Chapter 2 Flow of water through soils

2.1 Introduction

Due to the existence of interconnect pores, soils are permeable. Water flows through soils via the connected passage from points of higher energy to lower energy. The study of the flow of water through permeable soil is important as many engineering problems have close relation with it:

1) Quantity of seepage estimation. Many geotechnical problems need to conduct the quantity of seepage estimation. For example, the seepage loss of reservoir and water channel, dewatering of underground water during excavation, drainage of subgrade, flow of water into pumping well and many others.
2) Deformation and stability problems. The water flowing imposes an additional force (seepage force) on particles in the direction of flow. In practical engineering, there are mainly four types of ground failure: 1) Failure by heave of ground and hydraulic fracturing, which occurs when water pressure under a structure or soil layer with low permeability exceeds the overburden pressure (承压水含水层顶部平衡); 2) Failure due to "sand boiling" or "quick sand" in cohesionless soil, which is often seen as the opposite of subsidence or settlement. That phenomena occurs when upwards seepage force act against the weight of the soil, reducing the vertical effective stress to zero. The particles in this case seem to suspend in water and there is no more coherence in the particle skeleton. The shear strength of soil is completely lost. 3) Erosion between interface of soil layers or soil and structure. Due to the weakness of interface, the internal erosion is easy to take place due to flowing water. The regressive erosion may finally lead to collapse of the structure. 4) Failure due to piping, which is a particular form of seepage erosion. When the bases of water retaining structures are previous, the particles may be gradually removed and shallow pipes are formed in the direction flow. The structures fails as soon as the removal process reaches a certain degree.

Moreover, the permeability of soil plays a crucial role in consolidation, shear strength and compressibility. Therefore, to better manage the problems results from the seepage flow, it is necessary for engineers to have a deep understanding of how water flows and how soils affect the water flow.

It is worth pointing out that the degree of saturation has great impact on the flowing behavior and mechanisms are far more complex. In this chapter, only the water flow in saturated soil is considered.

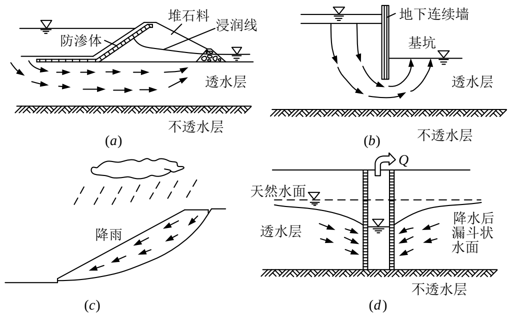


图2－2 一些典型渗流问题

（a）坝身及坝基的渗流；（b）支护结构下的基坑抗渗；（c）降雨引起的滑坡及泥石流；（d）水井渗流

Fig. 2-1 Typical seepage problems

2.2 Preliminaries

2.2.1 Steady flow and unsteady flow

The **steady flow** refers to a flow whose flow related parameters at any point, e.g., velocity, pressure, density, are time-independent. If any of flow parameters is time-dependent, the flow is termed as **unsteady flow**.

The flow through a pipe of variable diameter under constant pressure head can be considered as steady flow, while the pressure varies with time due to increasing or decreasing water level can be considered as unsteady flow.

2.2.2 Uniform flow and Non-uniform flow

The **uniform flow** refers to a flow whose velocity and hydrodynamic parameters do not change from point to point at any instant of time. If any of hydrodynamic parameters are different from point to point at any instant of time is termed as **non-uniform flow**.

2.2.3 Laminar flow and turbulent flow

The **laminar flow or stream flow** occurs when water flows at relatively low velocity without any disruption between layers, as figure shows. The motion of the fluid particles is very orderly. In perpendicular to the flow direction, the velocity decreases non-linearly from the center due to the friction of boundary. For **turbulent flow**, occurs when water flows at relatively high velocity. In this case, water no longer travels in layers and mixing across the tube, as figure shows.

For a given dimeter of a straight tube D, the flow pattern can be determined according to the Reynolds number R_e:

$$R_e = \frac{vD}{\nu} = \frac{\rho v D}{\mu} \tag{0-1}$$

ρ is the density of the fluid. v is the velocity of the fluid. ν is the kinematic viscosity and u is the dynamic viscosity. The correspondence between flow pattern and Reynolds number R_e is as Tab. 2-1 shows:

In this chapter, only steady, uniform and laminar flow is considered.

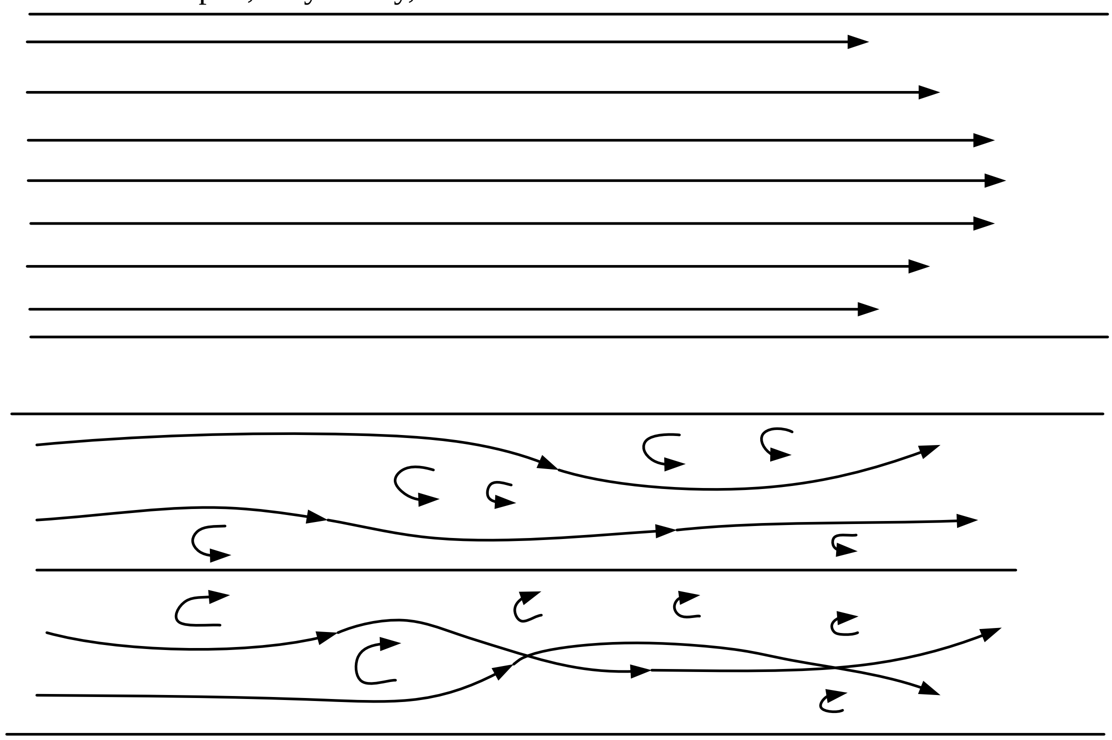


Fig. 2-2 a) Laminar flow; b) Turbulent flow

Tab. 2-1 Correspondence between flow pattern and Reynolds number R_e

Reynolds number R_e	Flow pattern
0~2000	Laminar flow
2000~4000	Transition flow (from laminar to turbulent)
4000~	Turbulent flow

2.2.4 Hydraulic heads and hydraulic gradient

It is widely accepted that the energy difference results in the flow of water. As Fig. 2-3 shows, due to the difference in water-level between the reservoirs, water flows from left to right.

According to Bernoulli's equation, the energy possessed by unit weight of water can represented by total water head, which consists of of:

$$h = h_z + h_p + h_v = z + \frac{u}{\gamma_w} + \frac{v^2}{2g} \tag{0-2}$$

h is the total head. h_z, h_p and h_v are **elevation head**, **pressure head** and **velocity head**, respectively. u is the pore water pressure and v is velocity of the fluid. The first and second terms in Eq. (0-2) are usually combined into a single term called as **piezometric head**, which represents the height of water within a piezometry. As the water flow in soils are usually low in most soil mechanics problems, the contribution of the third term is negligible in comparison with other two items and therefore it is ignored. In this case, Eq. (0-2) reduces to:

$$h = z + \frac{u}{\gamma_w} \tag{0-3}$$

The datum in Fig can be chosen at any elevation. As long as the datum is selected, all elevation heads are defined relative to the datum. Among the above mentioned heads, it is the difference of total heads that determines the water flow.

