Stress in Soils

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Outline

- General introduction
- Effective geostatic stress due to self-weight
- Gross pressure
- Vertical stress increment due to surface load

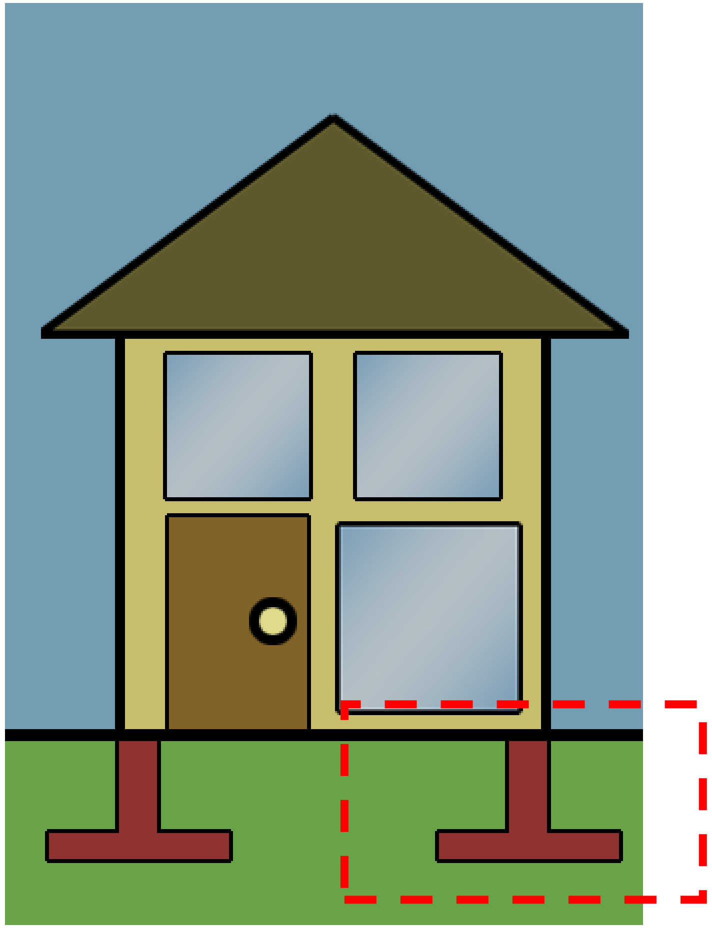
General introduction

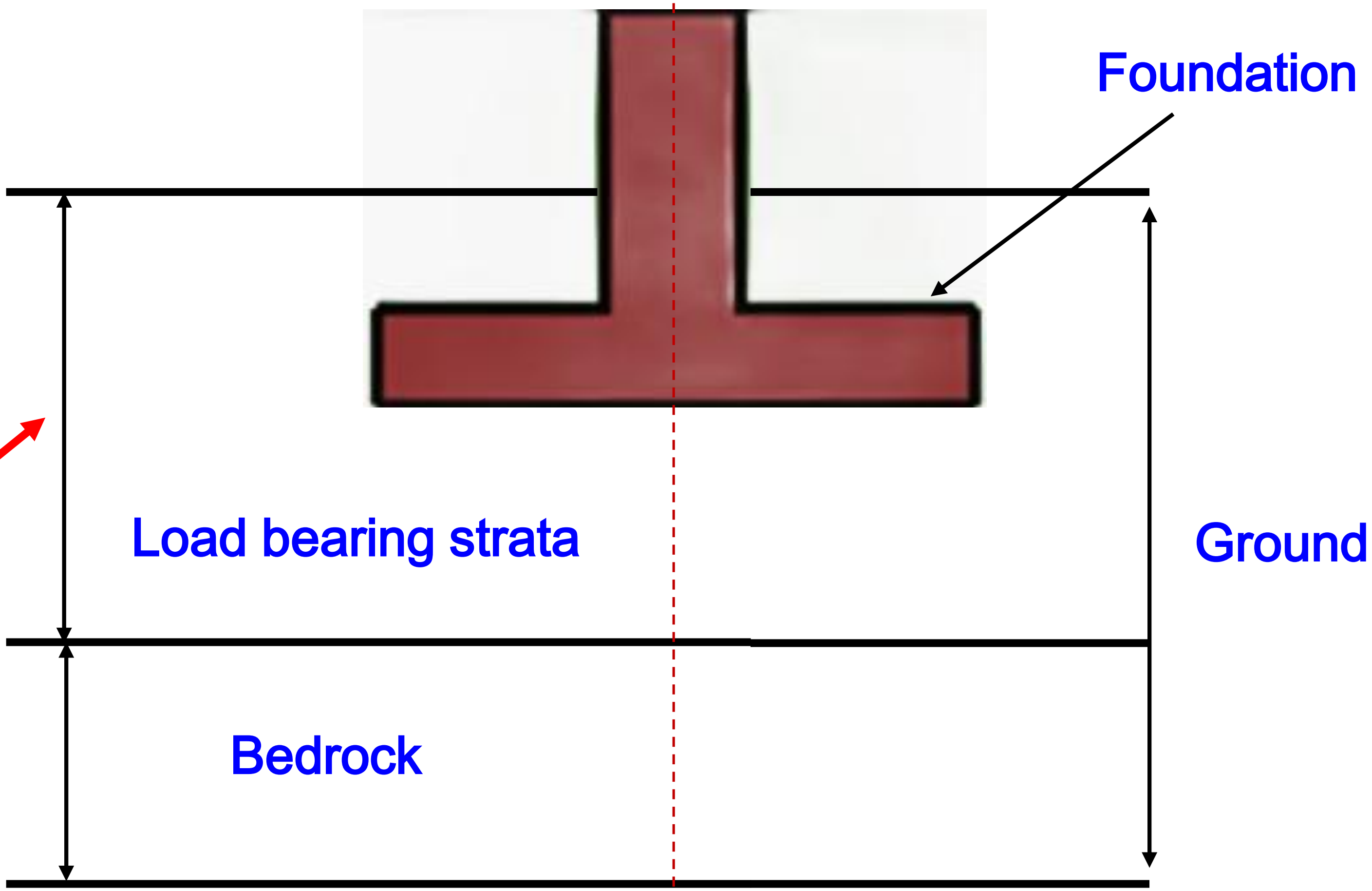
When the structure is constructed on the superficial soil layer, its self-weight (permeant) and live loads are transmitted by their foundation to ground.

If the properties of the ground are not good enough, technical improvements are need. Such kind of ground is called an artificial ground, otherwise, it is called a natural ground

Various ground treatment

General introduction





General introduction

Foundation: **the structural elements** that form the base of a building and transmit loads of it to the underlying ground. Foundations are generally considered either **shallow or deep.**

Load bearing strata: refers to the underlying soil or rock that can bear the weight of a building or structure without shifting or compressing too much.

Soft substrata: comparing with load bearing strata, the soft substrata normally has **low strength and high compressibility**.

General introduction

Geostatic stress: also called initial stress. It is a kind of stress which forms during the formation of soils due to their self-weight.

Vertical stress increment: When external load is applied to the soil surface, it increases the vertical stresses within soils. The increased stress is the vertical stress increment.

These two kinds of stress in together will result in two affects: 1) deformation problem, namely the compressibility of soil and thus the settlement of the top building; 2) strength problem. If the external load is large enough and the induced disturbance (additional stress) exceeds the bearing capacity of soil stratum. The failure of ground may also leads to the failure of the top building.

General introduction

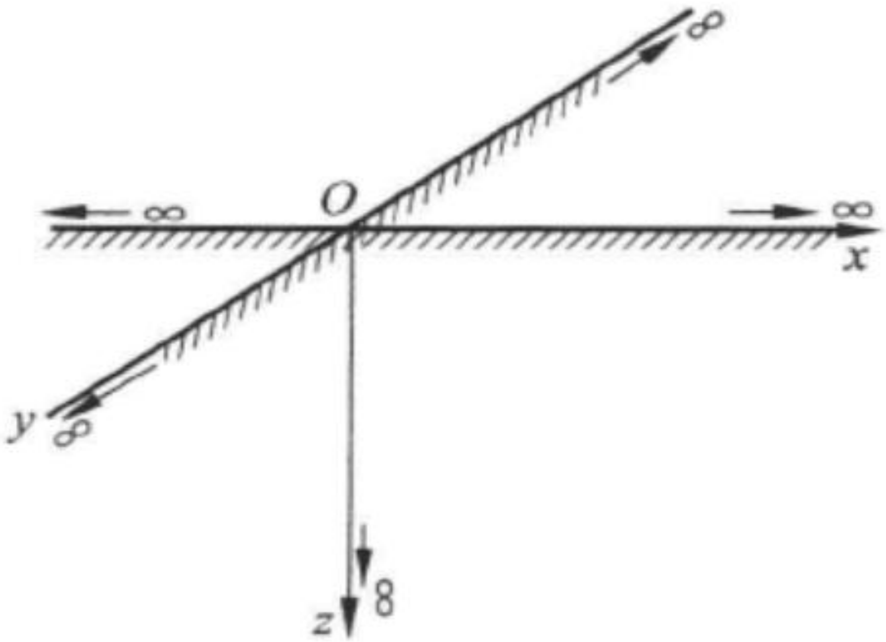
Basic assumptions in soil mechanics

Continuous medium: the theory of the continuum mechanics requires that soil is continuous medium.

Linear deformation body: the stress-strain relationship is linear

Homogeneous and isotropic body: when variation of the property of soil layer is not too large, soil is homogeneous and isotropic body

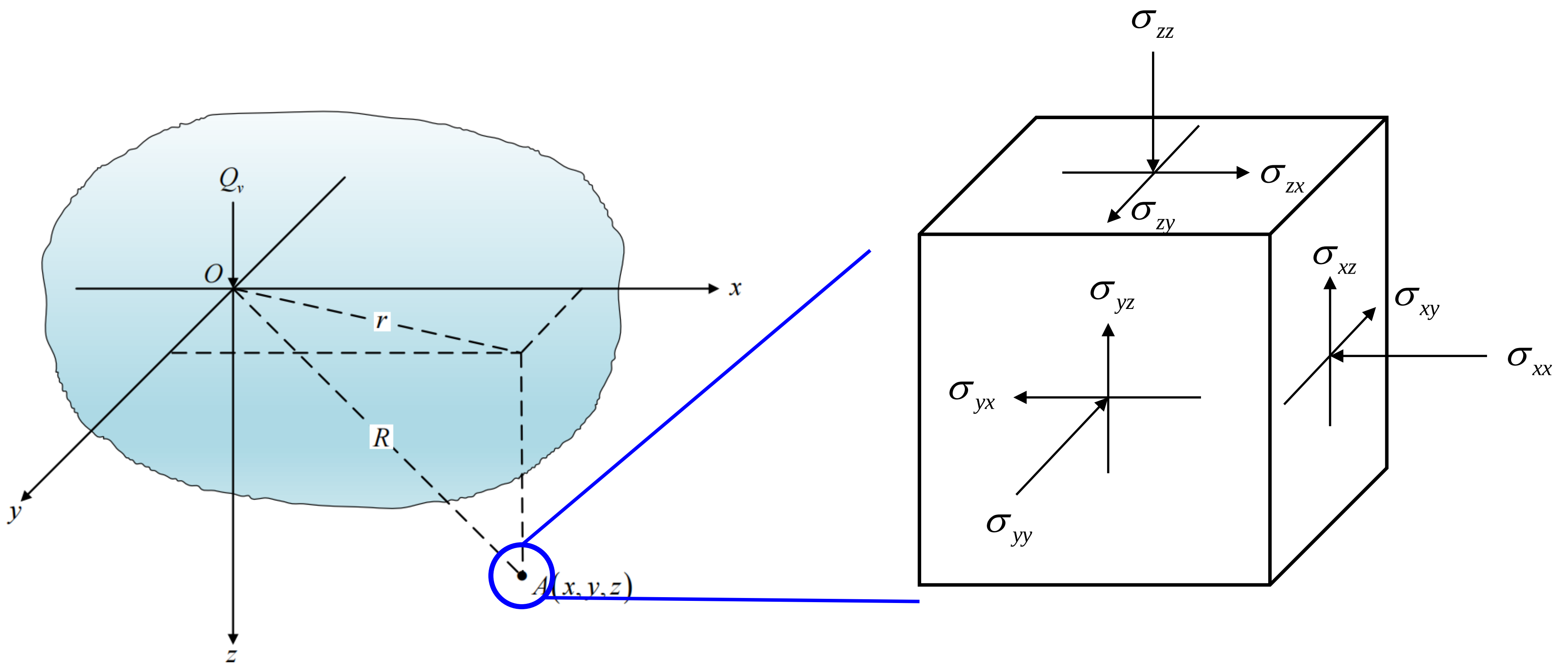
General introduction



Semi-infinite elastic region

The **ground** in soil mechanics is described as a **semi-infinite elastic region**. Its dimensions in x, y and z directions are extended to infinity and its mechanical behavior (stress-strain relation) is linear. As most structures related to soils are built underground, **the z direction preferably points downward**.

