

Behavioral Economics

Exercise 4 Time Preferences

経済学研究科

23A18014 Reio TANJI 丹治伶峰

Question 1 (a) Solve by backward induction.

- Period 2

By $\beta = \delta = 1, t = 2$ agent's intertemporal utility is :

$$-c(e_2) + B(e_1, e_2)$$

where e_1 is given

Then, the best-response of the $t = 2$ agent, depending on the parameters is derived as follows :

$$\text{If } B \geq kc \Leftrightarrow c \leq \frac{B}{k}$$

$$\text{BR}(e_1) = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

$$\text{and if } kc > B > 2c \Leftrightarrow c > \frac{B}{k}$$

$$\text{BR}(e_1) = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

- Period 1

Now by $\beta = \delta = 1, t = 1$ agent's intertemporal utility is :

$$-c(e_1) - c(\text{BR}(e_1)) + B(e_1, \text{BR}(e_1))$$

By the assumption $kc > 2c$ and $B > 2c$ and the derived best-response of the $t = 2$ agent, e_1 is the dominant strategy for each case of the parameters.

Therefore, the perception-perfect equilibria is :

$$\text{If } c \leq \frac{B}{k}$$

$$(e_1^*, e_2^*) = (1, e_2)$$

$$\text{where } e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

$$\text{If } c > \frac{B}{k}$$

$$(e_1^*, e_2^*) = (1, e_2)$$

$$\text{where } e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

(b) • Period 2

The intertemporal utility is :

$$-c(e_2) + \beta B(e_1, e_2)$$

– When $e_1 = 0$

For $t = 2$ agent, $e_2 = 1$ is strictly dominated by $e_2 = 0$, since both of them result in $B(0, e_2) = 0$.

Then, the agent prefer $e_2 = 2$ iff

$$\begin{aligned} -kc + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{k} \end{aligned}$$

Otherwise, $e_2 = 0$ is chosen.

– When $e_1 = 1$

$e_2 = 2$ is strictly dominated by $e_2 = 1$, since $e_2 = 1$ is enough to satisfy $B(., .) = B$.

The agent prefer $e_2 = 1$ iff

$$\begin{aligned} -c + \beta B &\geq 0 \\ c &\leq \beta B \end{aligned}$$

- Period 1

- When $c \leq \frac{\beta B}{k}$

Even if $e_1 = 0$, $t = 2$ agent choose $e_2 = 2$ and so $B(e_1, e_2) = B$ realizes.

Then, $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

$$(1 - \beta)kc \leq 0$$

which never hold since $\beta < 1$, so $e_1 = 2$ is stricly dominated by $e_1 = 0$.

Then, $e_1 = 1$ is preferred iff

$$-c - \beta c + \beta B \geq -kc + \beta B$$

$$(1 + \beta - \beta k)c \leq 0$$

$$\text{which holds iff } k \geq \frac{1 + \beta}{\beta}$$

Otherwise, $e_1 = 0$ is chosen.

- When $\frac{\beta B}{k} < c \leq \beta B$

$e_2 = 2$ is never chosen, which implies that if $e_1 = 0$, $B(.,.) = 0$.

$t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

$$-kc + \beta B \geq 0$$

$$c \leq \frac{\beta B}{k}$$

which never holds, by $c > \frac{\beta B}{k}$.

$e_1 = 1$ is preferred to $e_1 = 0$ iff

$$-c - \beta c + \beta B \geq 0$$

$$c \leq \frac{\beta B}{1 + \beta}$$

Note that $\left(\frac{\beta B}{k}, \frac{\beta B}{1 + \beta}\right]$ is not empty interval.

- $\beta B < c$

Both $t = 1, 2$ agent choose $e_t = 0$, rather than any positive e_t .

