Chapter 4 Compaction of soils

4.1 Introduction

The self-weight of the structure and live loads imposed over the structure are transmitted to the ground through foundation. Accord to the studies, the effect of the imposed load is felt by the ground normally up to a depth of about two or three times the width of the foundation. The ground within this depth is the bearing strata mentioned in the previous chapter. As a three-phase composite, the compressibility of the soil mass due to the imposed load results in the decrease in its volume and accordingly, the settlement of the top structure.

Under the action of the imposed load, the soil mass undertakes both volume deformation and shear deformation. The volume deformation, which is due to the normal stress, only shrinks and densifies the soil mass. In this case, the failure of the soil mass won't occur. While for the shear deformation, which is mainly due to the shear stress, will result in the shear failure as long as the shear stress exceeds certain limit. In this chapter, the settlement of the top structure mainly attributes to the volume deformation.

The excessive settlement can lead to distortion and damage to structures. In practical engineering, there are two types of settlement: uniform settlement and differential settlement. When foundation settles at nearly the same rate throughout all portions of a building, it is called uniform settlement. In general, the uniform settlement has no detrimental influence on the building safety. However, it influences utility of the building, e.g. damaging sewer, water supply; jamming doors and windows. The differential settlement refers to the settlement that occurs at differing rates between different portions of a building. In this case, the building may become distorted, floors may slope, walls and glass may crack, and doors and windows may not work properly. Majority of structure failures are attributable to severe differential settlement. To ensure the safety of buildings during and after construction, it is necessary to estimate its settlement during the design phase.

The magnitude of the settlement mainly depends on two factors: 1) the compressibility of soil. The higher compressibility of the soil strata, the larger settlement of the structure; 2) The magnitude and distribution of external load. The higher the external load, the larger settlement of the structure. Moreover, comparing with the concentrated load, the eccentric load leads to larger differential settlement.

4.2 Compressibility of soils

4.2.1 Compressibility characteristics of soils

The compressibility of soils refers to the reduction in its volume under the action of a compressive load. Strictly speaking, the volume decrease of a three-phase composite as soil may attribute to:

- Volume decrease of solid grains
- Volume decrease of water and air
- Expulsion of water and air

In practical engineering, the applied load is usually less than 600 kPa. In this case, the volume decrease of grain and water is actually small comparing with the total volume decrease. The former usually accounts for 1/400 of the latter. Accordingly, grain and water in most cases are assumed imcompressible for simplicity.

For saturated soil, only the expulsion of water contributes to the total volume decrease. For partially saturated soil, the situation become much more complex. The expulsion of water and air as well as the volume decrease of the entrapped air and dissolution of air in the water, contribute to the total volume decrease. For simplicity, only saturated soils are considered in this chapter.

4.2.2 Consolidation

Upon application of a static or quastatic compressive load on a saturated soil, the load is instantly carried by pore water because the skeleton of the soil needs to defome to take the load. The excess pore pressure[1], which initially has the same magnitude as the external load, is induced.

As the soil is premeable, the build-up excess pore pressure causes the water to be be drained out, which in turn leads to a decrease in the excess pore pressure. According to the principle of effective stress, the reduction in excess pore pressure is simutaneously transferred from water to soil skeleton as effective stress. This allows the soil skeleton to be compressed and stressed. The process wont end until the excess pore pressure is fully dissipated and thus no more water flows out. Finally, it is the soil skeleton that sustains the whole external load.

The whole process, which involves volume decrease of soils due to expulsion of

[1] The excess pore pressure is the excess over hydrostatic pressure, which is mainly induced by external actions including loading and change of boundary conditions.

water and a simutaneous transfer of pressure from water to soil skeleton, is called consolidation.

The consolidation process may be almost immediate or time consuming, which depends on the permeability characteristics of the corresponding soils. In general, the consolidation process of cohesionless soils finishes in a relatively short period of time as they usually possess high hydraulic conductivity. Therefore, the structures built on cohesionless soils suffer less damage due to settlement during and after construction.

For cohesive soils, the consolidation process of cohesionsive soils normally lasts several months or yeas due to low hydraulic conductivity. The reduction in volume of voids and the rearrangement of solid grains usually lead to large settlement. As the consolidation process of cohesive soil is more susceptible to play a detrimental role in the safety of a building, further studies are conducted in the following sections.

4.2.3 Indices for compressibility of saturated cohesive soils

To quantify the compressibility of saturated cohesive soils, several indices are necessary. These indices could be measured either in the laboratory or in the field. In this section, only the indoor consolidometer test and the associated indices are presented.

4.2.3.1 Consolidometer test

In the laboratory, the consolidometer test (oedometer test, or one-dimensional consolidation test) can be used to obtain several indices. The device is show in Fig 4.1. A consolidometer is fundamentally made out of three basic apparatus: 1) A loading frame to apply the compressive load; 2) A compression cell to hold the soil sample; 3) A measuring system to obtain the change in sample's thickness. Accessories including weight set, vernier calipers, spatula, stop watch, are also necessary.

An enlarged schematic view of the compression cell is also shown in Fig 4.1. A soil sample is placed in the rigid confining ring with paper filters and porous stones on both ends. The assembly is then installed at the center of the compression cell. Water is added to the reservoir to keep the soil sample saturated throughout the test. The consolidation pressure[2] by means of weight set is transferred to the top of the soil sample through a stiff metal cap. The vertical deformation of the soil sample is monitored with a dial gauge.

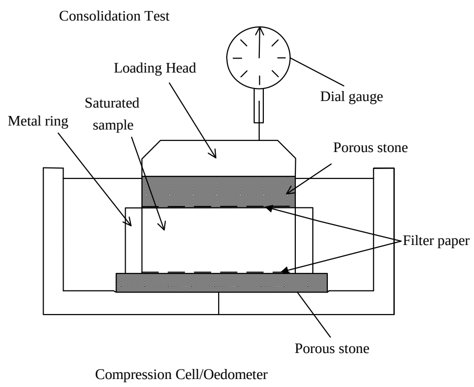
During the test, the consolidation pressure is applied incrementally. The number of loading stages and the maximum consolidation pressure applied depends on the

[2] Consolidation pressure refers to a compressive load which results in the consolidation of soils.

loading conditions in the field. Upon any loading, the vertical deformations are recorded at a series of specified time points. In general, almost 24 hours are necessary for each loading. After finishing the current consolidation process, the subsequent consolidation pressure, which is always the twice of the previous one, is applied. The vertical deformations at the similar time intervals are recorded.

At the end of the loading test with the maximum consolidation pressure, or at the end of certain loading stage, an unloading process can be performed to evaluate the loading-unloading behavior of soils. In this stage, loads should be also removed in steps. For each unloading stage, rebound on the specimen occurs and the final vertical deformation are recorded. After the unloading process, the soil sample could be further compressed (consolidated) by following the foregoing mentioned process.

The initial void ratio e_0 should be determined ahead of the test. When each loading stage i finishes, the vertical displacment of the sample Δh_i is recorded. With these data, the corresponding void ratio e_{1i} is determined. For more detailed information, refer to the design code ASTM D2435.



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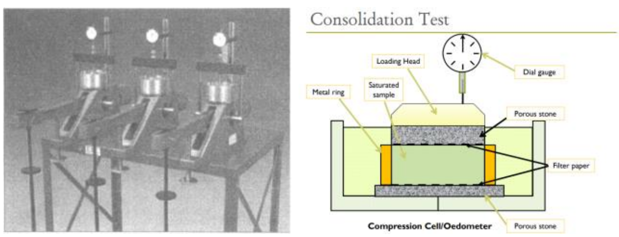


Fig 4.1 Consolidometer test

4.2.3.2 Variation of the void ratio

The variation of void ratio at the end of any load stage can be determined by the vertical displacment of the sample s_{vi}. Before the derivation, several assumptions are made as follows:

1) The volume decrease of the soil sample is mainly due to the volume decrease of the void. The volume decrease of the soil particles is negligible;

2) The deformation occurs only in the axial direction (zero lateral strain);

3) The soil sample is homogeneous and the induced stress is uniform.

