Propositional Logic: Logical Agents (Part I)

This lecture topic:

Propositional Logic (two lectures)

Chapter 7.1-7.4 (this lecture, Part I)

Chapter 7.5 (next lecture, Part II)

Next lecture topic:

First-order logic (two lectures)

Chapter 8

Outline

- Basic Definitions:
  - Syntax, Semantics, Sentences, Propositions, Entails, Follows, Derives, Inference, Sound, Complete, Model, Satisfiable, Valid (or Tautology)
- Syntactic Transformations:
  - E.g., $(A \Rightarrow B) \Leftrightarrow (\neg A \vee B)$
- Semantic Transformations:
  - E.g., $(KB \models \alpha) \equiv (\models (KB \Rightarrow \alpha))$
- Truth Tables
  - Negation, Conjunction, Disjunction, Implication, Equivalence (Biconditional)
  - Inference by Model Enumeration

You will be expected to know:

- Basic definitions (section 7.1, 7.3)
- Models and entailment (7.3)
- Syntax, logical connectives (7.4.1)
- Semantics (7.4.2)
- Simple inference (7.4.4)

Complete architectures for intelligence?

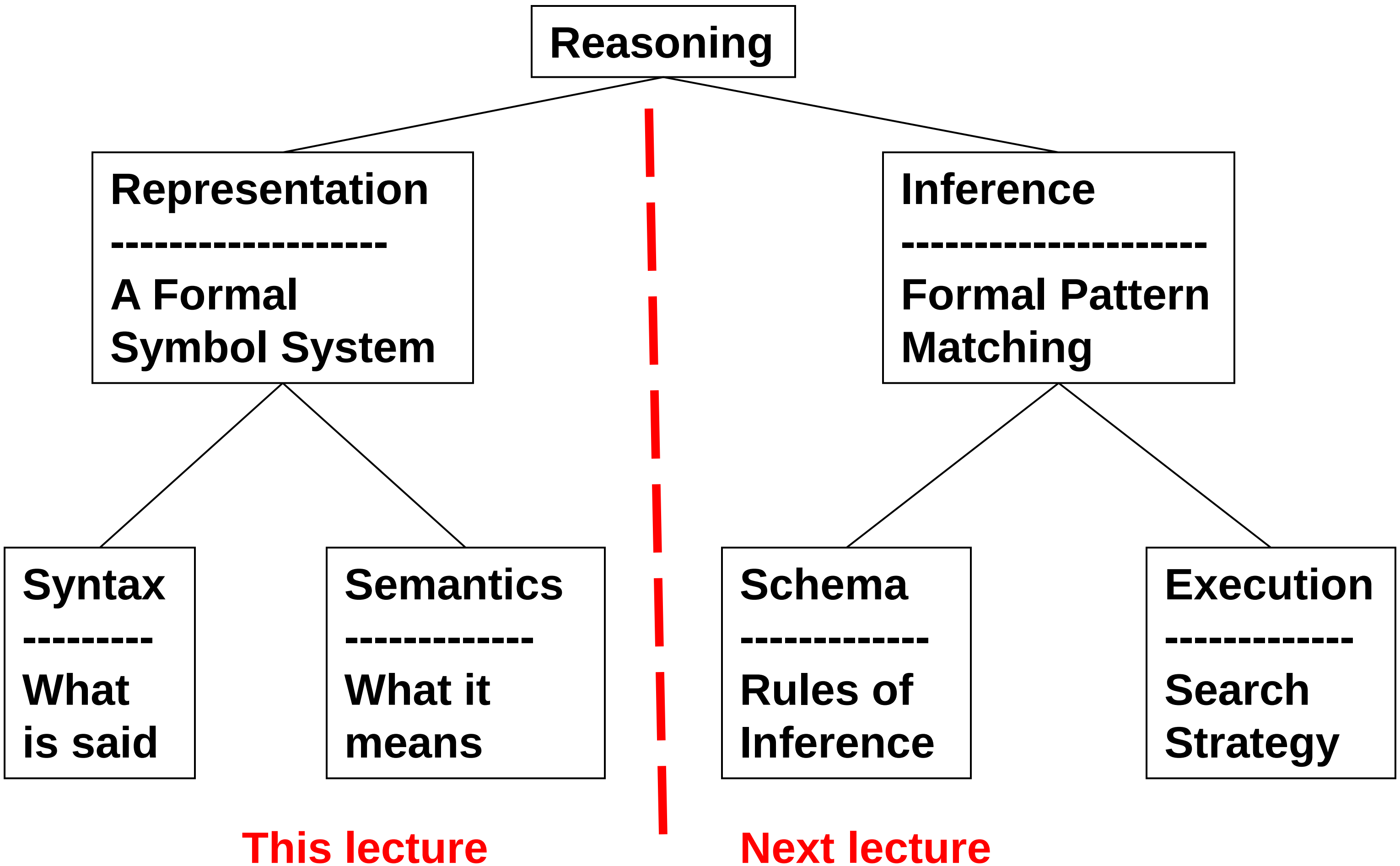
- Search?
  - Solve the problem of what to do.
- Learning?
  - Learn what to do.
- Logic and inference?
  - Reason about what to do.
  - Encoded knowledge/”expert” systems?
    - Know what to do.
- Modern view: It’s complex & multi-faceted.

