Labor Economics

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9.1 Principal-Agent Model with Moral Hazard

pp115-118
: ex post asymmetric information
$\rightarrow$ workers choose her/his effort level after the contract.

1. Agent

Assume negatibe exponential utility:

$$U\left(W-\frac{ce^2}{2}\right)=-\exp\left\{-a\left(W-\frac{ce^2}{2}\right)\right\}$$

where

U :CARA function

a : coefficient of absolute risk-aversion

W :wage schedule

c :effort cost constant

W is defined by the fixed wage w, performance wage b and productivity y as follows:

$$W=w+by$$

where

$$y=e+\epsilon,\epsilon\sim N(0,\sigma^2)$$

The second equation implies that principal cannot monitor the effort level of the agent e. Substituting this into the first one, $W=w+b(e+\epsilon)$.

The object function of maximization problem of the agent is:

$$\begin{aligned}\max_e EU &= E\left\{-\exp\left[-a\left(w+b(e+\epsilon)-\frac{ce^2}{2}\right)\right]\right\}\\ &= E\left\{-\exp\left[-a\left(w+be-\frac{ce^2}{2}\right)-ab\epsilon\right]\right\}\\ &= E\left\{-\exp\left[-a\left(w+be-\frac{ce^2}{2}\right)\right]\exp(-ab\epsilon)\right\}\\ &= -\exp\left[-a\left(w+be-\frac{ce^2}{2}\right)\right]E[\exp(-ab\epsilon)]\end{aligned}$$

Since $\exp(-ab\epsilon)$ follows log-normal distribution, the mean value is $\exp\left(-\frac{a^2b^2\sigma^2}{2}\right)$.

Thus, the maximization problem is rewritten as follows:

$$\max_e -\exp\left[-a\left(w+be-\frac{ce^2}{2}-\frac{ab^2\sigma^2}{2}\right)\right]$$

$$\Longrightarrow e=\frac{b}{c}$$

If b goes up, then the effort level goes down. When c increases, e decreases.

2. Principal

   The xProfit of the firm:

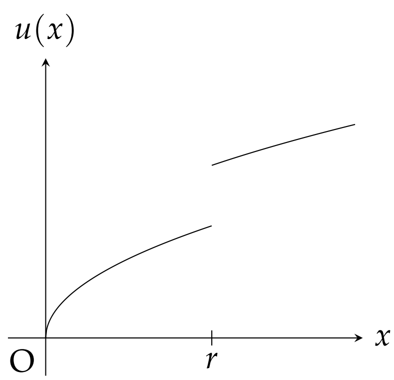
$$\begin{aligned} E(y - W) &= E[y - (w + by)] \\ &= E[(1-b)(e+\epsilon) - w] \\ &= (1-b)e - w \qquad (\because E(\epsilon) = 0) \end{aligned}$$

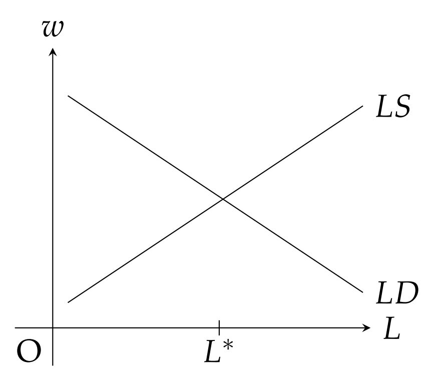
9.2 Double Moral Hazard

pp119-124

9.3 Shirking Model

pp125-130
$\rightarrow$ deferred payment (Lazear, 1979, 1981) $\rightarrow$ efficiency wage model (Shapiro & Stiglitz, 1984, AER)





w^* denotes the market clearing wage. When $w = w_E$, then, there exists involuntary worker.