General introduction



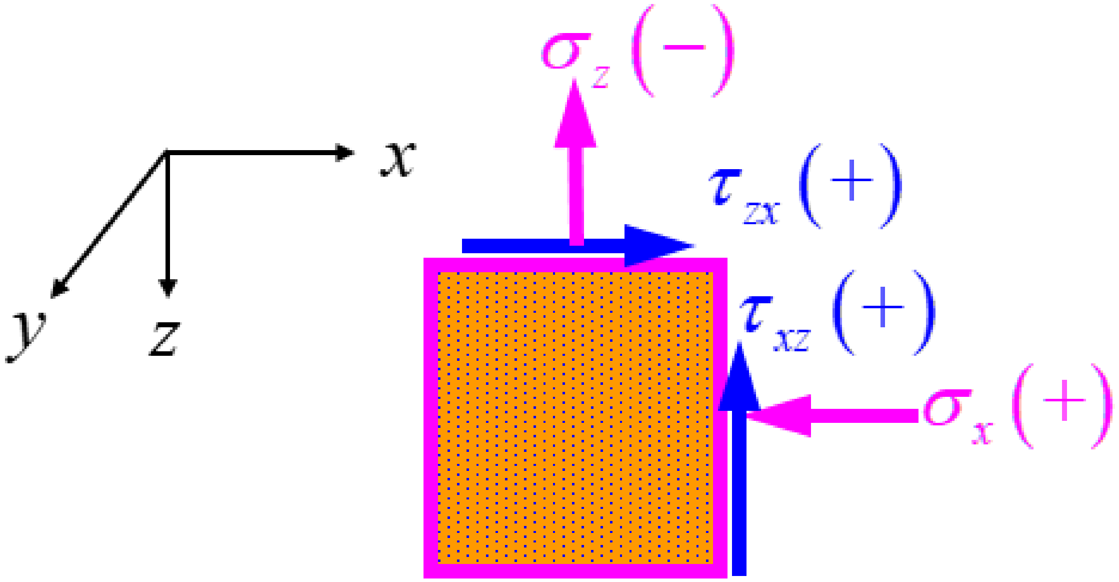
Stress tensors

$$\sigma = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

General stress state at any point in soils

General introduction

Stress convention in soil mechanics



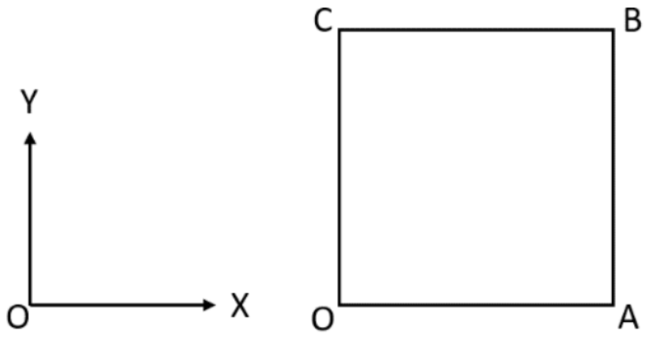
Positive stress on any surface

The surface of the 3D-stress element whose **outward normal points to the positive direction** of the coordinate axis is defined as the **positive surface**

The stress (normal stress or shear) is **positive** (negative) if it acts on the **positive surface** in a **negative (positive) direction**. All the stress components in the following 3D-stress element are positive.

General introduction

The reference coordinate system OXY is as figures shows. According to the stress convention in soil mechanics, the directions of positive normal stress and positive shear stress on face AB toward____________( )



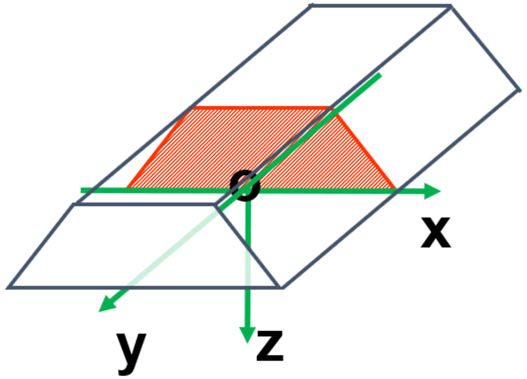
A) right, upward B) left, upward C) left, downward

General introduction

Plane strain condition

- The structure along certain direction is sufficiently large
- All sections perpendicular to y-axis is the symmetrical plane
- All stress or stain components are in the xz-plane

$$\sigma = \begin{bmatrix} \sigma_{xx} & 0 & \sigma_{xz} \\ 0 & \sigma_{yy} & 0 \\ \sigma_{zx} & 0 & \sigma_{zz} \end{bmatrix}$$

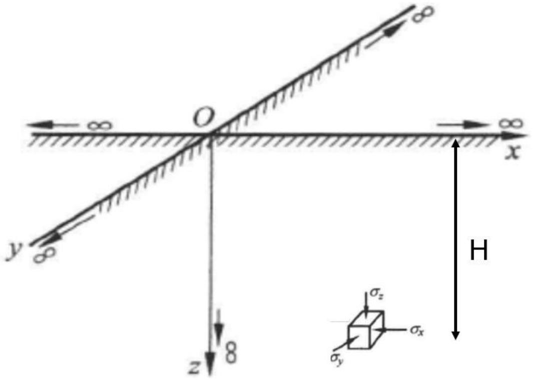


General introduction

Confined strain condition

The confined strain state refers to a special stress state where there is no lateral deformation.

The geostatic stress within the strata is in confined strain state. The deformation only occurs in the vertical direction. As all the vertical plane are symmetric plane, there is no shear stress on the lateral surfaces of the 3D-stress element.



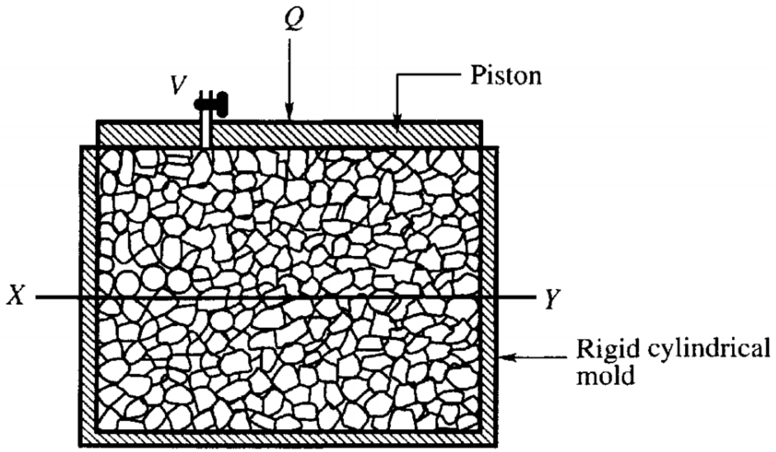
Schematic diagram of confined strain condition

$$\sigma = \begin{bmatrix} \sigma_{xx} & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix} \quad \text{and} \quad \sigma_{yy} = \sigma_{xx} = K_0 \sigma_{zz}$$

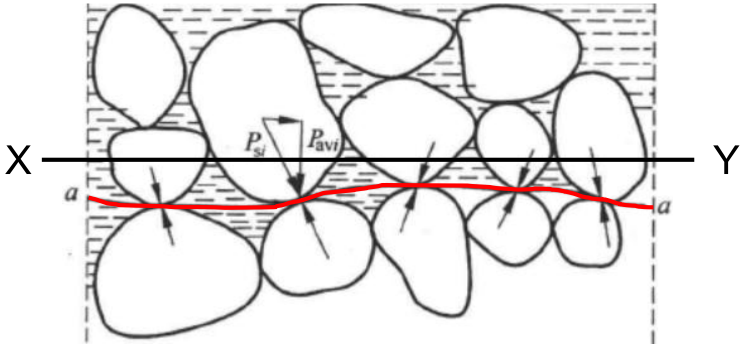
General introduction

Principle of effective stress

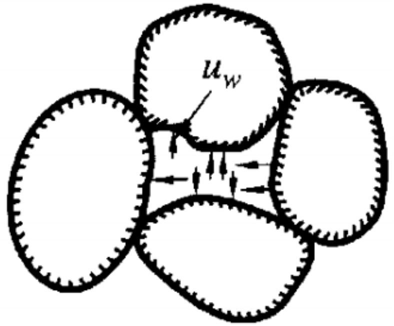
Von Karl Terzaghi



Soil under confined strain condition



Intergranular pressure



Pore pressure

General introduction

Principle of effective stress

Averaged vertical stress

$A_w = A$

$$\sigma A = \sum P_{svi} + u_w A_w \rightarrow \sigma = \frac{\sum P_{svi}}{A} + u_w \frac{A_w}{A}$$

$$\sigma = \sigma' + u_w$$

General introduction

$$\sigma = \sigma' + u_w$$

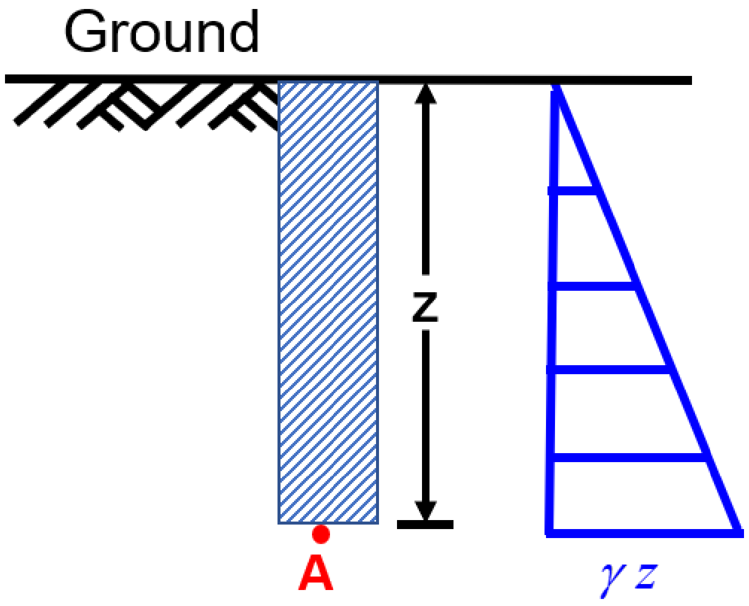
Effective stress (intergranular) is the pressure transmitted through grain to grain at the contact points, which is responsible for deformation and strength of soils.

For saturated soil, pressure is instantaneously induced in the water after loading. During the application of the external load, the time-dependent transition process from pressure to effective stress always exists.

Only the saturated soil is considered

Effective geostatic stress due to self weight

Homogeneous ground without groundwater



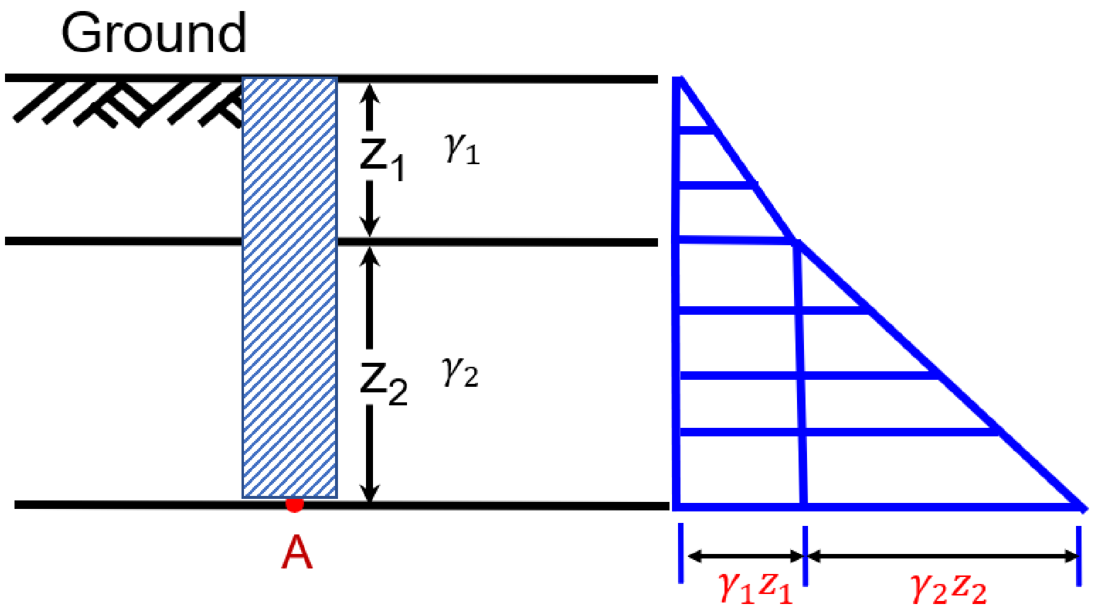
Schematic diagram of homogeneous ground

$$\sigma'_{cz} = \sigma_{cz} = \frac{W}{A} = \frac{\rho g V}{A} = \rho g z = \gamma z$$

