Chapter 5 Shear strength of soils

5.1 Introduction

Stress develops within ground under the reaction of its self-weight and external loading. One of the main characteristics of soils is that soils may eventually fail for sufficiently large shear stress. In engineering practice, the main failure mode for soils is shear failure, namely the soil mass sliding over the soil below it along a probable surface of slippage. The failures of a natural slope of hillside, a slope of an earth-fill dam, the ground beneath the foundation of structures and other geotechnical structures, as fig shows, all belong to this kind of failure.

Soils have capability to resist the sliding and stay in place. The maximum shearing resistance that soils are capable of developing is called the shear strength. As soon as the shear stress equals the shear strength at a point within the ground, the local failure occurs. With the increase of external loads, the local failure may further prorogates, coalesces and finally leads to the global failure.

Several factors influence the shear strength of soils: 1) Soil composition. The solid phase (mineralogy of grains, grain size and grain size distribution, grain shape) and liquid phase (content, type) could affect the shear strength; 2) State of soils. For cohesionless soils, the state is mainly described by the terms like loose or dense, while for cohesive soils, it is described by overconsolidated, normally consolidated, stiff or soft; 3) Structure of soils. It refers to the manner the particles are packed or distributed. In general, dense single grain structure for cohesionless soils and flocculated structure for cohesive soils possess higher shear strength; 4) Loading conditions. The loading conditions mainly refer to the loading rate, drainage condition and loading history (cyclic or monotic). All of them influence the consolidation process and hence the effective stress, which plays an imporant role in shear strength.

The real stress state for a point within ground is complex. To judge whether the point fails or not, a failure criterion, which is a functions in stress or strain space, is necessary. In geotechnical engineering, the Mohr-Coulomb failure crterion is the most widely used one.

In a failure criterion, apartment from stress components or stress variants, there are extra parameters and some of them are measured either in the laboratory or in the filed. Moreover, to simulate field conditions as far as possible, particular tests are necessary to be carried out, e.g., the drainage condition is the deciding factor in choosing a special test.

After studying this chapter, you are capable of:

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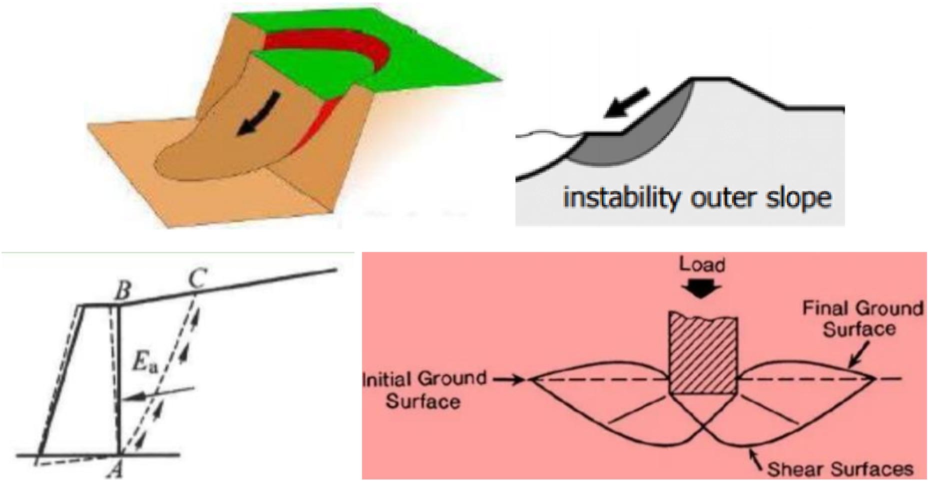


Fig 5.1

5.2 Mohr-Coulomb failure criterion

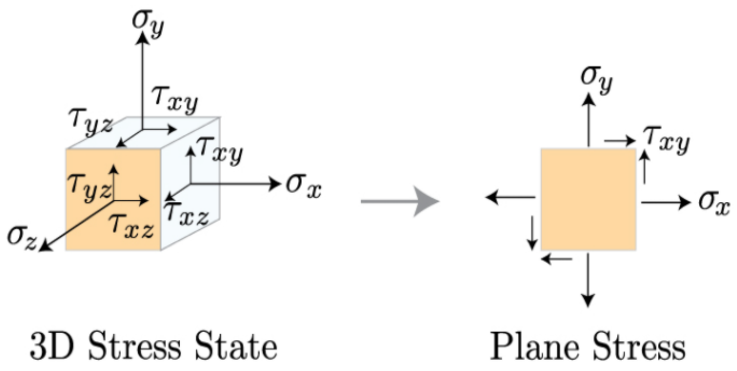
The Mohr–Coulomb failure criterion is named in honour of Coulomb's contribution on frictional law and linear representation of its relation and Mohr's contribution on defining failure with a unique combination of normal stress σ and shear stress τ .

5.2.1 Mohr's circle for plane stress

In the late 1800s, Mohr presented a graphic solution to represent the transformation law for the Cauchy stress tensor in 2D space.

5.2.1.1 General stress state within a soil mass

As a reaction to self-weight and external load, internal forces develop within a soil mass. A measure of the intensity of these internal forces is called stress. According to Cauchy's theory, the general stress state of a point in a continuum object is completely defined by three normal stresses (in parallel with the outward normal of the surface) and six shear stresses (in parallel with the axes) as shown in fig. For a 2D case, the general stress state of a point is reduced to two normal stresses and one shear stress as fig shows, where the components in the y direction have been kicked off. The sign conventions of stresses have already been discussed in section XXX.



5.2.1.2 Stress transformation

According to the failure characterstics of soils, two quantities on the probable surface of slippage are necessary to be known to judge whether the failure occurs or not. One is the shear stress produced by external load and the other is the resistance provided by soils. The former will be discussed in this section and the latter will be discussed in section 5.2.2.

The stress state of a point is shown in Fig. Consider a plane passes through the point, making an angle θ with the horizontal plane. On the inclined plane, σ is termed as normal stress, which is perpendicular (normal) to the plane; while τ is termed as shear stress, which is in parallel with the plane. The sign of σ, τ and θ are defined as: 1) compressive normal stress is considered as positive; 2) The shear stress, which has the tendence to rotate the material element in the counterclockwise direction, is considered to be positive; 3) Counterclockwise rotate from the horizontal plane θ is treated as positive.

The magnitude of σ and τ are determined by establishing force equilibrium equations on an infinitesimal element ABC as fig shows. The force equilibrium in the horizontal direction is:

$$\sigma \cdot l \cdot \sin\theta - \tau \cdot l \cdot \cos\theta - \tau_{zx} \cdot l \cdot \cos\theta - \sigma_x \cdot l \cdot \sin\theta = 0 \tag{1.1}$$

The force equilibrium in the vertical direction is:

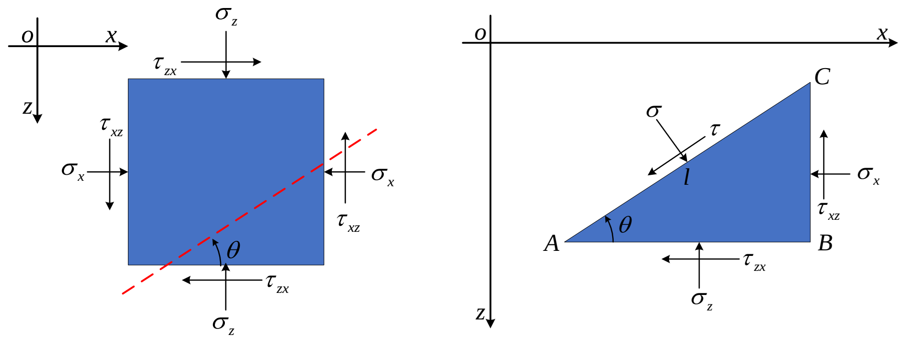
$$\sigma \cdot l \cdot \cos\theta + \tau \cdot l \cdot \sin\theta - \tau_{xz} \cdot l \cdot \sin\theta - \sigma_y \cdot l \cdot \cos\theta = 0 \tag{1.2}$$

Taking into account the condition $\tau_{xz} = \tau_{zx}$, σ and τ are solved as:

$$\sigma = \frac{\sigma_x + \sigma_z}{2} + \frac{\sigma_z - \sigma_x}{2}\cos 2\theta + \tau_{xz}\sin 2\theta \tag{1.3}$$

$$\tau = \frac{\sigma_z - \sigma_x}{2}\sin 2\theta - \tau_{xz}\cos 2\theta \tag{1.4}$$

With Eq. (1.3) and Eq. (1.4), the normal stress σ and shear stress τ on any plane with the inclined angle θ can be determined.



