Chapter 3 Stress in soils

Stresses in soils （应力（类型，符号规定)，）

Principle of effective stress （）

Geostatic stress

Vertical stress increment

3.1 Introduction

When the structure is constructed on the superficial layer of the earth, its self-weight and live loads imposed over structure are transmitted to its foundation. The load transmitted to the foundation is then directly transferred to ground. If the properties of the ground are not good enough, technical improvements are need. Such kind of ground is called an artificial ground. For the ground whose properties are not artificially modified in the project, it is called a natural ground,

The load transferred by the foundation will induced stresses in the ground. The induced stress is also called additional stress in this book. According to the studies, the additional stress decreases with the increase of the depth. The strata where the additional stress is non-negligible are called bearing strata. And the strata beneath the bearing strata are called subjacent bed. Although the effect due to the construction can be ignored, the validation of bearing capacity is still necessary if its properties are poor.

Meanwhile, before the construction, there is already the initial stress (geostatic stress) within the ground, which is established during the formation of the strata. These two kinds of stress in together will result in two affects: 1) deformation problem, namely the compressibility of soil and thus the settlement of the top building; 2) strength problem. If the external load is large enough and the induced disturbance (additional stress) exceeds the bearing capacity of soil stratum, the settlement may too large for top buildings to be further used. The failure of ground may also lead to the failure of the top building.

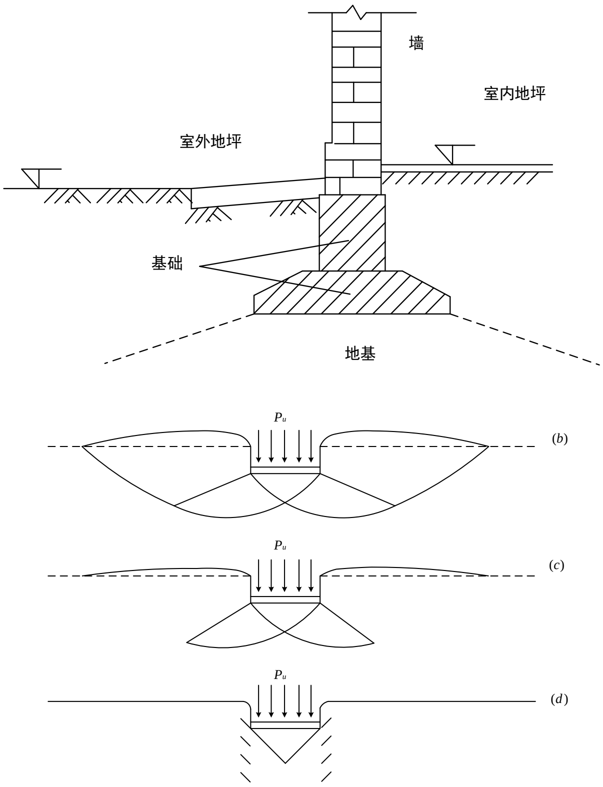
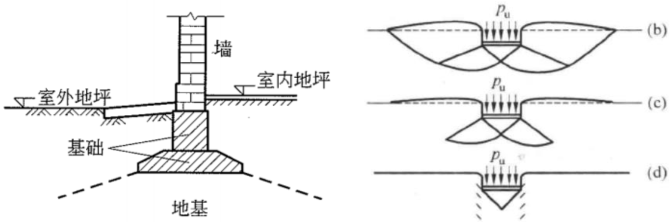


Fig. 3-1 Foundation and types of ground failure



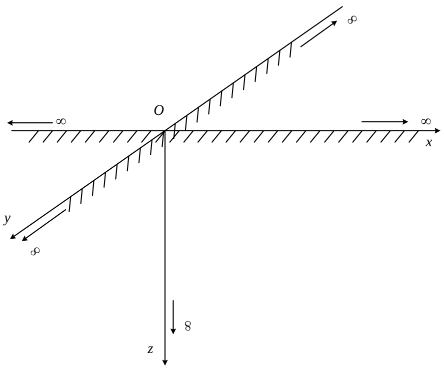
增加沉降或者破坏的图

3.2 Types of stress states in the ground

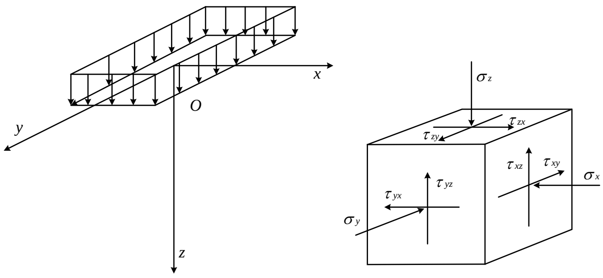
The ground in soil mechanics is described as a semi-infinite elastic region, as Fig shows. Its dimensions in x, y and positive z directions are extended to infinity and stress-strain relation is linear. With respect to various conditions, there are mainly four types of stress states in the ground:

1) **General stress state**: In most of the projects, the stress at any point is in general stress state, namely the stress has nine components (six of them are independent). Any component is the function its coordinate. To illustrate the positive conventions for stress, A 3D-stress element as fig shows is utilized. The surface of the 3D-stress element whose outward normal points to the positive direction of the coordinate axis is defined as the positive surface. The

stress (normal stress or shear) is positive (negative) if it acts on the positive surface in a negative (positive) direction. All the stress components in the following 3D-stress element are positive.



$$\sigma_{ij} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$



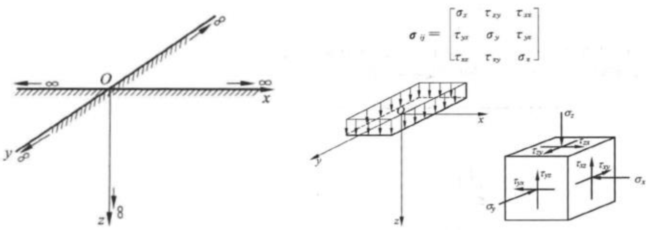


Fig. 3-2 Foundation and types of ground failure

2) **Plane strain state**: For certain geotechnical structures, such as tunnel, retaining wall, earth fill dam, leave, engineers deal with a situation in which the dimension of the structure in y -direction, as fig show, is very large in comparison with the dimensions in other directions. In this case, the stress state in the ground is in a plane strain state, namely, there are only stress components in the x-y plane and do not vary in the y direction.

$$\sigma_{ij} = \begin{bmatrix} \sigma_x & 0 & \tau_{xz} \\ 0 & \sigma_y & 0 \\ \tau_{zx} & 0 & \sigma_z \end{bmatrix}$$

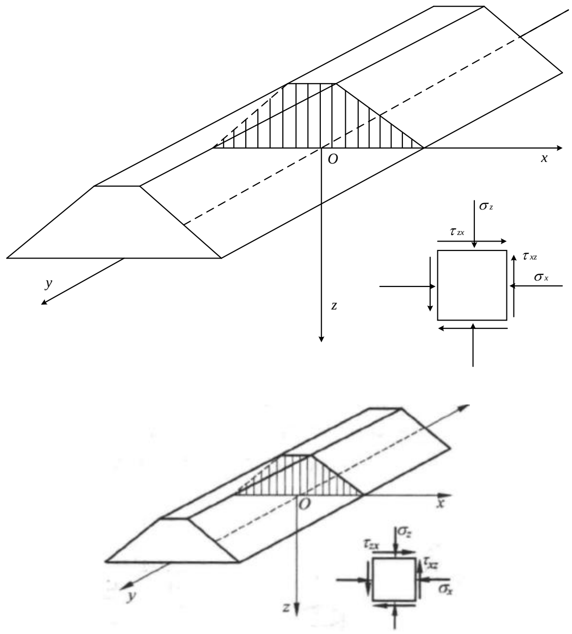


Fig. 3-3 Foundation and types of ground failure

3) **Lateral deformation confined (zero lateral strain) stress state**: The confined strain state refers to a special stress state where the lateral deformation of the structure is assumed to be zero. The stress state of the overburden pressure within soil layers is in a confined strain state. The deformation only occurs in the vertical direction. As all the vertical planes are symmetric planes, there are no shear stresses on the lateral surfaces of the 3D-stress element.

$$\sigma_{ij} = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_z \end{bmatrix}$$

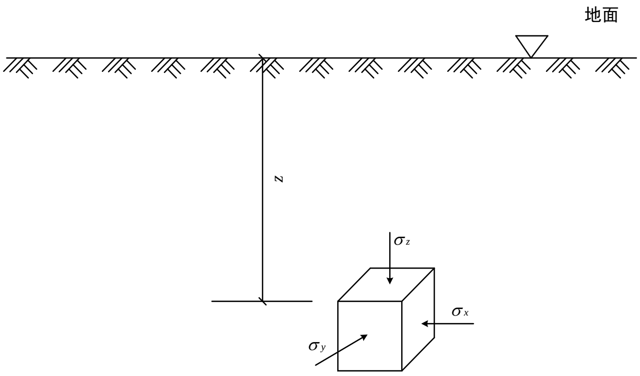


Fig. 3-4 Foundation and types of ground failure

4) **Axial symmetrical stress state**: The stress increment induced by any axial symmetrical structure, e.g., structure with circular foundation, is in axial symmetrical stress state. The cylindrical coordinate system is found to be convenient for analyzing such problems.