Inference in Formal Symbol Systems: Ontology, Representation, Inference

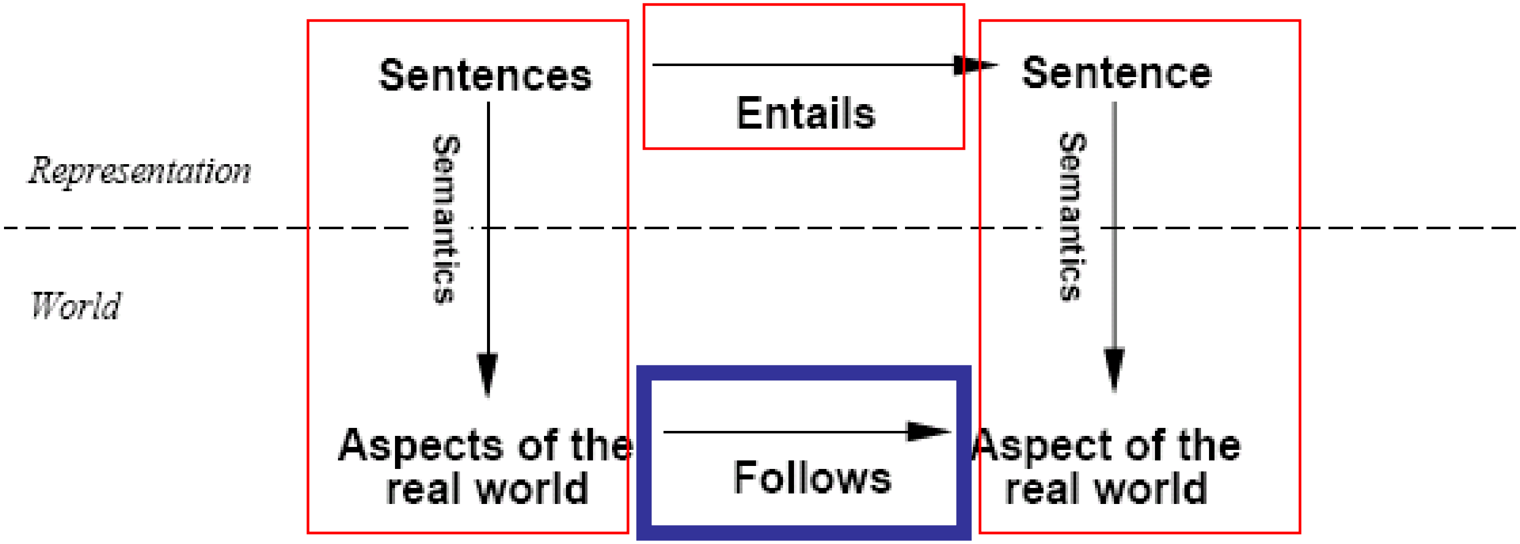
- **Formal Symbol Systems**
  - **Symbols** correspond to **things/ideas** in the world
  - **Pattern matching & rewrite** corresponds to **inference**

- **Ontology:** What exists in the world?
  - What must be represented?
- **Representation:** Syntax vs. Semantics
  - What’s Said vs. What’s Meant
- **Inference:** Schema vs. Mechanism
  - Proof Steps vs. Search Strategy

Ontology:
What kind of things exist in the world?
What do we need to describe and reason about?



Schematic perspective



If KB is true in the real world,

then any sentence α entailed by KB

is also true in the real world.

Why Do We Need Logic?

- Problem-solving agents were very inflexible: hard code every possible state.
- Search is almost always exponential in the number of states.
- Problem solving agents cannot infer unobserved information.
- We want an algorithm that reasons in a way that resembles reasoning in humans.

Knowledge-Based Agents

- **KB = knowledge base**
  - A set of sentences or facts
  - e.g., a set of statements in a logic language

- **Inference**
  - Deriving new sentences from old
  - e.g., using a set of logical statements to infer new ones

- **A simple model for reasoning**
  - Agent is told or perceives new evidence
    - E.g., A is true
  - Agent then infers new facts to add to the KB
    - E.g., KB = { A -> (B OR C) }, then given A and not C we can infer that B is true
    - B is now added to the KB even though it was not explicitly asserted, i.e., the agent inferred B

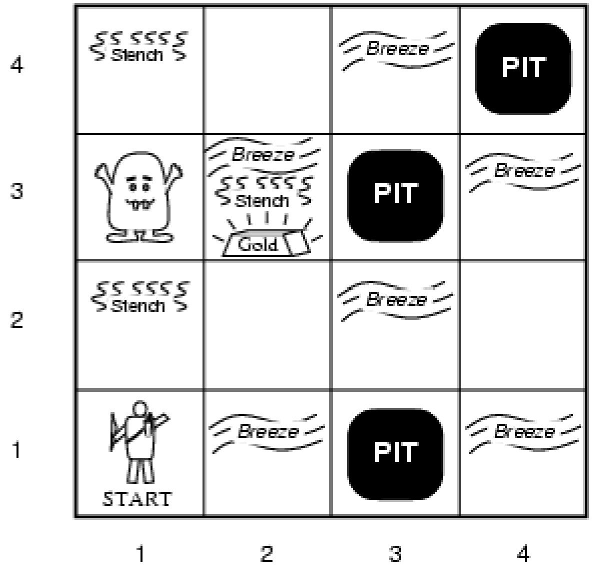
Types of Logics

- **Propositional logic** deals with specific objects and concrete statements that are either true or false
  - E.g., John is married to Sue.
- **Predicate logic (also called first order logic, first order predicate calculus)** allows statements to contain variables, functions, and quantifiers
  - For all X, Y: If X is married to Y then Y is married to X.
- **Fuzzy logic** deals with statements that are somewhat vague, such as this paint is grey, or the sky is cloudy.
- **Probability** deals with statements that are possibly true, such as whether I will win the lottery next week.
- **Temporal logic** deals with statements about time, such as John was a student at UC Irvine for four years.
- **Modal logic** deals with statements about belief or knowledge, such as Mary believes that John is married to Sue, or Sue knows that search is NP-complete.

