Behavioral Economics
Exercise 4 Time Preferences

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Question 1（a）Solve by backward induction.

- Period 2

  By $\beta = \delta = 1$, $t = 2$ agent's intertemporal utility is :

$$-c(e_2) + B(e_1, e_2)$$
$$\text{where } e_1 \text{ is given}$$

Then, the best-response of the $t = 2$ agent, depending on the parameters is derived as follows :
If $B \geq kc \Leftrightarrow c \leq \dfrac{B}{k}$

$$\text{BR}(e_1) = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

and if $kc > B > 2c \Leftrightarrow c > \dfrac{B}{k}$

$$\text{BR}(e_1) = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

- Period 1

  Now by $\beta = \delta = 1$, $t = 1$ agent's intertemporal utility is :

$$-c(e_1) - c(\text{BR}(e_1)) + B(e_1, \text{BR}(e_1))$$

By the assumption $kc > 2c$ and $B > 2c$ and the derived best-response of the $t = 2$ agent, e_1 is the dominant strategy for each case of the parameters.

Therefore, the perception-perfect equilibria is :

If $c \leq \dfrac{B}{k}$

$$(e_1^*, e_2^*) = (1, e_2)$$
$$\text{where } e_2 = \begin{cases} 2 & \text{if } e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 2 \end{cases}$$

If $c > \dfrac{B}{k}$

$$(e_1^*, e_2^*) = (1, e_2)$$
$$\text{where } e_2 = \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{if } e_1 = 0, 2 \end{cases}$$

(b)
- Period 2

  The intertemporal utility is :

$$-c(e_2) + \beta B(e_1, e_2)$$

  - When $e_1 = 0$

    For $t = 2$ agent, $e_2 = 1$ is strictly dominated by $e_2 = 0$, since both of them result in $B(0, e_2) = 0$.

    Then, the agent prefer $e_2 = 2$ iff

$$-kc + \beta B \geq 0$$
$$c \leq \frac{\beta B}{k}$$

    Otherwise, $e_2 = 0$ is chosen.

  - When $e_1 = 1$

    $e_2 = 2$ is strictly dominated by $e_2 = 1$, since $e_2 = 1$ is enough to satisfy $B(.,.) = B$.

    The agent prefer $e_2 = 1$ iff

$$-c + \beta B \geq 0$$
$$c \leq \beta B$$

- Period 1
  - When $c \leq \dfrac{\beta B}{k}$
    Even if $e_1 = 0$, $t = 2$ agent choose $e_2 = 2$ and so $B(e_1, e_2) = B$ realizes.
    Then, $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

$$(1 - \beta)kc \leq 0$$

    which never hold since $\beta < 1$, so $e_1 = 2$ is stricly dominated by $e_1 = 0$.
    Then, $e_1 = 1$ is preferred iff

$$\begin{aligned} -c - \beta c + \beta B &\geq -kc + \beta B \\ (1 + \beta - \beta k)c &\leq 0 \\ \text{which holds iff } k &\geq \frac{1+\beta}{\beta} \end{aligned}$$

    Otherwise, $e_1 = 0$ is chosen.
  - When $\dfrac{\beta B}{k} < c \geq \beta B$
    $e_2 = 2$ is never chosen, which implies that if $e_1 = 0, B(.,.) = 0$.
    $t = 1$ agent prefer $e_1 = 2$ to $e_1 = 0$ iff

$$\begin{aligned} -kc + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{k} \end{aligned}$$

    which never holds, by $c > \dfrac{\beta B}{k}$.
    $e_1 = 1$ is preferred to $e_1 = 0$ iff

$$\begin{aligned} -c - \beta c + \beta B &\geq 0 \\ c &\leq \frac{\beta B}{1+\beta} \end{aligned}$$

    Note that $\left(\dfrac{\beta B}{k}, \dfrac{\beta B}{1+\beta}\right]$ is not empty interval.
  - $\beta B < c$
    Both $t = 1, 2$ agent choose $e_t = 0$, rather than any positive e_t.

Therefore, the perception-perfect equilibria is:

$$(e_1^*, e_2^*) = (e_1, e_2)$$

If $c \in (\frac{\beta B}{k}, \frac{\beta B}{1+\beta})$,

$$\begin{aligned} e_1 &= 1 \\ e_2 &= \begin{cases} 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

If $(c \leq \frac{\beta B}{k}) \wedge (k \geq \frac{1+\beta}{\beta})$,

$$\begin{aligned} e_1 &= 1 \\ e_2 &= \begin{cases} 2 & \text{if} e_1 = 0 \\ 1 & \text{if } e_1 = 1 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

If $(c \leq \frac{\beta B}{k}) \wedge (k < \frac{1+\beta}{\beta})$,

$$\begin{aligned} e_1 &= 0 \\ e_2 &= \begin{cases} 2 & \text{if} e_1 = 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

If $\beta B < c$,

$$\begin{aligned} e_1 &= 0 \\ e_2 &= 0 \end{aligned}$$