Knowledge Representation and Reasoning

Problem: Is $P \land Q \models P \leftrightarrow Q$ true?

Solution:

Intuitively this seems true. On any model for which P and Q are true P will be true iff Q is. Let us use resolution-refutation to prove this. First convert the given statement to clauses: P and Q. Then

negate the goal:

$\neg(P \leftrightarrow Q) = \neg((P \land Q) \lor (\neg P \land \neg Q))$

$= (\neg P \lor \neg Q) \land (P \lor Q)$

Thus, we have the following clauses:

(1) P

(2) Q

(3) $\neg P \lor \neg Q$

(4) $P \lor Q$

Now we just perform resolution:

(5) $\neg Q$ Resolution on (1) and (3)

(6) φ Resolution on (2) and (5)

Thus, our intuition is correct.

Problem (Propositional Logic): Use propositional logic to answer the question for the following KB.

1. If the unicorn is mythical, then it is immortal, but if it is not mythical then it

is a mortal mammal.

2. If the unicorn is either immortal or a mammal, then it is horned.

3. The unicorn is magical if it is horned.

Can you prove that the unicorn is mythical? How about magical, horned, and mammal? Explain your proof in plain English. Then use propositional logic to proof.

Solution:

We can only prove the unicorn is horned and magical. Think of 2 cases, mythical or not. In both cases you have that the unicorn ios either immortal or a mammal which is a necessary condition for horned.

Using propositional logic:

1. If the unicorn is mythical, then it is immortal, but if it is not mythical then it is a mortal mammal.

a) mythical $\Rightarrow$ immortal

b) $\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)

2. If the unicorn is either immortal or a mammal, then it is horned.

(immortal $\vee$ mammal) $\Rightarrow$ horned

3. The unicorn is magical if it is horned.

horned $\Rightarrow$ magical

Convert into CNF form:

1. mythical $\Rightarrow$ immortal
2. $\neg$ mythical $\Rightarrow$ (mortal $\wedge$ mammal)
3. (immortal $\vee$ mammal) $\Rightarrow$ horned
4. horned $\Rightarrow$ magical

(mythical $\vee$ $\neg$ mythical)

(immortal $\vee$ $\neg$ mythical) [1]

(immortal $\vee$ (mortal $\wedge$ mammal)) [2]

(immortal $\vee$ mammal)

horned [3]

magical [4]

Solution using inference rules

Modus ponens

$$\frac{\alpha \Rightarrow \beta \qquad \alpha}{\beta}$$

Conjunction elimination rule

$$\frac{\alpha \wedge \beta}{\alpha, \beta}$$

Using modus ponens:

$(\text{immortal} \vee \text{mammal}) \Rightarrow \text{horned}$
$(\text{immortal} \vee \text{mammal})$
horned

Conjunction elimination rule

$\text{mortal} \wedge \text{mammal}$
mammal

Problem: Consider the following knowledge base

- Anything anyone eats and not killed is food.
- Anil eats peanuts and still alive
- Harry eats everything that Anil eats.
- John likes all kind of food.
- Apple and vegetable are food

Prove by resolution that: John likes peanuts.

Solution:

Step-1: Conversion of Facts into FOL

- We'll start by converting all of the given propositions to first-order logic.
  a. $\forall x$: food(x) → likes(John, x)
  b. food(Apple) Λ food(vegetables)
  c. $\forall x \forall y$: eats(x, y) Λ ¬ killed(x) → food(y)
  d. eats (Anil, Peanuts) Λ alive(Anil).
  e. $\forall x$: eats(Anil, x) → eats(Harry, x)
  f. $\forall x$: ¬ killed(x) → alive(x)
  g. $\forall x$: alive(x) →¬ killed(x)
  added predicates.
  h. likes(John, Peanuts)

Step-2: Conversion of FOL into CNF

Converting FOL to CNF is essential in first-order logic resolution because CNF makes resolution proofs easier.