Wumpus World PEAS description

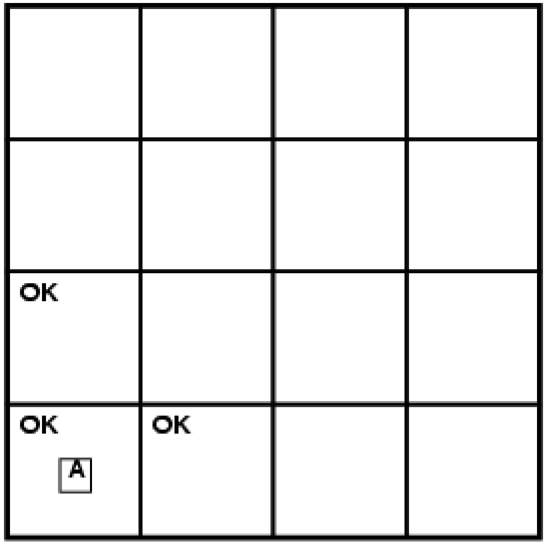
- Performance measure
  - gold: +1000, death: -1000
  - -1 per step, -10 for using the arrow
- Environment
  - Squares adjacent to wumpus are smelly
  - Squares adjacent to pit are breezy
  - Glitter iff gold is in the same square
  - Shooting kills wumpus if you are facing it
  - Shooting uses up the only arrow
  - Grabbing picks up gold if in same square
  - Releasing drops the gold in same square

Would DFS work well? A*?

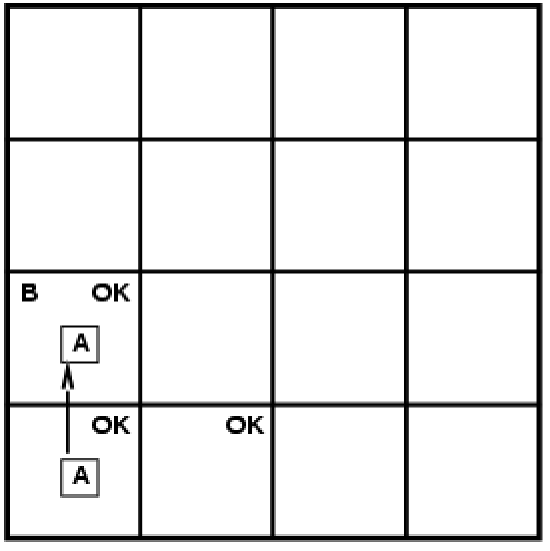


- Sensors: Stench, Breeze, Glitter, Bump, Scream
- Actuators: Left turn, Right turn, Forward, Grab, Release, Shoot

