Compressibility of Soils

Instructor Tao ZENG
School of Civil Engineering

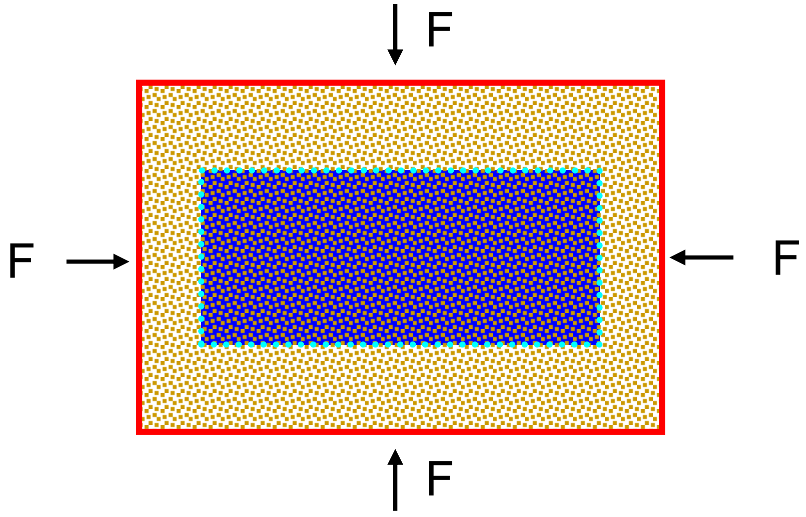
Outline

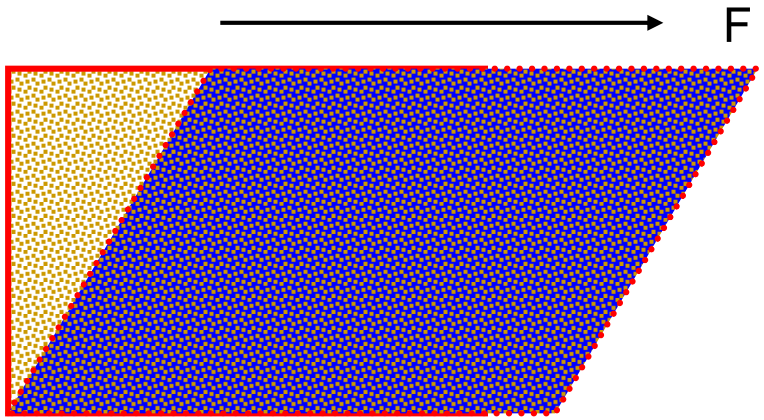
- General introduction
- Compressibility characteristics of soils
- Effect of history on compressibility of soils
- Settlement of ground with layer-wise summation method
- One-dimensional consolidation theory

General introduction

Deformation of soils

- Volumetric deformation — Normal stress
- Shape deformation — Shear stress





General introduction

Settlement: uniform and differential

Schematic diagram of uniform settlement

Schematic diagram of differential settlement

General introduction

Causes of differential settlements

- Different lithological characteristics in the horizontal direction
- Additional vertical stress increment due to newly built building
- Piping leaks, sewer drainage, …
- Inappropriate excavations near the structure
- Vibration

General introduction

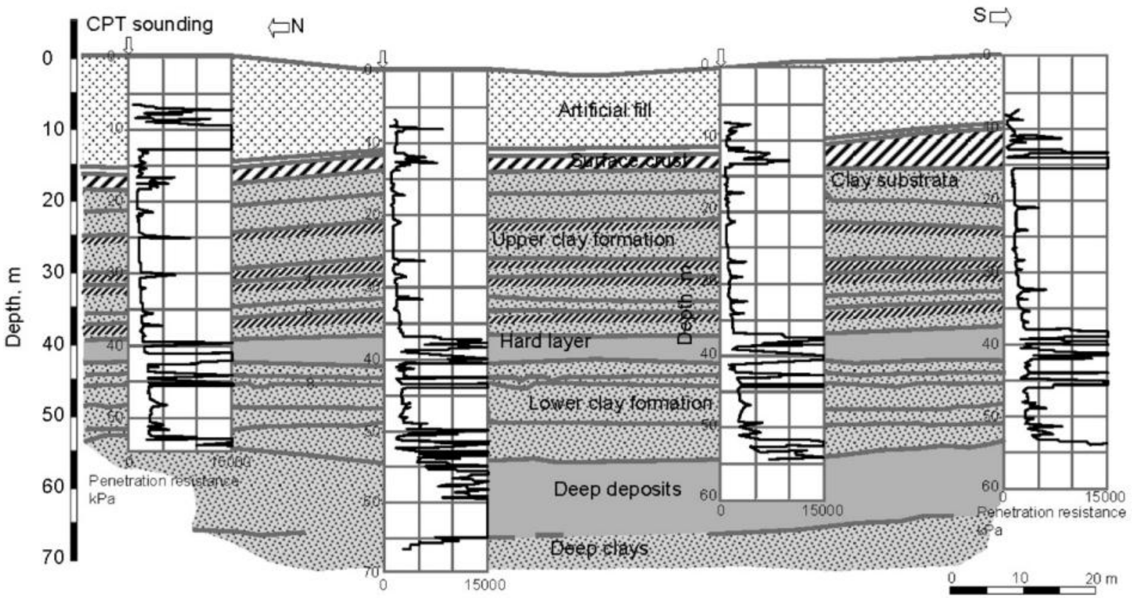
The sinking city of Mexico

Mexico City is mostly located over **extremely soft lacustrine clays** that have been undergoing a consolidation process due to the exploitation of the aquifers underlying these soils. Therefore, **the city has been sinking and will continue to do so in the foreseeable future**; resulting **differential settlements** have constantly damaged most of the city's architectural heritage.

From 1900 to 1920 the settlement rate in downtown Mexico City was 3 cm/year; by the 1940s the rate was 13 cm/year and in the early 1950s it reached 26 cm/year. In the late 1970s and early 1980s new wells were put into operation. Settlement rates increased again and in central Mexico City they now amount to 7-10 cm/year but at some sites near the newer wells they exceed 30 cm/year. The total subsidence **over the last 100 years** with respect to a reference point outside the lake zone is now more than **8m** in some areas.

General introduction

The sinking city of Mexico



Soil profile of central Mexico City

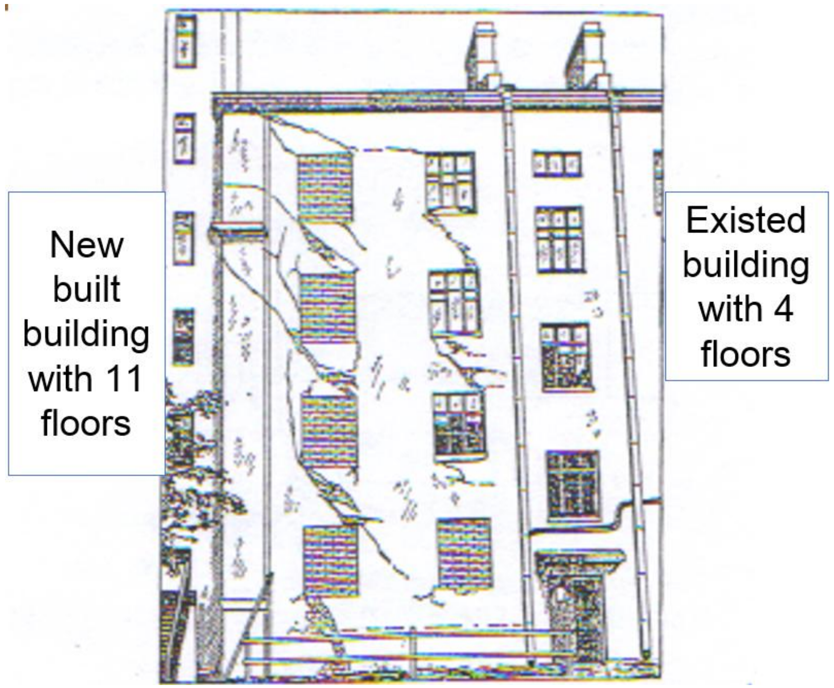
The monumento a la Independcia

A cathedral in Mexico City

General introduction

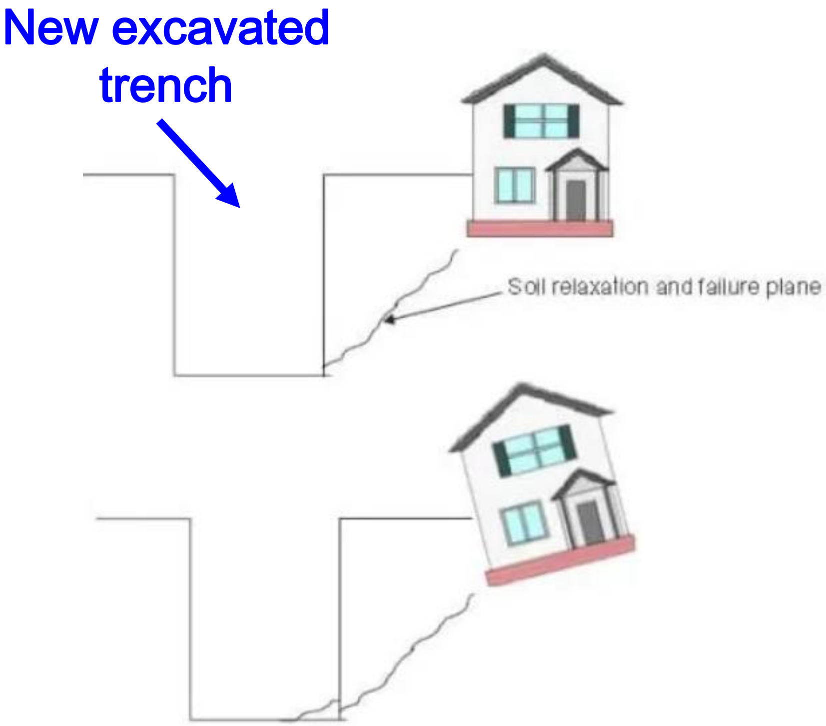
Signs of foundation damage due to nearby construction:

- Cracked tiles
- Hairline cracks in walls or ceilings
- Door and window frames out of alignment
- Cracked wood beams
- Nails protruding from drywall

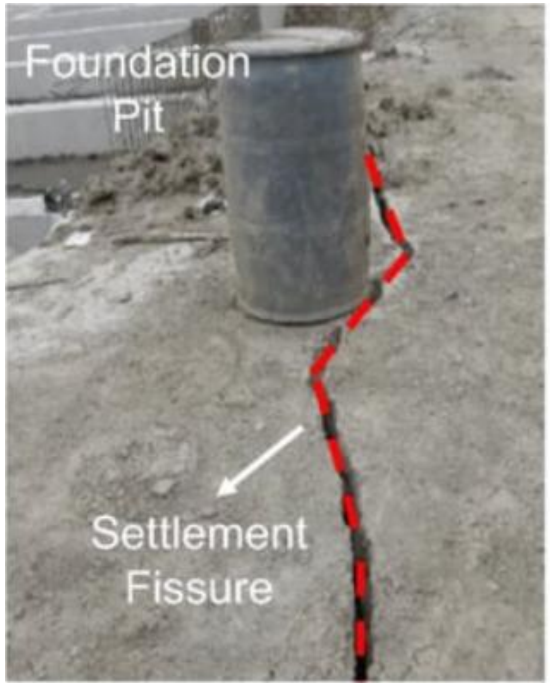


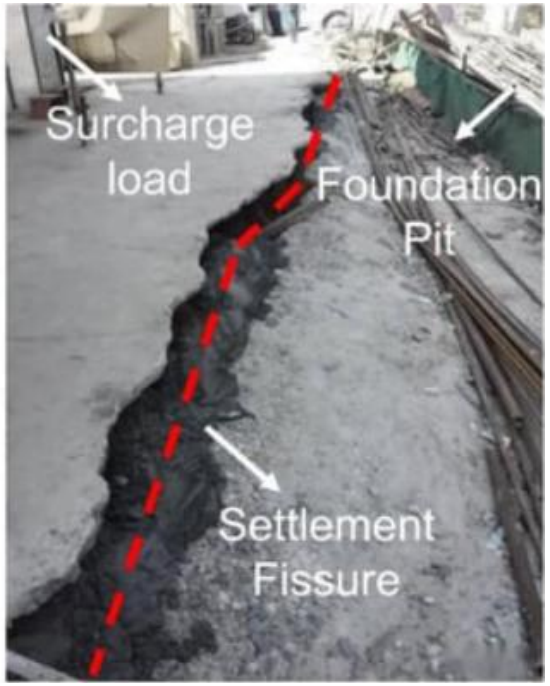
Damage due to height difference

General introduction



Effect of trench excavation on nearby buildings





Ground surface settlements induced by deep excavation

Compressibility characteristics of soils

Compressibility: the decrease of the volume under external loading

The decrease of the volume attribute to:

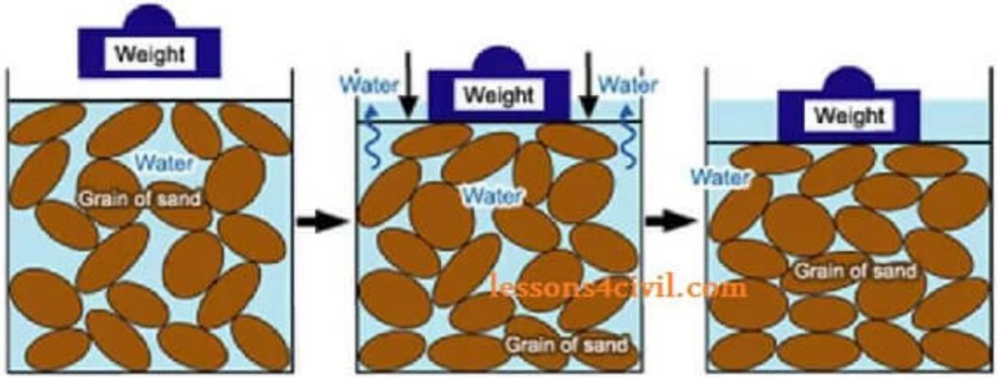
- The compressibility of solid particles and free water
- The compressibility of the gaseous phase
- Certain quantity of water and gas are expelled out

As the content of the gaseous phase is limited, the contribution of the entrapped air to the volume decrease of soil is also negligible.

