Labor Economics Term Paper

Reio TANJI 23A18014

Due date: 2/1

Q1
(1) The value functions of the worker are:

$$\begin{cases} rW = w + \lambda(\bar{u} + f_e - W) & \text{for employment} \\ r\bar{u} = z + \theta q(\theta)(W - \bar{u}) & \text{for unemployment} \end{cases}$$

W and $\bar{u}$ are the values of employment and unemployment. w and z are the instantaneous utility of employment and unemployment, respectively. r is the discount factor, and $q(\theta)$ is the matching function of $\theta = \frac{v}{u}$. v and u is the vacancy rate and the unemployment rate of the workers, satisfying $q'(\theta) < 0$. When a worker was dismissed, they receive f_e as the severance payment.

(2) The value functions of the firm are:

$$\begin{cases} rJ = (p - w) + \lambda(V - f_e - f_a J) & \text{for having a vacancy filled} \\ rV = -c + q(\theta)(J - W) & \text{for vacancy} \end{cases}$$

J and V are the values of vacancy filled and vacancy. p is the the instantaneous productivity of the worker and w is the instantaneous wage. c is the expected hiring cost. When a firm fire a worker, then it have to pay the severance payment f_e to the worker, and f_a as the related cost for firing.

(3) By the free-entry/exit condition, $V = 0$ holds. Then, the value function of vacancy is rewritten as follows:

$$J = \frac{c}{q(\theta)}$$

Then, substituting this into the value function of having vacancy filled,

$$
\begin{aligned}
rJ &= p - w + \lambda(V - f_e - f_a - J) \\
p - w &= (r+\lambda)\left(\frac{c}{q(\theta)}\right) - \lambda(f_e + f_a) \\
p - w &= \frac{(r+\lambda)c - \lambda(f_e + f_a)}{q(\theta)},
\end{aligned}
$$

which corresponds to the job creation condition.

(4) First, we derive the equilibrium conditions: wage condition and steady state condition. First, we derive the wage curve.
Given the value functions in (1) and (2), the wage is determined by Nash bargaining rule:

$$
w = \arg\max_{w}(W - \bar{u})^{\beta}(J - V)^{1-\beta},
$$

note that β stands for the bargaining power of the worker.

Q2
(1)"Hazard Function" is an instantaneous rate of an individual moving from a certain state to another. In the settings of job searching, it corresponds to that of leaving from unemployment pool.
Let $T \geq 0$ denote the duration and the cumulative distribution function $F(.)$ is defined as follows:

$$
F(t) = \Pr(T \leq t).
$$

Then, the hazard function $\lambda(.)$ is:

$$
\lambda(t) \equiv \lim_{h \to 0} \frac{\Pr(t \leq T \leq t + h | T \geq t)}{h},
$$

where $\Pr(t \leq T \leq t + h | T \geq t)$ is given as:

$$
\Pr(t \leq T \leq t + h | T \geq t) = \frac{F(t+h) - F(t)}{1 - F(t)}.
$$

Then, we can rewrite the hazard function by $F(t)$ and $f(t) = F'(t)$,

$$
\lambda(t) = \frac{f(t)}{1 - F(t)}.
$$

Now suppose the duration is distribution according to the Weibull distribution. Then, the cumulative distribution $F(t)$ is given as follows.

$$
F(t) = 1 - \exp(-\gamma t^{\alpha})
$$

γ and α are parameters. Then the density function is obtained as

$$
\begin{aligned}
f(t) = F'(t) &= \frac{d(1-\exp(-\gamma t^{\alpha}))}{dt} \\
&= \frac{d(1-\exp(-\gamma t^{\alpha}))}{d(-\gamma t^{\alpha})} \cdot \frac{d(-\gamma t^{\alpha})}{dt} \\
&= \gamma \alpha t^{\alpha - 1}
\end{aligned}
$$

Substituting this into $\gamma(t)$, we obtain the hazard function.

$$\lambda(t) = \gamma \alpha t^{\alpha-1}$$

Positive/Negative dependence of the hazard function on the duration is found by checking the first-order differential of the hazard function: if $\lambda'(.) > 0$, then it has positive dependence, and negative if $\lambda(.) < 0$. When the duration has Weibull distribution, by $\lambda'(t) = (\alpha-1)\gamma\alpha t^{\alpha-2}$. Since $t > 0$ and $\gamma \geq 0$ is assumed, it depends on the value of α. If $\alpha > 1$, then the hazard ratio has the positive dependence on the duration, and $\alpha < 1$ implies negative dependence.

(2) Suppose observation of the worker who started job-searching after time 0. We observe unemployed workers who started job-searching until time b, and we stop observation at the cetain time: after then, we do not observe the worker, even though they have not finished searching. We define the time a worker i enters the initial state (start searching) a_i, and time left for her/him until we stop the observation c_i.
Then, we cannot observe the true duration t_i if the worker have not finished searching at the censoring time. "Right-censoring" stands for possible bias caused by this problem.
On the other hand, when we start observation of the workers who are searching at the time b: If s/he have finished searching at time b, then we cannot observe her/him, even if s/he started job searching after time 0. Thus, obtained sample can be selected, that is, "left-censoring" problem occurs.

(3) Denote the "observed" duration to be t^*. Assume t^* is distributed independent of a_i and c_i, then the conditional cumulative distribution function $F(.)$ on the vector of observed covariates X_i is defined as:

$$F(t^*|X_i, a_i, c_i|\theta) = F(t_i|X_i;\theta).$$

θ is the vector of parameters.
If there is no censoring problem discussed in (2), then the conditional density function of duration is $f(t_i^*|X_i;\theta)$. When an observation is completed, we can obtain the likelihood function of this.
Then, we argue the observation with incomplete duration. First, we consider the right-censoring problem. The probability of right-censored observation is given by $1 - F(c_i|X_i;\theta)$. Then, the conditional likelihood function is obtained as follows:

$$f(t_i^*|X_i;\theta)^{d_i}[1 - F(c_i|X_i;\theta)]^{1-d_i}$$

Second, we consider the left-censoring. All the observed samples satisfy that the individual is still unemployed at time b. That is, $t_i^* \geq b_i - a_i$. The probability of this is expressed by $\Pr(t^* \geq b - a_i | X_i, a_i, c_i) = 1 - F(b - a_i | X_i)$.
By the Bayes Rule, the likelihood function including right and left-censoring is:

$$L(\theta) = \prod_i^N \frac{f(t_i^* | X_i; \theta)^{d_i} [1 - F(c_i | X_i, \theta)]^{1-d_i}}{1 - F(b - a_i | X_i; \theta)}$$

Maximizing this (or log-likelihood function) with θ, we obtain MLE: the consistent asymptotical normal estimated parameter $\hat{\theta}$.

(4) Holmås. 2002. Keeping nurses at work: a duration analysis *Health Economics*. 11: 493 – 503 (2002)

Q3
(1)

(2)

(3)

(4)