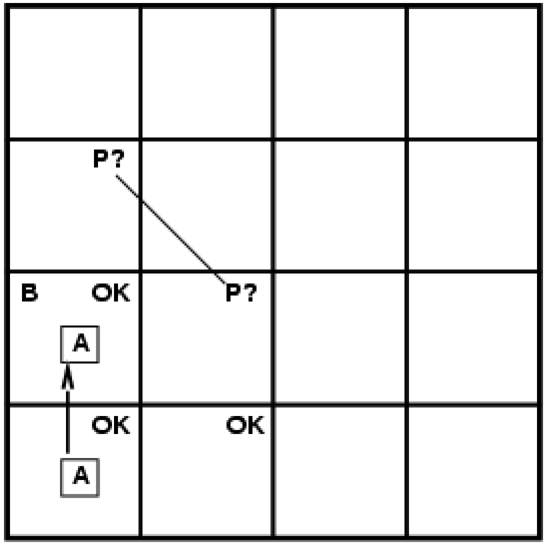
Exploring a wumpus world



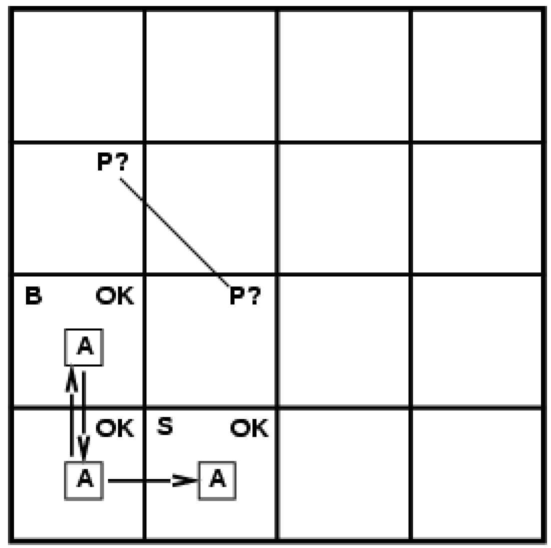
Exploring a wumpus world



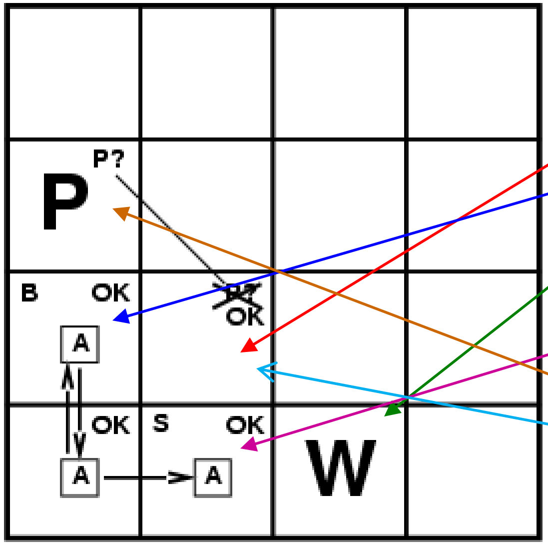
Exploring a wumpus world



Exploring a wumpus world



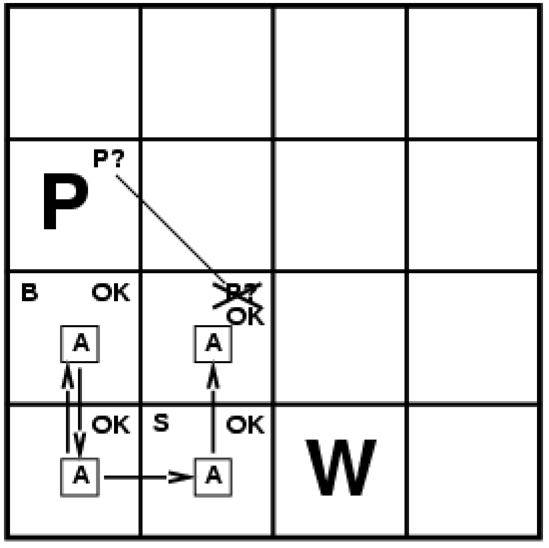
Exploring a Wumpus world



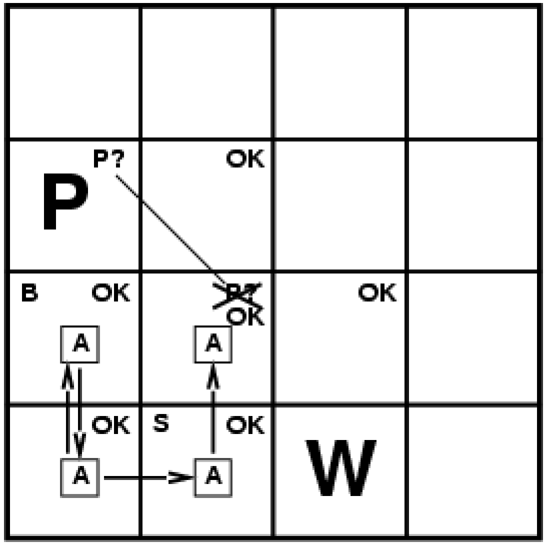
If the Wumpus were here, stench should be here. Therefore it is here.
Since, there is no breeze here, the pit must be there, and it must be OK here

We need rather sophisticated reasoning here!