9.3.1 Value Function

Agents(workers)' value function is:

- No shirking

$$V_t = w_t - c_t + \delta[(1-q)\max(V_{t+1}, V_{t+1}^s) + q\bar{V}_t]$$

where

$$\begin{aligned} V_t &: \text{ the value of working hard at time } t \\ w_t &: \text{ wage} \\ c_t &: \text{ effort cost} \\ \delta &: \text{ discont factor} \\ q &: \text{ exogeneous probability separation} \\ \bar{V}_t &: \text{ value of outside opportunity} \end{aligned}$$

- Shirking

$$V_t^s = w_t + (1-p)\{\delta'(1-q)\max(V_{t+1}, V_{t+1}^s) + q\bar{V}_t\} + P\delta\bar{V}_t$$

where

p : probability of being detected and fired

1. Incentive Compatible

   To prevent workers from shirking, the following condition must hold:

   $$V_t \geq V_t^s \text{ for } \forall t \geq 0$$

   The inequation is rewritten as:

   $$\begin{aligned} V_t - V_t^s = -c + p\delta[(1-q)\max(V_{t+1}, V_{t+1}^s) + q\bar{V}_{t+1}] - p\delta\bar{V}_{t+1} &\geq 0 \\ p\delta(1-q)(V_{t+1} - \bar{V}_{t+1}) &\geq c \\ \Longrightarrow V_{t+1} - \bar{V}_{t+1} \geq \frac{c}{p\delta(1-q)}, &\forall t \geq 0 \end{aligned}$$

   - Incentive mechanism is forward looking.
   - To give the worker an incentive to work hard today (t), the worker expects the positive rent tomorrow ($t+1$).
   - w_t does not affect effort level at t ($\because$ w_t is canceled out by $V_t - V_t^s$). The effort level at t comes from the prospect of the gain at $t+1$.
   - The wage is NOT important for the incentive, but for the contract.

2. Participation Constraint

   $$V_t \geq \bar{V}_t$$
   $$endalign{*} \forall t \geq 0$$

By the incentive compatible, $V_k \geq \bar{V}_k \forall k > 0$ is satisfied($\because \frac{c}{p\delta(1-q)} > 0$ by definition)

Thus, Participation only requires $V_0 \geq \bar{V}_0$

In sum, the set of feasible contract is:

$$\mathbb{P} = \left\{ \Pi_t, V_t | \pi_t \geq \bar{\pi}_t, V_{t+1} - \bar{V}_{t+1} \geq \frac{c}{p\delta(1-q)}, V_0 \geq \bar{V}_0, \forall \geq 0 \right\}$$
$$\Pi_t = y_t - w_t + \delta[(1-q)\pi_{t+1} + q\bar{\Pi}_{t+1}]$$

Π_t denotes the value of operation at period t.
Now we define the total surplus.

$$S_t \equiv (V_y - \bar{V}_t) + (\Pi_t - \bar{\Pi}_t)$$

Firms make a contract iff

$$\Pi_t - \bar{\Pi}_t = S_t - (V_t - \bar{V}_t) \geq 0$$

the set of feasible contract can be rewritten as follows:

$$\mathbb{P} = \left\{ \pi_t, V_t | S_{t+1} \geq V_{t+1} - \bar{V}_{t+1} \geq \frac{c}{p\delta(1-q)}, s_0 \geq V_0 - \bar{V}_0 \geq 0, \forall t \geq 0 \right\}$$
$$\Longrightarrow \begin{cases} S_0 \geq 0, \\ S_{t+1} \geq \dfrac{c}{p\delta(1-q)} \quad \forall t \geq 0 \end{cases} \quad : \text{ Implicit self-enforcing contract.}$$

To maximize Π_t: minimize V_t subject to the implicit self-enforcing contract.

$$V_0 - \bar{V}_0 = 0 \Rightarrow \text{then, } \Pi_0 - \bar{\Pi}_0 = S_0$$
$$V_{t+1} - \bar{V}_{t+1} = \frac{c}{p\delta(1-q)} \Rightarrow \text{ then, } \Pi_{t+1} - \bar{\Pi}_{t+1} = S_{t+1} - \frac{c}{p\delta(1-q)}$$

