Calculus 1 Formulas

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MATH

Limits & Continuity

Limits

Suppose $f(x)$ is defined when x is near the number a (f is defined on some open interval that contains a, except possibly at a itself). Then we write

$$\lim_{x\to a} f(x) = L$$

and say "the limit of $f(x)$ as x approaches a equals L" if we can make the values of $f(x)$ arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to a (on either side of a) but not equal to a.

One-sided limits

Left-hand limit (the limit as x approaches a from the left)

$$\lim_{x\to a^-} f(x) = L$$

Right-hand limit (the limits as x approaches a from the right)

$$\lim_{x\to a^+} f(x) = L$$

The general limit exists if and only if the left- and right-hand limits both exist and are equal to each other

$$\lim_{x\to a} f(x) = L \qquad \text{only if} \qquad \lim_{x\to a^-} f(x) = L \qquad \text{and} \qquad \lim_{x\to a^+} f(x) = L$$

Limit laws

$$\lim_{x\to a} c = c$$

$$\lim_{x\to a}[cf(x)] = c\lim_{x\to a} f(x)$$

$$\lim_{x\to a} x = a$$

$$\lim_{x\to a} x^n = a^n$$

$$\lim_{x\to a}[f(x)+g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)-g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[f(x)\cdot g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

$$\lim_{x\to a}\sqrt[n]{x} = \sqrt[n]{a}$$

$$\lim_{x\to a}\sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$$

$$\lim_{x\to a}[f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$$

Trigonometric limits

$$\lim_{\theta\to 0}\sin\theta = 0$$

$$\lim_{\theta\to 0}\cos\theta = 1$$

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

Solving limits with substitution

$$\lim_{x \to a} f(x) = f(a)$$

Squeeze theorem

If $f(x) \le g(x) \le h(x)$ when x is near a (except possibly at a itself) and if $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$ then $\lim_{x \to a} g(x) = L$

Precise definition of the limit

Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say that the limit of $f(x)$ as x approaches a is L, and we write

$$\lim_{x \to a} f(x) = L$$

if for every $\epsilon > 0$ there is a $\delta > 0$ such that

if $\quad 0 < |x - a| < \delta \quad$ then $\quad |f(x) - L| < \epsilon$

Continuity

A function f is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a)$$

One-sided continuity

A function f is continuous from the right at a number a if $\lim_{x \to a^+} f(x) = f(a)$

A function f is continuous from the left at a number a if $\lim_{x \to a^-} f(x) = f(a)$

Continuity of elementary functions

The following functions are continuous everywhere in their domain:

Polynomials

Rational functions

Root functions

Trigonometric functions

Inverse trigonometric functions

Exponential functions

Logarithmic functions

Continuity of composites

If g is continuous at a and f is continuous at $g(a)$, then the composite function $f \circ g$ given by $(f \circ g)(x) = f(g(x))$ is continuous at a.

Intermediate value theorem

Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

Derivatives

Definition of the derivative

The definition of the derivative, also called the difference quotient, is the slope of the tangent line, which is given by

$$f'(a) = \lim_{h \to a} \frac{f(a+h) - f(a)}{h} \qquad \text{or} \qquad f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Differentiability

- A function is differentiable at a if $f'(a)$ exists
- A function is differentiable on an open interval (a, b) if it is differentiable at every number in the interval
- If f is differentiable at a, then f is continuous at a

Derivative rules

Power rule $\quad \frac{d}{dx}\left(x^n\right) = nx^{n-1}$

Product rule $\quad \frac{d}{dx}\left[f(x)g(x)\right] = f'(x)g(x) + f(x)g'(x)$

Quotient rule $\quad \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{\left[g(x)\right]^2}$

Chain rule
$$\frac{d}{dx}\left[f(g(x))\right] = f'(g(x))g'(x)$$

Derivative of exponential functions

$$\frac{d}{dx}\left(e^x\right) = e^x \ln e = e^x(1) = e^x$$

$$\frac{d}{dx}\left(a^x\right) = a^x \ln a$$

Derivatives of logarithmic functions

$$\frac{d}{dx}\left(a \log_b x\right) = \frac{a}{x \ln b}$$

$$\frac{d}{dx}\left(a \ln x\right) = \frac{a}{x}$$

Derivatives of trig functions

$$\frac{d}{dx}\left[a \sin\left(f(x)\right)\right] = a \cos\left(f(x)\right) \cdot f'(x)$$