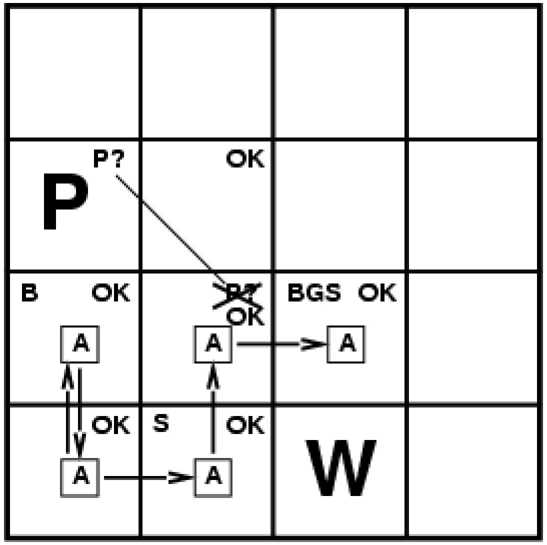
Exploring a wumpus world



Exploring a wumpus world



Exploring a wumpus world



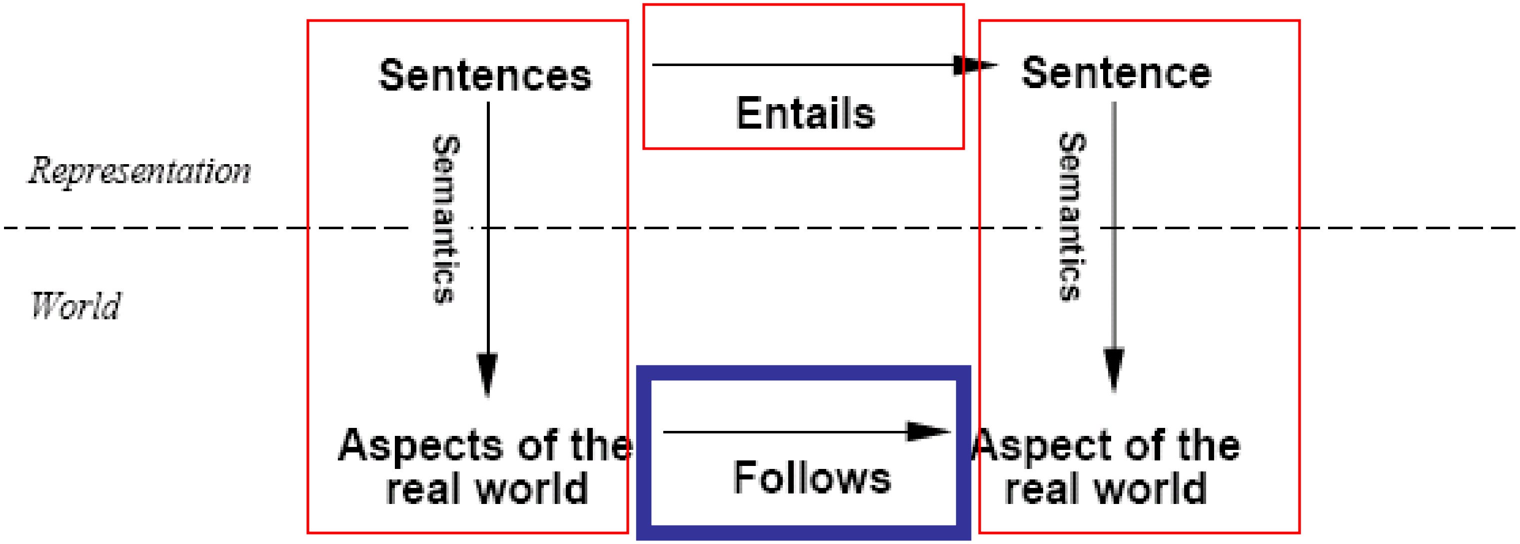
Logic

- We used logical reasoning to find the gold.
- Logics are formal languages for representing information such that conclusions can be drawn
- Syntax defines the sentences in the language
- Semantics define the "meaning" or interpretation of sentences;
  - connects symbols to real events in the world,
  - i.e., define truth of a sentence in a world

- E.g., the language of arithmetic
  - $x+2 \geq y$ is a sentence; $x2+y > \{\}$ is not a sentence; → syntax
  -
  - $x+2 \geq y$ is true in a world where $x = 7, y = 1$
  - $x+2 \geq y$ is false in a world where $x = 0, y = 6$

semantics

Schematic perspective



If KB is true in the real world,
then any sentence α entailed by KB
is also true in the real world.

Entailment

- Entailment means that one thing follows from another:

$$KB \models \alpha$$

- Knowledge base *KB* entails sentence α if and only if α is true in all worlds where *KB* is true
  - E.g., the KB containing "the Giants won and the Reds won" entails "The Giants won".
  - E.g., x+y = 4 entails 4 = x+y
  - E.g., "Mary is Sue's sister and Amy is Sue's daughter" entails "Mary is Amy's aunt."

Models

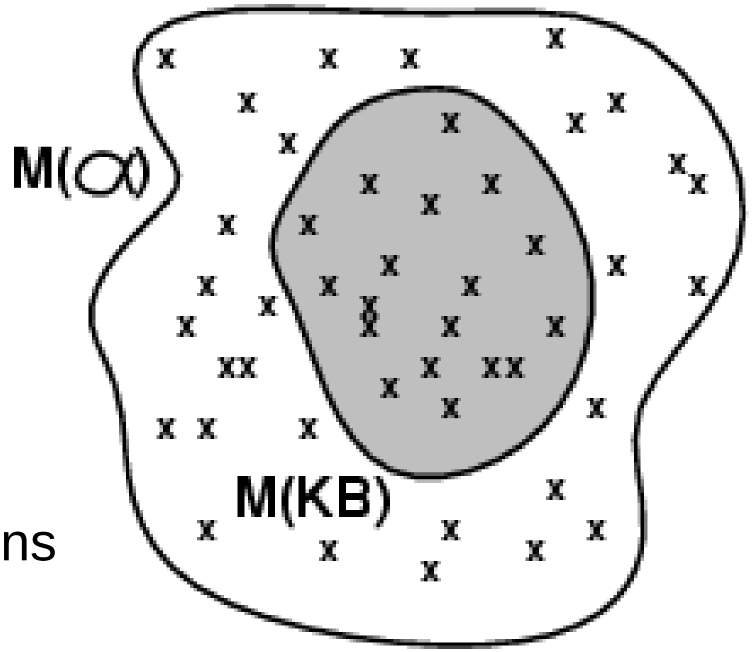
- Logicians typically think in terms of models, which are formally structured worlds with respect to which truth can be evaluated