The initial height and void ratio of the soil sample are h_0 and e_0 respectively. Assuming at the i^{th} loading stage, the soil sample is consolidated under the action of p_i, the change in height is Δh_i. As the lateral deformation is restricted, the change in volume ΔV per unit of original volume V_0 equals the change in height Δh per unit of original height h_0 as the cross section of the soil sample is kept a constant:

$$\frac{\Delta V_i}{V_0}=\frac{\Delta h_i \cdot A}{h_0 \cdot A}=\frac{\Delta h_i}{h_0} \tag{0-1}$$

A is area of the cross section. According to the phase diagram, as fig shows, the initial volume of soil sample before compaction equals:

$$V_0 = V_{s0} + V_{v0} = V_s\left(1+e_0\right) \tag{0-2}$$

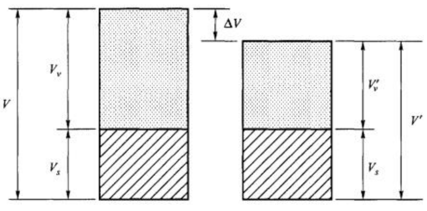
And the volume of sample after compaction equals

$$V_i = V_{si} + V_{vi} = V_s\left(1+e_i\right) \tag{0-3}$$

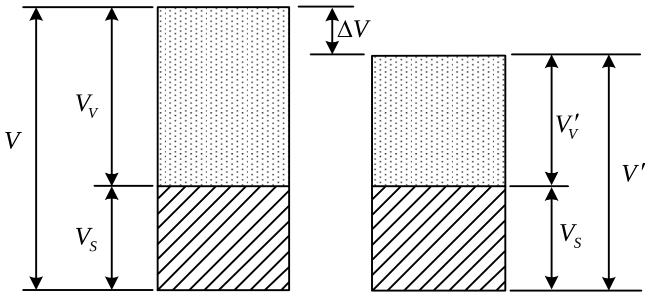
Substituting Eq. (0-2) and Eq. (0-3) into Eq. (0-1), we obtain:

$$\frac{\Delta h_i}{h_0}=\frac{\Delta V_i}{V_0}=\frac{V_i-V_0}{V_0}=\frac{e_i-e_0}{1+e_0}=\frac{\Delta e_i}{1+e_0} \rightarrow \Delta e_i=\frac{1+e_0}{h_0}\Delta h \tag{0-4}$$

Δe_i is the change in void ratio corresponds to the i^{th} loading stage.



(a) Initial condition (b) Compressed condition



(a)Initial condition (b)Compressed condition

Fig 4.2 Consolidometer test

Comments

1) Special attention should be paid to the sign of Δe_i. In fact, as the height of the soil sample decreases under compression, Δh is negative and thus Δe_i is also negative. In this case,

$$e_i = e_0 + \Delta e_i \tag{0-5}$$

Eq. (0-5) is slightly different from most of textbooks, where the plus sign is replaced by minus sign.

4.2.3.3 Coefficient of compressibility a_v

A *e-p* curve is plotted with a series of (e_i, p_i), as fig shows, and the coefficient of compressibility a_v can be determined therein. The coefficient of compressibility a_{vi} at the interval $[p_i, p_{i+1}]$ is defined as the ratio of the change in the void ratio to the change in effective stress as:

$$a_{vi} = -\frac{\Delta e_i}{\Delta p_i} = -\frac{e_{i+1} - e_i}{p_{i+1} - p_i} = \frac{e_i - e_{i+1}}{p_{i+1} - p_i} \tag{0-6}$$

The unit of a_{vi} is kPa^{-1} or Mpa^{-1}. The negative sign is used to indicate that the void ratio decreases with the increase in effective stress. Obviously, a_{vi} is not a constant.

It is found that the steeper (gentler) of the *e-p* curve, the higher (lower) the compressibility of the soil sample. To evaluate the compressibility of soils, a_{vi} is customarily defined at the interval $[0.1, 0.2]$ (unit, MPa). The degree of compressbility is shown in Tab.

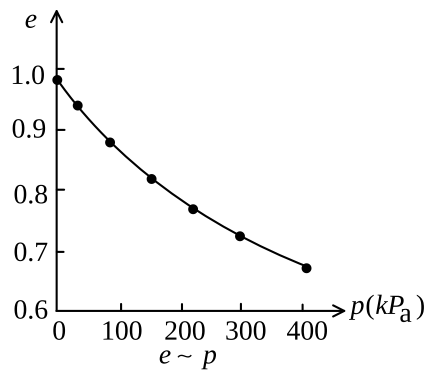


Fig. 4-1 *e-p* curve

Tab. 4-1 Classification of compressibility

Type of soil	$a_{1\text{-}2}$ (MP_a^{-1})
High compressibility	>=0.5
Medium compressibility	0.1-0.5
Low compressbility	<0.1

4.2.3.4 Compression index C_c

Figure shows the more commonly used void ratio-pressure relationship, where the applied pressure is plotted on a logarithmic scale. It is found that the e-$\log p$ curve tends to be linear at high pressure. The compression index is defined as the slope of the linear part, namely, the ratio of change in the void ratio to the corresponding change in the pressure as:

$$C_c = -\frac{\Delta e}{\Delta(\log p)} = -\frac{e_2 - e_1}{\log p_2 - \log p_1} \tag{0-7}$$

C_c is dimensionless index. e_1 and e_2 are the void ratios at the consolidation pressures p_1 and p_2, respectively, as Fig shows. The negative sign is used to indicate that the void ratio decreases with the increase in effective stress. It is found that the steeper (gentler) of the e-$\log p$ curve, the higher (lower) the compressibility of the soil sample.

Compression index can be also evaluated by some empirical laws:

1) In 1944, Skempton proposed the following equation:

$$C_c = 0.007(\omega_L - 10)$$

2) In 1967, Tezaghi and Peck proposed the following equation for low sensitive and medium sensitive undisturbed clays.

$$C_c = 0.009(\omega_L - 10)$$

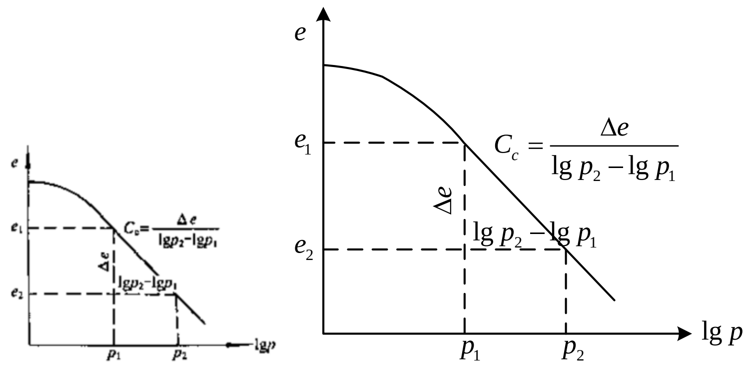
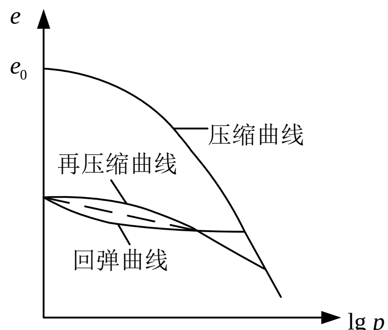


Fig. 4-2 *e-p* curve

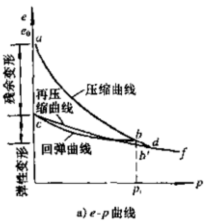
4.2.3.5 Rebound-recompression index C_s

The loading and unloading test can be used to quantify the behavior of the soil under cyclic loading. As fig shows, the loading procedure is as follows: 1) incrementally unloaded to pressure B from pressure A; 2) incrementally loaded to pressure C. After the loading-unloading cycle, it is found that: 1) the rebound curve does not coincide with the initial compression curve; 2) The recompression curve is different from the rebound curve, which forms a hysteresis loop; 3) both the rebound curve and recompression curve are much gentler than the virgin compression curve, which indicates that the soil sample become stiffer after loading-unloading cycle. 4) As long as the reloading pressure exceeds the pressure where the unloading process occurs, the recompression curve coincides with the initial compression curve.

C_s can be calculated according to the slope of the rebound-recompression curve. For inorganic soils, C_s is $(0.1 \sim 0.2)\ C_c$ value. It is worth pointing out that if the pressure that the soil experienced at present is less than the pressure it has been experienced in the past, namely the soil sample is at the rebound or recompression stage, the compressibility of the soil sample decreases and the corresponding settlement of the ground reduces. The widely used ground improvement technique, preloading method, utilizes this principle.