$$\frac{d}{dx}\left[a \csc\left(f(x)\right)\right] = -a \csc\left(f(x)\right) \cot\left(f(x)\right) \cdot f'(x)$$

$$\frac{d}{dx}\left[a \cos\left(f(x)\right)\right] = -a \sin\left(f(x)\right) \cdot f'(x)$$

$$\frac{d}{dx}\left[a \sec\left(f(x)\right)\right] = a \sec\left(f(x)\right) \tan\left(f(x)\right) \cdot f'(x)$$

$$\frac{d}{dx}\left[a \tan\left(f(x)\right)\right] = a \sec^2\left(f(x)\right) \cdot f'(x)$$

$$\frac{d}{dx}\left[a \cot\left(f(x)\right)\right] = -a \csc^2\left(f(x)\right) \cdot f'(x)$$

Derivatives of inverse trig functions

$$\frac{d}{dx}\left[a \sin^{-1}\left(f(x)\right)\right] = \frac{a \cdot f'(x)}{\sqrt{1 - \left[f(x)\right]^2}}$$

$$\frac{d}{dx}\left[a \csc^{-1}\left(f(x)\right)\right] = -\frac{a \cdot f'(x)}{\left|f(x)\right| \sqrt{\left[f(x)\right]^2 - 1}}$$

$$\frac{d}{dx}\left[a\cos^{-1}\left(f(x)\right)\right]=-\frac{a\cdot f'(x)}{\sqrt{1-\left[f(x)\right]^2}}$$

$$\frac{d}{dx}\left[a\sec^{-1}\left(f(x)\right)\right]=\frac{a\cdot f'(x)}{\left|f(x)\right|\sqrt{\left[f(x)\right]^2-1}}$$

$$\frac{d}{dx}\left[a\tan^{-1}\left(f(x)\right)\right]=\frac{a\cdot f'(x)}{1+\left[f(x)\right]^2}$$

$$\frac{d}{dx}\left[a\cot^{-1}\left(f(x)\right)\right]=-\frac{a\cdot f'(x)}{1+\left[f(x)\right]^2}$$

Derivatives of hyperbolic trig functions

$$\frac{d}{dx}\left[a\sinh\left(f(x)\right)\right]=a\cosh\left(f(x)\right)\cdot f'(x)$$

$$\frac{d}{dx}\left[a\operatorname{csch}\left(f(x)\right)\right]=-a\operatorname{csch}\left(f(x)\right)\coth\left(f(x)\right)\cdot f'(x)$$

$$\frac{d}{dx}\left[a\cosh\left(f(x)\right)\right]=a\sinh\left(f(x)\right)\cdot f'(x)$$

$$\frac{d}{dx}\left[a\operatorname{sech}\left(f(x)\right)\right]=-a\tanh\left(f(x)\right)\operatorname{sech}\left(f(x)\right)\cdot f'(x)$$

$$\frac{d}{dx}\left[a\tanh\left(f(x)\right)\right]=a\operatorname{sech}^2\left(f(x)\right)\cdot f'(x)$$

$$\frac{d}{dx}\left[a\coth\left(f(x)\right)\right]=-a\operatorname{csch}^2\left(f(x)\right)\cdot f'(x)$$

Derivatives of inverse hyperbolic trig functions

$$y=\sinh^{-1}\left[g(x)\right]\qquad y'=\frac{g'(x)}{\sqrt{\left[g(x)\right]^2+1}}$$

$$y=\cosh^{-1}\left[g(x)\right]\qquad y'=\frac{g'(x)}{\sqrt{\left[g(x)\right]^2-1}}\qquad g(x)>1$$

$$y=\tanh^{-1}\left[g(x)\right]\qquad y'=\frac{g'(x)}{1-\left[g(x)\right]^2}\qquad |g(x)|<1$$

$y = \coth^{-1}\left[g(x)\right]$ $\quad$ $y' = \dfrac{g'(x)}{1 - \left[g(x)\right]^2}$ $\quad$ $|g(x)| > 1$

$y = \text{sech}^{-1}\left[g(x)\right]$ $\quad$ $y' = -\dfrac{g'(x)}{g(x)\sqrt{1 - \left[g(x)\right]^2}}$ $\quad$ $0 < g(x) < 1$