- We say *m* is a model of a sentence α if α is true in *m*

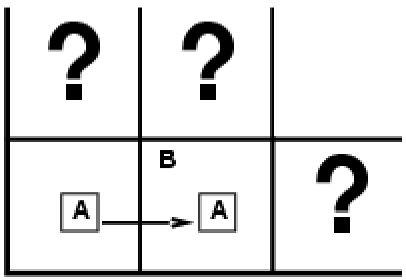
- *M(α)* is the set of all models of α

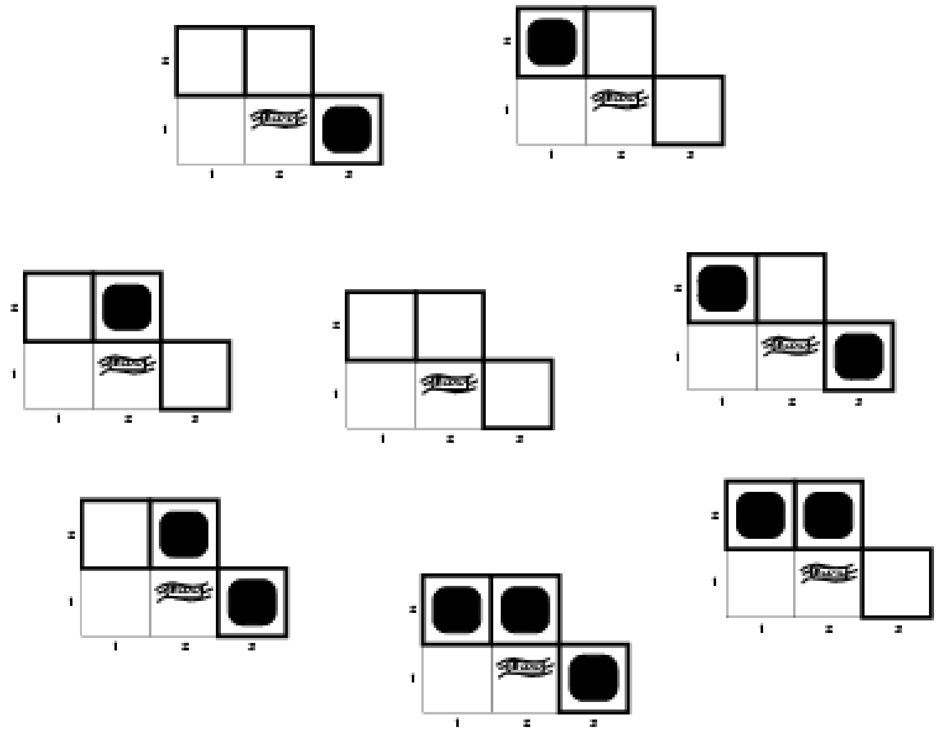
- Then KB $\models$ α iff *M(KB)* $\subseteq$ *M*(α)
  - E.g. *KB* = Giants won and Reds won α = Giants won

- Think of KB and α as collections of constraints and of models m as possible states. M(KB) are the solutions to KB and M(α) the solutions to α. Then, KB $\models$ α when all solutions to KB are also solutions to α.