When water flows through a saturated soils, the interaction (resistance) between water and grains leads to the decrease of flow energy. The loss of total head per unit length of flow path is termed as hydraulic gradient:

$$i = -\frac{\Delta h}{\Delta L} \text{ or } i = -\frac{dh}{dL} \tag{0-4}$$

The negative sign indicates the hydraulic head decreases with the increase of the flow path.

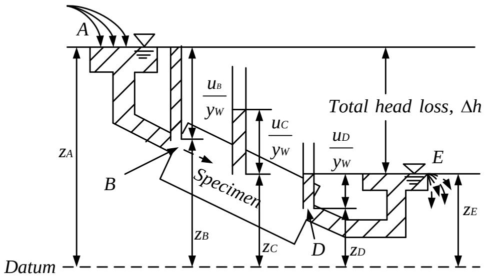


FIGURE 6.1 *Water flow through a pipe.*

$\tau_f = \sigma \tan\varphi + c$

Fig. 2-3 Water flow through a pipe

<u>Example 2.2-1</u>

The instrument for seepage test is shown in Fig. 2-4. Determine the elevation head, pressure head, total head of point B, C, D and F, as well as the loss of head from point B to C, point B to D and point B to F.

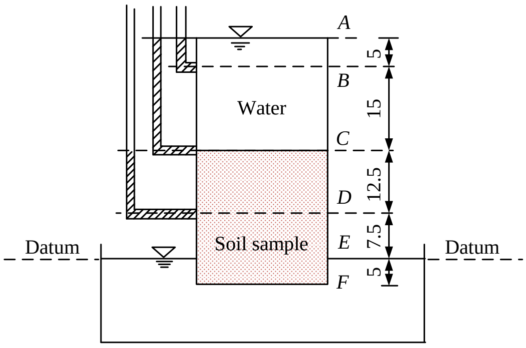


Fig. 2-4 Schematic diagram of the instrument (unit of length: cm)

Solution

The heads at each point and the corresponding loss of head are shown in Tab. 2-2.

Tab. 2-2 Corresponding heads and loss of heads

Point	Elevation head (cm)	Pressure head (cm)	Total head (cm)	Loss of head (cm)
B	35	5	40	
C	20	20	40	0
D	7.5	12.5	20	20
F	-5	5	0	40

2.3 Water flow through saturated soils

2.3.1 Darcy's law

In 1956, French engineer H. Darcy conducted a series of water flow experiments on granular soil and found that there exists a relation between the total discharge of water Q (unit, m^3/s), the cross-section area of a soil sample A (unit, m^2), the hydraulic conductivity k_h (unit, m/s) and the hydraulic gradient i as:

$$Q = k_h \cdot i \cdot A \tag{0-5}$$

The velocity of the fluid can be written as

$$v = \frac{Q}{A} = k_h \cdot i = -k_h \frac{dh}{dl} \tag{0-6}$$

Where the definition of hydraulic gradient Eq. (0-4) is used. The velocity defined by Eq. (0-6) is also called **Darcian velocity**.

2.3.2 Seepage velocity

It is worthy pointing that the velocity defined in Eq. (0-6) is not the true velocity of water through saturated soils, but is rather an average velocity over entire cross-section. The **seepage velocity** is an average velocity over pore space. Suppose the porosity is n, the seepage velocity is defined as:

$$v_s = \frac{v}{n} \tag{0-7}$$

The above equation is derived accordingly to the continuity equation of fluid. As the porosity is within the range 0~1, the seepage velocity is normally larger than the Darcian velocity.

2.3.3 Determination of the hydraulic conductivity

The hydraulic conductivity can be determined either in the laboratory or in the field. In the laboratory, two types of methods are available: 1) Constant head test; 2) Falling head test. The former is suited for coarse grained soils (gravel and sand) while the latter is suited for fine grained soil (silt and clay). In the field, the pumping test is the most commoly used.

2.3.3.1 Constant head test-Darcy's test

The apparatus which Darcy used similar as fig shows. The cylindrical container with the cross section area A is installed with a cohesionless soil sample, whose height is L. Two pieces of porous stones are attached to the top and bottom surfaces of the soil sample. Two piezometers are also installed to measure the corresponding head loss. The head and tail of water levels are kept constants by overflows.

The test is performed for a period of time Δt by allowing the water passing through the sample. Meanwhile, the quantity of water overflowed Q is recorded. With the quantities mentioned above, the hydraulic conductivity is determined as:

$$Q = k_h \cdot \frac{\Delta h}{L} \cdot A \cdot \Delta t \rightarrow k_h = \frac{Q \cdot L}{A \cdot \Delta t \cdot \Delta h} \tag{0-8}$$

<u>Comments</u>

1) The hydraulic conductivity is related to intrinsic permeability κ and mobility coefficient k as:

$$k_h = k \cdot \rho \cdot g \tag{0-9}$$

$$k_h = \frac{\kappa \cdot \rho \cdot g}{\mu} \tag{0-10}$$

ρ is the density of water. g is acceleration of gravity. μ is the dynamic viscosity.

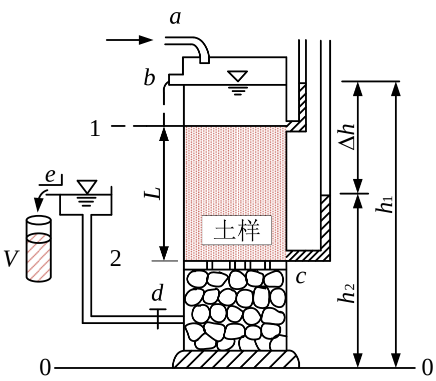


Fig. 2-5 Schematic diagram of the instrument (unit of length: cm)

Example

Example 4.1

A constant head permeability test was carried out on a cylindrical sample of sand 4 in. in diameter and 6 in. in height. 10 in^3 of water was collected in 1.75 min, under a head of 12 in. Compute the hydraulic conductivity in ft/year and the velocity of flow in ft/sec.

Solution

The formula for determining k is

$$k = \frac{Q}{Ait}$$

$$Q = 10 \text{ in}^3, \ A = 3.14 \times \frac{4^2}{4} = 12.56 \text{ in.}^2$$

$$i = \frac{h}{L} = \frac{12}{6} = 2, \quad t = 105 \text{ sec.}$$

$$\text{Therefore} \quad k = \frac{10}{12.56 \times 2 \times 105} = 3.79 \times 10^{-3} \text{ in./sec} = 31.58 \times 10^{-5} \text{ ft/sec} = 9960 \text{ ft/year}$$

$$\text{Velocity of flow} = ki = 31.58 \times 10^{-5} \times 2 = 6.316 \times 10^{-4} \text{ ft/sec}$$

2.3.3.2 Falling head test

Contrary to the constant head test, the water level within a falling head test varies constantly during the whole test. The apparatus is similar as fig shows. A standpipe is connected to the bottom of the soil sample and the tail of water level is kept constant by overflow. The geometry of the container and soil sample are the same as that of the constant head test.

The standpipe is filled with water up to a height of h_0 and the starting time is recorded as t_0. When the test finishes, the height of water level h_1 and the finishing time t_1 are recorded. The hydraulic conductivity k_h can be determined on the basis of the drop in head $h_1 - h_0$ and the elapsed time $t_1 - t_0$. The detailed procedure is explained as follows.