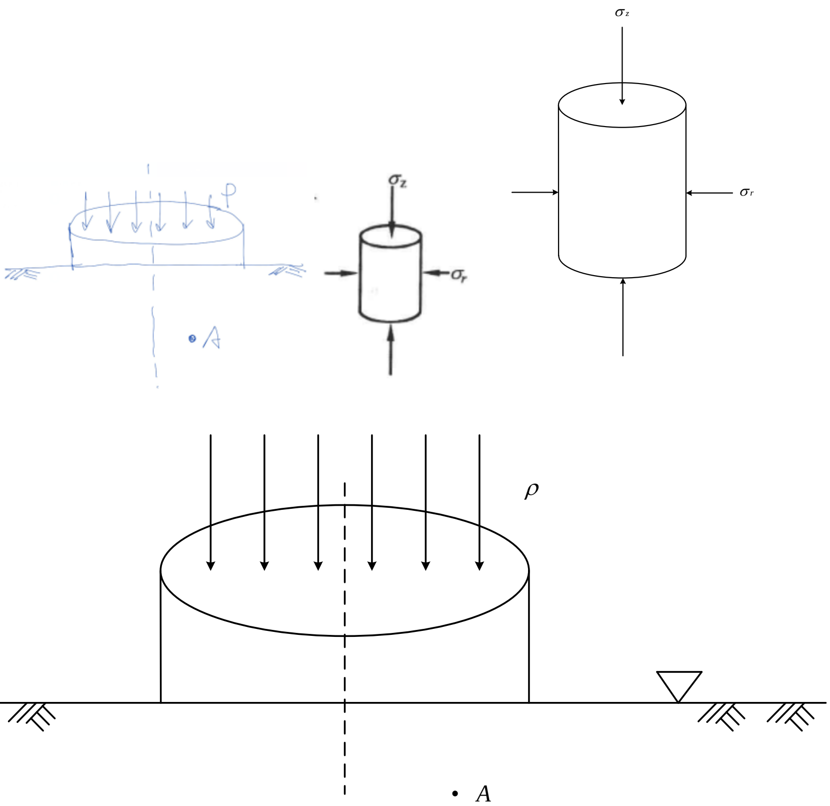


Fig. 3-5 Foundation and types of ground failure

3.3 Principle of effective stress

As forgoing mentioned, the induced stress results in the deformation and stability problems of the ground. One point needs to be clarified is that not the whole part of the additional stress controls the strain or strength behavior of soil. As soil is a three-phase composite, the additional stress is in fact shared by soil, liquid and gaseous phase. For simplicity, only the saturated soil is considered in this book.

When taking into account the complex interaction between particles and water, two problems arise. The first problem is how the additional stress is shared by two phases. The second one is how the transformation process is performed between two phases. Karl von Terzaghi did a lot of pioneering work on these two problems and finally proposed the principle of effective stress and the consolidation theory. The latter will be described in the next section.

3.3.1 Total stress, water pressure and effective stress

The principle of effective stress establishes the relation between total stress, water pressure and effective stress. The total stress the external load applied on the ground. The pressure transmitted through grain to grain at the contact points through a soil mass is termed as intergranular or effective pressure, which is responsible for the variation of void ratio or the frictional resistance of a soil mass. For saturated soil, a pressure is instantaneously induced in the water as the compressibility of soil is comparatively small comparing with the solid skeleton. During the application of the external load, the transformation of pressure between particles and water always exists. Unlike the effective stress, the effect of the water pressure is to increase the volume of void (void ratio) or decrease the frictional resistance of a soil mass.

A simple example is ultilized to illustrate the principle of effective stress. Consider a rigid cylindrical mold, as Fig shows, in which saturated sand is placed. Assume that there is no side friction between the porous piston and the mold. Load Q is applied at the surface of the soil through piston. At any curved section of the soil sample, e.g. section a-a, the force of equilibrium in the vertical direction satisfy,

$$Q=\sum_{i=1}^{n} P_{svi} + uA_w$$

Assume the projection area of the curved section is A, the projection area of all contact parts between particles is As and the projection area of void part is Aw. Psvi is the vertical component of the contact pressure at particle i. u is the pore pressure at the time level t. Divide the above equation by A,

$$\frac{Q}{A} = \frac{\sum_{i=1}^{n} P_{svi}}{A} + + \frac{uA_w}{A}$$

Define the total stress $\sigma = Q/A$, the effective stress $\sigma' = \sum_{i=1}^{n} P_{svi} \Big/ A$. As the contact parts is relatively small comparing with the whole curved section, then

$$\frac{A_w}{A}=\frac{A-A_s}{A}=1-\frac{A_s}{A}\approx 1$$

With the above definition, the equation is recast into

$$\sigma=\sigma'+u$$

The above equation applies to any time level during the application of the load. According to the derived equation, the decrease of the pore pressure due to the expulsion of water through porous piston will be compensated by the increase of the effective stress.

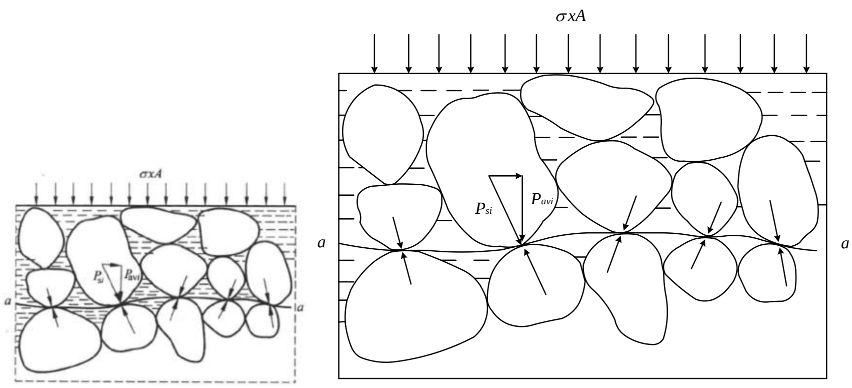


Fig. 3-6 Foundation and types of ground failure

3.4 Initial vertical stress in soil mass

Initial vertical stress at any point in a soil mass is induced due to weight of the layers above the point. This kind of vertical stress exists during the formation of the layers. In some textbooks it is also called geostatic stress.

The accurate determination of overburden pressure, either total or effective, is complex. For irregular ground surface or irregular interface of soil layers, e.g. moutain area, it may be impossible to obtain a “correct one”.

As foregoing mentioned, it is the effective stress that controls the characterstics and strength behavior of soils. In general, the greater depth beneath the ground surface, the higher the effective overburden pressure and thus the more compacted of soils. In this case, the lower compressibility and the higher strength of soils are obtained.

In the following sections, the effective vertical stress is determined for various situations.

3.4.1 Effective vertical stress of uniform dry ground

Fig shows the profile of uniform dry ground. As there is no pore pressure, the total overburden pressure equals the effective one. According to force equilibrium in the vertical direction, the total and effective overburden pressure of point A at a depth z is expressed as:

$$\sigma_z = \sigma_z' = \gamma z \tag{0-1}$$

γ is the bulk unit weight of the soil layer. According to Eq. (0-1), the overburden pressure is proportional to the depth z. The pressure diagram is plotted alongside as fig shows.

The horizontal effective stresseses are related to the overburden pressure by:

$$\sigma_x' = \sigma_y' = K_0 \sigma_z = K_0 \gamma z \tag{0-2}$$

K_0 is the coefficient of earth pressure at rest. Under lateral confined (zero lateral strain) stress state, K_0 equals:

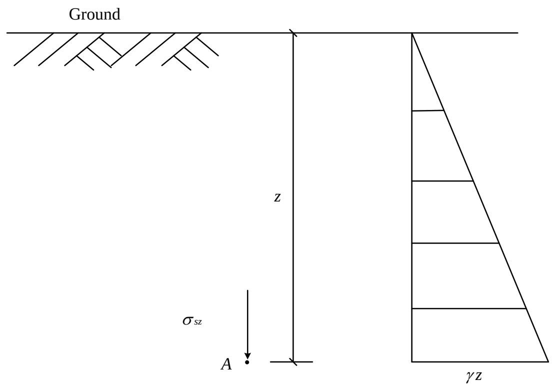
$$K_0 = \frac{\nu}{1-\nu}$$

ν is Poisson's ratio, which theoretically equals the ratio of lateral to vertical strain. However, the soil in natural could be idealized into a perfectly elastic medium. K_0 is in fact a function of soil types, soil stress state and the stress history that the soil has been experienced.

For stratified dry ground, as fig shows, the total and effective overburden pressure of point A at a depth z is the generalization of Eq. (0-1)

$$\sigma_z = \sigma_z' = \sum_{i=1}^{n} \gamma_i z_i \tag{0-3}$$

The pressure diagram is also plotted alongside.



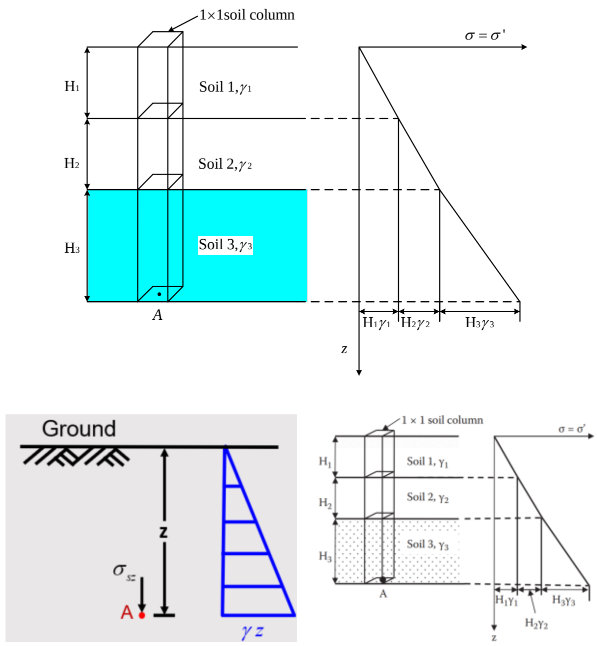


Fig. 3-7 Foundation and types of ground failure

3.4.2 Effective vertical stress of stratified ground with a steady water table

Fig shows a stratified ground with a steady water table. In this case, the effective geostatic stress σ_z' at point A is calculated as follows:

5) Calculating the total stress at point A with Eq. (0-3). The specific weight should be replaced by saturated specific weight for the layers beneath the water table:

$$\sigma_z = \gamma_1 H_1 + \gamma_2 H_2 + \gamma_{3,sat} H_3 + \gamma_{4,sat} H_4$$

6) Calculating hydrostatic pressure for the submerged soil layers:

$$u = \gamma_w H_3 + \gamma_w H_4$$

7) Calculating the effective overburden pressure according to the principle of effective stress:

$$\begin{aligned}\sigma_z' = \sigma_z - u &= \gamma_1 H_1 + \gamma_2 H_2 + \gamma_{3,sat} H_3 + \gamma_{4,sat} H_4 - \gamma_w H_3 - \gamma_w H_4 \\ &= \gamma_1 H_1 + \gamma_2 H_2 + \left(\gamma_{3,sat} - \gamma_w\right) H_3 + \left(\gamma_{4,sat} - \gamma_w\right) H_4 \\ &= \gamma_1 H_1 + \gamma_2 H_2 + \gamma_3' H_3 + \gamma_4' H_4\end{aligned} \tag{0-4}$$

