Flow of Water Through Soils

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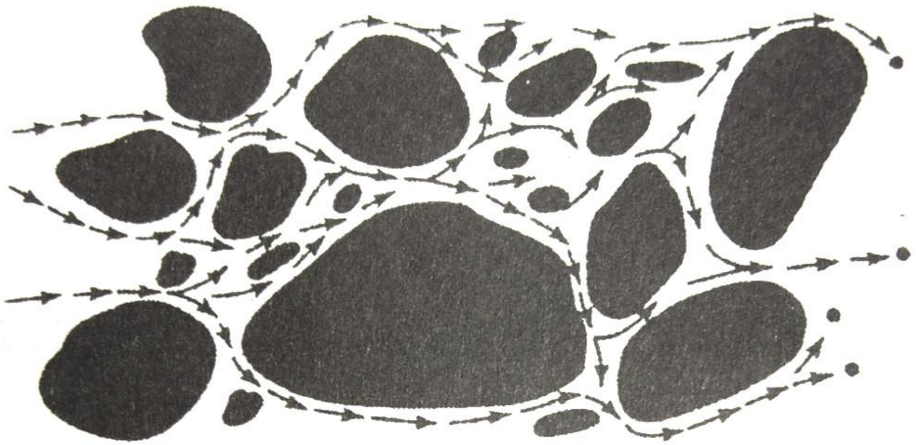
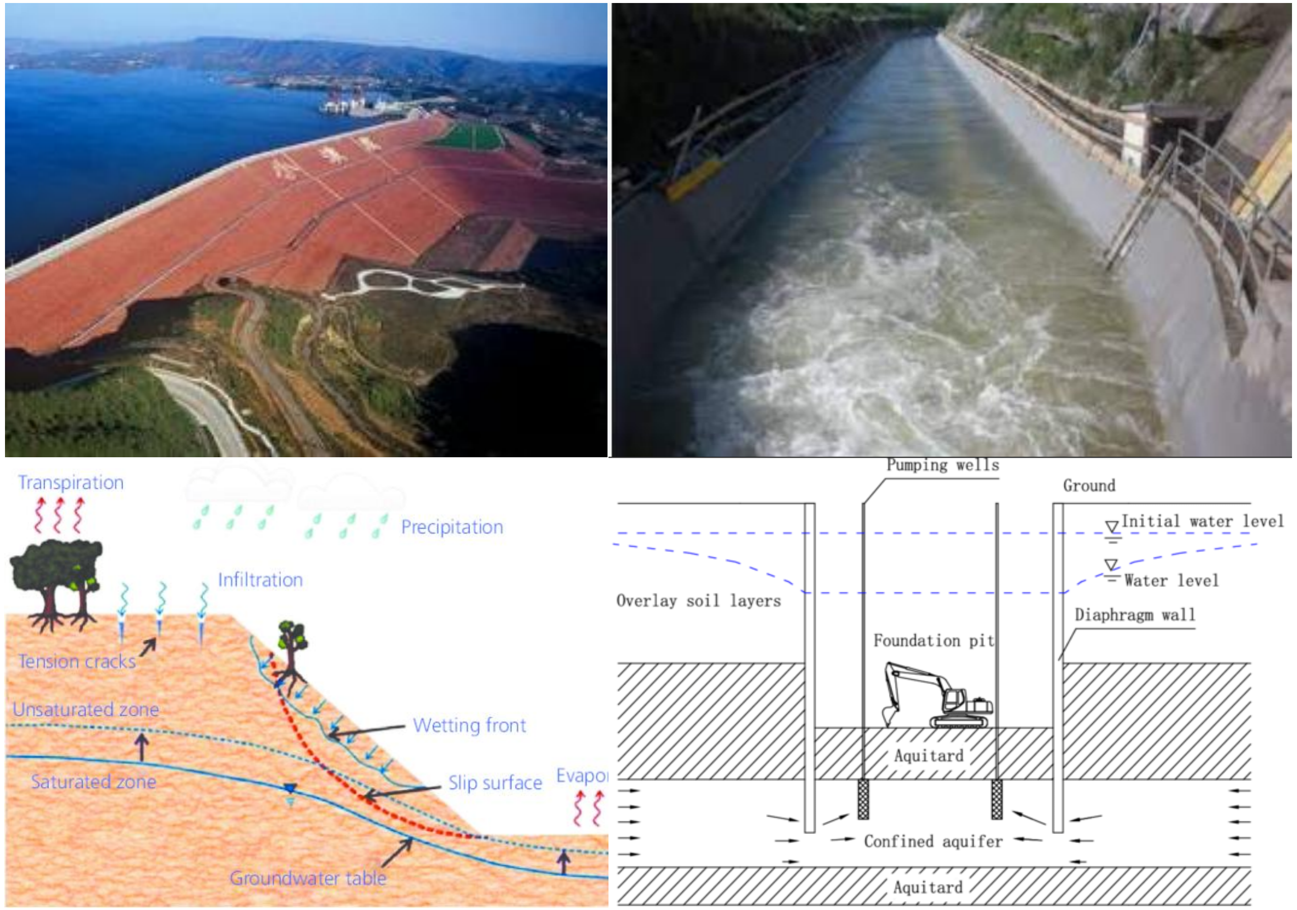
Outline

- General introduction
- Permeability of soils
- 2-D seepage and flow net
- Seepage force and related problems

General introduction

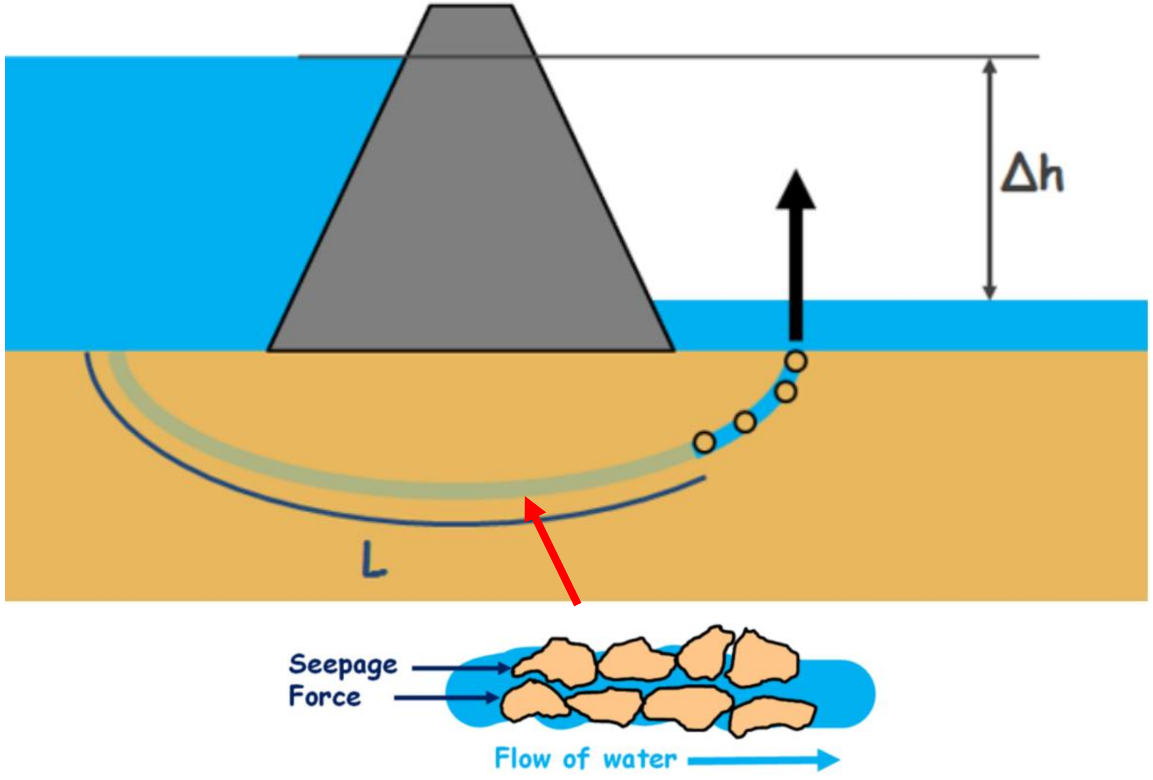


Schematic diagram of water flow through soils



Problems related to water flow in soils

General introduction



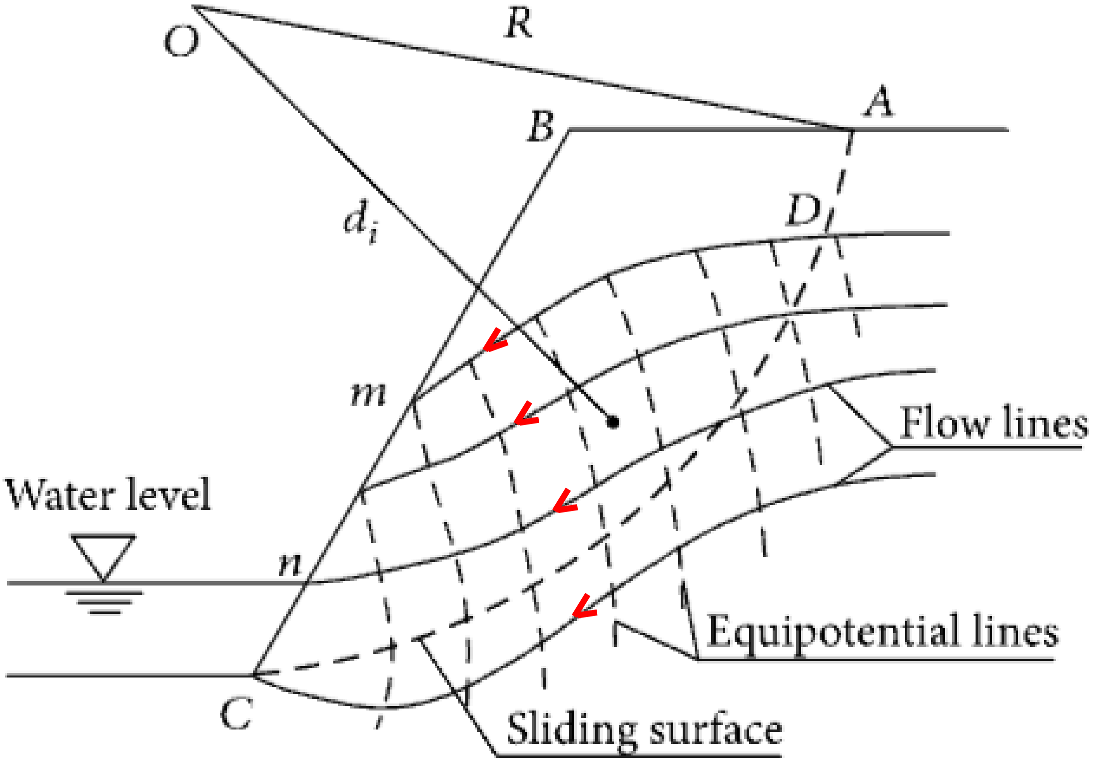
Schematic diagram for piping

The **seepage force of flowing water has the capability to displace the soil particles on its way**. Generally, the higher of the velocity of water, the stronger of the seepage force.

With the particles being washed away, **failure may occur due to progressively internal erosion**. This phenomenon is called **piping**.

