2 Dimensional Unsteady Heat Diffusion

Nikhil Thota

1 Method and Results

The 2-D unsteady heat diffusion equation : $\dfrac{\partial T(i,j)}{\partial t} = \alpha \dfrac{\partial^2 T(i,j)}{\partial x^2} + \alpha \dfrac{\partial^2 T(i,j)}{\partial y^2}$

The finite difference method was used to convert the partial differential equation into a difference equation. $\dfrac{\partial T(i,j)}{\partial t}$ was converted using **First-order forward difference** while $\dfrac{\partial^2 T(i,j)}{\partial x^2}$ and $\dfrac{\partial^2 T(i,j)}{\partial y^2}$ were converted to their difference forms using **Second-order central difference**. This is shown below :

First-order forward difference : $\dfrac{\partial T(i,j)}{\partial t} = \dfrac{T^{n+1}(i,j) - T^n(i,j)}{\Delta t}$

Second-order central difference :

$$\frac{\partial^2 T(i,j)}{\partial x^2} = \frac{T^n(i+1,j) - 2T^n(i,j) + T^n(i-1,j)}{\Delta x^2}$$

$$\frac{\partial^2 T(i,j)}{\partial y^2} = \frac{T^n(i,j+1) - 2T^n(i,j) + T^n(i,j-1)}{\Delta y^2}$$

Substituting in the 2-D unsteady heat diffusion equation and rearranging we get :

$$T^{n+1}(i,j) = T^n(i,j) + \frac{\alpha \Delta t}{\Delta x^2}[T^n(i+1,j) - 2T^n(i,j) + T^n(i-1,j)] + \frac{\alpha \Delta t}{\Delta y^2}[T^n(i,j+1) - 2T^n(i,j) + T^n(i,j-1)]$$

The x and y axis was divided into 21 grid points and since both the axis have limits from 0 to 1 then $\Delta x = \dfrac{1-0}{21-1} = \Delta y = \dfrac{1-0}{21-1} = 0.05$. α was chosen as 0.0001 m2/s and Δt was chosen as 0.001 s so that the following stability criterion held true :

$$\frac{\alpha \Delta t}{\Delta x^2} = 0.00004 < 0.5$$

The matrix output from C++ program was stored in a .txt file which was then passed to gnuplot to plot the contour and image plots. The x and y axis coordinates have been numbered according to the indices of the Temperature matrix.

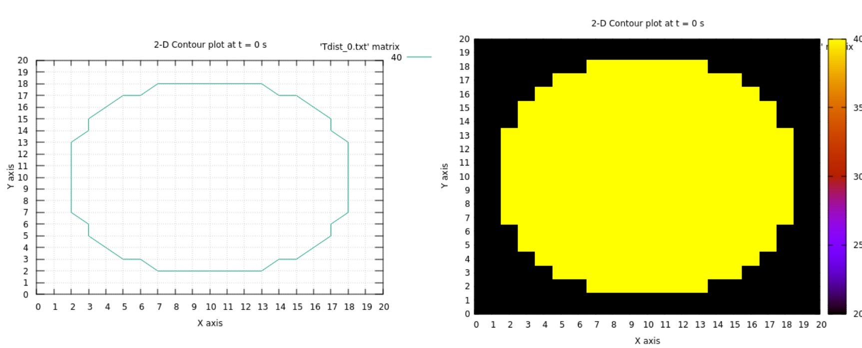

Figure 1: Temperature contour and image plots at t=0

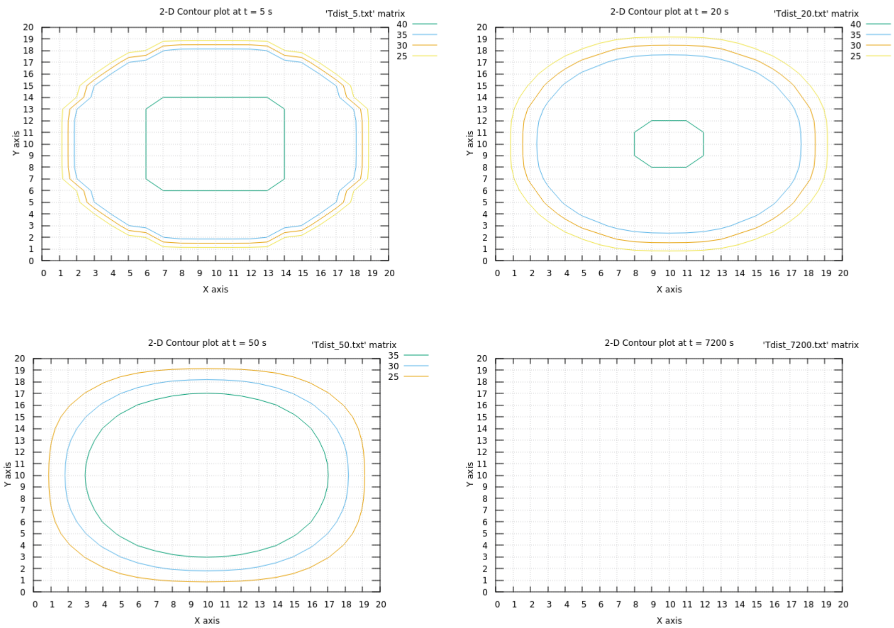

Figure 2: Temperature contour plots

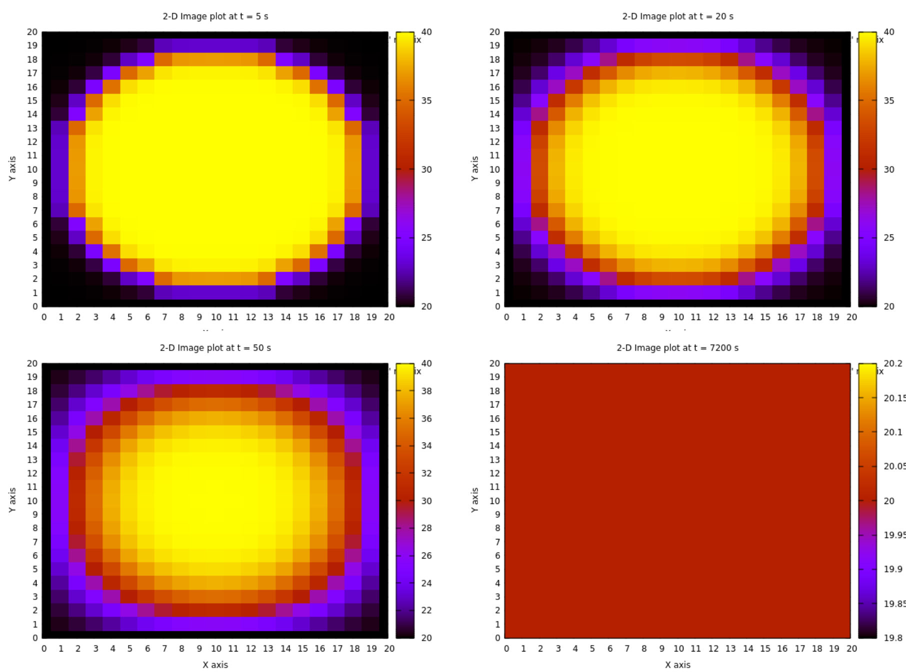

Figure 3: Temperature image plots

Note 1 : At 7200 s there is no contour plot as the plate has reached steady state condition of 20 C.
Note 2 : The C++ program was compiled using gcc 7.5.0 and was run from the terminal in Ubuntu OS.