Compressibility characteristics of soils

The expulsion of pore water is a time dependent process

1. Upon the application of external load, it is initially taken by water and induce excessive water pressure
2. The resulting excessive water pressure initiates a flow of water out of the soil mass, which in turn causes a reduction in the excess pore pressure.



Schematic diagram of expulsion of water

Compressibility characteristics of soils

3. According to the principle of effective stress, the reduction of excessive pore pressure is transferred to the soil skeleton and further leads to the compression of the soil mass. This process won't end until all the excess pore pressure dissipates. The expulsion of water from soil due to excess pore pressure with gradual reduction in soil volume accompanied by transfer of pressure from water to soil skeleton is called **consolidation**.

Compressibility characteristics of soil

Compaction V.S. consolidation

Compaction

- It is a instant process, it can be used for all type of soil.
- It is an artificial process.
- Dynamic loads by rapid mechanical methods like tamping, rolling and vibration are applied for a short interval in soil compaction.
- In compaction, soil changes from partially to fully saturation condition.
- Compaction is due to expulsion and compression of air in soil mass under short duration by moving or vibratory loads.
- Dry density of soil increases but water content will remains same.

Consolidation

- It is a slow process, it can be used for only clayey type of soil.
- It is a natural process.
- Static and sustained loading is applied for a long interval in soil consolidation.
- In consolidation, soil remains in fully saturation condition.
- Consolidation is due to expulsion of pore water from voids under steady, static, long term load.
- Dry density of soil increases but water content will decreases.

Compressibility characteristics of soil

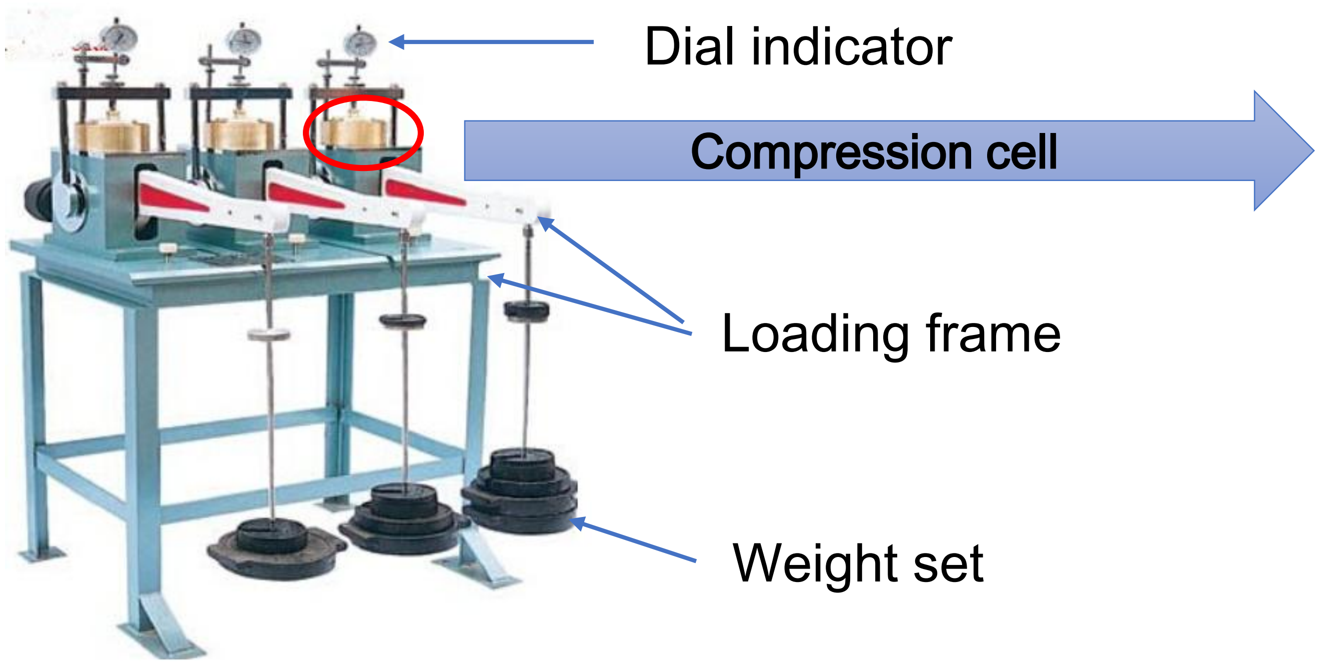
Factors influencing the process of consolidation

- **Permeability characteristics.** Factors influencing the hydraulic conductivity will also affect the process of consolidation. **In general**, cohesionless soils can be compressed in a relatively short period of time as compared to cohesive soils

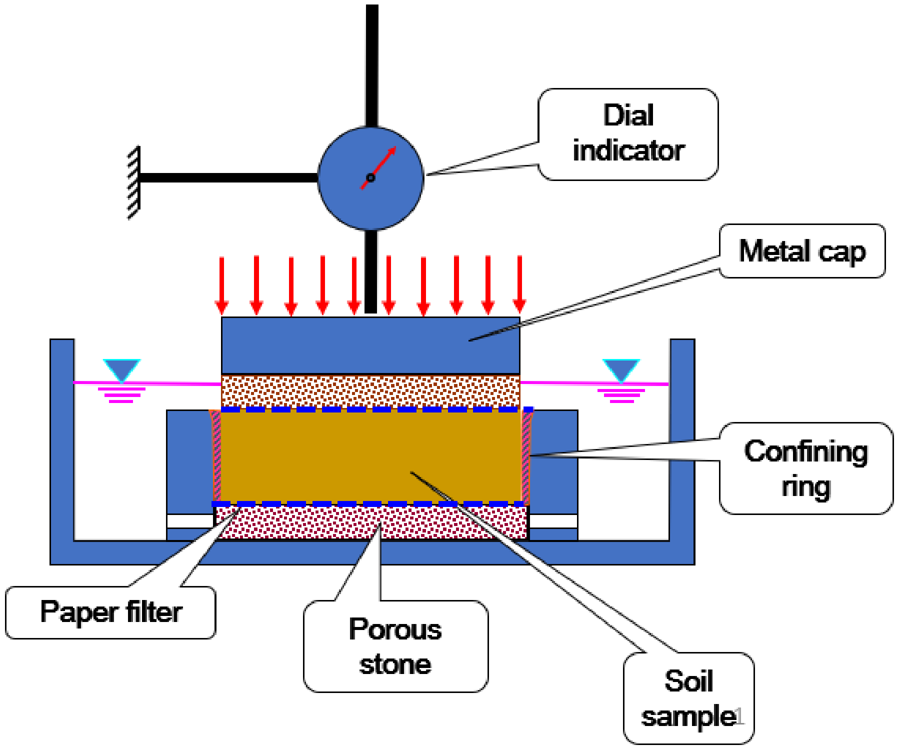
- **Rigidity of the soil skeleton.** The rigidity is dependent on the **structural arrangement** of particles and on the degree to which adjacent particles are bonded together. Soils which possess a honeycombed structure possess high porosity and as such are more compressible

Compressibility characteristics of soil

Indices describing the compressibility of saturated cohesive soils



Setup for Oedometer test



Setup for compression cell

General introduction

Basic procedures:

Sample preparation

- A stiff confining ring with a sharp edge is used to cut a sample of soil directly from a larger block of soil.
- Excess soil is carefully carved away
- Porous stones are placed on the top and bottom of the sample to provide drainage.
- A rigid loading cap is then placed on top of the upper porous stone (For saturated soil samples, it is important to submerge the entire sample ring in water to prevent the sample from drying out)

General introduction

Incremental loading

- This assembly is then placed into a loading frame. Weights are placed on the frame, imposing a load on the soil.
- Compression of the sample is measured over time by a dial indicator. The stability is determined according to the deflection value over time data.
- Another load is then immediately placed on the soil and this process is repeated (if the rebound behavior of the soil sample is the concern, the load on the sample is decreased incrementally).

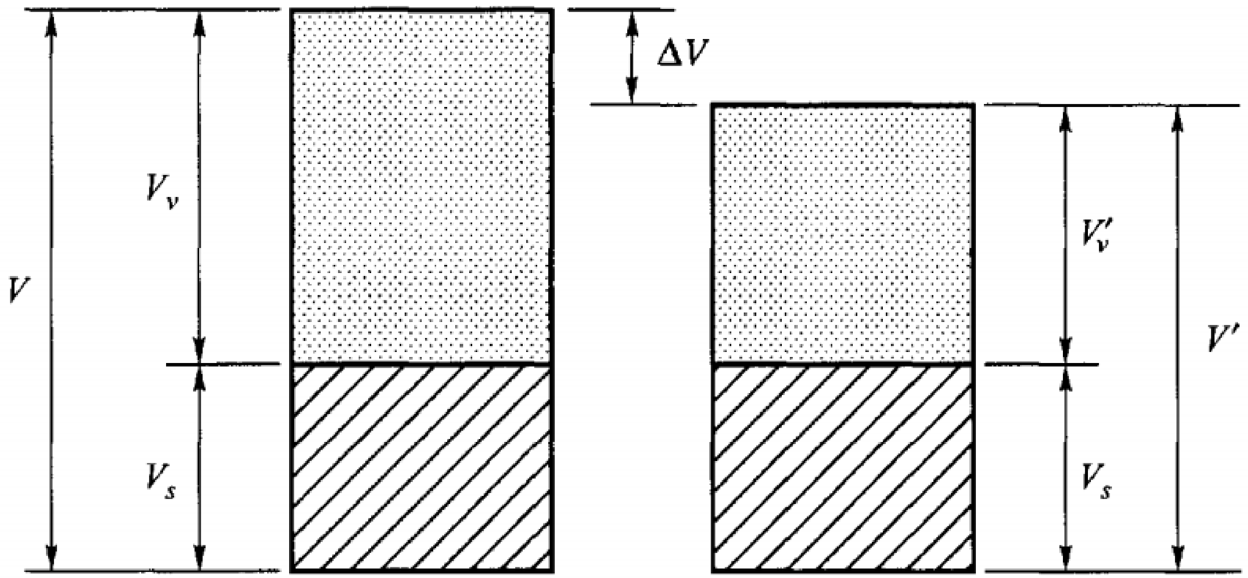
General introduction

Features of oedometer test

- Comparing to the soil sample, the metal ring is assumed rigid. **There is no lateral deformation of the soil sample**
- The water can be expelled out through the top and bottom surfaces
- Water and soil particles are incompressible. **The decrease of volume of soil sample is only attribute to the expulsion of water**

Compressibility characteristics of soil

Variation of void ratio--variation of height



Soil sample before and after test

Before loading

$$V_v = eV_s \qquad V = V_s\left(1+e_0\right)$$

After loading

$$V_v = eV_s \qquad V = V_s\left(1+e_1\right)$$

Volume decrease

$$\frac{\Delta V}{V} = \frac{V'-V}{V} = \frac{V_s\left(1+e_1\right)-V_s\left(1+e_0\right)}{V_s\left(1+e_0\right)} = \frac{e_1-e_0}{1+e_0}$$

Rigid container

$$\frac{\Delta h}{h_0} = \frac{e_1-e_0}{1+e_0} \quad \Rightarrow \quad e_1 = e_0 + \frac{\Delta h}{h_0}\left(1+e_0\right)$$