W – weight of the soil column
V – volume of the soil column
ρ – bulk density
γ – bulk specific weight

Effective geostatic stress due to self weight

Stratified ground without groundwater



Schematic diagram of stratified ground

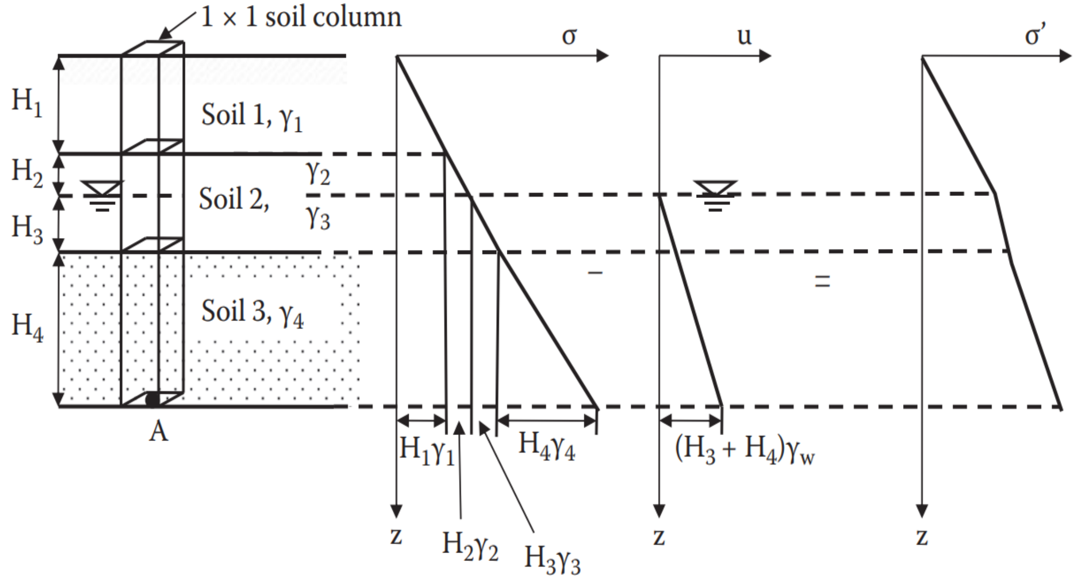
$$\sigma'_{cz} = \sigma_{cz} = \gamma_1 z_1 + \gamma_2 z_2$$

Generalization

$$\sigma_z = \sum_{i=1}^{n} \gamma_i z_i$$

Effective geostatic stress due to self weight

Stratified soil with steady water table



Stratified ground and stead groundwater

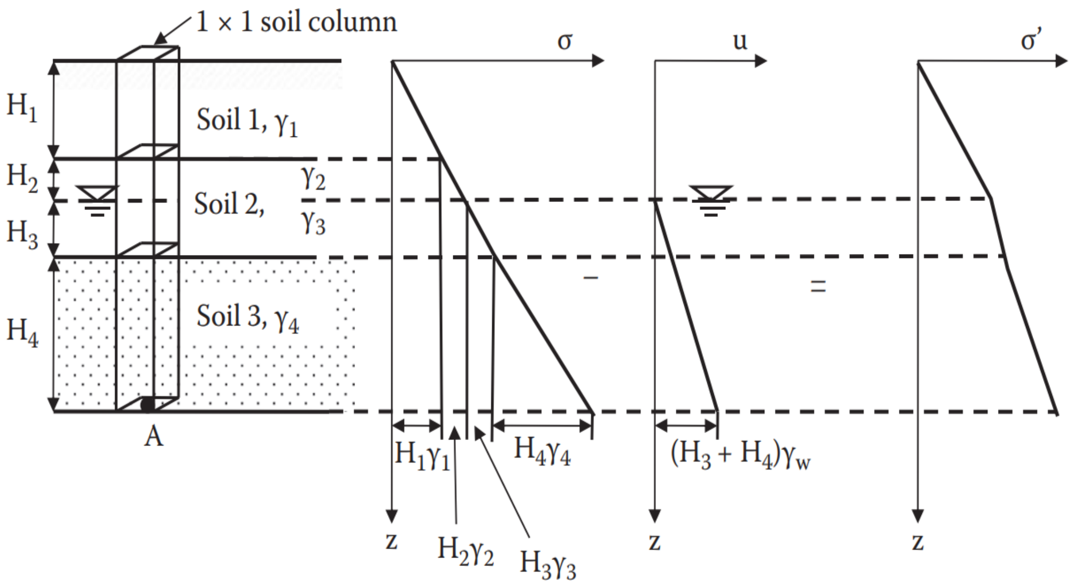
For a stratified ground with steady water table, the effective geostatic stress is calculated as follows

1) Calculate the total stress at point A. The bulk specific weight should be replaced by saturated specific weight for the layers which are below the water table

2) The hydrostatic pressure is calculated for submerged soil layers

Effective geostatic stress due to self weight

Stratified soil with steady water table



Stratified ground and stead groundwater

$$\sigma_{cz} = \gamma_1 H_1 + \gamma_2 H_2 + \gamma_{3,sat} H_3 + \gamma_{4,sat} H_4$$

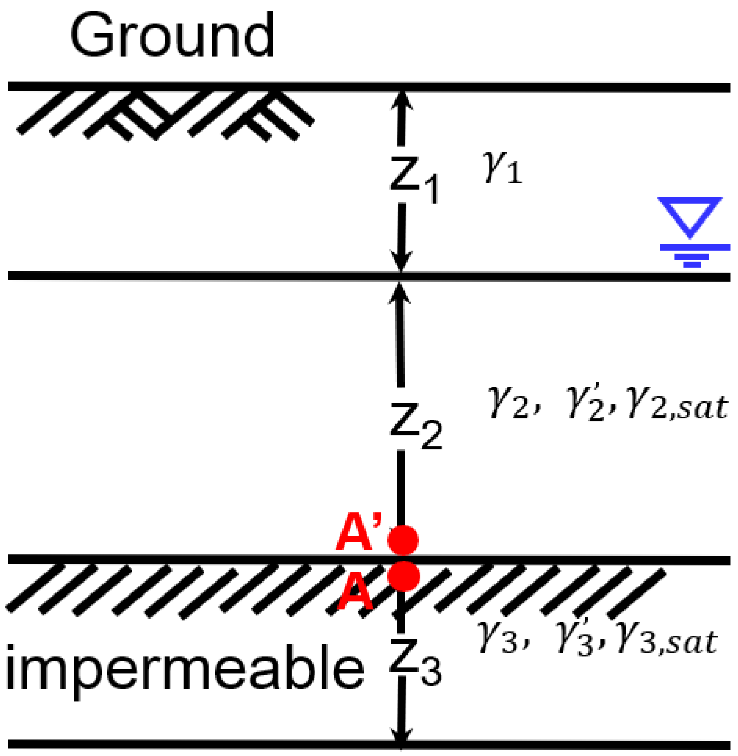
$$u_w = \gamma_w H_3 + \gamma_w H_4$$

Principle of effective stress

$$\begin{aligned}\sigma'_{cz} = \sigma_{cz} - u_w &= \gamma_1 H_1 + \gamma_2 H_2 + \gamma_{3,sat} H_3 + \gamma_{4,sat} H_4 - \gamma_w H_3 - \gamma_w H_4 \\ &= \gamma_1 H_1 + \gamma_2 H_2 + \left(\gamma_{3,sat} - \gamma_w\right) H_3 + \left(\gamma_{4,sat} - \gamma_w\right) H_4 \\ &= \gamma_1 H_1 + \gamma_2 H_2 + \gamma'_3 H_3 + \gamma'_4 H_4\end{aligned}$$

Effective geostatic stress due to self weight

Stratified soil with impermeable layer



Stratified ground with impermeable layer

For point A'

$$\sigma_{cz}' = \sigma_{cz} - u_w = \gamma_1 z_1 + \gamma_2' z_2$$

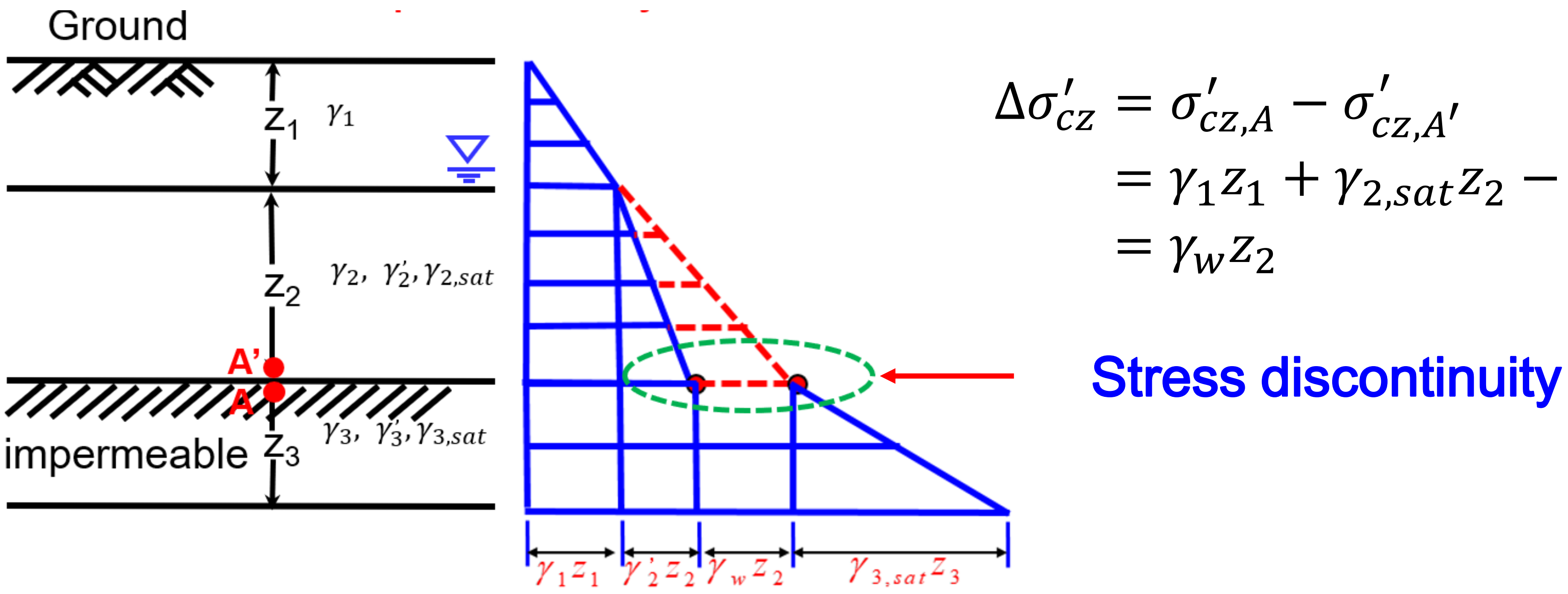
For point A

$$\sigma_{cz}' = \sigma_{cz} - u_w = \gamma_1 z_1 + \gamma_{2,sat} z_2$$

For point within the impermeable layer $u = 0$

Effective geostatic stress due to self weight

Stratified soil with impermeable layer

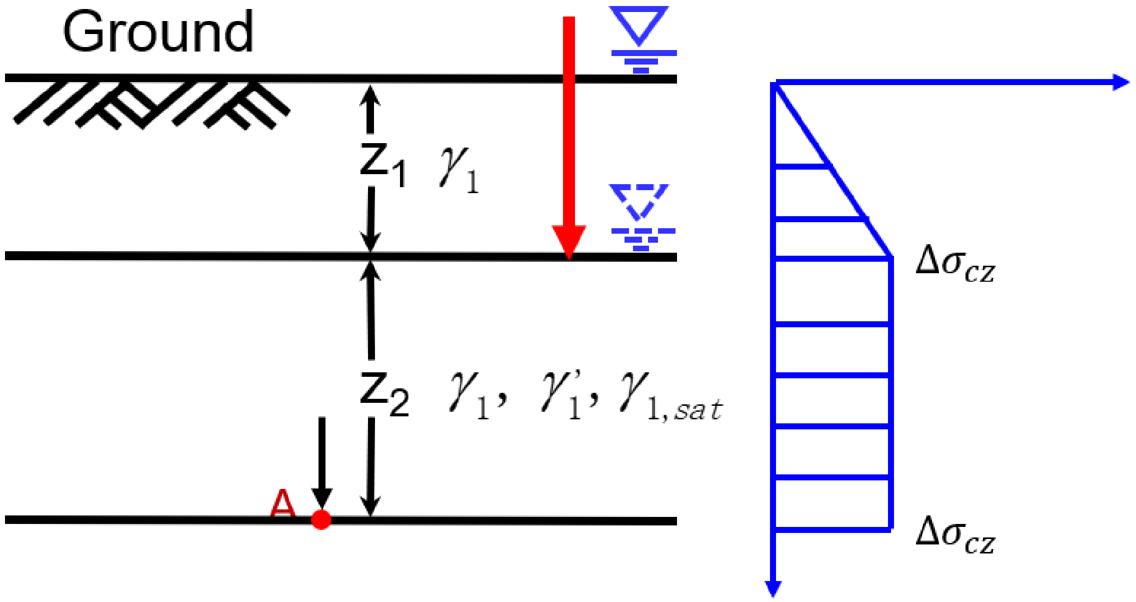


$$
\begin{aligned}
\Delta\sigma'_{cz} &= \sigma'_{cz,A} - \sigma'_{cz,A'} \\
&= \gamma_1 z_1 + \gamma_{2,sat} z_2 - (\gamma_1 z_1 + \gamma_2' z_2) \\
&= \gamma_w z_2
\end{aligned}
$$

