Labor Economics Term Paper

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Q1
(1) The value functions of the worker are:

$$\begin{cases} rW = w + \lambda(\bar{u} + f_e - W) & \text{for employment} \\ r\bar{u} = z + \theta q(\theta)(W - \bar{u}) & \text{for unemployment} \end{cases}$$

W and $\bar{u}$ are the values of employment and unemployment. w and z are the instantaneous utility of employment and unemployment, respectively. r is the discount factor, and $q(\theta)$ is the matching function of $\theta = \frac{v}{u}$. v and u is the vacancy rate and the unemployment rate of the workers, satisfying $q'(\theta) < 0$. When a worker was dismissed, they receive f_e as the severance payment.

(2) The value functions of the firm are:

$$\begin{cases} rJ = (p - w) + \lambda(V - f_e - f_a J) & \text{for having a vacancy filled} \\ rV = -c + q(\theta)(J - W) & \text{for vacancy} \end{cases}$$

J and V are the values of vacancy filled and vacancy. p is the the instantaneous productivity of the worker and w is the instantaneous wage. c is the expected hiring cost. When a firm fire a worker, then it have to pay the severance payment f_e to the worker, and f_a as the related cost for firing.

(3) By the free-entry/exit condition, $V = 0$ holds. Then, the value function of vacancy is rewritten as follows:

$$J = \frac{c}{q(\theta)}$$

Then, substituting this into the value function of having vacancy filled,

$$
\begin{aligned}
rJ &= p - w + \lambda(V - f_e - f_a - J) \\
p - w &= (r+\lambda)\left(\frac{c}{q(\theta)}\right) - \lambda(f_e + f_a) \\
p - w &= \frac{(r+\lambda)c - \lambda(f_e + f_a)}{q(\theta)},
\end{aligned}
$$

which corresponds to the job creation condition.

(4) First, we derive the equilibrium conditions: wage condition and steady state condition. First, we derive the wage curve.
Given the value functions in (1) and (2), the wage is determined by Nash bargaining rule:

$$w = \arg\max_{w} (W - \bar{u})^{\beta}(J - V)^{1-\beta},$$

note that β stands for the bargaining power of the worker.

Q2
(1)"Hazard Function" is an instantaneous rate of an individual moving from a certain state to another. In the settings of job searching, it corresponds to that of leaving from unemployment pool.
Let $T \geq 0$ denote the duration and the cumulative distribution function $F(.)$ is defined as follows:

$$F(t) = \Pr(T \leq t).$$

Then, the hazard function $\lambda(.)$ is:

$$\lambda(t) \equiv \lim_{h \to 0} \frac{\Pr(t \leq T \leq t + h | T \geq t)}{h},$$

where $\Pr(t \leq T \leq t + h | T \geq t)$ is given as:

$$\Pr(t \leq T \leq t + h | T \geq t) = \frac{F(t+h) - F(t)}{1 - F(t)}.$$

Then, we can rewrite the hazard function by $F(t)$ and $f(t) = F'(t)$,

$$\lambda(t) = \frac{f(t)}{1 - F(t)}.$$

Now suppose the duration is distribution according to the Weibull distribution. Then, the cumulative distribution $F(t)$ is given as follows.

$$F(t) = 1 - \exp(-\gamma t^{\alpha})$$

γ and α are parameters. Then the density function is obtained as

$$
\begin{aligned}
f(t) = F'(t) &= \frac{d(1-\exp(-\gamma t^{\alpha}))}{dt} \\
&= \frac{d(1-\exp(-\gamma t^{\alpha}))}{d(-\gamma t^{\alpha})} \cdot \frac{d(-\gamma t^{\alpha})}{dt} \\
&= \gamma \alpha t^{\alpha-1}
\end{aligned}
$$

Substituting this into $\gamma(t)$, we obtain the hazard function.

$$\lambda(t) = \gamma \alpha t^{\alpha-1}$$

Positive/Negative dependence of the hazard function on the duration is found by checking the first-order differential of the hazard function: if $\lambda'(.) > 0$, then it has positive dependence, and negative if $\lambda'(.) < 0$. When the duration has Weibull distribution, by $\lambda'(t) = (\alpha - 1)\gamma\alpha t^{\alpha-2}$. Since $t > 0$ and $\gamma \geq 0$ is assumed, it depends on the value of α. If $\alpha > 1$, then the hazard ratio has the positive dependence on the duration, and $\alpha < 1$ implies negative dependence.