$y = \text{csch}^{-1}\left[g(x)\right]$ $\quad$ $y' = -\dfrac{g'(x)}{|g(x)|\sqrt{\left[g(x)\right]^2 + 1}}$ $\quad$ $g(x) \neq 0$

Definitions of hyperbolic trig functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\text{csch} x = \frac{1}{\sinh x}$$

$$\text{sech} x = \frac{1}{\cosh x}$$

$$\coth x = \frac{\cosh x}{\sinh x}$$

Hyperbolic trig identities

$\sinh(-x) = -\sinh x$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

$\cosh(-x) = \cosh x$

$1 - \tanh^2 x = \text{sech}^2 x$

$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

Applications of Derivatives

Linear approximation and linearization

The linear approximation, or tangent line approximation, of f at a is

$$f(x) \approx f(a) + f'(a)(x - a)$$

To find the linearization of f at a, evaluate at a specific point a.

$$L(x) = f(a) + f'(a)(x - a)$$

Optimization and graph sketching

$f(x)$	$f'(x)$	$f''(x)$
Cubic (x^3)	Parabolic (x^2)	Linear (x)
Increasing	Positive (above the x-axis)	
Decreasing	Negative (below the x-axis)	
Concave up	Increasing	Positive (above the x-axis)
Concave down	Decreasing	Negative (below the x-axis)
	Concave up	Increasing
	Concave down	Decreasing
Critical point (extrema)	0 (x intercepts)	
Inflection point	Critical point (extrema)	0 (x intercepts)
	Inflection point	Critical point (extrema)

$f(x)$ is...

	$f'(x) = +$	$f'(x) = -$	$f'(x) = 0$
$f''(x) = +$	inc/concave up	dec/concave up	min/concave up
$f''(x) = -$	inc/concave down	dec/concave down	max/concave down
$f''(x) = 0$	inc/inflection point	dec/inflection point	possible inflection point

Rolle's theorem

If f is a function that satisfies these:

1. f is continuous on the closed interval $[a, b]$.

2. f is differentiable on the open interval (a, b).

3. $f(a) = f(b)$

Then there's a number c in the interval (a, b) where $f'(c) = 0$.

In other words, there's a point c in the interval (a, b) at which the derivative of f is 0, which means the slope of the graph is 0, the tangent line is horizontal, the point $x = c$ represents a critical point, and the graph changes direction there.

Mean value theorem

If f satisfies these:

1. f is continuous on the closed interval $[a, b]$.

2. f is differentiable on the open interval (a, b).

Then there is a number c in the interval (a, b) where

$$f'(c) = \frac{f(b) - f(a)}{b - a} \qquad \text{or} \qquad f(b) - f(a) = f'(c)(b - a)$$

Newton's method

If the nth approximation is x_n and $f'(x_n) \neq 0$, then the next approximation is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

L'Hospital's rule

If $$h(x) = \frac{f(x)}{g(x)},$$

and if $\lim_{x \to a} \frac{f(a)}{g(a)}$ gives an indeterminate form like $0/0$ or ∞/∞,

then $$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

if the limit on the right side exists (or is ∞ or $-\infty$). If the limit on the right side dos not exist, continue applying L'Hospital's rule as many times as necessary until the limit on the right side gives a real number answer or ∞ or $-\infty$.

Position, velocity and acceleration

Position $s(t)$ is the position function of an object that moves in a straight line

Velocity $v(t)$ is the velocity of the object

$$v(t) = s'(t)$$

Acceleration $a(t)$ is the acceleration of the object

$$a(t) = v'(t) = s''(t)$$

Marginal cost, revenue and profit

Cost

$C(x)$ is the cost function (the cost of producing x units)

$C'(x)$ is the marginal cost function (the derivative of $C(x)$, which is the rate of change of C with respect to x)

Revenue

$R(x)$ is the revenue function if

$R(x) = xp(x)$ where

x is the number of units sold and

$p(x)$ is the price per unit (the demand function, or price function, which is a decreasing function of x)

$R'(x)$ is the marginal revenue function (the derivative of $R(x)$, which is the rate of change of R with respect to x)

Profit

$P(x)$ is the profit function if

$P(x) = R(x) - C(x)$ where

$R(x)$ is the revenue function and

$C(x)$ is the cost function

$P'(x)$ is the marginal profit function (the derivative of $P(x)$, which is the rate of change or P with respect to x)

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