（b）$e - \lg p$曲线



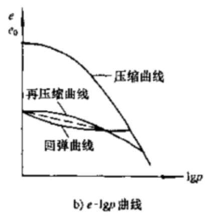


Fig. 4-3 *e-p* curve

4.2.3.6 Coefficient of volume compressibility

The coefficient of volume compressibility m_v at the interval $[p_i, p_{i+1}]$ is defined as the ratio of unit volume change per unit increase in effective stress:

$$m_{vi} = -\frac{\frac{\Delta V_i}{V_0}}{\Delta p_i} \qquad (0\text{-}8)$$

Taking into account the relation: $V_i = (1+e_i)V_s$, $V_{i+1} = (1+e_{i+1})V_s$, $\Delta V_i = V_{i+1} - V_i$, and the definition of a_v, m_v can be recast into:

$$m_{vi}=-\frac{\frac{\Delta V_i}{V_0}}{\Delta p_i}=-\frac{\frac{\Delta e_i V_s}{(1+e_0)V_s}}{\Delta p_i}=-\frac{\Delta e_i}{\Delta p_i}\frac{1}{1+e_0}=\frac{a_{vi}}{1+e_0} \tag{0-9}$$

4.2.3.7 Constrained modulus (E_s) and deformation modulus (E)

The constrained modulus at the interval $[p_i, p_{i+1}]$ is defined as the ratio of the change in axial stress to the change in axial strain as:

$$E_{si}=-\frac{\Delta p_i}{\Delta \varepsilon_{zi}}=-\frac{\Delta p_i}{\Delta \varepsilon_{vi}}=-\frac{\Delta p_i}{\frac{\Delta V_i}{V_0}}=\frac{1}{m_{vi}} \tag{0-10}$$

$\Delta \varepsilon_{vi}$ is volumetric strain increment. As the lateral deformation is restricted, the volumetric strain increment and axial strain increment are equal, namely $\Delta \varepsilon_{vi}=\Delta \varepsilon_{xi}+\Delta \varepsilon_{yi}+\Delta \varepsilon_{zi}=\Delta \varepsilon_{zi}$.

The defomation modulus at the interval $[p_i, p_{i+1}]$ is defined to relate stress increment and strain increment. According to the Hooke's law:

$$\Delta \varepsilon_{zi}=\frac{\Delta \sigma_{zi}}{E_i}-\frac{v_i}{E_i}\left(\Delta \sigma_{xi}+\Delta \sigma_{yi}\right) \tag{0-11}$$

v_i is the poison ratio defined as the ratio of the change in lateral strain to the change in axial strain when the lateral constraints are removed. For a consolidometer test, taking the following relations into account:

$$\begin{cases} \Delta \sigma_{xi}=\Delta \sigma_{yi}=K_{0i}\Delta \sigma_{zi} \\ K_{0i}=v_i/1-v_i \end{cases} \tag{0-12}$$

Substituting Eq. (0-12) into Eq. (0-11) leads to a theoretical relation between the deformation modulus E_{si} and the constrained modulus E_i:

$$E_i=E_{si}\left(1-\frac{2v_i^2}{1-v_i}\right) \tag{0-13}$$

As the poison ratio is usually no greater than 0.5, the constrained modulus E_{si} is greater than the deformation modulus E_i. The typical deformation modulus of certain soils are shown in Tab.

Tab. 4-2 Classification of compressibility

表 4-1　不同土类的变形模量经验值

土的类型	变形模量(kPa)	土的类型	变形模量(kPa)
泥炭	100～500	松砂	10 000～20 000
塑性粘土	500～4 000	密实砂	50 000～80 000
硬塑粘土	1 000～8 000	密实砂砾、砾石	100 000～200 000
较硬粘土	8 000～15 000		

4.2.3.8 Coefficient of consolidation and secondary compression index

For each loading stage, a set of vertical displacement Δh and the elapsed time t are available. A $\log t$-Δh curve is plotted in Fig. There are two linear parts on the curve, as shown in Fig.. One is the middle section and the straight line is considered as a primary consolidation curve. The other is the asymptotic part (later section) of the curve and the straight line is considered as a secondary compression curve. The intersection of the two straight lines is considered as the end of the primary consolidation and the corresponding vertical displacement is marked as Δh_{100}, where the subscript "100" indicates that the consolidation process completely finishes. After the determination of Δh_{50}, which implies half of the consolidation process finishes, the coefficient of consolidation C_v is obtained as:

$$C_v = 0.196\frac{H^2}{t_{50}} \tag{0-14}$$

t_{50} is the elapsed time to obtain Δh_{50}. H is the longest drainage path discussed in section 4.4.

The slope of the secondary compression curve is the so-called secondary compression index C_α, which can be used to estimate the creep settlement as described in section 4.5.

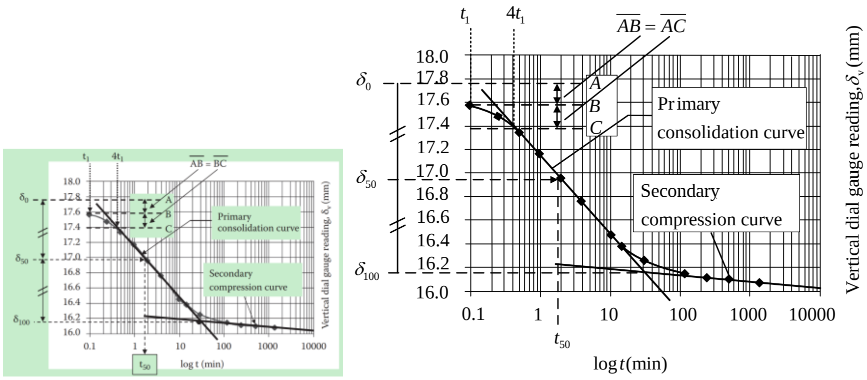


Fig. 4-4 *e-p* curve

4.3 Effect of stress history on the compressibility of soil

The history of stress state that the soil has experienced plays an important role on the compressibility of soil. Just we discussed in section 4.2.3.5, both the rebound curve and recompression curve are much gentler than the virgin compression curve, which indicates that the soil sample become stiffer after loading-unloading cycle. <u>The maximum effective stress that a soil sample has sustained in its geological history is</u>

called the pre-consolidation pressure (denote as P_c). The ratio of the pre-consolidation pressure to the current effective vertical stress (P_0) is defined as the over-consolidation ratio (OCR). The soil is termed as overconsolidated soil if P_0 is less than P_c, namely $OCR > 1$, while it is called as normally consolidated soil if P_0 equals P_c, namely $OCR = 1$.

More specially, for newly deposited soils or the soils on which a new external load is just applied, the consolidation process won't finish until the excess pore pressure completely dissipates. In this case, the soil is regarded as "under consolidated" or "un-consolidated". Before the gravitational force or the external load is fully transferred to the soil skeleton, the pre-consolidation pressure and the current effective vertical stress are equal and accordingly has an $OCR = 1$ of one.

Fig illustrates three strata with identical properties. Excerpt the layer 3, the consolidation processes in layer 1 and layer 2 has finished. For layer 2, the initial ground surface is marked with dot line. The erosion process may cause the fall of the ground surface. The soil in layer 1 is the normal consolidated soil as at any depth $P_0 = P_c = \gamma z$. For soil in layer 2, Pc= rh> P0=rz. Therefore, it is the over consolidated soil, For soil in layer 3, Pc=P0<rz, it is the unconsolidated soil.

The soils experiencing different stress states will show different compressibility characteristics. This can be illustrated with the help of e-p compression curve accompanied with loading and unloading cycle. Three points (ABC) within the corresponding soil strata have the same depth h'. Although the consolidation pressure are the same, they may locate at different place of the compression curve. For point A within the NC soil, it is on the virgin compression curve. For point B within the OC soil, it initially located at the point B as the pc is larger than the current consolidation pressure. After erosion due to geological process, the consolidation pressure decreases. Therefore, the soil in layer 2 experienced a rebound (increase in void ratio) process indicated by line B'B. The soil at point P, which is represented by point C in Figis now referred to as unconsolidated as the effective stress is less than the consolidation pressure. In this case, the excess pore pressure does not fully dissipate. Imaging a load increment is applied to three different layers, the corresponding compressibility characteristics are as follows: 1) For layer1 and layer3, it is compressed along the virgin compression curve to point D; 2) For layer 2, it is firstly compressed along the recompression curve B'B and then along the virgin compression curve to point D. Although the load increment is the same, the magnitude of compression are different.

Layer 3 has the largest the deformation, while the layer 2 has the least. Th building on these three layers certainly has different settlement.