At any given instant of time $t \in [t_1, t_2]$, the level of water in the standpipe is $h \in [h_1, h_2]$. Assume within an infinitesimal time increment dt, the velocity of water is a constant and the quantity of water overflowed is dQ, according to Eq. (0-8):

$$dQ = v \cdot a \cdot dt = k \cdot \frac{h}{L} \cdot a \cdot dt \tag{0-11}$$

a is the cross-section area of the standpipe. The quantity of water overflowed $\mathrm{d}Q$ is expressed as:

$$dQ = -a \cdot dh \tag{0-12}$$

The negative sign in Eq. (0-12) is due to the increase of the overflowed water as head decreases $\mathrm{d}h$. Combing Eq. (0-11) and Eq. (0-12) leads to:

$$-a \cdot dh = k\frac{h}{L} A \cdot dt \tag{0-13}$$

Rearranging and integrating the above equation in corresponding intervals as:

$$\int_{h_0}^{h_1} \frac{a}{h} dh = \int_{t_0}^{t_1} -k\frac{A}{L} dt \rightarrow k = \frac{aL}{A} \frac{\ln \frac{h_0}{h_1}}{(t_1 - t_0)} \tag{0-14}$$

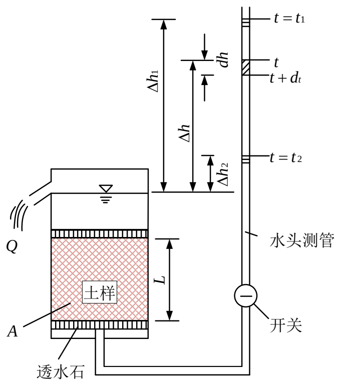
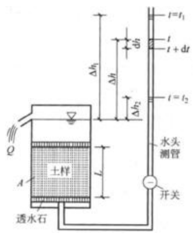


Fig. 2-6 Water flow through a pipe



2.3.3.3 Pumping test in the field

The accuracy of the determination of hydraulic conductivity in the laboratory is unavoidably influenced by the quality of soil sample and distubrance due to sample collection, transportation and preparation. To obtain an accurate result as far as pssible, the hydraulic conductivity can be measured directly pumping tests in the field, as fig shows.

A pumping test consists of a test well and a series of observation wells. Both the test and observation wells are sunk through the permeable soil layers up to the impermeable layer. The test well is pumped at a controlled rate and water-level response (drawdown) is measured in one or more surrounding observation wells. After the water level in the test and observation wells remain stationary, which indicates the water being pump out and the inflow into the well are equal, the data acquired can be used to determine the hydraulic conductivity.

Assuming at certain radial distance $r \in [r_1, r_2]$, the water level is h. Within an infinitesimal time increment dt, the quantity of water passing through the area of the vertical cylindrical surface of radius r and depth h equals:

$$\mathrm{d}Q = 2\pi rh\frac{\mathrm{dh}}{dr}\mathrm{dt} \to q = \frac{dQ}{\mathrm{dt}} = 2\pi rh\frac{\mathrm{dh}}{dr} \tag{0-15}$$

q is the stationary discharge rate for a test well. Rearranging and integrating the above equation in corresponding intervals as:

$$\int_{r_1}^{r_2}\frac{q}{2\pi}\frac{\mathrm{d}r}{r} = \int_{h_1}^{h_2} h\mathrm{d}h \to k = \frac{q}{\pi}\frac{\ln\frac{r_2}{r_1}}{h_2^2 - h_1^2} \tag{0-16}$$

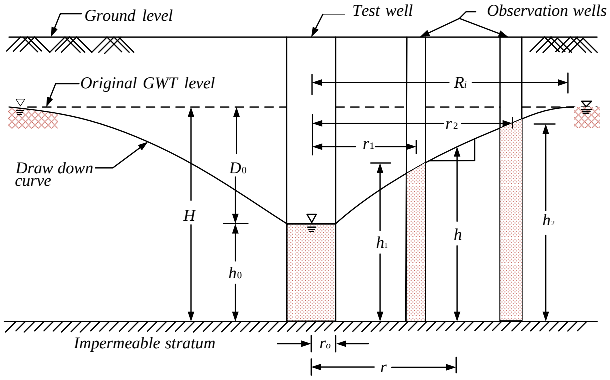


Fig. 2-7 Water flow through a pipe

Comments

1) As the viscosity of water changes with temperature, the hydraulic conductivity obtained either in the laboratory or in the field should be standardized at certain temperature T_s as

$$k_T = \frac{k_{T_s}\mu_{T_s}}{\mu_T} \tag{0-17}$$

It is customary to standardize he hydraulic conductivity at a temperature of $T_s = 20°$, then Eq. (0-17) becomes:

$$k_T = \frac{k_{20}\mu_{20}}{\mu_T}$$

2) Several groups of hydraulic conductivity are calculated and their mean value are adopted for practical design. Some typical hydraulic conductivity are listed in Tab.

Tab. 2-3 Correspondence between flow pattern and Reynolds number R_e

Table 4.1 Hydraulic conductivity of some soils
(after Casagrande and Fadum, 1939)

k (cm/sec)	Soils type	Drainage conditions
10^1 to 10^2	Clean gravels	Good
10^1	Clean sand	Good
10^{-1} to 10^{-4}	Clean sand and gravel mixtures	Good
10^{-5}	Very fine sand	Poor
10^{-6}	Silt	Poor
10^{-7} to 10^{-9}	Clay soils	Practically impervious

2.3.4 Factors influencing the hydraulic conductivity

The value of hydraulic conductivity depends on a number of factors. The factors related to soil are: 1) Particle size, shape and gradation; 2) Porosity; 3) Chemical properties of soil; 4) Structure; 5) Degree of saturation. The factors related to water are: 1) viscosity; 2) type of flow.

2.3.4.1 Factors related to soil

1) **Particle size, shape and gradation.** Soils dominated by large particles tend to have relatively large pore spaces and thus large values of hydraulic conductivity. It is found that the hydraulic conductivity is approximately proportional to the square of the grain size. Rounded Particles will have more permeability than angular shaped. It is due to specific surface area of angular particles is more compared to rounded particles. For a poorly graded soil, the particles either have the similar size or certain particle sizes are absent. As there are less smaller particles to fill the voids formed by larger particles, it will therefore have large values of hydraulic conductivity.

2) **Porosity.** According to the Kozeny-Carman formular:

$$\kappa = \mathrm{c}d^2 \frac{n^3}{(1-n)^3}$$

d is a measure for the grain size, and c is a coefficient related to the shape of the particles. κ is the intrinsic permeability, which depends upon the geometry of the skeleton. Taking Eq. (0-10) into account, the hydraulic conductivity is further cast into:

$$k_h ==cd^2 \frac{n^3}{(1-n)^3}\frac{\rho \cdot g}{\mu} \tag{0-18}$$

From Eq. (0-18), the effect of porosity is obviously observed.

3) **Chemical composition.** For soils mainly made up of clays which are susceptible to swell when adsorbing water, the hydraulic conductivity decreases.

4) **Structure.** The structure of soil plays an import role in hydraulic conductivity, especially for cohesive soil. From a microscopic point of view, those with flocculated structures normally have larger hydraulic conductivity than the clays with dispersed soil structure. For flocculated structures, the flat particles are in random orientation, while for dispersed structures, the particles are in face-to-face orientation and hence, permeability is very low. From a macroscopic point of view, for stratified soil deposits without subjected to any tectonic disturbance, the flow in parallel with the stratified layer have higher hydraulic conductivity while the flow in perpendicular to the stratified layer has lower hydraulic conductivity.

5) **Degree of saturation.** In a partially saturated soil, the hydraulic conductivity is no longer a constant. In general, it is a function of degree of saturation. A decrease in the degree of saturation leads to a further decrease in the pore volume occupied by the water. The increase of void space results in an increase in entrapped air, which will block the flow path thereby reduces the hydraulic conductivity.

2.3.4.2 Factors related to water

1) **Temperature.** According to the Kozeny-Carman formula, hydraulic conductivity is inversely proportional to the viscosity of the fluid, which varies inversely to the temperature, as fig shows.

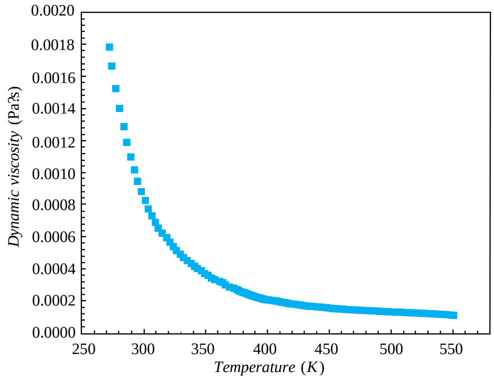
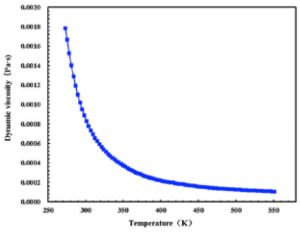


Fig. 2-8 Water flow through a pipe



2) **Chemical properties of water.** Chemical properties of the solution flowing through the soil can also impact the hydraulic conductivity. These chemical effects arise when solution characteristics promote chemical dispersion or flocculation of clay presented in the soil. Chemical dispersion is the process in which soil particles, which previously were held together in close contact within soil aggregates, respond to a changed chemical environment by expanding and separating from one another, breaking down the soil aggregates. This may due to irrigation with sodic water; low electrical conductivity of the flowing solution; or high exchangeable sodium percentage in the soil. Chemical flocculation is the process in which dispersed soil particles come together, often due to a change in the chemical environment. A high proportion of polyvalent cations, such as Ca2+, Mg2+, and Al3+, promotes flocculation, while a high proportion of monovalent cations, particularly Na+, promotes chemical dispersion.