Therefore, the perception-perfect equilibria is:

$$(e_1^*, e_2^*) = (e_1, e_2)$$

$$\text{If } (c \leq \frac{\beta B}{k}) \wedge (k \geq \frac{1+\beta}{\beta}),$$

$$\begin{aligned} e_1 &= 1 \\ e_2 &= \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{If } (c \leq \frac{\beta B}{k}) \wedge (k < \frac{1+\beta}{\beta}),$$

$$\begin{aligned} e_1 &= 0 \\ e_2 &= \begin{cases} 2 & \text{if } e_1 = 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{If } c \in (\frac{\beta B}{k}, \frac{\beta B}{1+\beta}),$$

$$\begin{aligned} e_1 &= 1 \\ e_2 &= \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

$$\text{If } \beta B < c,$$

$$\begin{aligned} e_1 &= 0 \\ e_2 &= 0 \end{aligned}$$

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- (c) $t = 2$ agent decides her/his behavior with $\beta < 1$, while $t = 1$ agent predicts it with $\hat{\beta} = 1$. Then, the agent's prediction and the actual strategy depending on the parameters are summarized as follows :

Parametrical condition			
Belief	Actual choice	e_1	$BR(e_1)$
$c \leq \frac{B}{k}$	$c \leq \frac{\beta B}{k}$	0	2
		1	1
		2	0
$\frac{B}{k} < c \leq B$	$\frac{\beta B}{k} < c \leq \beta B$	1	1
		0,2	0
$B < c$	$\beta B < c$	any	0

If $k \leq \frac{1}{\beta}$ holds, then $\frac{B}{k} \leq \beta B$; otherwise, $\frac{B}{k} > \beta B$.

- $k \geq \frac{1}{\beta}$

- i. $c \leq \frac{\beta}{k}$

Same as (b), $t = 1$ agent choose $e_1 = 1$ with belief that $t = 2$ agent choose the following strategy

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

iff

$$k \geq \frac{1 + \beta}{\beta}$$

and by $c \leq \frac{\beta}{k}$, the strategy above is realized as perception-perfect equilibrium.

If $c > \frac{\beta}{k}$, then $t = 1$ agent choose $e_1 = 0$, and the strategy of $t = 2$ agent is same, and this is perception-perfect equilibrium.

- ii. $\frac{\beta B}{k} < c \leq \frac{B}{k}$

By $c < \frac{B}{k}$, $t = 1$ agent predicts that $t = 2$ agent choose the following strategy :

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

The realized strategy of $t = 2$ agent, however, is below :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

Thus, there does not exist any perception-perfect equilibrium.

- iii. $\frac{B}{k} < c \leq \beta B$
 $t = 1$ agent predicts $t = 2$ agent's strategy as follows, and it will be realized.

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

$(e_1 = 0) \succsim (e_1 = 2)$, since

$$\begin{aligned} 0 &\geq -kc + \beta B \\ c &\geq \frac{\beta B}{k} \end{aligned}$$

and $\frac{B}{k} > \frac{\beta B}{k}$.

$t = 1$ agent prefer $e_1 = 1$ to $e_1 = 0$ iff

$$\begin{aligned} -c - \beta c + B &\geq 0 \\ c &\leq \frac{B}{1 + \beta} \end{aligned}$$

$\frac{B}{1 + \beta} < \beta B$ if $\frac{1}{1 + \beta} < \beta$.

A. $\frac{1}{1 + \beta} < \beta$

The perception-perfect equilibrium depends on the parameter.

If $\frac{B}{k} < c \leq \frac{B}{1 + \beta}$,

$t = 1$ agent choose $e_1 = 1$, and $e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$.

Then, if $\frac{B}{1 + \beta} < c \leq \beta B$,

$e_1 = 0$ and $e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$.

B. $\frac{1}{1 + \beta} \geq \beta$

Then, for $\frac{B}{k} < c \leq \beta B$,

$t = 1$ agent choose $e_1 = 1$, and $e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$.

iv. $\beta B < c \leq \frac{B}{2}$

$t = 1$ agent predicts $t = 2$ strategy as :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

While the realized strategy is $e_2 = 0$ for any e_1 . Thus, there is no perception-perfect equilibrium.