From Eq. (0-4), it can be concluded that the effective overburden pressure can be calculated by a summation of soil layer thickness multiply the corresponding unit weight. For layers above the water table, the bulk unit weight is used while for the layers below the water table, the buoyant unit weight is used. For totally submerged soil layers, e.g., the soils under the lake or ocean, the effective overburden pressure is

$$\sigma'_z = \sum_{i=1}^{n} \gamma'_i z_i \tag{0-5}$$

As the particles within an impermeable layer does not subject to any water pressure, the total stress equals the effective geostatic stress. For points A (just above the roof of the impermeable layer) and A' (just below the roof of the impermeable layer), the effective stresses are respectively:

$$\sigma'_z = \sigma_z - u = \gamma_1 H_1 + \gamma_2 H_2 + \gamma'_3 H_3 + \gamma'_4 H_4 \tag{0-6}$$

$$\sigma'_{z,A'} = \sigma_z = \gamma_1 H_1 + \gamma_2 H_2 + \gamma_{3,sat} H_3 + \gamma_{4,sat} H_4 \tag{0-7}$$

The stress jump at the interface between permeable and impermeable layers equals:

$$\Delta\sigma_z = \sigma'_{z,A'} - \sigma'_{z,A} = \gamma_w H_3 + \gamma_w H_4 > 0 \tag{0-8}$$

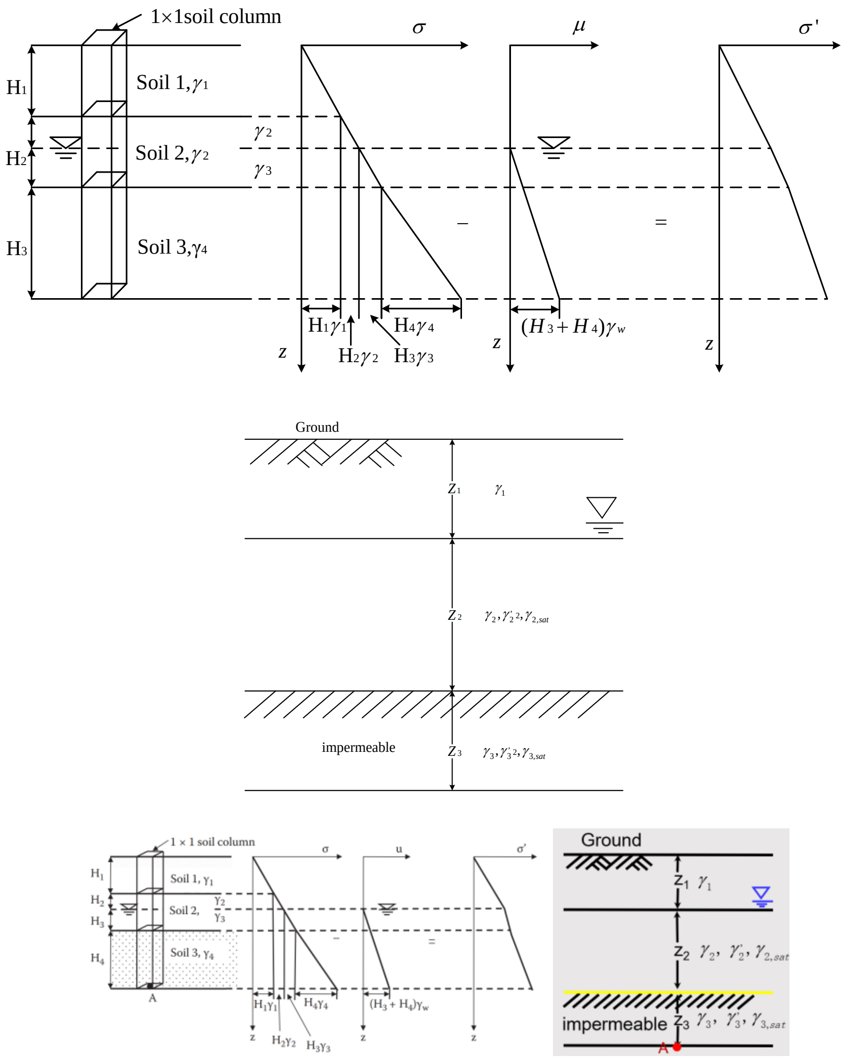


Fig. 3-8 Foundation and types of ground failure

Example

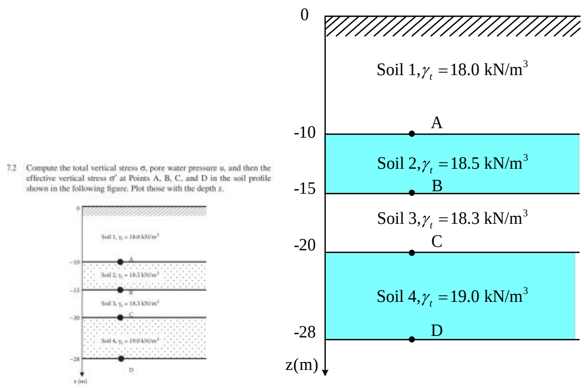


Fig. 3-9 Foundation and types of ground failure

7.3 Compute the total vertical stress σ, pore water pressure μ, and then the effective vertical stress σ' at Points A, B, C, and D in the soil profile shown in the following figure. Plot those with the depth z.

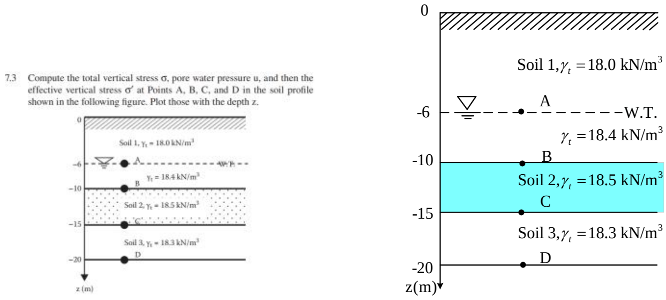


Fig. 3-10 Foundation and types of ground failure

3.4.3 Change of effective vertical stress due to fall of water table

When the water table changes, e.g., groundwater extraction, petroleum extraction, the effective overburden pressure in certain layers changes as the buoyant unit weight γ' for initially submerged layers will be replaced by bulk unit weight γ. Their difference is $\Delta\gamma = \gamma - \gamma' > 0$, which indicates that the effective overburden pressure increases after the drop of water table in certain soil layers. The increase of effective overburden pressure leads to volume decrease of void space. The consequence is the ground surface settlement.

To minimize the dewatering-induced ground settlement, artificial recharging is necessary. In contrast to the water table decreases, its raise causes a reduction in effective stress. In this case, the swelling of the ground surface is possible but may not be as severe as in the case of settlement.

Example

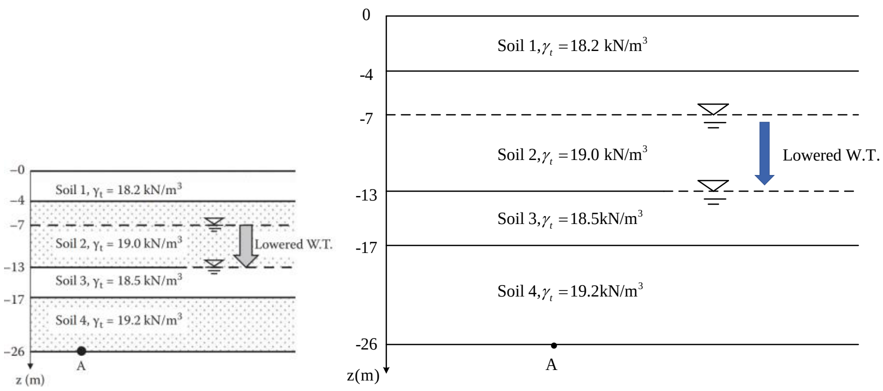


Fig. 3-11 Foundation and types of ground failure

3.4.4 Effective vertical stress with vertical water flow

As mentioned in chapter 3, the flow of water may impose a seepage force on particles in the direction of flow. Meanwhile, in different parts of the ground, the water flow plays different roles.

3.4.4.1 Totally submerged ground with upward water flow

For totally submerged ground with upward flow, e.g. the downstream of a earth-fill dam, the bottom of an excavated foundation pit, the effective overburden pressure is calculated as follows. The total overburden pressure at point A equals:

$$\sigma_z = \gamma_1 H_1 + \gamma_2 H_2 + \gamma_{3,sat} H_3 + \gamma_{4,sat} H_4$$

Taken the plane passing through the point A as the datum plane, the total heads at point A equals:

$$h_A = z_A + \frac{u_A}{\gamma_w} = \frac{u_A}{\gamma_w}$$

The total heads at the point B, which is on the phreatic surface, equals

$$h_B = z_B + \frac{u_B}{\gamma_w} = H_1 + H_2$$

The hydraulic gradient i due to head loss equals:

$$i = \frac{h_A - h_B}{l_{AB}} = \frac{\frac{u_A}{\gamma_w} - (H_1 + H_2)}{H_1 + H_2} \rightarrow u_A = (i+1)(H_1 + H_2)\gamma_w$$

According to the principle of effective stress, the effective overburden pressure at point A equals:

$$\begin{aligned} \sigma_z' = \sigma_z - u_A &= \gamma_1 H_1 + \gamma_2 H_2 + \gamma_{3,sat} H_3 + \gamma_{4,sat} H_4 - (i+1)(H_1 + H_2)\gamma_w \\ &= \gamma_1 H_1 + \gamma_2 H_2 + (\gamma_{3,sat} - \gamma_w) H_3 + (\gamma_{4,sat} - \gamma_w) H_4 - i(H_1 + H_2)\gamma_w \qquad (0\text{-}9) \\ &= \gamma_1 H_1 + \gamma_2 H_2 + \gamma_3' H_3 + \gamma_4' H_4 - i \cdot (H_3 + H_4) \cdot \gamma_w \end{aligned}$$