2.3.5 Overall hydraulic conductivity for stratified ground

In pratical engineering, the ground is stratified in layers with different hydraulic conductivities. For quick evaluation of the geology condition, engineers prefer to obtain an overall hydraulic conductivity either in horizontal or vertical direction.

Suppose the thickness and the isotropic hydraulic conductivity of any soil layer are known, the overall hydraulic conductivities in the horizontal and vertical directions are computed in the following sections.

2.3.5.1 Overall vertical hydraulic conductivity

Fig shows a vertical flow through a stratified ground. Our object is to derive an overall vertical hydraulic conductivity for an equivalent homogeneous ground with the same geometry, as fig shows.

Suppose the total head loss for the whole ground is Δh, while the total head loss for each soil layer is Δh_1, Δh_2 and Δh_3. The features for the flow in perpendicular to the soil layers are as follows:

1) According to the continuity of flow, the velocity of water in each soil layers, as well as the velocity of water the equivalent homogeneous ground are equal, namely:

$$v = v_1 = v_2 = v_3 \tag{0-19}$$

2) The total head loss for the equivalent homogeneous ground equals the sum of the total loss of each soil layer, namely:

$$\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 \tag{0-20}$$

According to the Darcy's law:

$$\begin{cases} k_{h1}\dfrac{\Delta h_1}{H_1} = v_1 \\ k_{h2}\dfrac{\Delta h_2}{H_2} = v_2 \\ k_{h3}\dfrac{\Delta h_3}{H_3} = v_3 \end{cases} \rightarrow \begin{cases} \Delta h_1 = \dfrac{v_1 H_1}{k_{h1}} \\ \Delta h_2 = \dfrac{v_2 H_2}{k_{h2}} \\ \Delta h_3 = \dfrac{v_3 H_3}{k_{h3}} \end{cases} \tag{0-21}$$

For the equivalent homogeneous ground:

$$v = k_v \frac{\Delta h}{H} \rightarrow \Delta h = \frac{vH}{k_v} \tag{0-22}$$

Substituting Eq. (0-21) and Eq. (0-22) into Eq. (0-20) and taking Eq. (0-19) into account, the overall vertical hydraulic conductivity equals

$$k_v = \frac{H}{\dfrac{H_1}{k_{h1}} + \dfrac{H_2}{k_{h2}} + \dfrac{H_3}{k_{h3}}} \tag{0-23}$$

Eq. (0-23) can be generalized to the ground consisting of n soil layers as:

$$k_v = \frac{H}{\sum_{i=1}^{n} \dfrac{H_i}{k_{hi}}} \tag{0-24}$$

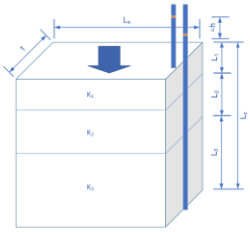


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Fig. 2-9 Water flow through a pipe

2.3.5.2 Overall horizontal hydraulic conductivity

Fig shows a horizontal flow through a stratified ground. Our object is to derive an overall horizontal hydraulic conductivity for an equivalent homogeneous ground with the same geometry, as fig shows.

Suppose the total head loss for the whole ground is Δh, while the total head loss for each soil layer is Δh_1, Δh_2 and Δh_3. The features for the flow in parallel with soil layers are as follows:

1) The hydraulic gradient of each soil layers, as well as the hydraulic gradient of the equivalent homogeneous ground are equal, namely:

$$i = i_1 = i_2 = i_3 \tag{0-25}$$

2) The total discharge of the equivalent homogeneous ground equals the sum of the total discharge of each soil layer, namely:

$$Q = Q_1 + Q_2 + Q_3 \tag{0-26}$$

According to the Darcy's law:

$$\begin{cases} k_{h1} \cdot i_1 \cdot H_1 = Q_1 \\ k_{h2} \cdot i_2 \cdot H_2 = Q_2 \\ k_{h3} \cdot i_3 \cdot H_3 = Q_3 \end{cases} \tag{0-27}$$

For the equivalent homogeneous ground:

$$Q = k_h \cdot i \cdot H \tag{0-28}$$

Substituting Eq. (0-26) and Eq. (0-27) into Eq. (0-26) and taking Eq. (0-25) into account, the overall horizontal hydraulic conductivity equals

$$k_h = \frac{H_1 k_{h1} + H_2 k_{h2} + H_3 k_{h3}}{H} \tag{0-29}$$

Eq. (0-29) can be also generalized to the ground consisting of n soil layers as:

$$k_v = \frac{\sum_{i=1}^{n} k_{hi} \cdot H_i}{H} \tag{0-30}$$

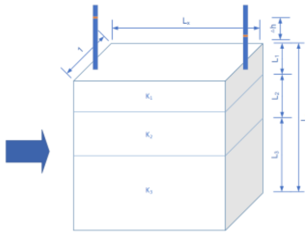

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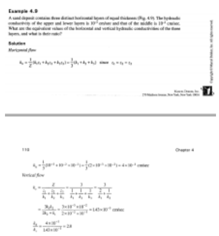
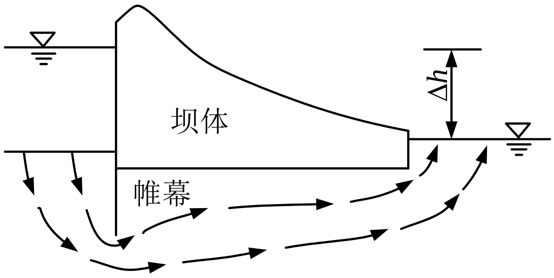
Fig. 2-10 Water flow through a pipe

<u>**Example**</u>



2.4 2D seepage problem and flow net

Till now, the problem encountered belongs to the one dimensional seepage problem with simple boundary condition. However, the practical engineering problems are two or three dimensional seepage problems with complex boundary conditions. For example, the water flow through the pervious foundation of dam or embankment can be considered as a two-dimensional seepage problem, while the water fall of an open pit belongs to a three-dimensional seepage problem. To solve such kind of problems, the multi-dimensional governing equations with corresponding boundary conditions are needed to be firstly established.



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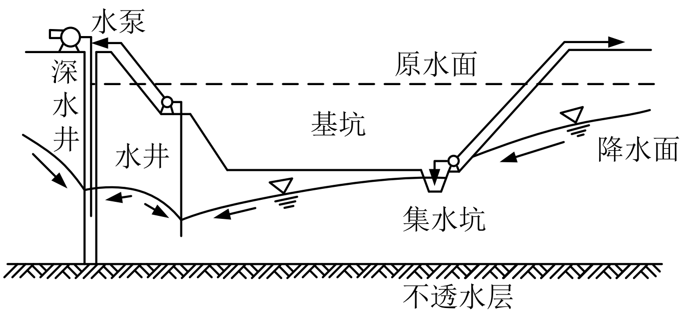

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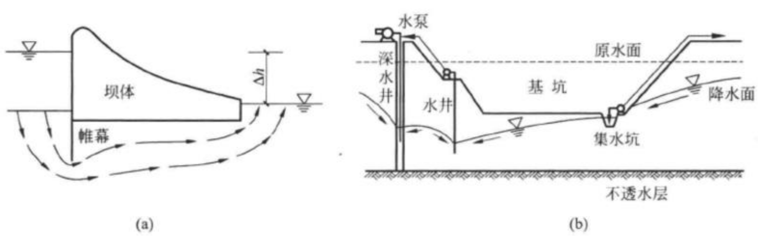


Fig. 2-11 Water flow through a pipe

2.4.1 Darcy's law in 2D

The one dimensional Darcy's Law can be extended to two dimensional such that the hydraulic head h becomes a function of the space coordinates, x and y. In this case, the derivative in Eq. (0-6) will be replaced by partial derivatives. Moreover, assuming soils are isotropic porous medium, the Darcian velocity along two axes equals:

$$\begin{cases} v_x = -k_x \dfrac{\partial h}{\partial x} \\ v_y = -k_y \dfrac{\partial h}{\partial y} \end{cases} \rightarrow \begin{bmatrix} v_x \\ v_y \end{bmatrix} = -\begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{bmatrix} \dfrac{\partial h}{\partial x} \\ \dfrac{\partial h}{\partial y} \end{bmatrix} \tag{0-31}$$

2.4.2 Governing equation of water flow through isotropic saturated soils

The governing equation of water flow in saturated soil can be derived with the help of an infinitesimal square as fig shows, whose dimensions are dx and dz, respectively. To make the derivation more clear, assumptions are firstly given as follows:

1) The hydraulic conductivity is isotropic, namely $k_x = k_y = k$;
2) The steady flow is considered;
3) Water is incompressible.