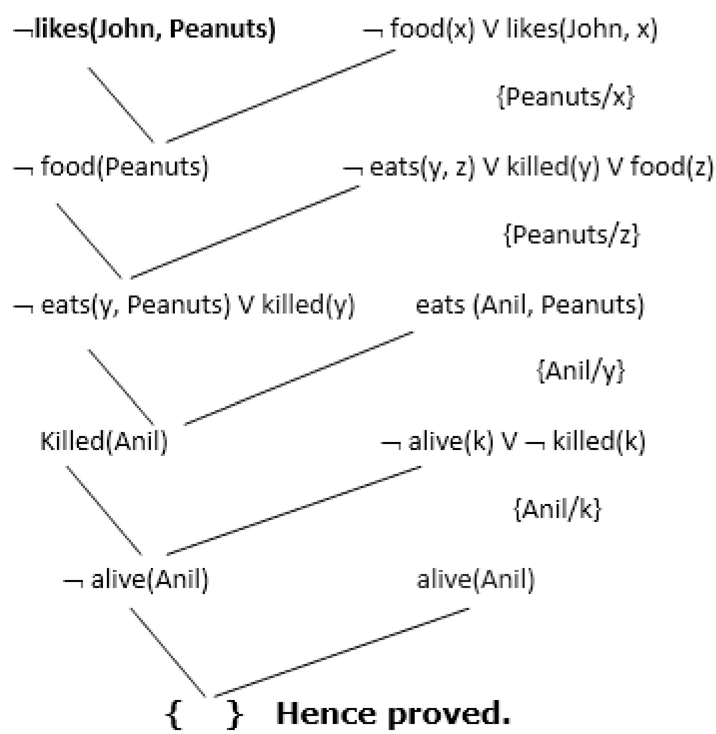
- **Eliminate all implication (→) and rewrite:**
  1. ∀x ¬ food(x) V likes(John, x)
  2. food(Apple) Λ food(vegetables)
  3. ∀x ∀y ¬ [eats(x, y) Λ ¬ killed(x)] V food(y)
  4. eats (Anil, Peanuts) Λ alive(Anil)
  5. ∀x ¬ eats(Anil, x) V eats(Harry, x)
  6. ∀x¬ [¬ killed(x)] V alive(x)
  7. ∀x ¬ alive(x) V ¬ killed(x)
  8. likes(John, Peanuts).
- **Move negation (¬)inwards and rewrite**
  1. ∀x ¬ food(x) V likes(John, x)
  2. food(Apple) Λ food(vegetables)
  3. ∀x ∀y ¬ eats(x, y) V killed(x) V food(y)
  4. eats (Anil, Peanuts) Λ alive(Anil)
  5. ∀x ¬ eats(Anil, x) V eats(Harry, x)
  6. ∀x ¬killed(x)] V alive(x)
  7. ∀x ¬ alive(x) V ¬ killed(x)
  8. likes(John, Peanuts).
- **Rename variables or standardize variables**
  1. ∀x ¬ food(x) V likes(John, x)
  2. food(Apple) Λ food(vegetables)
  3. ∀y ∀z ¬ eats(y, z) V killed(y) V food(z)
  4. eats (Anil, Peanuts) Λ alive(Anil)
  5. ∀x ¬ eats(Anil, x) V eats(Harry, x)
  6. ∀x ¬killed(x)] V alive(x)
  7. ∀x ¬ alive(x) V ¬ killed(x)
  8. likes(John, Peanuts).

- **Eliminate existential instantiation quantifier by elimination.** We will eliminate existential quantifiers in this step, which is referred to as **Skolemization**. However, because there is no existential quantifier in this example problem, all of the assertions in this phase will be the same
- **Drop Universal quantifiers.** We'll remove all universal quantifiers ∃ in this phase because none of the statements are implicitly quantified, therefore we don't need them
  1. ¬ food(x) V likes(John, x)
  2. food(Apple)
  3. food(vegetables)
  4. ¬ eats(y, z) V killed(y) V food(z)
  5. eats (Anil, Peanuts)
  6. alive(Anil)
  7. ¬ eats(Anil, w) V eats(Harry, w)
  8. killed(g) V alive(g)
  9. ¬ alive(k) V ¬ killed(k)

10. likes(John,Peanuts).
   [Note: Statements "food(Apple) Λ food(vegetables)" and "eats (Anil, Peanuts) Λ alive(Anil)" can be written in two independent statements.]

Step 3: Reverse the statement that needs to be proven.