Effective geostatic stress distribution in stratified ground with impermeable layer

Effective geostatic stress due to self weight

Stress change due to change of ground water



Effective geostatic stress distribution with fall of groundwater

Before water fall

$$\sigma'_{cz} = \gamma_1' z_1 + \gamma_2' z_2$$

After water fall

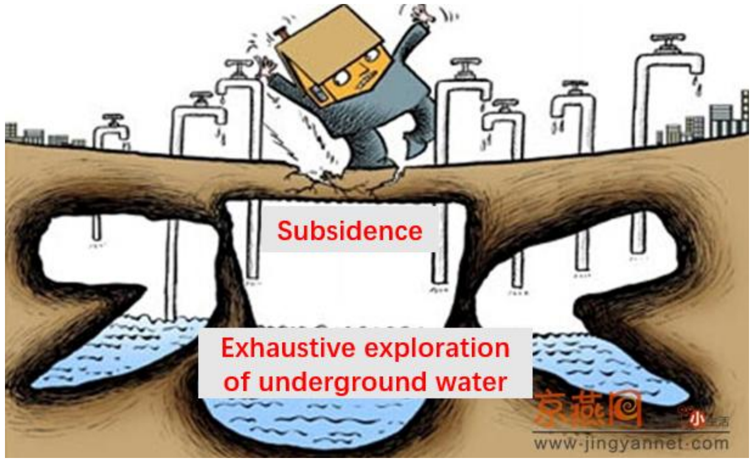
$$\sigma'_{cz} = \gamma_1 z_1 + \gamma_2' z_2$$

Stress change

$$\Delta\sigma'_{cz} = \gamma_1 z_1 + \gamma_2' z_2 - \gamma_1' z_1 - \gamma_2' z_2$$
$$= (\gamma_1 - \gamma_1') z_1$$

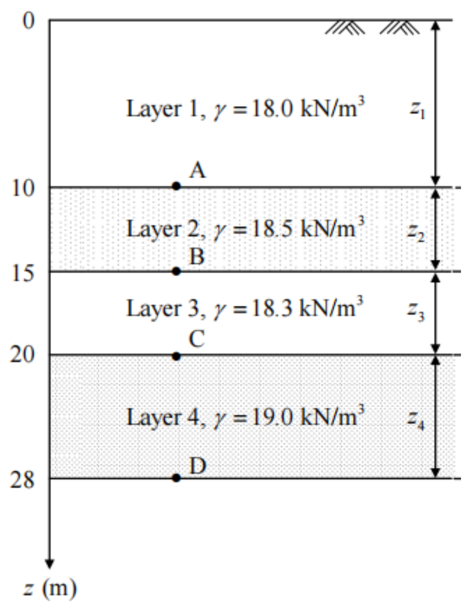
Exercises

Explain what will happen if the fall of water level happens?



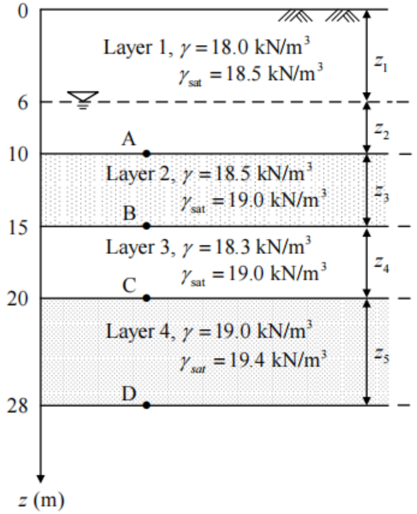
Exercises

According to the profile of the ground as Fig. 1 shows, compute the effective geostatic stress at Points A, B, C, D and plot the effective geostatic stress with the depth z .



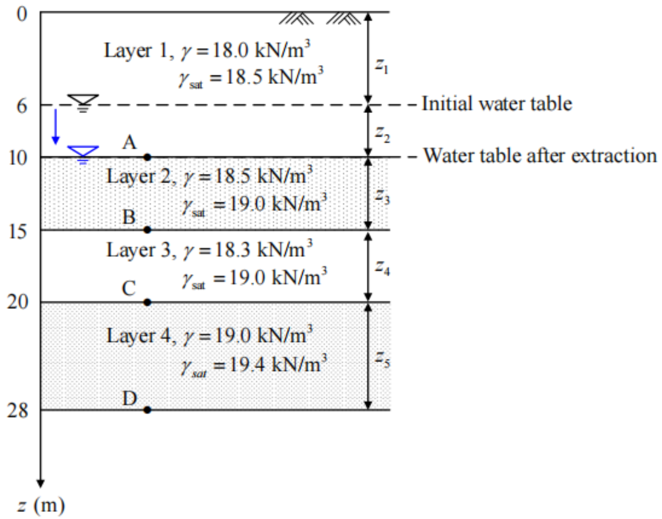
Exercises

The profile of the ground is shown in Fig. 2. The steady ground water table is at a depth of 6 m below the ground surface. 1) Compute the total geostatic stress σ_{cz}, pore pressure u, and then the effective geostatic stress σ'_{cz} at Points A, B, C, D; 2) Plot the total geostatic stress, pore pressure, as well as effective geostatic stress with the depth z.



Exercises

The profile of the ground is shown in Fig. 3. Originally, the steady ground water table is at a depth of 6 m below the ground surface. Due to heavy water extraction, the original ground water table is lowered to 10 m below the ground surface. Compute the change of the effective geostatic stress σ'_{cz} at Points A, B, C, D.



Effective geostatic stress due to self weight

Horizontal geostatic stresses (σ_x and σ_y): for semi-infinite half-space, the horizontal effective geostatic stresses under the action of self-weight are equal and expressed as

$$\sigma_x = \sigma_y = K_0 \sigma_z$$

$\boldsymbol{K_0}$ – coefficient of earth pressure at rest or lateral stress ratio at rest, which is related to the stress history and types of soils

Effective geostatic stress due to self weight

Specified values of K_0

Theoretical value: $$K_0 = \frac{\nu}{1-\nu}$$

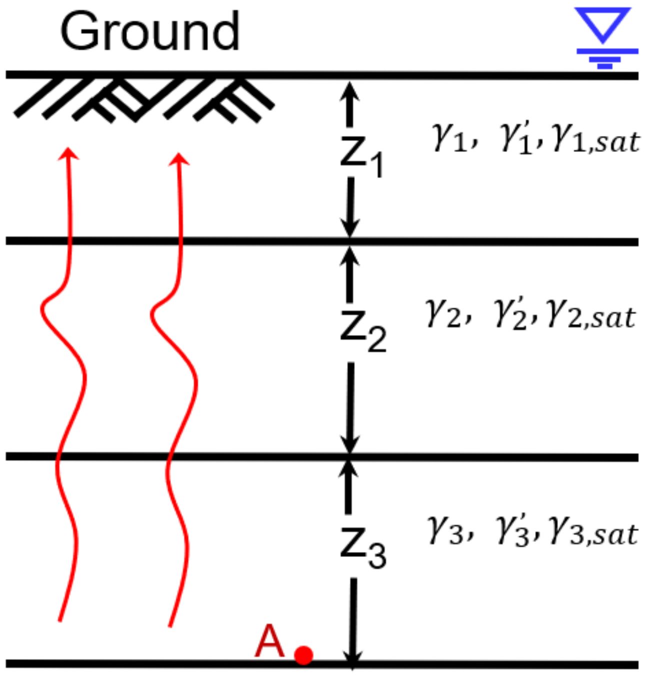
Empirical relation: $K_0 = 1 - \sin\varphi'$; φ' is the effective frictional angle

Code:

Type of soils	K_0
Loose sand	0.40~0.45
Tight sand	0.45~0.50
Compacted backfill soils	0.80~1.50
Normal consolidated soil	0.50~0.60
Over consolidated soil	1.00~4.00

Effective stress due to self weight

Stratified soil with vertical water flow (bottom to top)



Stratified ground with upward water flow

$$\sigma_{cz} = \gamma_{1,sat} z_1 + \gamma_{2,sat} z_2 + \gamma_{3,sat} z_3$$

When there is upward water flow

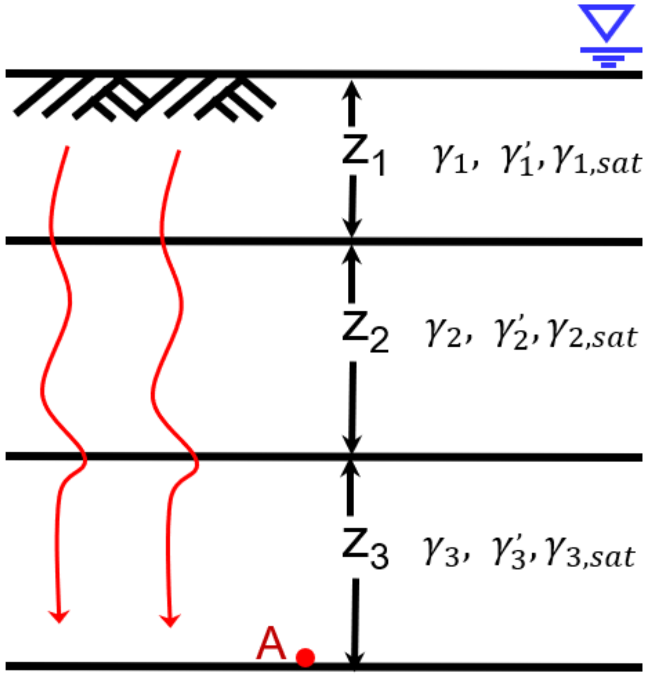
$$h_{bot} = h_p \quad h_{top} = h_z + h_p = h_z = z_1 + z_2 + z_3$$

$$\frac{h_{bot} - h_{top}}{z_1 + z_2 + z_3} = i \longrightarrow h_p = (z_1 + z_2 + z_2) \cdot (1 + i)$$

$$\sigma_{cz}' = \sigma_{cz} - u_w = \sigma_{cz} - h_p \gamma_w = \sum_{i=1}^{3} (\gamma' - i\gamma_w) z_i$$

Effective stress due to self weight

Stratified soil with vertical water flow (top to bottom)



Stratified ground with downward water flow

$$\sigma_{cz} = \gamma_{1,sat} z_1 + \gamma_{2,sat} z_2 + \gamma_{3,sat} z_3$$

When there is downward water flow

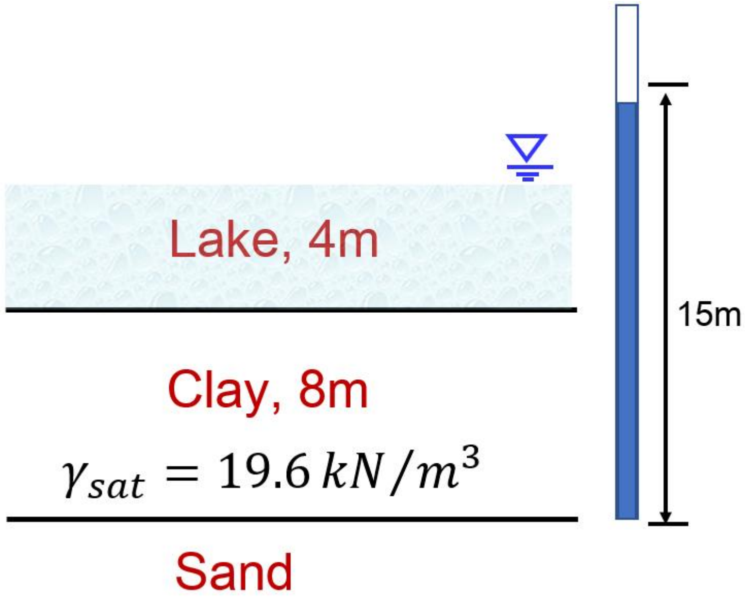
$$h_{bot} = h_p \quad h_{top} = h_z + h_p = h_z = z_1 + z_2 + z_3$$

$$\frac{h_{top} - h_{bot}}{z_1 + z_2 + z_3} = i \longrightarrow h_p = (z_1 + z_2 + z_2) \cdot (1 - i)$$

$$\sigma'_{cz} = \sigma_{cz} - u_w = \sigma_{cz} - h_p \gamma_w = \sum_{i=1}^{3} (\gamma' + i\gamma_w) z_i$$

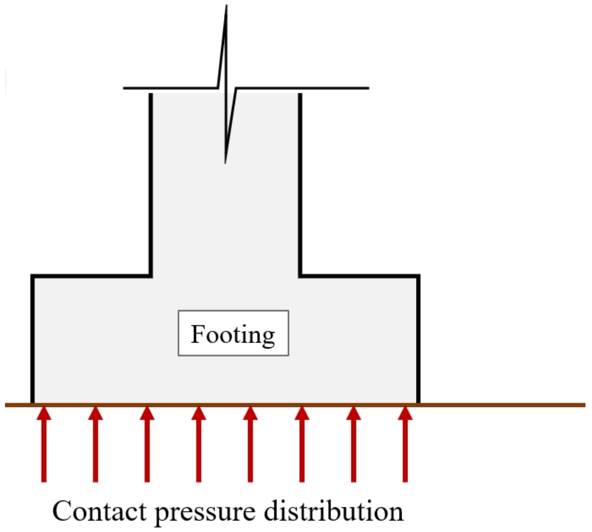
Exercises

Calculate the effective stress at mid of the sand layer.



