Behavioral Economics
Exercise 3 Social Preferences

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Question 1（a）$(s_1^*, s_2^*) = \{(D, D)\}$ where s_i denotes the strategy of the player i.

（b）The rewritten matrix is :

1/2	C	D
C	2,2	$-3\alpha, 3-3\beta$
D	$3-3\beta, -3\alpha$	1,1

Let $\mathrm{BR}_i(s_j)$ be the best response of player i when her opponent j chooses s_j, then for $i = 1, 2$,

$$\mathrm{BR}_i(C) = \begin{cases} C & \text{if } \beta \in \left[\frac{1}{3}, 1\right] \\ D & \text{if } \beta \in \left[0, \frac{1}{3}\right] \end{cases}$$

by solving $2 = 3 - \beta$, and

$$\mathrm{BR}_i(D) = D$$

since $-3\alpha < 1$ holds for all $\alpha \geq \beta \geq 0$.

Thus, the equilibria are derived as follows :

$$(s_1^*, s_2^*) = \begin{cases} \{(D, D)\} & \text{if } \beta < \frac{1}{3} \\ \{(D, D), (C, C)\} & \text{if } \beta \geq \frac{1}{3} \end{cases}$$

Note that $\alpha \geq \beta \geq 0$.

（c）The rewritten matrix is :

1/2	C	D
C	2,2	$3\sigma, 3(1-\rho)$
D	$3(1-\rho), 3\sigma$	1,1

(i) $\sigma < 0 < \rho < 1$

Define $\rho = \beta$ and $\sigma = -\alpha$, then by $0 < \rho < 1$ and $3\sigma < 1$, the rewritten matrix is identical as that in the question (b).

Therefore, equilibria are :

$$(s_1^*, s_2^*) = \begin{cases} \{(D, D)\} & \text{if } \rho < \frac{1}{3} \\ \{(D, D), (C, C)\} & \text{if } \rho \geq \frac{1}{3} \end{cases}$$

for any $\sigma < 0$.

(ii) $\sigma \leq \rho < 0$

For each player, C is stictly dominated by D, since $2 < 3(1 - \rho)$ for all $\rho < 0$, and $3\sigma < 1$ for all $\sigma < 0$.

Thus, equilibrium is derived as:

$$(s_1^*, s_2^*) = (D, D)$$

for any σ and ρ satisfying the assumption.

(iii) $\sigma = 0, \rho = 1$

The matrix is rewritten as follows :

1/2	C	D
C	2,2	0,0
D	0,0	1,1

Then the best response for player i is :

$$\text{BR}_i(s_j) = \begin{cases} \text{C} & \text{if } s_j = C \\ \text{D} & \text{if } s_j = D \end{cases}$$

Therefore, equilibria are :

$$(s_1^*, s_2^*) = \{(C, C), (D, D)\}$$

(iv) $\sigma = \rho = \frac{1}{2}$

Again, the matrix is rewritten as follows :

1/2	C	D
C	2,2	$\frac{3}{2}, \frac{3}{2}$
D	$\frac{3}{2}, \frac{3}{2}$	1,1

The best response for player i given her opponent's strategy is

$$\text{BR}_i = C$$

by $2 > \frac{3}{2}$ and $\frac{3}{2} > 1$.

The equilibrium is :

$$(s_1^*, s_2^*) = (C, C)$$