With these assumptions, the governing equation is derived according to the the principle of conservation of mass.

The amount of flow into the infinitesimal square from left side and bottom side are respectively:

1) In the x direction:

$$v_x \cdot dz$$

2) In the z direction:

$$v_z \cdot dx$$

The amount of flow out of the infinitesimal square from right side and top side are respectively:

1) In the x direction

$$\left(v_x + \frac{\partial v_x}{\partial x} \right) dz$$

2) In the z direction

$$\left(v_z + \frac{\partial v_z}{\partial z} \right) dx$$

As there is no source or sink within the infinitesimal square, the principle of conservation of mass requires that the net excess of mass flux must be zero (the density of water is assumed a constant), namely:

$$\left(v_z + \frac{\partial v_z}{\partial z}\right)dx + \left(v_x + \frac{\partial v_x}{\partial x}\right)dz - v_x \cdot dz - v_z \cdot dx = 0 \rightarrow \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \qquad (0\text{-}32)$$

Substitution Eq. (0-31) into Eq. (0-32) leads to the different equation:

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad (0\text{-}33)$$

Eq. (0-33) is the so-called two dimensional Laplace equation, which could be solved with various methods as described below.

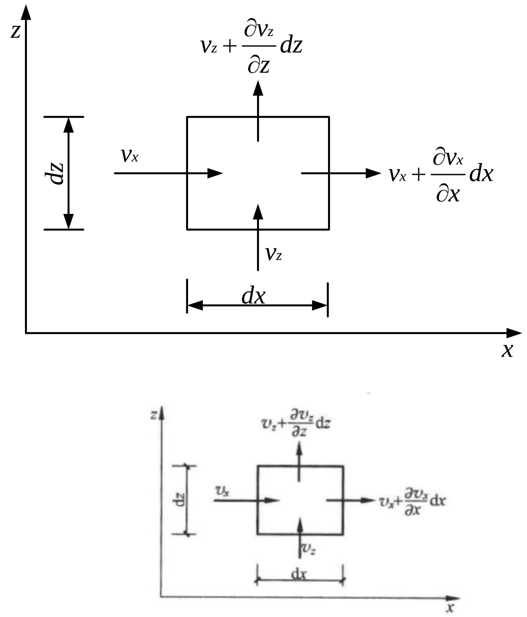


Fig. 2-12 Water flow through a pipe

2.4.3 Boundary conditions

To solve the Laplace equation, certain constraints named boundary conditions are needed to be applied. In practical engineering, engineer will meet four types of boundary conditions:

1) Along certain boundary, the total head is known. This is a typical Dirichlet boundary condition.
2) Along certain boundary, the distribution of velocity is known. This is a typical Neumann boundary condition, which specifies the values that the derivative of a total head is going to take. The typical Neumann boundary is the impermeable boundary, where the flux the zero.
3) The phreatic surface. Along this boundary, two conditions need to be satisfied: 1) the pressure head is zero and accordingly, the total head equals the evaluation head; 2) the velocity in normal direction is zero. There is only the tangential flow.

4) Seepage face. A special boundary condition is needed if the phreatic surface reaches an open, freely draining surface, as indicated on surface S_5. Along this boundary, In such a case the pore fluid can drain freely down the face of the dam, and pressure head is zero at all points on this surface below its intersection with the phreatic surface.

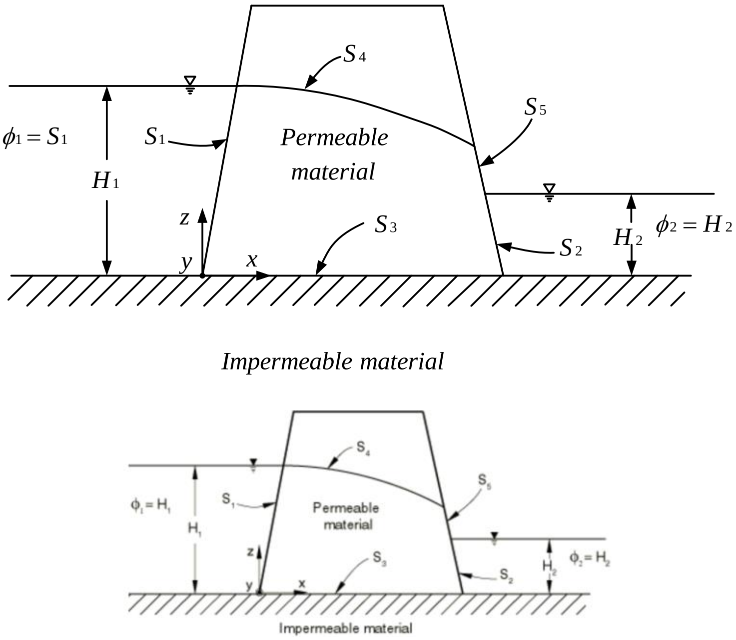


Fig. 2-13 Water flow through a pipe

2.4.4 Solutions to Laplacian equation

There are many methods that are in use for the construction of flow nets. Some of the important methods are

1) **Analytical method**: The analytical method, based on the Laplace equation although rigorously precise, is not universally applicable in all cases because of the complexity of the problem involved. The mathematics involved even in some elementary cases is beyond the comprehension of many design engineers. Although this approach is sometimes useful in the checking of other methods, it is largely of academic interest;

2) **Numerical method**: With the revolution of computational capacity, more and more engineering scale problems with complex boundary conditions can be solved. Moreover, variety of interaction, e.g. interaction between solid and liquid, can be taken into account. Most of the numerical methods used in geotechnical engineering are the finite difference method (FDM), finite element method (FEM), boundary element method (BEM), discontinuous deformation analysis (DDA) method, discrete element method (DEM),

particle flow method (PFM), etc. Some of these method can be even coupled to solve certain special problem, such as the application of the FEM-DEM coupling method to the local place where the special concern deserves;

3) Scaled model method: Scaled models are very useful to solve seepage flow problems. Soil models can be constructed to depict flow of water below concrete dams or through earth dams. These models are very useful to demonstrate the fundamentals of fluid flow, but their use in other respects is limited because of the large amount of time and effort required to construct such models;
4) Graphical method. The graphical method developed by Forchheimer (1930) has been found to be very useful in solving complicated flow problems. A. Casagrande (1937) improved this method by incorporating many suggestions. The main drawback of this method is that a good deal of practice and experience are essential to produce a satisfactory flow net. In spite of these drawbacks, the graphical method is quite popular among engineers.

2.4.5 2D flow net and its application

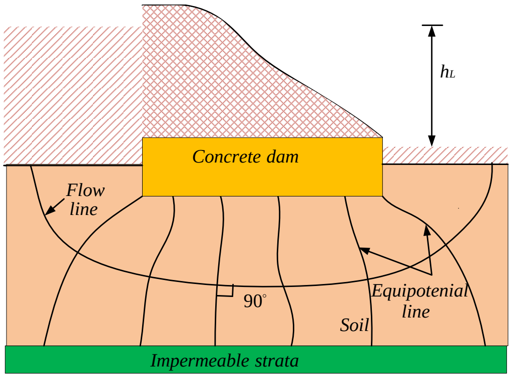
Although the seepage problem can be feasibly solve by numerical method now, it is still helpful to get a little knowledge about the graphical method. A flow net is a graphical representation of two-dimensional steady-state groundwater flow through aquifers, as fig shows. With the flow net, certain hydraulic properties such as the total discharge of water flow, water pressure, etc. can be calculated.