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5.2.1.3 Construction of a Mohr's cicle

Squaring Eqs. (1.3) and Eq. (1.4) and adding them together, we have:

$$\left(\sigma-\frac{\sigma_x+\sigma_z}{2}\right)^2+\tau^2=\left(\frac{\sigma_z-\sigma_x}{2}\right)^2+\tau_{xz}^2 \tag{1.5}$$

Eq. (1.5) represents an equation of a circle. The coordinates of the center of the circle are $\left(\sigma_x+\sigma_z/2,0\right)$. The radius of the circle is $r=\sqrt{\left(\sigma_z-\sigma_x/2\right)^2+\tau_{xz}^2}$. The corresponding circle is called Mohr's circle (1887). Any point on the circle represents an inclined plane and the corresponding coordinates represent the normal and shear stresses on that plane.

The procedures of constructing Mohr's circle are: 1) Drawing the Cartesian coordinate system σ-τ with a horizontal σ-axis and a vertical τ-axis; 2) Ploting the two points $A\left(\sigma_x,\ \tau_{xz}\right)$ and $B\left(\sigma_z,\ -\tau_{zx}\right)$, which correspond to the stress components on the planes BC and BA, respectively; 3) Connecting the points A and B with a straight line. As $\left|\tau_{xz}\right|=\left|-\tau_{zx}\right|$, the straight line must pass througth the center of the circle o' and hence it is the diameter of the circle; 3) Drawing the Mohr's circle with the center and diameter of the circle.

The maximum and minimum values of σ correspond to the rightmost point and leftmost point, which are the intersections of the circle with the σ-axis. The values are:

$$\begin{cases}\sigma_{\max}=\dfrac{\sigma_x+\sigma_z}{2}+\sqrt{\left(\dfrac{\sigma_x-\sigma_z}{2}\right)^2+\tau_{xz}^2}\\[2ex]\sigma_{\min}=\dfrac{\sigma_x+\sigma_z}{2}-\sqrt{\left(\dfrac{\sigma_x-\sigma_z}{2}\right)^2+\tau_{xz}^2}\end{cases} \tag{1.6}$$

It is worth noting that the shear stress equals zero when the normal stress attains the extreme values (maxima or minima). By the definition in continuum mechanics, the plane corresponds to the rightmost point or the leftmost point is called principal plane.

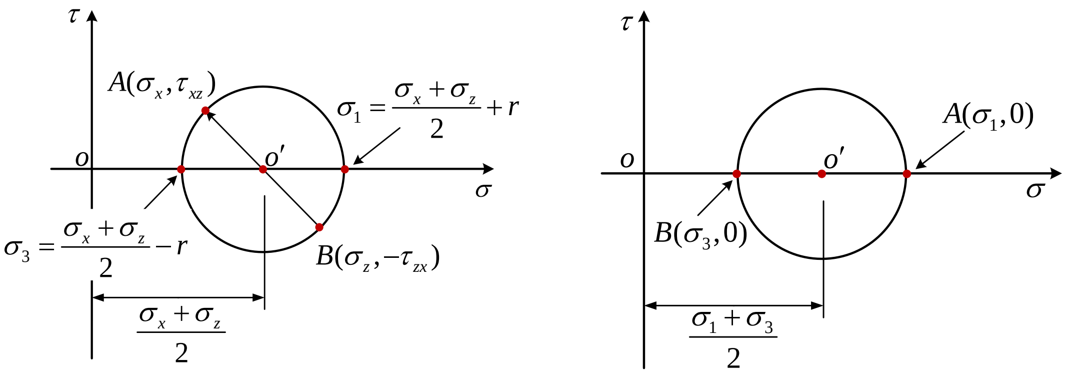
The normal stress on the principal plane is termed as principal stress. By definition, the maximum value $\sigma_{\max}$ is designated as the major principal stress σ_1 and the minimum one as the minor principal stress σ_3, namely:

$$\begin{cases} \sigma_1 = \sigma_{\max} \\ \sigma_3 = \sigma_{\min} \end{cases} \tag{1.7}$$

If the shear stresses on two mutually orthogonal planes are zero, which indicates that the planes are themselves principal planes, the equation for the Mohr's circle can be expressed by principal stresses as:

$$\left(\sigma - \frac{\sigma_1 + \sigma_3}{2}\right)^2 + \tau^2 = \left(\frac{\sigma_1 - \sigma_3}{2}\right)^2 \tag{1.8}$$

The Mohr's circle is as Fig shows.



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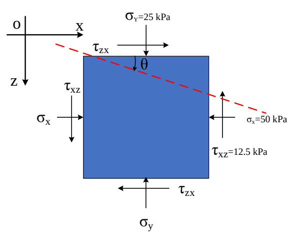
Exercise

1) Boundary stresses σ_x and τ_{xy} on the x-plane and σ_x and τ_{yx} on the y-plane are given in Figure. Compute σ and τ on the θ plane, which inclines 20° clockwise from the y-plane (horizontal plane).



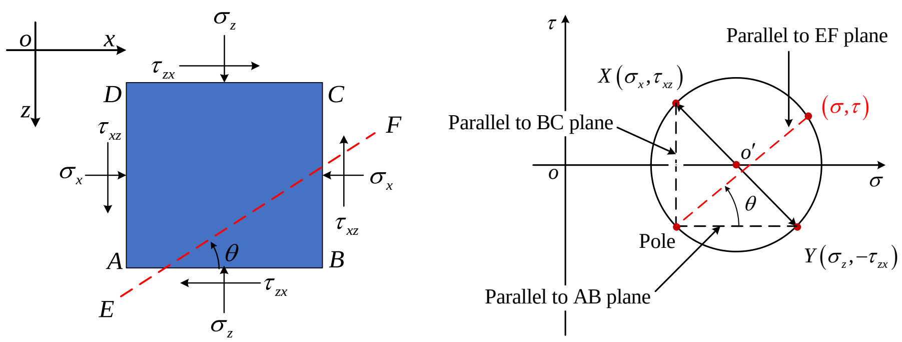
2) Major and minor principal stresses are given as $\sigma_1 = 120$ kPa and $\sigma_3 = 50$ kPa as shown in Figure 10.7(a). Determine the normal stress and shear stress on the plane, which is inclined 20° counterclockwise from the horizontal plane.

5.2.1.4 Pole (origin of plane) of Mohr's circle

The pole (or origin of planes) of Mohr's circle is a point so special that it can help to readily find stress components on any specified plane by using graphical technique instead of complicated computation. Starting from the pole, a straight line, which is in parallel with the plane, is draw to intersect with the Mohr's circle. The coordinates of the intersection point represents the stress components on that plane.

In figure, the pole of Mohr's circle is located as follows: Starting from the Point *Y*, draw a parallel line to the direction of the plane *AB*. The intersection of the line with the Mohr's circle is the pole. This is a unique point on the circle. In figure, starting from the Point *X*, the parallel line to the plane *BC* also passes through the pole.

After the pole is found on Mohr's circle, starting from the pole, draw a parallel line to any particular plane (i.e., the plane *EF* in Figure), and find the intersection on Mohr's circle. The coordiantes of the intersection are the stress components which act on the plane *EF*.



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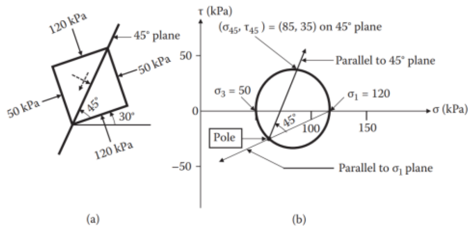
Exercise

1) Figure shows principal stresses on an element. By using the pole, determine stresses on the plane with 45° counterclockwise inclined from the major principal stress plane.



5.2.2 Coulomb's law and shear strength of soil

The resistance of soils to sliding may be attributed to two distinctly different mechanisms: one is the frictional resistance and the other is the cohesive resistance along the sliding surface. The frictional resistance follows Coulomb's law as []:

$$\tau_{friction} = \sigma \cdot \tan\varphi \tag{1.9}$$

σ is the normal stress perpendicular to the sliding surface and φ can be interpreted as the intergranular friction angle along the shear surface. In general, the rougher the surface of grains (irregularity), the higher the value of frictional angle.

Coulomb's law was later extended to include the effect of intergranular cohesion. For cohesive soils, the intergranular cohesion comes from close-range interactive forces like electrostatic force, chemical bonds, apparent force. Moreover, the intergranular cohesion is independent of the normal stress, which even when the normal stress σ is zero, soils still have capacity to resist the shear failure.