(note that $s_{t+1} \equiv (\Pi_{t+1} - \Pi_{t+1}) + V_{t+1} - \bar{V}_{t+1}))$

$$\begin{aligned}
&\forall t \geq 0, \\
&\quad V_0 = \bar{V}_0 \\
&\quad V_{t+1} = \bar{V}_{t+1} + \frac{c}{p\delta(1-q)} \\
&\quad V_t = w_t - c + \delta[(1-q)\max(V_{t+1}, V^s_{t+1}) + q\bar{V}] \\
&\quad w_0 = \bar{V}_0 - \delta\bar{V}_1 + c\frac{c}{p} \\
&\quad w_{t+1} = \bar{V}_{t+1} - \delta\bar{V}_{t+2} + c + \frac{c}{p}\left[\frac{1}{\delta(1-q)} - 1\right]
\end{aligned}$$

9.3.2 Two Period

$$
\begin{aligned}
w_0 &= \bar{V}_0 - \delta \bar{V}_1 + c\frac{c}{p} \\
w_1 &= \bar{V}_1 - \delta \bar{V}_2 + c + \frac{c}{p}\left[\frac{1}{\delta(1-q)} - 1\right]
\end{aligned}
$$

Assume no capital accumulation and

$$
\begin{aligned}
\bar{V}_t &= \sum_{i=0}^{\infty} (\bar{w}_{t+1}), \forall t \geq 0 \\
\bar{V}_0 - \delta \bar{V}_1 &= \bar{w}_0 - c \\
\bar{V}_1 - \delta \bar{V}_2 &= \bar{w}_1 - c
\end{aligned}
$$

Suppose $\bar{w}_0 = \bar{w}_1 = \bar{w}_2 = \ldots = \bar{w}$,

$$
\begin{aligned}
w_0 &= \bar{w} - \frac{c}{p} \\
w_1 &= \bar{w} + \frac{c}{p}\left[\frac{1}{\delta(1-q) - 1}\right]
\end{aligned}
$$

9.4 Efficiency wage Model

pp130-136
Shapiro and Stiglitz (1984, AER).

- Involuntary unemployment
- wage downward rigidity
- keep the wage high to prevent the employee from shirking.

9.4.1 stationary case

$$
\begin{aligned}
V_0 = V_1 = V_2 = \ldots = V_t = V \\
\bar{V}_0 = \bar{V}_1 = \bar{V}_2 = \ldots = \bar{V}_t = \bar{V}
\end{aligned}
$$

By $w_{t+1} = \bar{V}_{t+1} - \delta \bar{V}_{t+2} + c + \frac{c}{p}\left[\frac{1}{\delta(1-q)} - 1\right]$,

$$
w = (1-\delta)\bar{V} + c + \frac{c}{p}\left[\frac{1}{\delta(1-q) - 1}\right]
$$

By $V - \bar{V} = \frac{c}{p\delta(1-q)} > 0$, $V > \bar{V}$

: involuntary unemployment occurs.
Define $\bar{V} = z + \delta[sV + (1-s)\bar{V}]$, where z and s is linear utility per period: unemployment benefit, and probability of finding a job, respectively.
Then,

$$(1-\delta)\bar{V} = z + \frac{sc}{p(1-q)}$$

$$w = z + c + \frac{c}{p}\left[\frac{1}{1-q}\left(s + \frac{1}{\delta} - 1\right)\right]$$

Under steady state, $s(N-L) = qL$ holds: # of the worker fired and that of those who find a job is equal. Then,

$$s = \frac{qL}{N-L} < 1$$

$$w = z + c + \frac{c}{p}\left[\left(\frac{1}{1-q}\right)\left(\frac{qL}{N-L} + \frac{1}{\delta}\right) - 1\right] \text{ : incentive curve}$$

Firm side:

$$\Pi = y - w + \delta[(1-q)\Pi + q\bar{\Pi}]$$

Free entry condition:$\Pi = \bar{\Pi} \equiv c_k$ Fixed entry cost.