A flow net is a curvilinear net formed by the combination of flow lines (streamline) and equipotential lines, which are everywhere perpendicular to each other, as fig shows. Flow lines represent the path of flow along which the water will seep through the soil. Equipotential lines are formed by connecting the points with equal total head.

To draw a flow net with good quality, the following rules should be followed:

1) Flow line and equipotential line intersect with each other at a $90°$ angle;
2) Flow lines or equipotential lines themselves never merge;
3) Equal quantity of seepage occurs in each flow channel. A flow channel is the space between two flow lines;
4) Head loss is the same between two neighboring equipotential lines;
5) The space (also called grid) formed between two flow lines and two equipotential should be in a square form or close to it;

6) The flow lines and equipotential lines should be smooth curves.



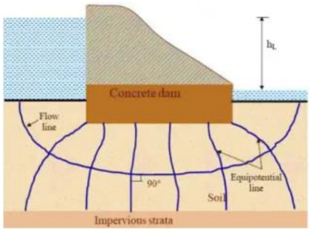


Fig. 2-14 Water flow through a pipe

2.4.5.1 Brief rules for the construction of a flow net

Based on the foregoing rules, the following steps are recommended for a flow net construction:

1) Draw a real model with designed scale on a piece of grid paper;

2) Make a trial for the number of intervals for equipotential lines N_f;

3) Properly identify the boundary flow lines and boundary equipotential lines;

4) Draw flow lines based on how water flows. Make sure that the interval of the flow channel should be equal;

5) Starting from the upstream site, draw the first equipotential line to have all net openings squares or near-squares with 90° intersections;

6) Continue the foregoing step for the rest of equipotential lines until it reaches the downstream;

7) Determine the number of intervals for equipotential lines N_d.

2.4.5.2 Total discharge

1) For each flow channel, the head loss between two neighboring equipotential lines is: $\Delta h = \Delta H / N_d$;

2) The hydraulic gradient is: $\Delta h / l$, l is the distance between two neighboring equipotential lines;

3) According to Darcy's law, the total discharge of each flow channel is:

$k \cdot \Delta h \cdot s / l$, s is the distance between two neighboring flow lines

4) The total discharge through each equipotential line is: $k \cdot \Delta h \cdot s \cdot N_f / l$

5) As the shape of each grid is approximately equal, namely $s \approx l$. Therefore, the total discharge finally recasts into: $k \cdot \Delta H \cdot N_f / N_d$.

2.4.5.3 Estimation of the pressure head

The pressure head on any equipotential line can be evaluated as follows:

1) Determine the total head after n potential drops: $\Delta H - n \cdot \Delta h$;

2) Determine the elevation head z;

3) If the velocity of water is low and thus the velocity head is dropped, the pressure head equals $\Delta H - n \cdot \Delta h - z$.

2.4.5.4 Exit gradient

The exit gradient is the hydraulic gradient of a flow line at the downstream exit point, which can be expressed as:

$$i_{exit} = \frac{\Delta H}{L} \tag{0-34}$$

L is the length of a flow path.

<u>Example</u>

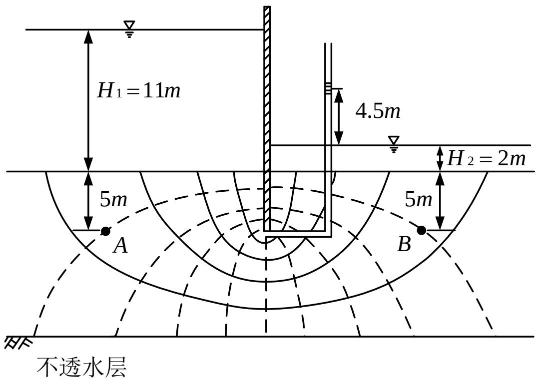


Fig. 2-15 Water flow through a pipe

2.5 Seepage related problems

The flow of water in a saturated soil imposes the additional force on soil particles. Therefore, the original stress state within soils will be altered. This may lead to two types of problems:

1) The erosion takes place at certain part of base or soil-structure interface as particles are displaced, which may lead to local failure or deformation of

structures

2) The complete failure of structures. The water flow may greatly reduce the strength of soil and a complete failure occurs, for example, the collapse of slop due to rain water infiltration. It is worth pointing that the progressive local failure may also ultimately lead to complete failure and the piping is a representative example.

2.5.1 Seepage force and critical hydraulic gradient

As foregoing mentioned, the flowing water may impose the dynamic force on particles in the direction of flow. The dynamic force, which we usually called seepage force is the term indicating the hydraulic force per unit volume of soil. It is worth noting that the seepage force is expressed in an average sense over the whole soil, although it actually acts on particles.

The effect of seepage force can be show through the following device. At the beginning, the water levels in the left reservoir and right reservoir are the same. The water levels in two piezometers are equal as there is no water flow through soil and thus no head loss. If the left reservoir is gradually lifted to certain height, the phenomena "sand boil" or "quick sand" occurs.

To gain a full insight into the phenomena, the soil is separated from the container for further analysis. The types of forces act on the soil sample are as follows:

1) In the horizontal direction (left side and right side of the soil sample), the water pressure and the reaction force by container are in equilibrium.
2) In the vertical direction, there are three types of forces: 1) The total weight of the soil sample $\gamma_{sat} L \cdot A$; 2) The water pressure acts on the top and bottom surface of soil are respectively $\gamma_w h_1$ and $\gamma_w h_w$; 3) The reaction force R due to the support of the filter

According to equilibrium state in the vertical direction,

$$\gamma_{sat} L \cdot A + \gamma_w h_w \cdot A = R + \gamma_w h_1 \cdot A \rightarrow R = \gamma' L \cdot A - \gamma_w \left(h_1 - h_w - L\right) \cdot A$$

As $\Delta h = h_1 - h_w - L$, the above equation changes to:

$$R = \gamma' L \cdot A - \gamma_w \Delta h \cdot A$$

The second item is due to action of seepage force. Define the total seepage face and seepage face as J and j respectively as:

$$J = \gamma_w \Delta h \cdot A \quad \text{and} \quad j = \frac{J}{A \cdot L} = \gamma_w \frac{\Delta h}{L} = \gamma_w i$$

When the "sand boiling" or "quick sand" occurs, the particles suspend in water.

Therefore, reaction force R equals zero and the corresponding hydraulic gradient is named as critical hydraulic gradient:

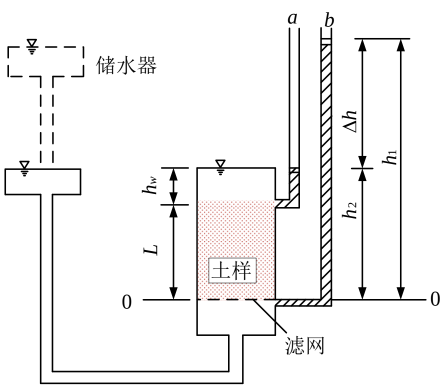
$$R=\gamma' L\cdot A-\gamma_w \Delta h\cdot A=0\rightarrow i_{crit}=\frac{\Delta h}{L}=\frac{\gamma'}{\gamma_w}$$

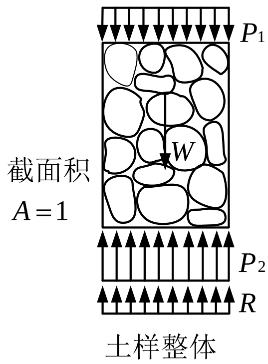
As $\gamma'=(G_s-1)\gamma_w/(1+e)$ and $n=e/(1+e)$, alternative expressions for the critical hydraulic gradient can be derived as:

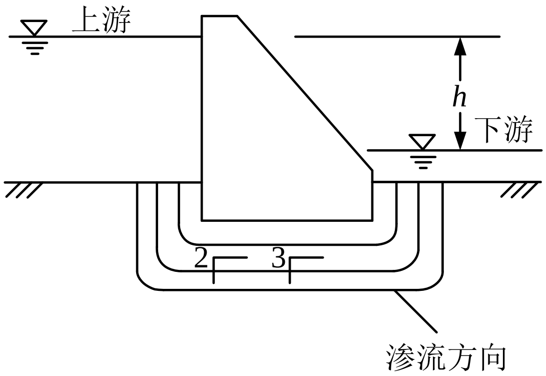
$$i_{crit}=\frac{\gamma'}{\gamma_w}=\frac{G_s-1}{1+e} \quad \text{or} \quad i_{crit}=(G_s-1)(1-n)$$

When there is no water flow, namely $\Delta h=0$, the reaction force exactly equals the net weight of the soil sample (Total weight minus the buoyant force).