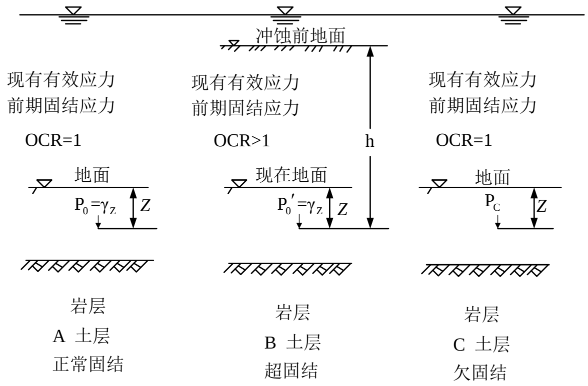
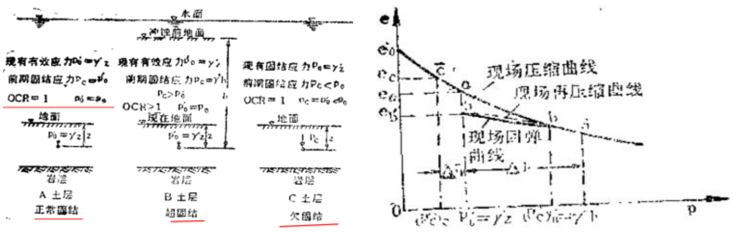


Fig. 4-5 *e-p* curve



4.3.1 Determination of the pre-consolidation pressure by Casagrande's graphical method

The pre-consolidation pressure can be estimated with the e-lop compression curve. Although there are a number of different ways, the earliest and the most widely used method was the one proposed by Casagrande (1936). The basic procedures are as follows:

1) Choose the point of maximum curvature on the consolidation curve. This is could be done more precisely with the help curve fitting. Based on the fitted curve, the point corresponding to maximum curvature can be more conveniently located
2) Draw a horizontal line through that point and a line tangent to the curve at that point.
3) Bisect the angle made from the horizontal line and the tangent line.
4) Extend the "straight portion" of the virgin compression curve up to the bisector line.
5) The abscissa of the point of intersection corresponds to the pre-consolidation pressure pc

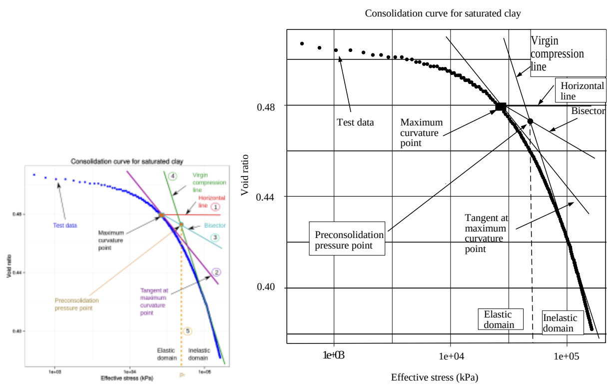


Fig. 4-6 *e-p* curve

4.3.2 The compression curve in-situ

Due to disturbance to the soil sample during sample collection and transportation, the compression curve obtained with the oedometer test may differ from in some manner from that in the field. To estimate the settlement of the ground more precisely, it is better to utilize the compression curve which more or less truly reflect the *in-situ* condition of the soil. This can be done with the e-logp compression curve.

4.3.2.1 *In-situ* compression curve for NC soil

For normal consolidated (NC) soil, the in-situ compression curve is determined as follows:

1) Draw a horizontal line passing through e0 and a vertical line passing through the pre-consolidation pressure pc. The intersection point A is determined
2) Determine the point C corresponds to the void ratio 0.42e0. The straight line AC gives the *in-situ* compression curve for normally consolidated clay soil

4.3.2.2 *In-situ* compression curve for OC soil

The *in-situ* compression curve for over-consolidated soil consists of two straight lines, represented by AB and BC in Fig. 7.9(b). The procedures are as follows:

1) Draw a horizontal line passing through e0 and a vertical line passing through the current effective pressure p0. The intersection point A is determined
2) Starting from point A, draw an inclined line in parallel with MN (mean slope of the rebound-recompression laboratory curve). The inclined line intersect with a vertical line passing through the pre-consolidation pressure pc at point B.
3) Determine the point C corresponds to the void ratio 0.42e0. The straight line

BC gives the second part *in-situ* compression curve for over-consolidated clay soil

4.3.2.3 *In-situ* compression curve for UC soil

The under consolidated soil is a special normal consolidated soil. Therefore, the procedure to determine its *in-situ* compression curve is almost the same as that of the normal consolidated (NC) soil.

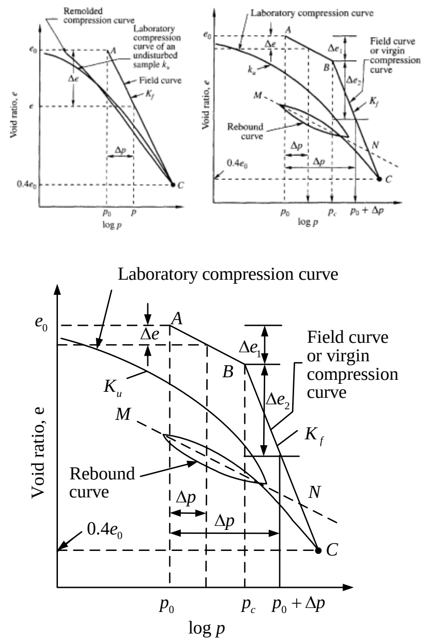


Fig. 4-7 *e-p* curve

4.3.3 Settlement of the ground beneath the foundation

With the *in-situ* compression curve, the settlement of the ground can be estimated. Assuming the following the data for i^{th} layer are already in hand:

- The original thickness of each layer H;
- The initial void ratio e_0;
- The pre-consolidation pressure p_c (or current effective stress p_0') and the additional stress increment (Δp). The assumption that the stress is uniformly distributed is made;
- The *in-situ* e-log p curve and the corresponding C_c is determined therein;

According to Eq. (0-4), the change in thickness per unit of original thickness H may be related through:

$$\frac{\Delta h}{H}=\frac{\Delta e}{1+e_0}\rightarrow \Delta h = H\frac{\Delta e}{1+e_0} \tag{0-15}$$

From the above equation, it is easy to know that as long as the variation of the void ratio due to the induced addition stress increment is known, the decrease in the thickness of the layer Δh (or the settlement) can be calculated. The detailed procedures are discussed below.

4.3.4 Ground with a single thin soil layer

When ground is composed of single thin soil layer, one-step computation is suitable as the current effective stress p_0' and induced stress increment Δp can be considered as constant values throughout the layer. p_0' is the effective geostatic stress at its mid-depth as seen in the Figure.

4.3.4.1 Normal consolidated (NC) soil

For normal consolidated soil, the decrease in the void ratio equals:

$$\Delta e = -C_c\left(\lg p_2 - \lg p_1\right) = -C_c \lg\left(\frac{p_1+\Delta p}{p_1}\right) \tag{0-16}$$

Note that C_c is determined from the *in-situ* e-$\log p$ curve but not the laboratory e-$\log p$. Subsisting Eq. (0-16) into Eq. (0-15), the settlement equals:

$$\Delta h = -C_c H \frac{\lg\left(\frac{p_1+\Delta p}{p_1}\right)}{1+e_0} \tag{0-17}$$

The negative sign indicates the decrease in the original thickness.

4.3.4.2 Over-consolidated (OC) soil

For over-consolidated soil, an additional attempt should be made to check whether $p_0'+\Delta p$ is larger than p_c or not.

