && (2.9) \end{aligned}$$

## 2.2 Subproblem B

**Problem: Given that the charges where free to move between A and B, what would the resulting state be? What's the total amount of energy in the resulting state?**

Since we're dealing with negative charges, we know that they will gather at the point with the most positive electric potential. In this case, it is point B. So charge $Q_A$ moves to point B. By moving from point a to b, work equal to $W_m$ is done.

$$\begin{aligned} W_m &= \phi_{AB} \cdot Q_A && (2.10) \\ &= 55[V] \cdot -1[C] && (2.11) \\ &= -55[J] && (2.12) \end{aligned}$$

Adding this (negative) work to the potential energy the charge had at point A will yield the new potential energy of $Q_A$ at point B.

$$\begin{aligned} W'_A &= W_A + W_m && (2.13) \\ &= 60[J] - 55[J] && (2.14) \\ &= 5[J] && (2.15) \end{aligned}$$

And the total Energy of this new state is

$$\begin{aligned} W_{total} &= W'_A + W_B && (2.16) \\ &= 5[J] + 10[J] && (2.17) \\ &= 15[J] && (2.18) \end{aligned}$$

## 2.3 Subproblem C

**Problem: How much work in [J] has to be done to get 2.5 trillion electrons from very far away to point A?**

The trick here is to use the conservation of energy. Charge $Q_A$ has a potential energy of 60[J] because 60[J] of work was done to get it there (pushing it against the implicit static electric field).

We observe that it took 60[J] to get a charge of -1[C] to point A. since the work necessary to move a charge in a electric field is proportional to the amount of charge, we come to the conclusion that the work necessary to get a charge of -0.4[C] to point A must be 24[J] (ignoring the field from the charge $Q_A$).

## 2.4 Subproblem D

**Problem: How much is the electric power if half of the charge at point B was moved infinitely far away?**

There is a Charge of -2[C] at point B. Half of it is -1[C]. This half of the charge also owns half of the potential energy, i.e 5[J]. The electric power is just the rate of work per second

$$P = -\frac{W_B}{2 \cdot t} \tag{2.19}$$

$$= -\frac{5[J]}{10[s]} \tag{2.20}$$

$$= -0.5[Ws] \tag{2.21}$$

The electric power is negative because the Work done is negative. There was no force added to the system. The charge moves because of the static electric field. The potential energy went to 0 and the only thing capable of doing this is negative work. And negative work leads to negative electrical power.