Accordingly, the shear strength of soils or the shear stress on the failure surface is expressed as:

$$\tau_f = \tau_{\text{friction}} + c = \sigma \cdot \tan\varphi + c \tag{1.10}$$

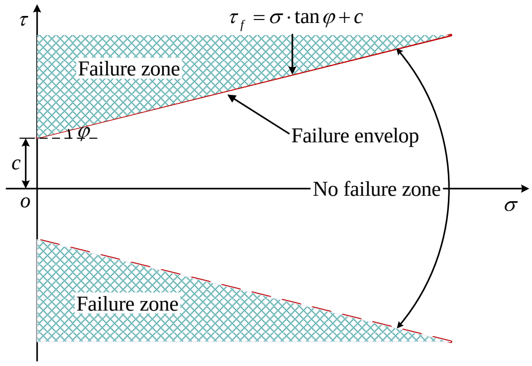
The subscript f indicates that the associated quantity is at failure. Eq. (1.10) shows a linear relation between τ_f and σ and it is plotted as a straight line on σ-τ coordinate system, as Fig shows. The line defined by Equation (11.1) is called the failure envelope, which implies that if any stress combination of σ and τ on any arbitrary plane (as shown in Figure 11.2(b)) below the failure envelope, there is no failure on that plane. On the other hand, if the stress combination of σ and τ goes above the envelope, the failure occurs on that plane []. In practice, combinations of σ and τ cannot go beyond the envelope. Therefore, the failure envelop defines the boundary of stress combination of σ and τ.

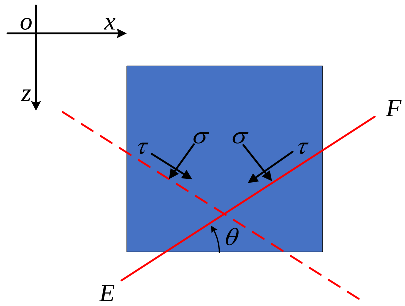
A mirror image of the failure envelope is also plotted with a dotted line, as as Fig shows. As the negative shear stress merely means the same magnitude of shear stress in the opposite direction, and thus these two failure envelopes completely separate the "No failure zone" from the "Failure zone", as fig shows.

Terzaghi (1925) modified Eq. (1.10) to include his effective stress concept as the strength of soil is acutally controlled by the effective stress in the solid skeleton rather than the total stress:

$$\tau'_f = \sigma' \cdot \tan\varphi' + c' \tag{1.11}$$

c' and φ' are effective strength parameters. There are many ways to determine c (c') and φ (φ') in the laboratory as well as in the field. This will be detailed discussed in section 5.3.





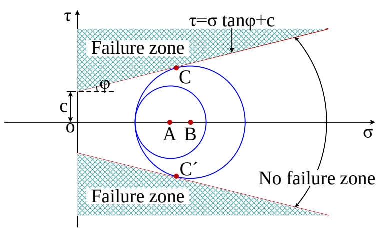
5.2.2.1 Mohr-Coulomb failure theory

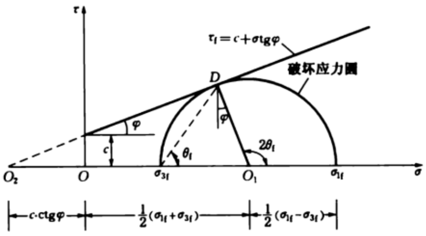
The Mohr circle and the Coulomb's law represented by Eq. (1.10) can be plotted in the same figure to determine the state of a point within a soil mass, as fig shows. In the figure, two Mohr's circles are drawn. For the smaller circle, it is completely below the envelopes and accordingly no failure will occur the planes passing through the point. For the larger circle, it intersects the envelopes at the points C and C', which implies the shear stresses on the planes represented by the points C and C' equal the shear strength and thus the failure takes place. The plane where the shear failure occurs is called the failure plane.

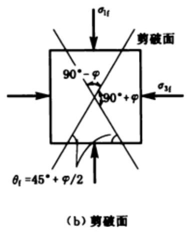
It worthy pointing out that the real angle θ_f between any failure plane and the major principal plane (where the major principal stress acts) appeared $2\theta_f$ on Mohr's circle. For the plane represented by the point D on Mohr's circle, the relation between θ_f and the frictional angle φ is

$$2\theta_f = 90^\circ + \varphi \rightarrow \theta_f = 45^\circ + \frac{\varphi}{2} \tag{1.12}$$

The real angle θ_f between any failure plane and the direction of the major principal stress σ_1 is $90^\circ - \theta_f$ or $45^\circ - \varphi/2$.







5.2.2.2 The relation of major and minor principal stresses at failure

In continuum mechanics, it is customary to express the failure criterion in principal stresses. According to the geometry relation shown in fig,

$$\sin\varphi = \frac{\frac{\sigma_{1f}-\sigma_{3f}}{2}}{\frac{\sigma_{1f}+\sigma_{3f}}{2}+c\cdot\cot\varphi} \tag{1.13}$$

After several operations on trigonometric functions, the major principal stresses σ_{1f} is expressed by σ_{3f} as:

$$\sigma_{1f}=\sigma_{3f}\tan^2\left(45°+\frac{\varphi}{2}\right)+2c\cdot\tan\left(45°+\frac{\varphi}{2}\right) \tag{1.14}$$

On the other hand, the minor principal stresses σ_{3f} can be expressed by σ_{1f} as:

$$\sigma_{3f}=\sigma_{1f}\tan^2\left(45°-\frac{\varphi}{2}\right)-2c\cdot\tan\left(45°-\frac{\varphi}{2}\right) \tag{1.15}$$

The failure criterion can be also cast into effective from, where the principal stresses and strength parameters are replaced by σ'_{1f}, σ'_{3f}, c' and φ' as:

$$\begin{cases}\sigma'_{1f}=\sigma'_{3f}\tan^2\left(45°+\frac{\varphi'}{2}\right)+2c'\cdot\tan\left(45°+\frac{\varphi'}{2}\right)\\ \sigma'_{3f}=\sigma'_{1f}\tan^2\left(45°-\frac{\varphi'}{2}\right)-2c'\cdot\tan\left(45°-\frac{\varphi'}{2}\right)\end{cases} \tag{1.16}$$

Exercise

1) The major principal stresses at certain point within a cohesionless soil mass is $\sigma_1 = 300\text{kPa}$. The shear strength parameters $\varphi = 20°$. Determine the major principal stress at failure.
2) The major principal stress and the minor principal stress at certain point within a soil mass are respectively $\sigma_1 = 480\text{kPa}$ and $\sigma_3 = 210\text{kPa}$. The shear strength parameters are $c = 20\text{kPa}$ and $\varphi = 20°$. Determine the state at the point.

5.3 Laboratory shear test

Laboratory and *in-situ* shear tests can be employed to determine the shear strength parameters. For the former, direct shear test, triaxial compression test and unconfined compression test are the most commonly used. For the latter, vane shear test is widely used.

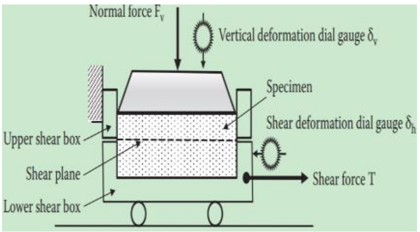
5.3.1 Direct shear test

The direct shear test is considered one of the most common and simple tests to derive the strength of undisturbed or remolded soils. The principle is to fail a soil sample along a predetermined shear plane (usually at mid-height) at a constant rate. The Mohr-

Coulomb failure envelop is based on a relationship between shear stress and normal stress on the failure plane.

A schematic diagram of the apparatus used in direct shear test is as fig shows. The soil sample is placed in a shear box which is split at mid-height. In most devices, the upper part is fixed while the bottom part is moveable and located on low-friction roller. A constant vertical load F_v is applied through a load frame and uniformly distributed by a metal cap. The vertical displacement s_v is recorded by a dial gauge to accounts for the specimen's volume change during shearing. The horizontal displacement s_h is measured by another dial gauge. The applied horizontal force T is calculated through the stiffness of a proving ring and the corresponding horizontal displacment.

Each test consists of two loading stages: application of F_v and T. The normal stress σ and shear stress τ are calculated as F_v/A and T/A, respectively. A is the cross section area of the soil, which is assumed a constant throughout the test. The change in volume of the specimen ΔV equals $s_v \times A$. To obtain the shear strength parameters, several tests under different vertical loads are necessary.



