IFT 603-712 : TP1

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1/ Mq $H[x,y] = H[y|x] + H[x]$

Rappel:

$$\begin{cases} H[x] = -\sum_x p(x) \log p(x) \\ H[y|x] = -\sum_{x\in X} \sum_{y\in Y} p(x,y) \log p(y|x) \\ H[x,y] = -\sum_{x\in X} \sum_{y\in Y} p(x,y) \log p(x,y) \end{cases}$$

$$H[x,y] = -\sum_{x\in X} \sum_{y\in Y} p(x,y) \log p(x,y) \quad \text{tq } p(x,y) = p(x).p(y|x)$$

$$= -\sum_{x\in X} \sum_{y\in Y} p(x,y) \log p(y|x) + \Big[-\sum_{x\in X} \underbrace{\sum_{y\in Y} p(x,y)}_{p(x)} \log p(x)\Big]$$

$$= -\sum_{x\in X} \sum_{y\in Y} p(x,y) \log p(y|x) + \Big[-\sum_{x\in X} p(x) \log p(x)\Big]$$

$$\boxed{H[x,y] = H[y|x] + H[x]}$$

2/ Mq $I[x,y] = H[x] - H[x|y]$

Rappel:

$$I[x,y] = \sum_{x,y} P(x,y) \times \log\left(\frac{P(x,y)}{P(x).P(y)}\right)$$

$$I[x,y] = \sum_{x\in X} \sum_{y\in Y} P(x,y) [\log(P(x,y)) - \log[P(x).P(y)]]$$

$$= \sum_{x\in X} \sum_{y\in Y} p(x,y) \log P(x,y) - \sum_{x\in X} \sum_{y\in Y} p(x,y) \log P(x) - \sum_{x\in X} \sum_{y\in Y} p(x,y) \log P(y)$$

$$= -\sum_{x\in X} p(x) \log p(x) + \sum_{x\in X} \sum_{y\in Y} p(x,y) \log\left(\frac{p(x,y)}{P(y)}\right)$$

$$= H[x] + \sum_{x\in X} \sum_{y\in Y} p(x,y) \log p(x|y)$$

$$\boxed{I[x,y] = H[x] - H[x|y]}$$

3/ Mq : $\text{cov}[x,y] = E_{xy}[xy] - E[x]E[y]$

Rappel:

$$\begin{cases} E(x) = \sum_x x\, p(x) \\ \text{cov}[x,y] = E_{xy}[(x - E_x(x))(y - E_y(y))] \quad (\text{definition cours}) \end{cases}$$

$$\text{Cov}[x,y] = E_{xy}[(xy - xE_y(y) - yE_x(x) + E_x(x)E_y(y))]$$

$$= E_{xy}[xy] - E_{xy}[x.E_y(y)] - E_{xy}[yE_x(x)] + E_{xy}(E_x(x).E_y(y))$$

$$= E_{xy}[xy] - E_x(x).E_y(y) - E_y(y)E_x(x) + E_x(x)E_y(y)$$

$$= E_{xy}(xy) - 2E_x(x)E_y(y) + E_y(y).E_x(x)$$

$$\boxed{\text{cov}[x,y] = E_{xy}(xy) - E(x)E(y)}$$

4- Soit $r = \{0,0,0,1,0,1,0,0,0,1\}$

(a) Distribution :

n	0	1
$P(n)$	7/10	3/10

(b) Espérance :

$E(n) = \sum_n n\,p(n) = 0 \times 0,7 + 1 \times 0,3 = 0,3 \Rightarrow \boxed{E(n) = 0,3}$

(c) Variance

$Var[n] = \sum_n p(n)\,(n - E(n))^2$

$= 0,7\,(0-0,3)^2 + 0,3\,(1-0,3)^2$

$\boxed{Var[n] \simeq 0,21}$

(d) Entropie :

$H[n] = -\sum_n p(n) \log p(n)$

$= -\left[0,3 \log(0,3) + 0,7 \log(0,7)\right]$

$\boxed{H[n] \simeq 0,88}$