Comparing Eq. (0-9) with Eq. (0-6), the upward seepage force decreases the effective overburden pressure at point A by:

$$\Delta\sigma_z = i\cdot\left(H_3+H_4\right)\cdot\gamma_w \tag{0-10}$$

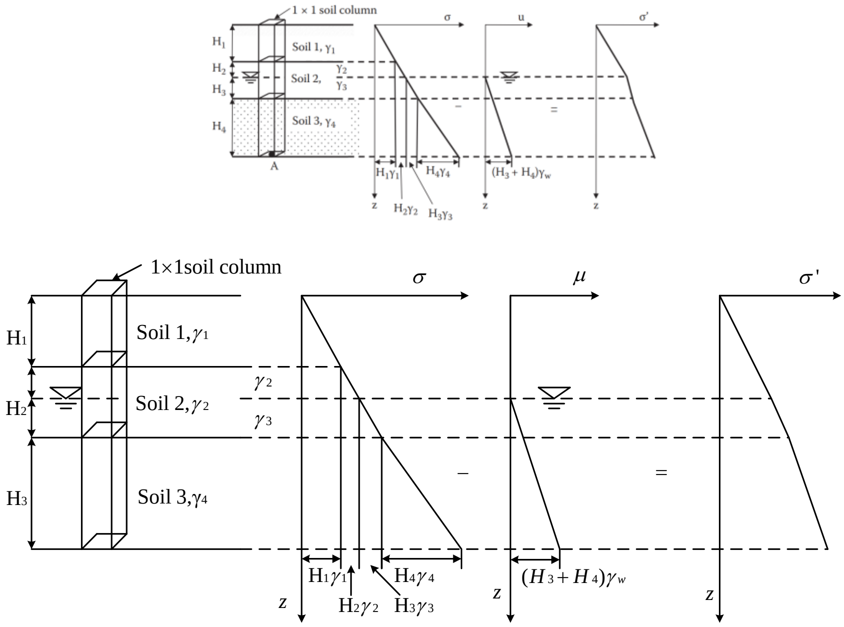


Fig. 3-12 Foundation and types of ground failure

3.4.4.2 Totally submerged ground with downward water flow

For totally submerged ground with downward flow, e.g., the upstream of a earth-fill dam, the hydraulic gradient i due to head loss equals:

$$i=\frac{h_B-h_A}{l_{AB}}=\frac{\left(H_1+H_2\right)-\dfrac{u_A}{\gamma_w}}{H_1+H_2}\rightarrow u_A=\left(1-i\right)\left(H_1+H_2\right)\gamma_w$$

According to the principle of effective stress, the effective overburden pressure at point A equals:

$$\begin{aligned}\sigma_z' &= \sigma_z-u_A=\gamma_1H_1+\gamma_2H_2+\gamma_{3,sat}H_3+\gamma_{4,sat}H_4+\left(1-i\right)\left(H_1+H_2\right)\gamma_w\\ &=\gamma_1H_1+\gamma_2H_2+\left(\gamma_{3,sat}-\gamma_w\right)H_3+\left(\gamma_{4,sat}-\gamma_w\right)H_4+i\left(H_1+H_2\right)\gamma_w\\ &=\gamma_1H_1+\gamma_2H_2+\gamma_3'H_3+\gamma_4'H_4+i\cdot\left(H_3+H_4\right)\cdot\gamma_w\end{aligned} \tag{0-11}$$

Comparing Eq. (0-11) with Eq. (0-6), the downward seepage force increases the effective overburden pressure at point A by:

$$\Delta\sigma_z = i\cdot\left(H_3+H_4\right)\cdot\gamma_w \tag{0-12}$$

3.5 Vertical stress increment in a soil mass

Estimation of vertical stresses at any point in a soil mass due to external vertical loadings are of great significance in the prediction of settlements of buildings, bridges, embankments and many other structures.

3.5.1 Contact pressure distribution under a footing

Section 3.4 studied the effective overburden pressure under various situations. In fact, soils are stable under the existing effective overburden pressure. During the long formation of soil layers, their deformations (settlement) have already finished. However, when additional loads are applied, such as new constructions, traffic loads, etc., stress increment is induced in the ground. Estimation of the stress increment is of great significance in the prediction of deformation or strength problems of soils.

The self-weight and live loads imposed over the structures are transferred to the supporting soils through a footing. In response to the applied loads, the supporting soils exert an upward normal pressure on the bottom surface of a footing which is termed as **contact pressure**. According to the studies, the magnitude and distribution of the contact pressure are mainly influenced by the rigidity of the foundation and the types of soils

- For a flexible footing, e.g. earth embankment is considered to have flexible foundation, it will not show strong resistance against deflection. When column loads are applied at its center, as fig shows, the deformation is maximum at the center and minimum at the edges which forms a bowl like shape. The contact pressure is almost uniform along the deformed boundary.

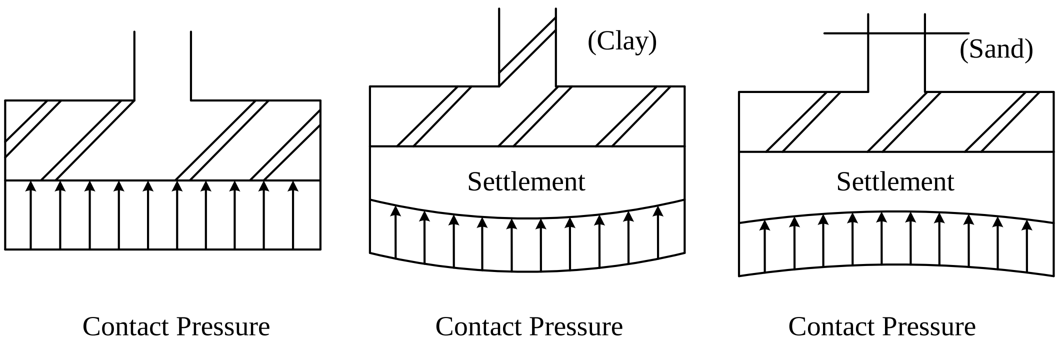
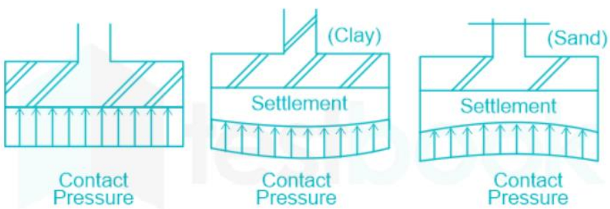


Fig. 3-13 Foundation and types of ground failure



- For a rigid footing, it has an infinite stiffness, i.e., a concrete pad footing having significant thickness. In this case, the foundation settles as a rigid element, namely the uniform settlement, as fig shows. The contact pressure distribution depends on the types of soils, as fig shows.

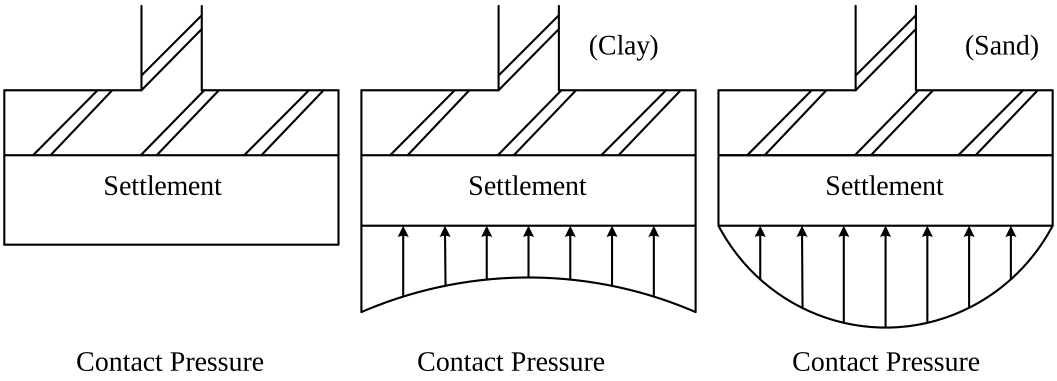
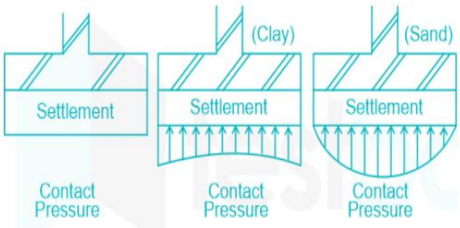


Fig. 3-14 Foundation and types of ground failure

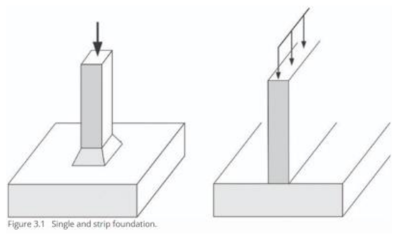


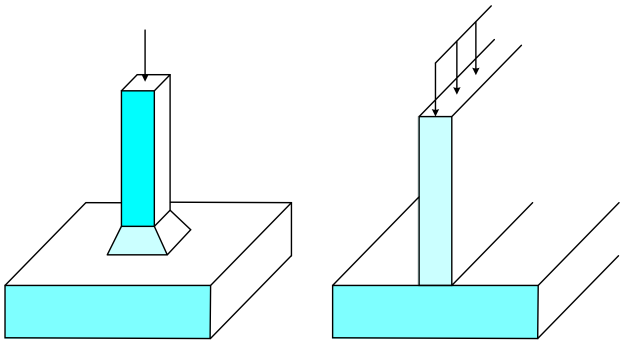
3.5.2 A simplified model for contact pressure

Although the contact pressure distributioin is complex, the contact pressure distribution is simply assumed linear in pratical design.

3.5.2.1 Single and strip footing

For point loads (loads transferred by columns), a single foot is used, as fig shows, while for line load (loads transferred by walls), strip footing is preferred. The design of a single or strip footing mainly depends on the bearing capacity of the supporting soil. Detailed discussion on the bearing capacity will be made on Chapter. If the soils underneath a footing have insufficient bearing capacity, ground improvment should be carried out ahead of construction.