5.3.1.1 Direct shear test on cohesive soil

For cohesive soils, special attention should be paid to the drainage conditions during the application of F_v and T. The reason is that the pore pressure has a major influence on the effective stress and thus on the shear strength of cohesive soils. Depending on whether the water is allowed to escape or not during the two loading processes, the direct shear test can be further categorized into quick shear test, consolidated-quick shear test and consolidated-slow shear test.

- **Quick shear test (Q test):** During the application of F_v, weight sets are consecutive applied till the desired value. The shear test is conducted immediately. Moreover, the rate of shear load should be quickly exerted to ensure that no water drains from the soil sample. In these two loading stages, no consolidation is considered to be occurred.
- **Consolidated-quick (R test) test:** The procedure is almost the same as that of UU test except that after the application of F_v, the evolution of s_v with time

should be recorded. The shear test should be conducted only after complete consolidation has occurred. In the shear test, the shear load is also applied quickly to ensure no water is allowed to be escaped from the soil sample.

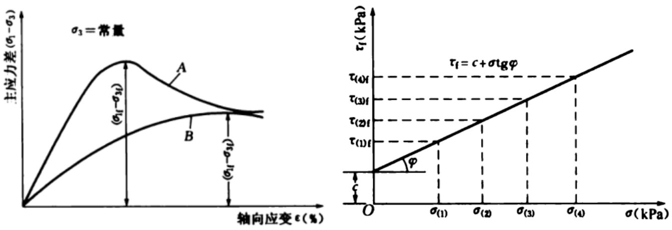
- **Consolidated-slow shear (S test) test:** After the application of F_v, the soil sample should be allowed to completely consolidate. During the second stage, the shear test should be carried out at such a slow rate that almost all pore pressure could dissipate.

5.3.1.2 Direct shear test on cohesionless soils

As cohesionless soils possess relatively high hydraulic conductivity, consolidation finishes rapidly during the application of F_v and T. The effect of pore pressure on the shar strength is negligible.

5.3.1.3 General results

Several samples under different vertical loads F_v should tested. The axial strain s_h/H (H is the initial heigh of the soil sample) corresponds to the application of the deviatoric stress is plotted. The shear stress at failure τ_f corresponds to the peak value if it exists or certain defined s_h if it does not exist (e.g. s_h=4mm). With a series of tests, the shear stress at failure τ_f and the corresponding σ are plotted on the $\sigma - \tau$ coordinate system. For simplicity, a linear regression technique is employed to obtain a straight line which best fits the experimental data. The shear strength parameters are determined as follows: The y-intercept is the cohesion c, and the slope of the line is the friction angle φ.



5.3.1.4 Advantages and disadvantages

Advantages:

- Almost all soil types can be tested;
- Direct measurement of shear strength parameters;
- The principles are easy to understand and the procedures are relatively easy to be carried out;

- The strength parameters taking into account the effect of consolidation process can be roughly determined.

Disadvantages:

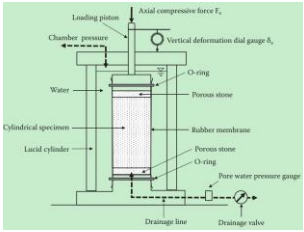
- The failure plane is predetermined;
- The drainage and measurement of the pore pressure could not be precisely controlled;
- The assumption that the normal stress and shear stress are uniformly distributed on the failure plane is not reasonable.

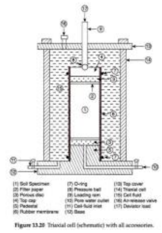
5.3.2 Triaxial compression test

The pore pressure can have a major influence on the effective stress and thus on the shear strength of soils. This is especially important for cohesive soils since excess pore pressure need relatively long time to dissipate when comparing with cohesionless soils. To properly reflect the *in-situ* drainage conditions and obtain more realistic shear strength parameters, the equipment must have the capability to measure pore pressure.

The main apparatus for triaxial compression test is the triaxial cell shown in Fig. It is a high-pressure cylindrical cell made of transparent materials. The accessories are also shown schematically in Fig. The soil sample is cylindrical in shape with 38 mm diameter and 76 mm height, enclosed in a rubber membrane between porous stones at top and bottom. The triaxial cell is filled with fluid (e.g. oil or water) to cause 3D consolidation of the soil sample. The pore water from the soil specimen escapes through the bottom drainage tube during consolidation process. The O-rings prevent entry of cell fluid into the voids of the soil specimen. Additional axial load is applied through the loading piston to fail the soil sample.

The whole test generally consists of two loading stages: 1) The cell pressure (also confining pressure) is increased to the desired value σ_3 by puming in the liquid; 2) By fixing cell pressure, additional axial load, usally at a constant rate, is applied to fail the soil sample. <u>If the vertical stress at failure is σ_1, the additional stress induced in the second stage is $\Delta\sigma_1 = \sigma_1 - \sigma_3$, which is known as the deviatoric stress</u>. A series of tests should be carried out on a number of samples with different confining pressures and the results obtained could be ultilized to draw Mohr's circles as in Fig shows. A straight line, which is tangent to the circles, gives the failure envelop and represents Coulomb's shear strength equation from which the strength parameters, c and φ, can be obtained. Detailed discussion will be given in section 5.3.2.3.





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5.3.2.1 Triaxial compression test on cohesionless soils

Similar to the direct shear test, three types of triaxial tests are commonly conducted in the laboratory. Depending on the particular drainage conditions during the two loading stages, there are unconsolidated undrained test (UU test), consolidated undrained test (CU test) and consolidated drained test (CD test). The introduction of the drainage valve and the pore water pressure gauge provide us more control on drainage conditions and more precise measurement on pore pressure.

- **Unconsolidated undrained test (UU test):** In a UU test, no drainage is allowed during the application of cell pressure σ_3 and deviatoric stress $\sigma_1 - \sigma_3$. As the test usually takes short time, UU test is also called as Quick test or Q-test. This is done simply by keeping the drainage valve closed throughout the test. The strength parameters by UU test are c_u and $\varphi_u = 0°$;
- **Consolidated undrained test (CU test):** In a CU test, drainage is allowed to take place only during the application of cell pressure σ_1. It is also referred to as Rapid test or R-test. When the consolidation of the specimen under the cell pressure is completed, the drainage valve is closed and the additional deviatoric stress $\sigma_1 - \sigma_3$ is applied. The soil specimen fails in shear under undrained conditions. The strength parameters by CU test are c_{cu} and φ_{cu}.
- **Consolidated drained test (CD test):** In a CD test, drainage is allowed to take place during the application of cell pressure σ_1 and the additional deviatoric stress $\sigma_1 - \sigma_3$. As the consolidation is allowed to completely finish, it usually cost considerably long time and hence is also referred to as slow test or S-test. The strength parameters by CD test are c_d and φ_d

5.3.2.2 Pore pressure coefficients under triaxial compression test

It is extremely useful to predict the induced excess pore pressure within ground due to external loading. To achieve this, two dimensionless pore pressure coefficients, A and B, which characters the change in pore pressure caused by a change in applied stress are necessay. These two coefficients were firstly proposed by Skempton (1954) and thus they are also named as Skempton's pore pressure parameters.

The parameter B is defined as:

$$B = \frac{u_c}{\sigma_3} \tag{1.17}$$

The subscript c indicates the induced pore pressure due to the application of the confining pressure σ_3. The parameter B is a function of the degree of saturation of soils. As water and grains are usually assumed incompressible, $B = 1$ for fully saturated soils, $B = 0$ for dry soils and $0 < B < 1$ for partially saturated soil.

The parameters A is defined through $\bar{A}$ as

$$\bar{A} = B \cdot A = \frac{u_d}{\sigma_1 - \sigma_3} \tag{1.18}$$

The subscript d indicates the induced pore pressure due to the application of the deviatoric stress $\sigma_1 - \sigma_3$. The parameter A is also not a constant, and it varies with the over-consolidation ratio and the magnitude of deviatoric stress.

The total induced pore pressure u is the sum of the induced pore pressure in the two loading stages:

$$u = u_c + u_d = B\sigma_3 + B \cdot A(\sigma_1 - \sigma_3) \tag{1.19}$$

For a UU test on a saturated soil sample, Eq. (1.19) changes to:

$$u = u_c + u_d = \sigma_3 + A(\sigma_1 - \sigma_3) \tag{1.20}$$

For a CU test on a saturated soil sample, Eq. (1.19) changes to:

$$u = u_d = A(\sigma_1 - \sigma_3) \tag{1.21}$$

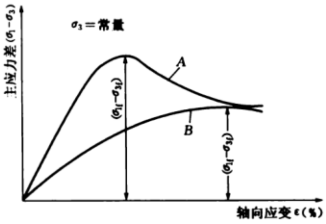
5.3.2.3 General results

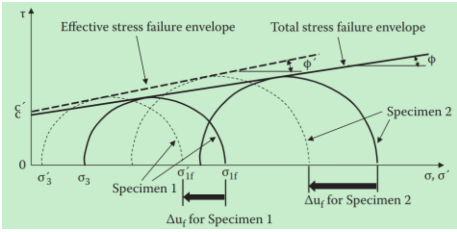
For any type of triaxial compression test (UU, CU, CD), several samples under different confing pressurees should be tested. The axial strain corresponds to the application of the deviatoric stress is plotted. The deviatoric stress at failure corresponds to the peak value or certain defined axial strain ε_v (e.g., 15%).