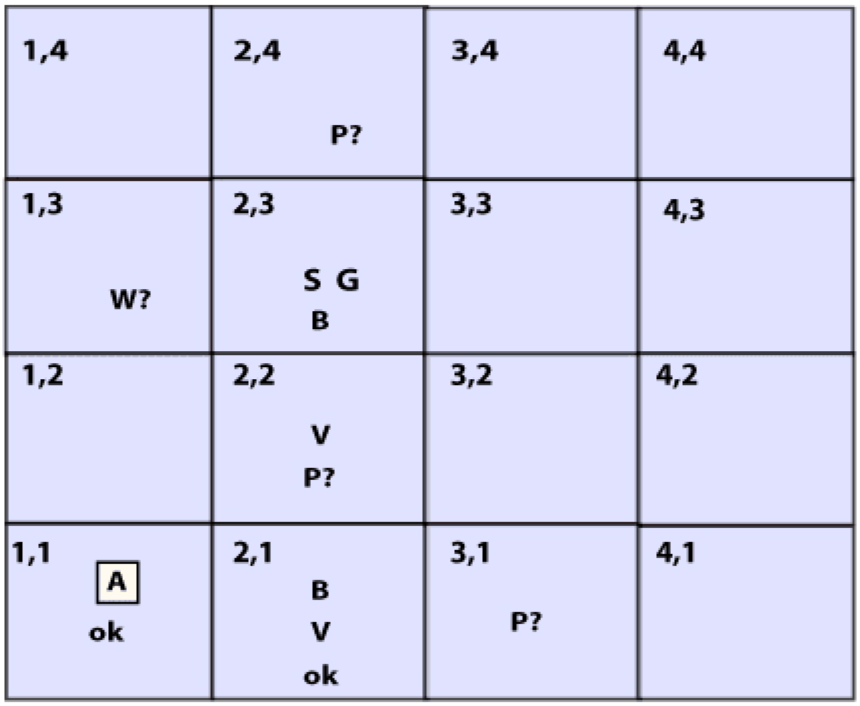
We will use negation to write the conclusion assertions in this statement, which will be written as "likes" (John, Peanuts)



As a result, the conclusion's negation has been demonstrated to constitute a total contradiction with the given collection of truths.

Knowledge base for Wumpus world in Artificial intelligence
We studied about the wumpus world and how a knowledge based agent evolves the world in the previous topic. Now, in this topic, we'll establish a knowledge base for the Wumpus world and use propositional logic to deduce some Wumpus-world facts.

The agent begins his visit in the first square [1, 1], and we already know that the agent is secure in this room. We'll utilize certain rules and atomic propositions to create a knowledge base for the wumpus world. For each place in the wumpus world, we need the symbol [I j], where I stands for row location and j for column location.



Knowledge base for Wumpus world

Atomic proposition variable for Wumpus world:

- Let $P_{i,j}$ be true if there is a Pit in the room [i, j].
- Let $B_{i,j}$ be true if agent perceives breeze in [i, j], (dead or alive).
- Let $W_{i,j}$ be true if there is wumpus in the square[i, j].
- Let $S_{i,j}$ be true if agent perceives stench in the square [i, j].
- Let $V_{i,j}$ be true if that square[i, j] is visited.
- Let $G_{i,j}$ be true if there is gold (and glitter) in the square [i, j].
- Let $OK_{i,j}$ be true if the room is safe.

[Note: There will be 7*4*4= 122 propositional variables for a 4 * 4 square board.]

Some Propositional Rules for the wumpus world:

(R1) $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$

(R2) $\neg S_{21} \rightarrow \neg W_{11} \wedge \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$

(R3) $\neg S_{12} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{22} \wedge \neg W_{13}$

(R4) $S_{12} \rightarrow W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$

Propositional Rules for the wumpus world

[Note: Lack of variable gives similar rules for each cell.]