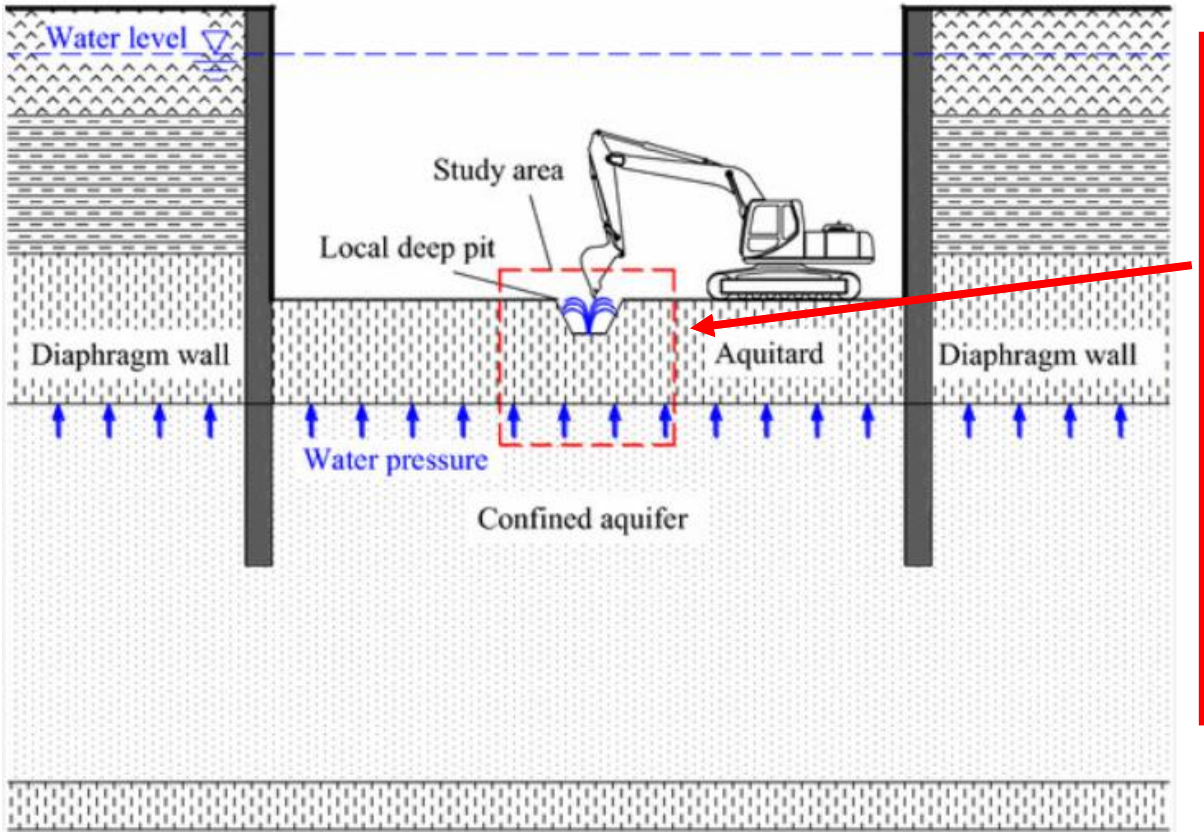
General introduction

Three gorges reservoir

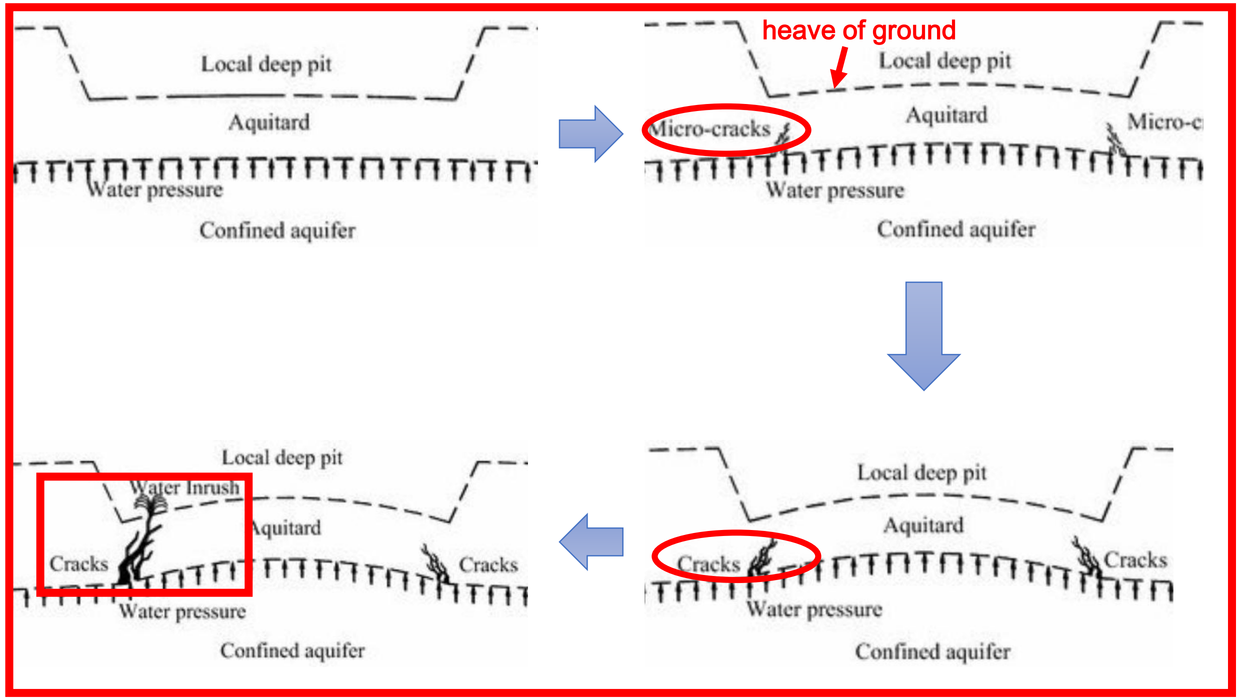


Schematic diagram of slope stability analysis

General introduction



Conceptual model for heave of ground or water inrush



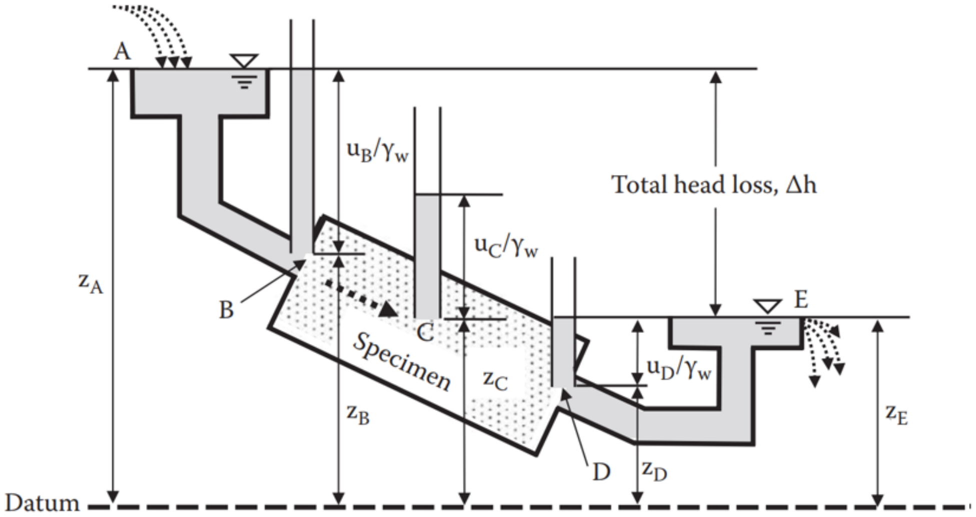
Progressive failure due to excavation

Permeability of soils

Total water head

$$h = h_z + h_p + h_v = z + \frac{u}{\gamma_w} + \frac{v^2}{2g} \xrightarrow{\text{Low velocity}} h \approx z + \frac{u}{\gamma_w}$$

$$\frac{v^2}{2g} \approx 0$$



Water flow through a pipe

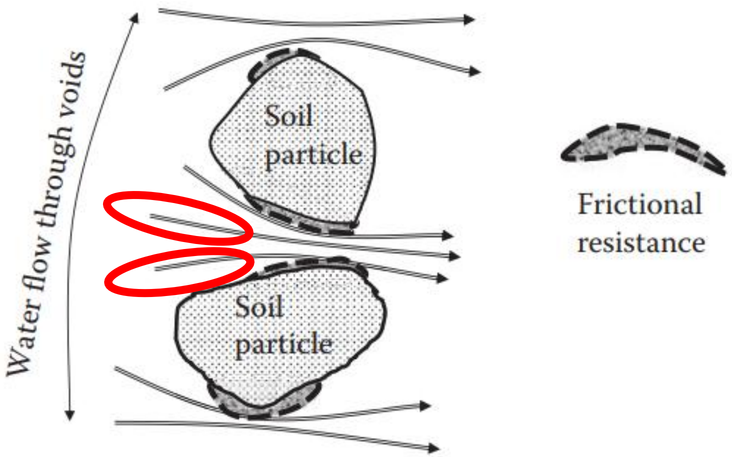
z: elevation head

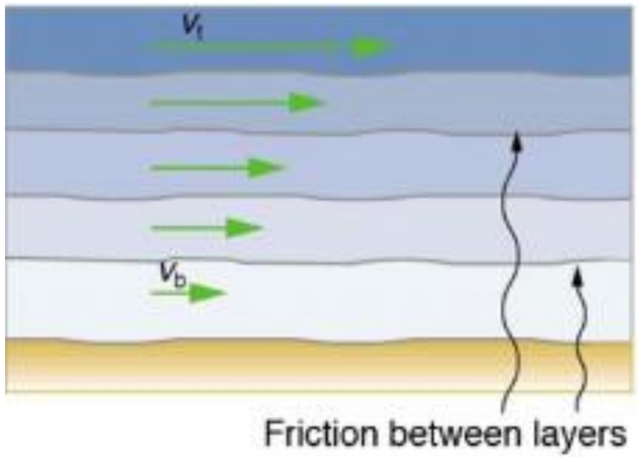
$h_p = \frac{u}{\gamma_w}$: pressure head

$h_v = \frac{v^2}{2g}$: velocity head

Permeability of soils

Head loss occur when the flow of water through seepage pipes of soils. This is due to **the friction between soil particles and water**, as well as the friction (viscosity) inside the fluid.





Schematic diagram of head loess

Permeability of soils

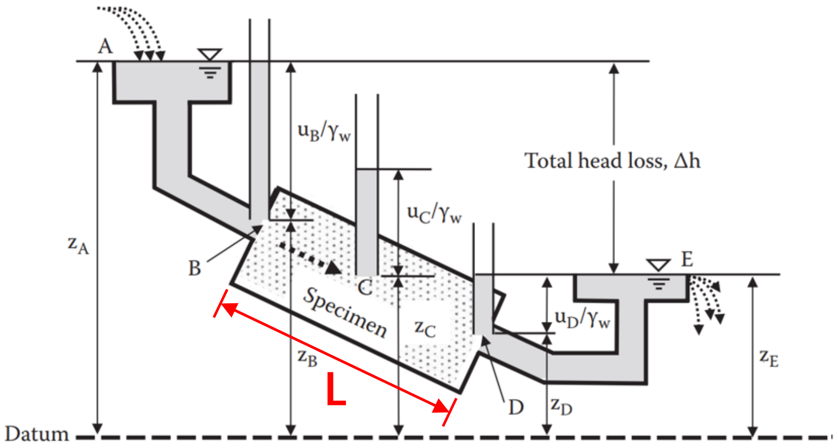
Hydraulic gradient: total head loess over the length of flow path. It is a dimensionless variable.

$$i = \frac{\Delta h}{L} = \frac{h_B - h_D}{L}$$

h_B: total head at point A

h_D: total head at point B

L: length of flow path



Water flow through a pipe

Permeability of soils

Laminar Flow V.S. Turbulent Flow

Laminar flow occurs when the **water flows at low velocity in parallel layers**, **with no disruption between the layers**. The motion of the particles of the fluid is very orderly and there are no cross-currents perpendicular to the direction of flow. **While for turbulent flow,** the water no longer travels in layers and mixing across the tube.

Schematic diagram of laminar and turbulent flow

Permeability of soils

Reynolds number

For a given dimeter of a straight tube, **the flow pattern of the water** in saturated soil at a given temperature can be determined according to Reynolds number

$$R_e = \frac{vL}{u} = \frac{\rho vL}{\mu}$$

ρ: density of the fluid (kg/m^3)

v: velocity of water flow (m/s)

L: Characteristic linear dimension (m)

μ: dynamic viscosity (N·s/m^2)

u: kinematic viscosity (m^2/s)

Flow type classification

Re	Classification
<2000	laminar
2000~4000	Unstable
>4000	Turbulent

Permeability of soils

Darcy's law

For laminar flow in saturated soil, the Darcy velocity (v) of flow is proportional to the hydraulic gradient:

$$v = ki$$

k is the hydraulic conductivity (m/s).

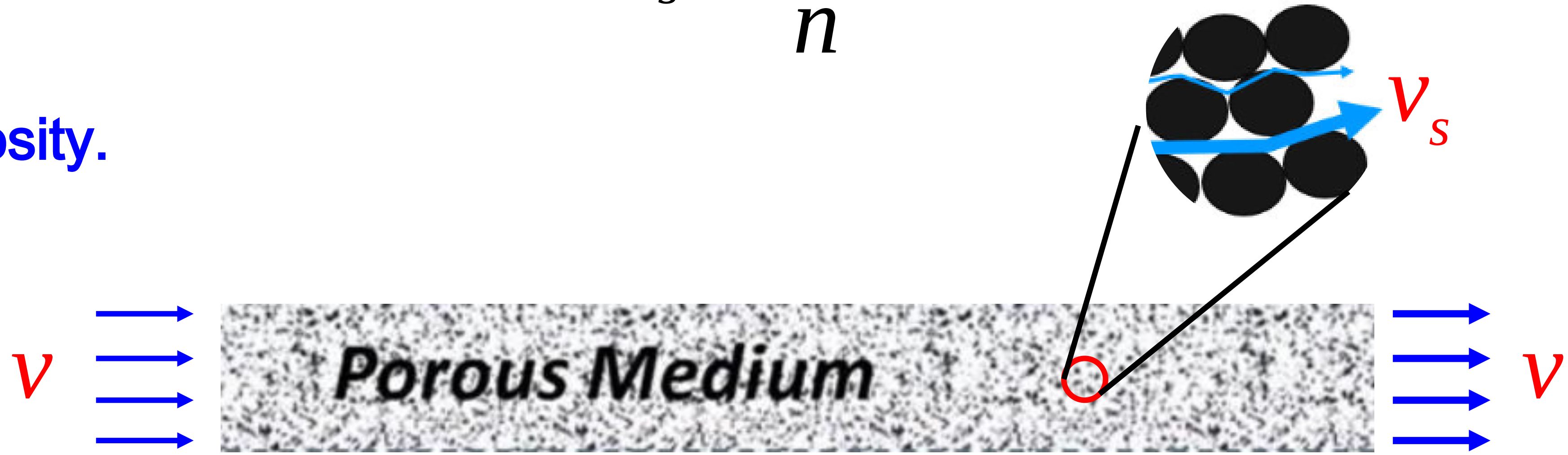
Darcy velocity not the velocity of water flow through seepage pipes of soils, but is rather an average velocity in the flow direction through the porous media!!!

Permeability of soils

Seepage velocity (v_s): the velocity of water through seepage pipe of soils

$$v_s = \frac{v}{n}$$

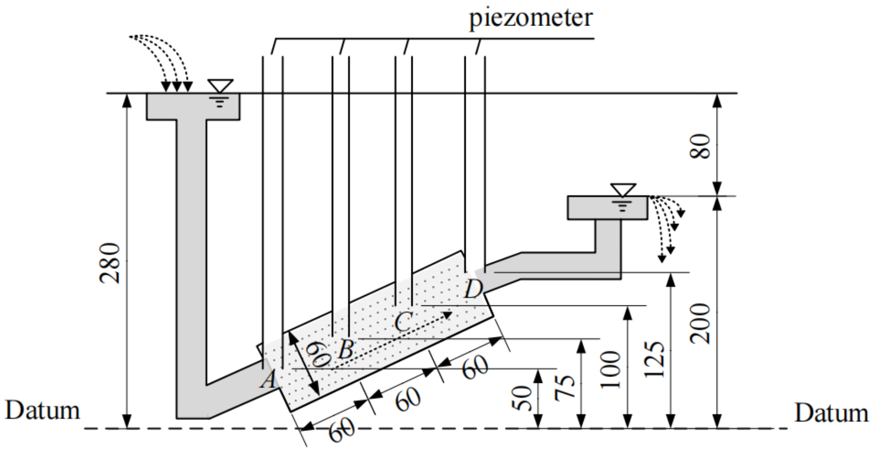
n is porosity.



