1) Solve:

a. $x(n) = x(n-1) + 5$, $n \geq 1$, $x(1) = 0$

Forward subs^n:

$x(n) = x(n-1) + 5$

$n=1$:

$x(1) = 0$

$n=2$:

$x(2) = x(2-1) + 5$
$= x(1) + 5 = 0+5 = 5$

$n=3$

$x(3) = x(3-1) + 5$
$= x(2) + 5 = 5+5 = 10$

$n=4$,

$x(4) = x(4-1) + 5$
$= x(3) + 5 = 10+5 = 15$

$n=5$

$x(5) = x(5-1) + 5$
$= x(4) + 5 = 15+5 = 20$

$\therefore x(n) = 0 + 5 + 10 + 15 + 20 + \ldots$

$$\Rightarrow \left.\begin{aligned} x(2) &= 5 \times 1 \\ x(3) &= 5 \times 2 \\ x(4) &= 5 \times 3 \\ x(5) &= 5 \times 4 \end{aligned}\right\} 5(n-1)$$

$\therefore x(n) = 5(n-1)$ $\quad\therefore O(n)$ is Time Complexity

Proof: $f(n) \le c \cdot g(n) \Rightarrow O(n)$

$\therefore x(n) \le c \cdot n$

$x(n-1) \le c \cdot 5((n-1)-1)$

$x(n-1) \le c \cdot 5(n-2)$

$\therefore$

$\therefore 5(n-2)+5 \Rightarrow 5n-10+5$

$\Rightarrow 5n-5$

$\Rightarrow 5(n-1)$

Hence Proved $O(n)$.

b) $x(n) = 3x(n-1)$ for $n>1$, $x(1)=4$

Frwd subn:

$x(n) = 3x(n-1)$

n=1:

$x(1) = 4$

n=2:

$x(2) = 3(x)(2-1) = 3(x)(1) = 12$

n=3:

$x(3) = 3(x)(3-1) = 3x(2) = 36$

n=4:

$x(4) = 3(x)(4-1) = 3(x)(3) = 108$

n=5:

$x(5) = 3(x)(5-1) = 3x(4) = 324$

$\Rightarrow x(n) = 4+12+36+108+324+\dots$

$x(1) \Rightarrow 4 \times 3^0$

$x(2) = 4 \times 3^1$

$x(3) = 4 \times 3^2$

$x(4) = 4 \times 3^3$

$x(5) = 4 \times 3^4$

$\Big\} \; 4 \times 3^{n-1}$

$\therefore x(n) = 4 \times 3^{n-1}$

$\therefore O(3^n)$

Parents signature

Proof:

(1) initial condition $\Rightarrow x(1) = 4$

$\Rightarrow 4 \times 3^{1-1} \Rightarrow 4 \times 3^0 \Rightarrow 4$

(2) Apply n-1

$\Rightarrow x(n) = 3x(n-1) \Rightarrow (a)$

$\Rightarrow x(n) = 4 \cdot 3^{(n-1)} \Rightarrow (n = n-1)$

$\therefore x(n-1) = 4 \cdot 3^{(n-1)-1} = 4 \cdot 3^{n-2} \Rightarrow (1)$

apply (1) in (a)

$3x(n-1) = 3(4 \cdot 3^{n-2}) \Rightarrow 4 \cdot 3^{n-1}$

Hence proved.

c) $x(n) = x(n/2) + n$ for $n > 1$, $x(1) = 1$

Solve for $n = 2k$.

~~Recursive~~ method $\Rightarrow$ Master's theorem:

$x(n) = 1x(n/2) + n$

Formula: $T(n) = aT(n/b) + f(n)$

$a = 1, b = 2, f(n) = n$

$2k \Rightarrow x(2k/2) + 2k$

$f(n) = n^k \log_a b \Rightarrow$ formula.

$\therefore$ ~~fn~~ $n = n^k \log_a b$

$= n^1 \log_1 2 \Rightarrow n^1 \log_2 1$

$\therefore k = 1, \log_2 1 = 0$

$\boxed{\log_b^a < k}$

$\therefore \theta(n^k \log_n^p)$

Assume $P=1$,

$\therefore \theta(n^{(1)} \log_n^{(1)})$

$\therefore \theta(n)$

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d) $x(n) = x(n/3)+1$, $n>1$, $x(1)=1$, Solve for $n=3k$.

Master theorm

$x(n) = 1x(\frac{n}{3})+1$

$T(n) = aT(n/b) + f(n)$

$a=1$, $b=3$, $f(n)=1$

$\Rightarrow 1 = n^k \log_3^1$

$= n^0 \cdot 0$ $\therefore k=0$, $\log_3^1 = 0$

$\boxed{k = \log_b^a}$

$\therefore P=1$

$\Rightarrow \theta(n^k \log_n^{p+1})$

$\Rightarrow \theta(n^0 \log_n 2)$

$\Rightarrow \theta(\log n)$.

2) $T(n) = T(n/2)+1$, $n=2k$

$T(n/2) = T(n/2/2)+1$

$= T(n/2^2)+1$

① $T(n) = T(n/2^2)+1+1 \Rightarrow T(n/2^2)+2 \Rightarrow$ ②

$T(n/2^2) = T(n/2^3)+1$

② $T(n) = T(n/2^3)+3$

DATE.....................

$T(n/n^3) = T(n/2^4)+1$

Parents signature

(3) $T(n) = T(n/2^4)+4$

$\Rightarrow T(n) = T(n/2^i)+i$

$n/2^i k = 2^i$, where $i = \log_2 n$

$T(n) = T\left(\frac{n}{\log_2 n}\right) + \log_2 n$

$T(2k) \Rightarrow T(1)+\log(2k)+\ldots$

$T(2k) \Rightarrow T(1)+\log_2 2k$

$\Rightarrow O(\log n)$

(ii) $T(n) = T(n/3)+T(2n/3)+cn$

$T(n) = aT(n/b)+f(n)$

$a=2,\ b=3 \quad f(n) = cn$

$f(n) = n^k \log_a b$

$cn = n^k \log_a b$

$k<1$

$\Rightarrow T(n) = \theta(f(n)$

$\therefore T(n) = \theta(n)$

3) Recursion Algorithm:

a) What does Algorithm compute

⇒ It returns the smallest value / element in the array A and consists a base condn of $(n==1)$, and this snippet executes recursively when the size of Array (n) is greater than 1.

⇒ Thereby it gives smallest element of the Array "A".

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b) Recurrence relation:

⇒ $T(n) = T(n-1)+1$

⇒ The snippet runs recursively $(n-1)$ tim

$\therefore \; T(1) = T(1-1)+1 = 0$

$T(2) = T(2-1)+1 = 1$

$T(3) = T(3-1)+1 = 2$

$T(4) = T(4-1)+1 = 3$

$\therefore T(n) = n-1$

$\therefore O(n)$

4) Analyze:

i) $F(n) = 2n^2+5$

$g(n) = 7n$ $\quad \Omega(g(n))$

	$F(n)$	$g(n)$
	$2n^2+5$	$7n$
$n=1$	7	7
$n=2$	13	14
$\boxed{n=3}$	23	21
$n=4$	37	28

$\therefore f(n) > g(n) \cdot c$

when $n \geq 3$

$\therefore \Omega(7n)$ is the recurrence relation

$\therefore \Omega(n)$