Although the water flow alters the original stress state within soils, this does not mean that the disturbance is always harmful. In different parts of the structure, the seepage force plays different roles. Take the dam as an example. At point 1, the directions the seepage force and the weight of particles are in the same direction. The existence of the seepage force increases the effective stress between particles and thus increase the compaction and strength, which is good for stability. At points 2 and 3, the directions the seepage force and the weight of particles are mutually orthogonal, the small particles have the tendency to be displaced. At points 4, the seepage force and the weight of particles are in the opposite direction. The existence of the seepage force decreases the effective stress and thus decreases the compaction and strength, which is bad for stability. Especially, when the seepage force exceeds the weight of particles, the “sand boiling” occurs. If the special attention is not paid, progressive local failure may lead to backward erosion and ultimately piping failure of the whole structure.







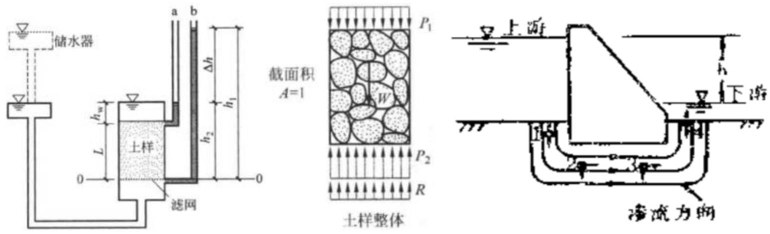


Fig. 2-16 Water flow through a pipe

2.5.2 Seepage related local failure problems

The water flow in saturated soil leads to variety types of failures. Main failure are as follows:

1) Failure by heave of ground and hydraulic fracturing

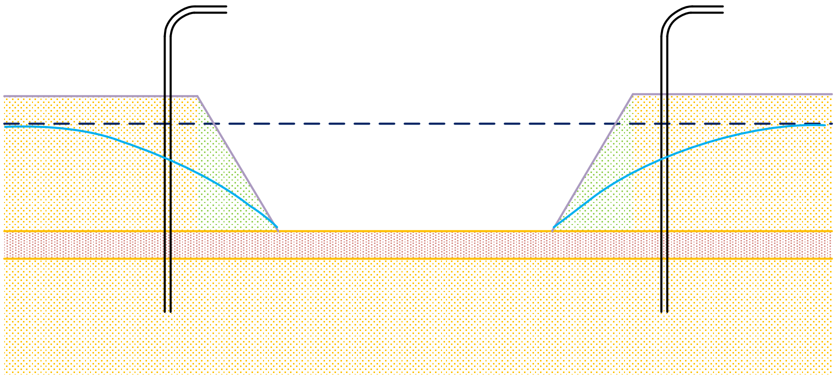
Figure 8.3 shows an excavation pit. A thin layer with poor permeability is sandwiched by two permeable sand layers. The groundwater level in the upper sand layer is lowered by a drainage system in the excavation, the shape of the phreatic surface may be of the form sketched in the figure by the fully drawn curves. Water in the upper layer will flow into the excavation, and may be drained away by pumping at the bottom of the excavation. As the permeability of the clay layer is sufficiently small, the groundwater level in the lower layer will hardly be affected by this drainage system, and very little water will flow through the clay layer. The potential phreatic level in the lower sand layer is indicated in the figure by the dotted line. The situation drawn in the figure is very dangerous. Only a thin clay layer separates the deep sand from the excavation can be a very dangerous. The water pressures in the lower layer are far too high to be in equilibrium with the weight of the clay layer. This layer will certainly collapse, and the excavation will be flooded.

Tezgahi has studied this problem. With his theory, the critical excavation depth can be roughly estimated. According to the balance at the roof of the lower sand layer

$$\gamma_{sand}(H-h)+\gamma_{clay}h_{clay}=\gamma_w\Delta h$$

Here, H is the thickness of the upper sand layer. h is the depth of excavation. h_{clay} is the thickness of clay layer. Δh_{clay} is the pressure head of the confined layer

To prevent this problem, the groundwater level in the lower layer may be lowered artificially, by pumping wells. A disadvantage of this solution is that large amounts of water must be pumped to lower the groundwater level in the lower layer sufficiently, and this entails that over a large region the groundwater is affected. IF the failure occurs, an emergency measure is to dump water into the pit, which will increase the overburden pressure and thus balance the upward water pressure.



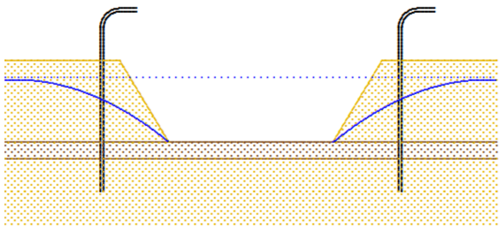


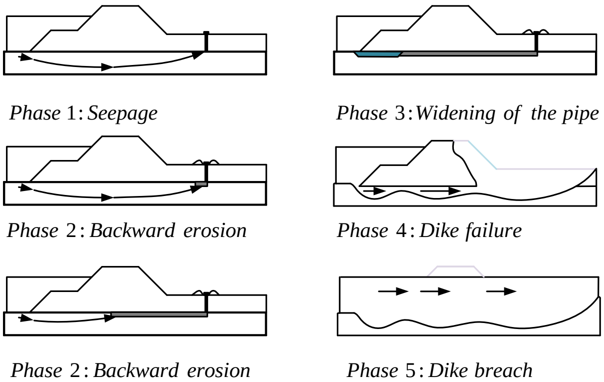
Fig. 2-17 Water flow through a pipe

(是不是还有一种情形，上面是不透水层，下面是透水层)

2) Failure due to internal erosion

Internal erosion is the formation of voids within a soil caused by the removal of material by seepage. It is the second most common cause of failure in levees and one of the leading causes of failures in earth dams. Internal erosion occurs when the hydraulic forces exerted by water seeping through the pores and cracks of the material in the dam and/or foundation are sufficient to detach particles and transport them out of the dam structure. Internal erosion is especially dangerous because there may be no external evidence, or only subtle evidence, that it is taking place. Usually, a sand boil can be found, but the boil might be hidden under water. A dam may breach within a few hours after evidence of internal erosion becomes obvious.

Piping is a related phenomenon and is defined as the progressive development of internal erosion by seepage. Its development process is as fig shows. Starting with the sand boiling, the backward erosion causes particle detachment and transport. A pipe is induced by regressive erosion of particles along flow path. The pipe is widened with continuously internal erosion and further initiate the seepage at a higher rate. If no measures were taken, piping will ultimately leads to breach or collapse of the top structure.