Schematic diagram for 1D flow in porous medium

Exercises

The following figure shows water flow though the soil specimen contained in a cylindrical tube. The specimen's hydraulic conductivity k is 3.4×10^{-4} cm/s . Determine: 1) pressure heads at point A, B, C, and D in each piezometer; 2) the volumetric flow rate through the soil specimen.



Exercises

The instrument for seepage test is shown in Fig. 1. Determine the elevation head, pressure head, total head of point B, C, D and F, as well as the loss of head from point B to C, point B to D and point B to F.

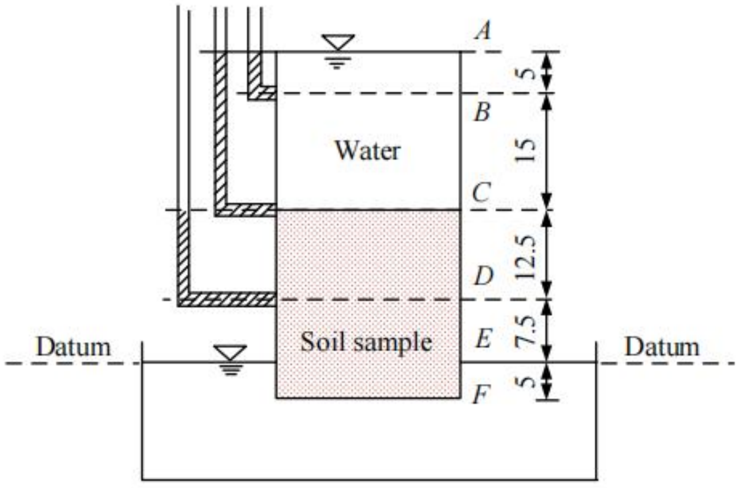


Fig. 1 Schematic diagram of the instrument (unit of length: cm)

Permeability of soils

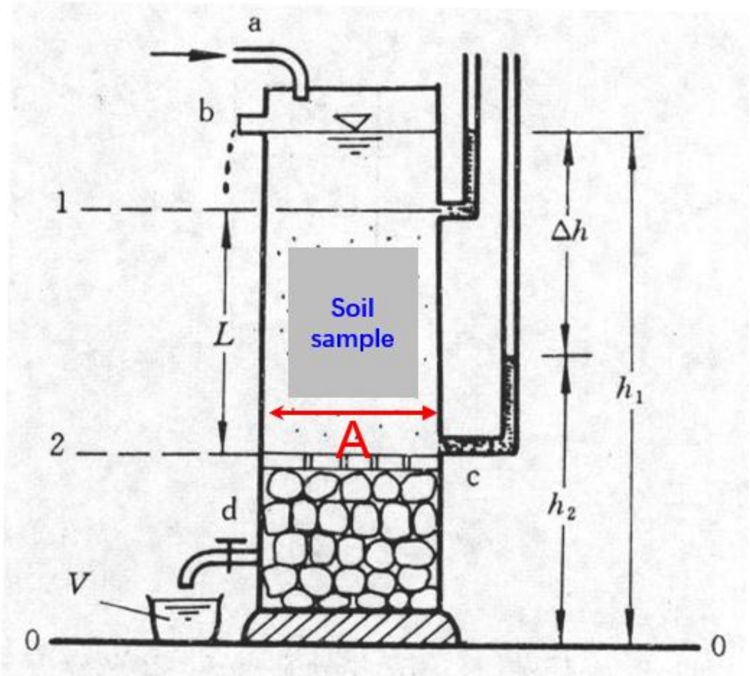
Hydraulic conductivity determination—constant head test

H. Darcy

$$\frac{V}{t} \propto A \times \frac{\Delta h}{L}$$

$$Q = \frac{V}{t} = kA\frac{\Delta h}{L} = kAi$$

$$v = \frac{Q}{A} = ki$$



Setup for constant head test

Permeability of soils

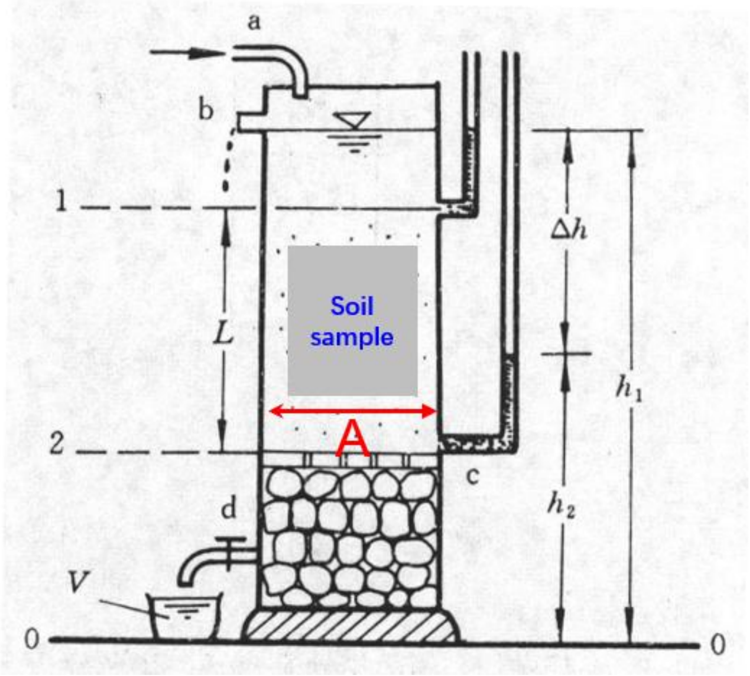
The constant head test is suitable for coarse grain soil (gravel and sand).

Variables to be measured: V, t

Geometry parameters: $A, \Delta h, L$

$$k = \frac{VL}{A\Delta h t}$$

Unit consistency is necessary



Setup for constant head test

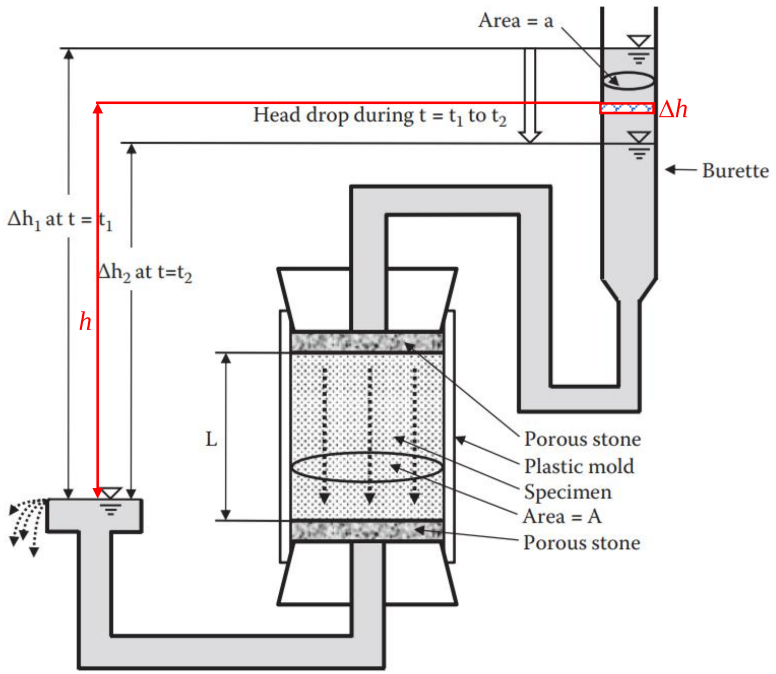
Permeability of soils

Hydraulic conductivity determination—falling head test (fine grain soils)

$$\left.\begin{aligned} dQ = vA\Delta t = k\frac{h}{L}Adt \\ dQ = -adh \end{aligned}\right\} \quad -adh = k\frac{h}{L}Adt$$

Integration over $[t_1, t_2]$ and $[\Delta h_1, \Delta h_2]$

$$\int_{\Delta h_1}^{\Delta h_2} \frac{a}{h} dh = \int_{t_2}^{t_2} -k\frac{A}{L} dt \rightarrow k = \frac{aL}{A} \frac{\ln\dfrac{\Delta h_1}{\Delta h_2}}{(t_2 - t_1)}$$



Setup for falling (variable) head test

Permeability of soils

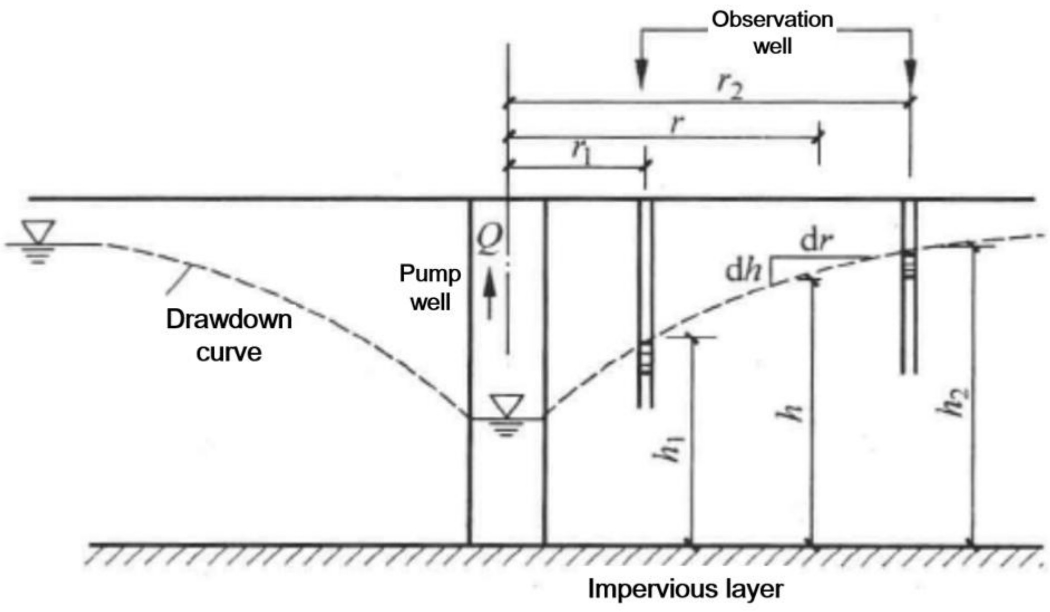
Hydraulic conductivity determination *in-situ* — pump test

Continuity condition:

$$Q=2\pi hr\cdot k\cdot\frac{dh}{dr}\rightarrow Q\frac{dr}{r}=2\pi hk\cdot dh$$

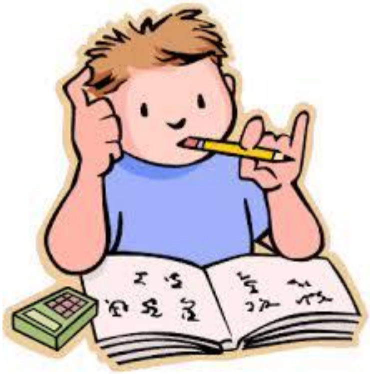
Integration over [r_1, r_2] and [h_1, h_2]

$$Q\int_{r_1}^{r_2}\frac{\mathrm{d}r}{r}=2\pi k\int_{h_1}^{h_2}h\mathrm{d}h\rightarrow k=\frac{Q}{\pi}\frac{\ln\frac{r_2}{r_1}}{h_2^2-h_1^2}$$



Schematic diagram for *in-situ* pumping

Exercises



A sand sample of $35\ \mathrm{cm}^2$ cross sectional area and $20\ \mathrm{cm}$ long was used in a constant head hydraulic conductivity test. Under a head of loss $60\ \mathrm{cm}$, $120\ \mathrm{ml}$ of water was collected in $6\ \mathrm{min}$. The dry weight of sand used for the test was $1120\ \mathrm{g}$, and $G_s = 2.68$. Determine: 1) the hydraulic conductivity (in unit $\mathrm{cm/s}$); 2) the Darcy velocity, and 3) the seepage velocity.

Exercises

A falling head test was conducted and the following data were obtained. $L = 15$ cm, sample diameter $D = 7.2$ cm, Δh_1 (at $t = 0$) = 40.0 cm, Δh_2 (at $t = 10$ min) = 22.9 cm, burette diameter $d = 1.2$ cm. Compute the hydraulic conductivity.

Permeability of soils

Factors influencing the hydraulic conductivity

Factors related to soils:

1) Particle size, shape and gradation
2) Porosity
3) Chemical composition
4) Structure
5) Degree of saturation

Factors related to fluid:

1) Temperature

Permeability of soils

Particle size, shape and gradation

Soils dominated by large particles tend to have relatively large pore spaces and thus large values of hydraulic conductivity. According to Hazen's formula, the hydraulic conductivity is approximately proportional to the square of the typical grain size.

$$k = cd_{10}^2$$

Particle size

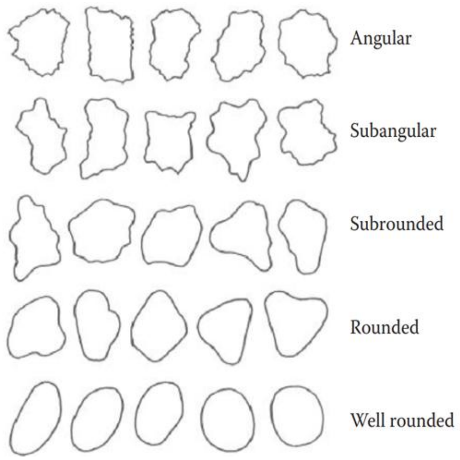
Type of soil	k (m/s)
gravel	$10^{-3} - 10^{-1}$
sand	$10^{-6} - 10^{-3}$
silt	$10^{-8} - 10^{-6}$
clay	$10^{-10} - 10^{-8}$

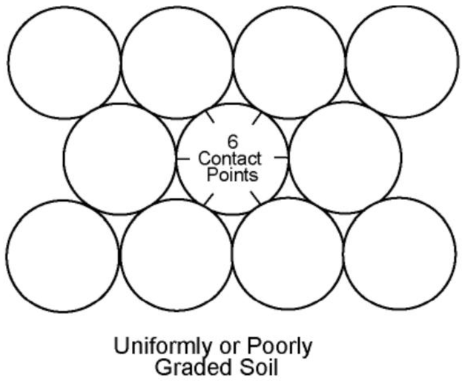
Permeability of soils

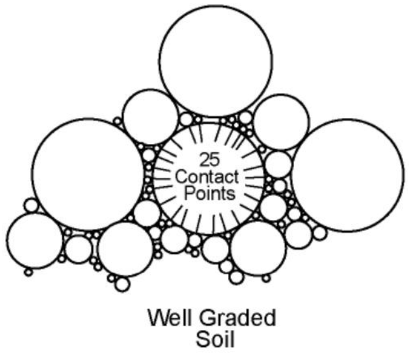
Particle size, shape and gradation

Rounded Particles are more permeable than angular shaped. It is due to specific surface area of angular particles is more compared to rounded particles.