In UU and CU tests, the pore pressure u_f at failure could be accurately measured by the pressure gauge. Therefore, the effective principal stresses at failure are:

$$\sigma'_{1f} = \sigma_{1f} - u \text{ and } \sigma'_{3f} = \sigma_3 - u \tag{1.22}$$

By drawing Mohr's circles with the total principal stresses and effective principal stresses at failure, the total strength parameters c and φ as well as effective strength parameters c' and φ' are determined.





5.3.2.4 Advantages and disadvantages

Advantages:

- The state of stress at all intermediate stages up to failure is known and hence it is possible to draw Mohr's circles at any shear stage;
- The failure plane need not to be predefined
- Complete control over the drainage conditions. Pore pressure and volumetric change are possible to be directly measured;
- The behavior of soils under different loading conditions could be simulated and hence it is more adaptable to the research work.

Disadvantages:

- Only axis-symmetrical problems could be simulated;
- It is suitable for the problems whose principal stresses are in the vertical direction and horizontal direction.

5.3.3 Unconfined compression test

The unconfined compression test is a relatively simple method to determine the unconfined compressive strength (UCS) q_u of an undisturbed or a remoulded cohesive soil sample. As show in the figure, a soil sample is trimmed to have a cylindrical shape and placed on a loading platform. The sample's height-to-diameter ratio should be at least 2.0 or more to avoid the end boundary effect.

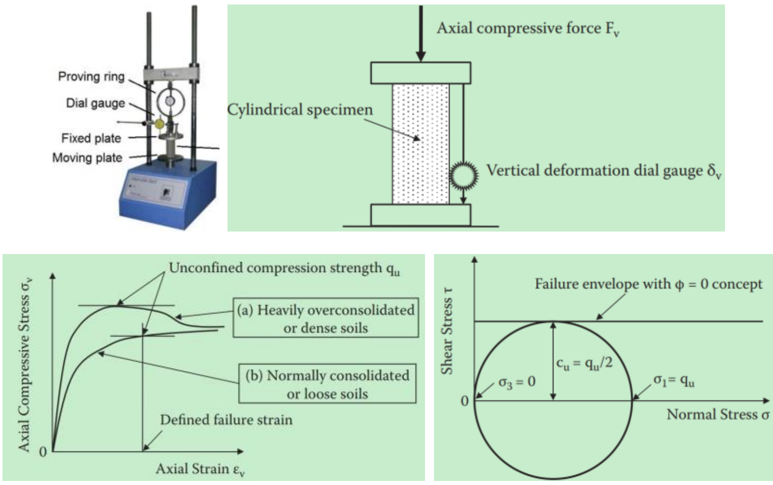
The test only consists of one stage. The vertical load F_v is gradually increased until failure with a measurement of the axial displacement s_v. As the loading process normally ends within 10~20 minutes, the induced excess pore pressure may not have sufficient time to dissipate. Therefore, the unconfined compression test can be regarded as undrained shear test.

The relation between normal stress $\sigma = F_v/A$ and axial strain $\varepsilon_v = s_v/h$ are plotted in Figure. A and h are the cross section area and height of the soil sample in its initial state. Two typical curves, one with clear peak value and the other without, are shown in Fig.

The UCS (q_u) corresponds to the peak value or certain defined axial strain ε_v (e.g., 20%). As there is no lateral stress, the minor principal stress σ_3 is zero. The Mohr's circle is drawn as in Fig with σ_1 and σ_3 known. With only one Mohr circle, it is not possible draw the failure envelop as discussed in seciton 5.3.2.3. In view of the fact that the friction angle obtained from an undrained test would be always zero, the unconfined compression test could be an alternative method to determine its undrained strength:

$$\tau_f = c_u = \frac{q_u}{2} \tag{1.23}$$

Namely, the undrained strength is half the unconfined compressive strength.



5.3.4 Vane shear test

Vane shear testing is one of the most common *in-situ* methods for the estimation of the undrained shear strength of soils. The significant advantage of *in-situ* tests over the tests in the laboratory is that it avoids the disturbrance (e.g., sample collection, transporation and preparation) to soils and thus more accurate results could be obtained.

Typical appratus for vane shear test is as fig shows. A straight rigid rod is equipped with four cross-shaped vanes on one end and a combined handle/torque gauge on the other end. During the test, the end with vanes is firstly pushed into the desried depth and a torque is then applied to make the soil fails in shear. The failure body of soil is assumed a cylinder as fig shows, and the torque at failure is obtained from the torque gauge.

To calucated the undrained shear strength, assumptions are made as follows: 1) The failure surface is made up of peripheral, top and bottom surfaces of the cylinder, whose diameter and height are the length and height of the vane; 2) The shear strength is uniform at any place of the failure suface. Accoridng to the equlibrium of driving torque and resisting torque:

$$T = M_p + M_b + M_t \tag{1.24}$$

T is the driving torque. M_p, M_b and M_t represent the resisting torque from the peripheral, top and bottom surfaces of the cylinder respectively. By definition:

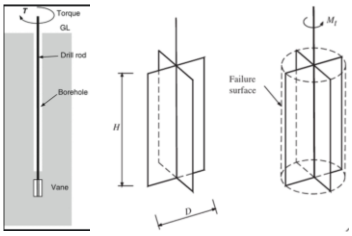
$$M_p = \tau_f \cdot \pi \cdot D \cdot H \cdot \left(\frac{D}{2}\right) \tag{1.25}$$

$$M_b = M_t = \int_0^{\frac{D}{2}} \tau_f \cdot 2 \cdot \pi \cdot r \cdot \mathrm{d}r \cdot r = \tau_f \frac{\pi D^3}{6} \tag{1.26}$$

Subsisting eq and eq into eq, the undrained shear strength τ_f equals:

$$\tau_f = \frac{T}{\frac{\pi D^2}{2}\left(H + \frac{D}{3}\right)} \tag{1.27}$$

The undrained strength parameters, $c_u = \tau_f$ and $c_u = 0$



5.4 Shear characteristics of cohesionless soils and cohesive soils

The shear characteristics of cohesionless soils and cohesive soils have repectively distinctive features, which will be detailed described in the following sections.

5.4.1 Cohesionless soils

5.4.1.1 Strength parameters

Because of relatively high hydraulic conductivity of cohesionless soils, the consolidation process finishes rapidly during loading. As cohesionless soils normally have no cohesive resistance, the strength envelop of cohesionless soils passes through the origin point and it is expressed as:

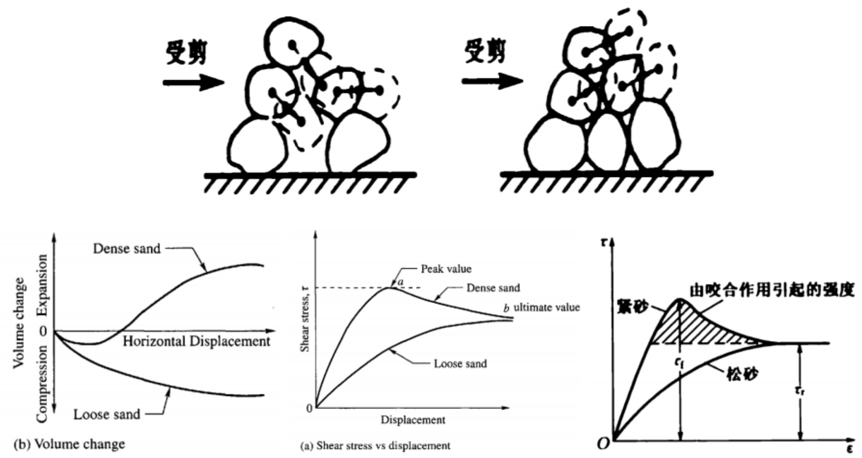
$$\tau_f = \sigma \cdot \tan\varphi_d \text{ or } \tau_f = \sigma' \cdot \tan\varphi_d \tag{1.28}$$

φ_d is the frictional angle measured in drained condition. The relative density is the key factor in determining the strength of cohesionless soils. In general, the higher the relative density, the higher strength of cohesionless soils. Other factors, e.g., mineralogy, grain size and gradation, shape of particle, also play an important part.