Figue3.1 Single and strip foundation

Fig. 3-15 Foundation and types of ground failure

3.5.2.2 The contact pressure for a centric load

For a single footing, the contact pressure equals:

$$p = \frac{P_v}{A} = \frac{P_v}{b \times l} \tag{0-13}$$

p is the contact pressure. P_v is the vertical central load acting on the bottom surface of a single footing. A is the area of a rectangle footing. b and l are the width and length, respectively.

For a strip foundation, the length l is usually taken as unit. Therefore, the contact pressure equals:

$$p = \frac{P_v}{b \times 1} = \frac{P_v}{b} \tag{0-14}$$

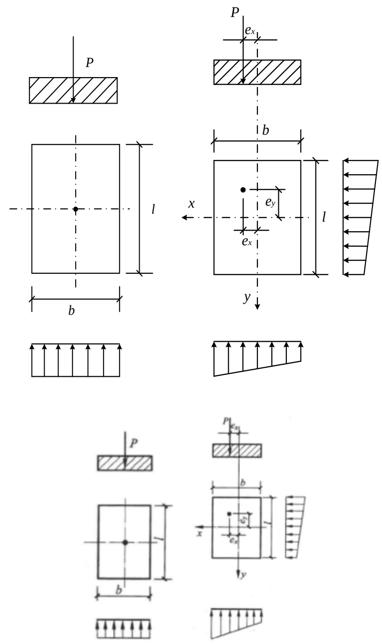


Fig. 3-16 Foundation and types of ground failure

3.5.2.3 The contact pressure for an eccentric load

Eccentric load results in a non-uniform contact pressure distribution. According to the mechanics of material, the structure subjected to any eccentric load can be equivalently transformed to a centric load combined with a correspondingly moment. For the eccentric load show in fig. The contact pressure equals:

$$p(x,y)=\frac{P_v}{A}\pm\frac{M_x\cdot y}{I_x}\pm\frac{M_y\cdot x}{I_y} \tag{0-15}$$

$$M_x=P\cdot e_x \text{ and } M_y=P\cdot e_y$$

$p(x,y)$ is contact pressure at any point. M_x and M_y are the bending moments. I_x and I_y are the momentum of inertia. e_x and e_y are the eccentricity.

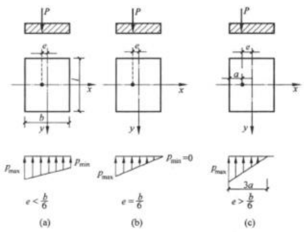
A much simpler case is that an eccentric load is applied along the x-axis. In this case $M_x=0$, $e_y=0$, the contact pressures at the edges of a footing are:

$$p_{\substack{\max\\ \min}}=\frac{P_v}{A}\left(1\pm\frac{6e}{b}\right) \tag{0-16}$$

If $e<b/6$, the contact pressure distribution is a trapezoid, as fig shows. If $e=b/6$, the distribution of contact pressure is a triangle as fig shows. If $e>b/6$, the minimum contact pressure become negative. According to the stress convention, the negative stress means tension, which is impossible as soils almost have no resistance to the tensile force. If the pressure is in tension, it means the contact is gone and the pressure is really zero. In this case, the pressure distribution should be adjusted to balance the eccentric load. According to the force and moment equilibrium, the pressures at the edges of the footing equals:

$$p_{\max}=\frac{2P_v}{3al} \text{ and } p_{\min}=0 \tag{0-17}$$

Where $a=b/2-e$. For a strip footing, $l=1$



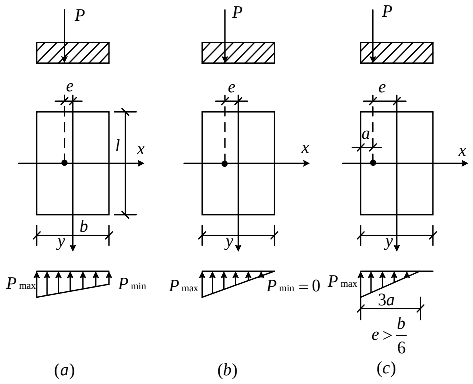


Fig. 3-17 Foundation and types of ground failure

3.5.2.4 The contact pressure for a horizontal load

When a horizontal load P_h is applied at the bottom of a footing, as fig shows, the contact pressure changes to:

- For a single footing

$$p = \frac{P_h}{A} = \frac{P_h}{b \times l}$$

- For a strip footing

$$p = \frac{P_h}{b}$$

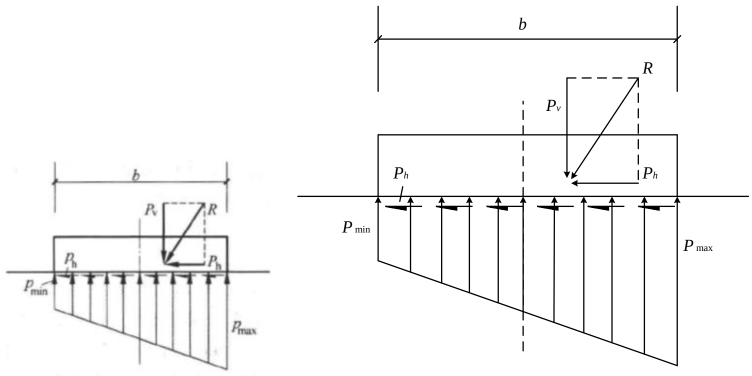


Fig. 3-18 Foundation and types of ground failure

3.5.2.5 The contact pressure for an inclined load

When an inclined load P is applied at the bottom of a footing, as fig shows, it can be equivalently resolved into a vertical force P_v and a horizontal one P_h. The contact pressures due to each component are calculated as section 3.5.2.2 and section 3.5.2.4 did. Then, the contact pressure due to the inclined load is the superposition of each part.

3.5.2.6 The net pressure

As foregoing mentioned, the settlement of the ground is mainly caused by stress increment. To obtain the stresses increment distribution within the ground, it is necessary to know the net pressure at the bottom of a footing.

The vertical load component P_v mentioned in section 3.5.2.2 is calculated as:

$$P_v = F + G$$

F is the self-weight and live loads transmitted to the toe of a column (at ground surface level), as shown in fig. G is the weight of the footing plus the weight of the backfill soil, namely the soil within the zone ABCD, which equals:

$$G = \gamma_{ave} \cdot d$$

Here γ_{ave} is the weighted average of the unit weight of the backfill soil and footing. For simplicity, it equals $20\,\mathrm{kN/m^3}$. d is the burial depth.

The **net pressure** is the part of the contact pressure which is in excess of the effective overburden pressure, namely:

$$p_0 = p - \sigma_z' = p - \gamma_m \cdot d$$

γ_m is the weighted average of the unit weight of the layers above the base of the footing. If the layer is submerged, the buoyant unit weight should be used.

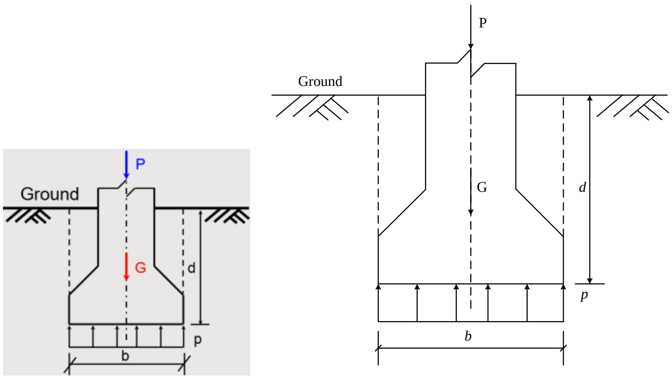


Fig. 3-19 Foundation and types of ground failure

3.5.3 Vertical stress increment for various situations

3.5.3.1 Vertical stress increment due to a vertical point load

Fig. 3-20 shows a semi-infinite solid with a vertical point load Q_v acting at point O on the surface. Boussinesq (1885) solved that problem by assuming soil is an elastic, isotropic, homogeneous and weightless medium. The expression given by Boussinesq for computing vertical stress increment σ_z at the point which has a radial distance r and a depth z to the loading point is:

$$\sigma_z = \frac{3Q_v}{2\pi}\frac{z^3}{R^5} = \frac{3}{2\pi}\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}} \cdot \frac{Q_v}{z^2} = K\frac{Q_v}{z^2} \tag{0-18}$$

$$K = \frac{3}{2\pi}\frac{1}{\left[1+\left(\frac{r}{z}\right)^2\right]^{\frac{5}{2}}}$$

The stress distribution of induced stress along the radial and vertical directions are as fig shows. It is found that:

1) It is obvious that with the increase of z, the induced stress decrease. When the depth is infinitely large, the induced stress tends to zero. As the induced stress is proportional to 1/z2, the stress decreases rapidly.
2) At a certain radial distance to the load point, the induced stress firstly increases and then decreases with the increase of the depth.
3) At certain depth, the maximum induced stress in the ground decreases with the increase of the depth. Along the radial direction, the induced stress tends to more uniform with the increase of the depth.

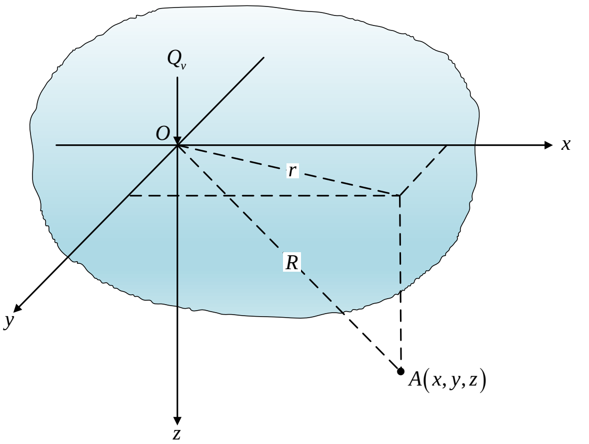
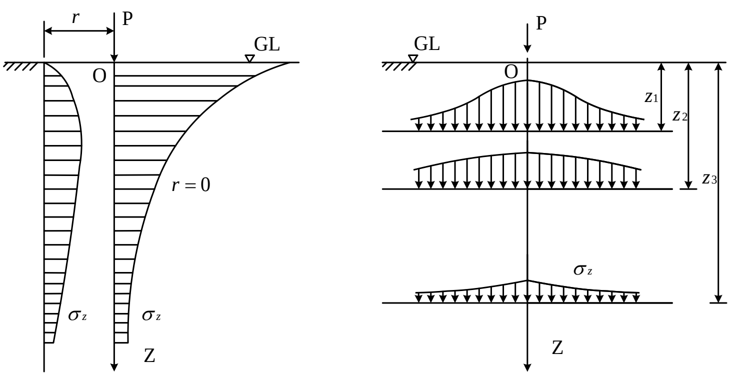


Fig. 3-20 Schematical diagram of Boussinesq's problem



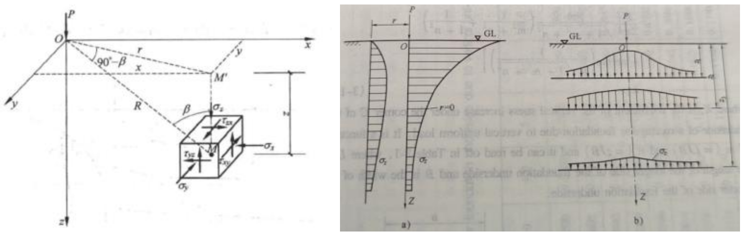


Fig. 3-21 Foundation and types of ground failure

Example 1

A vertical point load 1000 kN is applied at the ground surface. Determine the vertical stress increment: 1) at a depth of 4 m beneath the point load; 2) at a radial distance 3 m and a depth of 4 m.