For a poorly graded soil, the particles either have the similar size or certain range of particle sizes are absent. As there are less smaller particles to fill the voids formed by larger particles, it will therefore have large values of hydraulic conductivity.







Permeability of soils

Porosity

According to the Kozeny-Carman formula and the relation between intrinsic permeability (κ) and hydraulic conductivity (k)

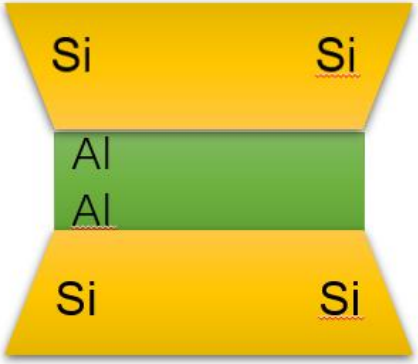
$$\left.\begin{aligned} \kappa &= cd^2 \frac{n^3}{(1-n)^3} \\ k &= \frac{\kappa \gamma_w}{\mu} \end{aligned}\right\} \quad k = \frac{cd^2 \gamma_w}{\mu} \frac{n^3}{(1-n)^3}$$

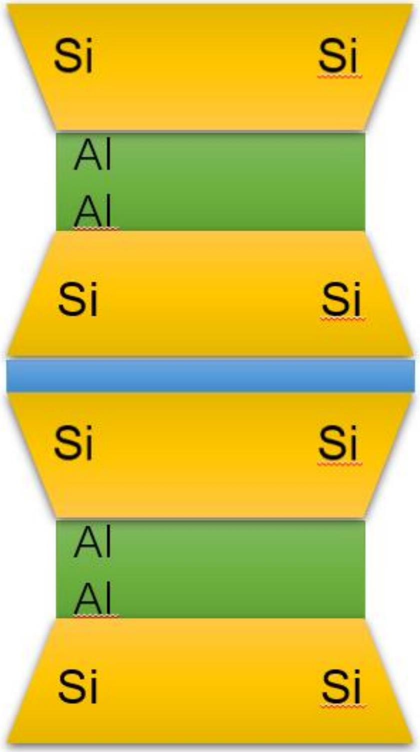
d is a measure for the grain size, and ***c*** is a coefficient related to the shape of the particles. μ is the dynamic viscosity.

Permeability of soils

Chemical composition

The effects arise when soil consist of minerals which have strong interaction with water, i.e. swelling of clay, particularly montmorillonite.



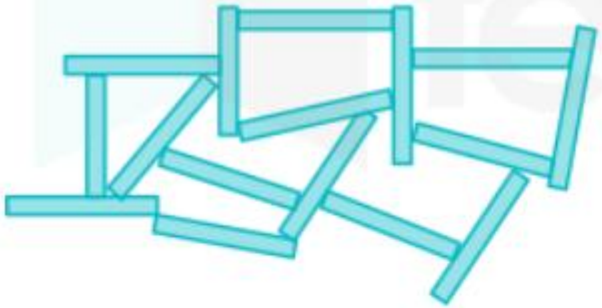


Schematic diagram of montmorillonite

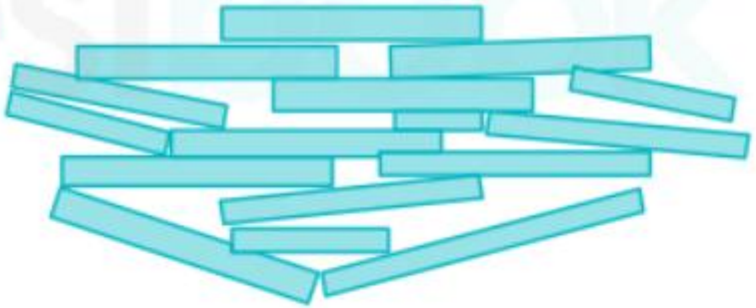
Permeability of soils

Structure

For clays with same void ratio, those with flocculated structure normally have larger hydraulic conductivity than the clays with dispersed structure.



Flocculated structure

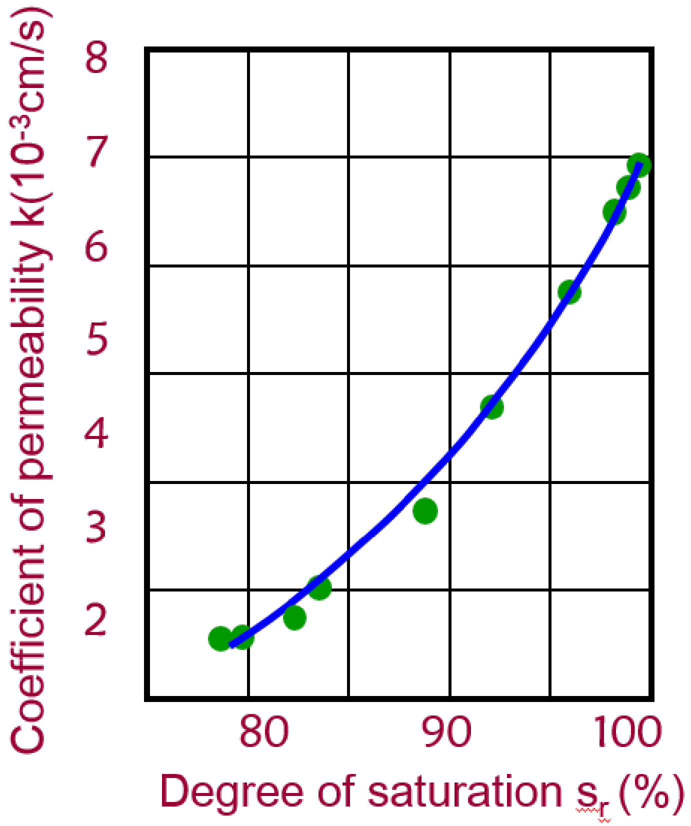


Dispersed structure

Permeability of soils

Degree of saturation

In a partially saturated soil, the hydraulic conductivity is no longer a constant. In general, it is a function of degree of saturation. A decrease in the degree of saturation leads to a further decrease in the pore volume occupied by the water. The increase of void space results in an increase in entrapped air, which will block the flow path thereby reduces the hydraulic conductivity

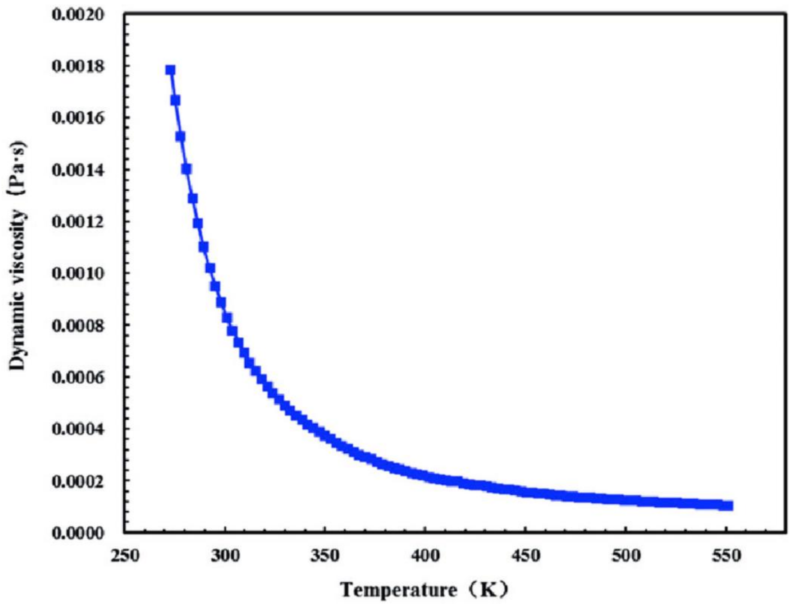


Relationship between k and Sr

Permeability of soils

$$k = \frac{cd^2\gamma_w}{\mu}\frac{n^3}{(1-n)^3}$$

Dynamic Viscosity of water



Relationship between T and u

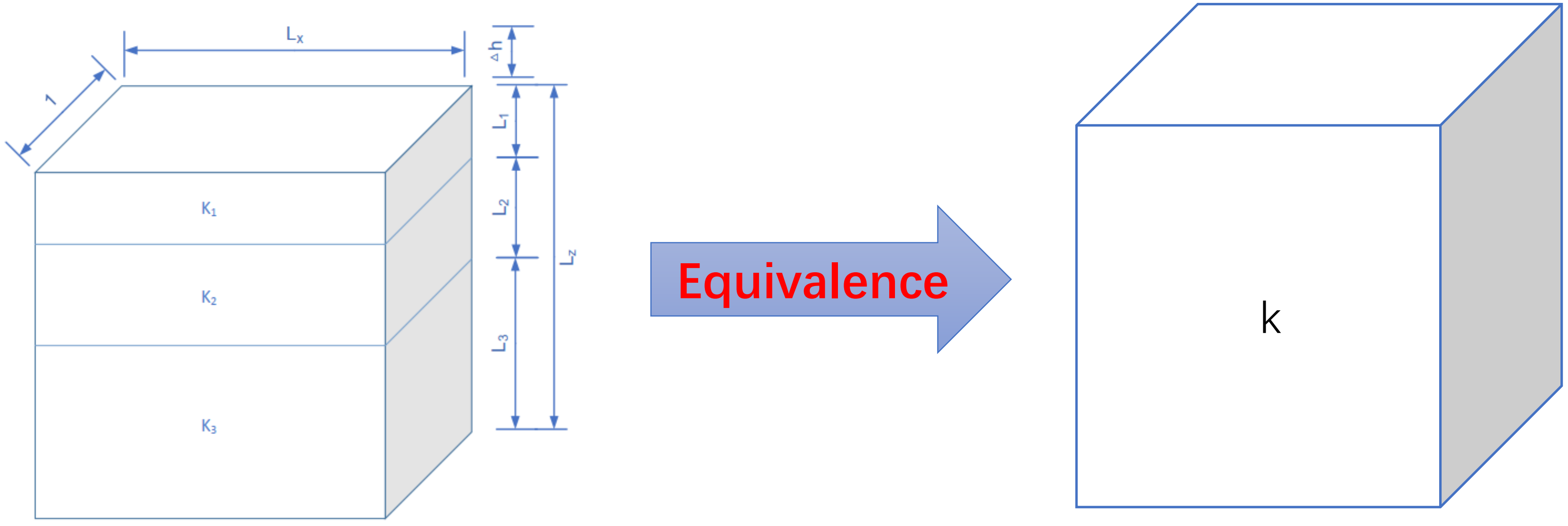
The higher the temperature, the lower the dynamic viscosity, and then the higher value of hydraulic conductivity.

The dynamic viscosity of water at 20°C has a value of around 1.0 mPa·s. The hydraulic conductivity measured under other temperature needs **to be standardized**

Permeability of soils

Equivalent hydraulic conductivity

For quick evaluation of the hydraulic conductivity of the stratified ground, engineers prefer to obtain the hydraulic conductivity in an average sense **either in horizontal or vertical direction**.