- If $p_0'+\Delta p \le p_c$, the decrease in the void ratio is along the line AB as shown in the figure. In this case, the decrease in the void ratio equals:

$$\Delta e = -C_s\left(\lg p_2 - \lg p_1\right) = -C_s \lg\left(\frac{p_1+\Delta p}{p_1}\right) \tag{0-18}$$

  C_s is rebound-recompression index determined from the *in-situ* e-$\log p$ curve. The final settlement equals:

$$\Delta h = -C_s H \frac{\lg\left(\frac{p_1+\Delta p}{p_1}\right)}{1+e_0} \tag{0-19}$$

- If $p_0'+\Delta p > p_c$, the decrease in the void ratio is firstly along the line AB and then along the line BC as shown in the figure. In this case, the decrease in the void ratio are the contribution of two items:

The decrease in the void ratio along the recompression curve AB equals:

$$\Delta e_1 = -C_s\left(\lg p_c - \lg p_1\right) = -C_s \lg\left(\frac{p_c}{p_1}\right) \tag{0-20}$$

Then the decrease in the void ratio along the recompression curve BC equals:

$$\Delta e_2 = -C_c\left(\lg\left(p_1 + \Delta p\right) - \lg p_c\right) = -C_c \lg\left(\frac{p_1 + \Delta p}{p_c}\right) \tag{0-21}$$

Therefore, the total decrease in the void ratio equals:

$$\Delta e{=}\Delta e_1{+}\Delta e_2 = -\left(C_s \lg\left(\frac{p_c}{p_1}\right){+}C_c \lg\left(\frac{p_1+\Delta p}{p_c}\right)\right) \tag{0-22}$$

Subsisting Eq. (0-22) into Eq. (0-15), the settlement equals:

$$\Delta h = -\frac{H}{1+e_0}\left(C_s \lg\left(\frac{p_c}{p_1}\right){+}C_c \lg\left(\frac{p_1+\Delta p}{p_c}\right)\right) \tag{0-23}$$

4.3.4.3 Under consolidated soil

The consolidation process under the action of self-weight is still ongoing. The current effective stress equals the pre-consolidation pressure but less than the consolidation pressure. According to the fig, the under consolidated soil is compressed firstly under the action of self-weight and then the addition stress increment along the virgin compression curve. In this case, the decrease in the void ratio are also the contribution of two items as that of the over-consolidated soil.

Under the action of self-weight, namely from $p_0' \rightarrow p_0$, the decrease in the void ratio equals:

$$\Delta e_1 = -C_c\left(\lg p_0 - \lg p_0'\right) = -C_c \lg\left(\frac{p_0}{p_0'}\right) \tag{0-24}$$

Under the action of the induced addition stress increment, namely from $p_0 \rightarrow p_0{+}\Delta p$, the decrease in the void ratio equals:

$$\Delta e_1 = -C_c\left(\lg\left(p_0{+}\Delta p\right) - \lg p_0\right) = -C_c \lg\left(\frac{p_0{+}\Delta p}{p_0}\right) \tag{0-25}$$

Therefore, the total decrease in the void ratio equals:

$$\Delta e = \Delta e_1{+}\Delta e_2 = -\left(C_c \lg\left(\frac{p_0}{p_0'}\right){+}C_c \lg\left(\frac{p_0+\Delta p}{p_0}\right)\right) = -C_c \lg\left(\frac{p_0+\Delta p}{p_0'}\right) \tag{0-26}$$

Subsisting Eq. (0-26) into Eq. (0-15), the settlement equals:

$$\Delta h = -\frac{H}{1+e_0} C_c \lg\left(\frac{p_0+\Delta p}{p_0'}\right) \tag{0-27}$$

4.3.5 Ground with multilayers or a single thick soil layer

When the ground consists of multilayers or a single thick soil layer, the one-step computation strategy by Equations (9.28) and (9.29) is not suitable, as the current effective stress p_0' and induced stress increment Δp are not considered to be constant values throughout the depth that engineers are interested in. In this case, the widely used layer-wise summation method is utilized to estimate the settlement of the ground.

The essential of the layer-wise summation method is to divide the ground into several sublayers as fig shows. For each sublayer, the one-step computation technique is used to obtain Δh_i. The total settlement Δs is then the summation of Δh_i.

Two problems arise immediately when applying the layer-wise summation to the settlement of the ground. One is the depth needs to taken into account and the other is the criterion to make the division. In different design codes, the rules are different. Take a Chinese design code, e.g. code for design of building foundation, as an example. For the former problem, the depth it is determined by the ratio of the induced stress to the effective geostatic stress. The depth is measured form the bottom of the foundation to a certain depth where the ratio of the induced stress to the effective geostatic stress equals 0.2 if there is no soft clay layer, otherwise, the depth should be further enlarged to make the ratio of the induced stress to the effective geostatic stress equals 0.1. For the latter problem, the divisions should be made at the interfaces of different soil layers and the water table. Moreover, the thickness of each layer should be no greater than $0.4b$ (b is the width of the single foundation).

4.3.6 Layer-wise summation method

The essential points of layer-wise summation method are as follows:

1) Determine the location where special attention should be paid to based on the shape of foundations, lithological characteristics, et al.. Determine the contact pressure and additional pressure as described in section;
2) Make the division of the ground as describe in the section 4.3.5 or according to the code of design;
3) Calculate the effective geostatic stress at the mid-depth of the i^{th} sublayer, which the average (arithmetic mean) of the corresponding values at the roof and bottom surface of the i^{th} sublayer;
4) Calculate the addition stress Δp_i at the mid-depth of i^{th} sublayer along the depth of the specified location, which is also the average (arithmetic mean) of

the corresponding values at the roof and bottom surface of the i^{th} sublayer;

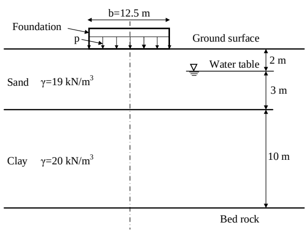
5) Calculate the defamation (settlement) of each sublayer Δh_i;

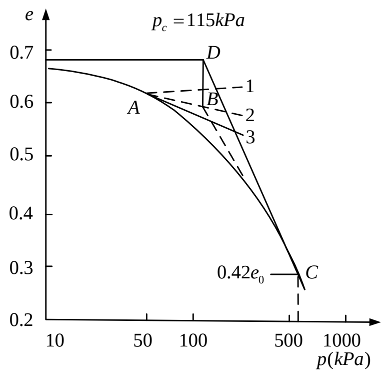
6) Add each Δh_i up to obtain the final settlement.

$$\Delta s = \sum_{i=1}^{n} \Delta h_i$$

Example

1) The layouts of the foundation and ground are as fig shows. The dimension of the square foundation is 12.5×12.5 m. The contact pressure at bottom of the foundation equals 100 kPa. To estimate the settlement of clay soil (the settlement of the sand and the bedrock is ignored), corresponding tests are conducted. The *in-situ* and laboratory compression curves are as fig shows. The initial void ratio e_0 is 0.67 (constant value throughout the clay layer). Determine the settlement of the ground beneath the center of the foundation ($\gamma_w = 10kN/m^3$).





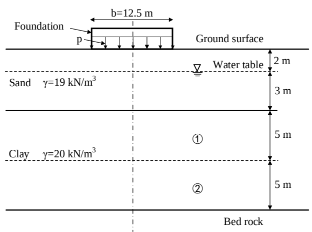


Fig. 4-8 *e-p* curve

According the procedure of the layer wise summation method:

1) The location is the center of foundation and the contact pressure $p = 100\text{kPa}$. As the burial depth is zero, the additional pressure and the contact pressure are equal, namely $p = p_0 = 100\text{kPa}$;

2) As the settlement of the sandy layer and the bed rock are ignored, the division is only made at the clay layer. The thickness of each sublayer

$H_i = 0.4b = 0.4 \times 12.5 = 5\text{m}$;

3) The effective geostatic stress at the mid-depth of the clay layer equals:

$$\sigma'_{cz} = 19 \times 2 + 9 \times 3 + 10 \times 5 = 115\text{kPa}$$

It is found that $\sigma'_{cz} = p_c$. Therefore, the soil in the clay layer is the normal consolidated soil. The compression index is 0.53;

4) For sublayer 1, the effective geostatic stress at the depth 5 m and 10 m are respectively 65kPa and 115kPa. Therefore, the average value of the effective geostatic stress within the sublayer 1 is 90kPa;

5) For sublayer 2, the effective geostatic stress at the depth 10 m and 15 m are respectively 115kPa and 165kPa. Therefore, the average value of the effective geostatic stress within the sublayer 2 is 140kPa;

6) The distribution of the additional stress beneath the center of foundation can be calculated with the "corner method". For the center point of a square shape foundation, the induced stress $\sigma_z = 4K_s\left(1/1, z/1\right)p_0$;

7) For sublayer 1, the additional stress at the depth 5 m and 10 m are respectively 80kPa and 45kPa. The corresponding average value of the additional stress within the sublayer 1 is 62.5kPa;

8) For sublayer 2, the additional stress at the depth 10 m and 15 m are respectively 45kPa and 26kPa. The corresponding average value of the additional stress within the sublayer 1 is 35.5kPa;

9) The settlement of each sublayer Δh_i is calculated as

◆ For sublayer 1

$$\Delta h_1 = -\frac{C_c H}{1+e_0}\log\left(\frac{\sigma_{sz,i}+\sigma_{z,i}}{\sigma_{sz,i}}\right) = -\frac{0.53\times 5}{1+0.67}\log\left(\frac{90+62.5}{90}\right) \approx -0.363\text{m}$$

◆ For sublayer 2

$$\Delta h_2 = -\frac{C_c H}{1+e_0}\log\left(\frac{\sigma_{sz,i}+\sigma_{z,i}}{\sigma_{sz,i}}\right) = -\frac{0.53\times 5}{1+0.67}\log\left(\frac{140+35.5}{140}\right) \approx -0.156\text{m}$$

10) The final settlement is calculated as:

$$\Delta s = \sum_{i=1}^{n}\Delta h_i = -0.363 + \left(-0.156\right) = -0.519\text{m}$$

Comments:

1) The more division are made, the more accurate the result. However, the more curbstone of the calculation;

2) As the distribution of the additional stress is nonlinear, the additional stress

could be calculated by calculating the corresponding coefficient. The arithmetic mean of the additional stress at the roof and at the bottom of the layer to represent the additional stress at the mid-depth is just an approximation

3) The void ratio may vary according to the depth. Therefore, it is more reasonable to calculate the void ratio at the mid-depth according to the e-logp *in-situ* compression curve with eq.
4) If the effect of stress history on the compressibility of soil is taken into account, the e-logp laboratory compression curve is used instead of the e-logp *in-situ* compression curve.

