労働経済 I 期末レポート

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1. (1) For any (L_a, L_b), marginal rate of technical rate of substitution of L_a to L_b is :

$$
\begin{aligned}
MRTS &= \frac{\partial y/\partial L_a}{\partial y/\partial L_b} \\
&= \frac{\alpha A(\frac{L_b}{L_a})^{1-\alpha}}{(1-\alpha)A(\frac{L_a}{L_b})^{\alpha}} \\
&= \frac{\alpha L_b}{(1-\alpha)L_a}
\end{aligned}
$$

(2)
For arbitrary target output, $\bar{y}$, the cost minimization problem of the planner is

$$
\begin{aligned}
&\min_{L_a, L_b} = w_a L_a + w_b + L_b \\
&\text{sub to } \bar{y} \leq y
\end{aligned}
$$

Since $y = A(L_a)^{\alpha}(L_b)^{1-\alpha}$ produce no output if either L_a or L_b is zero, there cannot be any corner solution.
Then, the Lagrangian function and Kuhn-Tucker condition are described as follows.

$$
\mathcal{L} = w_a L_a + w_b L_b + \lambda(\bar{y} - y)
$$

$$
\begin{aligned}
w_a - \lambda \alpha A \left(\frac{L_b}{L_a}\right)^{1-\alpha} &= 0 \\
w_b - \lambda(1-\alpha) A \left(\frac{L_b}{L_a}\right)^{\alpha} &= 0 \\
A(L_a)^{\alpha}(L_b)^{1-\alpha} &= \bar{y}
\end{aligned}
$$

Solving this, the conditional demand funcion for L_a and L_b are derived as follows
:

$$
\begin{aligned}
\bar{L_a}(w_a, w_b, \bar{y}) &= \frac{\bar{y}}{A}\left(\frac{\alpha w_b}{(1-\alpha)w_a}\right)^{1-\alpha} \\
\bar{L_b}(w_a, w_b, \bar{y}) &= \frac{\bar{y}}{A}\left(\frac{(1-\alpha)w_a}{\alpha w_b}\right)^{\alpha}
\end{aligned}
$$

(3)
By the conditional demand function obtained in (3), $\bar{L_a}(w_a, w_b, \bar{y})$ is decreasing in w_a, while $\bar{L_b}(w_a, w_b, \bar{y})$ increasing in w_a, since $\bar{y}, A > 0$,$\alpha \in (0,1)$, $\left(\frac{w_b}{w_a}\right) > 0$ and $\left(\frac{w_a}{w_b}\right) > 0$ for any positive w_a and w_b.
Thus, for $\bar{w_a} > w_a$,

$$
\begin{aligned}
\bar{L_a}(\bar{w_a}, w_b, \bar{y}) &< \bar{L_a}(w_a, w_b, \bar{y}) \\
\bar{L_b}(\bar{w_a}, w_b, \bar{y}) &> \bar{L_b}(w_a, w_b, \bar{y})
\end{aligned}
$$

(4)
Planner's profit maximization problem is :

$$
\begin{aligned}
\max_y \Pi(y) &= p(y) \cdot y - w_a L_a - w_b L_b \\
&= y^{\frac{1+\beta}{\beta}} - \frac{y}{A} \cdot \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} w_a^{\alpha} w_b^{1-\alpha} - \frac{y}{A} \cdot \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} w_a^{\alpha} w_b^{1-\alpha}
\end{aligned}
$$

FOC yields

$$
\frac{1+\beta}{\beta} y^{\frac{1}{\beta}} - \frac{1}{A}\left(\frac{\alpha^{1-\alpha}}{(1-\alpha)^{1-\alpha}} + \frac{(1-\alpha)^{\alpha}}{\alpha^{\alpha}}\right) w_a^{\alpha} w_b^{1-\alpha} = 0
$$

$$
y^*(w_a, w_b) = \left[\frac{\beta}{A(1+\beta)(1-\alpha)^{1-\alpha}\alpha^{\alpha}} w_a^{\alpha} w_b^{1-\alpha}\right]^{\beta}
$$

(5)
When y is normal goods, $p(y)$ should be decreasing in y, which requires that $p'(y) = \frac{1}{\beta} y^{\frac{1-\beta}{\beta}} < 0$. Also, by the optimal output obtained in (4), $y^* \geq 0$ iff $\beta < -1$ or $\beta \geq 0$.