Solution

According to Eq. (0-18):

1) $r = 0$, $z = 4$ m, then $K \approx 0.48$

$$\sigma_z = 0.48\frac{Q_v}{z^2} = 0.48 \times \frac{1000}{4^2} = 30.0 \text{ kPa}$$

2) $r = 3$, $z = 4$ m, then $K \approx 0.16$

$$\sigma_z = 0.16\frac{Q_v}{z^2} = 0.16 \times \frac{1000}{4^2} = 10.0 \text{ kPa}$$

3.5.3.2 Vertical stress increment beneath any corner of a rectangular area

An uniformly distributed load with intensity q (unit: Pa or kPa) is applied on a rectangular area, as Fig. 3-22 shows. The vertical stress increment for point A at a depth of z beneath certain corner of the rectangle can be calculated by utilizing the result given in section 3.5.3.1. Consider an infinitesimal rectangular area with the dimension $\mathrm{d}x \times \mathrm{d}y$, as Fig. 3-22 shows. The pressure acting on that area can be replaced by a concentrated load $dQ_v = q\mathrm{d}x\mathrm{d}y$ applied at the center of the area. According to Eq.(0-18), the vertical stress increment due to that concentrated load equals:

$$\mathrm{d}\sigma_z = \frac{3q}{2\pi}\frac{z^3}{R^5}\mathrm{d}x\mathrm{d}y \tag{0-19}$$

The vertical stress increment induced by the uniformly distributed load on the entire area can then be obtained through the integration over x from 0 to L and y from 0 to B:

$$\sigma_z = \int_0^L \int_0^B \frac{3q}{2\pi} \frac{z^3}{R^5} \mathrm{d}x\mathrm{d}y$$
$$= \frac{q}{2\pi}\left[\arctan\frac{m}{n\sqrt{1+m^2+n^2}} + \frac{m\cdot n}{\sqrt{1+m^2+n^2}}\left(\frac{1}{m^2+n^2}+\frac{1}{1+n^2}\right)\right] \qquad \text{(0-20)}$$
$$= K_s(m,n)q$$

K_s is the influence factor depending on $m = L/B$ and $n = z/B$. It is worthy pointing out that B is always the smaller dimension of the rectangle. Table 3.1 shows K_s values as a function of m and n.

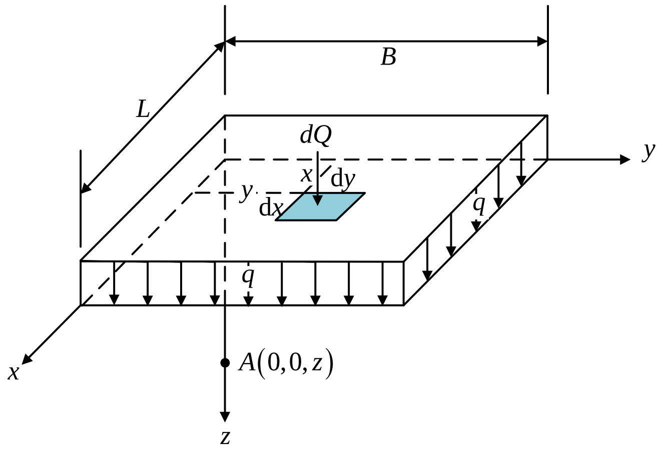


Fig. 3-22 Vertical stress increment under any corner of a rectangular area

Table 3.1 Influence factor K_s by Eq. (0-20)

z/B \ L/B	1.0	1.2	1.4	1.6	1.8	2.0	3.0	4.0	5.0	6.0	10.0
0	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500	0.2500
0.2	0.2486	0.2489	0.249	0.2491	0.2491	0.2491	0.2492	0.2492	0.2492	0.2492	0.2492
0.4	0.2401	0.242	0.2429	0.2434	0.2437	0.2439	0.2442	0.2443	0.2443	0.2443	0.2443
0.6	0.2229	0.2275	0.2301	0.2315	0.2324	0.233	0.2339	0.2341	0.2342	0.2342	0.2342
0.8	0.1999	0.2075	0.212	0.2147	0.2165	0.2176	0.2196	0.22	0.2202	0.2202	0.2202
1	0.1752	0.1851	0.1914	0.1955	0.1981	0.1999	0.2034	0.2042	0.2044	0.2045	0.2046
1.2	0.1516	0.1628	0.1705	0.1757	0.1793	0.1818	0.187	0.1882	0.1885	0.1887	0.1888
1.4	0.1305	0.1423	0.1508	0.1569	0.1613	0.1644	0.1712	0.173	0.1735	0.1738	0.174
1.6	0.1123	0.1241	0.1329	0.1396	0.1445	0.1482	0.1566	0.159	0.1598	0.1601	0.1604
1.8	0.0969	0.1083	0.1172	0.124	0.1294	0.1334	0.1434	0.1463	0.1474	0.1478	0.1482
2	0.084	0.0947	0.1034	0.1103	0.1158	0.1202	0.1314	0.135	0.1363	0.1368	0.1374
2.2	0.0732	0.0832	0.0915	0.0983	0.1039	0.1084	0.1205	0.1248	0.1264	0.1271	0.1277
2.4	0.0642	0.0734	0.0813	0.0879	0.0934	0.0979	0.1108	0.1156	0.1175	0.1184	0.1192
2.6	0.0566	0.0651	0.0725	0.0788	0.0842	0.0886	0.102	0.1073	0.1096	0.1106	0.1116
2.8	0.0502	0.058	0.0649	0.0709	0.076	0.0805	0.0941	0.0999	0.1024	0.1036	0.1048
3	0.0447	0.0519	0.0583	0.064	0.0689	0.0732	0.087	0.0931	0.0959	0.0973	0.0987
3.2	0.0401	0.0467	0.0526	0.0579	0.0627	0.0668	0.0806	0.087	0.0901	0.0916	0.0932
3.4	0.0361	0.0421	0.0477	0.0527	0.0571	0.0611	0.0747	0.0814	0.0847	0.0864	0.0882
3.6	0.0326	0.0382	0.0433	0.048	0.0523	0.0561	0.0694	0.0763	0.0798	0.0816	0.0837
3.8	0.0296	0.0348	0.0395	0.0439	0.0479	0.0516	0.0646	0.0717	0.0753	0.0773	0.0796
4	0.027	0.0318	0.0362	0.0403	0.0441	0.0475	0.0603	0.0674	0.0712	0.0733	0.0758

L/B \ z/B	1.0	1.2	1.4	1.6	1.8	2.0	3.0	4.0	5.0	6.0	10.0
4.2	0.0247	0.0291	0.0332	0.0371	0.0407	0.0439	0.0563	0.0634	0.0674	0.0696	0.0724
4.4	0.0227	0.0268	0.0306	0.0342	0.0376	0.0407	0.0526	0.0598	0.0639	0.0662	0.0692
4.6	0.0209	0.0247	0.0283	0.0317	0.0348	0.0378	0.0493	0.0564	0.0606	0.063	0.0663
4.8	0.0193	0.0228	0.0262	0.0294	0.0324	0.0352	0.0463	0.0533	0.0575	0.0601	0.0635
5	0.0179	0.0212	0.0243	0.0273	0.0301	0.0328	0.0435	0.0504	0.0547	0.0573	0.061
6	0.0127	0.0151	0.0174	0.0196	0.0217	0.0238	0.0325	0.0388	0.0431	0.046	0.0506
7	0.0094	0.0112	0.013	0.0147	0.0164	0.018	0.0251	0.0306	0.0347	0.0376	0.0428
8	0.0073	0.0087	0.0101	0.0114	0.0127	0.014	0.0198	0.0246	0.0283	0.0312	0.0367
9	0.0058	0.0069	0.008	0.0091	0.0102	0.0112	0.0161	0.0202	0.0235	0.0262	0.0319
10	0.0047	0.0056	0.0065	0.0074	0.0083	0.0092	0.0132	0.0168	0.0198	0.0222	0.0279

3.5.3.3 Vertical stress increment beneath a rectangular area

The problem discussed in section 3.5.3.2 can be extended to calculate the vertical stress increment for point O at a depth of z beneath a rectangular area by using the principle of superposition. The projection of point O may lie either inside or outside of the loaded area, as Fig. 3-23 shows. The key point is to make point O' the corner point of the rectangles by drawing straight lines parallel to the axes. The procedures are detailed as follows:

1) The point O' is inside the loaded area

The loaded area is divided into four smaller rectangular areas by drawing straight lines passing through O', as Fig. 3-23(a) shows. For each smaller rectangular area, compute the influence factor K_s. As the whole loaded area is the superposition of the four smaller ones, namely:

$$\text{Area of } ABCD = AA_1O'A_2 + A_1BB_2O' + O'B_2CD_1 + A_2O'D_1D \tag{0-21}$$