Gross pressure

Gross pressure: loads from the structure are transferred to the soil through foundation. A reaction to this load, soil exerts an upward pressure on the bottom surface of the foundation. Quantitatively, gross pressure is due to column loads, footing self weight and soil pressure above the footing



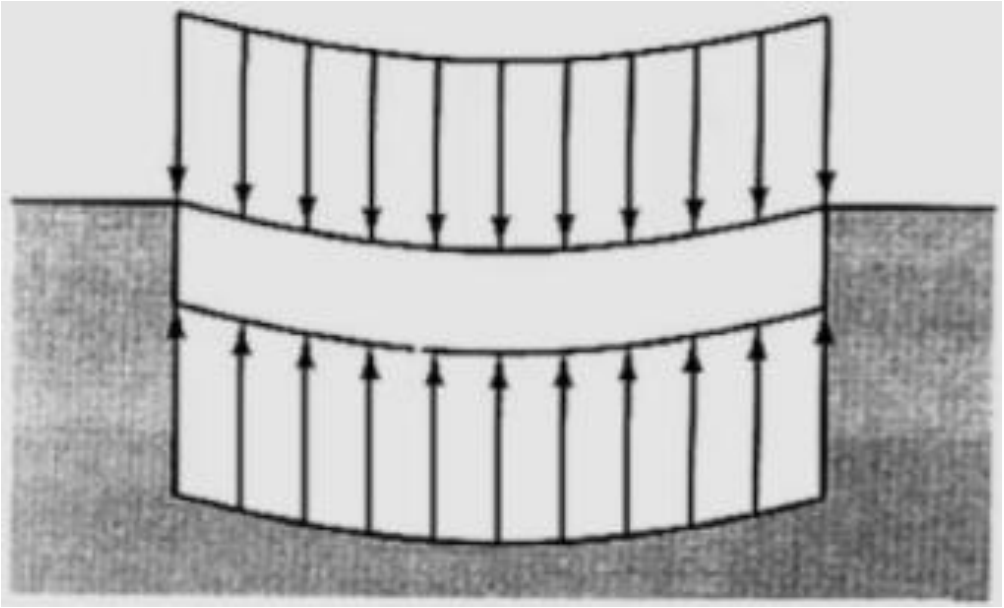
Gross pressure

Main factors affecting gross pressure distribution

- ◆ Rigidity of footing
- ◆ Types of soils
- ◆ Types loading

Gross pressure

Rigidity of footing and types of soil

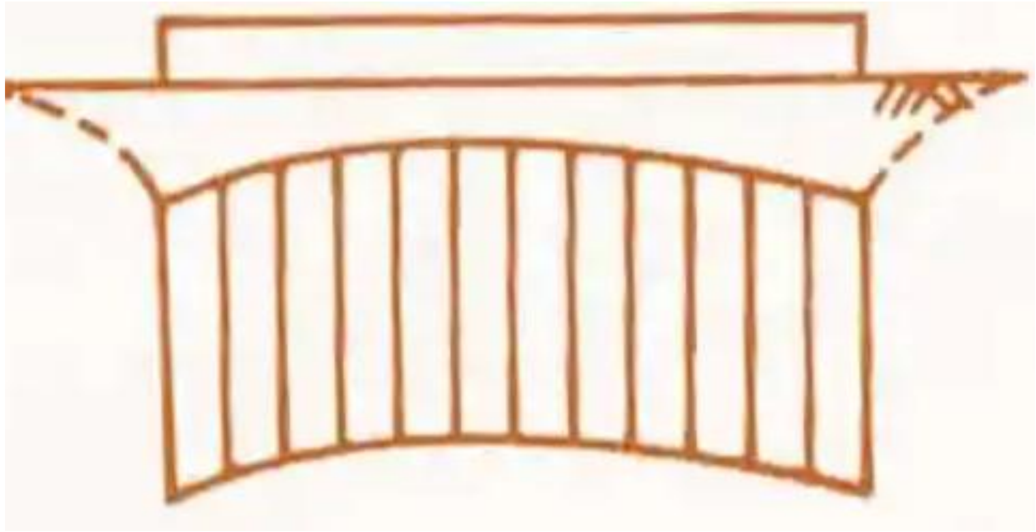


Schematic diagram of flexible footing

For flexible footing on cohesive soil with uniformly distributed load. Settlement is maximum at center of footing and minimum at the edges which forms bowl like shape. The distribution of the gross pressure is also uniform.

Gross pressure

Rigidity of footing and types of soil

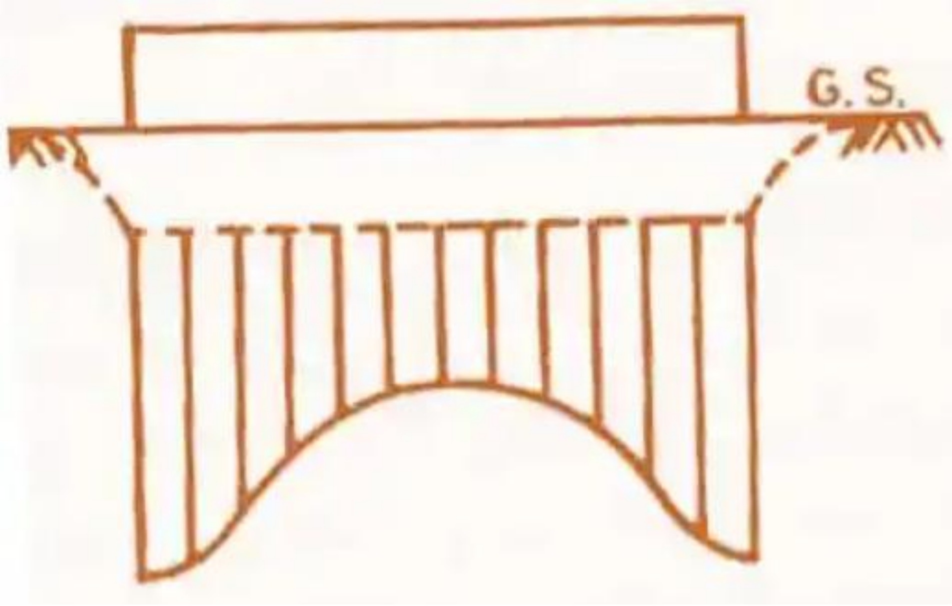


Schematic diagram of flexible footing

For flexible footing on cohesionless soil with uniformly distributed load. Settlement at center becomes minimum while at edges it is maximum which exact opposite case of the settlement of flexible footing over cohesive soil. However, the distribution of the gross pressure is still uniform.

Gross pressure

Rigidity of footing and types of soil

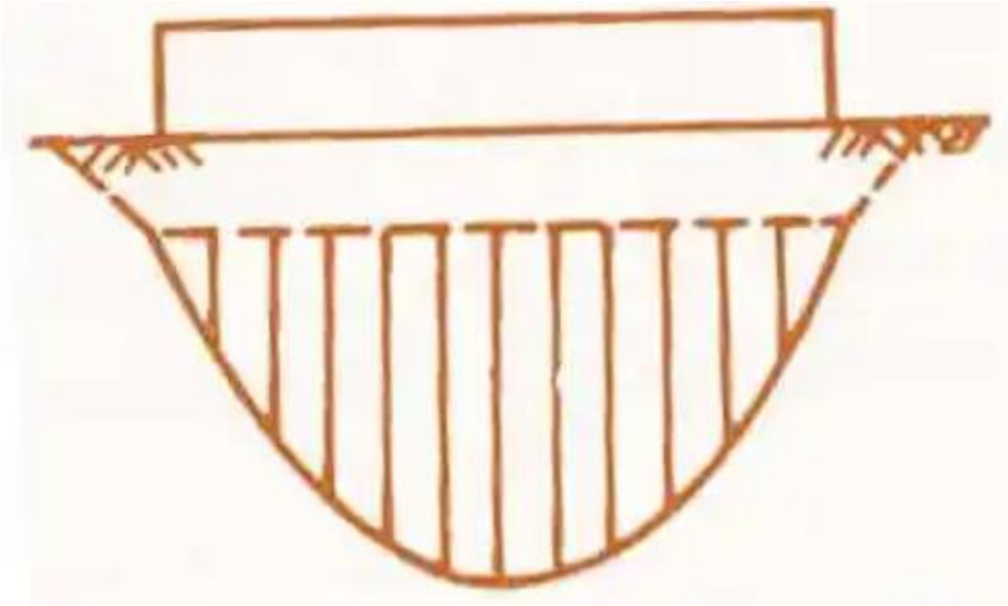


Schematic diagram of rigid footing

For rigid footing on cohesive soil with uniformly distributed load. Settlement is uniform but gross pressure varies. At edges pressure is maximum and at center it is minimum which forms inverted bowl shape. The values of stresses at edges becomes finite when plastic flow occurs in real soils.

Gross pressure

Rigidity of footing and types of soil

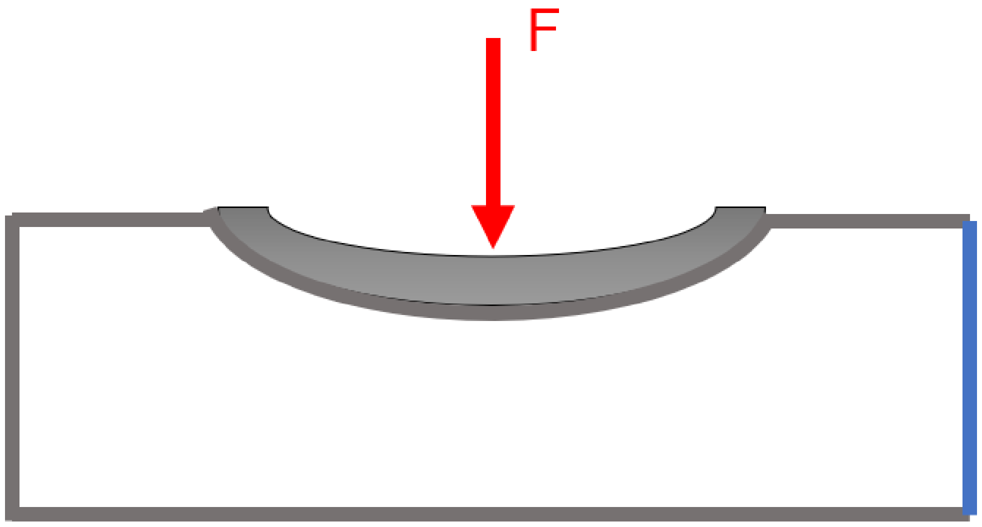


Schematic diagram of rigid footing

For rigid footing on cohesionless soil with uniformly distributed load. Gross pressure is maximum at center and gradually reduces to zero towards edges. Settlement is uniform. If the footing is embedded, then there may be some amount of gross pressure at the edges of rigid footing.