So, I assume that $\beta < -1$. Then, as $\beta < 0$, y^* is decreasing in w_a. By (2), conditional demand for both L_a and L_b is increaseing in $\bar{y}$, which indicate that for any $\bar{w_a} > w_a$, $\bar{y^*} < y^*$ holds and so

$$\bar{L_a}(\bar{w_a}, w_b, \bar{y^*}) < \bar{L_a}(w_a, w_b, y^*)$$

Note that the relationship between $\bar{L_b}(\bar{w_a}, w_b, \bar{y^*})$ and $\bar{L_b}(w_a, w_b, y^*)$ is parametrical.

(6)

Summing up (2) and (4), the unconditional demand for L_a and L_b is obtained as follows :

$$\begin{aligned} L_a^*(w_a, w_b) &= \bar{L_a}(w_a, w_b, y^*) \\ &= \frac{y^*}{A}\left(\frac{\alpha w_b}{(1-\alpha)w_a}\right)^{1-\alpha} & L_b^*(w_a, w_b) &= \bar{L_b}(w_a, w_b, y^*) \\ &= \frac{y^*}{A}\left(\frac{(1-\alpha)w_a}{\alpha w_b}\right)^{\alpha} \end{aligned}$$

Note that $y^* = y^*(w_a, w_b) = \left[\frac{\beta}{A(1+\beta)(1-\alpha)^{1-\alpha}\alpha^{\alpha}} w_a^{\alpha} w_b^{1-\alpha}\right]^{\beta}$.

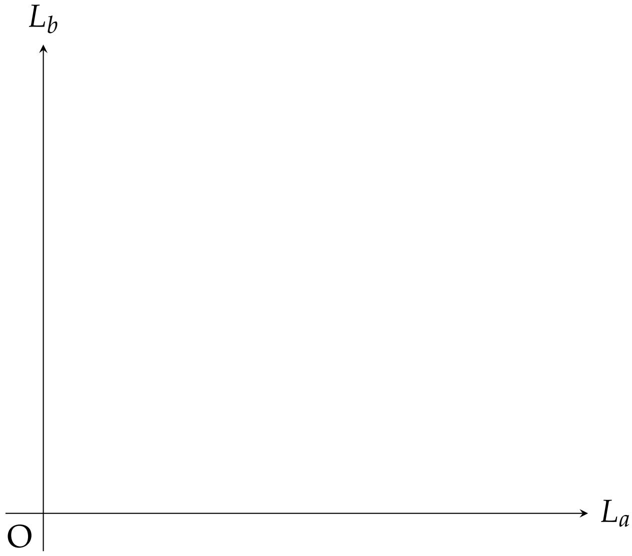
(7)

There are two effects by the change in w_a.

- Scale effect decreases both L_a and L_b.
- Substitution effect decreases L_a, while raising L_b.

The Illustration below shows the case that scale effect on L_b exceeds the substitution effect.