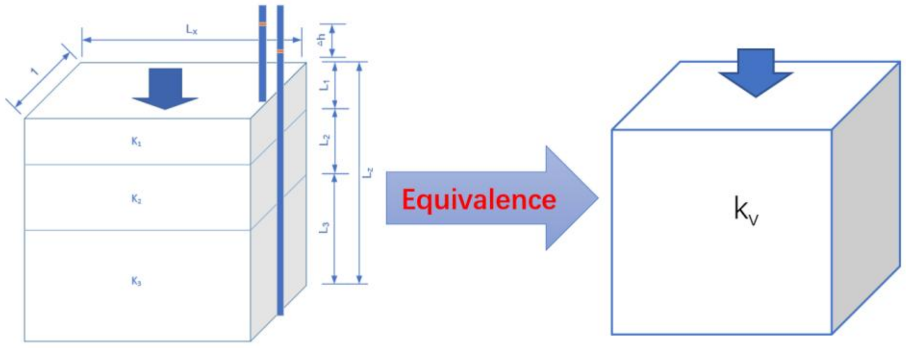


Stratified ground　　Homogeneous ground

Permeability of soils

Equivalent vertical hydraulic conductivity (k_v)

$$\left.\begin{aligned} Q = k_1 \frac{\Delta h_1}{L_1} L_x = k_2 \frac{\Delta h_2}{L_2} L_x = k_3 \frac{\Delta h_3}{L_3} L_x \\ Q = k_v \frac{\Delta h}{L} L_x \\ \Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3 \end{aligned}\right\} \quad k_v = \frac{L}{\dfrac{L_1}{k_1} + \dfrac{L_2}{k_2} + \dfrac{L_3}{k_3}} \xrightarrow{\text{Generalization}} k_v = \frac{L}{\sum_i \dfrac{L_i}{k_i}}$$



Stratified ground　　　Homogeneous ground

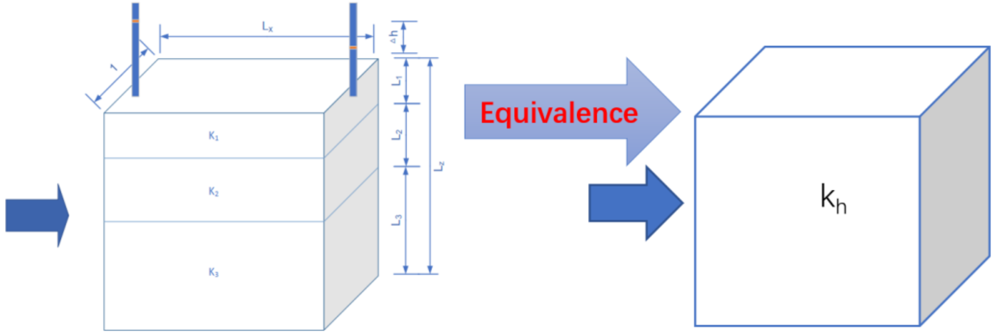
Permeability of soils

Equivalent horizontal hydraulic conductivity

$$\left.\begin{aligned} k_1 \frac{\Delta h}{L_x} L_1 = Q_1,\ k_2 \frac{\Delta h}{L_x} L_2 = Q_2,\ k_3 \frac{\Delta h}{L_x} L_3 = Q_3 \\ Q = k_h \frac{\Delta h}{L_x} L \\ Q = Q_1 + Q_2 + Q_3 \end{aligned}\right\} \quad k_h = \frac{L_1 k_1 + L_2 k_2 + L_3 k_3}{L}$$

Generalization

$$k_h = \frac{\sum_i L_i k_i}{L}$$



Stratified ground

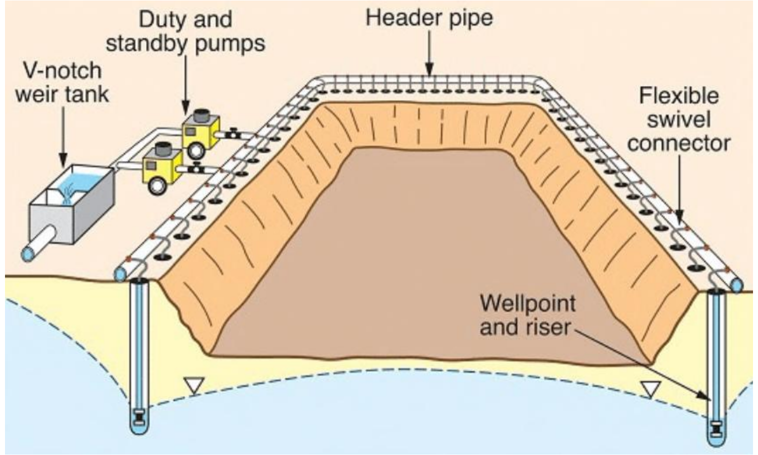
Homogeneous ground

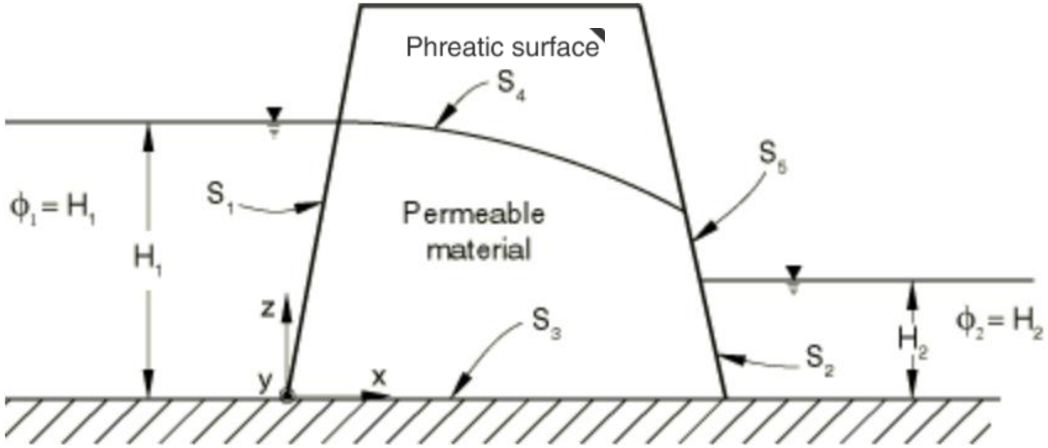
Exercises

Certain ground contains three distinct horizontal layers of equal thickness. The hydraulic conductivity of the upper and lower layers is 10^{-3} cm/s and that of the middle is 10^{-2} cm/s. What are the equivalent values of the horizontal and vertical hydraulic conductivities of the three layers, and what is their ratio?

2D seepage and flow net

The practical engineering problems are **two- or three-dimensional seepage problems** with complex boundary conditions. To solve such kind of problems, the multi-dimensional governing equations with corresponding boundary conditions need to be established.





2D seepage and flow net

General Darcy's law in 2D

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = -\begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \begin{bmatrix} \frac{\partial h}{\partial x} \\ \frac{\partial h}{\partial y} \end{bmatrix}$$

The total head h is a function of x, y, i.e. h(x, y). The Darcy velocity is a vector with 2 components: v_x, v_y. The hydraulic conductivity is a 2X2 symmetrical tensor, with 3 independent variables: k_{xx}, k_{xy}, k_{yy}

2D seepage and flow net

- The seepage direction coincide with the reference frame, then $k_{xy}=k_{yx}=0$

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = -\begin{bmatrix} k_{xx} & 0 \\ 0 & k_{yy} \end{bmatrix} \begin{bmatrix} \dfrac{\partial h}{\partial x} \\ \dfrac{\partial h}{\partial y} \end{bmatrix} \rightarrow \begin{array}{l} v_x = -k_{xx} \dfrac{\partial h}{\partial x} \\ v_y = -k_{yy} \dfrac{\partial h}{\partial y} \end{array}$$

- The above condition is satisfied and the material is isotropic, then $k_{xy}=k_{yx}=0$, $k_{xy}=k_{yx}=k$,

$$\begin{bmatrix} v_x \\ v_y \end{bmatrix} = -\begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} \begin{bmatrix} \dfrac{\partial h}{\partial x} \\ \dfrac{\partial h}{\partial y} \end{bmatrix} \rightarrow \begin{array}{l} v_x = -k \dfrac{\partial h}{\partial x} \\ v_y = -k \dfrac{\partial h}{\partial y} \end{array}$$

2D seepage and flow net

Governing equation of 2D seepage problem

Assumptions:

1) The soil has an isotropic property
2) Only the steady flow is considered,
3) The fluid is incompressible

2D seepage and flow net

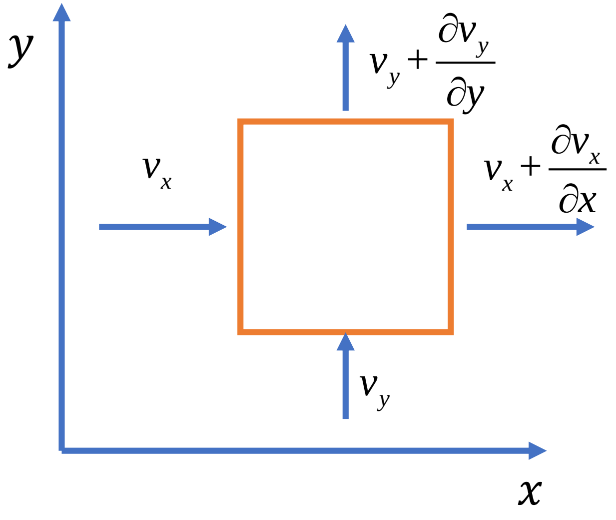
Governing equation of 2D seepage problem

Continuity condition:

$$v_x + v_y = v_x + \frac{\partial v_x}{\partial x} + v_y + \frac{\partial v_y}{\partial y} \rightarrow \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

$$v_x = -k\frac{\partial h}{\partial x}, \quad v_y = -k\frac{\partial h}{\partial y}$$

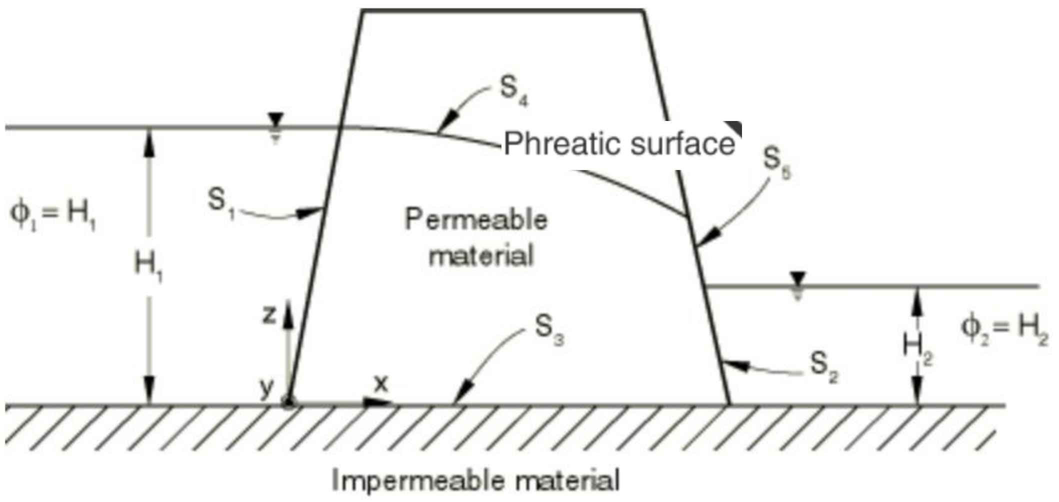
Laplacian Equation $\longrightarrow$ $$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$



Arbitrary infinitesimal element for derivation

2D seepage and flow net

Boundary conditions



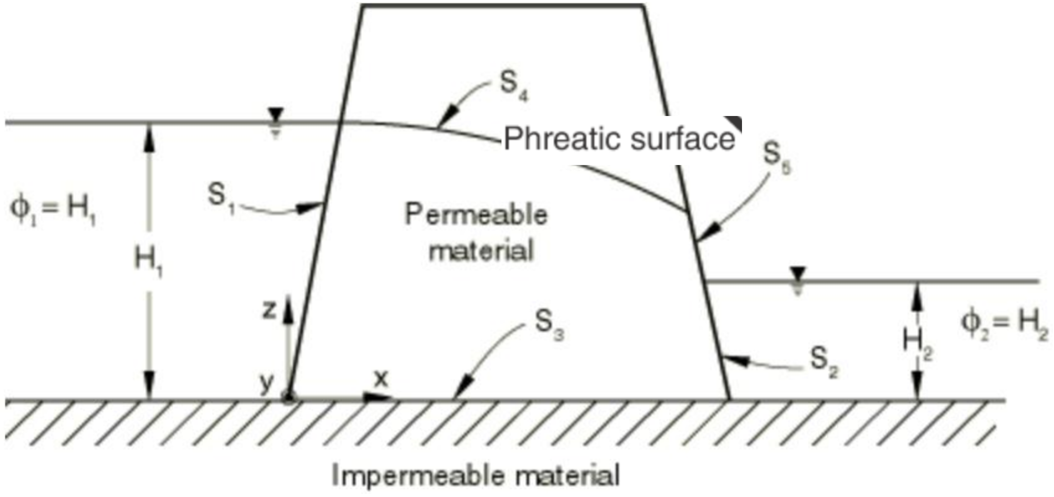
Various boundary conditions

Four types of boundary conditions:

1) Along certain boundary, the total head is known. This is a typical **Dirichlet boundary condition** (S_1 and S_2).
2) Along certain boundary, **the distribution of velocity is known**. This is a typical **Neumann boundary condition**, which specifies the values that the derivative of a total head is going to take. **The typical Neumann boundary is the impermeable boundary**, where the flux is zero (S_3).

2D seepage and flow net

Boundary conditions



Various boundary conditions

3) The phreatic surface. Along this boundary, two conditions need to be satisfied: 1) the pressure head is zero and accordingly, the total head equals the evaluation head; 2) the velocity in normal direction is zero. There is only the tangential flow (S_4).

4) Seepage face. A special boundary condition is needed if the phreatic surface reaches an open, freely draining surface. Along this boundary, the pore fluid can drain freely down the face of the dam, and pressure head is zero at all points on this surface below its intersection with the phreatic surface (S_5).

2D seepage and flow net

Solutions to Laplacian equation

Four types of methods:

1) Analytical method
2) Numerical method
3) Scaled model
4) Graphical method

The analytical method is rigorous and precise. However, it is not universally applicable in all cases because of the complexity of the problem involved. The mathematics involved even in some elementary cases is beyond the comprehension of many design engineers. Although this approach is sometimes useful in the checking of other methods, it is largely of academic interest.

2D seepage and flow net

Solutions to Laplacian equation

Four types of methods:

1) Analytical method

2) **Numerical method**

3) Scaled model

4) Graphical method

With the revolution of computational capacity, more and more engineering scale problems with complex boundary conditions can be solved. Moreover, variety of interaction, e.g. interaction between solid and liquid, can be taken into account. **Most of the numerical methods** used in geotechnical engineering are the finite difference method (FDM), finite element method (FEM), boundary element method (BEM). **Some of these method can be even coupled** to solve certain special problem.

2D seepage and flow net

Solutions to Laplacian equation

Four types of methods:

1) Analytical method

2) Numerical method

3) Scaled model

4) Graphical method

Scaled models can be constructed to depict flow of water below concrete dams or through earth dams. These models are very useful to demonstrate the fundamentals of fluid flow, but their use in other respects is limited because of the large amount of time and effort required to construct such models.