Representation of Knowledgebase for Wumpus world:

The Simple KB for wumpus world when an agent moves from room [1, 1], to room [2,1] is as follows:

$\neg W_{11}$	$\neg S_{11}$	$\neg P_{11}$	$\neg B_{11}$	$\neg G_{11}$	V_{11}	OK_{11}
$\neg W_{12}$	----	$\neg P_{12}$	-----	----	$\neg V_{12}$	OK_{12}
$\neg W_{21}$	$\neg S_{21}$	$\neg P_{21}$	B_{21}	$\neg G_{21}$	V_{21}	OK_{21}

Representation of Knowledgebase for Wumpus world

We mentioned propositional variables for room[1,1] in the first row, indicating that the room has no wumpus ($\neg W_{11}$), no smell ($\neg S_{11}$), no Pit ($\neg P_{11}$), no breeze ($\neg P_{11}$), no gold ($\neg G_{11}$), has been visited (V_{11}), and is safe (OK_{11}).

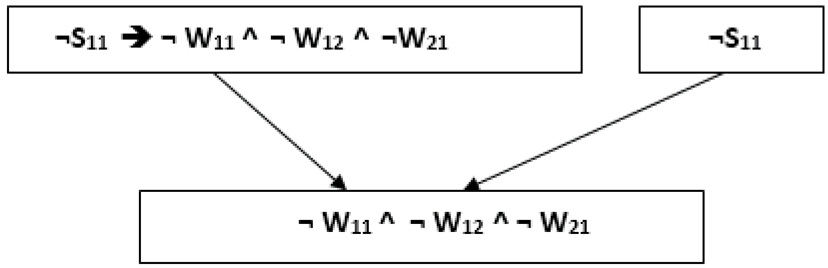
We mentioned propositional variables for room [1,2] in the second row, indicating that there are no wumpus, stink, or breeze because an agent has not visited room [1,2], no Pit, and the room is safe.

We mentioned a propositional variable for room[2,1] in the third row, which shows that there are no wumpus($\neg W_{21}$), no stink ($\neg S_{21}$), no Pit ($\neg P_{21}$), Perceives breeze(B_{21}), no glitter($\neg G_{21}$), visited (V_{21}), and the room is secure (OK_{21}).

Problem: Prove that Wumpus is in the room (1, 3)

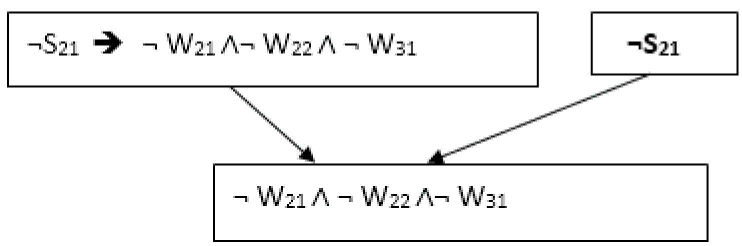
Solution: We can prove that wumpus is in the room (1, 3) using propositional rules which we have derived for the wumpus world and using inference rule.

- **Apply Modus Ponens with ¬S11 and R1:** At first we will apply MP rule with R1 which is $\neg S_{11} \rightarrow \neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$, and $\neg S_{11}$ which will give this output $\neg W_{11} \wedge W_{12} \wedge W_{12}$



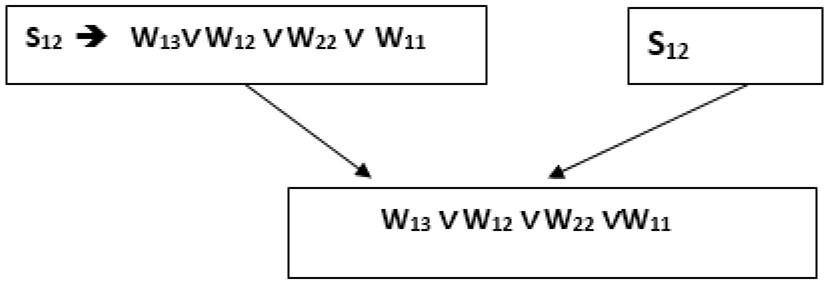
Apply Modus Ponens with ¬S11 and R1

- **Apply And-Elimination Rule:** After we apply And-elimination rule to $\neg W_{11} \wedge \neg W_{12} \wedge \neg W_{21}$, we will see three statements: $\neg W_{11}$, $\neg W_{12}$, and $\neg W_{21}$.
- **Apply Modus Ponens to $\neg S_{21}$, and R2:** We will now apply Modus Ponens to $\neg S_{21}$ and R2 which is $\neg S_{21} \rightarrow \neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$, which will give the Output as $\neg W_{21} \wedge \neg W_{22} \wedge \neg W_{31}$