• $k < \frac{1}{\beta}$

i. $c \leq \frac{\beta B}{k}$

$t = 1$ agent's belief :

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

which realizes in period 2. Then, the perception-perfect equilibrium is :

$$e_1 = 1 \text{ and } e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \cdot \text{if } k \geq \frac{1+\beta}{\beta} \\ 0 & \text{if } e_1 = 2 \end{cases}$$

$$e_1 = 0 \text{ and } e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \cdot \text{if } k < \frac{1+\beta}{\beta} \\ 0 & \text{if } e_1 = 2 \end{cases}$$

ii. $\frac{\beta B}{k} < c \leq \beta B$

$t = 1$ agent's belief is

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

while $t = 2$ agent's strategy is

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

Thus, there is no perception-perfect equilibrium.

iii. $\beta B < c \leq \frac{\beta B}{k}$

$t = 1$ agent predicts $t = 2$ agent's strategy:

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

$t = 2$ agent's strategy is, however,

$$e_2 = 0 \text{ for any } e_1$$

Again, no perception-perfect equilibrium occurs.

iv. $\frac{\beta B}{k} < c \leq \frac{B}{2}$

$t = 1$ agent's belief :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

while $e_2 = 0$ for any e_1 in period 2, which indicates there is no perception-perfect equilibrium.

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- (d) $t = 2$ agent decides her/his behavior with $\beta < 1$, while $t = 1$ agent predicts it with $\hat{\beta} \in (\beta, 1)$. Then, the agent's prediction and the actual strategy depending on the parameters are summerized as follows :

Parametrical condition			
Belief	Actual choice	e_1	$BR(e_1)$
$c \leq \frac{\hat{\beta}B}{k}$	$c \leq \frac{\beta B}{k}$	0	2
		1	1
		2	0
$\frac{\hat{\beta}B}{k} < c \leq \hat{\beta}B$	$\frac{\beta B}{k} < c \leq \beta B$	1	1
		0,2	0
$\hat{\beta}B < c$	$\beta B < c$	any	0

If $k \geq \frac{\hat{\beta}}{\beta}$, then $\beta B \geq \frac{\hat{\beta}B}{k}$; otherwise, $\beta B < \frac{\hat{\beta}B}{k}$.

- $k \geq \frac{\hat{\beta}}{\beta}$
 - i. $c \leq \frac{\beta B}{k}$
 $t = 1$ agent's belief is

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

and by $c \leq \frac{\beta B}{k}$, her/his belief is true.

$t = 1$ agent choose $e_1 = 1$ iff $k \geq \frac{1 + \hat{\beta}}{\hat{\beta}}$, and otherwise $e_1 = 0$, both of them being the perception-perfect equilibrium in each case.

- ii. $\frac{\beta B}{k} < c \leq \frac{\hat{\beta} B}{k}$

By $c < \frac{B}{k}$, $t = 1$ agent predicts that $t = 2$ agent choose the following strategy :

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

The realized strategy of $t = 2$ agent, however, is below :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

Thus, there does not exist any perception-perfect equilibrium.

- iii. $\frac{\hat{\beta} B}{k} < c \leq \beta B$

$t = 1$ agent predicts $t = 2$ agent's strategy as follows, and it will be realized.

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

$(e_1 = 0) \succsim (e_1 = 2)$, since

$$\begin{aligned} 0 &\geq -kc + \beta B \\ c &\geq \frac{\beta B}{k} \end{aligned}$$

and $\frac{B}{k} > \frac{\beta B}{k}$.

$t = 1$ agent prefer $e_1 = 1$ to $e_1 = 0$ iff

$$\begin{aligned} -c - \beta c + B &\geq 0 \\ c &\leq \frac{B}{1 + \beta} \end{aligned}$$

Note that $\frac{\beta B}{k} < \frac{\hat{\beta} B}{k}$.
 $\frac{B}{1 + \hat{\beta}} < \beta B$ if $\frac{1}{1 + \hat{\beta}} < \beta$.

A. $\frac{1}{1 + \hat{\beta}} < \beta$

The perception-perfect equilibrium depends on the parameter.

If $\frac{B}{k} < c \leq \frac{B}{1 + \hat{\beta}}$,

$t = 1$ agent choose $e_1 = 1$, and $e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$.