2D seepage and flow net

Solutions to Laplacian equation

Four types of methods:

1) Analytical method

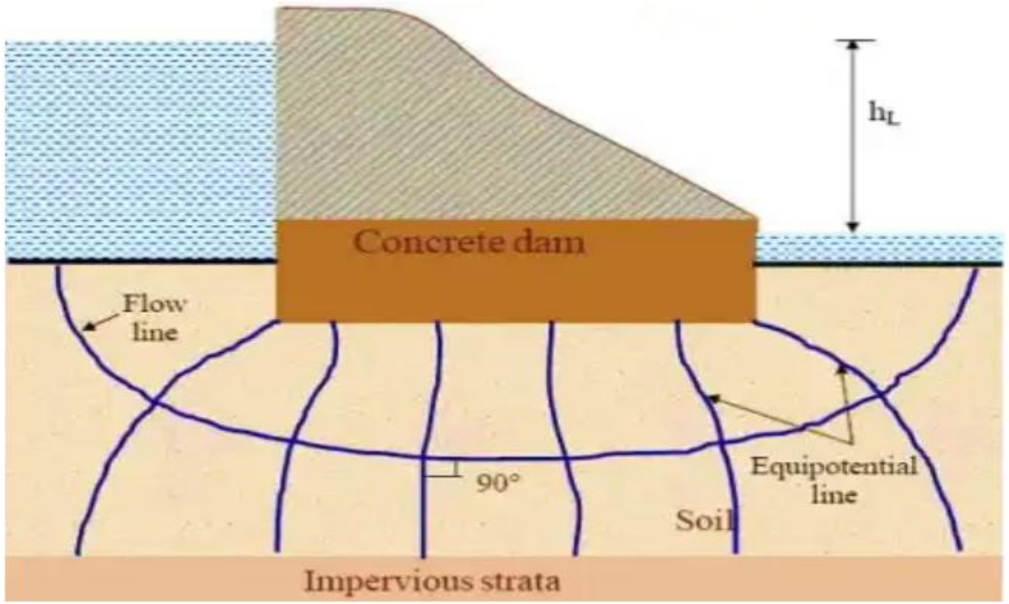
2) Numerical method

3) Scaled model

4) **Graphical method**

The graphical method developed by Forchheimer (1930) has been found to be very useful in solving complicated flow problems. A. Casagrande (1937) improved this method. **The main drawback** of this method is that a good deal of practice and aptitude are essential to produce a satisfactory flow net. In spite of these drawbacks, the graphical method is quite popular among engineers.

2D seepage and flow net

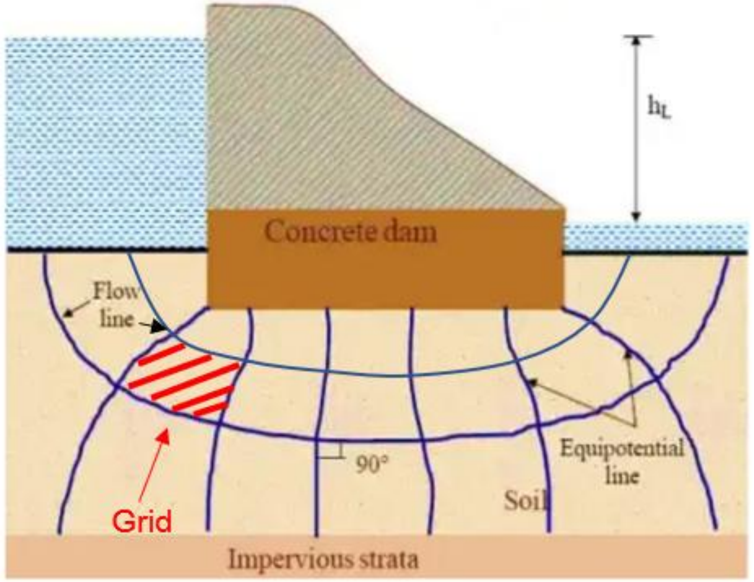


2D flow net of a concreter dam

A flow net is a **graphical representation of two-dimensional steady-state** groundwater flow through aquifers

The flow net is formed by the combination of **curved flow lines** and **curved equipotential lines**, which are everywhere **perpendicular to** each other. Flow lines represent the path of flow along which the water will seep through the soil. Equipotential lines are formed by connecting the points of equal total water head.

2D seepage and flow net



2D flow net of a concreter dam

Features of flow net:

1) The flow line and equipotential line are **mutually orthogonal**

2) Two flow lines or two equipotential lines **can never merge**

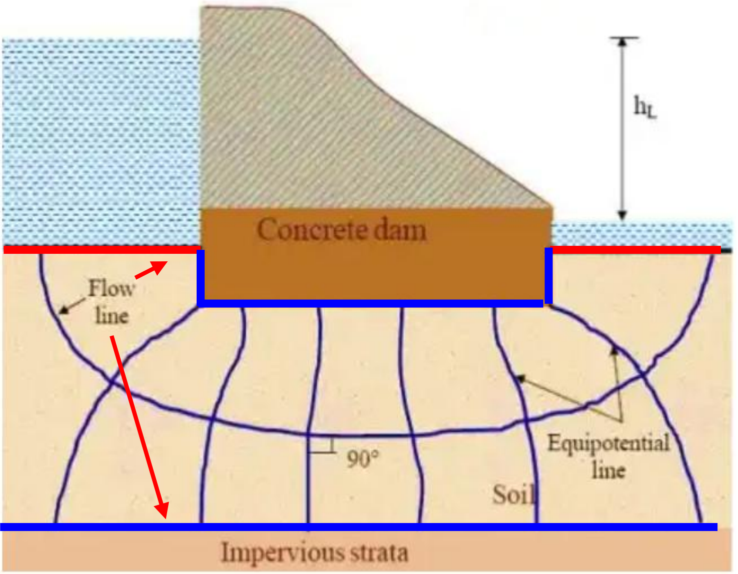
3) **Equal quantity** of seepage occurs in each flow channel. **A flow channel is a space between two flow lines.**

4) Head loss is the same between two adjacent potential lines.

6) The space formed between two flow lines and two equipotential lines is called a **grid**. It should be **in a square form or close to it**

7) The flow line and equipotential line should be smooth curves

2D seepage and flow net

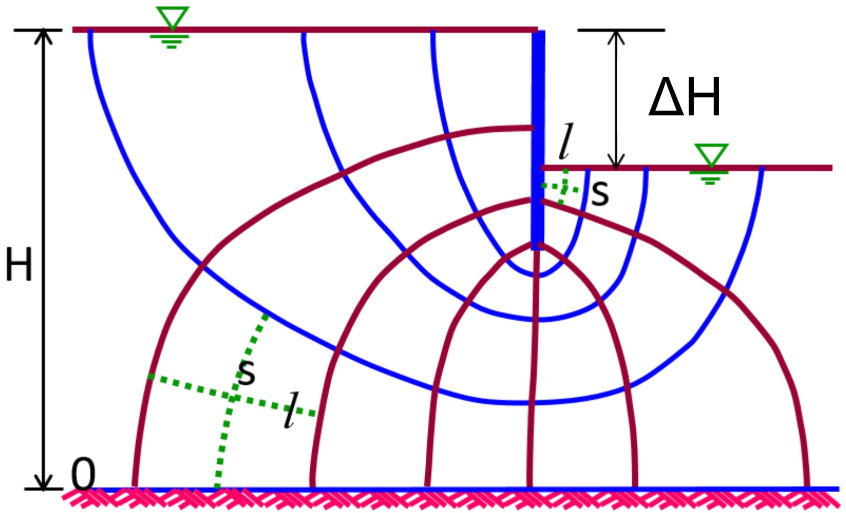


2D flow net of a concreter dam

Brief procedures to draw a 2D flow net:

1) Draw a real model with **a designed scale** on a piece of grid paper
2) Properly identify the boundary conditions, i.e., label the position of the water table, of any impermeable boundaries, of any points of known head or known pressure.
3) Make a trial for the number of flow channel (N_f)
4) Starting from the upstream site, draw the first equipotential line to have all square nets or near-square nets with 90° intersections.
5) Continue the foregoing steps for the rest of equipotential lines until it reaches the downstream. If not available, back to step 3 and modify (N_f)
6) Determine the number of intervals, namely number of equipotential drops (N_d)

2D seepage and flow net



2D flow net of a sheet pile

Application-1 Volumetric flow rate estimation

1) Total water head loss: ΔH
2) Equipotential drop: $\Delta h = \Delta H / N_d$
3) Darcian velocity in each channel:

$$v_i = k i_i = k \Delta h / l_i$$

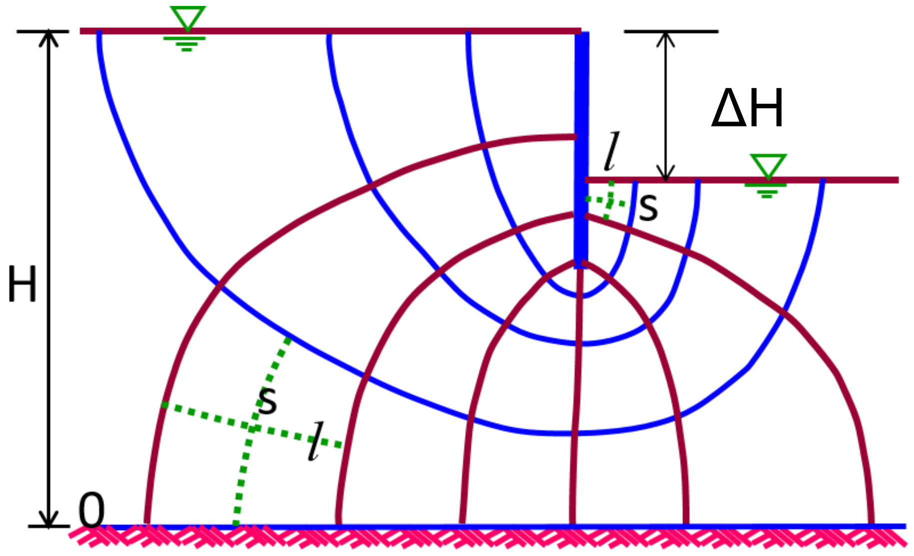
4) Volumetric flow rate of each channel:

$$q_i = v_i s_i = k \Delta h s_i / l_i$$

5) Volumetric flow rate ($s_i \approx l_i$):

$$Q = N_f q_i = k \Delta H \frac{N_f}{N_d}$$

2D seepage and flow net

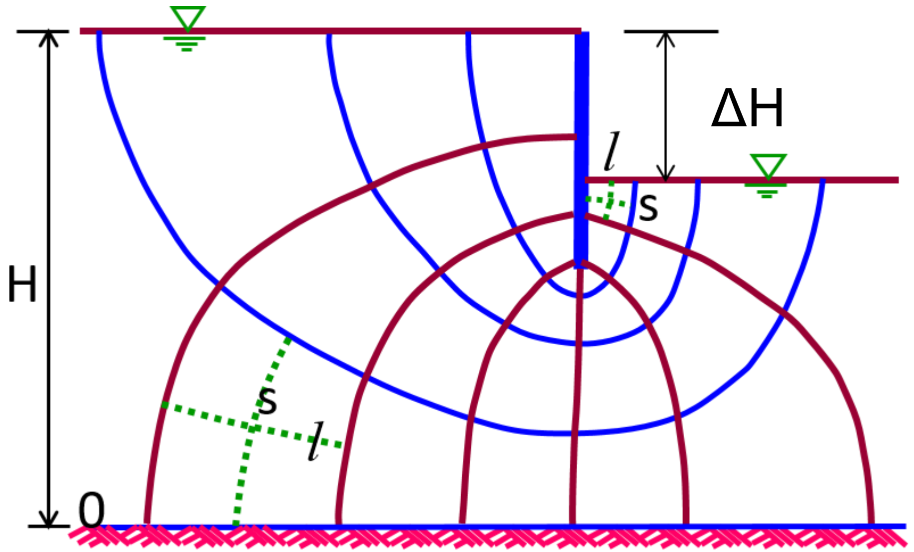


2D flow net of a sheet pile

Application-2 Pressure head estimation along certain equipotential line

1) The total head after n equipotential drops: $\Delta H - n\Delta h$
2) Determine the elevation head z
3) The pressure head equals: $\Delta H - n\Delta h - z$

2D seepage and flow net



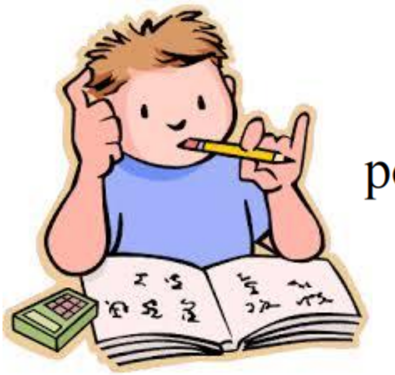
2D flow net of a sheet pile

Application-3 Exit (hydraulic) gradient

The exit (hydraulic) gradient is the hydraulic gradient at the end of flow line where seepage water from the soil mass joins with free water at the downstream. **Exit gradient can be expressed**

$$i_{exit} \approx \frac{\Delta H}{L} = \frac{\Delta h}{l} = \frac{\Delta H}{N_d l}$$

Exercises



According to the flow net as shown in Fig. 2, determine: 1) The pressure head at point A and Point B; 2) Volumetric flow rate ($k = 5\times10^{-4}$ cm/s)

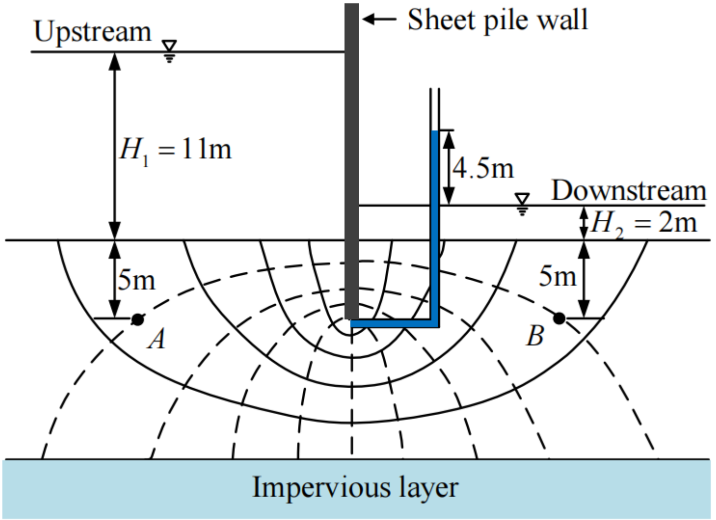


Fig. 2 Schematic diagram of the flow net

Exercises

Two lines of sheet piles were driven in a river bed as shown in Fig. 2. The depth of water overthe river bed is 2.5 m. The trench level within the sheet piles is 2 m below the river bed. The water level within the sheet piles is kept at trench level by resorting to pumping. If a quantity of waterflowing into the trench from outside is $0.3\ m^2/h$ per unit length of sheet pile, what is the hydraulic conductivity of the sand? What is the hydraulic gradient immediately below the trench bed?