Gross pressure

Types of load

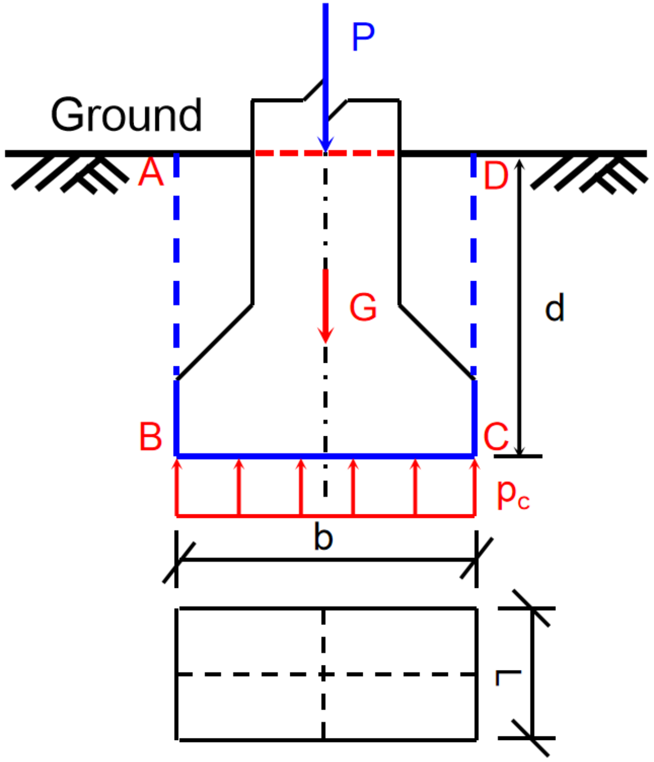


Schematic diagram of concentrated load

- If concentrated loading is applied at the center of foundation resting on cohesive soil, gross pressure is not uniform irrespective of stiffness of foundation.
- For flexible foundation, gross pressure is maximum exactly under the load application
- For rigid foundations, application of point load on rigid foundations can be comparable to the application of uniform loading on rigid foundation resting on cohesive soil.

Gross pressure

Gross pressure beneath a footing



Central load on a footing

The weight of the foundation and the backfill soil on it (area of ABCD)

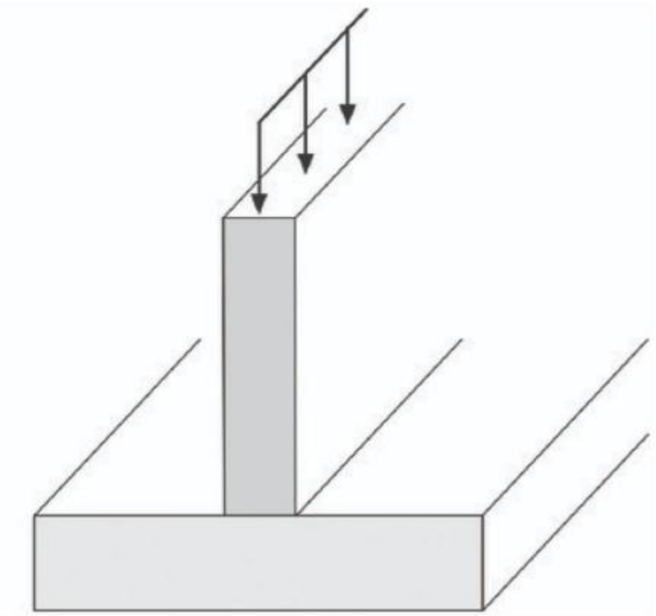
$$G=\gamma_G d \qquad \gamma_G=20\,\mathrm{kN/m^3}$$

Gross pressure

$$p_k=\frac{P+G}{A}=\frac{P+A\cdot d\cdot\gamma_G}{A}=\frac{P}{A}+\gamma_G\cdot d$$

Gross pressure

Gross pressure beneath a strip footing



Central load on a strip footing

$$p_k = \frac{P+G}{A} = \frac{P + A \cdot d \cdot \gamma_G}{A} = \frac{P}{b} + \gamma_G \cdot d$$

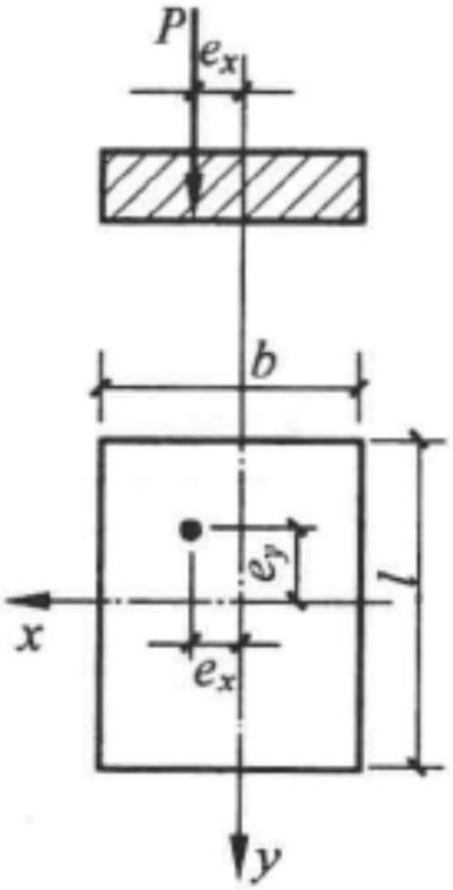
For a strip footing, the unit length is taken for analysis

Gross pressure

Gross pressure for an eccentric load

$$p_k = \frac{P+G}{A} \pm \frac{M_x \cdot e_y}{I_x} \pm \frac{M_y \cdot e_x}{I_y}$$

- M_x and M_y are the bending momentum around x and y-axis
- e_x and e_y are the eccentricity along x and y direction, I_x and I_y are the moment of inertia around x and y axis



Eccentric load on a footing

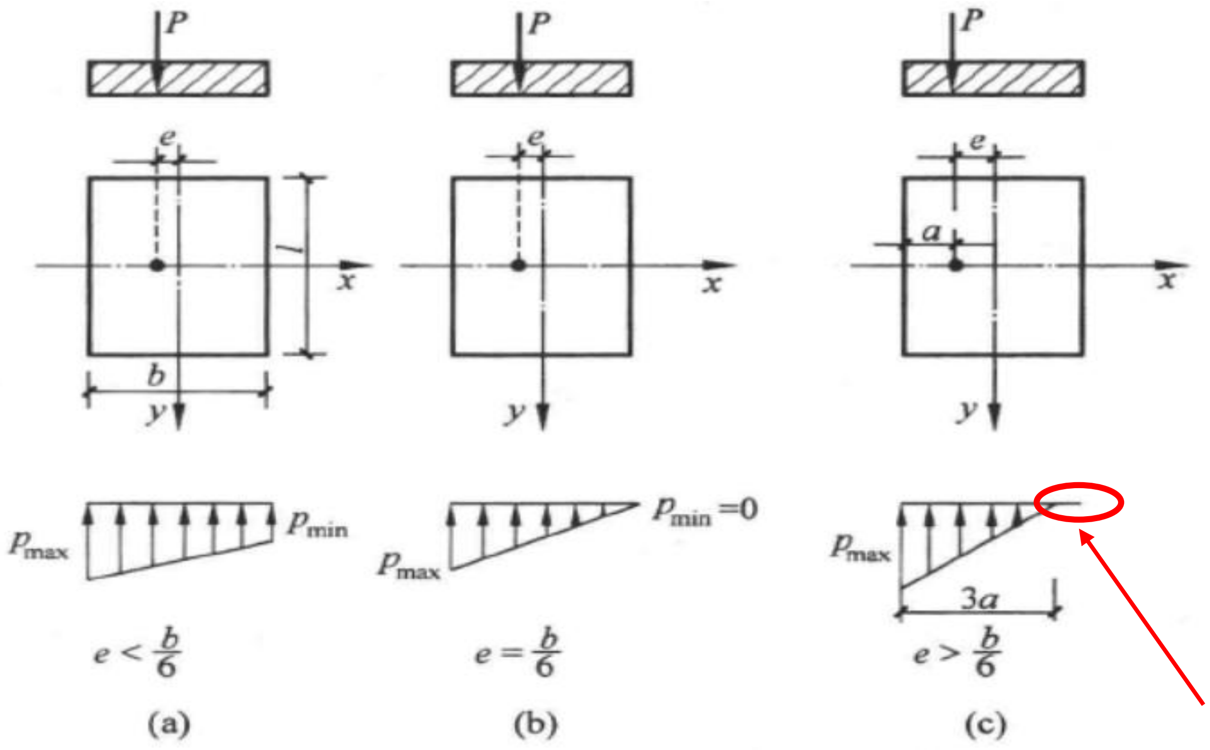
Gross pressure

Gross pressure for an eccentric load

Much simpler case is that the eccentric load is applied along certain axis (either x or y-axis). When the eccentric load is along the x-axis, M_x=0 , e_y=0 . In this case, the gross pressure beneath the footing simplifies to:

$$p_{k,\substack{\max \\ \min}} = \frac{P+G}{A}\left(1 \pm \frac{6e}{b}\right)$$

Gross pressure



Tensile force

If e<b/6, Trapezoidal distribution

$$p_{k,\substack{\max\\ \min}} = \frac{P+G}{A}\left(1 \pm \frac{6e}{b}\right)$$

If e=b/6, Triangle distribution

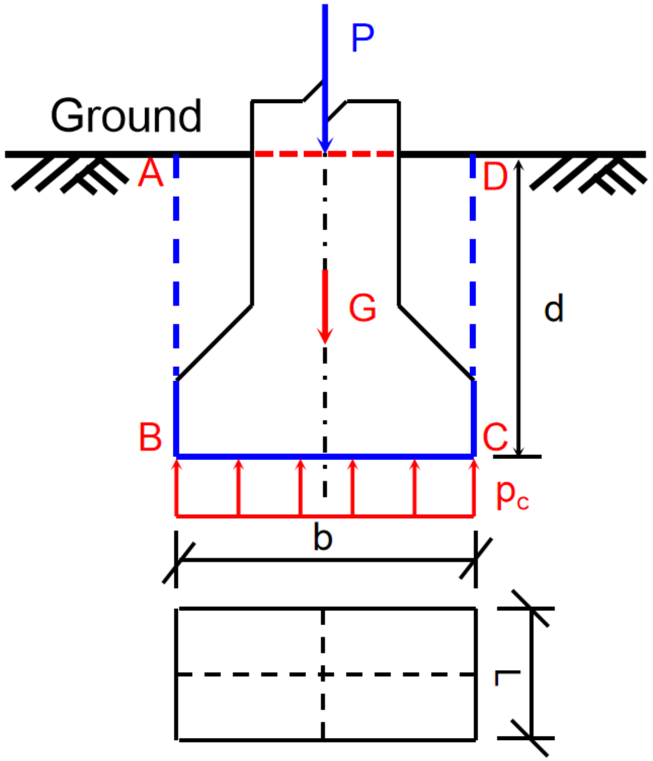
$$p_{k,\max} = \frac{P+G}{A}\left(1 + \frac{6e}{b}\right), p_{k,\min} = 0$$

If e>b/6, Triangle distribution

$$p_{k,\max} = \frac{2P}{3al}, a = \frac{b}{2} - e$$

Gross pressure

Vertical stress increment at the base of footing



Central load on a footing

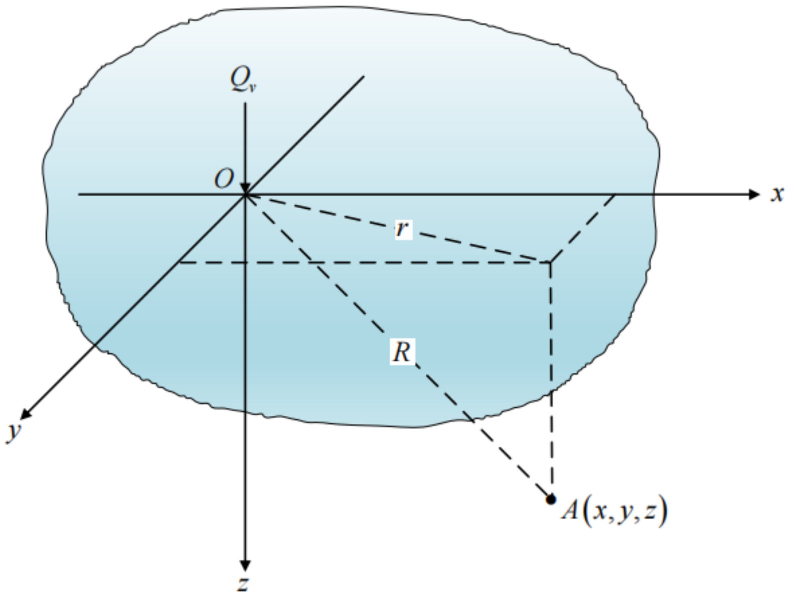
Averaged gross pressure regardless of eccentric load

$$\Delta p_k = p_k - \sigma'_{cz}$$

Effective geostatic stress at the base of footing

Vertical stress increment due to surface load

Vertical stress increment due to a vertical point load



Schematic diagram of Boussinesq's problem

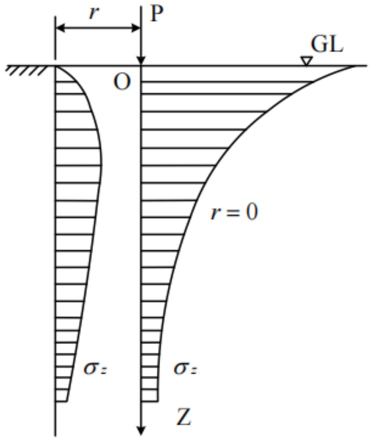
Boussinesq solution assumptions:

1) The ground is elastic, isotropic and homogeneous
2) The effect of geostatic stress is taken into account

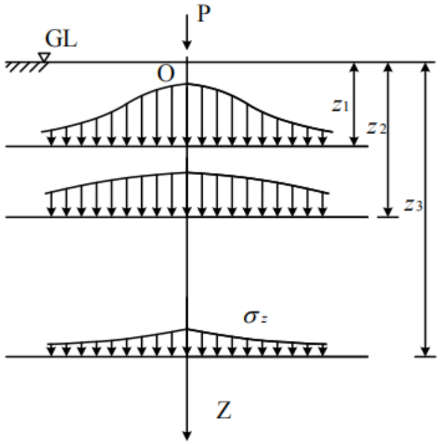
Vertical stress increment due to surface load

$$\sigma_z = \frac{3Q_v}{2\pi}\frac{z^3}{R^5} = \frac{3}{2\pi}\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}}\cdot\frac{Q_v}{z^2} = K\frac{Q_v}{z^2}$$

Stress increment distribution



Stress increment along depth



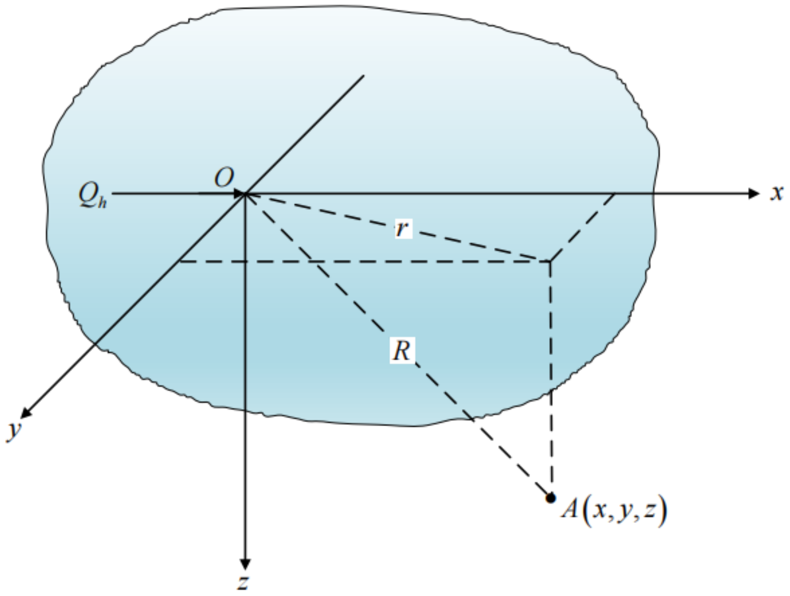
Radial stress at different depths

Exercises

A vertical point load 1000 kN is applied at the ground surface. Determine the vertical stress increment: 1) at a depth of 4 m beneath the point load; 2) at a radial distance 3 m and a depth of 4 m .

Vertical stress increment due to surface load

Vertical stress increment due to a horizontal point load



Schematic diagram of Cerruti's problem

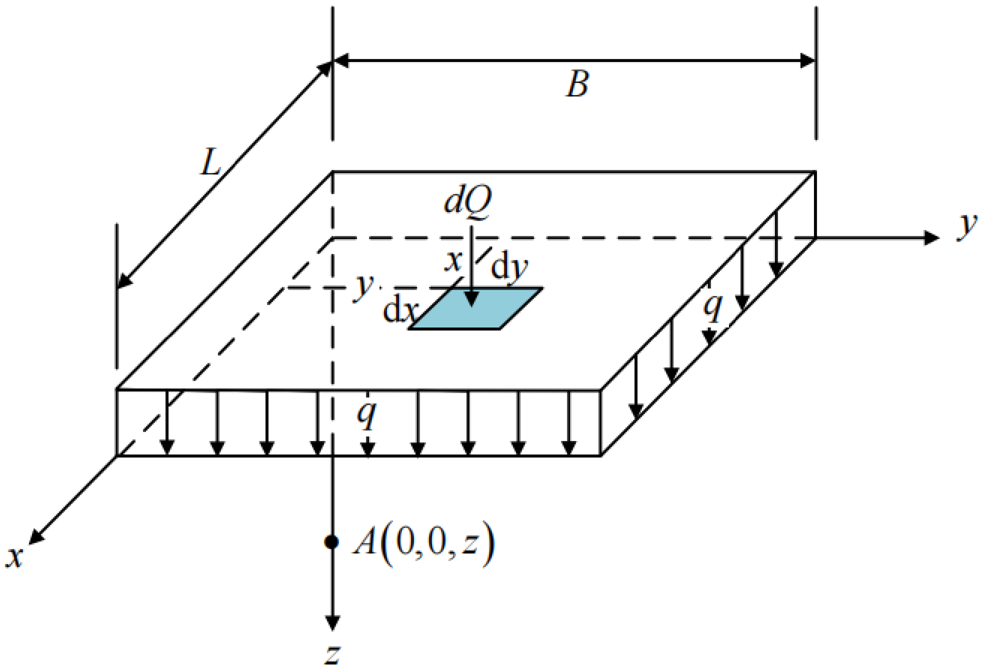
Assumptions: the ground is elastic, isotropic homogeneous, and weightless medium

Cerruti's solution

$$\sigma_z = \frac{3Q_h}{2\pi}\frac{x \cdot z^2}{R^5}$$

Vertical stress increment due to surface load

Vertical stress increment beneath any corner of a rectangular area



Vertical stress under the corner of a rectangular area

$$d\sigma_z = \frac{3q}{2\pi}\frac{z^3}{R^5}\,dxdy$$

Integration over rectangular area

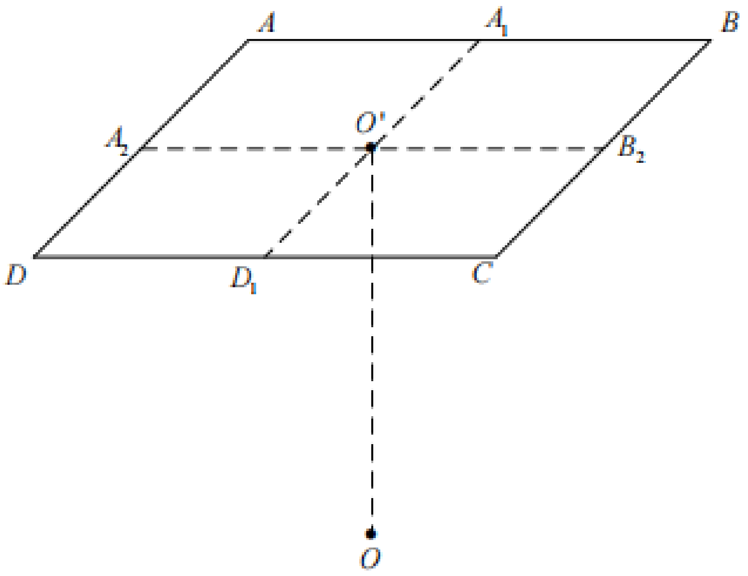
$$\sigma_z = \int_0^L\int_0^B \frac{3q}{2\pi}\frac{z^3}{R^5}\,dxdy$$

$$= \frac{q}{2\pi}\left[\arctan\frac{m}{n\sqrt{1+m^2+n^2}} + \frac{m\cdot n}{\sqrt{1+m^2+n^2}}\left(\frac{1}{m^2+n^2}+\frac{1}{1+n^2}\right)\right]$$

$$= K_s(m,n)\,q$$

Vertical stress increment due to surface load

Vertical stress increment due to a rectangular footing



Steps to determine the vertical stress increment

1. Divide the whole rectangular area into smaller ones. Make sure the projection point locates on any smaller rectangle.
2. Determine the influence factors of smaller rectangle

$$\text{Area of } ABCD = AA_1O'A_2 + A_1BB_2O' + O'B_2CD_1 + A_2O'D_1D$$

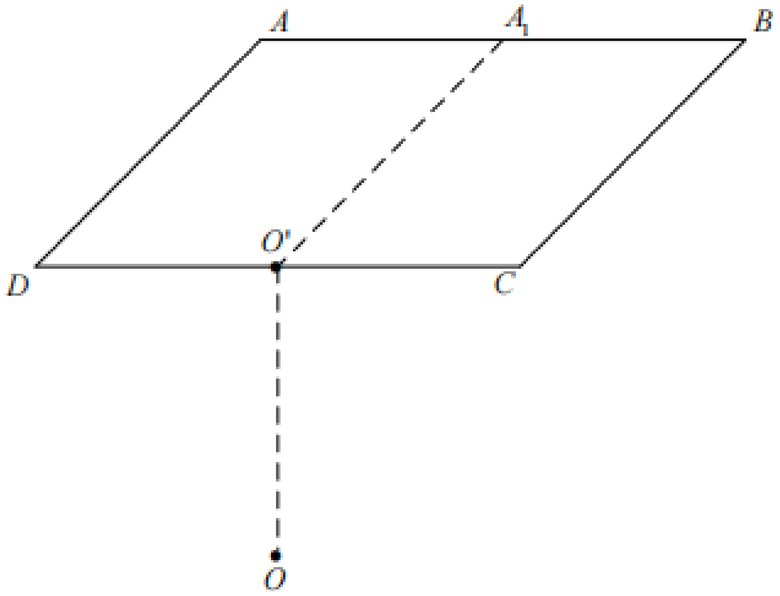
$$K_{sABCD} = K_{sAA_1O'A_2} + K_{sA_1BB_2O'} + K_{sO'B_2CD_1} + K_{sA_2O'D_1D}$$

$$\sigma_z = K_{sABCD}q = q\left(K_{sAA_1O'A_2} + K_{sA_1BB_2O'} + K_{sO'B_2CD_1} + K_{sA_2O'D_1D}\right)$$

