Behavioral Economics
Exercise 4 Time Preferences

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Question 1（a）Solve by backward induction.

- Period 2

  By $\beta = \delta = 1$, $t = 2$ agent's intertemporal utility is :

$$-c(e_2) + B(e_1, e_2)$$
$$\text{where } e_1 \text{ is given}$$

Then, the best-response of the $t = 2$ agent, depending on the parameters is derived as follows :

If $B \geq kc \Leftrightarrow c \leq \dfrac{B}{k}$

$$\mathrm{BR}(e_1) = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

and if $kc > B > 2c \Leftrightarrow c > \dfrac{B}{k}$

$$\mathrm{BR}(e_1) = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

- Period 1

  Now by $\beta = \delta = 1$, $t = 1$ agent's intertemporal utility is :

$$-c(e_1) - c(\mathrm{BR}(e_1)) + B(e_1, \mathrm{BR}(e_1))$$

By the assumption $kc > 2c$ and $B > 2c$ and the derived best-response of the $t = 2$ agent, e_1 is the dominant strategy for each case of the parameters.

Therefore, the perception-perfect equilibria is :
If $c \leq \dfrac{B}{k}$

$$(e_1^*, e_2^*) = (1, e_2)$$
$$\text{where } e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

If $c > \dfrac{B}{k}$

$$(e_1^*, e_2^*) = (1, e_2)$$
$$\text{where } e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

(b)
- Period 2

  The intertemporal utility is :

$$-c(e_2) + \beta B(e_1, e_2)$$

  - When $e_1 = 0$

    For $t = 2$ agent, $e_2 = 1$ is strictly dominated by $e_2 = 0$, since both of them result in $B(0, e_2) = 0$.

    Then, the agent prefer $e_2 = 2$ iff

$$-kc + \beta B \geq 0$$
$$c \leq \frac{\beta B}{k}$$

    Otherwise, $e_2 = 0$ is chosen.
  - When $e_1 = 1$

    $e_2 = 2$ is strictly dominated by $e_2 = 1$, since $e_2 = 1$ is enough to satisfy $B(.,.) = B$.

    The agent prefer $e_2 = 1$ iff

$$-c + \beta B \geq 0$$
$$c \leq \beta B$$

- Period 1
  - When $c \leq \dfrac{\beta B}{k}$
    Even if $e_1 = 0$, $t = 2$ agent choose $e_2 = 2$ and so $B(e_1, e_2) = B$ realizes.
    Then, $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

    $$(1 - \beta)kc \leq 0$$

    which never hold since $\beta < 1$, so $e_1 = 2$ is stricly dominated by $e_1 = 0$.
    Then, $e_1 = 1$ is preferred iff

    $$\begin{aligned} -c - \beta c + \beta B &\geq -kc + \beta B \\ (1 + \beta - \beta k)c &\leq 0 \\ \text{which holds iff } k &\geq \frac{1 + \beta}{\beta} \end{aligned}$$

    Otherwise, $e_1 = 0$ is chosen.
  - When $\dfrac{\beta B}{k} < c \leq \beta B$
    $e_2 = 2$ is never chosen, which implies that if $e_1 = 0$, $B(.,.) = 0$.
    $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

    $$\begin{aligned} -kc + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{k} \end{aligned}$$

    which never holds, by $c > \dfrac{\beta B}{k}$.
    $e_1 = 1$ is preferred to $e_1 = 0$ iff

    $$\begin{aligned} -c - \beta c + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{1 + \beta} \end{aligned}$$

    Note that $\left(\dfrac{\beta B}{k}, \dfrac{\beta B}{1 + \beta}\right]$ is not empty interval.
  - $\beta B < c$
    Both $t = 1, 2$ agent choose $e_t = 0$, rather than any positive e_t.

Therefore, the perception-perfect equilibria is:

$$(e_1^*, e_2^*) = (e_1, e_2)$$

If $(c \leq \frac{\beta B}{k}) \wedge (k \geq \frac{1+\beta}{\beta})$,

$$e_1 = 1$$
$$e_2 = \begin{cases} 2 & \text{if} e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $(c \leq \frac{\beta B}{k}) \wedge (k < \frac{1+\beta}{\beta})$,

$$e_1 = 0$$
$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 0 & \text{otherwise} \end{cases}$$

If $c \in (\frac{\beta B}{k}, \frac{\beta B}{1+\beta})$,

$$e_1 = 1$$
$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $\beta B < c$,

$$e_1 = 0$$
$$e_2 = 0$$

(c) $t = 2$ agent decides her/his behavior with $\beta < 1$, while $t = 1$ agent predicts it with $\hat{\beta} = 1$. Then, the agent's prediction and the acual strategy depending on the parameters are summerized as follows :

Parametrical condition			
Belief	Actual choice	e_1	BR(e_1)
$c \leq \frac{B}{k}$	$c \leq \frac{\beta B}{k}$	0	2
		1	1
		2	0
$\frac{B}{k} < c \leq B$	$\frac{\beta B}{k} < c \leq \beta B$	1	1
		0,2	0
$B < c$	$\beta B < c$	any	0

If $k \leq \frac{1}{\beta}$ holds, then $\frac{B}{k} \leq \beta B$; otherwise, $\frac{B}{k} \leq \beta B$.