$$w^* = y - (1-\delta)c_k$$

10 Search Model

10.1 Sequential Search Model: Individual's decision making

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10.1.1 Settings

Consider a representative agent (infinite living) to maximize the lifetime utility

$$E\sum_{t=0}^{\infty}\delta^t u_t.$$

Workers realize the time-invariant wage offer distribution (Firm's strategy is given).

$$\text{draw } w \text{ with probability of } \alpha \Rightarrow \begin{cases} \text{accept } (w \geq w_R) & \rightarrow \text{ finish searching} \\ \text{reject } (w < w_R) & \rightarrow \text{ go to the next round} \end{cases}$$

10.1.2 Value Function

1. Employment

$$W(w) = w + \delta[(1-\lambda)W(w) + \lambda\bar{u}]$$

   where *lambda* denotes the prob. of unemployment.
   $\bar{u}$ is the value of unemployment

2. Unemployment

$$\bar{u} = z + \delta\{\alpha E \max[W(w), \bar{u}] + (1-\alpha)\bar{u}\}$$

   z is the value of this period with unemployed.
   α denotes the prob. of get offer.

- Reservation Wage Property

$$W(w_R) = \bar{u}$$

  Thus, the worker accept the offer if

$$W(w) > \bar{u} \Leftrightarrow w > w_R$$

  And reject otherwise.

By (1), when $w = w_R$,

$$\begin{aligned} W(w_R) &= w_R + \delta[(1-\lambda)W(w_R) + \lambda\bar{u}] \\ \bar{u} &= w_R + \delta\bar{u} \qquad (\because W(w_R) = \bar{u}) \\ \Rightarrow W(w_R) &= \bar{u} = \frac{w_R}{1-\delta} \end{aligned}$$

By (2),

$$\bar{u} = z + \alpha\delta\left\{\int_{\underline{w}}^{w_R} \bar{u}dF(w) + \int_{w_R}^{\bar{w}} W(w)dF(w)\right\} + \delta(1-\alpha)\bar{u}$$

$(\because E\max[W(w), \bar{u}]$ is the expected value of the maximized utility$)$

$$\begin{aligned} \Longrightarrow (1-\delta)\bar{u} &= z + \alpha\delta\int_{w_R}^{\bar{w}} [W(w) - \bar{u}]dF(w) \\ &\left(\because \int_{\underline{w}}^{w_R} \bar{u}dF(w) = \bar{u}\left[1 - \int_{w_R}^{\bar{w}} dF(w)\right]\right) \end{aligned}$$

Note that $(1-\delta)\bar{u} = w_R$, and

$$\begin{aligned} W(w) - \bar{u} &= \frac{w + \delta\lambda\bar{u}}{1-\delta(1-\lambda)} - \bar{u} \\ &= \frac{w - (1-\delta)\bar{u}}{1-\delta(1-\lambda)} \\ &= \frac{w - w_R}{1-\delta(1-\lambda)}, \end{aligned}$$

we obtain

$$w_R - z = \frac{\alpha\delta}{1-\delta(1-\lambda)} \int_{w_R}^{\bar{w}} (w - w_R) dF(w)$$

The left-hand side indicates the marginal cost of additional search, while the right-hand side denotes the expected capital gain: expected benefit of additional search.

10.1.3 Competitive Static Analysis

$$\frac{dw_R}{dz} > 0 \qquad \frac{dw_R}{d\alpha} > 0$$
$$\frac{dw_R}{d\delta} > 0 \qquad \frac{dw_R}{d\lambda} > 0$$

10.1.4 Expected Duration of Search

Define the Hazard rate $\tau \equiv \alpha[1 - F(w_R)]$
The probability of finishing search at period t is $(1-\tau)^{t-1}\tau$
Then, the expected time to get a job is:

$$\begin{aligned} T &= \sum_{t=1} \infty t \times (1-t)^{t-1} \times \tau \\ &= \frac{1}{\tau} && T \text{ follows the negative binominal distribution.} \end{aligned}$$

By the static analysis, When z goes up, then w_R goes up. $F(w_R)$ is increasing in w_R, so by definition, τ decreases. As a result, T: the expected duration gets longer.