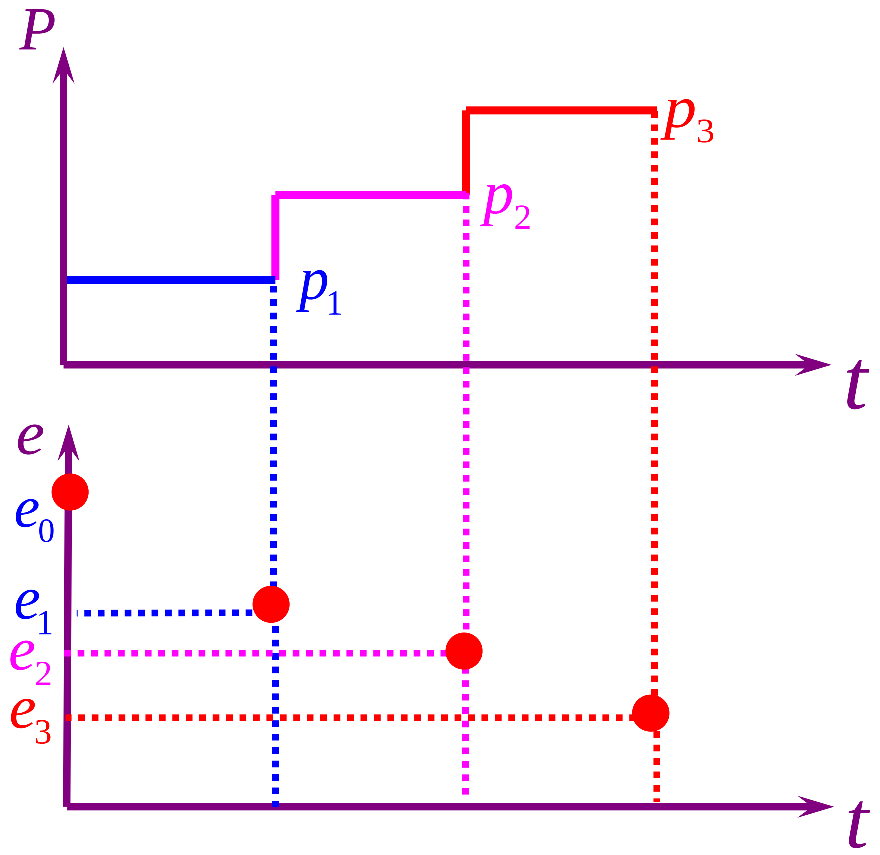
Compressibility characteristics of soil

Increment loading

For loading stage *i*,

$$e_i = e_0 + \frac{\Delta h_i}{h_0}(1+e_0)$$

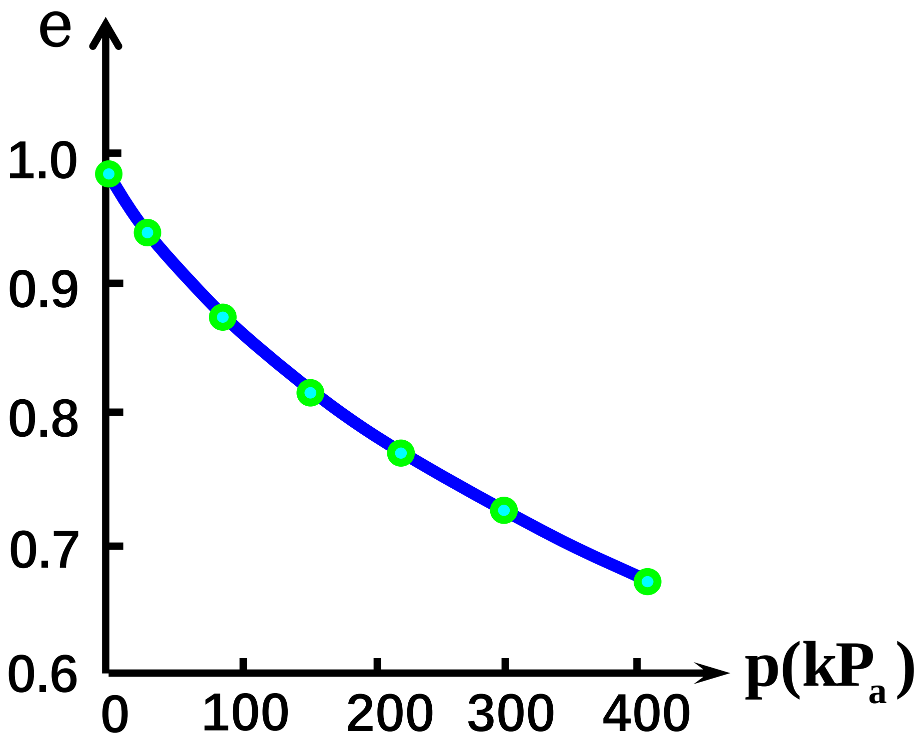
e_i is void ratio at stage i and Δh_i is the variation of height



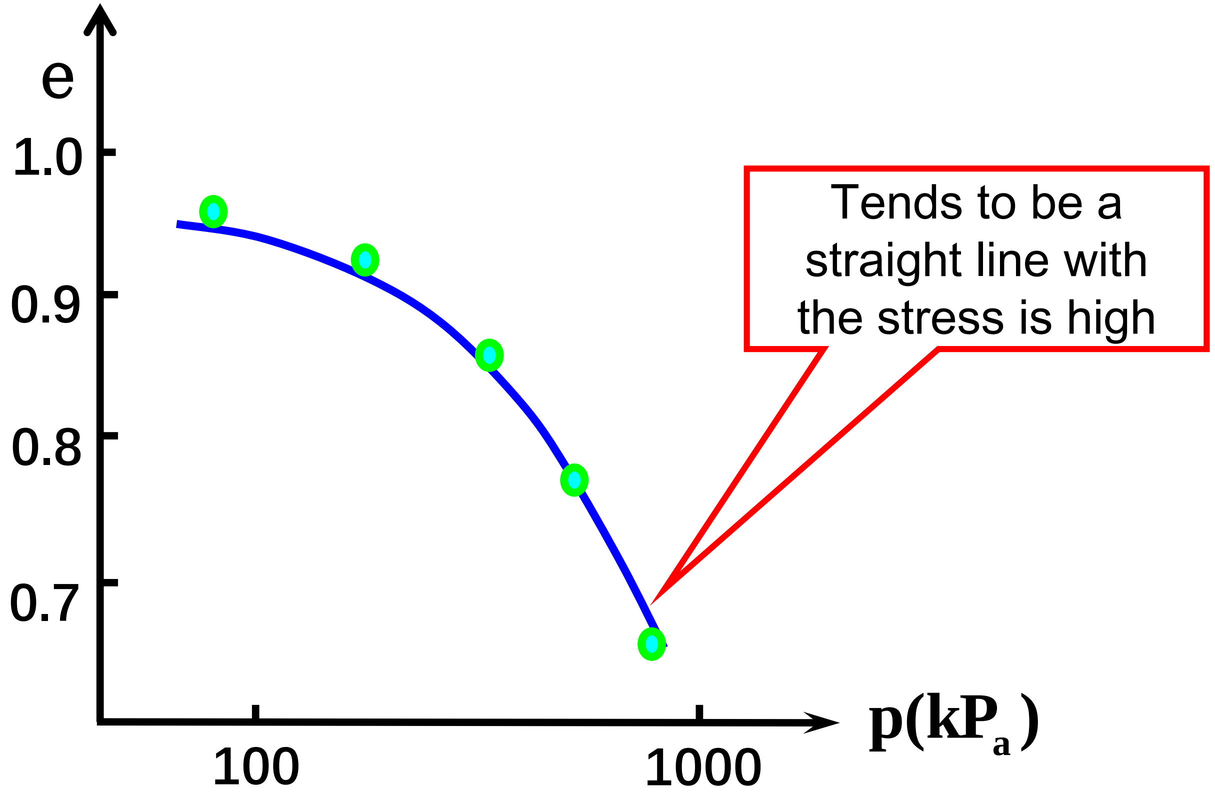
Schematic diagram of incremental loading

Compressibility characteristics of soil

The virgin compression curve



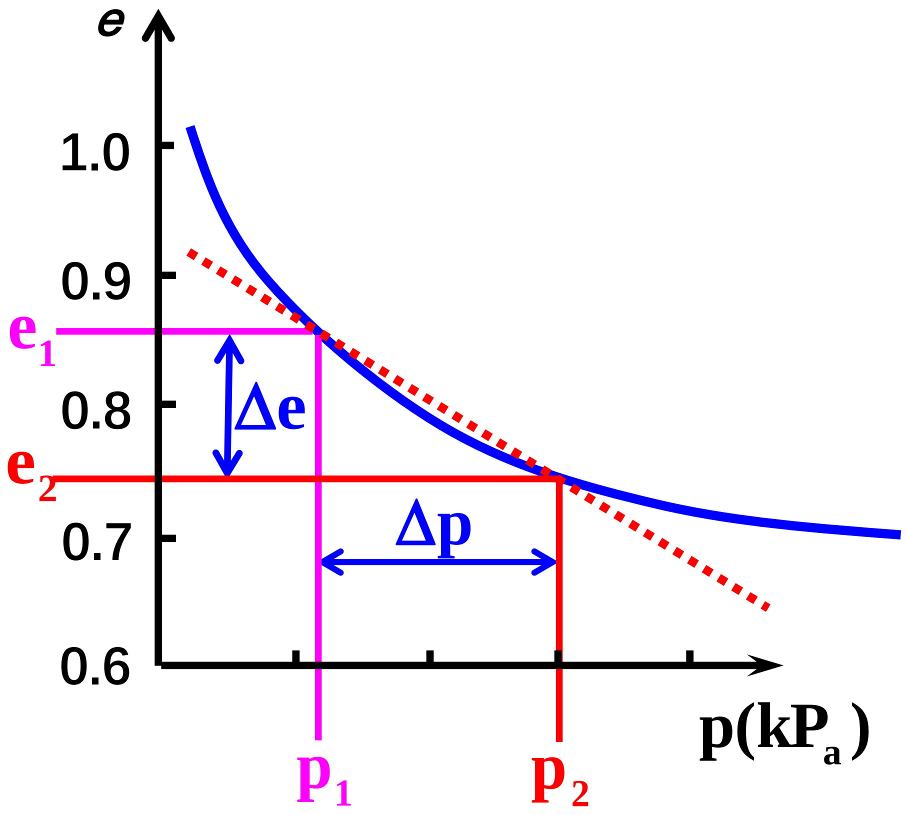
Typical diagram of e~p curve



Typical diagram of e~logp curve

Compressibility characteristics of soil

Coefficient of compressibility



$$a_v = -\frac{\Delta e}{\Delta p} = -\frac{e_2 - e_1}{p_2 - p_1}$$

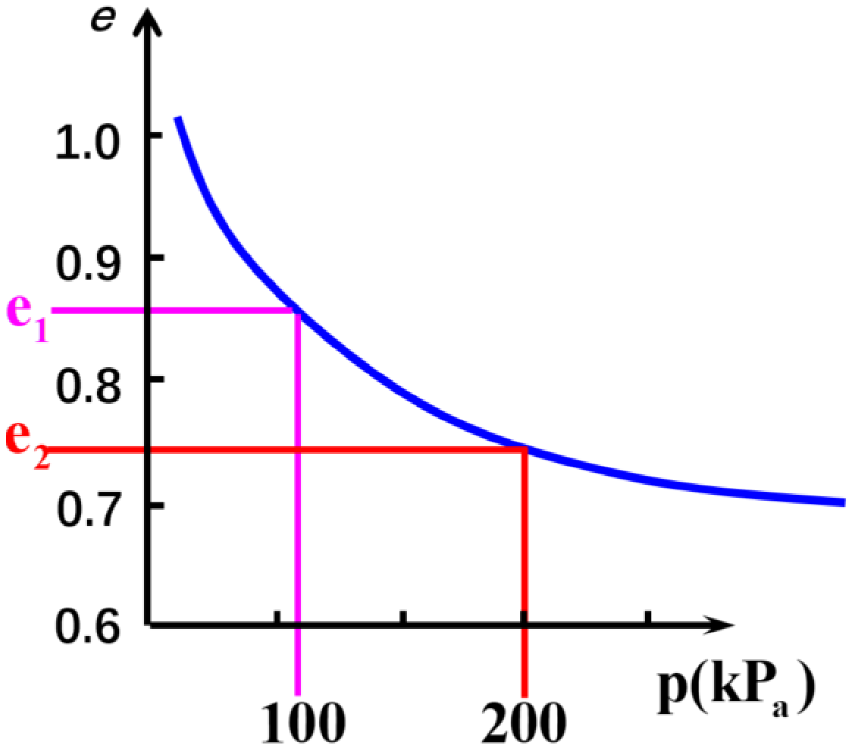
The larger the coefficient of compressibility, the higher the compressibility of soils