The influence factor is therefore the summation of the corresponding ones of those rectangular areas. Thus, the total vertical stress increment σ_z equals:

$$\sigma_z = K_{sABCD}q = q\left(K_{sAA_1O'A_2} + K_{sA_1BB_2O'} + K_{sO'B_2CD_1} + K_{sA_2O'D_1D}\right) \tag{0-22}$$

2) The point O' locates at any edge of the rectangle

The loaded area is divided into two smaller rectangles, as Fig. 3-23(b) shows. As the whole loaded area is the superposition of the two smaller ones, namely:

$$\text{Area of } ABCD = AA'O'D + A'BCO' \tag{0-23}$$

The influence factor is therefore the summation of the corresponding ones of those rectangular areas. Thus, the total vertical stress increment σ_z equals:

$$\sigma_z = K_{sABCD}q = q\left(K_{sAA'O'D} + K_{sA'BCO'}\right) \tag{0-24}$$

3) The point O' is outside the loaded area

Construct the rectangles as Fig. 3-23(c) shows. The real loaded area is colored while the imaginary ones are drawn with dotted lines. From the figure, it can be seen that:

$$\text{Area of } ABCD = AB_1O'D_1 - BB_1O'C_1 - DC_2O'C_1 + CC_2O'C_1 \quad (0\text{-}25)$$

Similarly, the total vertical stress increment σ_z equals:

$$\sigma_z = K_{sABCD}q = q\left(K_{sAB_1O'D_1} - K_{sBB_1O'C_1} - K_{sDC_2O'C_1} + K_{sCC_2O'C_1}\right) \quad (0\text{-}26)$$

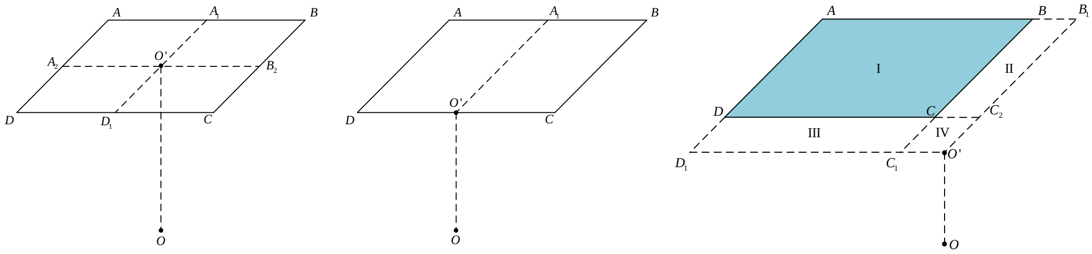


Fig. 3-23 a) Projection point O' is inside the loaded area; b) Projection point O' is at the edge of the loaded area; c) Projection point O' is outside the loaded area.

Example 1

A rectangular footing $ABCD$ with net pressure $q = 200$ kPa on the ground is shown in Fig. 3-24. Compute σ_z beneath points E, F, B, and G at a depth of 5 m.

Solution

(a) Point E locates inside rectangle $ABCD$. Therefore, the rectangle $ABCD$ is divided into four smaller one to make point E locates on the corner.

For smaller rectangle $LEID$, $B = 1$ m, $L = 1.5$ m; thus $m = L/B = 1.5/1 = 1$, $n = z/B = 5/1 = 5$. According to Table 3.1, the influence factor $K_{sLEID} \approx 0.0258$:

According to Eq. (0-21), the influence factor of rectangle $ABCD$ is the superposition of the four smaller ones, which are exactly the same. Namely:

$$K_{sABCD} = K_{sLEID} + K_{sEMCI} + K_{sFBME} + K_{sAFEL} = 4K_{sLEID} = 4 \times 0.0258 = 0.1032$$

And the vertical stress increment equals:

$$\sigma_{cz} = K_{sABCD}q = 0.1032 \times 200 = 20.64 \text{ kPa}$$

(b) Point F locates on the edge of rectangle $ABCD$. Therefore, the rectangle $ABCD$ is divided into two smaller one to make point F locates on the corner.

For smaller rectangle $AFID$, $B = 1.5$ m, $L = 2$ m; thus $m = L/B = 1.5 \approx 1.33$, $n = z/B = 5/1.5 \approx 3.33$. According to Table 3.1, the influence factor $K_{sAFID} \approx 0.0474$.

According to Eq. (0-23), the influence factor of rectangle $ABCD$ is the superposition of the two smaller ones, which are exactly the same. Namely:

$$K_{sABCD} = K_{sAFID} + K_{sFBCI} = 2K_{sAFID} = 2 \times 0.0474 = 0.0948$$

And the vertical stress increment equals:

$$\sigma_z = K_{sABCD} q = 0.0948 \times 200 = 18.96 \text{ kPa}$$

(c) Point B locates at the corner of rectangle $ABCD$. In this case, $B = 1.5\text{ m}$, $L = 2\text{ m}$; thus $m = L/B = 1.5 \approx 1.33$, $n = z/B = 5/1.5 \approx 3.33$. According to Table 3.1, the influence factor $K_{sABCD} \approx 0.0801$:

$$\sigma_z = K_{sABCD} q = 0.0801 \times 200 = 16.02 \text{ kPa}$$

(d) Point G locates outside rectangle. The imaginary rectangle $AGHD$ is drawn. For rectangle $AGHD$, $B = 2\text{ m}$, $L = 5\text{ m}$; thus $m = L/B = 5/2 = 2.5$, $n = z/B = 5/2 = 2.5$. According to Table 1, the influence factor $K_{sAGHD} \approx 0.0998$. For rectangle $BGCH$, $B = 2\text{ m}$, $L = 2\text{ m}$; thus $m = L/B = 2/2 = 1$, $n = z/B = 5/2 = 2.5$. According to Table 1, the influence factor $K_{sBGCH} \approx 0.0604$

The influence factor of rectangle $ABCD$ is the difference of rectangle $AGHD$ and $BGCH$, Namely:

$$K_{sABCD} = K_{sAGHD} - K_{sBGCH} = 0.0998 - 0.0604 = 0.0394$$

And the vertical stress increment equals:

$$\sigma_z = K_{sABCD} q = 0.0394 \times 200 = 7.88 \text{ kPa}$$

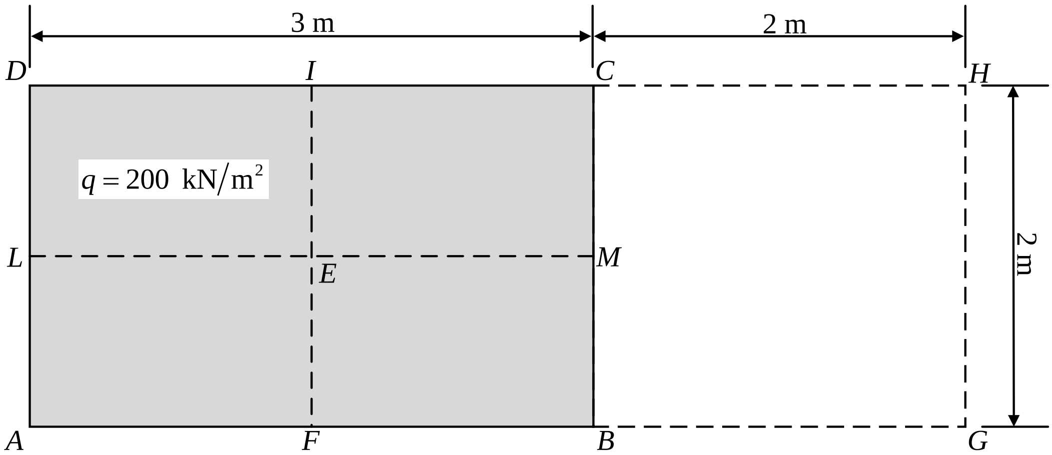


Fig. 3-24 Schematic diagram of Example

3.5.3.4 Vertical stress increment due to a line load

The vertical stress increment for point A at a depth of z due to a line load of intensity q (unit: N/m or kN/m) with infinite extent, as Fig. 3-25 shows, can be obtained by integrating Boussinesq's solution over y from $-\infty$ to ∞:

$$\sigma_z = \int_{-\infty}^{\infty} \mathrm{d}\sigma_z = \int_{-\infty}^{\infty} \frac{3q}{2\pi} \frac{z^3}{\left(x^2 + y^2 + z^2\right)^{\frac{5}{2}}} \mathrm{d}y = \frac{2qz^3}{\pi\left(x^2 + z^2\right)^2} \tag{0-27}$$

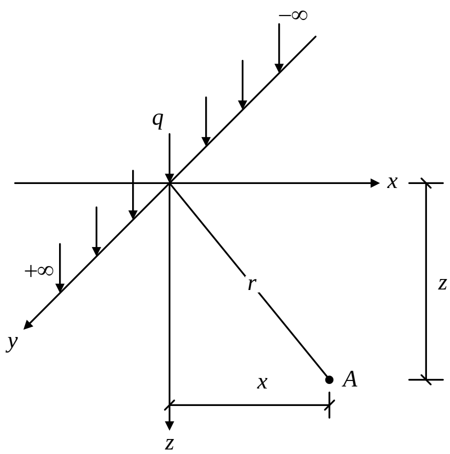


Fig. 3-25 Vertical stress increment due to a vertical line load

3.5.3.5 Vertical stress increment due to a strip load

The vertical stress increment due to a strip load, as Fig. 3-26 shows, can be obtained by utilizing the result presented in section 3.5.3.4. The vertical stress increment for a point A at a depth of z due to an infinite extent line load in y direction with intensity $q\mathrm{d}\xi$ acting at $x=\xi$ equals:

$$d\sigma_z = \frac{2q}{\pi}\frac{z^3}{\left[\left(x-\xi\right)^2+z^2\right]}\mathrm{d}\xi \tag{0-28}$$

Then the vertical stress increment due to a strip load can be obtained by an integration of Eq. (0-28) over x from $-B/2$ to $B/2$:

$$\begin{aligned}\sigma_z &= \frac{2q}{\pi}\int_{-\frac{B}{2}}^{+\frac{B}{2}}\frac{z^3}{\left[\left(x-\bar{x}\right)^2+z^2\right]}\mathrm{d}\xi \\ &= \frac{q}{\pi}\left\{\arctan\frac{\frac{2z}{B}}{\frac{2x}{B}-1}-\arctan\frac{\frac{2z}{B}}{\frac{2x}{B}+1}-\frac{\frac{2z}{B}\left[\left(\frac{2x}{B}\right)^2-\left(\frac{2z}{B}\right)^2-1\right]}{2\left\{\frac{1}{4}\left[\left(\frac{2x}{B}\right)^2+\left(\frac{2z}{B}\right)^2-1\right]^2+\left(\frac{2z}{B}\right)^2\right\}}\right\} \\ &= K_s\left(m,n\right)q\end{aligned} \tag{0-29}$$

K_s is the influence factor depending on $m=2x/B$ and $n=2z/B$. Table 3.1 shows the values of K_s as a function of m and n. For point A locates inside the strip load width, the item $2x/B-1$ becomes negative. In this case, π should be added to it, namely, the first term in the second line of Eq.(0-29) should be replaced by:

$$\text{For } 2x/B<1 \quad \arctan\frac{\frac{2z}{B}}{\frac{2x}{B}-1}\to\arctan\frac{\frac{2z}{B}}{\frac{2x}{B}-1}+\pi \tag{0-30}$$

The result given here is suitable for certain structure extended very much in one direction, such as strip foundation, wall foundation, foundation of retaining wall and the like.