4.4 Terzaghi's one dimension consolidation theory

For saturated soils under compression, only the expulsion of water contributes to the volume decrease of soil. The time for the end of the consolidation process has close relation to the flow rate of water. In general, the higher (lower) the hydraulic conductivity, the shorter (longer) the time for the finish of the consolidation process.

Although the final settlement is the most concern in practical engineering, it is desirable to estimate the settlement at certain time level. This can be done with Terzaghi's one dimensional consolidation theory.

From the discussion in section, as long as the effective stress is known, the settlement of the ground could be calculated with the layer-wise summation method. As the principle of effective stress is always applicable during the compression, the problem of determining the effective stress can be equivalently transferred to the problem of determining the excess pore pressure.

To develop Terzaghi's one dimension consolidation theory, the following assumptions are made:

- The soil is homogenous (uniform in composition throughout), isotropic (show same physical property in each direction) and full saturated
- The solid particle and water are incompressible. Only the expulsion of water contributes to the volume decrease of soil
- Compression and flow only occur in vertical direction (one-dimensional). The lateral deformation of the soil sample is restricted
- Darcy's Law is strictly valid
- The deformation is infinitesimal. The derivation could be made on the unreformed geometry
- The coefficient of permeability and the coefficient of volume compressibility

remain constant throughout the process.

4.4.1 Differential equation for one dimensional consolidation theory

Consider a soil layer infinite in extent in the horizontal direction (Fig. 7.14) and a thick H in the vertical direction, the consolidation process under the action of self-weight has already finished. Upon the application of additional pressure σ_z, the drainage of water solely occurs at the top surface of the soil layer. Assuming the induced stress is uniform thought, the volume decrease of any infinitesimal soil element $\left(\mathrm{d}V\text{=dxdy}\right)$ equals:

$$\delta\left(\mathrm{d}V\right)=-m_v \mathrm{d}\sigma_z'\mathrm{dxdy}=-m_v\frac{\partial\sigma_z'}{\partial t}\mathrm{dtdxdy} \tag{0-28}$$

According to the principle of effective stress

$$\sigma_z'+p=\sigma_z\rightarrow\frac{\partial\sigma_z'}{\partial t}+\frac{\partial p}{\partial t}=0\rightarrow\frac{\partial\sigma_z'}{\partial t}=-\frac{\partial p}{\partial t} \tag{0-29}$$

$$\delta\left(\mathrm{d}V\right)=m_v\frac{\partial p}{\partial t}\mathrm{dtdxdy}\ \text{(0-30)} \tag{0-31}$$

The total flow of water through the top surface of the infinitesimal soil element equals:

$$\delta\left(\mathrm{dV}_w\right)=Q_{\mathrm{out}}-Q_{\mathrm{in}} \tag{0-32}$$

According to Darcy's Law

$$Q=v\mathrm{dtdx}=ki\mathrm{dtdx} \tag{0-33}$$

At level xy

$$i_{xy}=-\frac{\partial h}{\partial z}=-\frac{1}{\gamma_w}\frac{\partial p}{\partial y} \tag{0-34}$$

At level x'y'

$$i_{x'y'}=i_{xy}+\frac{\partial i_{xy}}{\partial y}\mathrm{dy}=-\frac{1}{\gamma_w}\frac{\partial p}{\partial y}-\frac{1}{\gamma_w}\frac{\partial^2 p}{\partial y^2}\mathrm{dy} \tag{0-35}$$

Subsisting eq and eq into results in

$$\delta\left(\mathrm{dV}_w\right)=Q_{\mathrm{out}}-Q_{\mathrm{in}}=-\frac{k}{\gamma_w}\frac{\partial p}{\partial y}\mathrm{dtdx}-\left(-\frac{k}{\gamma_w}\frac{\partial p}{\partial y}-\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial y^2}\mathrm{dy}\right)\mathrm{dtdx}=\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial y^2}\mathrm{dtdydx} \tag{0-36}$$

Therefore,

$$\delta\left(\mathrm{d}V\right)=\delta\left(\mathrm{dV}_w\right)\rightarrow m_v\frac{\partial p}{\partial t}\mathrm{dtdxdy}=\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial y^2}\mathrm{dtdydx}\rightarrow m_v\frac{\partial p}{\partial t}=\frac{k}{\gamma_w}\frac{\partial^2 p}{\partial y^2} \tag{0-37}$$

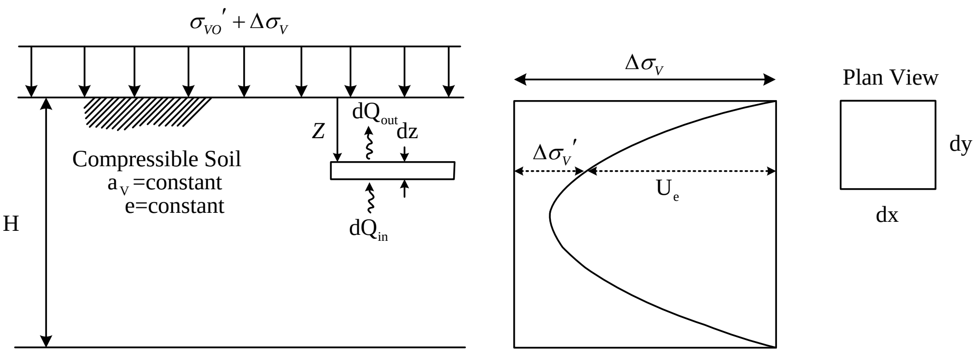
Define the coefficient of consolidation c_v as:

$$c_v=\frac{k}{\gamma_w m_v}=\frac{k\left(1+e_0\right)}{\gamma_w a_v} \tag{0-38}$$

Equation is recast into:

$$\frac{\partial p}{\partial t} = c_v \frac{\partial^2 p}{\partial y^2} \tag{0-39}$$

Eq is the differential equation for one dimensional flow. By solving it, the distribution of excessive pore pressure at any time level (or the distribution of effective stress at any time level) is available.



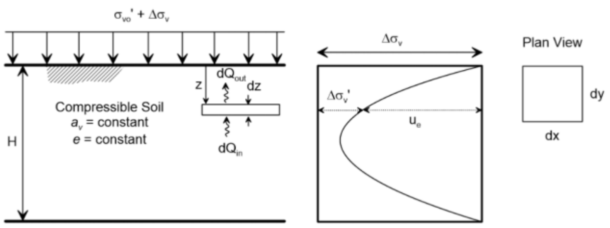


Fig. 4-9 *e-p* curve

4.4.2 Solution for the equation of one dimensional consolidation process.

The differential equation can be solved with the following initial and boundary conditions:

- $t=0$, $0 \le y \le H$ $\quad p = \sigma_z$
- $0 < t < \infty$, z=0 $\quad p = 0$
- $0 < t < \infty$, z=*H* $\quad \partial p / \partial z = 0$
- $t \to \infty$, $0 \le z \le H$ $\quad p = 0$

Combing with the above mentioned boundary conditions, Eq can be solved with the Fourier method (separation of variables). The solution expressed in Fouries series form is:

$$p(z,t) = \frac{4p}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} \sin \frac{m\pi z}{2H} e^{-m^2\left(\frac{\pi^2}{4}\right)T_v} \tag{0-40}$$

$$\text{with} \quad T_v = \frac{C_v t}{H^2}$$

Where m is the odd positive number. H is the longest length of the drainage path. If the water is allowed to escape only through the top surface of the clay layer (one-way drainage case), H is the whole thickness of the clay layer. On the other hand, if the water is allowed to escape through both the top and bottom surfaces of the clay layer (two-way drainage case), H is half the thickness of the clay layer. T_v is a dimensionless time factor. t is the elapsed time of the consolidation process.