Generalization:

$$a_v = -\frac{\Delta e_i}{\Delta p_i} = -\frac{e_{i+1} - e_i}{p_{i+1} - p_i}$$

Compressibility characteristics of soil

Compressibility of soils by a_v



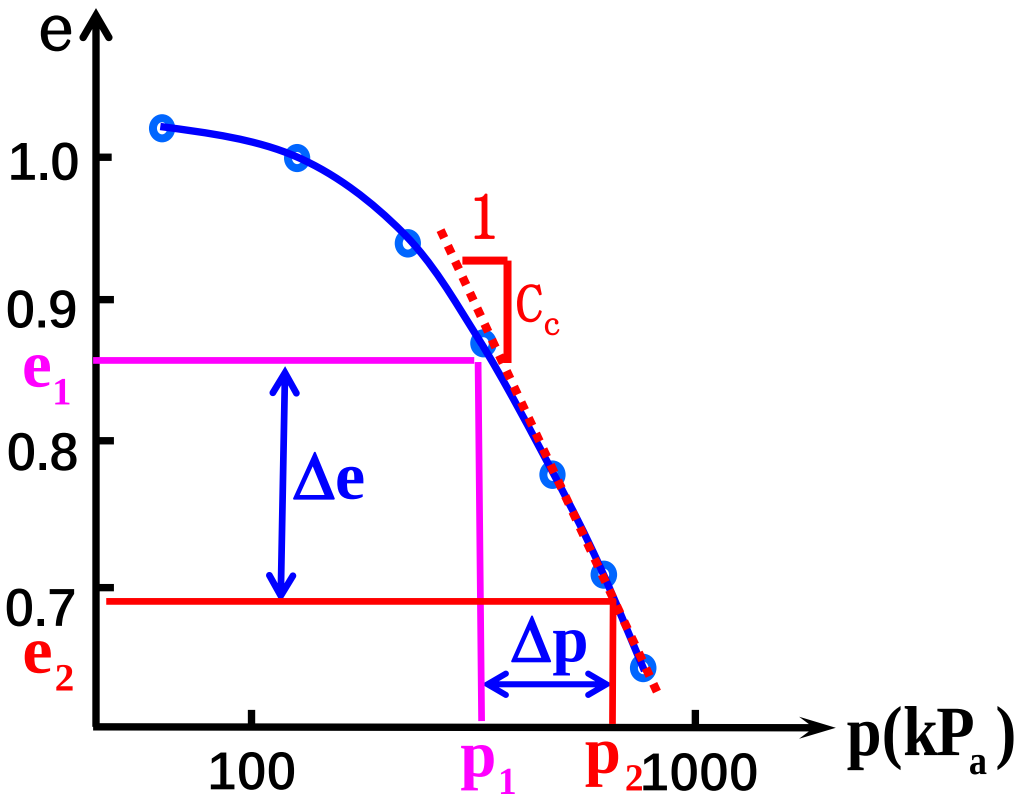
To evaluate the compressibility characteristics, two points: P1=100kPa and P2=200kPa are taken as

$$a_{v1-2} = -\frac{e_2 - e_1}{p_2 - p_1} = -\frac{\Delta e}{100}$$

Compressibility of soils	a_{v1-2} (MP_a^{-1})
High compressibility	>=0.5
Medium compressibility	0.1-0.5
Low compressbility	<0.1

Compressibility characteristics of soil

Compression index（C_c）



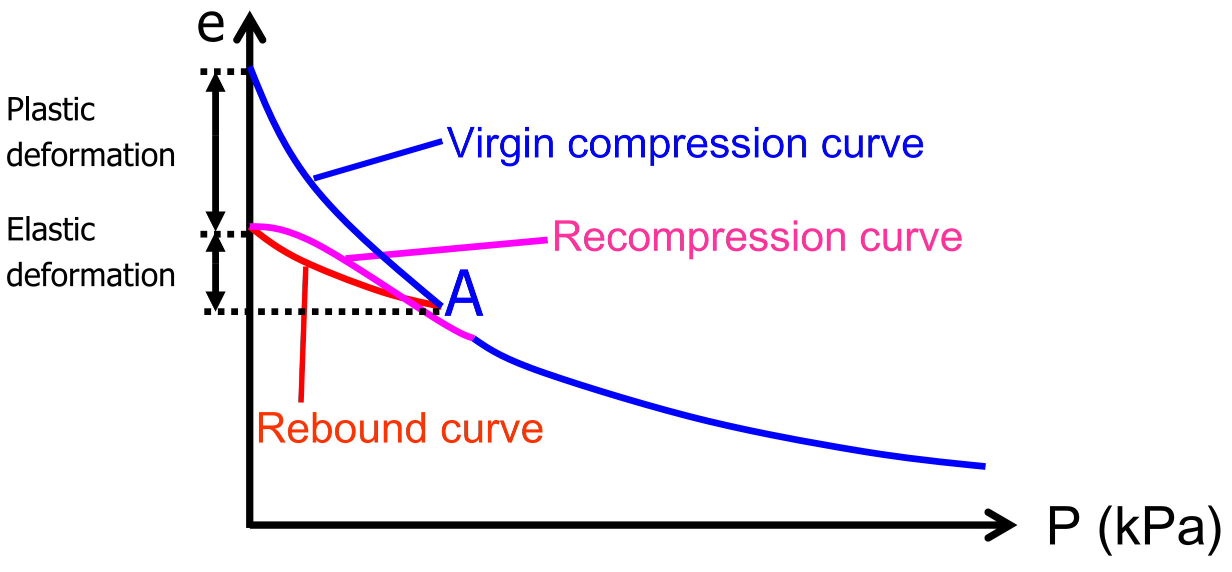
$$C_c = -\frac{e_2 - e_1}{\log p_2 - \log p_1} = -\frac{\Delta e}{\log\left(\dfrac{p_1 + \Delta p}{p_1}\right)}$$

The larger the compression index, the higher the compressibility.

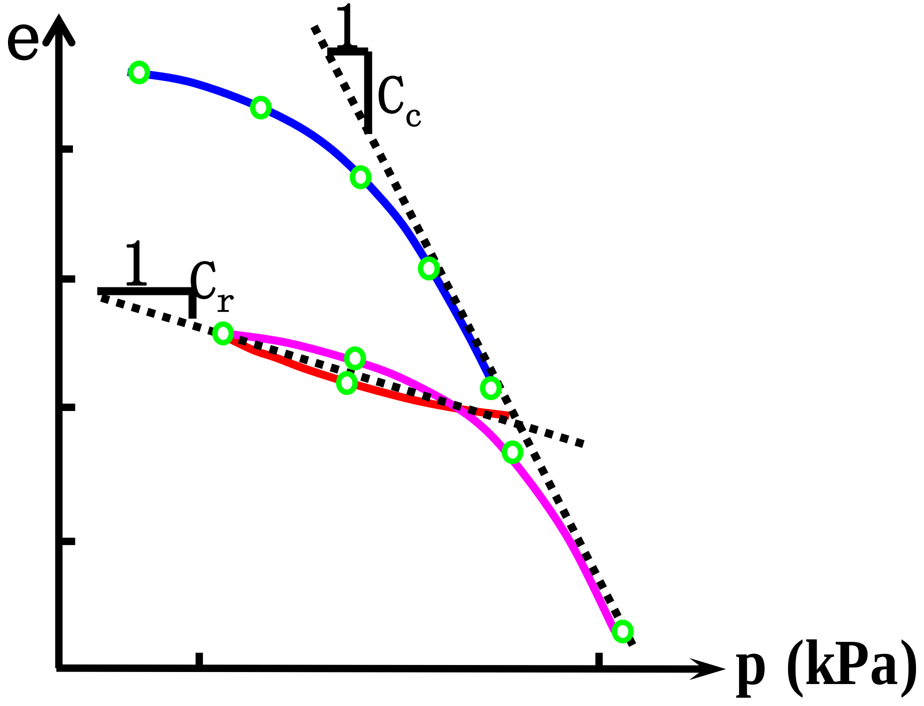
Compressibility of soils	C_c
High compressibility	>=0.4
Low compressbility	<0.2

Compressibility characteristics of soil

Recompression index (C_r)



e~p curve with loading and unloading



e~logp curve with loading and unloading

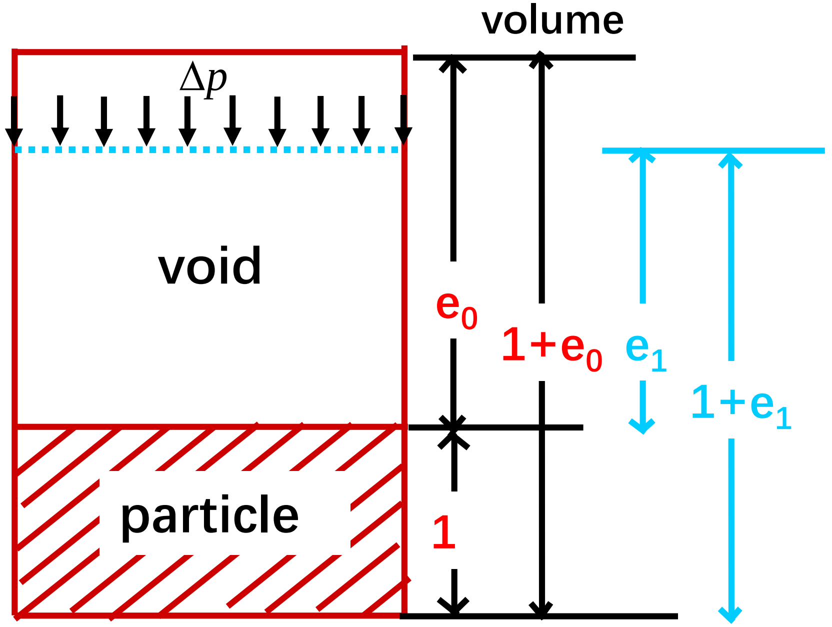
Compressibility characteristics of soil

Coefficient of volume compressibility (m_v)： the ratio of unit change in volume (change in the volume of soil to unit original volume) to the change in the effective stress **under confined condition**

$$m_V = \frac{-\Delta V}{\Delta p V} = -\frac{(1+e_1)-(1+e_0)}{\Delta p(1+e_0)}$$

$$= -\frac{e_1 - e_0}{\Delta p(1+e_0)} = -\frac{\Delta e}{\Delta p}\frac{1}{(1+e_0)}$$

$$= \frac{a_v}{(1+e_0)}$$

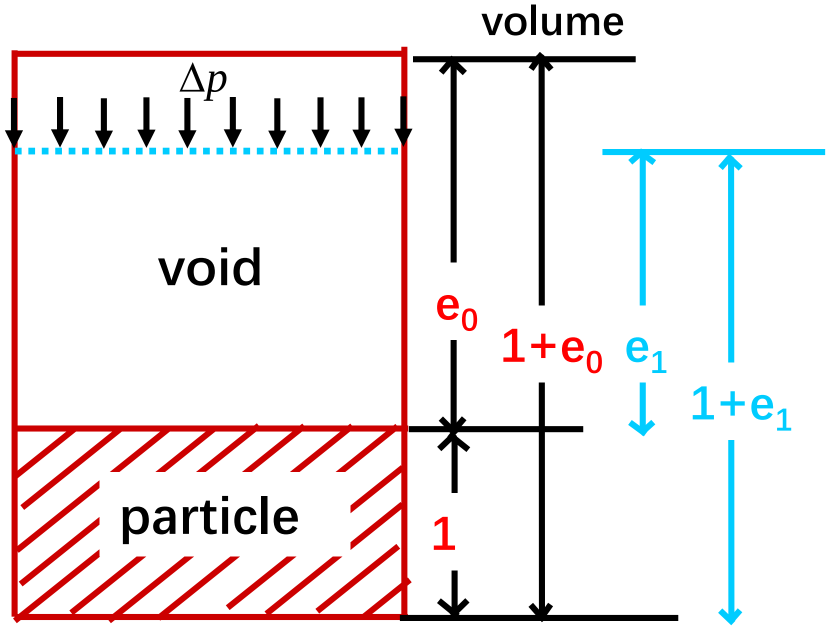


Compressibility characteristics of soil

Constrained modulus (E_s) : the ratio of the change in the **vertical stress** to the change in the **vertical strain** under the confined condition

$$E_s = -\frac{\Delta\sigma_z}{\Delta\varepsilon_z} = -\frac{\Delta p}{\Delta\varepsilon_v} = -\frac{\Delta p}{\frac{\Delta V}{V_0}}$$

$$= -\frac{\Delta p}{\frac{\Delta e}{1+e_0}} = -\frac{\Delta p\left(1+e_0\right)}{\Delta e} = \frac{1+e_0}{a_v} = \frac{1}{m_v}$$



Compressibility characteristics of soil

Constrained elastic modulus (E)： the ratio of the change in the vertical stress to the change in the vertical strain under the confined condition

Stress-strain relation in elastic stage (Hooke's law)

$$\begin{cases} \Delta\varepsilon_x = \dfrac{\sigma_x}{E} - \dfrac{v}{E}\left(\sigma_y + \sigma_z\right) \\ \Delta\varepsilon_y = \dfrac{\sigma_y}{E} - \dfrac{v}{E}\left(\sigma_x + \sigma_z\right) \\ \Delta\varepsilon_z = \dfrac{\sigma_z}{E} - \dfrac{v}{E}\left(\sigma_x + \sigma_y\right) \end{cases}$$