(2) Suppose observation of the worker who started job-searching after time 0. We observe unemployed workers who started job-searching until time b, and we stop observation at the cetain time: after then, we do not observe the worker, even though they have not finished searching. We define the time a worker i enters the initial state (start searching) a_i, and time left for her/him until we stop the observation c_i.
Then, we cannot observe the true duration t_i if the worker have not finished searching at the censoring time. "Right-censoring" stands for possible bias caused by this problem.
On the other hand, when we start observation of the workers who are searching at the time b: If s/he have finished searching at time b, then we cannot observe her/him, even if s/he started job searching after time 0. Thus, obtained sample can be selected, that is, "left-censoring" problem occurs.

(3) Denote the "observed" duration to be t^*. Assume t^* is distributed independent of a_i and c_i, then the conditional cumulative distribution function $F(.)$ on the vector of observed covariates X_i is defined as:

$$F(t^*|X_i, a_i, c_i|\theta) = F(t_i|X_i;\theta).$$

θ is the vector of parameters.
If there is no censoring problem discussed in (2), then the conditional density function of duration is $f(t_i^*|X_i;\theta)$. When an observation is completed, we can obtain the likelihood function of this.
Then, we argue the observation with incomplete duration. First, we consider the right-censoring problem. The probability of right-censored observation is given by $1 - F(c_i|X_i;\theta)$. Then, the conditional likelihood function is obtained as follows:

$$f(t_i^*|X_i;\theta)^{d_i}[1 - F(c_i|X_i;\theta)]^{1-d_i}$$

Second, we consider the left-censoring. All the observed samples satisfy that the individual is still unemployed at time b. That is, $t_i^* \geq b_i - a_i$. The probability of this is expressed by $\Pr(t^* \geq b - a_i | X_i, a_i, c_i) = 1 - F(b - a_i | X_i)$.
By the Bayes Rule, the likelihood function including right and left-censoring is:

$$L(\theta) = \prod_i^N \frac{f(t_i^* | X_i; \theta)^{d_i} [1 - F(c_i | X_i, \theta)]^{1-d_i}}{1 - F(b - a_i | X_i; \theta)}$$

Maximizing this (or log-likelihood function) with θ, we obtain MLE: the consistent asymptotical normal estimated parameter $\hat{\theta}$.

(4) I review the following paper.
Holmås. 2002. Keeping nurses at work: a duration analysis *Health Economics*. 11: 493 – 503 (2002)

This paper estimated the hazard function of nurses: instantaneous rate of leaving public health sector, using duration analysis. The contribution of this is a better understanding of nurses' labor supply and to answer a policy relevant question to increase it.

Their dataset consists of two administrative data in Norway. Information about wages, working time or specific occupation is obtained from Association of Local and Regional Authorities (NALRA). They restricted their sample to female workers worked for 34 public hospitals, which report information to the NALRA register. The dataset also includes the hospital-specific information, such as the number of beds or the average length of stay of the patients. Importantly, they restrict the samples to the hospitals that provide information including the shift work: duties at late, night and weekend. In addition, the obtained dataset includes the samples both right-censored and left censored, because they cannnot know the actual start time of working, and the maximum tracking period is five years (1993 to 1997). The final sample size is 5284 nurses. Other individual-specific information, such as children and spouse characteristics, and the location of the hospitals they work for, is obtained from Statistics Norway. They merged these two datasets. Hourly wages are calculated using basic income and all bonuses.

