Topic: Completing the square

Question: Complete the square to solve for the variable.

$$x^2 + 4x + 2 = 0$$

Answer choices:

A $x = -2 \pm \sqrt{2}$

B $x = 2 \pm \sqrt{2}$

C $x = -2 \pm \sqrt{3}$

D $x = 2 \pm \sqrt{3}$

Solution: A

Whenever we can't factor the equation (we can't factor this one), we can either use the quadratic formula or the method of completing the square. Since we've been asked to complete the square, we'll choose that method.

We'll take the coefficient on the first-degree term, 4, and divide it by 2.

$$\frac{4}{2} = 2$$

Now we'll take that result and square it.

$$(2)^2 = 4$$

This is the value we have to add to the first two terms on the left in order to complete the square. Since we'll be adding 4 to the left, we'll have to also add 4 to the right in order to keep the equation balanced.

$$\left(x^2 + 4x + 4\right) + 2 = 0 + 4$$

$$\left(x^2 + 4x + 4\right) + 2 = 4$$

$$\left(x^2 + 4x + 4\right) + 2 - 2 = 4 - 2$$

$$x^2 + 4x + 4 = 2$$

Now we can factor the left-hand side as a perfect square.

$$(x + 2)(x + 2) = 2$$

$$(x + 2)^2 = 2$$

$$\sqrt{(x+2)^2} = \pm\sqrt{2}$$

$$x+2 = \pm\sqrt{2}$$

$$x+2-2 = \pm\sqrt{2}-2$$

$$x = -2 \pm \sqrt{2}$$

We can't reduce $\sqrt{2}$ at all, so the roots of our equation are

$$x = -2+\sqrt{2}$$

and

$$x = -2-\sqrt{2}$$

Topic: Completing the square

Question: Complete the square to solve for the variable.

$$u^3 - 4u^2 + 3u = 0$$

Answer choices:

A $u = 0, \ -1, \ -3$

B $u = 1, \ 3$

C $u = 0, \ 1, \ 3$

D $u = 0, \ -1, \ 3$

Solution: C

We can factor this quadratic equation directly as

$$u^3 - 4u^2 + 3u = 0$$

$$u\left(u^2 - 4u + 3\right) = 0$$

$$u\,(u-3)\,(u-1) = 0$$

So the solutions are

$$u = 0$$

or

$$u - 3 = 0$$

$$u = 3$$

or

$$u - 1 = 0$$

$$u = 1$$

but since we've been asked to complete the square to find the solutions, we'll solve it that way.

We have to first factor out a u so that we can get a standard quadratic equation.

$$u^3 - 4u^2 + 3u = 0$$

$$u\left(u^2-4u+3\right)=0$$

We'll take the coefficient on the first-degree term, -4, and divide it by 2.

$$\frac{-4}{2}=-2$$

Now we'll take that result and square it.

$$(-2)^2=4$$

This is the value we have to add to the first two terms inside the parentheses in order to complete the square. Since we'll be adding 4 to the left, we'll have to also add 4 to the right in order to keep the equation balanced. But be careful here. If we add 4 inside the parentheses, that 4 is actually multiplied by the u outside of the parentheses, so it's like we're adding $4u$ to the left, which means we have to add $4u$ to the right as well.

$$u\left(u^2-4u+4+3\right)=0+4u$$

$$u\left(u^2-4u+4\right)+3u=4u$$

$$u\left(u^2-4u+4\right)=u$$

Now we can factor the left-hand side as a perfect square.

$$u(u-2)(u-2)=u$$

$$u(u-2)^2=u$$

Before we divide both sides of the equation by u to get the perfect square by itself, we have to realize at this point that $u=0$ is a solution to the equation.

$$(u-2)^2 = 1$$

$$\sqrt{(u-2)^2} = \pm\sqrt{1}$$

$$u - 2 = \pm 1$$

$$u = 2 \pm 1$$

So the solutions are

$$u = 2 + 1$$

$$u = 3$$

or

$$u = 2 - 1$$

$$u = 1$$

Putting both of these together with $u = 0$ that we found earlier, the solutions to the equation are $u = 0, 1, 3$.