Illustration : $\bar{w}_a > w_a$



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2. (1)

The individual's maximization problem is :

$$\max U(C, l)$$

sub to

$$\begin{aligned} C &= c_D + c_M \\ c_D &= f(h_D) \\ c_M &\leq wh_M + R \end{aligned}$$

Note that for the third restriction, equation holds in the static case : Saving is irrational choice.
Then, define R_0 be the potential income

$$R_0 \equiv wL_0 + R$$

where $L_0 \equiv= l + h_D + h_M$ stands for net time available.
Then, by $c_M = wh_M + R$,

$$\begin{aligned} c_M &= w(L_0 - l - h_D) + R_0 - wL_0 \\ c_M &= R_0 - wl - wh_D \end{aligned}$$

By $C = c_D + c_M$ and $c_D = f(h_D)$,

$$\begin{aligned} c_M + wl &= R_0 - wh_D \\ C - f(h_D) + wl &= R_0 - wh_D \\ C + wl &= [f(h_D) - wh_D] + R_0 \end{aligned}$$

These discussion rewrite the UMP as follows

$$\max u(C, l)$$

$$\begin{aligned} &\text{sub to} \\ &C + wl = [f(h_D) - wh_D] + R_0 \end{aligned}$$

Finally, we derive the Kuhn-Tucker condition. The Lagrangian function is :

$$\mathcal{L} = u(C, l) + \lambda\{R_0 + [f(h_D) - wh_D] - c - wl\}$$

λ is the Lagrangian multiplier.
Kuhn-Tucker condition is derived as follows :

$$\begin{aligned} \frac{\partial u}{\partial C} - \lambda \leq 0, &\quad C\left(\frac{\partial u}{\partial C} - \lambda\right) = 0 \\ \frac{\partial u}{\partial l} - \lambda w \leq 0, &\quad l\left(\frac{\partial u}{\partial l} - \lambda w\right) = 0 \\ \lambda[f'(h_D) - wh_D] \leq 0, &\quad \lambda[f'(h_D) - w] = 0 \\ R_0 + [f(h_D) - wh_D] - c - wl \geq 0, &\quad \lambda\{R_0 + [f(h_D) - wh_D] - c - wl\} = 0 \end{aligned}$$

where $R_0 = w(l + h_D + h_M) + R$.
(2)
By the assumption, $l > 0$ and $h_M > 0$,

$$\begin{aligned} \frac{\partial u}{\partial l} &= 0 \\ \frac{\partial u}{\partial C} &= 0 \end{aligned}$$

holds. Then, assume u(.) is monotonic in l,

$$0 < \frac{\partial u}{\partial l} \leq \lambda$$

which indicates that

$$R_0 + [f(h_D) - wh_D - C - wl] = 0$$

and

$$f'(h_D) - w = 0$$

Deriving marginal rate of substitution,

$$MRS \equiv \frac{\partial u / \partial l}{\partial u / \partial C} = \frac{\lambda w}{\lambda} = w$$

First, by $f'(h_D) - w = 0$, optimal h_D is determined, and then, $R_0 + [f(h_D) - wh_D - C - wl] = 0$ and $MRS = w$ yields the optimal h_M and l.
(3)
By the assumption $h_M = 0$,

$$\begin{aligned} L_0 &= l + h_D \\ R_0 &= wl + wh_D + R \end{aligned}$$

Then, the maximization problem is rewritten as follows :

$$\max u(C, L_0 - h_D)$$

$$\begin{aligned} &\text{sub to} \\ &C + wl = [f(h_D) - wh_D] + R_0 \\ &\Rightarrow C = f(h_D) + R \end{aligned}$$

Lagrangian function and Kuhn-Tucker condition are :

$$\mathcal{L} = u(C, L_0 - h_D) + \lambda[f(h_D) + R - C]$$

$$\begin{aligned}
\frac{\partial u}{\partial C} - \lambda \leq 0 \quad & c\left(\frac{\partial u}{\partial C} - \lambda\right) = 0 \\
-\frac{\partial u}{\partial (h_D)} + \lambda f'(h_D) \leq 0 \quad & (L_0 - h_D)\left(-\frac{\partial u}{\partial h_D} + \lambda f'(h_D)\right) = 0 \\
f(h_D) + R - C \geq 0 \quad & \lambda[f(h_D) + R - C] = 0
\end{aligned}$$