- Unemployment Insurance

  If w_R goes up, then the accepted wage also increases and it varies over serching time. With the existance of unemployment insurance, even when the worker gets a new job early, s/he tries to put off the start time, in order to receive the full payment (90 days).

10.1.5 Continuous-Time Model

We begin with an environment whose multiple jobs arrive at unemployed with probabilty of $a(n, \Delta t)$, for Δ.

$$a(n, \Delta t) = \frac{e^{-\alpha \Delta t}(\alpha \Delta t)^n}{n!} \text{ : Poisson Procedure}$$

- Assumption

  $\Delta t' \perp$ the arrival probability of the nth job offer.

Then, we can say

$$a(n, \Delta t) = \begin{cases} \alpha \Delta t + O(\Delta t) & \text{if } n = 1 \\ O(\Delta t) & \text{if } n \geq 2 \end{cases}$$

$$\text{where } \left(\lim_{\Delta t \to \infty} \frac{O(\Delta t)}{\Delta t} = 0 \right), O(\Delta t) + O(\Delta t) = O(\Delta t)$$

$O(.)$ denotes a tiny change.

- Value Function

  $$\bar{u} = \left(\frac{1}{1 + r\Delta t} \right) \left\{ z\Delta t + \sum_{n=1}^{\infty} a(n, \Delta t) E \max[W(w), \bar{u}] + a(0, \Delta t) \right\}$$

  RHS is rewritten as follows.

  $$\left(\frac{r\Delta t}{1 + r\Delta t} \right) \bar{u} = \left(\frac{1}{1 + r\Delta t} \right) \left\{ z\Delta t + \sum_{n=1}^{\infty} a(n, \Delta t) \times [E \max[W(w), \bar{u}] - \bar{u}] \right\}$$

  1. Multiply the both-hand side by $(1 + r\Delta t)$
  2. Devide by Δt
  3. $\Delta t \to 0$

  yields

  $$r\bar{u} = z + \alpha \left\{ E \max[W(w), \bar{u}] - \bar{u} \right\}$$

  or

  $$r\bar{u} = z + \alpha \int_{w_R}^{\bar{w}} [W(w) - \bar{u}] dF(w)$$

$$
\begin{aligned}
W(w) &= \left(\frac{1}{1+r\Delta t}\right)\{w\Delta t + \lambda\Delta t\bar{u} + (1-\lambda\Delta t)W(w)\} \\
rW(w) &= w + \lambda[\bar{u} - W(w)]
\end{aligned}
$$

$$
\begin{aligned}
W(w) &= \frac{w+\lambda\bar{u}}{r+\lambda} \\
W(w_R) &= \bar{u}
\end{aligned}
$$

$$
\begin{aligned}
W(w) - \bar{u} &= \frac{w - r\bar{u}}{r+\lambda} \\
&= \frac{w - w_R}{r+\lambda}
\end{aligned}
$$

$$
w_R = z + \frac{\alpha\delta}{1-\delta(1-\lambda)}\int_{w_R}^{\bar{w}}(w-w_R)dF(w)
$$

(cf. instantaneous reservation wage)

$$
w_R = z + \frac{\alpha\delta}{1-\delta(1-\lambda)}\int_{w_R}^{\bar{w}}(w-w_R)dF(w)
$$

10.2 Search Effort

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1. reseavation wage: constant (endogeneous)
2. search effort exists

10.2.1 Value Functions

- No search

$$rW(w) = w + \lambda[\bar{u} - W(w)]$$

where

$r:$ discount factor

$w:$ linear instantaneous utility

$\lambda:$ probability of separation

$\bar{u} - W(w)$ stands for the capital loss.

Note that there is no search cost.