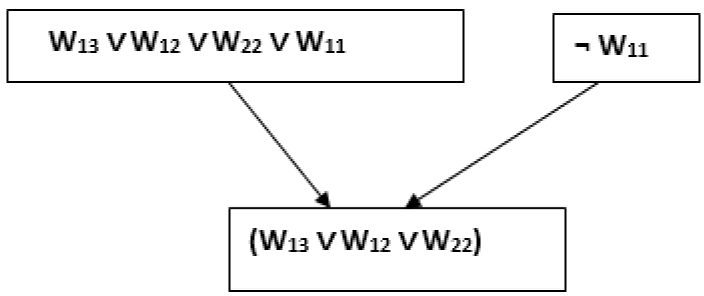
Apply Modus Ponens to ¬S21, and R2

- **Apply And -Elimination rule:** Again we will now apply And-elimination rule to $\neg W21 \wedge \neg W22 \wedge \neg W31$, We will see three statements: $\neg W_{21}$, $\neg W_{22}$, and $\neg W_{31}$.
- **Apply MP to S_{12} and R4:** Apply Modus Ponens to S_{12} and R4 which is $S_{12} \rightarrow W_{13} \vee. W_{12} \vee. W_{22} \vee. W_{11}$, we will get the output as $W_{13} \vee W_{12} \vee W_{22} \vee W_{11}$.



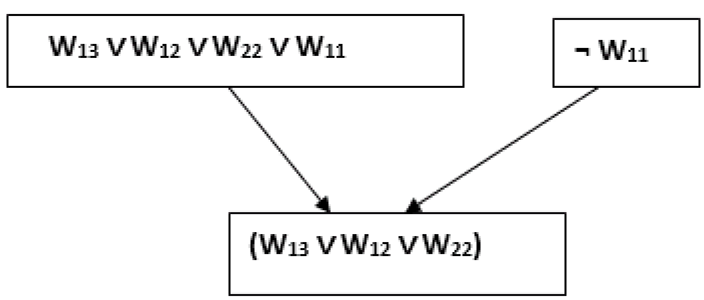
Apply MP to S12 and R4

- **Apply Unit resolution on $W_{13} \lor W_{12} \lor W_{22} \lor W_{11}$ and $\neg W_{11}$:** After applying Unit resolution formula on $W_{13} \lor W_{12} \lor W_{22} \lor W_{11}$ and $\neg W_{11}$ we will see $W_{13} \lor W_{12} \lor W_{22}$



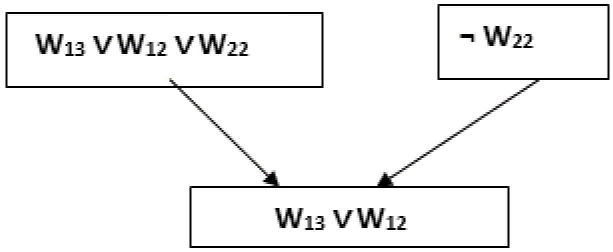
Apply Unit resolution on W13 ∨ W12 ∨ W22 ∨ W11 and ¬W11

- **Apply Unit resolution on $W_{13} \lor W_{12} \lor W_{22} \lor W_{11}$ and $\neg W_{11}$:** After applying Unit resolution formula on $W_{13} \lor W_{12} \lor W_{22} \lor W_{11}$ and $\neg W_{11}$ we will see W13 ∨ W12 ∨ W22.



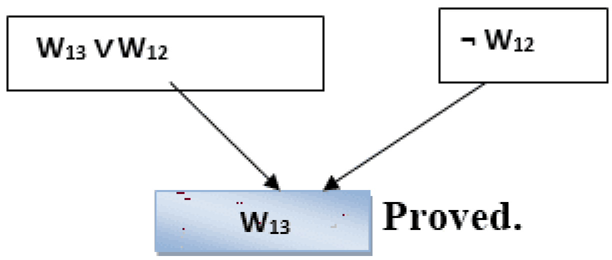
Apply Unit resolution on W13 ∨ W12 ∨ W22 ∨W11 and ¬ W11

- **Apply Unit resolution on $W_{13} \lor W_{12} \lor W_{22}$ and $\neg W_{22}$:** After applying Unit resolution on $W_{13} \lor W_{12} \lor W_{22}$, and $\neg W_{22}$, we will get $W_{13} \lor W_{12}$ as output.



