Soit la fct $R = \| (w_1, w_2, w_3, w_4, w_5) \cdot \sum_{\text{lines}_j \in I^{k}_{\sim j}} Z^{k}_{*} \|$. On pose :

$$\begin{cases} W = (w_1, \dots, w_5) \\ Z = \sum_{\text{lines}_j \in I^{k}_{\sim j}} Z^{k}_{*} = [Z_1, Z_2, \dots, Z_p] = \begin{bmatrix} z_{1,1} & \dots & z_{p,p} \\ \vdots & \ddots & \vdots \\ z_{p,1} & \dots & z_{p,p} \end{bmatrix} \end{cases}$$

$$\Rightarrow R = \| W \cdot Z \| = \sqrt{\sum_{i=1}^{p} (W \cdot Z_i)^2}$$ avec $W \cdot Z_i = w_1 \cdot z_{1,i} + w_2 \cdot z_{2,i} + \dots + w_5 z_{5,i}$

Calculons la dérivée partielle $\frac{\delta R}{\delta w_j}$:

$$\begin{cases} \frac{\delta (W \cdot Z_i)}{\delta w_j} = w_j \cdot z_{j,i} \\ \frac{\delta (W \cdot Z_i)^2}{\delta w_j} = 2 w_j \cdot z_{j,i} \, (W \cdot Z_i) \end{cases}$$

$$\frac{\delta \left(\sum_{i=1}^{p} (W \cdot Z_i)^2 \right)}{\delta w_j} = \sum_{i=1}^{p} \frac{\delta (W \cdot Z_i)^2}{\delta w_j} = 2 \sum_{i=1}^{p} w_j \cdot z_{j,i} \, (W \cdot Z_i)$$

Par conséquent : $$\frac{\delta R}{\delta w_j} = \frac{\sum_{i=1}^{p} w_j \cdot z_{j,i} \, (W \cdot Z_i)}{\sqrt{\sum_{i=1}^{p} (W \cdot Z_i)^2}}$$

D'où :

$$\frac{\delta E_{val}}{\delta w_k} = \frac{1}{np} \sum_{i=1}^{n} \sum_{j=1}^{p} \frac{\delta (|R - r_{i,j}|)}{\delta w_k}$$ avec $$\frac{\delta |R - r_{i,j}|}{\delta w_k} = \begin{cases} -\frac{\delta R}{\delta w_k} & \text{si } R - r_{i,j} < 0 \\ \frac{\delta R}{\delta w_k} & \text{sinon.} \end{cases}$$

Ainsi :

$$\nabla_W E_{val} = \begin{pmatrix} \frac{\delta E_{val}}{\delta w_1} \\ \vdots \\ \frac{\delta E_{val}}{\delta w_5} \end{pmatrix} = \frac{1}{np} \sum_{i=1}^{n} \sum_{j=1}^{p} \frac{Z_{i,j}^{T} \cdot (W \cdot Z_{i,j})}{\| W \cdot Z_{i,j} \|} \times \text{sign}$$

avec $\text{sign} = \begin{cases} -1 & \text{si } R - r_{i,j} < 0 \\ +1 & \text{sinon.} \end{cases}$