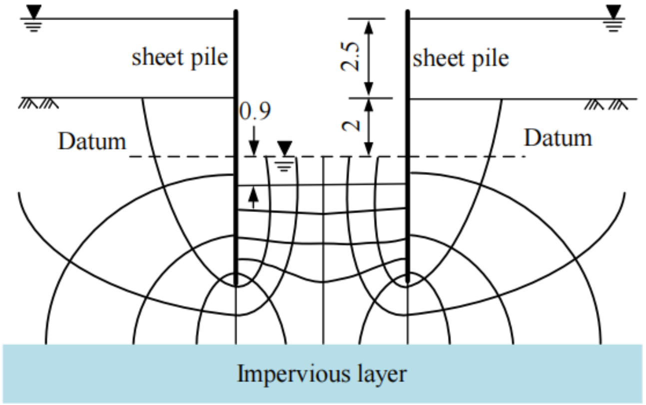
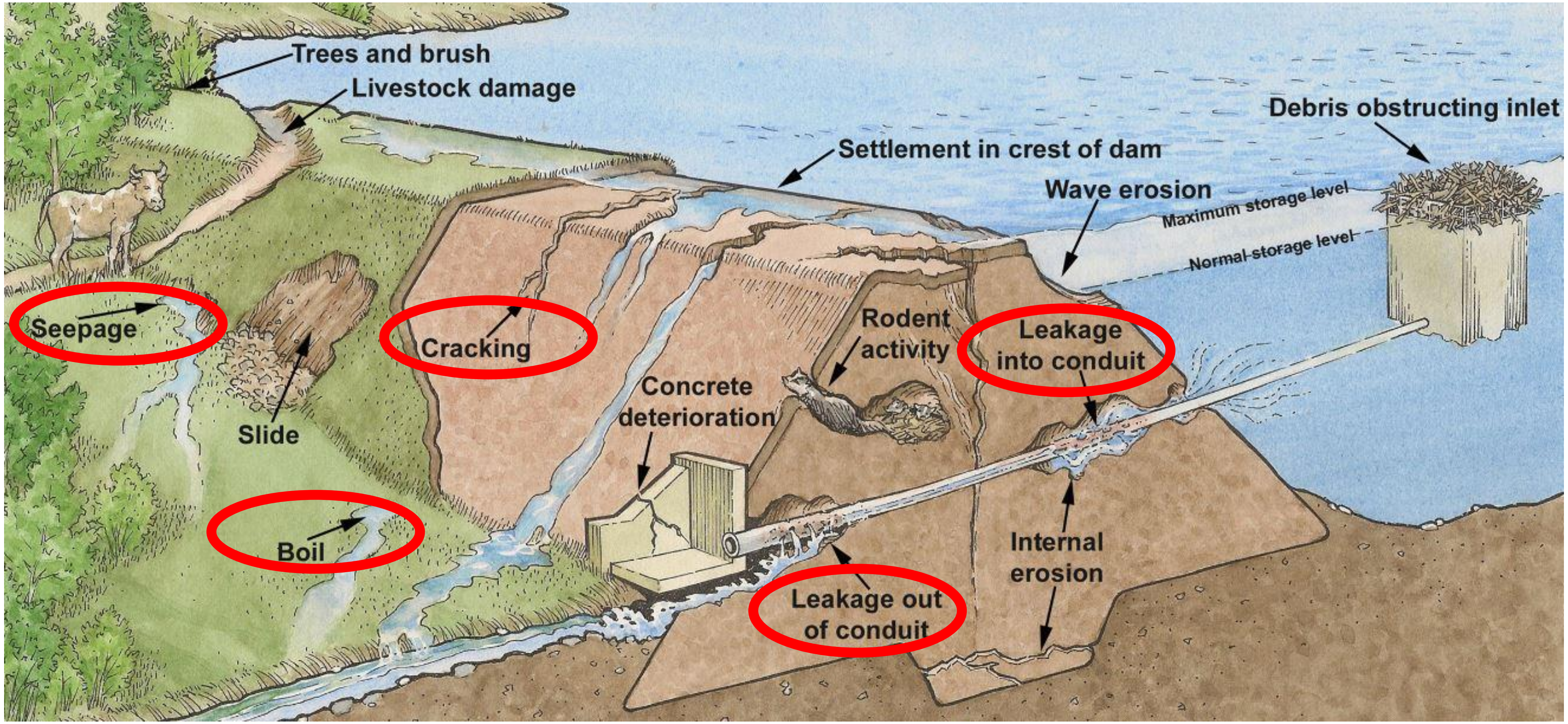


Fig. 2 Schematic diagram of the flow net (unit of length: m)

Seepage force and related problems



Seepage related problems for a earth-fill dam

Seepage force and related problems

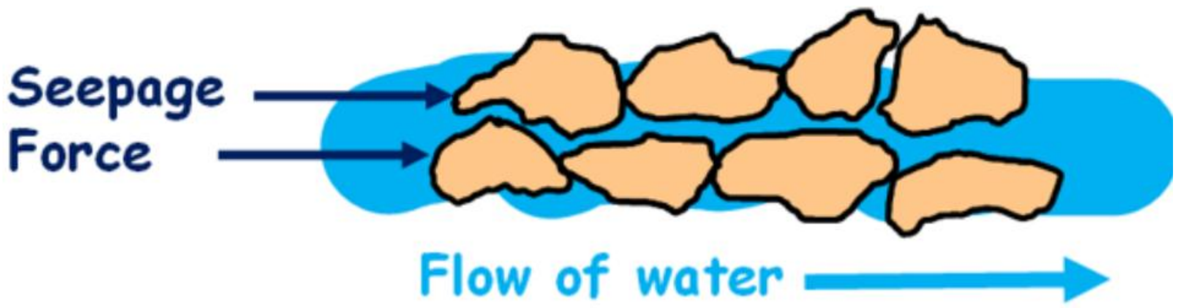
The flow of water in a saturated soil imposes additional force on soil particles. Therefore, the original stress state within soils will be altered. This may lead to two types of problems:

1) The erosion takes place at certain part of base or soil-structure interface as particles are displaced, which may lead to local failure or deformation of structures
2) The complete (global) failure of structures. The water flow may greatly reduce the strength of soil and a complete failure occurs. It is worth pointing that the progressive local failure may also ultimately lead to complete (global) failure, piping is a representative example.

Seepage force and related problems

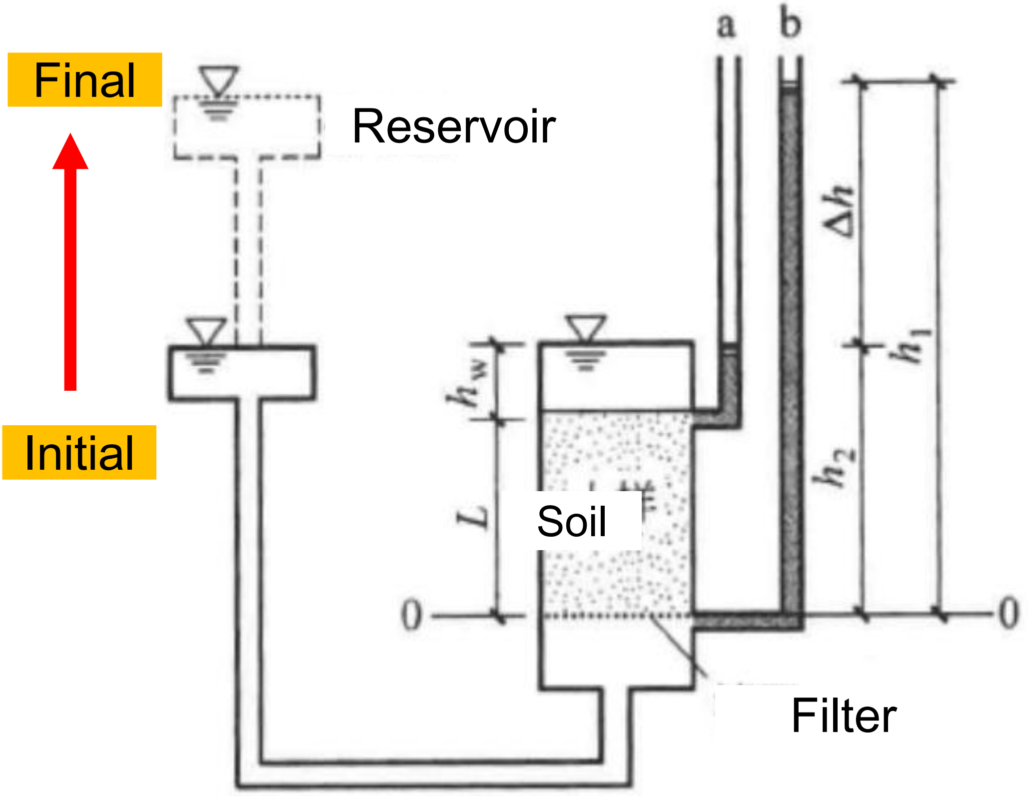
Seepage force

As foregoing mentioned, the flowing water may impose the dynamic force on particles in the direction of flow. **The dynamic force, which we usually called seepage force** is the term indicating the force per unit volume of soil (**body force**).



Schematic diagram of seepage force

Seepage force and related problems



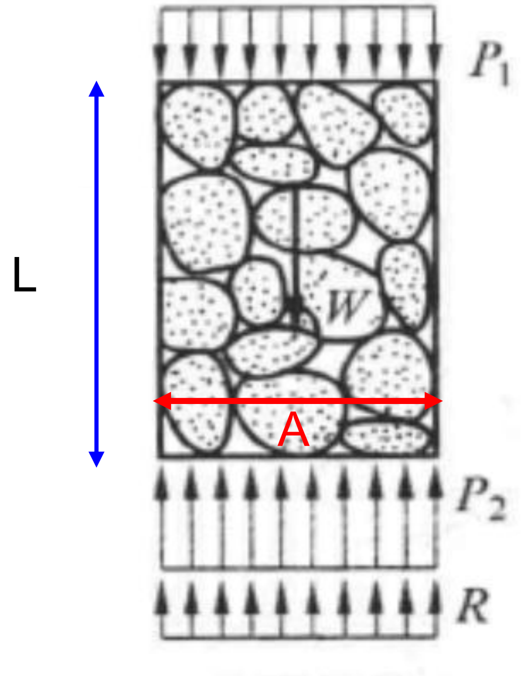
Setup for seepage force demonstration

The effect of seepage force can be show through the following device.

1) The water in the left reservoir and the right container are at the same line. The water levels in two piezometers are equal as there is no water flow through soil and thus no head loss.
2) Left reservoir is gradually lifted to certain height, the phenomena “sand boil” or “quick sand” occurs.

Seepage force and related problems

Force analysis



Soil sample

1) In the horizontal direction (left side and right side of the soil sample), the water pressure and the reaction force by container are in equilibrium.
2) In the vertical direction, **there are three types of forces:** 1) The total weight of the soil sample $\gamma_{sat}LA$; 2) The water pressure acts on the top and bottom surface of soil are respectively $\gamma_w h_1$ and $\gamma_w h_w$; 3) The reaction force R due to the support of the filter

Seepage force and related problems

According to equilibrium state in the vertical direction

$$\gamma_{sat} L \bullet A + \gamma_w h_w \bullet A = R + \gamma_w h_1 \bullet A \rightarrow R = \gamma' L \bullet A - \gamma_w \left(h_1 - h_w - L \right) \bullet A$$

As $\Delta h = h_1 - h_w - L$ the above equation changes to:

$$R = \gamma' L \cdot A - \gamma_w \Delta h \cdot A$$

The **second item** is due to action of seepage force. Define the **total seepage force (J) and seepage force (j) as**

$$J = \gamma_w \Delta h \cdot A \quad and \quad j = \frac{J}{A \cdot L} = \gamma_w \frac{\Delta h}{L} = \gamma_w i$$

Seepage force and related problems

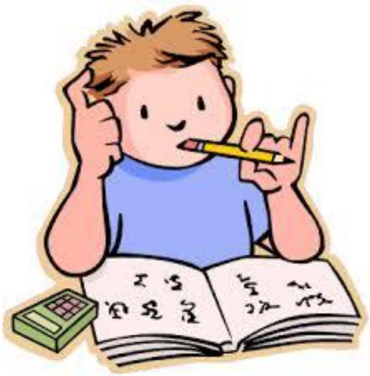
When the “sand boiling” or “quick sand” occurs, **the particles suspend in water. Therefore, the reaction force R equals zero** and the corresponding hydraulic gradient is named **as critical hydraulic gradient**:

$$R=\gamma' L\cdot A-\gamma_w \Delta h\cdot A=0 \rightarrow i_{crit}=\frac{\Delta h}{L}=\frac{\gamma'}{\gamma_w}$$