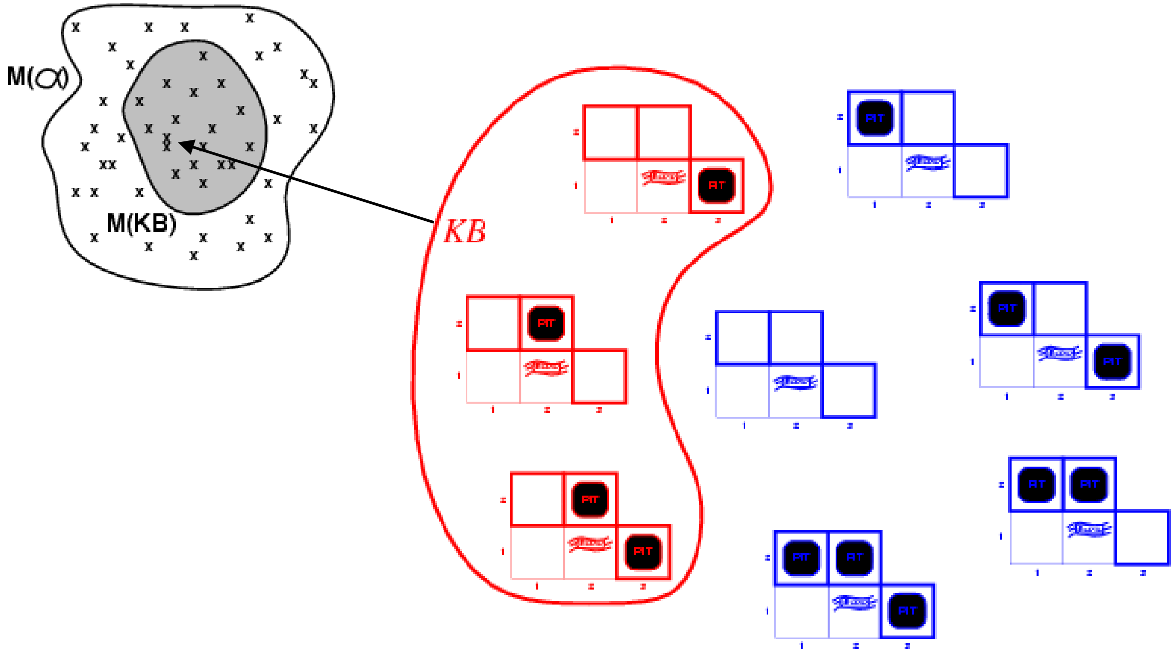
Wumpus models





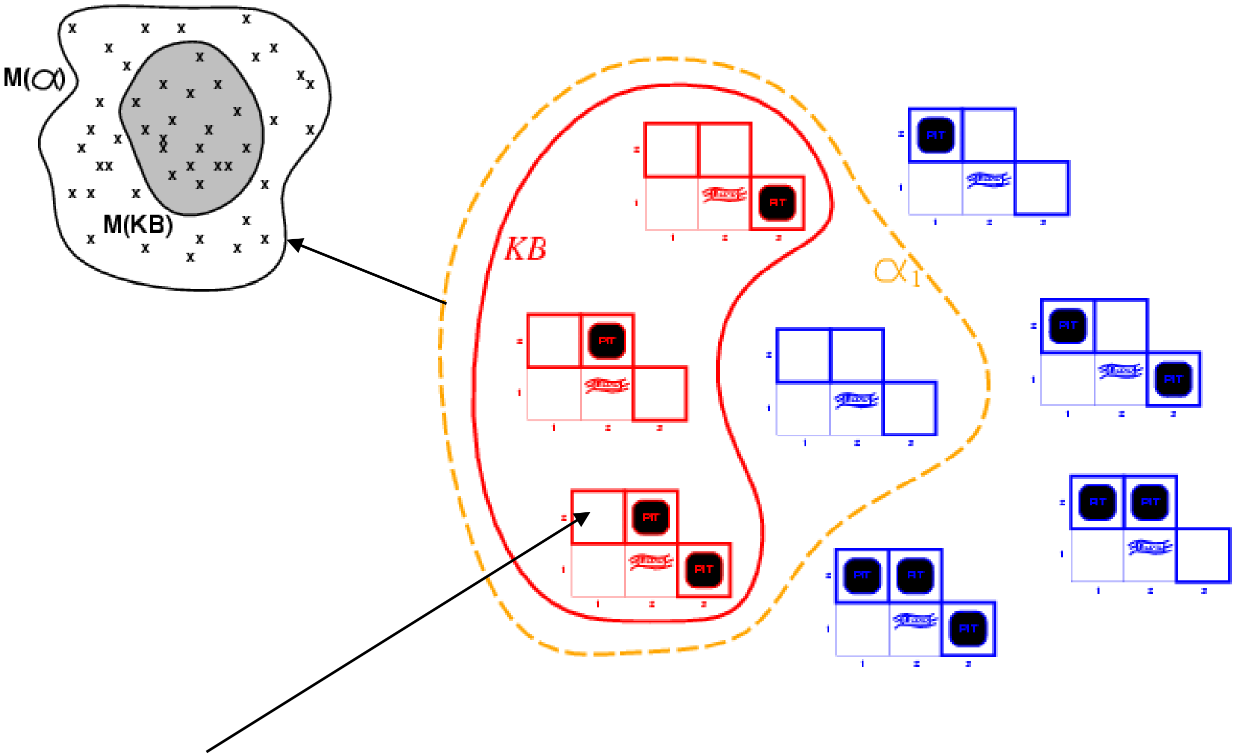
All possible models in this reduced Wumpus world.

Wumpus models



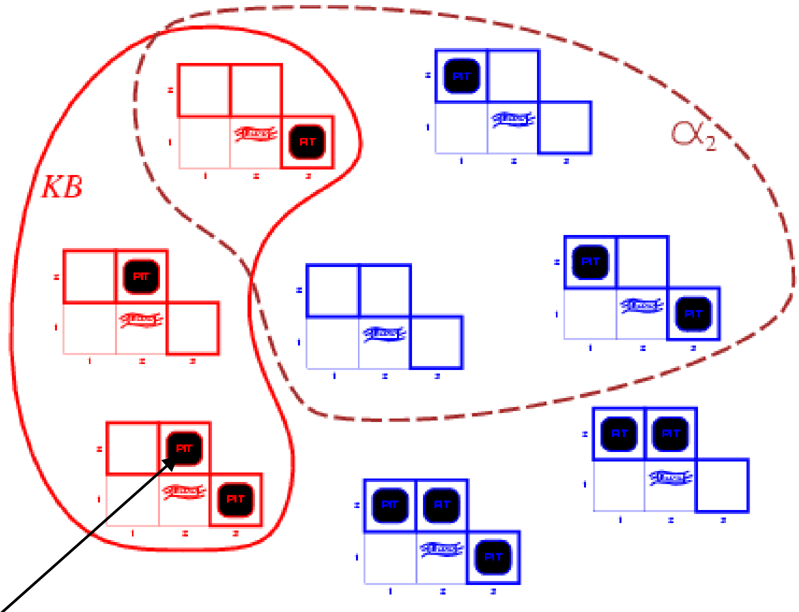
- *KB* = all possible wumpus-worlds consistent with the observations and the “physics” of the Wumpus world.