Compressibility characteristics of soil

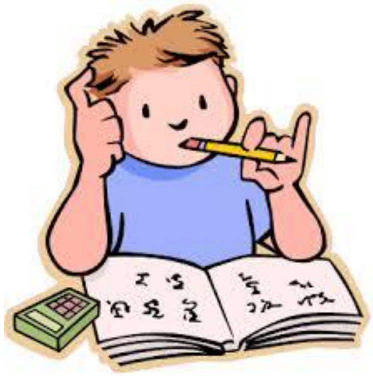
Under confined condition: $\Delta\varepsilon_x = \Delta\varepsilon_y = 0 \quad \Delta\sigma_x = \Delta\sigma_y = K_0\Delta\sigma_z$

$$\begin{cases} \Delta\varepsilon_x = \dfrac{\sigma_x}{E} - \dfrac{v}{E}\left(\sigma_y + \sigma_z\right) \\ \Delta\varepsilon_y = \dfrac{\sigma_y}{E} - \dfrac{v}{E}\left(\sigma_x + \sigma_z\right) \\ \Delta\varepsilon_z = \dfrac{\sigma_z}{E} - \dfrac{v}{E}\left(\sigma_x + \sigma_y\right) \end{cases}$$

$$E = \frac{\Delta\sigma_z}{\Delta\varepsilon_z}\left(1 - \frac{2v^2}{1-v}\right) = E_s\left(1 - \frac{2v^2}{1-v}\right)$$

Relation between E and E_s: $\boxed{E < E_s}$

Exercises



What is the mechanism for preloading method

Pre-loading method

Exercises

During an oedometer test, the porosity corresponding to the pressure 100kPa and 200kPa are 48.6% and 48.3%, respectively (the initial void ratio is 0.95).

1) Determine the coefficient of compressibility a_v, coefficient of volume compressibility m_v, constrained modulus E_s
2) Evaluate the compressibility characteristics

Exercises

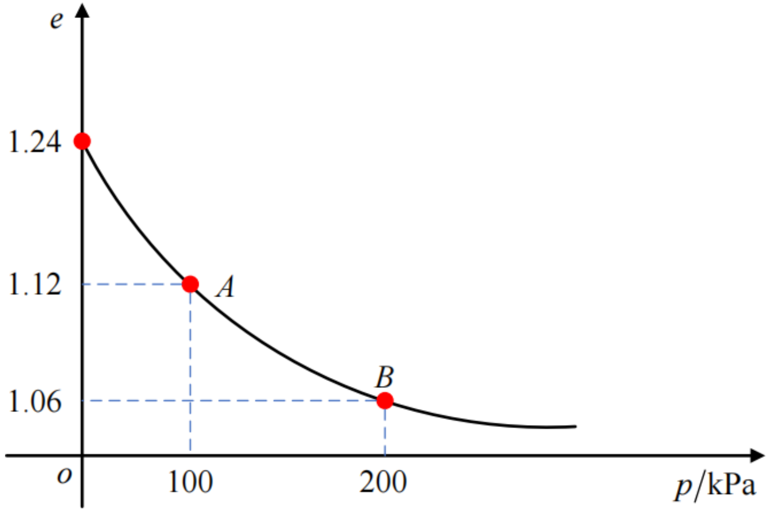
During an oedometer test, the thickness of the clay layer is 4m and its initial void ratio is 1.25. If the uniform load 100kPa is applied on its surface, the void ratio decreases to 1.12.

1) Determine the settlement of the clay layer
2) Determine the coefficient of volume compressibility m_v

Exercises

In an oedometer test, the $e-p$ compression curve is as Fig. 1 shows. Determine:

1) The coefficient of compressibility according two loading stages inicated by point A and B; 2) The coefficient of volume compressibility.



Exercises

During an oedometer test, a sample of fully saturated clay 3 cm thick is consolidated under a pressure increment of 200 kPa. When equilibrium is reached, the sample thickness is reduced to 2.6 cm. The pressure is then removed and the sample is allowed to expand and absorb water. The final thickness is observed as 2.8 cm and the water content is 24.9 %. Determine: 1) the void ratio after expansion; 2) the void ratio after consolidation; 3) the void ratio before consolidation (the specific gravity of the solid is 2.70).

Effect of stress history on the compressibility of soils

Stress history: the stress state to which the soil has experienced

Consolidation pressure (p_o): the effective vertical stress which results in the consolidation of soils

Pre-consolidation pressure (p_c): the maximum effective vertical stress to which the soil has sustained in the past

Over consolidation ratio (OCR): the ratio of p_c to the current effective stress p_0',

$OCR = p_c / p_0$

Effect of stress history on the compressibility of soils

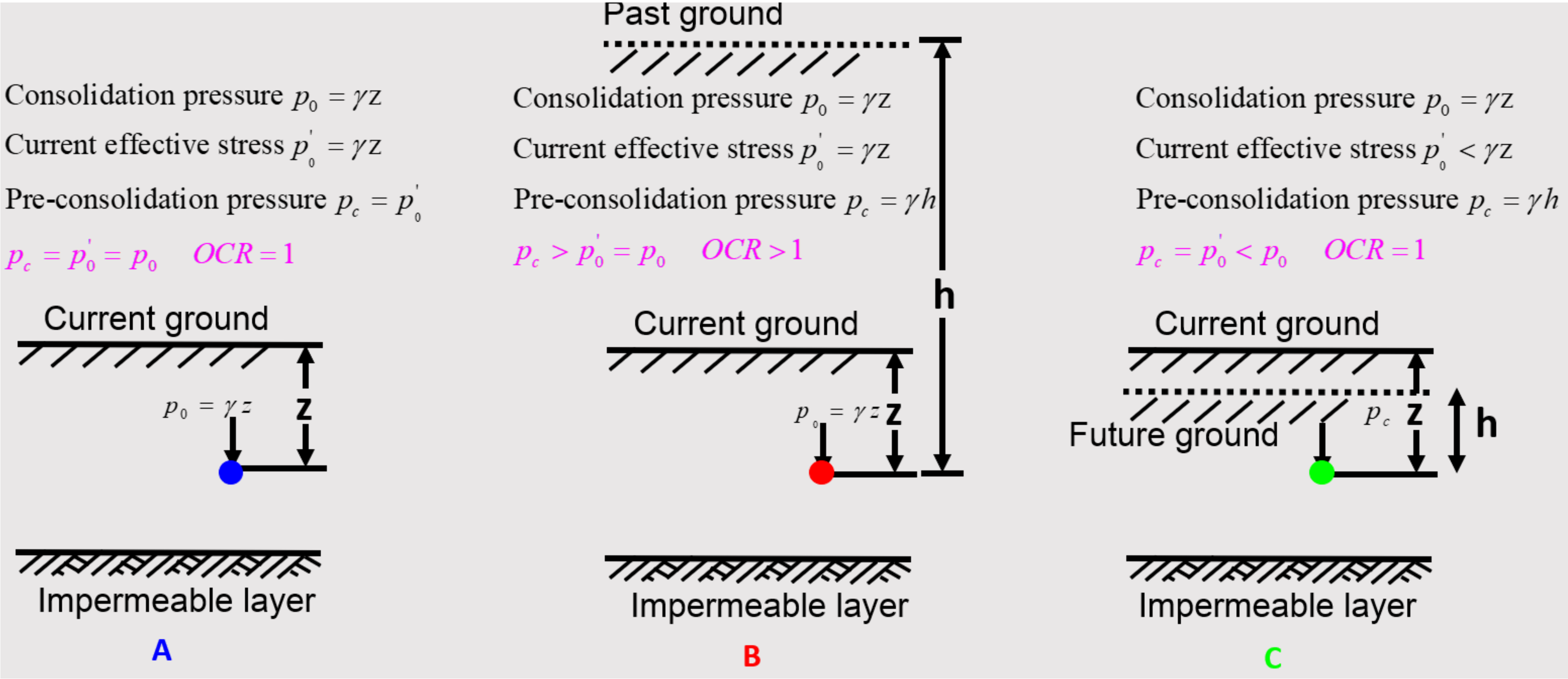
State of soils based on stress history

OCR>1, over-consolidated (OC) soil, end of consolidation process

OCR=1 and the soil is stable under the action of self-weight, normal consolidation (NC) soil, end of consolidation process

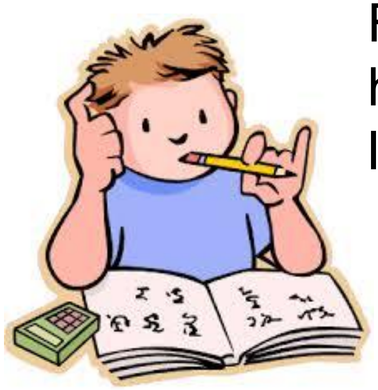
OCR<1, under consolidated (UC) or unconsolidated soil, consolidation process ongoing

Effect of stress history on the compressibility of soils

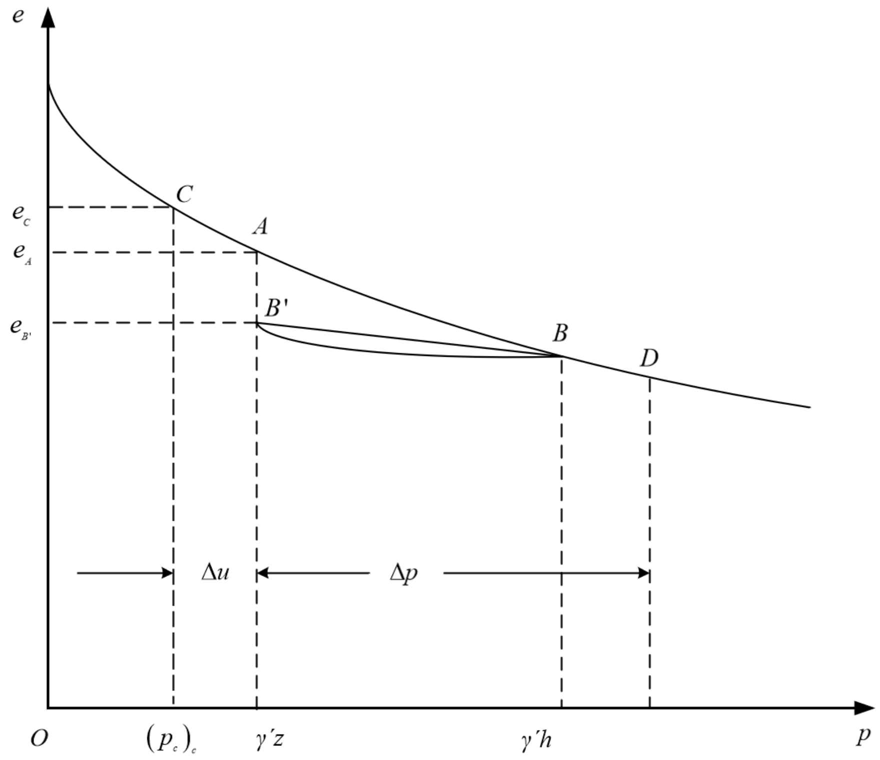


NC OC UC

Exercises



For three soil layers (NC, OC, UC, respectively), the point at the same depth z have same effective geostatic stress. Determine which one will have the largest settlement if they subjected to the same external loading.



Effect of stress history on the compressibility of soils

Pre-consolidation pressure (p_c) with Casagrande's graphical method

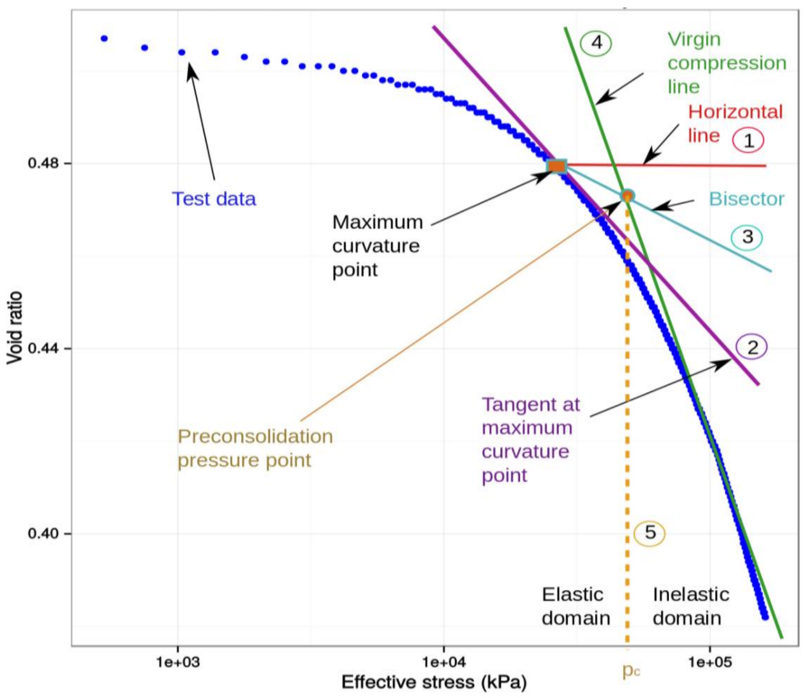


Illustration diagram for Casagrande's method

Procedures

1. Determine the point of maximum curvature on the consolidation curve. (This is could be done more precisely with the help curve fitting. Based on the fitted curve, the point corresponding to maximum curvature can be more conveniently located). Draw a horizontal line through that point
2. Draw a straight line tangent to the curve at that point.
3. Bisect the angle made from the horizontal line and the tangent line.