- Search

$$r\bar{u} = \max_e \left\{ z - c(e) + \alpha(e) \int_{w_R}^{\bar{w}} [W(w) - \bar{u}] dF(w) \right\}$$

where

$z:$ instantaneous utility

$c(e)$: search cost, note that $c'(.) > 0, c''(.) > 0, c(0) = 0.$

$\alpha(e)$: job arrival rate, note that $\alpha'(.) > 0, \alpha''(.) < 0, c(0) = 0$

FOC yields

$$(r + \lambda)c'(e) = \alpha'(e) \int_{w_R}^{\bar{w}} [W(w) - \bar{u}] dF(w)$$

Then, we obtain:

$$\begin{cases} w_R = z - c(e^*) + \dfrac{\alpha(e^*)}{r + \lambda} \int_{w_R}^{\bar{w}} (w - w_R) dF(w) \\ (r + \lambda)c'(e^*) = \alpha(e^*) \int_{w_R}^{\bar{w}} (w - w_R) dF(w) \end{cases}$$

10.2.2 Competitive Static Analysis

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \frac{dw_R}{dz} \\ \frac{de^*}{dz} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$\frac{dw_R}{dz} > 0, \frac{de^*}{dz} < 0$$

10.2.3 On-the-job Search

Both unemployed and employed workers search, with the different rate of arriving rate: α_0 for unemployed and α_1 for employed.

- If there is severe time constraint for the employed, then $\alpha_0 > \alpha_1$.
- If the employed worker have some connection about searching, then $\alpha_0 < \alpha_1$

The value function is:

$$
\begin{aligned}
r\bar{u} &= z + \alpha_0 \int_{w_R}^{\bar{w}} [W(w) - \bar{u}] dF(w) \\
r\bar{u} &= w + \alpha_1 \int_{w_R}^{\bar{w}} [W(w) - \bar{u}] dF(w) + \lambda[\bar{u} - W(w)]
\end{aligned}
$$

By the reservation wage property,

$$
\begin{aligned}
r\bar{u} = rW(w_R) = w_R + \alpha_1 \int_{w_R}^{\bar{w}} [W(w') - \bar{u}] dF(w') \\
w_R + \alpha_1 \int_{w_R}^{\bar{w}} [W(w') - \bar{u}] dF(w') = z + \alpha_0 \int_{w_R}^{\bar{w}} [W(w) - \bar{u}] dF(w)
\end{aligned}
$$

assume w and w' follow the same distribution. Then,

$$
w_R = z + (\alpha_0 - \alpha_1) \int_{w_R}^{\bar{w}} [W(w) - \bar{u}] dF(w)
$$

10.2.4 Implication

1. Relationship of α_i and z

$$
\begin{aligned}
\alpha_0 > \alpha_1 \Longrightarrow w_R > z \\
\alpha_0 < \alpha_1 \Longrightarrow w_R < z
\end{aligned}
$$

2. z and w_R

$$
z \text{ increases } \Longrightarrow w_R \text{ increases } \Longrightarrow \text{ average } w \text{ goes up}
$$

   Then, the probability that a higher wage offer than the current wage w goes down. As a result, turnover of the employed workers gets less likely to occur.

3. positive assotiation between the wage level and the firm size:

   High wage firms are more attractive to the workers, so those in lower wage firms move to them. Then, the size of the firm with high wage gets larger.

10.3 Duration Analysis

pp146 - 152
Let $T(\geq 0)$ denote the search duration. The cumulative distribution function of the duration is:

$$F(t) = \Pr(T \leq t)$$

Then, the survivor function: the probability of surviving after time t is

$$s(t) \equiv 1 - F(t)$$

10.3.1 Hazard function

Instantaneous rate of leaving from unemployment pool.

$$\lambda(t) = \lim_{h \to 0} \frac{\Pr(t \leq T \leq t + h | T \geq t)}{h}$$

(If $\lambda'(t) < 0$, the harzard rate has negative dependence on search time, otherwise if $\lambda'(t) > 0$.)

$$\begin{aligned}
&\Pr(t \leq T \leq t + h | T \geq t) \\
&= \frac{\Pr(t \leq T \leq t + h)}{\Pr(T \geq t)} \\
&= \frac{F(t + h) - F(t)}{1 - F(t)}
\end{aligned}$$

