# Behavioral Economics
# Exercise 4 Time Preferences

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Question 1（a）Solve by backward induction.

- Period 2

  By $\beta = \delta = 1$, $t = 2$ agent's intertemporal utility is :

$$-c(e_2) + B(e_1, e_2)$$
$$\text{where } e_1 \text{ is given}$$

  Then, the best-response of the $t = 2$ agent, depending on the parameters is derived as follows :
  If $B \geq kc \Leftrightarrow c \leq \dfrac{B}{k}$

$$\mathrm{BR}(e_1) = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

  and if $kc > B > 2c \Leftrightarrow c > \dfrac{B}{k}$

$$\mathrm{BR}(e_1) = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

- Period 1

  Now by $\beta = \delta = 1$, $t = 1$ agent's intertemporal utility is :

$$-c(e_1) - c(\mathrm{BR}(e_1)) + B(e_1, \mathrm{BR}(e_1))$$

  By the assumption $kc > 2c$ and $B > 2c$ and the derived best-response of the $t = 2$ agent, $e_1$ is the dominant strategy for each case of the parameters.

Therefore, the perception-perfect equilibria is :
If $c \leq \dfrac{B}{k}$

$$(e_1^*, e_2^*) = (1, e_2)$$
$$\text{where } e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

If $c > \dfrac{B}{k}$

$$(e_1^*, e_2^*) = (1, e_2)$$
$$\text{where } e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

(b) • Period 2
The intertemporal utility is :

$$-c(e_2) + \beta B(e_1, e_2)$$

- When $e_1 = 0$
  For $t = 2$ agent, $e_2 = 1$ is strictly dominated by $e_2 = 0$, since both of them result in $B(0, e_2) = 0$.
  Then, the agent prefer $e_2 = 2$ iff

$$-kc + \beta B \geq 0$$
$$c \leq \frac{\beta B}{k}$$

  Otherwise, $e_2 = 0$ is chosen.
- When $e_1 = 1$
  $e_2 = 2$ is strictly dominated by $e_2 = 1$, since $e_2 = 1$ is enough to satisfy $B(.,.) = B$.
  The agent prefer $e_2 = 1$ iff

$$-c + \beta B \geq 0$$
$$c \leq \beta B$$

- Period 1
  - When $c \leq \dfrac{\beta B}{k}$
    Even if $e_1 = 0$, $t = 2$ agent choose $e_2 = 2$ and so $B(e_1, e_2) = B$ realizes.
    Then, $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

$$(1 - \beta)kc \leq 0$$

    which never hold since $\beta < 1$, so $e_1 = 2$ is stricly dominated by $e_1 = 0$.
    Then, $e_1 = 1$ is preferred iff

$$\begin{aligned} -c - \beta c + \beta B &\geq -kc + \beta B \\ (1 + \beta - \beta k)c &\leq 0 \\ \text{which holds iff } k &\geq \frac{1+\beta}{\beta} \end{aligned}$$

    Otherwise, $e_1 = 0$ is chosen.
  - When $\dfrac{\beta B}{k} < c \geq \beta B$
    $e_2 = 2$ is never chosen, which implies that if $e_1 = 0, B(.,.) = 0$.
    $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

$$\begin{aligned} -kc + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{k} \end{aligned}$$

    which never holds, by $c > \dfrac{\beta B}{k}$.
    $e_1 = 1$ is preferred to $e_1 = 0$ iff

$$\begin{aligned} -c - \beta c + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{1+\beta} \end{aligned}$$

    Note that $\left(\dfrac{\beta B}{k}, \dfrac{\beta B}{1+\beta}\right]$ is not empty interval.
  - $\beta B < c$
    Both $t = 1, 2$ agent choose $e_t = 0$, rather than any positive $e_t$.

Therefore, the perception-perfect equilibria is:

$$(e_1^*, e_2^*) = (e_1, e_2)$$

If $(c \leq \frac{\beta B}{k}) \wedge (k \geq \frac{1+\beta}{\beta})$,

$$e_1 = 1$$
$$e_2 = \begin{cases} 2 & \text{if} e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $(c \leq \frac{\beta B}{k}) \wedge (k < \frac{1+\beta}{\beta})$,

$$e_1 = 0$$
$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 0 & \text{otherwise} \end{cases}$$