By the assumption, the individual never save, so $C > 0$ as long as $R > 0$. Moreover, by $l > 0$ and $h_D > 0$, $L_0 - h_D > 0$. Moreover, when $u(.)$ is monotonically increasing in C, $\frac{\partial u}{\partial C} > 0$, indicating that $\lambda > 0$.
Therefore, each inequalty condition holds equation,

$$\begin{aligned}
\frac{\partial u}{\partial C} - \lambda = 0 \\
-\frac{\partial u}{\partial (h_D)} + \lambda f'(h_D) = 0 \\
f(h_D) + R - C = 0
\end{aligned}$$

Solving this, optimal h_D is derived, which also derives l by $l = L_0 - h_D$.

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3. (1)
   - Ability bias
   - Selection bias

   (2)
   - Ability bias
     Optimal wage depending on the worker's ability θ is :

$$\begin{aligned}
w(\theta) &= Ah_0 e^{\theta x(\theta)} \\
&\text{where} \\
h_0 &: \text{initial human capital} \\
x(.) &: \text{optimal length of education as a function of } \theta
\end{aligned}$$

     When θ goes up, there are two different effects : direct effect on $w(\theta)$ and indirect effect, that raises $x(\theta)$, and then affect on $w(\theta)$.
     Then, assume simplified form regression on loggarithm wage w_i by education length t_i,

$$\ln w_i = \beta_0 + \beta_1 t_i + \epsilon_i$$

t_i and error term ϵ_i are positively correlated, which causes bias by OLS.

- Selection bias
  Consider decision-making whether to go to college or not.
  Define B^i be the education effect for type $i \in \{C, H\}$ individual, then we would like to specify

$$B^C = E^C_C - E^C_H$$
$$B^H = E^H_C - E^H_H$$

  Note that E is return of education, superscript standing for the type of individual, and subscript denoting her/his actual choice (: C is to go to college, while H not).
  By observed sample, however, the "appeared" effect B is :

$$B = E^C_C - E^H_H$$

  since type C individual usually go to school and vice versa.
  Again, assume the wage regression

$$\ln w_i = \beta_0 + \beta_1 t_i + \epsilon_i$$

  If $E^C_H < E^H_H$, where the earnings of job for high school would have been lower for type C than for type H, then t_i and ϵ_i are negatively correlated, and so B^C is underestimated.
  Similarly, if $E^H_C < E^C_C$, then t_i and ϵ_i are positively correlated, so B^H is overestimated.

(3)
Exclusive instruments should have explanatory power to the independent variable you are interested in, but be independent of the error term of the original regression model.
Consider the regression model

$$\ln y_i = \beta_0 + \beta_1 t_i + \beta_2 + x_i + \epsilon_i$$

Then, excluded variable Z_i is introduced to the following regression :

$$t_i = \gamma_0 + \gamma_1 Z_i + \gamma_2 + e_i$$

For the excluded variable being valid, two conditions below are required :

- γ_1 is statistically significant, and not close to zero.
- Z_i is independent of any variables that determines y_i, except for t_i.
  : $Cov(Z_i, \epsilon_i) = 0$.

In order to find a valid instruments, we should pay attention to legal engagement that force individuals to apply some choice, regardless of charactaristics of each individual.

(4)

In this paper, they utilized birth-month of the students, to identify the effect of compulsory school attendance on their ability and wages.

In the U.S., those who were born early in the calender year receive compulsory education for shorter duration than those who were born late. This law is guaranteed by legal regulation about compulsory education, and there is no correlation between birth-month and their ability. Thus, using this as an instrument variable, they can limit the estimation bias.

(5)

This article specifies the marginal return to length of education on their wage, using dataset of identical twins.

They conducted a questionnaire, by twins, asking their schooling level. The important point is they requested each of them to answer the difference in education between the twins. Then, they used this sibling-report as instrument variable of the schooling level of the individuals. Correlation between self and sibling report, caused by person-specific component of mesurement error is controlled in this method, and we can see the actual effect of the difference in education on that of wage.