Wumpus models



α_1 = "[1,2] is safe", *KB* $\models \alpha_1$, proved by model checking

Wumpus models



α_2 = "[2,2] is safe", $KB \not\models \alpha_2$

Inference Procedures
(next lecture)

- $KB \vdash_i \alpha$ = sentence α can be derived from *KB* by procedure *i*
- Soundness: *i* is sound if whenever $KB \vdash_i \alpha$, it is also true that $KB \models \alpha$ *(no wrong inferences, but maybe not all inferences)*
- Completeness: *i* is complete if whenever $KB \models \alpha$, it is also true that $KB \vdash_i \alpha$ *(all inferences can be made, but maybe some wrong extra ones as well)*

Recap propositional logic: Syntax

- Propositional logic is the simplest logic – illustrates basic ideas

- The proposition symbols P_1, P_2 etc are sentences
  - If S is a sentence, $\neg S$ is a sentence (negation)
  - If S_1 and S_2 are sentences, $S_1 \wedge S_2$ is a sentence (conjunction)
  - If S_1 and S_2 are sentences, $S_1 \vee S_2$ is a sentence (disjunction)
  - If S_1 and S_2 are sentences, $S_1 \Rightarrow S_2$ is a sentence (implication)
  - If S_1 and S_2 are sentences, $S_1 \Leftrightarrow S_2$ is a sentence (biconditional)

Recap propositional logic: Semantics

Each model/world specifies true or false for each proposition symbol

E.g. $P_{1,2}$ $P_{2,2}$ $P_{3,1}$

false true false

With these symbols, 8 possible models, can be enumerated automatically.

Rules for evaluating truth with respect to a model *m*:

$\neg S$	is true iff	S is false	
$S_1 \wedge S_2$	is true iff	S_1 is true and	S_2 is true
$S_1 \vee S_2$	is true iff	S_1 is true or	S_2 is true
$S_1 \Rightarrow S_2$	is true iff	S_1 is false or	S_2 is true
i.e.,	is false iff	S_1 is true and	S_2 is false
$S_1 \Leftrightarrow S_2$	is true iff	$S_1 \Rightarrow S_2$ is true and	$S_2 \Rightarrow S_1$ is true

Simple recursive process evaluates an arbitrary sentence, e.g.,

$$\neg P_{1,2} \wedge (P_{2,2} \vee P_{3,1}) = \mathit{true} \wedge (\mathit{true} \vee \mathit{false}) = \mathit{true} \wedge \mathit{true} = \mathit{true}$$

Recap truth tables for connectives

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
$false$	$false$	$true$	$false$	$false$	$true$	$true$
$false$	$true$	$true$	$false$	$true$	$true$	$false$
$true$	$false$	$false$	$false$	$true$	$false$	$false$
$true$	$true$	$false$	$true$	$true$	$true$	$true$

OR: P or Q is true or both are true.
XOR: P or Q is true but not both.

Implication is always true when the premises are False!

Inference by enumeration
(generate the truth table)

- Enumeration of all models is sound and complete.
- For *n* symbols, time complexity is $O(2^n)$...
- We need a smarter way to do inference!
- In particular, we are going to infer new logical sentences from the data-base and see if they match a query.

Logical equivalence

- To manipulate logical sentences we need some rewrite rules.
- Two sentences are logically equivalent iff they are true in same models: $\alpha \equiv \beta$ iff $\alpha \models \beta$ and $\beta \models \alpha$

You need to know these !

$$
\begin{aligned}
(\alpha \land \beta) &\equiv (\beta \land \alpha) && \text{commutativity of } \land \\
(\alpha \lor \beta) &\equiv (\beta \lor \alpha) && \text{commutativity of } \lor \\
((\alpha \land \beta) \land \gamma) &\equiv (\alpha \land (\beta \land \gamma)) && \text{associativity of } \land \\
((\alpha \lor \beta) \lor \gamma) &\equiv (\alpha \lor (\beta \lor \gamma)) && \text{associativity of } \lor \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \lor \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \land (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \land \beta) &\equiv (\neg\alpha \lor \neg\beta) && \text{de Morgan} \\
\neg(\alpha \lor \beta) &\equiv (\neg\alpha \land \neg\beta) && \text{de Morgan} \\
(\alpha \land (\beta \lor \gamma)) &\equiv ((\alpha \land \beta) \lor (\alpha \land \gamma)) && \text{distributivity of } \land \text{ over } \lor \\
(\alpha \lor (\beta \land \gamma)) &\equiv ((\alpha \lor \beta) \land (\alpha \lor \gamma)) && \text{distributivity of } \lor \text{ over } \land
\end{aligned}
$$

Validity and satisfiability

A sentence is valid if it is true in all models,

e.g., *True*, $A \vee \neg A$, $A \Rightarrow A$, $(A \wedge (A \Rightarrow B)) \Rightarrow B$

Validity is connected to inference via the Deduction Theorem:

$KB \models \alpha$ if and only if $(KB \Rightarrow \alpha)$ is valid

A sentence is satisfiable if it is true in some model

e.g., $A \vee B$, C

A sentence is unsatisfiable if it is false in all models

e.g., $A \wedge \neg A$

Satisfiability is connected to inference via the following:

$KB \models \alpha$ if and only if $(KB \wedge \neg \alpha)$ is unsatisfiable

(there is no model for which KB=true and α is false)

Summary (Part I)

- Logical agents apply inference to a knowledge base to derive new information and make decisions
- Basic concepts of logic:
  - syntax: formal structure of sentences
  - semantics: truth of sentences wrt models
  - entailment: necessary truth of one sentence given another
  - inference: deriving sentences from other sentences
  - soundness: derivations produce only entailed sentences
  - completeness: derivations can produce all entailed sentences
  - valid: sentence is true in every model (a tautology)
- Logical equivalences allow syntactic manipulations
- Propositional logic lacks expressive power
  - Can only state specific facts about the world.
  - Cannot express general rules about the world (use First Order Predicate Logic)