Behavioral Economics
Exercise 5 Behavioral Game Theory

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Question 1（a）Every type of seller chooses to disclose her own type $i = L, M, H$, and $p = v_i$.
If the seller disclosed her type, the buyer purchases iff $v_i \geq p$, otherwise purchase iff $\epsilon \geq p$.

When the type is concealed, rational buyer evaluates lemon by its expected value :

$$E(v) = v_L \cdot \hat{q}_L + v_M \cdot \hat{q}_M + (1 - \hat{q}_L - \hat{q}_M) > v_H = 1$$

where $\hat{q}_i$ denotes her belief about each q_i, and buy only if $E(v) \geq p$.
Then, type-H seller raise her profit by disclosing and set $p = 1$, and the buyer revises her belief : If the seller did not disclose her type, then $\hat{q}_H = 0$, and so

$$E(v) = v_L \cdot \hat{q}_L + v_M \cdot \hat{q}_M > v_M$$

Again, there is an incentive for type-M seller to disclose and set $p = v_M$, and finally, the buyer predicts $\hat{q}_L = 1$, and the expected value $E(v) = v_L = \epsilon$, which makes type-L seller to disclose and set $p = \epsilon$.

（b） When the buyer is fully cursed, she predicts the expected value if the private information was hidden is : $\hat{q}_i = q_i$ for each $i \in L, M, H$

$$E(v) = v_L \cdot q_L + v_M \cdot q_M + (1 - q_L - q_M) \equiv E$$

and purchase iff $E(v) > p$.
Note that when the type was disclosed, the buyer purchase iff $v_i > p$.

In Stage 1, then,

- Type-H seller : $v_H = 1 > E$
  She discloses her type and set $p = 1$.
- Type-L seller : $v_L = \epsilon < E$
  She conceal her private information and set $p = E$.
- Type-M seller : $v_M \in (\epsilon, 1)$
  Her strategy is conditional on the value of v_M.
  If $v_M \geq E$, then she disclose her type and set $p = v_M$.
  Otherwise, $v_M < E$, she conceal and set $p = E$.

(c) By the assumption, in this question, χ-cursed buyer's belief and action is:

$$\begin{cases} \hat{q}_L = 1 \text{ and purchase iff } p \leq \epsilon & \text{if "rational"} \\ \hat{q}_i = q_i \text{ and purchase iff } p \leq E & \text{if "cursed"} \end{cases}$$

$$\text{where } p = E = v_L \cdot q_L + v_M \cdot q_M + (1 - q_L - q_M)$$

Note that if $p > v_L = \epsilon$, then "rational" buyer revises her belief same as "cursed."
Suppose the seller is type-L. Then, there are some possible strategies as follows :

- disclose, and set $p = \epsilon$
  The seller's expected payoff is:

$$(1-\chi)\epsilon + \chi\epsilon = \epsilon$$

- conceal, and $p = \epsilon$

$$(1-\chi)\epsilon + \chi\epsilon = \epsilon$$

- conceal, and $p = E = v_L \cdot q_L + v_M \cdot q_M + (1 - q_L - q_M)$
  The "rational" buyer behave as if she is "cursed," since $p = E$ is off-path. Then ,the seller's expected payoff is :

$$(1-\chi)E + \chi E = E$$

  which yields the best response for the seller. This strategy, however, cannot be a perfect Bayesian Nash equilibrium, since the buyer's bilief is not consistent.
  Thus, there is no equilibrium strategies.

(d) Buyer's belief and action is same as (c), except for that if $p < E$, "cursed" buyer behave as if she is "rational."
First, suppose the buyer is type-L.

- disclose and $p = \epsilon$
  The seller's expexted payoff is ϵ.
- conceal and $p = \epsilon$
  The "cursed" buyer revises her belief. Seller's expected payoff is :

$$(1-\chi)\epsilon + \chi\epsilon = \epsilon$$

- conceal and $p = E$
  The expexted payoff is :

$$(1-\chi) \cdot 0 + \chi E = \chi E$$

By the assumption that ϵ is sufficiently small, type-L seller choose to conceal and set $p = E$.

Next, suppose type-M seller.

- disclose and $p = v_M$
  The seller's expected payoff is v_M.
- conceal and $p = \epsilon$
  "Cursed" buyer revises her belief, and the expected payoff is :

$$(1-\chi)\epsilon + \chi\epsilon = \epsilon < v_M$$

- conceal and $p = v_M$
  Again, "cursed" buyer revises her belief, and the expected payoff is :

$$(1-\chi)\cdot 0 + \chi \cdot 0 = 0$$

- conceal and $p = E$

$$(1-\chi)\cdot 0 + \chi E = \chi E$$

Thus, type-M seller's decision is conditional :

$$\begin{cases} \text{disclose and set } p = v_M & \text{if } \chi > \frac{v_M}{E} \\ \text{conceal and set } p = E & \text{otherwise} \end{cases}$$

Finally, consider type-H seller.

- disclose and $p = 1$
  The seller's expected payoff is 1.
- conceal and $p = \epsilon$
  The buyer revises her belief, and so the expected payoff is :

$$(1-\chi)\epsilon + \chi\epsilon = \epsilon$$

- conceal and $p = v_M$
  Again, the buyer revises her belief, and the expected payoff is :

$$(1-\chi)\cdot 0 + \chi v_M = \chi v_M$$

- conceal and $p = E$
  The expected payoff is :

$$(1-\chi)\cdot 0 + \chi E = \chi E$$

- conceal and $p = 1$

$$(1-\chi)\cdot 0 + \chi \cdot 0 = 0$$

Summerizing this, type-H seller unconditionally discloses her own type, and set $p = 1$.