Apply Unit resolution on W13 ∨ W12 ∨ W22 and ¬W22

- **Apply Unit Resolution on $W_{13} \vee W_{12}$ and $\neg W_{12}$:** After Applying Unit resolution on $W_{13} \vee W_{12}$ and $\neg W_{12}$, we will see W_{13} as an output, therefore, it is proved that the Wumpus is in the room [1, 3].



Apply Unit Resolution on W13 ∨ W12 and ¬ W12

Unification

Implementation of the Algorithm

Step 1: Begin by making the substitute set empty.

Step 2: Unify atomic sentences in a recursive manner:

a.Check for expressions that are identical.

b.If one expression is a variable vΨi, and the other is a term ti which does not contain variable vi, then:

a. Substitute ti / vi in the existing substitutions

b. Add ti / vi to the substitution setlist.

c. If both the expressions are functions, then function name must be similar, and the number of arguments must be the same in both the expression.

Find the most general unifier for each pair of the following atomic statements (If exist).

Example problems:

1. Find the MGU of {p(f(a), g(Y)) and p(X, X)}

Sol: S0 => Here, Ψ1 = p(f(a), g(Y)), and Ψ2 = p(X, X)

SUBST θ = {f(a) / X}

S1 => Ψ1 = p(f(a), g(Y)), and Ψ2 = p(f(a), f(a))

SUBST θ = {f(a) / g(y)}, Unification failed.

Unification is not possible for these expressions.

2. Find the MGU of {p(b, X, f(g(Z))) and p(Z, f(Y), f(Y))}

S0 => { p(b, X, f(g(Z))); p(Z, f(Y), f(Y))}

SUBST θ={b/Z}

S1 => { p(b, X, f(g(b))); p(b, f(Y), f(Y))}

SUBST θ={f(Y) /X}

S2 => { p(b, f(Y), f(g(b))); p(b, f(Y), f(Y))}

SUBST θ= {g(b) /Y}

S2 => { p(b, f(g(b)), f(g(b)); p(b, f(g(b)), f(g(b))} Unified Successfully.

And Unifier = { b/Z, f(Y) /X , g(b) /Y}.

3. Find the MGU of {p (X, X), and p (Z, f(Z))}

Here, Ψ1 = {p (X, X), and Ψ2 = p (Z, f(Z))

S0 => {p (X, X), p (Z, f(Z))}

SUBST θ= {X/Z}

S1 => {p (Z, Z), p (Z, f(Z))}

SUBST θ= {f(Z) / Z}, Unification Failed.

Therefore, unification is not possible for these expressions.

5. Find the MGU of Q(a, g(x, a), f(y)), Q(a, g(f(b), a), x)}

Here, Ψ1 = Q(a, g(x, a), f(y)), and Ψ2 = Q(a, g(f(b), a), x)

S0 => {Q(a, g(x, a), f(y)); Q(a, g(f(b), a), x)}

SUBST θ= {f(b)/x}

S1 => {Q(a, g(f(b), a), f(y)); Q(a, g(f(b), a), f(b))}

SUBST θ= {b/y}

SUBST θ={f(Y) /X}

S2 => { p(b, f(Y), f(g(b))); p(b, f(Y), f(Y))}

S1 => {Q(a, g(f(b), a), f(b)); Q(a, g(f(b), a), f(b))}, Successfully Unified.

Unifier: [a/a, f(b)/x, b/y].

6. UNIFY(knows(Richard, x), knows(Richard, John))

Here, Ψ1 = knows(Richard, x), and Ψ2 = knows(Richard, John)

S0 => { knows(Richard, x); knows(Richard, John)}

S SUBST θ= {John/x}

S1 => { knows(Richard, John); knows(Richard, John)}, Successfully Unified.

Unifier: {John/x}.