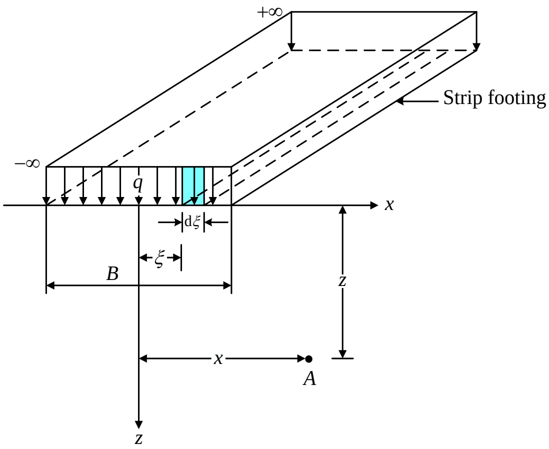


Fig. 3-26 Vertical stress increment due to a strip load

Table 3.2 Influence factor K_s by Eq.(0-29) or Eq.(0-30)

2z/B \ 2x/B	0	0.2	0.4	0.6	0.8	1	1.25	1.5	2	3	5	10
0	1	1	1	1	1	0.5	0	0	0	0	0	0
0.1	1.000	0.999	0.999	0.997	0.980	0.500	0.011	0.002	0.000	0.000	0.000	0.000
0.2	0.997	0.996	0.992	0.979	0.909	0.500	0.059	0.011	0.002	0.000	0.000	0.000
0.4	0.977	0.973	0.955	0.906	0.773	0.498	0.178	0.059	0.011	0.001	0.000	0.000
0.6	0.937	0.928	0.896	0.825	0.691	0.495	0.258	0.120	0.030	0.004	0.000	0.000
0.8	0.881	0.869	0.829	0.755	0.638	0.489	0.305	0.173	0.056	0.010	0.001	0.000
1	0.818	0.805	0.766	0.696	0.598	0.480	0.332	0.214	0.084	0.017	0.002	0.000
1.2	0.755	0.743	0.707	0.646	0.564	0.468	0.347	0.243	0.111	0.026	0.004	0.000
1.4	0.696	0.685	0.653	0.602	0.534	0.455	0.354	0.263	0.135	0.037	0.005	0.000
1.6	0.642	0.633	0.605	0.562	0.506	0.440	0.356	0.276	0.155	0.048	0.008	0.001
1.8	0.593	0.585	0.563	0.526	0.479	0.425	0.353	0.284	0.172	0.060	0.010	0.001
2	0.550	0.543	0.524	0.494	0.455	0.409	0.348	0.288	0.185	0.071	0.013	0.001
2.5	0.462	0.458	0.445	0.426	0.400	0.370	0.328	0.285	0.205	0.095	0.022	0.002
3	0.396	0.393	0.385	0.372	0.355	0.334	0.305	0.274	0.211	0.114	0.032	0.003
3.5	0.345	0.343	0.338	0.329	0.317	0.302	0.281	0.258	0.210	0.127	0.042	0.004
4	0.306	0.304	0.301	0.294	0.285	0.275	0.259	0.242	0.205	0.134	0.051	0.006
5	0.248	0.247	0.245	0.242	0.237	0.231	0.222	0.212	0.188	0.139	0.065	0.010
6	0.208	0.208	0.207	0.205	0.202	0.198	0.192	0.186	0.171	0.136	0.075	0.015
8	0.158	0.157	0.157	0.156	0.155	0.153	0.150	0.147	0.140	0.122	0.083	0.025
10	0.126	0.126	0.126	0.126	0.125	0.124	0.123	0.121	0.117	0.107	0.082	0.032
15	0.085	0.085	0.085	0.084	0.084	0.084	0.083	0.083	0.082	0.078	0.069	0.041
20	0.064	0.064	0.064	0.063	0.063	0.063	0.063	0.063	0.062	0.061	0.056	0.041
50	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.025	0.024
100	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.013	0.012

Example 1

Two parallel strip footings with 3 m wide and 5 m apart center to center are as Fig. 3-27 shows. The contact pressure are 200 kPa and 150 kPa respectively.

Calculate the vertical stress increment beneath the center of each strip footing at a depth of 3 m to the base (hints: the influence from adjacent strip footing should be taken into account).

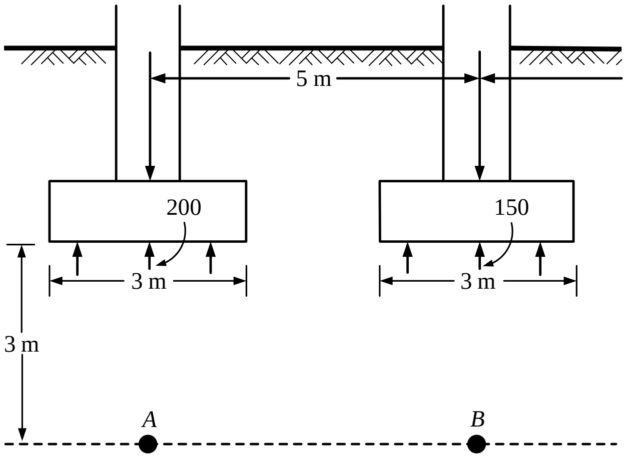


Fig. 3-27 Schematic diagram of two parallel footings

Solution

(a) For point A, the vertical stress increment by left strip footing: $m = 2x/B = 0$, $n = 2z/B = 2\times 3/3 = 2$. According to Table 2, the influence factor $K_{sLeft} \approx 0.550$.

The vertical stress increment by right strip footing: $m = 2\times 5/3 \approx 2.66$, $n = 2z/B = 2\times 3/3 = 2$. According to Table 2, the influence factor $K_{sRight} \approx 0.128$.

The vertical stress increment at point A due to two parallel strip footings equals:

$$\sigma_z = q_A K_{sLeft} + q_B K_{sRight} = 200\times 0.550 + 300\times 0.128 = 148.4 \text{ kPa}$$

(b) For point B, the vertical stress increment by right strip footing: $m = 2x/B = 0$, $n = 2z/B = 2\times 3/3 = 2$. According to Table 2, the influence factor $K_{sRight} \approx 0.550$.

The vertical stress increment by left strip footing: $m = 2\times 5/3 \approx 2.66$, $n = 2z/B = 2\times 3/3 = 2$. According to Table 2, the influence factor $K_{sLeft} \approx 0.128$.

The vertical stress increment at point B due to two parallel strip footings equals:

$$\sigma_z = q_A K_{sLeft} + q_B K_{sRight} = 200\times 0.128 + 300\times 0.550 = 190.6 \text{ kPa}$$

3.6 Some measures to alleviate the differential settlement

Exercises

3.1

［例题 2-1］ 设有图 2-3 所示的多种土层地基，各土层的厚度及重度示于图中，试求各土层交界面上的自重应力并绘出自重应力沿深度的分布图。

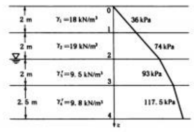


图 2-3 例题 2-1 附图

［解］ 根据图中所给资料，各土层交界面上的自重应力分别计算如下

$$\sigma_{cz0}=0$$
$$\sigma_{cz1}=\gamma_1 h_1=18\times 2=36\ \text{kPa}$$
$$\sigma_{cz2}=\gamma_1 h_1+\gamma_2 h_2=36+19\times 2=74\ \text{kPa}$$
$$\sigma_{cz3}=\gamma_1 h_1+\gamma_2 h_2+\gamma_3' h_3=74+9.5\times 2=93\ \text{kPa}$$
$$\sigma_{cz4}=\gamma_1 h_1+\gamma_2 h_2+\gamma_3' h_3+\gamma_4' h_4$$
$$=93+9.8\times 2.5=117.5\ \text{kPa}$$

3.3

2-1 如图2-28所示为某地基剖面图，各土层的重度及地下水位如图，试求土的自

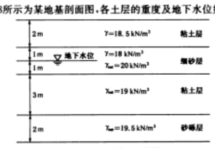
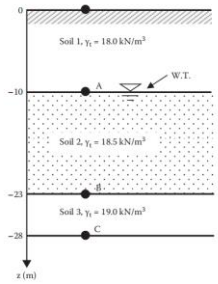


图 2-28 习题 2-1 附图

重应力和静孔隙水应力，并绘出它们的分布图。

3.2

Compute the vertical effective stress σ' at Points A, B, and C directly by using the submerged unit weight of soils γ' for the given soil's profile shown in the following figure. Plot those with the depth z.



3.2

2-2 如图2-29所示为一矩形基础，埋深 1 m，上部结构传至设计地面标高处的荷载为 P=2 106 kN，荷载为单偏心，偏心距 e=0.3 m。试求基底中心点 O，边点 A 和 B 下 4 m 深处的竖向附加应力。

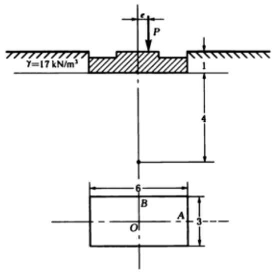


图 2-29 习题 2-2 附图