Effect of stress history on the compressibility of soils

Pre-consolidation pressure (p_c) with Casagrande's graphical method

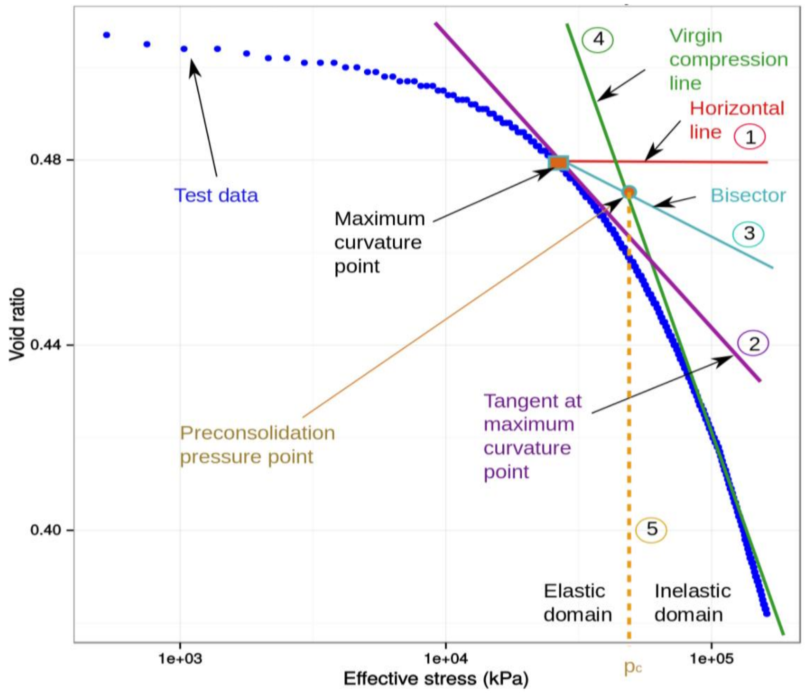
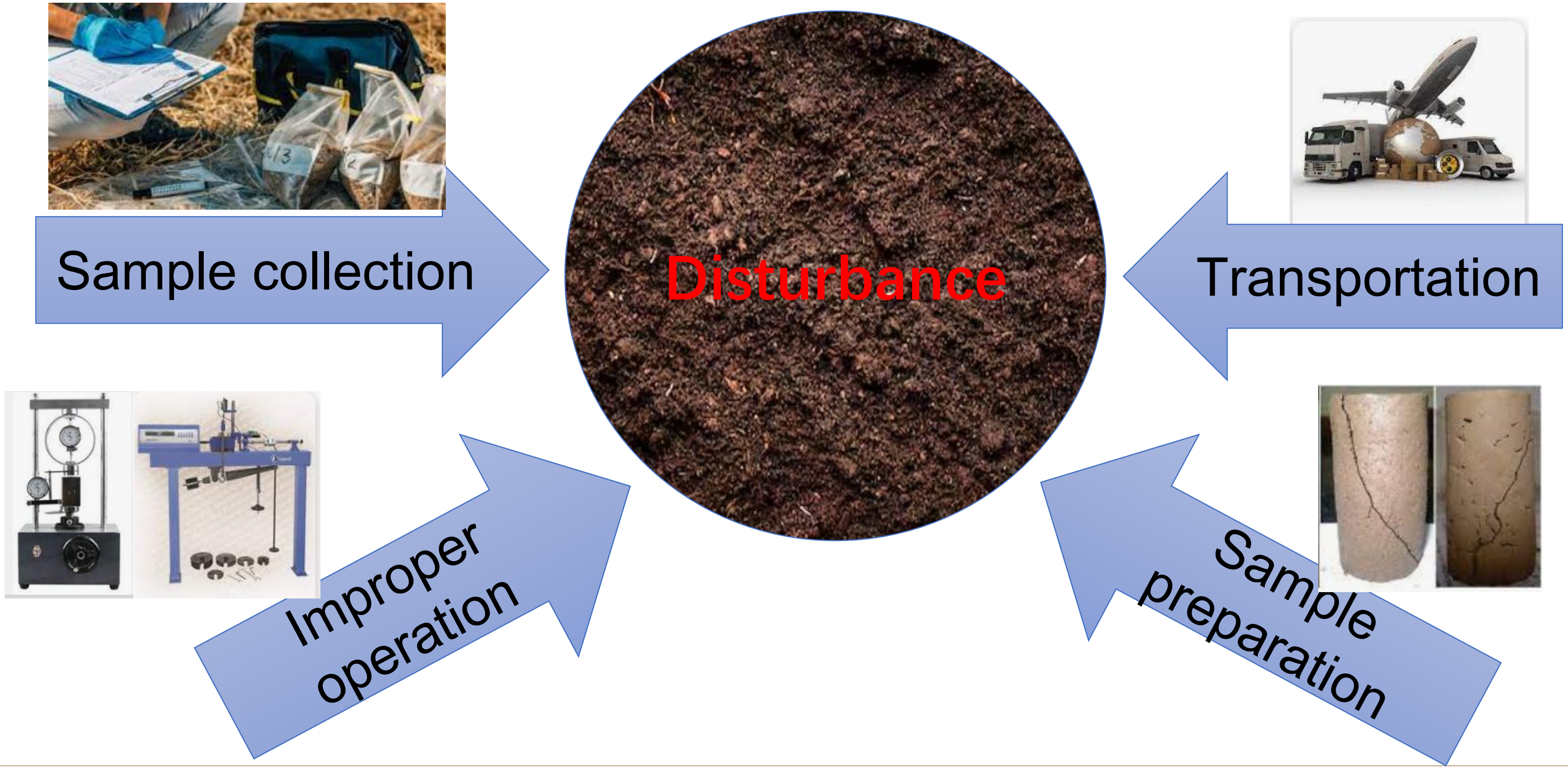


Illustration diagram for Casagrande's method

4. Extend the "straight portion" of the virgin compression curve up to intersect with the bisector line.
5. The abscissa of the point of intersection corresponds to the pre-consolidation pressure p_c

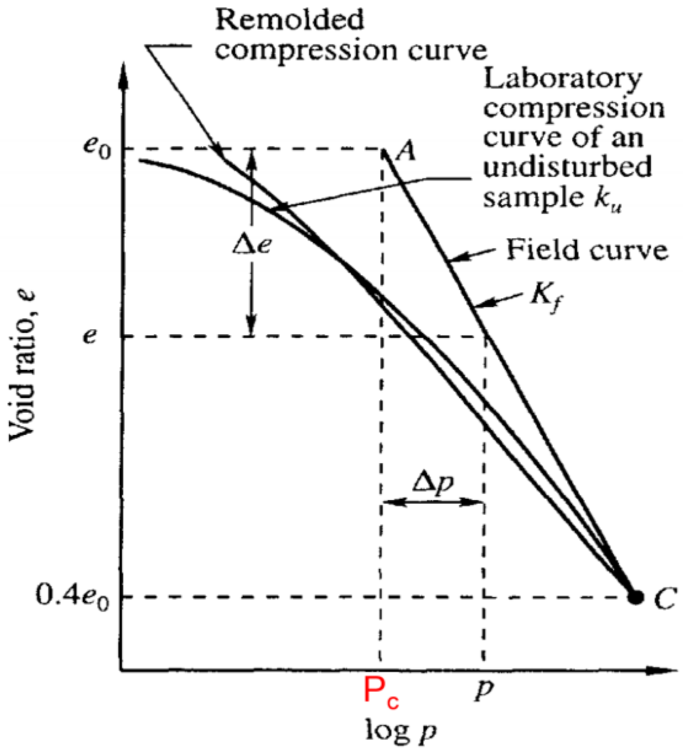
Settlement of ground with layer-wise summation method

Why is laboratory compression curve need to be modified?



Settlement of ground with layer-wise summation method

In-situ e-log(p) compression curve for NC soil



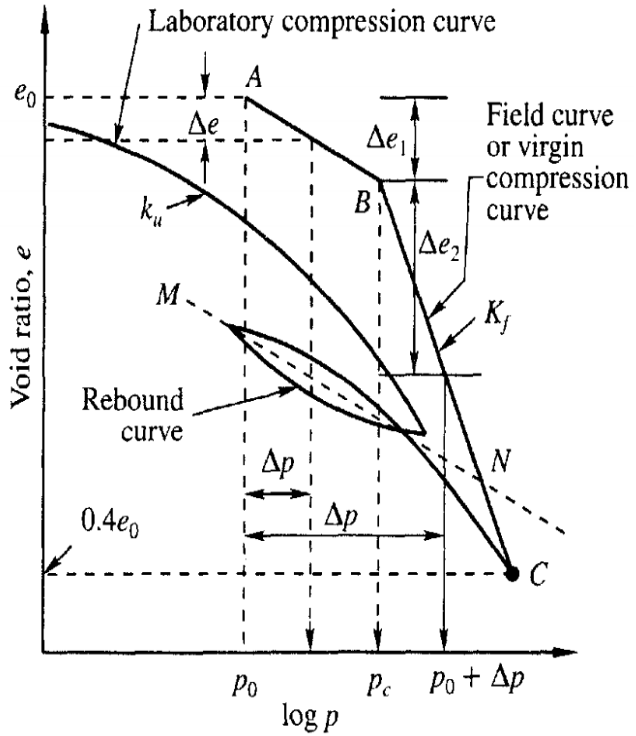
In-situ compression curve determination for NC soil

Procedures

1. Draw a horizontal line passing through the initial void ratio e_0 and a vertical line passing through the pre-consolidation pressure p_c . The intersection point A is determine.
2. Determine the point C corresponds to the void ratio $0.4e_0$.
3. The straight line AC gives the *in-situ* e-log(p) compression curve for normally consolidated clay soil

Settlement of ground with layer-wise summation method

In-situ e-log(p) compression curve for OC soil



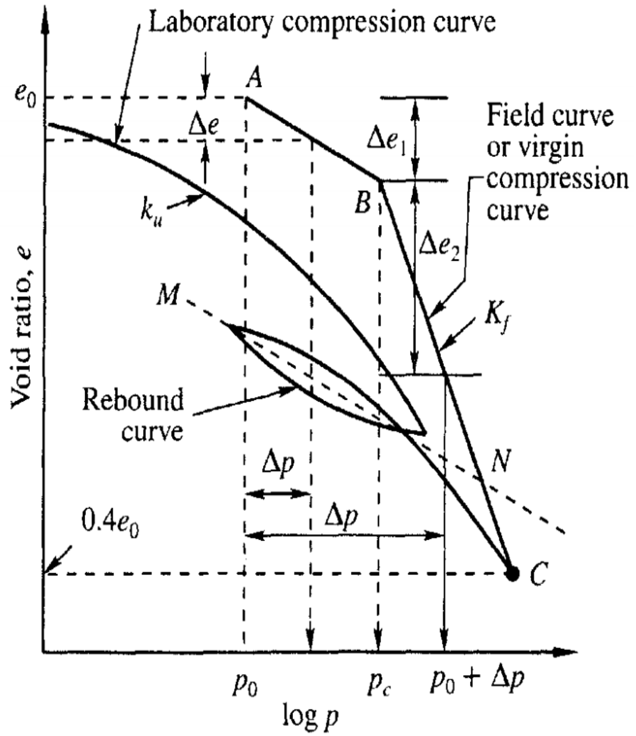
In-situ compression curve determination for OC soil

Procedures

1. Draw a horizontal line passing through the initial void ratio e_0 and a vertical line passing through the current effective pressure p_0 . The intersection point A is determined
2. Starting from point A, draw an inclined line in parallel with MN (mean slope of the rebound-recompression laboratory curve). The inclined line intersect with a vertical line passing through the pre-consolidation pressure p_c at point B. The straight line AB gives the first part in-situ compression curve for OC soil

Settlement of ground with layer-wise summation method

In-situ e-log(p) compression curve for OC soil



In-situ compression curve determination for OC soil

3. Determine the point C corresponds to the void ratio $0.4e_0$. The straight line BC gives the second part in-situ compression curve for OC soil

Settlement of ground with layer-wise summation method

In-situ e-log(p) compression curve for UC soil

The under consolidated soil is a special normal consolidated soil. Therefore, the procedure to determine its *in-situ* compression curve is the same as that of the normal consolidated (NC) soil.