4.4.3 Degree of consolidation and its application

With eq, the evolution of the excess pore pressure at any point within the clay layer is available. For an infinitesimal element at the depth z with a dz thick, its corresponding settlement can be calculated with the one-step computation method as:

$$\Delta h = -m_v \sigma_z' \mathrm{d}z = -m_v \left(\sigma_z - p \right) \mathrm{d}z \tag{0-41}$$

The settlement of the whole clay layer at a time t is obtained from an integration of eq over the toal clay layer thickness H as

$$\Delta s = \int_0^H \Delta h = \int_0^h -m_v \left(\sigma_z - p \right) \mathrm{d}z = -m_v \sigma_z H + \int_0^h m_v p \mathrm{d}z \tag{0-42}$$

Substituting eq into, the integration becomes

$$S_t = m_v \Delta \sigma H \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \frac{1}{2N+1} e^{-\frac{(2N+1)^2 \pi^2}{4} T_v} \right] \tag{0-43}$$

Define the degree of consolidation U as the percentage of settlement at an arbitrary time t to its final settlement at $t \to \infty$, which is calculated from eq and eq

$$U = \frac{S_t}{S} = 1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \frac{1}{2N+1} e^{-\frac{(2N+1)^2 \pi^2}{4} T_v} = f\left(T_v\right) \tag{0-44}$$

As seen from the above Equation, the degree of consolidation U is only a function of time factor Tv. Their relation are shown in Table 9.4 and plotted in Figure 9.7.

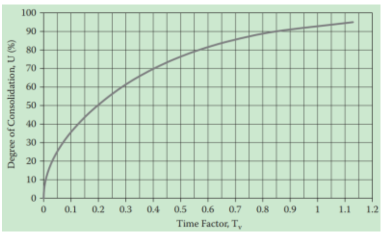


Fig. 4-10 *e-p* curve

Tab. 4-3 Classification of compressibility

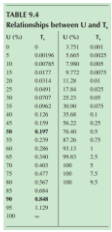


TABLE 9.4
Relationships between U and T_v

U (%)	T_v	U (%)	T_v
0	0	3.751	0.001
5	0.00196	5.665	0.0025
10	0.00785	7.980	0.005
15	0.0177	9.772	0.0075
20	0.0314	11.28	0.01
25	0.0491	17.84	0.025
30	0.0707	25.23	0.05
35	0.0962	30.90	0.075
40	0.126	35.68	0.1
45	0.159	56.22	0.25
50	**0.197**	76.40	0.5
55	0.239	87.26	0.75
60	0.286	93.13	1
65	0.340	99.83	2.5
70	0.403	100	5
75	0.477	100	7.5
80	0.567	100	9.5
85	0.684		
90	**0.848**		
95	1.129		
100	∞		

4.4.4 Application of degree of consolidation

Two kinds of problem can be solved with degree of consolidation:

1) With the known final settlement s, find the settlement s_t at a time t. The procedures are as follows:
   - Calculate the coefficient of consolidation C_v and the time factor T_v with the known parameters: hydraulic conductivity k, coefficient of compressibility a_v, initial void ratio e_0, the thickness of the clay layer H and the elapsed time t;
   - Determine the degree of consolidation U;
   - Determine the settlement s_t at a time t.
2) With the known final settlement s, find the elapsed time t to achieve a settlement s_t
   - Determine the degree of consolidation U;
   - Determine the time factor T_v;
   - Determine the elapsed time t according to the time factor T_v.

Example

1) In a laboratory odometer test, a 12.7 mm thick clay specimen is tested. According to the experimental data, it takes 15.8 minutes to complete 90% of the consolidation process. At the construction site, the same clay with a 6.5 m thick is sandwiched by sand and gravel layers. How long does the field clay take to accomplish 50% and 90% of the consolidation process, respectively?

SOLUTION

In the laboratory test, top and bottom are drainage layers, so the clay thickness 12.7 mm = 2H and T_{90} = 0.848 from Table 9.4. Inserting these values into Equation (9.15),

$$C_v = \frac{H^2}{t_{90}} T_{90} = \frac{\left(\frac{12.7}{2}\right)^2}{15.8} 0.848 = 2.164\ \text{mm}^2/\text{min}$$

From the field drainage condition, 2H = 6.5 m. Also T_{50} = 0.197 from Table 9.4. Utilizing Equation (9.15), 50% consolidation time, t_{50}, is

$$t_{50} = \frac{H^2}{C_v} T_{50} = \frac{\left(\frac{6.5 \times 1000}{2}\right)^2}{2.164} 0.197 = 9.615 \times 10^5\ \text{min} = \mathbf{667.7\ days} \leftarrow$$

Similarly, for 90% consolidation time t_{90},

$$t_{90} = \frac{H^2}{C_v} T_{90} = \frac{\left(\frac{6.5 \times 1000}{2}\right)^2}{2.164} 0.848 = 41.39 \times 10^5\ \text{min}$$

$$= 2874\ \text{days} = \mathbf{7.87\ years} \leftarrow$$

or, from Equation (9.15) and by using a common C_v value,

$$C_v = \frac{H^2}{t_{50}} T_{50} = \frac{H^2}{t_{90}} T_{90}, \quad \text{then,} \quad t_{90} = \frac{T_{90}}{T_{50}} t_{50} = \frac{0.848}{0.197} \times 667.7 = 4.305 \times 667.7$$

$$= 2874\ \text{days} = \mathbf{7.87\ years} \leftarrow$$

2) A clay layer had a thickness of 4.5 m. After 6 months, it settled to 30% of the final settlement and the corresponding settlement is 50 mm. For a similar clay layer and loading condition, if the thickness of clay is 20 m, how much settlement occurs at the end of 3 years? (Assume that the top of the clay layer is a drainage layer and the bottom is an impervious layer for both 4.5 m and 20 m thick clay layers.)

SOLUTION

For the 4.5 m thick clay, since 30% settlement is 50 mm, the final settlement will be

$$S_{f,4.5m} = 50/0.30 = 166.7\ \text{mm}$$

H = 4.5 m since the top is only a drainage layer in this case and thus,

$$C_v = \frac{H^2}{t_{30}} T_{30} = \frac{(4.5)^2}{6} 0.0707 = 0.239\ \text{m}^2/\text{month}$$

For the 20 m thick clay, the final settlement $S_{f,20m}$ is proportional to the one for 4.5 m clay; thus,

$$S_{f,\ 20m} = 166.7 \times (20/4.5) = 740\ \text{mm} \leftarrow$$

At the end of 3 years,

$$T_v = \frac{C_v t}{H^2} = \frac{0.239 \times (3 \times 12)}{20^2} = 0.0215$$

From the right two columns of Table 9.4 corresponding to T_v = 0.0215, U = 16.3% was obtained by a linear interpolation of data points. Thus, 20 m thick clay settles at the end of 3 years in the amount of

$$S_{3yrs,\ 20m} = S_{f,\ 20m} \times U_{3yrs} = 740 \times 0.163 = \mathbf{120.6\ mm} \leftarrow$$

4.5 Ground settlement

For the ground mainly composed of saturated cohesive soils, the total settlement s consists of three components:

$$s = s_i + s_c + s_s \tag{0-45}$$

s_i is the immediate settlement (commonly referred to as **elastic settlement**). s_c is the consolidation settlement due to consolidation process (commonly referred to as **primary consolidation**). s_s is the creep settlement due to rearrangement of fine particles (commonly referred to as **secondary consolidation**).