Substituting this into λ,

$$\begin{aligned}
\lambda(t) &= \lim_{h \to 0} \frac{F(t + h) - F(t)}{h} \times \frac{1}{1 - F(t)} \\
&= \frac{f(t)}{1 - F(t)} \\
&= \frac{f(t)}{s(t)} \\
&\text{or} \\
\lambda(t) &= -\frac{d \ln s(t)}{dt} \\
&\left(\because \frac{d \ln s(t)}{ds(t)} \times \frac{ds(t)}{dt} = \frac{-f(t)}{s(t)} \text{ by definition} \right)
\end{aligned}$$

$\lambda'(t) > 0$: positive duration dependence (Weibull function)

Table 1: Two Kinds of Hazard Functions

	$f(t)$	$F(t)$	$\lambda(t)$	
exponential	$\lambda \exp(-\lambda t)$	$1-\exp(-\lambda t)$	λ	constant hazard rate
Weibull	$\gamma \alpha t^{\alpha-1} \exp(-\gamma t^{\alpha})$	$1-\exp(-\gamma t^{\alpha})$	$\gamma \alpha t^{\alpha-1}$	positive duration dependence

- Parametric Specification

  Assume the searching duration follows the Weibull distribution.

$$\lambda(t : X) = \exp(X\beta)\alpha t^{\alpha-1}$$

X denotes the time-invariant covariate

$$\ln \lambda(t : X) = X\beta + \ln(\alpha t^{\alpha-1})$$

- Semi-Parametric estimation

  (Cox's proportional hazard): do not specify the distribution.

$$\lambda(t : X) = \kappa(x)\lambda_0(t)$$

$kappa(.)$ is the non-negative function of X: $\exp(X\beta)$.

λ_0 stands for the baseline function > 0

10.3.2 Data

1. Right-censoring

   An individual enters the initial state: visit the job-search office during the interval $a_i \in [0, b]$

   Then, those who does not finish her/his search until the end of the investigation cannot be observed: the obtained data is truncated.

   - MLE

$$\begin{aligned} a_i &\in [0, b] \\ t_i^* &: \text{duration of search} \\ X_i &: \text{vector of observed covariates} \\ c_i &: \text{censoring time: time left for observation for the individual } i \end{aligned}$$

Assume
$$F(t^*|X_i, a_i, c_i) = F(t|X_i)$$
the duration is not affected by the time starting search.

- When the observation is complete one, then $f(t_i^*|X_i : \theta)$
- If incomplete, $1 - F(c_i|X_i, \theta)$

The conditional likelihood function is:

$$f(t_i^*)^{d_i}[1 - F(c_i|X_i, \theta)]^{1-d_i}$$
$$d_i \text{ is the indicator}$$

Therefore, the log-likelihood function

$$L(\theta) = \sum^{N}\{d_i - F(c_i|X_i, \theta) + (1 - d_i)\ln[1 - F(c_i|X_i, \theta)]\}$$

Maximizing this, we obtain consistent and asymptotical normal estimator $\hat{\theta}$.

$$(\sqrt{\hat{\theta}} - \theta) \sim N\left(0, -\left[\frac{1}{N}\frac{\partial^2 L(\hat{\theta})}{\partial\hat{\theta}\partial\hat{\theta}'}\right]\right)$$

2. Left censoring

   We missed observation of individuals who are not searching for a job at the point of $b \rightarrow$ sample selection problem may occur

   We observe observations iff an individual is still unemployed at b

$$\begin{aligned} &a_i + t_i^* \geq b \\ &\text{or} \\ &t_i^* \geq b - a_i \end{aligned}$$

   The probability is obtained as follows:

$$\begin{aligned} &\Pr(t_i^* \geq b - a_i|X_i, a_i, c_i) \\ =&1 - F(b - a_i|X_i) \qquad \because \text{assumption} \end{aligned}$$

   Therefore, the likelihood function is:

$$\frac{f(t_i^*|X_i\theta)_i^d[1 - F_(c_i|X_i, \theta)]^{1-d_i}}{1 - F(b - a_i|X_i, \theta)}$$

10.3.3 Empirical Example

11 Equilibrium Matching Model