Projection of the point interested in is within the area

Vertical stress increment due to surface load

Vertical stress increment due to a rectangular footing

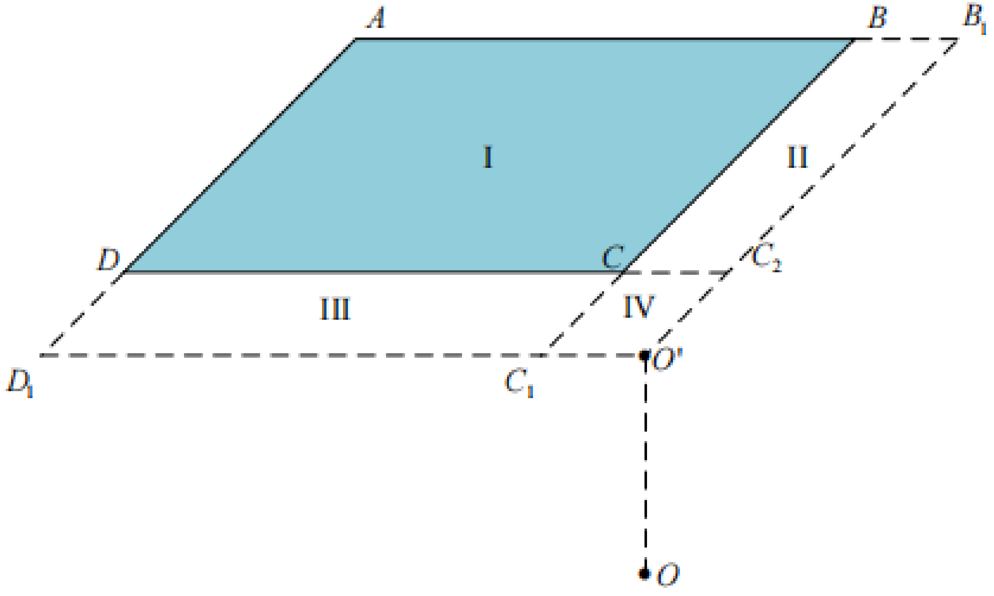


Projection of the point interested in is at any edge of loaded area

$$\sigma_z = K_{sABCD} q = q\left(K_{sAA'O'D} + K_{sA'BCO'}\right)$$

Vertical stress increment due to surface load

Vertical stress increment due to a rectangular footing

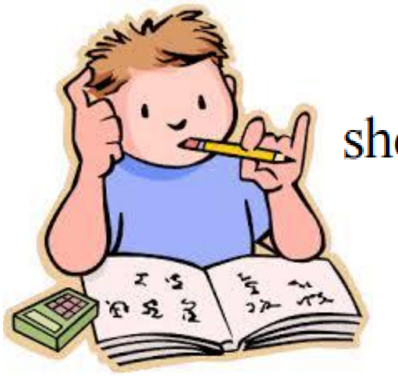


Projection of the point interested in is outside of the loaded area

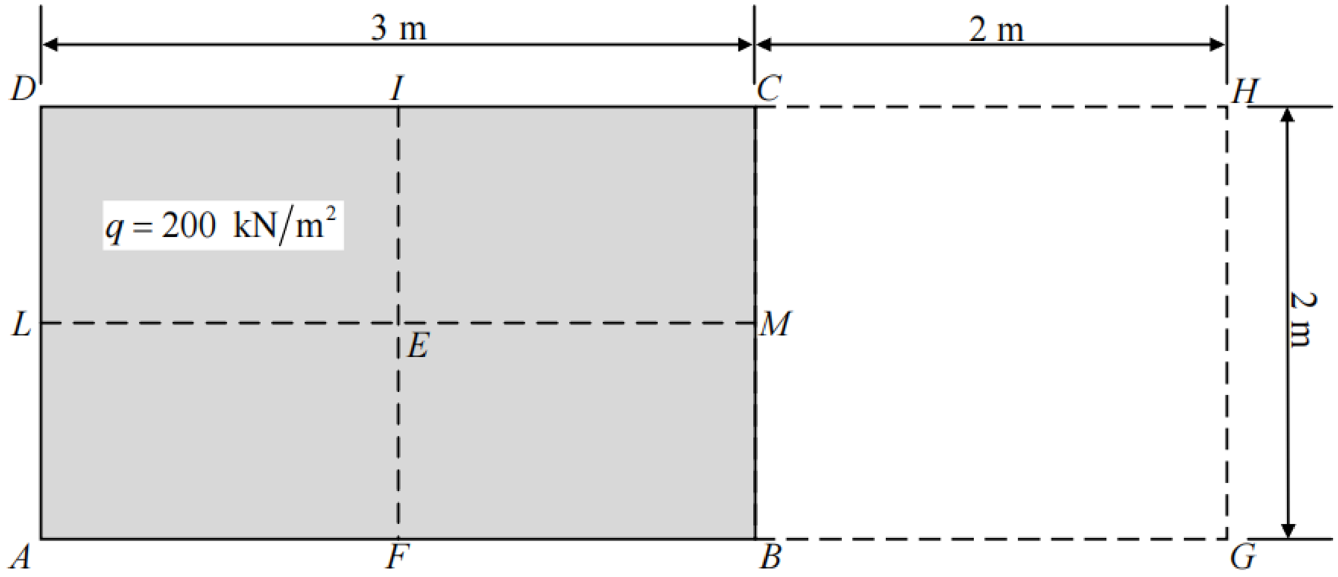
$$\text{Area of } ABCD = AB_1O'D_1 - BB_1O'C_1 - DC_2O'C_1 + CC_2O'C_1$$

$$\sigma_z = K_{sABCD}q = q\left(K_{sAB_1O'D_1} + K_{sBB_1O'C_1} + K_{sDC_2O'C_1} + K_{sCC_2O'C_1}\right)$$

Exercises

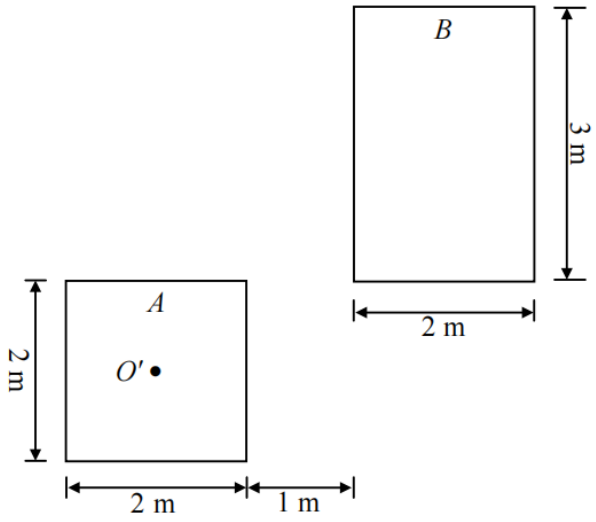


A rectangular footing $ABCD$ with net pressure $q = 200$ kPa on the ground is shown in Fig. 4. Compute σ_z under Points E, F, B, and G at a depth of 5 m.



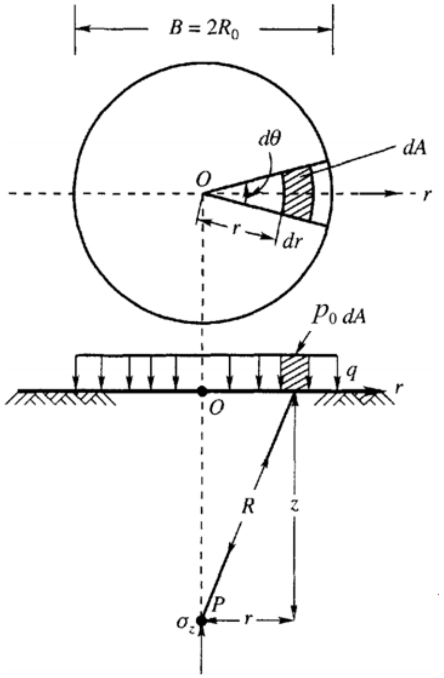
Exercises

The size and relative position of two rectangular footings are as Fig. 2 shows (bird veiw). The net pressure on footing A and footing B are $200\ \mathrm{kPa}$ and $300\ \mathrm{kPa}$, respectively. Determine the vertical stress increment at point O, which is directly under the center of footing A and at a depth of $2\ \mathrm{m}$.



Vertical stress increment due to surface load

Vertical stress increment beneath the center of a circular footing



Schematic diagram of circular footing

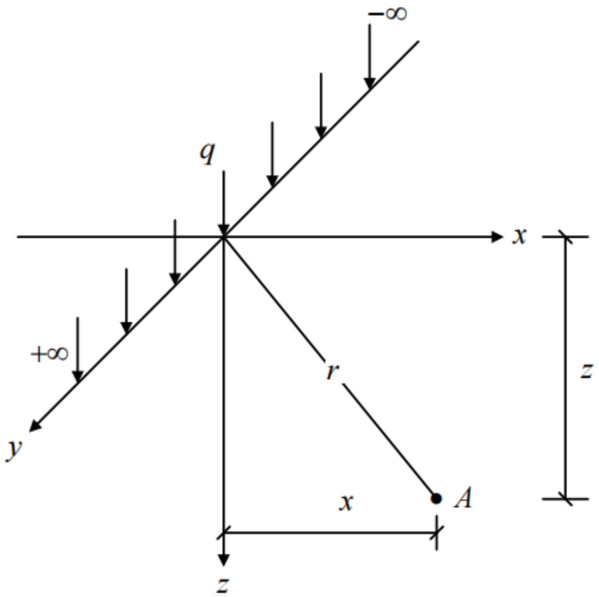
$$d\sigma_z = \frac{3q}{2\pi}\frac{z^3}{R^5}\mathrm{d}x\mathrm{d}y$$

Integration over circular area

$$\sigma_z = \int_A d\sigma_z = \int_0^{2\pi}\int_0^{R_0}\frac{3p_0}{2\pi}\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}z^2} r\mathrm{d}r\mathrm{d}\theta = p_0\left[1-\frac{1}{\left[\left(\frac{R_0}{z}\right)^2+1\right]^{\frac{5}{2}}}\right]$$

Vertical stress increment due to surface load

Vertical stress increment due to line load



Stress increment due to a line load

$$d\sigma_z = \frac{3q}{2\pi}\frac{z^3}{R^5}\,\mathrm{d}x\mathrm{d}y$$

Integration over loading area

$$\sigma_z = \int_{-\infty}^{\infty} d\sigma_z = \int_{-\infty}^{\infty} \frac{3q}{2\pi}\frac{z^3}{\left(x^2+y^2+z^2\right)^{\frac{5}{2}}}\,\mathrm{d}y = \frac{2qz^3}{\pi\left(x^2+z^2\right)^2}$$

Vertical stress increment due to surface load

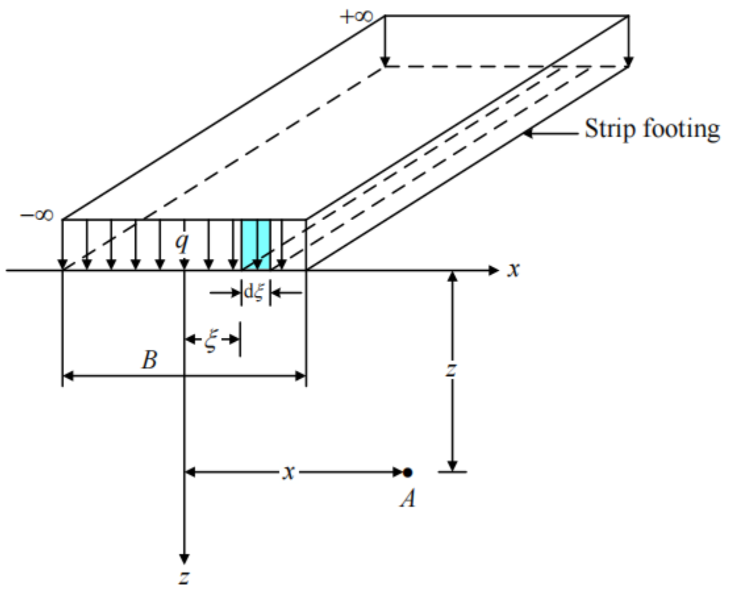
Additional stress due to line load

$$d\sigma_z = \frac{2q}{\pi}\frac{z^3}{\left[(x-\xi)^2+z^2\right]}\mathrm{d}\xi$$

Integration over loading area

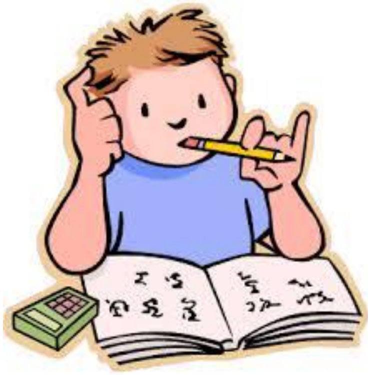
$$\sigma_z = \frac{2q}{\pi}\int_{-\frac{B}{2}}^{+\frac{B}{2}}\frac{z^3}{\left[(x-\bar{x})^2+z^2\right]}\mathrm{d}\xi$$

$$= \frac{q}{\pi}\left\{\arctan\frac{\frac{2z}{B}}{\frac{2x}{B}-1} - \arctan\frac{\frac{2z}{B}}{\frac{2x}{B}+1} - \frac{\frac{2z}{B}\left[\left(\frac{2x}{B}\right)^2-\left(\frac{2z}{B}\right)^2-1\right]}{2\left\{\frac{1}{4}\left[\left(\frac{2x}{B}\right)^2+\left(\frac{2z}{B}\right)^2-1\right]^2+\left(\frac{2z}{B}\right)^2\right\}}\right\}$$

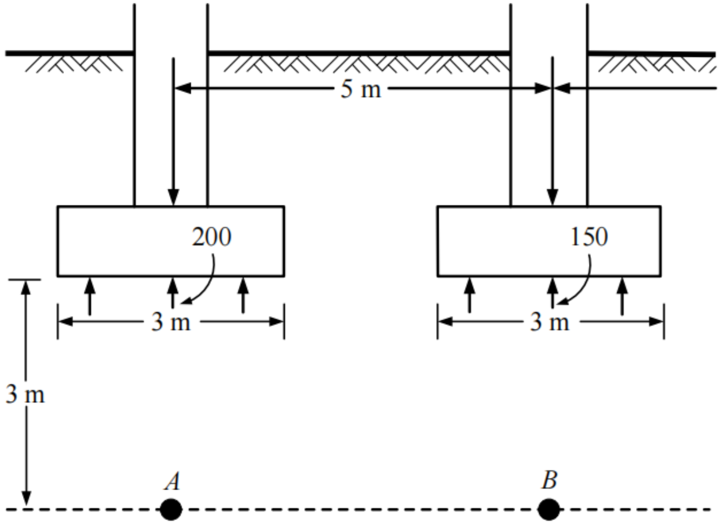


Vertical stress increment due to a strip load

Exercises



Two parallel strip footings with 3 m wide and 5 m apart center to center are as Fig. 7 shows. The contact pressure are 200 kPa and 150 kPa respectively. Calculate the vertical stress increment beneath the center of each strip footing at a depth of 3 m to the base (hints: the influence from adjacent strip footing should be taken into account).



The End