5.4.1.2 Stress and strain characteristics

The cohesionless soils behavior distinctivly with different degrees of compactness. For a dense cohesionless soil subjected to a shear load, the grains have to roll over the neighbouring grains to change their relative positions. In this case, the rearrangement leads to a level motion and produces a bulk exapansion. The expansion of soils due to shear is termed as dilatancy. While for a lose cohesionless soil, the grains continuously compact instead of dilating. Typical axial strain and volumetric strain curves for loose and dense cohesionless soils are shown in Fig.

Typical stress-strain curves for loose and dense cohesionless soils are shown in Fig. For dense cohesionless soils, the whole curve is divided into three parts: 1) The stress increases non-linearly from zero to a peak value represented by point a; 2) After the peak value, a reduction of stress is observed; 3) Finally, the stress approaches an asymptotic value represented by point b. For loose cohesionless soils, the stress increases monotonically to the asymptotic value. The asymptotic stress at a large deformation is regared as residual strength.



5.4.1.3 Liquefaction

When saturated loose cohesionless soils (e.g. silty sand) subjected to earthquake shaking or other rapid loading, the water within the soils may have not sufficient time to escape and the excess pore pressure increases rapidly. As seen from Eq. (1.28), when the excess pore pressure increases to a point, it results in the effective stress reducing to zero and accordingly zero strength of soils. In this case, soils behavior similar to liquid and can no longer bears any shear load.

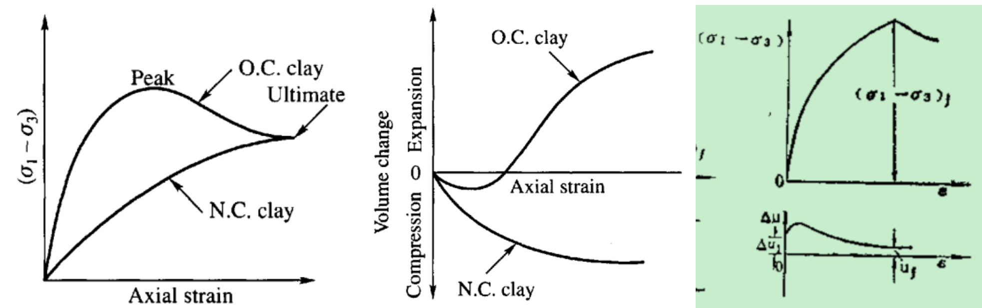
Liquefaction and related phenomena have been responsible for tremendous accidents in geotechnical engineering. When liquefaction occurs, the ground loses its ability to support foundations of top buildings. Liquefied soils may exert higher earth pressure on retaining walls, which cause them to tilt or slide. Increased water pressure within soils may also trigger landslides and collapse of dams.

5.4.2 Cohesive soils

Apart from the degree of consolidation and drainage conditions, the stress history also play a fundamental role in the strength of cohesive soils. According to the definitions in Chapter 4, the nomral consolidated soils (NC) in triaxial compression test is defined as if the ratio of the pre-consolidation pressure p_c over the confing pressure σ_3 equals one and the overconsolidated soils (OC) is defined as if the corresponding ratio is larger than one.

Typical stress-strain curves and axial strain-volumetric strain curves are plotted in Fig. Generally speaking, the shear characteristics of NC soils are similar to loose cohesionless soils, while the shear characteristics of OC soils are similar to dense cohesionless soils.

The volume of a NC soil keeps decreasing during the application of a shear load. Therefore, the pore pressure monotically increases. On the other hand, the volume of a OC soil firstly decreases and then keeps increasing till failure. Accordingly, the pore firstly increases and then keeps decreasing till failure. In certain cases, the pore pressure could even be negative.



5.4.2.1 Unconsolidated undrained test on NC soils

In a UU test, no drainage is allowed during the application of cell pressure σ_3 and deviatoric stress $\sigma_1 - \sigma_3$. As the test usually takes short time, UU test is also called as Quick test or Q-test. This is done simply by keeping the drainage valve closed throughout the test. Pore pressures are developed in the soil specimen during the two loading states. As the water content do not vary throughout the test, the soil sample would not gain any shear strength. No matter what magnitude of confining pressure applies, the deviatoric stress $\sigma_1 - \sigma_3$ keeps a constant, namely, the diameters of Mohr's circles under different cell pressures at failure are the same and the drawn failure envelope is horizontal, as fig shows. The strength parameters are

$$\varphi_u = 0 \text{ and } c_u = \frac{q_u}{2} \tag{1.29}$$

It is worth pointing out that the horizontal failure envelope only applies to fully saturated soils. For partially saturated soil, the voids filled with gas are easy to be compressed, which makes the soil sample stronger. Thus, the failure envelope forms a convex shape. With higher confining stresses, all air voids would be compressed, and it reaches a flat failure envelope to obey the φ = 0 concept.

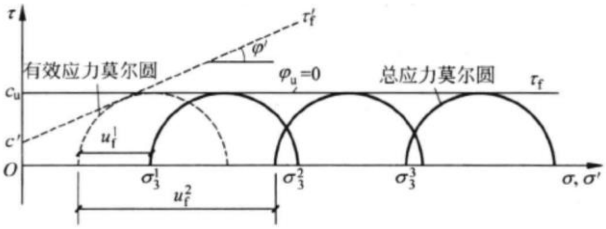
In fact, only a single test is enough to determine c_u, as the circle in dotted line in Fig. When there is no cell pressure, the loading conditions of the UU test are the same as those of the unconfined compression test. Therefore, the unconfined compression test could replace the UU test to a certain extent if it is run properly.

The pore pressure u_f at failure could be accurately measured by the pressure gauge. Therefore, the effective principal stresses at failure are:

$$\sigma'_{1f} = \sigma_{1f} - u \text{ and } \sigma'_{3f} = \sigma_3 - u \tag{1.30}$$

As the induced pore pressure for NC soil is always positive, the Mohr's circles drawn by effective principal stresses always locate to the left of the corresponding Mohr's circles drawn by total principal stresses. Moreover, no matter what magnitude of the confining pressure applies, there is only one Mohr's circle drawn by effective principal stresses, as indicated by dot line in fig. The reason is that any increment in minor principal stress results in an equal increment of the pore pressure as the pore pressure coefficient $B = 1$. With only one Mohr's circle, it is impossible to obtain the effective strength parameters.

The strength parameters, c_u is commonly used to assess the short-term strength of cohesive soil as: temporary excavation or quick construction. As cohesive soils will consolidate during or after construction, it will gradually gain strength. Therefore, the designs using strength parameters by UU test is relatively conservative.



5.4.2.2 Consolidated undrained test on NC soils (CU test)

In a CU test, drainage is allowed to take place only during the application of cell pressure σ_1. It is also referred to as rapid test or R-test. When the consolidation of the specimen under the cell pressure is completed, the drainage valve is closed and the additional deviatoricic stress $\sigma_1 - \sigma_3$ is applied. The soil specimen fails in shear under undrained conditions.

The pore pressure induced during the application of deviatoric stress is measured by a pressure gauge. As the induced pore pressure for NC soil is always positive, the Mohr's circles drawn by effective principal stresses always locate to the left of the corresponding Mohr's circles drawn by total principal stresses, as fig shows. With a group of σ_{3f} , σ_{1f} and u_f in hand, the Mohr's circles at failure in total principal stresses and in effective principal stresses can be drawn, as fig shows.

Due to the consolidation process in the first stage, strength of the soil sample is gained. Normally, the higher the consolidation pressure, the higher the strength. This is reflected by increasing diameter of the Mohr's circle. The effective stress failure envelope and total stress failure envelope, which are tangent to the corresponding Mohr's circles, are determined to obtain the total stress strength parameters c_{cu} and φ_{cu} as well as the effective stress parameters c'_{cu} and φ'_{cu} as seen in Fig.

It worthy point out the strength envelops in Fig passes through the origin point. The consolidated undrained strength of NC soils can be expressed as:

$$\tau_f = \sigma \cdot \tan\varphi_{cu} \text{ and } \tau_f = \sigma' \cdot \tan\varphi'_{cu} \tag{1.31}$$

Note that zero cohesion component c_{cu}=0 or c'_{cu}=0 in Eq. (1.31) does not necessarily mean that soil is resisted purely by friction. In fact, shear resistance of clays is mostly contributed from cohesive resistance, but its expression merely implies that, when consolidation pressure is zero, there will be no strength.