Settlement of ground with layer-wise summation method

Settlement of the ground beneath the foundation

With the *in-situ* e-log(p) compression curve, the settlement of the ground can be estimated with the following data:

- The thickness of soil layer (H)
- The initial void ratio (e_0)
- The pre-consolidation pressure (p_c), current effective geostatic stress (σ'_{cz}), and the vertical stress increment (σ_z).
- The *in-situ* e-log(p) compression curve and the corresponding C_c

Settlement of ground with layer-wise summation method

One step computational method

When ground is composed of **a single thin soil layer**, one-step computation is suitable. The change in thickness per unit of thickness H is related through

$$\frac{\Delta h}{H} = \frac{\Delta e}{1+e_0} \rightarrow \Delta h = \frac{H}{1+e_0}\Delta e$$

Δe is calculated under the action of vertical stress increment (σ_z) with the *in-situ* e-log(p) compression curve

For a thin soil layer, the effective geostatic stress and the vertical stress increment are **assumed uniform throughout and evaluated at the mid depth**

Settlement of ground with layer-wise summation method

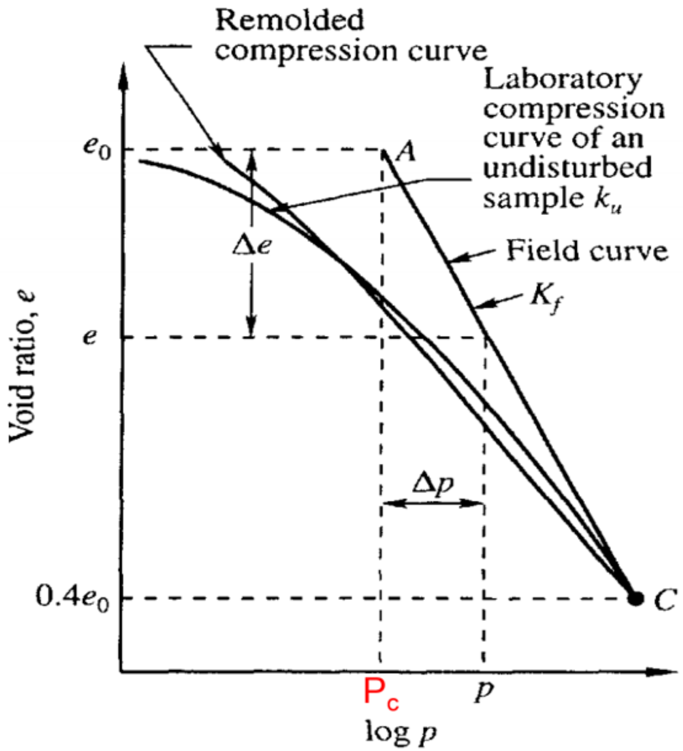
Normally consolidated (NC) soil

The decrease in void ratio equals

$$\Delta e = -C_c\left(\text{logp} - \log\sigma'_{cz}\right) = -C_c\left(\log\left(\sigma'_{cz}+\sigma_z\right) - \log\sigma'_{cz}\right) = -C_c\log\left(\frac{\sigma'_{cz}+\sigma_z}{\sigma'_{cz}}\right)$$

The settlement equals

$$\Delta h = \frac{-C_c \cdot H}{1+e_0}\log\left(\frac{\sigma'_{cz}+\sigma_z}{\sigma'_{cz}}\right)$$



In-situ compression curve determination for NC soil

Settlement of ground with layer-wise summation method

Over consolidated (OC) soil

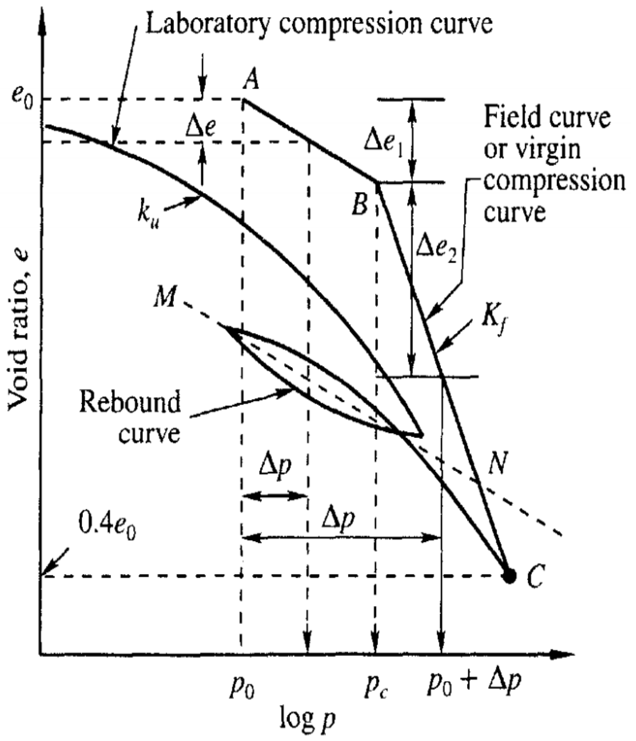
An additional step is needed to check the magnitude of $\sigma'_{cz} + \sigma_z$ and p_c

If $\boldsymbol{\sigma'_{cz} + \sigma_z \le p_c}$, compression is along AB

$$\Delta e = -C_s\left(\log\left(\sigma'_{cz} + \sigma_z\right) - \log \sigma'_{cz}\right) = -C_s \log\left(\frac{\sigma'_{cz} + \sigma_z}{\sigma'_{cz}}\right)$$

The settlement equals

$$\Delta h = -C_s H \frac{\log\left(\frac{\sigma'_{cz} + \sigma_z}{\sigma'_{cz}}\right)}{1 + e_0}$$



In-situ compression curve determination for OC soil

Settlement of ground with layer-wise summation method

If $\sigma'_{cz} + \sigma_z > p_c$, compression is firstly along AB, then along BC

When the compression is firstly along AB, the decrease in void ratio equals

$$\Delta e_1 = -C_s\left(\log\left(p_c\right) - \log \sigma'_{cz}\right) = -C_s \log\left(\frac{p_c}{\sigma'_{cz}}\right)$$

When the compression is along BC, the decrease in void ratio equals

$$\Delta e_2 = -C_c\left(\log\left(\sigma'_{cz} + \sigma_z\right) - \log p_c\right) = -C_c \log\left(\frac{\sigma'_{cz} + \sigma_z}{p_c}\right)$$

Settlement of ground with layer-wise summation method

The total decrease in void ratio equals

$$\Delta e = \Delta e_1 + \Delta e_2$$

$$= -C_s \log\left(\frac{p_c}{\sigma'_{cz}}\right) - C_c \log\left(\frac{\sigma'_{cz} + \sigma_z}{p_c}\right)$$

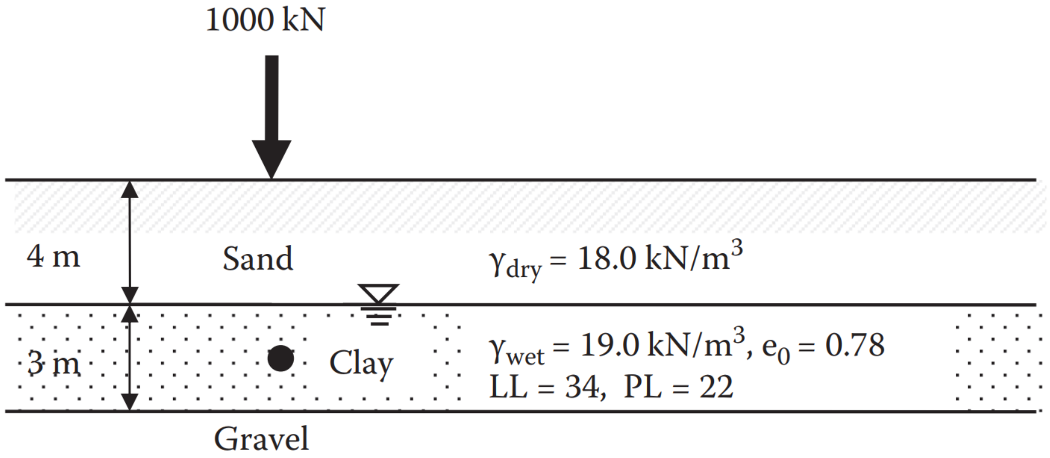
The settlement equals

$$\Delta h = -\frac{H}{1+e_0}\left(C_s \log\left(\frac{p_c}{\sigma'_{cz}}\right) + C_c \log\left(\frac{\sigma'_{cz} + \sigma_z}{p_c}\right)\right)$$

Exercises

A 3 m thick clay layer is sandwiched between dry sand on the top and saturated gravel on the bottom. On top of the sand layer, 1000 kN of a point load is applied.

1) Estimate settlement of the clay layer directly under the loading point. Handle the clay layer as a single layer and assume that it is normally consolidated (C_c=0.216, γ=10kN/m^3).
2) If it is the distributed load with same magnitude, estimate the settlement of the clay layer



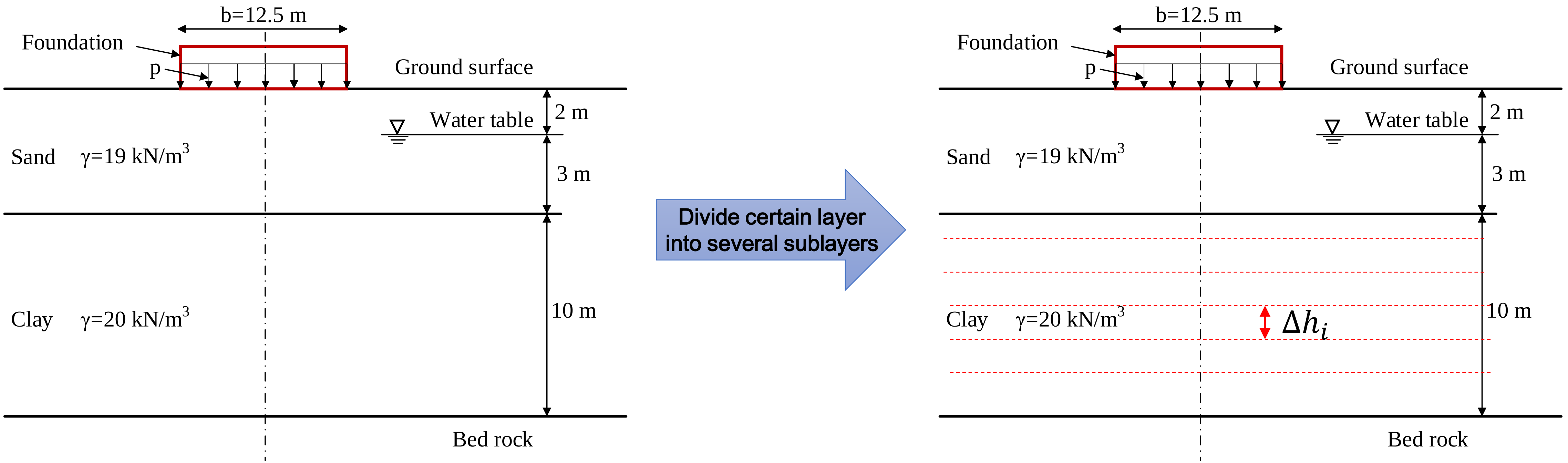
Settlement of ground with layer-wise summation method

Layer-wise summation method

When the ground consists of multilayers or a single thick soil layer, one-step computation is not suitable as the current effective stress and induced stress could no longer be considered as constant values

The essential of the layer-wise summation method is to divide the ground into several sub-layers. For each sub-layer, the one-step computation technique is applied to obtain Δh_i. The total settlement Δs is then the summation of Δh_i.

Layer-wise summation method



Schematic diagram of layer-wise summation method

Settlement of ground with layer-wise summation method

Layer-wise summation method: two critical problems

1. **The determination of compressive depth:**

   (According to the Chinese code) The compressive depth is determined by the ratio of the vertical stress increment (σ_z) to the effective geostatic stress (σ'_{cz}) . The depth is measured form the bottom of the foundation to a certain depth where the ratio of σ_z to σ'_{cz} equals 0.2 if there is no soft clay layer, otherwise, the depth should be further enlarged to make the ratio of the σ_z to the σ'_{cz} equals 0.1.

Settlement of ground with layer-wise summation method

Layer-wise summation method: two critical problems

2 The criterion to divide the layer of most concern:

- Interfaces of different soil layers
- The water table.
- The thickness of each layer should be no greater than 0.4b (b is the width of the single foundation).