Estimation used a semi-parametric procedure. Calender time is devided into three-months period, and set the period for nurse i to start working is $t = 1$. Defining $\tau_1 \geq 1$ to be the period they start observation(January 1993), then for period s_i, the conditional probability of observing a nurse who had quit until 1997 is:

$$\Pr(T_i = \tau_i + s_i | T_i > \tau_i - 1) = \left(\frac{h_{i,\tau_i + s_i}}{1 - h_{i,\tau_i + s_i}} \right) \prod_{t=\tau_i}^{\tau_i + s_i} (1 - h_{i,t})$$

$h_{i,t}$ is the hazard function. It considers the left-censoring. Then, the conditional probability of observing those who did not quit until 1997 (=right-censored sample) is:

$$\Pr(T_i > \tau_i + s_i | T_i > \tau_i - 1) = \prod_{t=\tau_i}^{\tau_i + s_i}$$

Therefore, the log-likelihood function is given as follows:

$$\log L = \sum_{i=1}^{n} \sum_{t=\tau}^{\tau_i+s_i} y_{i,t} \log\left(\frac{h_{i,t}}{1-h_{i,t}}\right) + \sum_{i=1}^{n} \sum_{t=\tau_i}^{\tau_i+s_i} \log(1-h_i)$$

$y_{i,t}$ indicates whether nurse i quit or not during the observation.
They specify the hazard function $h_{i,t}$ by the complementary log-log model.

$$h_{i,t} = 1 - \exp\{-\exp[\theta(t) + \beta' \mathbf{X}_{i,t}]\}$$

θ is the baseline hazard, or the dummy variable of each time-period. $\mathbf{X}_{i,t}$ is the observed characteristics of the individual. Additionally, they used alternative model that considers the unobserved heterogeneity by adding the random variable term ϵ_i, which follows the gamma distribution. They also tried Heckman and Singer(1984)'s specification about the distribution of the heterogeneity.

Estimation results support the effect of wage to keep them in the labor supply, which is controversial to the previous studies. They argue that this is because the previous researches did not take shift work into consideration. The more the rate of wage from shift work increases, the more the hazard rate goes up. Years of experience, on the other hand, show negative effect to the hazard rate: consistent with the previous ones. Part-time workers are likely to quit compared to the full-time workers. The hospital-specific characteristics are also important: high occupancy rate or the number of beds increase the exit rate, while the average length of stay of the patients are negatively affect the exit rate.
Moreover, family related variables influence on the hazard rate. Nurses with children older than seven years and those married show lower exit rate. They interpreted these are because they are allowed more flexible labor supply.

Q3
(1)Value functions are:

$$\begin{aligned} rV_e(w) &= w + q[V_u - V_e(w)] \\ rV_u &= \max\{b - c(e) + \lambda(e) \int_x^\infty [V_e - V_u] dF(w)\} \end{aligned}$$

By the reservation wage property, $V_e(x) = V_u$, when $w = x$, the first equation is:

$$\begin{aligned} rV_e(x) &= x + q[V_u - V_e(x)] \\ x &= rV_u \\ x &= \max\{b - c(e) + \lambda(e) \int_x^\infty [V_e - V_u] dF(w)\} \end{aligned}$$

Moreover, the first equation can be rewritten as follows.

$$V_e(w) = \frac{w + qV_u}{r + q}$$

Then,

$$\begin{aligned} V_e(w) - V_e(x) &= \frac{w - x}{r + q} \\ &= V_e(w) - V_u \quad (\because V_e(x) = V_u) \end{aligned}$$

Therefore, we obtain

$$x = \max_e \left\{ b - c(e) + \lambda(e) \int_x^\infty \left(\frac{w - x}{r + q} \right) dF(w) \right\}$$

FOC yields the optimal effort level of e, or e^* such that

$$-c'(e^*) + \frac{\lambda'(e^*)}{r + q} \int_x^\infty (w - x) dF(w) = 0$$

Then, we can obtain the reservation wage:

$$x = b - c(e^*) + \lambda(e^*) \int_x^\infty (w - x) dF(w)$$

(2)

(3)

(4)