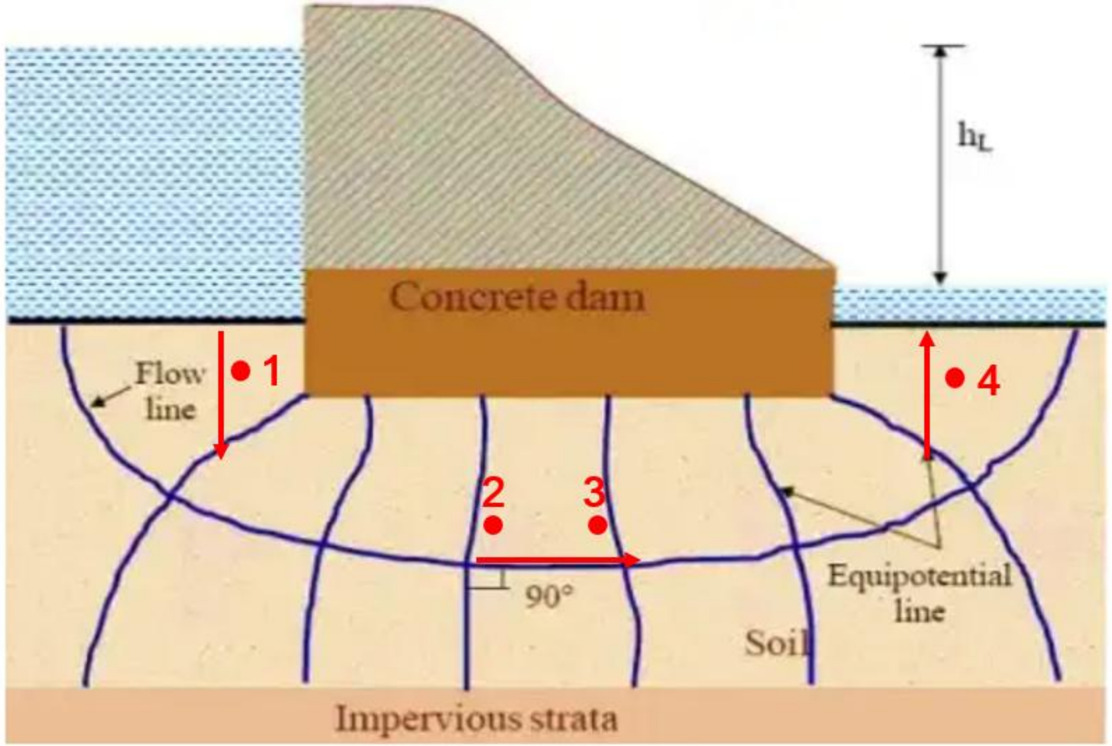
As and $\gamma' = (G_s - 1)\gamma_w/(1+e)$ and $\mathrm{n} = e/(1+e)$, alternative expressions for the critical hydraulic gradient can be written as:

$$i_{crit}=\frac{\gamma'}{\gamma_w}=\frac{G_s-1}{1+e} \quad or \quad i_{crit}=(G_s-1)(1-n)$$

Exercises

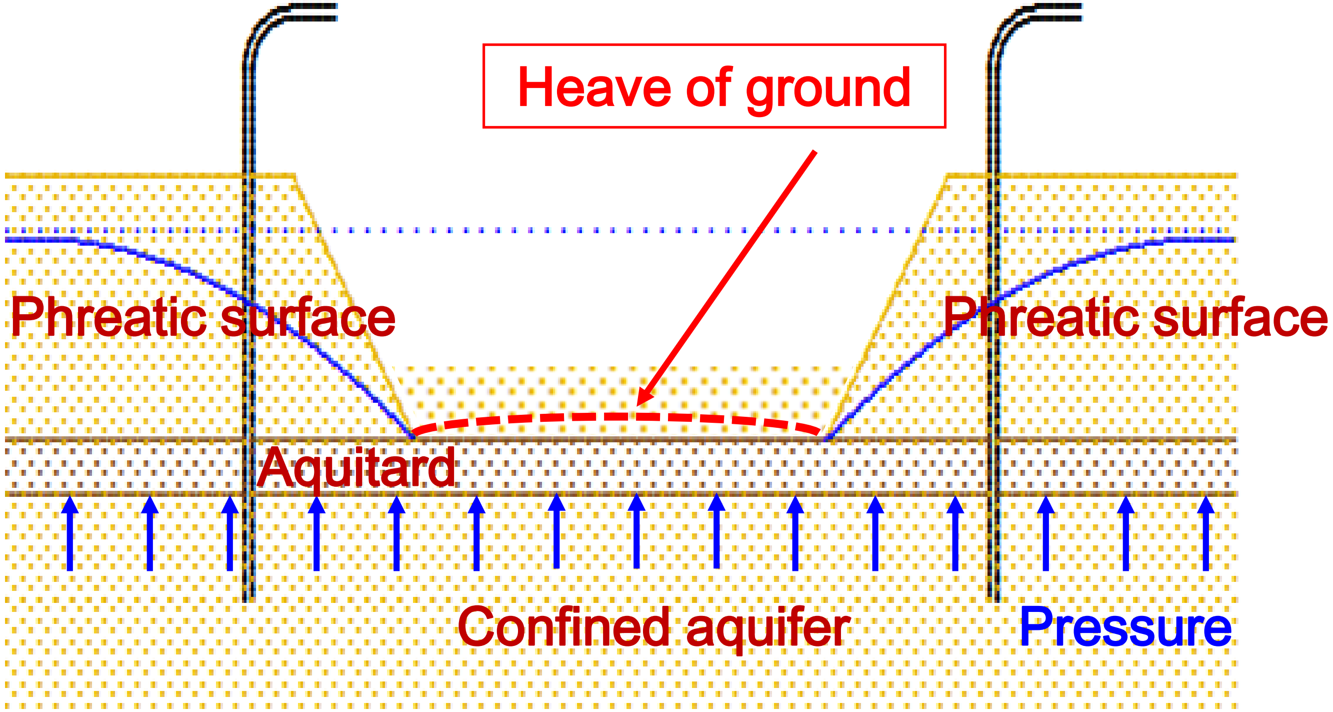


Does the seepage force at points 1, 2, 3 and 4 play the same role?



Seepage force and related problems

Failure by heave of ground and hydraulic fracturing

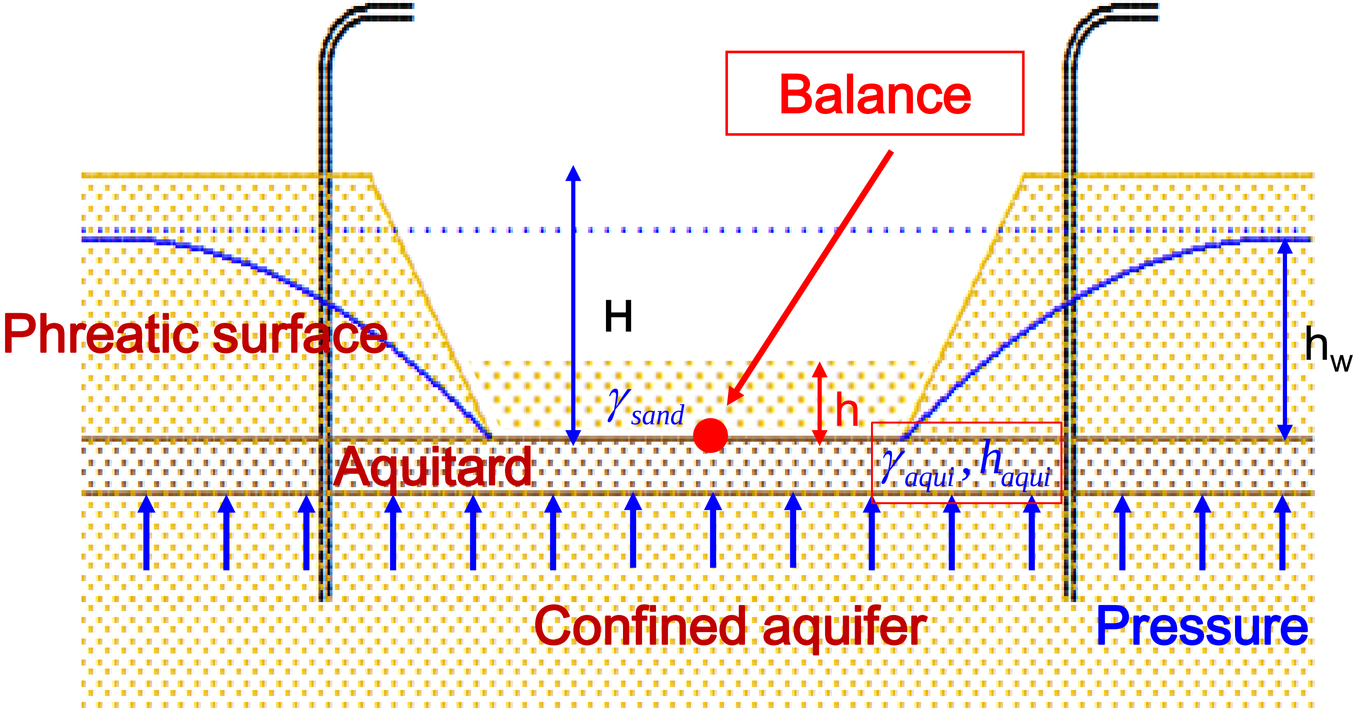


Conceptual model for heave of ground or water inrush

A thin impervious layer is sandwiched by two permeable sand layers. If no attention is paid to the excavation depth, the pressure in the confined aquifer may cause the **heave of the ground** and **thus the induced tension** in aquitard, which may further cause **hydraulic fracturing** (**water inrush**).

Seepage force and related problems

Failure by heave of ground and hydraulic fracturing



Conceptual model for heave of ground or water inrush

According to the balance **at the roof of the confined aquifer**

$$\gamma_{sand} h + \gamma_{aqui} h_{aqui} = \gamma_w h_w \rightarrow h = \frac{\gamma_w h_w - \gamma_{aqui} h_{aqui}}{\gamma_{sand}}$$

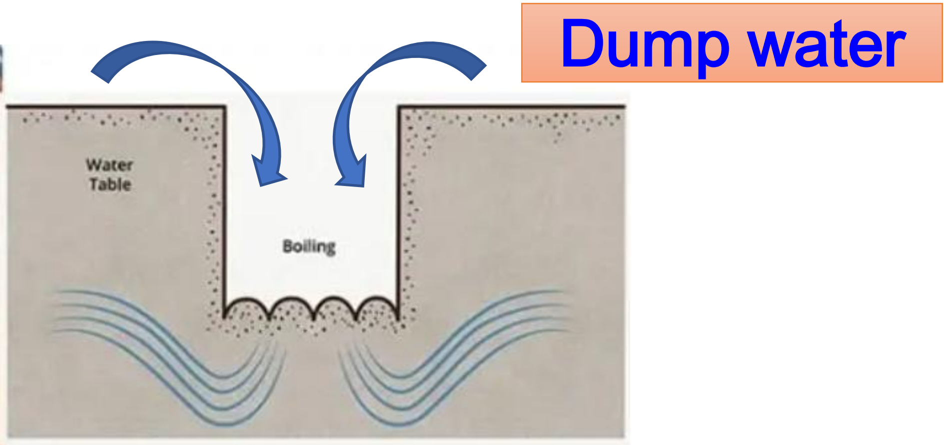
The depth of excavation

$$h_{exca} = H - h$$

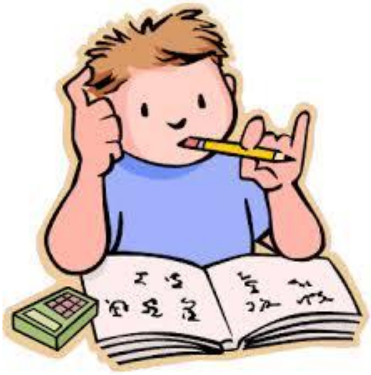
Seepage force and related problems

To prevent this problem, the groundwater level in the confined layer needs to be lowered by pumping wells. **A disadvantage of this solution** is that large amounts of water must be pumped. This affects a large region of the groundwater, which entails subsidence of the ground around and have cost

IF the failure occurs, **an emergency measure** is to dump water into the excavation pit, **which will increase the overburden pressure and thus balance the upward water pressure**.

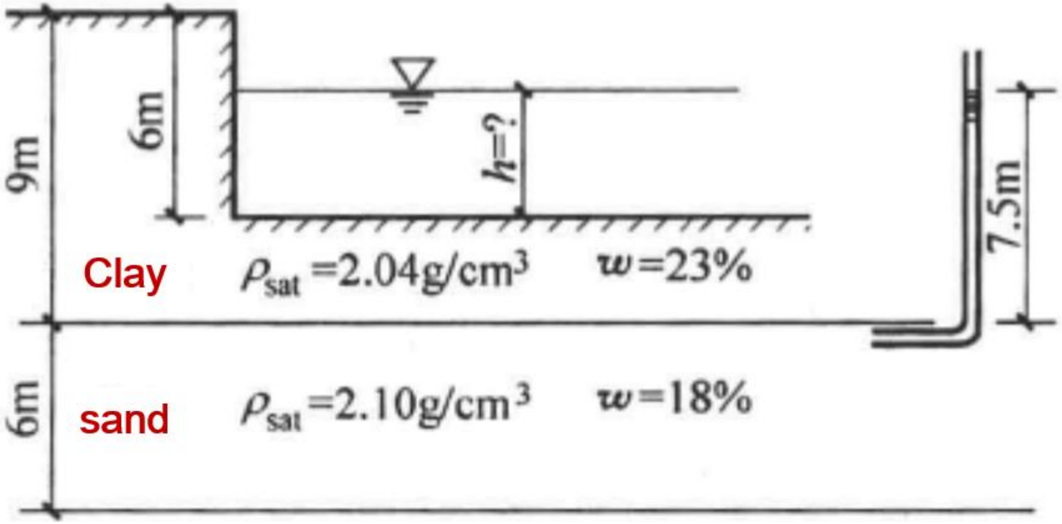


Exercises



The excavation is performed on a 9 m thick clay. Check:

1) Does the heave of the ground occurs if the excavation depth reaches 6 m.
2) If the accidents takes place, calculate the depth of the dumped water to avoid the further failure.



The End