Settlement of ground with layer-wise summation method

Procedures of Layer-wise summation method

1. Determine the location where special attention should be paid to based on the shape of foundations, lithological characteristics, et al.. Determine the contact pressure and additional pressure.
2. Make the division according to certain code of design
3. Calculate the effective geostatic stress at the mid-depth of the i^{th} sublayer, which is the average (arithmetic mean) of the corresponding values at the roof and bottom surface of the i^{th} sublayer

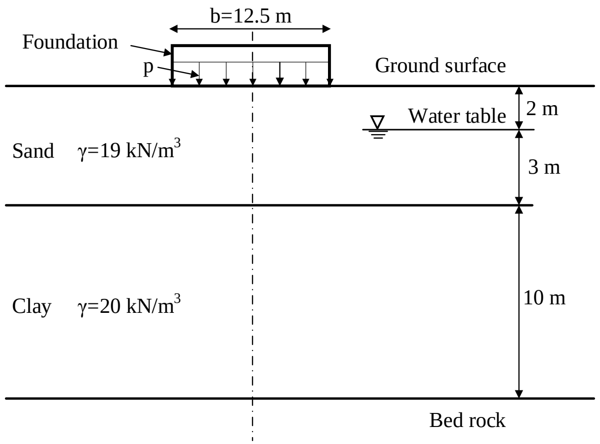
Settlement of ground with layer-wise summation method

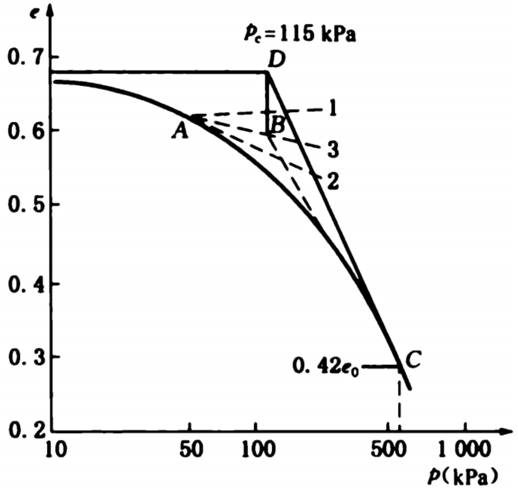
4. Calculate the addition stress at the mid-depth of i^{th} sublayer along the depth of the specified location, which is also the average (arithmetic mean) of the corresponding values at the roof and bottom surface of the i^{th} sublayer
5. Calculate the defamation (settlement) of each sublayer Δh_i.
6. Add each Δh_i up to obtain the final settlement.

$$\Delta s = \sum_{i=1}^{n} \Delta h_i$$

Exercises

The layouts of the foundation and ground are as figure show. The dimension of the square foundation is 12.5×12.5 m. The contact pressure at bottom of the foundation equals 100 kPa. To estimate the settlement of clay soil (the settlement of the sand and the bedrock is ignored), corresponding tests are conducted. The *in-situ* and laboratory compression curves are determined. The initial void ratio e_0 is 0.67 (constant value throughout the clay layer). Determine the settlement of the ground beneath the center of the foundation ($\gamma_w = 10\,kN/m^3$).





One-dimensional consolidation theory

Karl von Terzaghi

Consolidation refers to the process by which soils change volume in response to a change in pressure, encompassing both compaction and swelling

Although the final settlement is the most concern in practical engineering, it is also desirable to estimate the settlement at certain time level. This can be done with Terzaghi's one dimensional consolidation theory.

One-dimensional consolidation theory

Assumptions:

1) The soil is homogenous, isotropic and full saturated

2) The solid particle and water are incompressible. Only the expulsion of water contributes to the volume decrease of soil

3) Compression and flow only occur in vertical direction. The lateral deformation of the soil sample is restricted

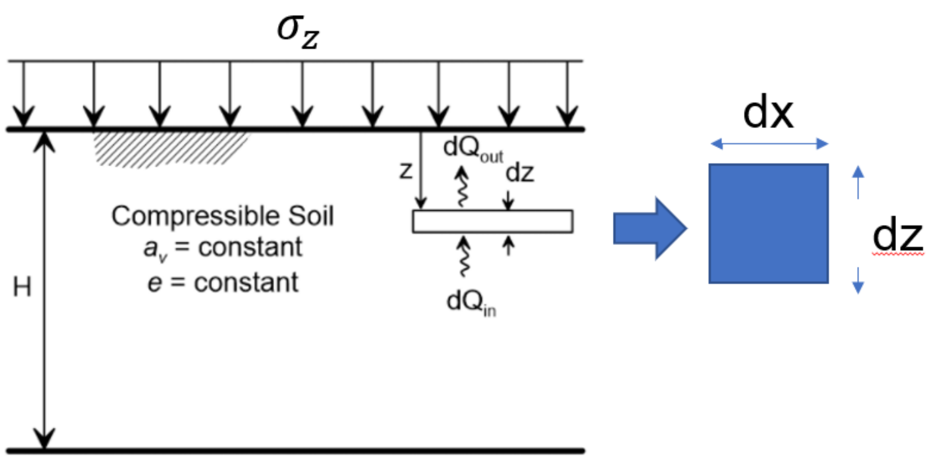
4) Darcy's Law is strictly valid

5) The deformation is infinitesimal. The derivation could be made on the undeformed configuration.

6) The coefficient of permeability and the coefficient of volume compressibility remain constant throughout the process.

One-dimensional consolidation theory

Governing equation for 1D consolidation



Schematic diagram of 1D consolidation

The volume decrease of any infinitesimal soil element

$$\delta\left(\mathrm{d}V\right) = -m_v \mathrm{d}\sigma_z' \mathrm{dxdz} = -m_v \frac{\partial \sigma_z'}{\partial t} \mathrm{dtdxdz}$$

According to the principle of effective stress

$$\sigma_z' + \mathrm{u} = \sigma_z \rightarrow \frac{\partial \sigma_z'}{\partial t} + \frac{\partial u}{\partial t} = 0 \rightarrow \frac{\partial \sigma_z'}{\partial t} = -\frac{\partial u}{\partial t}$$

$$\delta\left(\mathrm{d}V\right) = m_v \frac{\partial u}{\partial t} \mathrm{dtdxdz}$$

One-dimensional consolidation theory

For the liquid phase

$$\delta\left(\mathrm{d}V_w\right)=Q_{\text{out}}-Q_{\text{in}}=-\frac{k}{\gamma_w}\frac{\partial u}{\partial z}\mathrm{d}t\mathrm{d}x-\left(-\frac{k}{\gamma_w}\frac{\partial u}{\partial z}-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}\mathrm{d}z\right)\mathrm{d}t\mathrm{d}x=\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}\mathrm{d}t\mathrm{d}z\mathrm{d}x$$

The volume decrease of soil is due to the expulsion of water

$$\delta\left(\mathrm{d}V\right)=\delta\left(\mathrm{d}V_w\right)\rightarrow m_v\frac{\partial u}{\partial t}\mathrm{d}t\mathrm{d}x\mathrm{d}z=\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}\mathrm{d}t\mathrm{d}z\mathrm{d}x\rightarrow m_v\frac{\partial u}{\partial t}=\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}\rightarrow\frac{\partial u}{\partial t}=c_v\frac{\partial^2 u}{\partial z^2}$$

Coefficient of consolidation $c_v=\dfrac{k}{\gamma_w m_v}=\dfrac{k\left(1+e_0\right)}{\gamma_w a_v}$

One-dimensional consolidation theory

Solution to one dimensional consolidation theory

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2}$$

$$s.t.$$

$$t = 0,\ 0 \le z \le H \qquad u = \sigma_z$$

$$0 < t < \infty,\ z = 0 \qquad u = 0$$

$$0 < t < \infty,\ z = H \qquad \frac{\partial u}{\partial z} = 0$$

$$t \to \infty,\ 0 \le z \le H \qquad u = 0$$

$$u(z,t) = \frac{4p}{\pi} \sum_{m=1}^{\infty} \frac{1}{m} \sin \frac{m\pi z}{2h} e^{-m^2\left(\frac{\pi^2}{4}\right)\boxed{T_v}}$$

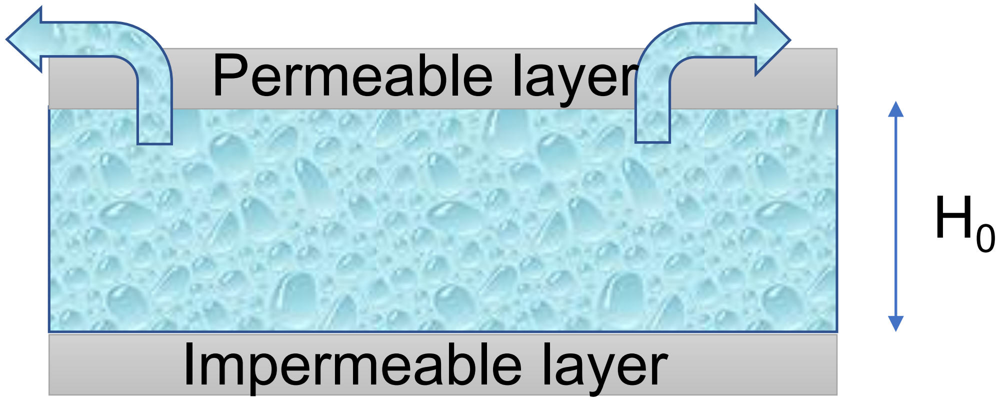
Time factor $T_v = \frac{c_v t}{H^2}$

One-dimensional consolidation theory

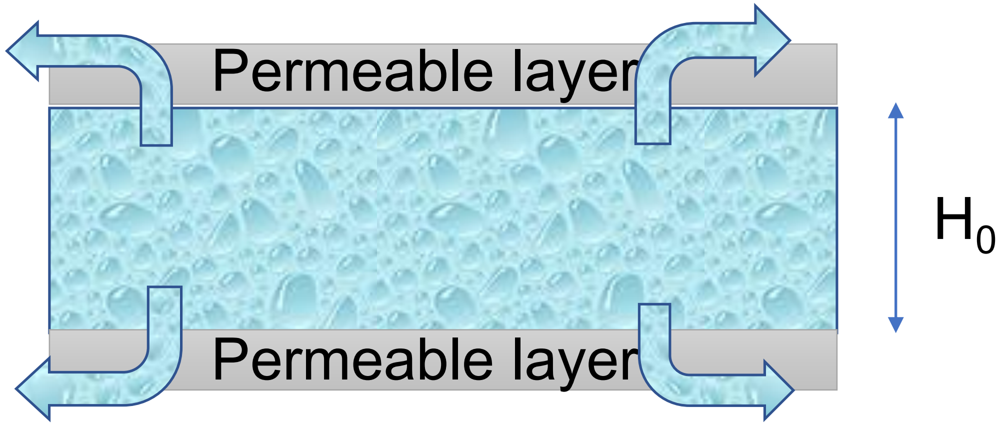
Drainage path

$$T_v = \frac{c_v t}{H^2}$$

H is the longest length of the drainage path



One way drainage $H=H_0$



Two way drainage $H=H_0/2$

One-dimensional consolidation theory

Degree of consolidation (U): it is termed as the percentage of settlement at an arbitrary time t, s_t, to its final settlement s_∞ or simply s at $t \to \infty$

Settlement at certain point

$$\Delta h = -m_v \sigma_z' \mathrm{dz} = -m_v \left(\sigma_z - u \right) \mathrm{dz}$$

Settlement of the whole layer

$$s_t = \int_0^H \Delta h = \int_0^h -m_v \left(\sigma_z - p \right) \mathrm{dz} = -m_v \sigma_z H + \int_0^h m_v p \mathrm{dz} = m_v \sigma_z H \left[1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \frac{1}{2N+1} \cdot e^{-\frac{(2N+1)^2 \pi^2}{4} T_v} \right]$$

Degree of consolidation

$$U = \frac{s_t}{s_\infty} = 1 - \frac{8}{\pi^2} \sum_{N=0}^{\infty} \frac{1}{2N+1} \cdot e^{-\frac{(2N+1)^2 \pi^2}{4} T_v}$$

One-dimensional consolidation theory

Application of degree of consolidation

Two kinds of problem can be solved with degree of consolidation:

1) With the known final settlement s, find the settlement s_t at time t
2) With the known final settlement settlement s , find the elapsed time t to achieve a settlement s_t

Exercises

The thickness of a clay layer is 4 m and it is an half-closed layer (water can escape through only one surface). The infinity uniform load is applied on the surface and the final settlement is 28 cm. After 100 days, the settlement of the clay layer is 18.5 cm. The relationship between the degree of consolidation and time factor is $U = 1.128(T_v)^{\frac{1}{2}}$.

Determine the coefficient of consolidation C_v (in unit: cm^2/s)

Exercises

A saturated clay layer of thickness 10 m overlays an impermeable hard rock. A vertical uniform pressure 200 kPa is applied on the foundation base. If the initial void ratio e_0 of the clay layer is 0.8, the coefficient of compressibility is $2.5 \times 10^{-4} kPa^{-1}$, and the coefficient of permeability $k = 2.0\ cm/year$.

1) Determine the settlement after 1 year loading
2) The elapsed time required for achieving a settlement of 20 cm

The End