4.5.1 Immediate settlement

Immediate settlement is a time-independent process. As grains and water are usually assumed imcompressible and water in this short stage does not have enough time to escape, the immediate settlement of ground is mainly due to elastic distortion of soils. Although this settlement component is not elastic, it is generally calculated using elastic theory. In 1926, Schleicher proposed his theory to estimate the immediate settlement of the ground beneath a circular or rectangular footing:

$$s_i = pB\left(1-v^2\right)\frac{I_s}{E} \tag{0-46}$$

p is the mean contact pressure. B is the smaller dimension of the footing. v is the passion ratio. E is the defamation modulus as Tab shows. I_s is the influence coefficient as Tab shows:

Tab. 4-4 Classification of compressibility

表 4-2 影响系数 I 值

		圆形	方形	矩形(l/b)										
		—	1.0	1.5	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	100.0
柔性基础	角点	0.64	0.56	0.68	0.77	0.89	0.98	1.05	1.12	1.17	2.21	1.25	1.27	2.00
	中心	1.00	1.12	1.36	1.53	1.78	1.96	2.10	2.23	2.33	2.42	2.49	2.53	4.00
	平均	0.85	0.95	1.15	1.30	1.53	1.70	1.83	1.96	2.04	2.12	2.19	2.25	3.69
刚性基础		0.79	0.88	1.08	1.22	1.44	1.61	1.72	—	—	—	—	2.12	3.40

4.5.2 Consolidation settlement

The consolidation settlment is a time dependent process resulting from void space decrease in response to the expulsion of water. Terzaghi's consolidation theory, which has been discussed in section 4.4, can be ultilized to estimate this settlement component.

4.5.3 Creep settlement

The creep settlement is also a time dependent process. After complete dissipation of excess pore pressure, the constant effective stress leads to the time-dependent rearrangement of fine particles and thus the creep settlement. In general, the creep characterstics of solid skeleton affect the rate and magnitude of the creep settlement.

The estimation of creep settlement sometimes is significant as this kind of settlement lasts long time. The creep settlement can be predicted using the secondary compression index C_α as:

$$\Delta e = C_\alpha \log\frac{t}{t_p} \tag{0-47}$$

t is any arbitrary time (usually taken as the service life of the structure) and t_p is the time when the primary consolidation ends. Substituting Eq. (0-47) into Eq. (0-15), the creep settlement s_c equals:

$$s_c = \frac{H}{1+e_0}C_\alpha \log\frac{t}{t_p} \tag{0-48}$$

Comments

1) In fact, there is no distinctive limits separating these settlement components. e.g. the creep settlement occurs in parallel with the consolidation settlement but with a small amount. With the dissipation of excess pore pressure, the creep settlement becomes more and more prominent.

Exercise

4.1

Example 7.2

A recently completed fill was 32.8 ft thick and its initial average void ratio was 1.0. The fill was loaded on the surface by constructing an embankment covering a large area of the fill. Some months after the embankment was constructed, measurements of the fill indicated an average void ratio of 0.8. Estimate the compression of the fill.

Solution

Per Eq. (7.7), the compression of the fill may be calculated as

$$\Delta H = \frac{\Delta e}{1+e_0} H_0$$

where ΔH = the compression, Δe = change in void ratio, e_0 = initial void ratio, H_0 = thickness of fill.

Substituting, $\Delta H = \dfrac{1.0-0.8}{1+1.0} \times 32.8 = 3.28$ ft.

4.2

Example 7.1

During a consolidation test, a sample of fully saturated clay 3 cm thick (= h_0) is consolidated under a pressure increment of 200 kN/m^2. When equilibrium is reached, the sample thickness is reduced to 2.60 cm. The pressure is then removed and the sample is allowed to expand and absorb water. The final thickness is observed as 2.8 cm (h_f) and the final moisture content is determined as 24.9%.

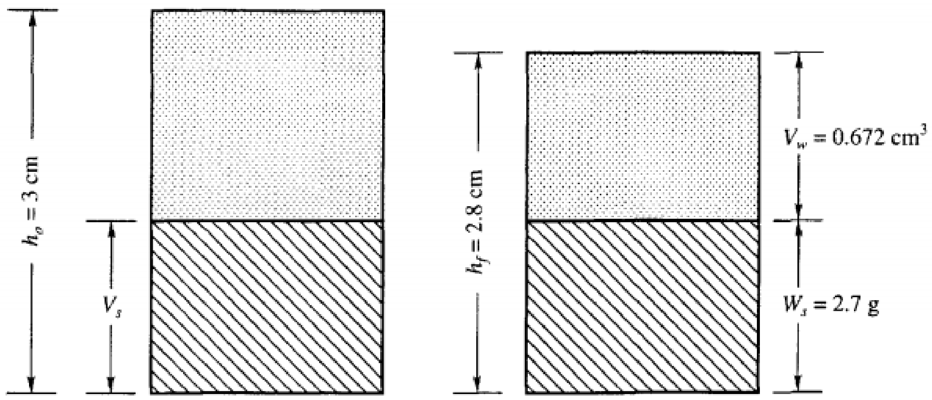


Figure Ex. 7.1

If the specific gravity of the soil solids is 2.70, find the void ratio of the sample before and after consolidation.

Solution

Use equation (7.3)

$$\Delta e = \frac{1+e}{h}\Delta h$$

1. *Determination of e_f*

   Weight of solids $= W_s = V_s G_s \gamma_\omega = 1 \times 2.70 \times 1 = 2.70$ g.

   $\frac{W_w}{W_s} = 0.249$ or $W_w = 0.249 \times 2.70 = 0.672$ gm, $e_f = V_w = 0.672$.

2. *Changes in thickness from final stage to equilibrium stage with load on*

   $\Delta h = 2.80 - 2.60 = 0.20$ cm, $\qquad \Delta e = \frac{(1+0.672)\,0.20}{2.80} = 0.119.$

   Void ratio after consolidation $= e_f - \Delta e = 0.672 - 0.119 = 0.553.$

3. *Change in void ratio from the commencement to the end of consolidation*

   $\Delta e = \frac{1+0.553}{2.6}(3.00-2.60) = \frac{1.553}{2.6}\times 0.40 = 0.239.$

   Void ratio at the start of consolidation = 0.553 + 0.239 = 0.792

4.3

Example 7.5

Soil investigation at a site gave the following information. Fine sand exists to a depth of 10.6 m and below this lies a soft clay layer 7.60 m thick. The water table is at 4.60 m below the ground surface. The submerged unit weight of sand γ_b is 10.4 kN/m^3, and the wet unit weight above the water table is 17.6 kN/m^3. The water content of the normally consolidated clay w_n = 40%, its liquid limit w_l = 45%, and the specific gravity of the solid particles is 2.78. The proposed construction will transmit a net stress of 120 kN/m^2 at the center of the clay layer. Find the average settlement of the clay layer.

Solution

For calculating settlement [Eq. (7.15a)]

$$S_t = \frac{C_c}{1+e_0} H \log \frac{p_0 + \Delta p}{p_0} \qquad \text{where } \Delta p = 120 \text{ kN/m}^2$$

From Eq. (7.17), $C_c = 0.009\ (w_l - 10) = 0.009(45 - 10) = 0.32$

From Eq. (3.14a), $e_0 = \frac{wG}{S} = wG = 0.40 \times 2.78 = 1.11 \quad$ since $S = 1$

γ_b, the submerged unit weight of clay, is found as follows

$$\gamma_{sat} = \frac{\gamma_w (G_s + e_0)}{1+e_0} = \frac{9.81(2.78+1.11)}{1+1.11} = 18.1 \text{ kN/m}^3$$

$$\gamma_b = \gamma_{sat} - \gamma_w = 18.1 - 9.81 = 8.28 \text{ kN/m}^3$$

The effective vertical stress p_0 at the mid height of the clay layer is

$$p_0 = 4.60 \times 17.6 + 6 \times 10.4 + \frac{7.60}{2} \times 8.28 = 174.8 \text{ kN/m}^2$$

Now $\qquad S_t = \frac{0.32 \times 7.60}{1+1.11} \log \frac{174.8+120}{174.8} = 0.26 \text{ m} = 26 \text{ cm}$

Average settlement $\quad = \quad$ 26 cm.

4.4

Example 7.8

A 2.5 cm thick sample of clay was taken from the field for predicting the time of settlement for a proposed building which exerts a uniform pressure of 100 kN/m² over the clay stratum. The sample was loaded to 100 kN/m² and proper drainage was allowed from top and bottom. It was seen that 50 percent of the total settlement occurred in 3 minutes. Find the time required for 50 percent of the

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total settlement of the building, if it is to be constructed on a 6 m thick layer of clay which extends from the ground surface and is underlain by sand.

Solution

T for 50% consolidation = 0.197.

The lab sample is drained on both sides. The coefficient of consolidation c_v is found from

$$c_v = \frac{TH_{dr}^2}{t} = 0.197 \times \frac{(2.5)^2}{4} \times \frac{1}{3} = 10.25 \times 10^{-2} \text{ cm}^2/\text{min.}$$

The time t for 50% consolidation in the field will be found as follows.

$$t = \frac{0.197 \times 300 \times 300 \times 100}{10.25 \times 60 \times 24} = 120 \text{ days.}$$