If $c \in (\frac{\beta B}{k}, \frac{\beta B}{1+\beta})$,

$$e_1 = 1$$
$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $\beta B < c$,

$$e_1 = 0$$
$$e_2 = 0$$

(c) $t=2$ agent decides her/his behavior with $\beta<1$, while $t=1$ agent predicts it with $\hat{\beta}=1$. Then, the agent's prediction and the acual strategy depending on the parameters are summerized as follows :

| Parametrical condition | | | |
|---|---|---|---|
| Belief | Actual choice | $e_1$ | BR($e_1$) |
| $c \leq \frac{B}{k}$ | $c \leq \frac{\beta B}{k}$ | 0 | 2 |
| | | 1 | 1 |
| | | 2 | 0 |
| $\frac{B}{k} < c \leq B$ | $\frac{\beta B}{k} < c \leq \beta B$ | 1 | 1 |
| | | 0,2 | 0 |
| $B < c$ | $\beta B < c$ | any | 0 |

If $k \leq \frac{1}{\beta}$ holds, then $\frac{B}{k} \leq \beta B$ ; otherwise, $\frac{B}{k} \leq \beta B$.

- $k \geq \frac{1}{\beta}$
  i. $c \leq \frac{\beta}{k}$
  ii. $\frac{\beta B}{k} < c \leq \frac{B}{k}$
  iii. $\frac{B}{k} < c \leq \beta B$
  iv. $\beta B < c \leq \frac{B}{2}$
- $k < \frac{1}{\beta}$
  i. $c \leq \frac{\beta B}{k}$
  ii. $\frac{\beta B}{k} < c \leq \beta B$
  iii. $\beta B < c \leq \frac{\beta B}{k}$
  iv. $\frac{\beta B}{k} < c \leq \frac{B}{2}$

(d) $t=2$ agent decides her/his behavior with $\beta<1$, while $t=1$ agent predicts it with $\hat{\beta} \in (\beta, 1)$. Then, the agent's prediction and the acual strategy depending on the parameters are summerized as follows :

| Parametrical condition | | | |
|---|---|---|---|
| Belief | Actual choice | $e_1$ | BR($e_1$) |
| $c \leq \frac{\hat{\beta} B}{k}$ | $c \leq \frac{\beta B}{k}$ | 0 | 2 |
| | | 1 | 1 |
| | | 2 | 0 |
| $\frac{\hat{\beta} B}{k} < c \leq \hat{\beta} B$ | $\frac{\beta B}{k} < c \leq \beta B$ | 1 | 1 |
| | | 0,2 | 0 |
| $\hat{\beta} B < c$ | $\beta B < c$ | any | 0 |

If $k \geq \frac{\hat{\beta}}{\beta}$, then $\beta B \geq \frac{\hat{\beta}B}{k}$ ; otherwise, $\beta B < \frac{\hat{\beta}B}{k}$.

- $k \geq \frac{\hat{\beta}}{\beta}$
  i. $c \leq \frac{\beta B}{k}$
  ii. $\frac{\beta B}{k} < c \leq \frac{\hat{\beta}B}{k}$
  iii. $\frac{\hat{\beta}B}{k} < c \leq \beta B$
  iv. $\beta B < c \leq \hat{\beta}B$
  v. $\hat{\beta}B < c$
- $k < \frac{\hat{\beta}}{\beta}$
  i. $c \leq \frac{\beta B}{k}$
  ii. $\frac{\beta B}{k} < c \leq \beta B$
  iii. $\beta B < c \leq \frac{\hat{\beta}B}{k}$
  iv. $\frac{\hat{\beta}B}{k} < c \leq \hat{\beta}B$
  v. $\hat{\beta}B < c$

(e)
- Period 2

  When $e_1 = 0$, then $t = 2$ agent has no incentive to take any choice other than $e_2 = 0$, since $B(0, e_2) = 0$ for any $e_2$.

  When $e_1 = 1$, $t = 2$ agent prefer $e_2 = 1$ to $e_2 = 0$ iff

$$
\begin{aligned}
-c + \beta B &\geq 0 \\
c &\leq \beta B
\end{aligned}
$$

  otherwise, $e_2 = 0$ for any $e_1$

- Period 1

  $t = 1$ agent predicts her/his behavior in period 2 with $\hat{\beta} \in (\beta, 1)$.

  Then, the agent's prediction and the acual strategy depending on the parameters are summerized as follows :

| Parametrical condition | | | |
|---|---|---|---|
| Belief | Actual choice | $e_1$ | BR($e_1$) |
| $c \leq \hat{\beta}$ | $c \leq \beta B$ | 1 | 1 |
| | | 0 | 0 |
| $\hat{\beta}B < c$ | $\beta B < c$ | any | 0 |

i. $c \leq \beta B$
ii. $\beta B < c \leq \hat{\beta}B$
iii. $\hat{\beta}B < c < \frac{B}{2}$