- $k \geq \frac{1}{\beta}$

  i. $c \leq \dfrac{\beta}{k}$

  Same as (b), $t = 1$ agent choose $e_1 = 1$ with belief that $t = 2$ agent choose the following strategy

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

  iff

$$k \geq \frac{1+\beta}{\beta}$$

  and by $c \leq \dfrac{\beta}{k}$, the strategy above is realized as perception-perfect equilibrium.

  $c > \dfrac{\beta}{k}$, then $t = 1$ agent choose $e_1 = 0$, and the strategy of $t = 2$ agent is same.

  ii. $\dfrac{\beta B}{k} < c \leq \dfrac{B}{k}$

  By $c < \dfrac{B}{k}$, $t = 1$ agent predicts that $t = 2$ agent choose the following strategy :

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

  The realized strategy of $t = 2$ agent, however, is below :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

  Thus, there does not exist any perception-perfect equilibrium.

iii. $\frac{B}{k} < c \leq \beta B$
$t=1$ agent predicts $t=2$ agent's strategy as follows, and it will be realized.

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

まだ完成じゃないぞ。

iv. $\beta B < c \leq \frac{B}{2}$
$t=1$ agent predicts $t=2$ strategy as :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

While the realized strategy is $e_2 = 0$ for any e_1. Thus, there is no perception-perfect equilibrium.

- $k < \frac{1}{\beta}$

i. $c \leq \frac{\beta B}{k}$
$t=1$ agent's belief :

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

which realizes in period 2. Then, the perception-perfect equilibrium is :

$$e_1 = 1 \text{ and } e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}.$$

ii. $\frac{\beta B}{k} < c \leq \beta B$
$t=1$ agent's belief is

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

while $t=2$ agent's strategy is

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

Thus, there is no perception-perfect equilibrium.

iii. $\beta B < c \leq \frac{\beta B}{k}$
$t=1$ agent predicts $t=2$ agent's strategy:

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

$t=2$ agent's strategy is, however,

$$e_2 = 0 \text{ for any } e_1$$

Again, no perception-perfect equilibrium occurs.

iv. $\frac{\beta B}{k} < c \leq \frac{B}{2}$

$t=1$ agent's belief :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

while $e_2 = 0$ for any e_1 in period 2, which indicates there is no perception-perfect equilibrium.

(d) $t = 2$ agent decides her/his behavior with $\beta < 1$, while $t = 1$ agent predicts it with $\hat{\beta} \in (\beta, 1)$. Then, the agent's prediction and the acual strategy depending on the parameters are summerized as follows :

Parametrical condition			
Belief	Actual choice	e_1	BR(e_1)
$c \leq \frac{\hat{\beta} B}{k}$	$c \leq \frac{\beta B}{k}$	0	2
		1	1
		2	0
$\frac{\hat{\beta} B}{k} < c \leq \hat{\beta} B$	$\frac{\beta B}{k} < c \leq \beta B$	1	1
		0,2	0
$\hat{\beta} B < c$	$\beta B < c$	any	0

If $k \geq \frac{\hat{\beta}}{\beta}$, then $\beta B \geq \frac{\hat{\beta} B}{k}$; otherwise, $\beta B < \frac{\hat{\beta} B}{k}$.

- $k \geq \frac{\hat{\beta}}{\beta}$
  - i. $c \leq \frac{\beta B}{k}$
  - ii. $\frac{\beta B}{k} < c \leq \frac{\hat{\beta} B}{k}$
  - iii. $\frac{\hat{\beta} B}{k} < c \leq \beta B$
  - iv. $\beta B < c \leq \hat{\beta} B$
  - v. $\hat{\beta} B < c$
- $k < \frac{\hat{\beta}}{\beta}$
  - i. $c \leq \frac{\beta B}{k}$
  - ii. $\frac{\beta B}{k} < c \leq \beta B$
  - iii. $\beta B < c \leq \frac{\hat{\beta} B}{k}$
  - iv. $\frac{\hat{\beta} B}{k} < c \leq \hat{\beta} B$

v. $\hat{\beta}B < c$

(e)
- Period 2

  When $e_1 = 0$, then $t = 2$ agent has no incentive to take any choice other than $e_2 = 0$, since $B(0, e_2) = 0$ for any e_2.

  When $e_1 = 1$, $t = 2$ agent prefer $e_2 = 1$ to $e_2 = 0$ iff

$$
\begin{aligned}
-c + \beta B &\geq 0 \\
c &\leq \beta B
\end{aligned}
$$

  otherwise, $e_2 = 0$ for any e_1

- Period 1

  $t = 1$ agent predicts her/his behavior in period 2 with $\hat{\beta} \in (\beta, 1)$.

  Then, the agent's prediction and the acual strategy depending on the parameters are summerized as follows :

Parametrical condition			
Belief	Actual choice	e_1	BR(e_1)
$c \leq \hat{\beta}$	$c \leq \beta B$	1	1
		0	0
$\hat{\beta}B < c$	$\beta B < c$	any	0

i. $c \leq \beta B$

ii. $\beta B < c \leq \hat{\beta}B$

iii. $\hat{\beta}B < c < \dfrac{B}{2}$