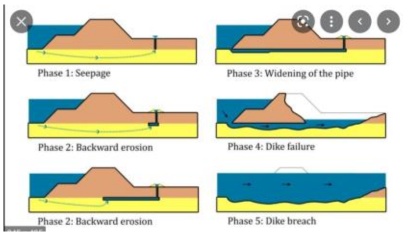


Fig. 2-18 Water flow through a pipe

Example

2-7 如图 2-42 所示，在 9m 厚的黏土沉积层中进行开挖，下面为砂土层。砂层顶面具有 7.5m 高的承压水头。试计算，当开挖深度为 6m 时，基坑中水深 h 至少保持多深才能防止发生流土现象？

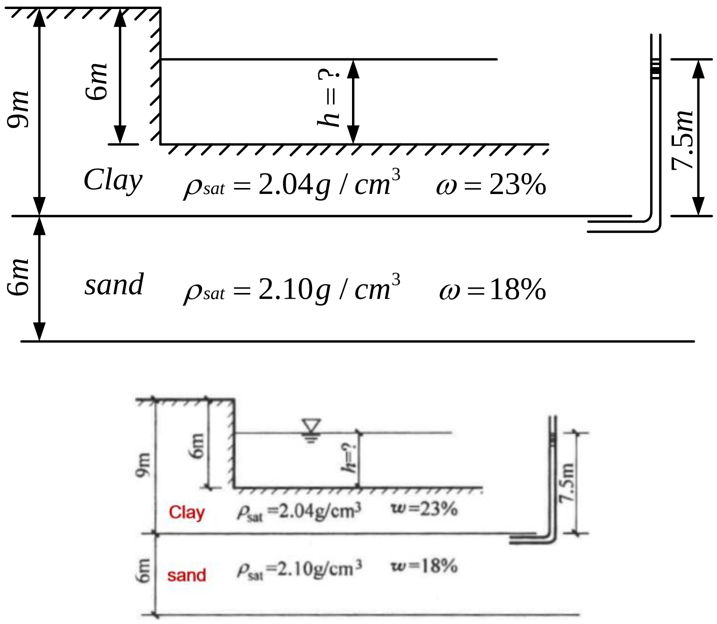


Fig. 2-19 Water flow through a pipe

2.5.3 Seepage control measures

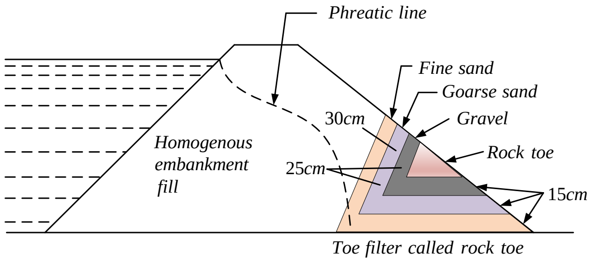
In practical engineering, several measures can be taken to prevent the sand boiling and piping. The most effective way to prevent sand boiling is to decrease the exit hydraulic gradient as follows:

1) Construct the impervious cut off wall at the upstream. The vertical barrier made of concrete or sheet pile can be built through the permeable stratum up to the impermeable layer to completely block the passage of water flow. If the depth is large, partial cut off is provide to increase the length of the flow path and thus decrease the exit hydraulic gradient;
2) Improve drainage conditions at downstream to lower the pressure within the acquirer. When a pervious sand stratum underlying an almost impermeable layer, the artesian pressure may exceeds the overburden pressure. Improve drainage conditions, i.e. relief wells, could mitigate the danger of hydraulic fracturing due to heave of the ground.
3) Install the permeable counterweight fill at the downstream side where water flushes out. Increasing the overburden pressure could lower the risk of the heave of the ground

To prevent piping, measures can be taken in terms of changing hydraulic conditions and installing filters.

1) Modify hydraulic conditions. As piping usually starts with the boiling sand, those measures applicable for boiling sand can be also utilized.

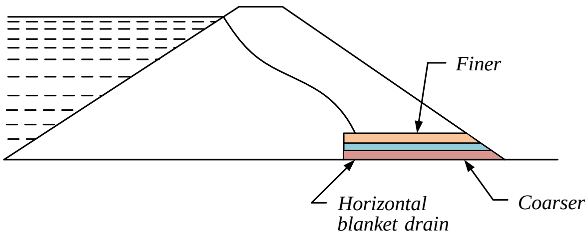
2) Install filters. Providing drainage filters is the best method to prevent piping and heaving. A multi-layered drainage system that consists of graded of both fine and coarse materials is adopted to prevent the loss of fine materials. The following figures show commonly used drainage systems in earth fill dam.



Rock Toe

(*Source*: *Garg*, 2011)

Rock Toe to prevent Seepage in Earth Dam



Horizontal filter

(*Source*: *Garg*, 2011)

Horizontal filters to control Seepage in Dam

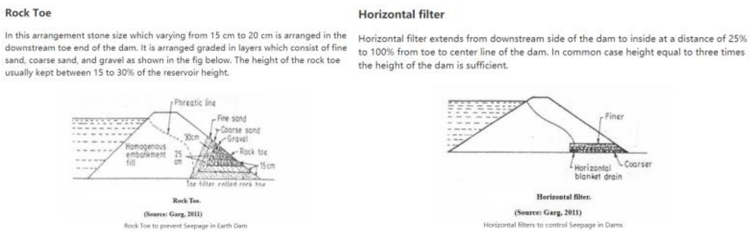


Fig. 2-20 Water flow through a pipe

Exercise

2.1

Example 4.2

A sand sample of 35 cm^2 cross sectional area and 20 cm long was tested in a constant head permeameter. Under a head of 60 cm, the discharge was 120 ml in 6 min. The dry weight of sand used for the test was 1120 g, and $G_s = 2.68$. Determine (a) the hydraulic conductivity in cm/sec, (b) the discharge velocity, and (c) the seepage velocity.

Solution

Use Eq. (4.9), $k = \dfrac{QL}{hAt}$

where $Q = 120$ ml, $t = 6$ min, $A = 35\ cm^2$, $L = 20$ cm, and $h = 60$ cm. Substituting, we have

$$k = \frac{120 \times 20}{60 \times 35 \times 6 \times 60} = 3.174 \times 10^{-3}\ \text{cm/sec}$$

Discharge velocity, $v = ki = 3.174 \times 10^{-3} \times \dfrac{60}{20} = 9.52 \times 10^{-3}$ cm/sec

Seepage velocity v_s

$$\gamma_d = \frac{W_s}{V} = \frac{1120}{35 \times 20} = 1.6\ \text{g/cm}^3$$

From Eq. (3.18a), $\gamma_d = \dfrac{\gamma_w G_s}{1+e}$ or $e = \dfrac{G_s}{\gamma_d} - 1$ since $\gamma_w = 1$ g/cm^3

Substituting, $e = \dfrac{2.68}{1.6} - 1 = 0.675$

$$n = \frac{e}{1+e} = \frac{0.675}{1+0.675} = 0.403$$

Now, $v_s = \dfrac{v}{n} = \dfrac{9.52 \times 10^{-3}}{0.403} = 2.36 \times 10^{-2}$ cm/sec

2.4

2-2 如图 2-38 所示，有 A、B、C 三种土体，装在断面为 10cm×10cm 的方形管中，其渗透系数分别为 $k_A = 1 \times 10^{-2}$ cm/s，$k_B = 3 \times 10^{-3}$ cm/s，$k_C = 5 \times 10^{-4}$ cm/s。问：

(1) 求渗流经过 A 土后的水头降落值 Δh；

(2) 若要保持上下水头差 $h = 35$cm，需要每秒加多少水？

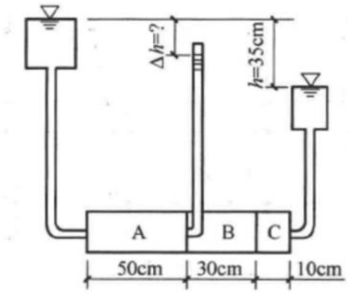


图 2-38 习题 2-2 图

2-5 对图 2-40 所示的基坑，其底面积为 20m×10m，粉质黏土层 $k = 1.5 \times 10^{-6}$ cm/s，如果忽略基坑周边水的渗流，假定基坑底部土体发生一维渗流：(1) 如果基坑内的水深保持 2m，求土层中 A、B、C 三点的测压管水头和渗透力。

(2) 试求当保持基坑中水深为 1m 时，所需要的排水量 Q。

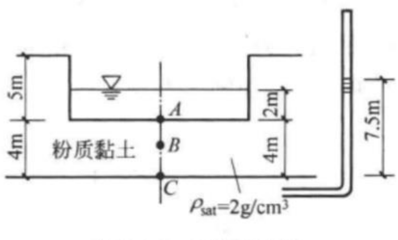


图 2-40 习题 2-5 图

2.3

2-3 一种黏性土的比重 $G_s = 2.70$，孔隙比 $e = 0.58$，试求该土发生流土的临界水力坡降。

2.4

有一粘土层位于两砂层之间，其中砂层的湿重度 γ 为 17.6 kN/m^3，饱和重度 γ_{sat} 为 19.6 kN/m^3，粘土层的饱和重度 γ_{sat} 为 20.6 kN/m^3，土层的厚度如图 3-29 所示。地下水位保持在地面以下 1.5 m 处，若下层砂中有承压水，其测压管水位高出地面3 m，试计算：