Then, if $\frac{B}{1 + \hat{\beta}} < c \leq \beta B$,

$e_1 = 0$ and $e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$.

B. $\frac{1}{1 + \hat{\beta}} \geq \beta$

Then, for $\frac{\hat{\beta} B}{k} < c \leq \beta B$,

$t = 1$ agent choose $e_1 = 1$, and $e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$.

iv. $\beta B < c \leq \hat{\beta} B$

$t = 1$ agent predicts $t = 2$ strategy as :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

While the realized strategy is $e_2 = 0$ for any e_1 . Thus, there is no perception-perfect equilibrium.

v. $\hat{\beta} B < c$

$t = 1$ agent's belief is $e_2 = 0$ for any e_1 , which is true. Therefore, there exists perception-perfect equilibrium s.t.

$$e_1 = 0, \text{ and } e_2 = 0 \text{ for all } e_1$$

- $k < \frac{\hat{\beta}}{\beta}$

- i. $c \leq \frac{\beta B}{k}$

Conditions are exactly same as i. in $k \geq \frac{\hat{\beta}}{\beta}$, so same equilibrium is derived.

- ii. $\frac{\beta B}{k} < c \leq \beta B$

$t = 1$ agent's belief is

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

while $t = 2$ agent's strategy is

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

Thus, there is no perception-perfect equilibrium.

- iii. $\beta B < c \leq \frac{\hat{\beta} B}{k}$

$t = 1$ agent predicts $t = 2$ agent's strategy:

$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

$t = 2$ agent's strategy is, however,

$$e_2 = 0 \text{ for any } e_1$$

Again, no perception-perfect equilibrium occurs.

- iv. $\frac{\hat{\beta} B}{k} < c \leq \hat{\beta} B$

$t = 1$ agent's belief :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

while $e_2 = 0$ for any e_1 in period 2, which indicates there is no perception-perfect equilibrium.

- v. $\hat{\beta} B < c$,

which is same as v. in $k \geq \frac{\hat{\beta}}{\beta}$, so again same equilibrium strategy derived.

(e) • Period 2

When $e_1 = 0$, then $t = 2$ agent has no incentive to take any choice other than $e_2 = 0$, since $B(0, e_2) = 0$ for any e_2 .

When $e_1 = 1$, $t = 2$ agent prefer $e_2 = 1$ to $e_2 = 0$ iff

$$\begin{aligned} -c + \beta B &\geq 0 \\ c &\leq \beta B \end{aligned}$$

otherwise, $e_2 = 0$ for any e_1

• Period 1

$t = 1$ agent predicts her/his behavior in period 2 with $\hat{\beta} \in (\beta, 1)$.

Then, the agent's prediction and the actual strategy depending on the parameters are summerized as follows :

Parametrical condition			
Belief	Actual choice	e_1	BR(e_1)
$c \leq \hat{\beta}B$	$c \leq \beta B$	1	1
		0	0
$\hat{\beta}B < c$	$\beta B < c$	any	0,2

i. $c \leq \beta B$

$t = 1$ agent predicts her/his strategy as follows :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

In the same way as (b), $e_1 = 2$ is dominated by $e_1 = 0$.

$e_1 = 1$ is chosen iff $k \geq \frac{1+\hat{\beta}}{\hat{\beta}}$ and then it will be the perceprion-perfect equilibrium.

Otherwise, $k < \frac{1+\hat{\beta}}{\hat{\beta}}$, $e_1 = 0$ and $e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$

is the perception-perfect equilibrium.

ii. $\beta B < c \leq \hat{\beta}B$

$t = 1$ agent's belief is :

$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

while, $t = 2$ agent's strategy is $e_2 = 0$ for any e_1 . Therefore, there exists no perceprion-perfect equilibrium.

iii. $\hat{\beta}B < c < \frac{B}{2}$

$t = 1$ agent predicts $e_2 = 0$ for any e_1 , which turn to be true.

Thus, the perception-perfect equilibrium is

$$e_1 = 0, \text{ and } e_2 = 0 \text{ for any } e_1$$