By definitions, NC soil indicates that the confining pressure applied for consolidation in the frist stage equals the pre-consolidation pressrue. From a theoretical point of view, if the applied confining pressure tends to zero, the pre-consolidation pressrue should also tends to zero. In this case, the soils could not have any strength as they have not experienced any consolidation process. However, soils in natual have already experienced certain magnitude of consolidation. Therefore, if the CU test was carried out by increasing the confining pressure from a small value to the desired one, the test may have a transition from OC soil to NC soil and the corresponding envelop may be composed of two linear portions, as fig shows.

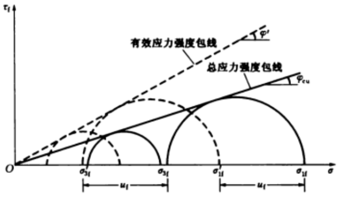
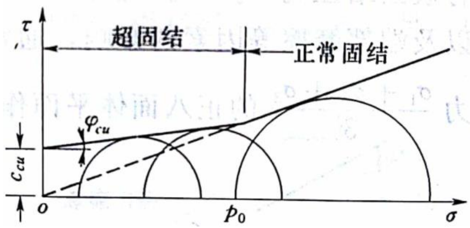


图 5-26 正常固结饱和粘土 CU 试验的强包线和有效应力强度包线



5.4.2.3 Consolidated drained test on NC soil (CD test)

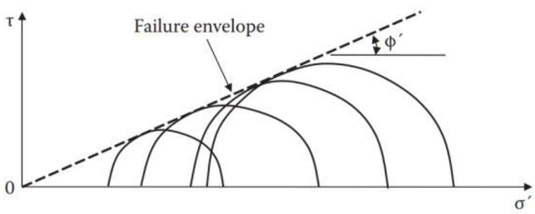
In a CD test, drainage is allowed to take place during the application of cell pressure (σ_1) and the additional deviatoric stress ($\sigma_1 - \sigma_3$). As the consolidation is allowed to completely finish, it usually cost considerably long time and hence is referred to as slow test or S-test.

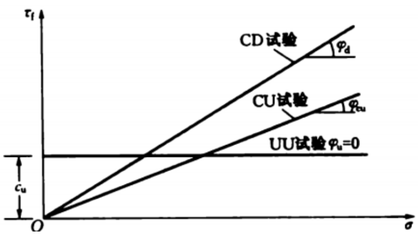
As the drainage is allowed throughout the test, the soil sample gains more strength due to its higher reduction in water content through the consolidation and drained shear processes. Accordingly, sizes of failure Mohr's circles nearly proportional to their consolidation pressures are drawn to define the failure envelope of the soil as seen in Figure.

With three or four more tests under different confining pressures, the Mohr's circles at failure can be drawn. The strength parameters gained from the CD test are denoted as c_d and φ_d. As the excess pore pressure is completely dissipated, the total stress equals the effective stress. The effective strength parameters $c'_d = c_d$ and $\varphi'_d = \varphi_d$. For NC soil, the strength envelop determined by CD test also passes through the origin point and its expression is:

$$\tau_f = \sigma' \cdot \tan \varphi'_d \tag{1.32}$$

The strength parameters, c_d and φ_d are commonly applied to the problems whose long-time stability needs special care, i.e., clay soil slop, retaining wall backfilled with clay soil, foundation or embankment on clay soil.





5.4.2.4 Comparisons of strength parameters by UU, CU and CD tests on NC soils

Fig shows the relation of strength envelops obtained from UU, CU and CD tests. The strength envelop from UU test is a horizontal line while the strength envelops from CU and CD tests are straight lines passing through the origin point. The relation of strength parameters by UU, CU and CD tests are: $c_u > c_{cu} = c_d = 0$ and $\varphi_d > \varphi_{cu} > \varphi_u = 0$.

5.4.2.5 Unconsolidated undrained test on OC soils

The procedure of UU test on OC soils is same as of UU test on NC soil. The main difference lies in the applied confining pressure should be larger than the pre-consolidation pressure. For heavy OC soil, the induced pore pressure is always negative. Therefore, the Mohr's circles drawn by effective principal stresses always locate to the right of the corresponding Mohr's circles drawn by total principal stresses. While for lightly OC soil, the induced pore pressure is postitive. In this case, the Mohr's circles drawn by effective principal stresses locate to the left of the corresponding Mohr's circles drawn by total principal stresses.

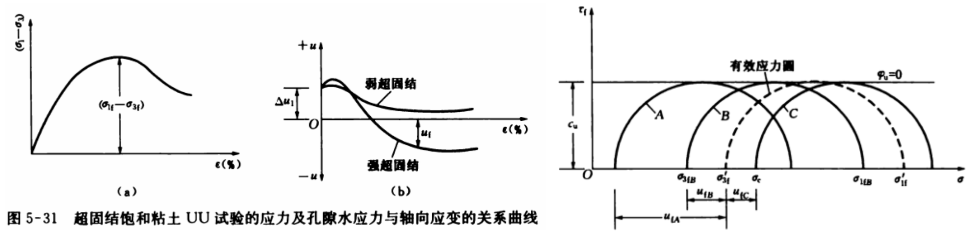


图 5-31 超固结饱和粘土 UU 试验的应力及孔隙水应力与轴向应变的关系曲线

5.4.2.6 Consolidated undrained test on OC soils (CU test)

The procedure of CU test on OC soils is same as of CU test on NC soil. The main difference lies in the applied confining pressure should be larger than the pre-consolidation pressure. The relative location between the Mohr's circles drawn by effective principal stresses and the Mohr's circles drawn by total principal stresses depends on OCR. Fig shows that, at lower confining pressure, the CU test is performed on a heavy OC soil. Therefore, the negative pore pressure leads to the Mohr's circles drawn by effective principal stresses locates to right the Mohr's circles drawn by total principal stresses. On the other hand, at lower confining pressure, the CU test is performed on a lightly OC soil, the positive pore pressure leads to the Mohr's circles drawn by effective principal stresses locates to left the Mohr's circles drawn by total principal stresses. The strength parameters determined therein have following relations $c_{cu} > c' \neq 0$ and $\varphi_{cu} < \varphi'$.

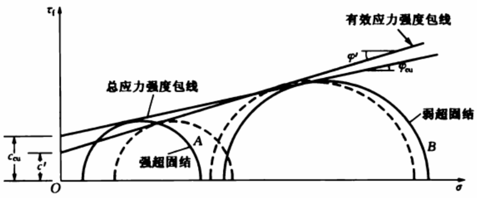


图 5-34 超固结饱和粘土 CU 试验的强度包线

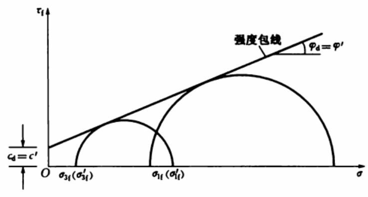


图 5-36 超固结饱和粘土 CD 试验时的强度包线

5.4.2.7 Consolidated drained test on OC soil (CD test)

The procedure of CD test on OC soils is same as of CD test on NC soil. The main difference lies in the applied confining pressure should be larger than the pre-consolidation pressure. As the drainage is allowed throughout the test, the excess pore pressure is completely dissipated and the total stress equals the effective stress. The effective strength parameters $c'_d = c_d$ and $\varphi'_d = \varphi_d$. For OC soil, the strength envelop determined by CD test does not pass through the origin point and its expression is:

$$\tau_f = \sigma' \tan \varphi'_d + c'_d \tag{1.33}$$

5.4.2.8 Comparisons of strength parameters by UU, CU and CD tests on OC soils

Fig shows the relation of strength envelops obtained from UU, CU and CD tests. The strength envelop from UU test is a horizontal line. Contrary to NC soils, the strength envelops from CU and CD tests no longer pass through the origin point. The relation of strength parameters by UU, CU and CD tests are: $c_u > c_{cu} > c_d$ and $\varphi_d > \varphi_{cu} > \varphi_u = 0$.

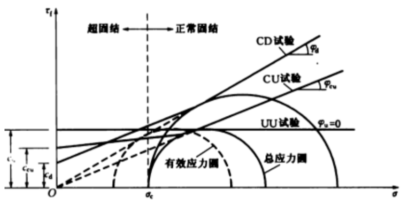


图 5-37 饱和粘土 UU,CU,CD 试验结果的强度包线

5.5 Stress path

Stress path is used to represent the successive variation of stresses at a point within soils during a test. Although the changes in stress states can be represented by a series of Mohr's circles in one diagram, the whole procedure is really cumbersom and the final result is confusing. To provide a simple graphical representation of the changes in stress states, use has been made of typical points on Mohr's circles. Usually, the point whose shear stress attains the maximum value is chosen, as fig shows. The locus of these points is called a stress path.

