Topic: Composite functions, domain

Question: What is the domain of $f \circ g$?

$$f(x) = x^2 - 5$$

$$g(x) = \sqrt{x + 4}$$

Answer choices:

A $x \leq -4$

B $x > -4$

C $x \geq -4$

D $x < -4$

Solution: C

First, find the domain of $g(x)$. The expression $\sqrt{x+4}$ is undefined if the radicand is negative. For example, if $x = -5$, then $x+4$ is -1. Likewise, if x is any number less than -4, $x+4$ will be negative. However, -4 itself is okay, because $\sqrt{x+4}$ will then be 0, which is not undefined.

The domain of $g(x)$ then is all reals x such that $x \geq -4$.

The composite function is

$$f \circ g = f(g(x)) = \left(\sqrt{x+4}\right)^2 - 5$$

$$f \circ g = f(g(x)) = x + 4 - 5$$

$$f \circ g = f(g(x)) = x - 1$$

For this simple binomial, no real numbers are excluded, so its domain is all reals. But because the domain of $g(x)$ excludes $x < -4$, those values also have to be excluded from the composite $f(g(x))$.

That means the domain of $f(g(x))$ is $x \geq -4$.

Topic: Composite functions, domain

Question: What is the domain of $f \circ g$?

$$f(x) = \frac{1}{x+3}$$

$$g(x) = \frac{x}{x-2}$$

Answer choices:

A $x \neq 2, \ -3$

B $x \neq \frac{3}{2}, \ 2$

C $x \neq -2, \ 3$

D $x \neq -\frac{3}{2}, \ 2$

Solution: B

First, find the domain of $g(x)$. The expression $x/(x-2)$ is undefined if the denominator is 0. That means $x = 2$ isn't in the domain of $g(x)$. Therefore, the domain of $g(x)$ is all reals x such that $x \neq 2$.

The composite function is

$$f \circ g = f(g(x)) = \frac{1}{\frac{x}{x-2} + 3}$$

$$f \circ g = f(g(x)) = \frac{1}{\frac{x}{x-2} + 3\frac{(x-2)}{(x-2)}}$$

$$f \circ g = f(g(x)) = \frac{1}{\frac{x + 3x - 6}{x-2}}$$

$$f \circ g = f(g(x)) = \frac{1}{\frac{4x-6}{x-2}}$$

$$f \circ g = f(g(x)) = \frac{x-2}{4x-6}$$

$$f \circ g = f(g(x)) = \frac{x-2}{2(2x-3)}$$

For this rational function, any numbers that make the denominator 0 are excluded from the domain.

$$2(2x-3) = 0$$

$$2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

Putting both exclusions together, the domain of the composite is all real numbers except

$$x \neq \frac{3}{2}, 2$$