Behavioral Economics
Exercise 4 Time Preferences

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Question 1（a）Solve by backward induction.

- Period 2
  By $\beta = \delta = 1$, $t = 2$ agent's intertemporal utility is :

$$-c(e_2) + B(e_1, e_2)$$
$$\text{where } e_1 \text{ is given}$$

  Then, the best-response of the $t = 2$ agent, depending on the parameters is derived as follows :
  If $B \geq kc \Leftrightarrow c \leq \dfrac{B}{k}$

$$\mathrm{BR}(e_1) = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

  and if $kc > B > 2c \Leftrightarrow c > \dfrac{B}{k}$

$$\mathrm{BR}(e_1) = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

- Period 1
  Now by $\beta = \delta = 1$, $t = 1$ agent's intertemporal utility is :

$$-c(e_1) - c(\mathrm{BR}(e_1)) + B(e_1, \mathrm{BR}(e_1))$$

  By the assumption $kc > 2c$ and $B > 2c$ and the derived best-response of the $t = 2$ agent, e_1 is the dominant strategy for each case of the parameters.

Therefore, the perception-perfect equilibria is :
If $c \leq \dfrac{B}{k}$

$$
\begin{aligned}
(e_1^*, e_2^*) &= (1, e_2) \\
\text{where } e_2 &= \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}
\end{aligned}
$$

If $c > \dfrac{B}{k}$

$$
\begin{aligned}
(e_1^*, e_2^*) &= (1, e_2) \\
\text{where } e_2 &= \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}
\end{aligned}
$$

(b) • Period 2
The intertemporal utility is :

$$-c(e_2) + \beta B(e_1, e_2)$$

- When $e_1 = 0$
For $t = 2$ agent, $e_2 = 1$ is strictly dominated by $e_2 = 0$, since both of them result in $B(0, e_2) = 0$.
Then, the agent prefer $e_2 = 2$ iff

$$
\begin{aligned}
-kc + \beta B &\geq 0 \\
c &\leq \frac{\beta B}{k}
\end{aligned}
$$

Otherwise, $e_2 = 0$ is chosen.
- When $e_1 = 1$
$e_2 = 2$ is strictly dominated by $e_2 = 1$, since $e_2 = 1$ is enough to satisfy $B(.,.) = B$.
The agent prefer $e_2 = 1$ iff

$$
\begin{aligned}
-c + \beta B &\geq 0 \\
c &\leq \beta B
\end{aligned}
$$

- Period 1
  - When $c \leq \dfrac{\beta B}{k}$
    Even if $e_1 = 0$, $t = 2$ agent choose $e_2 = 2$ and so $B(e_1, e_2) = B$ realizes.
    Then, $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

    $$(1-\beta)kc \leq 0$$

    which never hold since $\beta < 1$, so $e_1 = 2$ is stricly dominated by $e_1 = 0$.
    Then, $e_1 = 1$ is preferred iff

    $$\begin{aligned} -c - \beta c + \beta B &\geq -kc + \beta B \\ (1 + \beta - \beta k)c &\leq 0 \\ \text{which holds iff } k &\geq \frac{1+\beta}{\beta} \end{aligned}$$

    Otherwise, $e_1 = 0$ is chosen.
  - When $\dfrac{\beta B}{k} < c \geq \beta B$
    $e_2 = 2$ is never chosen, which implies that if $e_1 = 0, B(.,.) = 0$.
    $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

    $$\begin{aligned} -kc + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{k} \end{aligned}$$

    which never holds, by $c > \dfrac{\beta B}{k}$.
    $e_1 = 1$ is preferred to $e_1 = 0$ iff

    $$\begin{aligned} -c - \beta c + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{1+\beta} \end{aligned}$$

    Note that $\left(\dfrac{\beta B}{k}, \dfrac{\beta B}{1+\beta}\right]$ is not empty interval.
  - $\beta B < c$
    Both $t = 1, 2$ agent choose $e_t = 0$, rather than any positive e_t.

Therefore, the perception-perfect equilibria is:

$$(e_1^*, e_2^*) = (e_1, e_2)$$

If $c \in (\frac{\beta B}{k}, \frac{\beta B}{1+\beta})$,

$$e_1 = 1$$
$$e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $(c \leq \frac{\beta B}{k}) \wedge (k \geq \frac{1+\beta}{\beta})$,

$$e_1 = 1$$
$$e_2 = \begin{cases} 2 & \text{if} e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases}$$

If $(c \leq \frac{\beta B}{k}) \wedge (k < \frac{1+\beta}{\beta})$,

$$e_1 = 0$$
$$e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 0 & \text{otherwise} \end{cases}$$

If $\beta B < c$,

$$e_1 = 0$$
$$e_2 = 0$$

(c) $t = 2$ agent's intertemporal utility is same as (b). So her/his response would be same, too. Namely,
If $c \leq \dfrac{\beta B}{k}$,

$$BR(e_1) = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

and if $\dfrac{\beta B}{k} \leq \beta B$,

$$BR(e_1) = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

And otherwise, $BR(e_1) = 0$ for any e_1.
$t = 1$ agent, on the other hand, predicts as if $t = 2$ agent has intertemporal utility with $\beta = 1$: $\hat{\beta} = 1$. This is same as (a), so s/he predicts $t = 2$ agent's response as follows.

$$\begin{aligned} \hat{BR}(2) &= 0 \\ \hat{BR}(1) &= 1 \\ \hat{BR}(0) &= \begin{cases} 2 & \text{if } c \leq \frac{B}{k} \\ 0 & \text{if } c > \frac{B}{k} \end{cases} \end{aligned}$$

i. $c \leq \dfrac{\beta B}{k} < \dfrac{B}{k}$
$t = 1$ agent prefer $(e_1, e_2) = (2, 0)$ iff

$$\begin{aligned} &-kc + \beta B \geq -\beta kc + \beta B \\ &(1 - \beta)kc \leq 0 \end{aligned}$$

which never holds.
Then, s/he choose $e_1 = 1$ to realize (1,1) rather than (0,2) iff

$$\begin{aligned} -c + \beta c + \beta B &\geq -\beta kc + \beta B \\ k &\geq \frac{1+\beta}{\beta} \end{aligned}$$

and by $c \leq \dfrac{\beta B}{k}$, $e_2 = 1$ is chosen and (1,1) is the perception-perfect equilibrium.
If $k < \dfrac{1+\beta}{\beta}$, then (0,2) is the perception-perfect equilibrium.

ii. $\frac{\beta B}{k} \leq c \leq \frac{B}{k}$
Similarly to

iii. $\frac{B}{k} < c \leq \frac{\beta B}{1+\beta}$

iv. $\frac{\beta B}{1+\beta} < \frac{\beta}{k} < c$

（d）

（e）