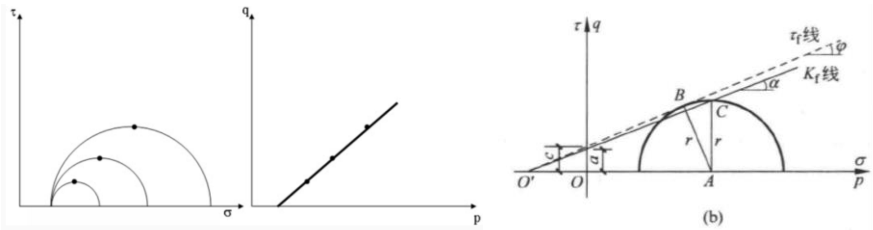
The evolution of stress states can be also plotted on p-q plane as fig shows, where:

$$\begin{cases} p = \frac{1}{2}(\sigma_1 + \sigma_3) \\ q = \frac{1}{2}(\sigma_1 - \sigma_3) \end{cases} \tag{1.34}$$

p is the mean of principal stress and q is the principal stress difference. The line, which joins the points of maximum shear stress for each test, is called k_f line. The failure envelop and k_f line are not independent. According to the geometrical relation:

$$\begin{cases} \alpha = \arctan(\sin\varphi) \\ a = c \cdot \cos\varphi \end{cases} \tag{1.35}$$

Therefore, it is possible to draw the k_f line from the failure envelop or vice versa.



5.5.1 Total and effective stress paths

As the Mohr's circle can be drawn according to total stresses and effective stresses, the stress paths defined accordingly are termed as total stress path (TSP) and effective stress path (ESP). After introducing the effect of pore pressure, Eq. (1.34) changes to:

$$\begin{cases} p' = \frac{1}{2}(\sigma_1' + \sigma_3') = \frac{1}{2}(\sigma_1 - u + \sigma_3' - u) = p - u \\ q' = \frac{1}{2}(\sigma_1' - \sigma_3') = \frac{1}{2}(\sigma_1 - u - \sigma_3' + u) = q \end{cases} \tag{1.36}$$

Eq. (1.36) indicates that the Mohr's circle drawn according to (p, q) and (p', q') have the same size while the locations of the center points are displaced by the pore pressure. Their relative position depends on the sign of the pore pressure as discussed in section 5.4.2. In other words, the points on TSP and ESP, which represent the total stress state and the effective stress state of the same point, have identical principal stress difference. The difference between the total mean stress and the mean effective stress is equal to the pore pressure.

5.5.2 TSP in triaxial compression test

In a triaxial compression test, the TSPs corresponding to the two loading stages are as follows:

1) Application of the confining pressure $\Delta\sigma_1 = \Delta\sigma_3 = \sigma_3$. The TSP starts from the point O to the point A and the increments of mean of the principal stress and the principal stress difference are respectively as:

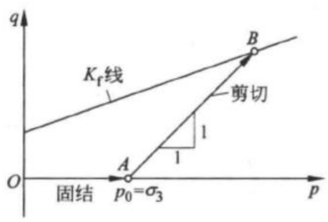
$$\begin{cases} \Delta p = \dfrac{1}{2}\left(\Delta\sigma_1 + \Delta\sigma_3\right) = \Delta\sigma_3 \\ \Delta q = \dfrac{1}{2}\left(\Delta\sigma_1 - \Delta\sigma_3\right) = 0 \end{cases} \Rightarrow \frac{\Delta q}{\Delta p} = 0 \tag{1.37}$$

In this stage, The TSP concides with the horizontal line, which starts from the original point O to point A.

2) Application of the principal stress difference $\Delta\sigma_1 = \sigma_1 - \sigma_3$ and $\Delta\sigma_3 = 0$. The TSP starts from the point A to point B and the increments of mean of the principal stress and the principal stress difference are respectively as:

$$\begin{cases} \Delta p = \dfrac{1}{2}\left(\Delta\sigma_1 + \Delta\sigma_3\right) = \dfrac{1}{2}\Delta\sigma_1 \\ \Delta q = \dfrac{1}{2}\left(\Delta\sigma_1 - \Delta\sigma_3\right) = \dfrac{1}{2}\Delta\sigma_1 \end{cases} \Rightarrow \frac{\Delta q}{\Delta p} = 1 \tag{1.38}$$

In this stage, the TSP is a straight line at an angle of $45°$ to the p-axis, as fig shows.



5.5.3 ESP in a triaxial compression test

If the water is allowed to escape, there would not be any pore pressure. The ESP and TSP are concide, so does the k_f line and the k_f' line.

If there is the pore pressure, the ESP and the corresponding k_f' should be determined as follows: 1) Determine p and q by Eq. (1.34) according to certain stress state; 2) Determine $p' = p - u$ and $q' = q - u$ by measured pore pressure u; 3) Plot ESP and k_f' line. In fact, the esstential is the determiantion of the induced pore pressure due to total stress.

The following picture shows ESP and k_f' line in a CU test on NC or lightly OC soils where the induced pore pressure might be positive. For comparison, the TSP and k_f line are also plotted. Generally speaking, the ESP can not be a straight line as the coiefficient of pore pressure A depends on the over-consolidation ratio and the applied stress. For a CU test, the induced pore pressure is determined by Eq. (1.21). Fig shows several ESPs with A equals 0, 0.5 and 1.

1) A=0

In this case u=0. The total stress and effective stress are equal. Therefore, the ESP and TSP are coincide, which is a straight line at an angle of 45° to the p-axis

2) A=0.5

In this case $u = \sigma_1 - \sigma_3/2 = \Delta\sigma_1/2$, the increments of mean of the principal stress and the principal stress difference are respectively as:

$$\begin{cases} \Delta p' = \Delta p' - u = \dfrac{1}{2}\Delta\sigma_1 - \dfrac{1}{2}\Delta\sigma_1 = 0 \\ \Delta q' = \Delta q' = \Delta\sigma_1 \end{cases} \Rightarrow \frac{\Delta q'}{\Delta p'} = \infty \tag{1.39}$$

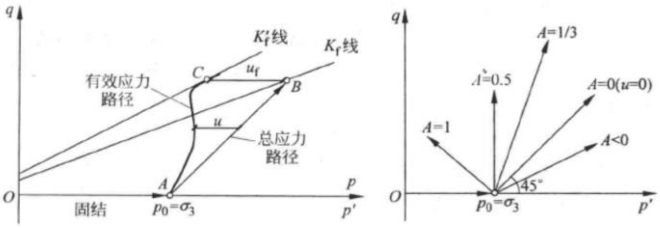
In this stage, the ESP is a vertical straight line.

3) A=1.0

In this case $u = \sigma_1 - \sigma_3 = \Delta\sigma_1$, the increments of mean of the principal stress and the principal stress difference are respectively as:

$$\begin{cases} \Delta p' = \Delta p' - u = \dfrac{1}{2}\Delta\sigma_1 - \Delta\sigma_1 = -\dfrac{1}{2}\Delta\sigma_1 \\ \Delta q' = \Delta q = \Delta\sigma_1 \end{cases} \Rightarrow \frac{\Delta q'}{\Delta p'} = -1 \tag{1.40}$$

In this stage, the ESP is a straight line at an angle of 135° to the p-axis.



【例题 5-3】 若土的泊松比 $\nu=0.3$，求侧限压缩条件下加载时土体中的应力路径。

【解】

在侧限压缩条件下加载时，水平向应力增量 $\Delta\sigma_3$ 与竖直向应力增量 $\Delta\sigma_1$ 之比为侧压力系数 K_0。根据式(3-6)有，

$$K_0 = \frac{\Delta\sigma_3}{\Delta\sigma_1} = \frac{\nu}{1-\nu} = \frac{0.3}{1-0.3} = 0.429$$

$$\Delta\sigma_3 = 0.429\Delta\sigma_1$$

又 $\Delta p = \frac{1}{2}(\Delta\sigma_1 + \Delta\sigma_3) = 0.715\Delta\sigma_1$

$\Delta q = \frac{1}{2}(\Delta\sigma_1 - \Delta\sigma_3) = 0.286\Delta\sigma_1$

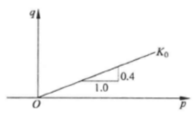


图 5-37 例题 5-3 图