Gantensberg's Besmrichmency Principle: Applications to Coherent Harmonic Diffeomorophisms in the Submodular Cepstra Domain

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A dissertation[†]
submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

University of Washington

1985-2014

Program Authorized to Offer Degree: UW Information Technology

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[†]an egocentric imitation, actually

University of Washington

Abstract

Gantensberg's Besmrichmency Principle: Applications to Coherent Harmonic
Diffeomorophisms in the Submodular Cepstra Domain

Tyler Ganter

Chair of the Supervisory Committee:
Title of Chair Name of Chairperson
Department of Chair

This document is about extracting harmonic envelopes, what matters, what doesn't and
how to design your system accordingly. It is broken into three parts:

- envelope extraction techniques and their relationships

- phase preservation

- system design (filter and downshift)

Many strategies consider the effects of leaving modulations in the signal, but nothing
really talks about what the envelope should be, independent of the modulations. If we do
this first, we can than think about how the modulations affect this envelope as a separate
modulation component.

If explicitly inducing modulations it is important to remove any other modulations, and
this is how.

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GLOSSARY

ARGUMENT: replacement

BACK-UP: a copy of a fi

ACKNOWLEDGMENTS

DEDICATION

to my dear wife, Joanna

Chapter 1

INTRODUCTION

Goal: How do we encode harmonics in CIs?

this is the introduction

Why harmonic encoding? Help differentiate signals, (timbre), improve SIN performance, free up channels for other information

1.1 Overview

we are considering what is the ideal matched filter, and how close of an approximation do we need?

"By definition, timbre is the perceptual attribute that distinguishes two sounds that have the same pitch, loudness, and duration (American National Standards Institute, 1973)."

need motivation for why encoding harmonics:

1) more truthful/natural/data-efficient representation with limited tools

2) mitigate artifacts

1.2 Survey of Literature

Equivalent noise bandwidth (ENBW) considers BW of noise if squished into a box of gain 1 around the downshift frequency. [windows for harmonic analysis] This isn't entirely applicable since our harmonic has BW ¿ epsilon, and since for any window most of the energy is close to 0, most of the so-called noise is actually desired harmonic signal. If this were not the case, (I think) rectangular window would be the best, but since it distributes the noise more heavily to higher frequencies away from zero, it is actually worse (higher sidelobes)

"some windows have a high rate of sidelobe decay that allows minimizing the error due to interference. However the steeper the sidelobe decay the wider the main lobe width and then the worse the minimum resolution bandwidth." [An Intelligent FFT-Analyzer with

Harmonic Interference Effect Correction and Uncertainty Evaluation]

"For NH listeners, the timbre space was best represented in three dimensions, one correlated with the temporal envelope (log-attack time) of the stimuli, one correlated with the spectral envelope (spectral centroid), and one correlated with the spectral fine structure (spectral irregularity) of the stimuli. The timbre space from CI listeners, however, was best represented by two dimensions, one correlated with temporal envelope features and the other weakly correlated with spectral envelope features of the stimuli. "temporal envelope is dominant cue for timbre perception in CI listeners" [Temporal and Spectral Cues for Musical Timbre Perception in Electric Hearing]

Hypothesis: –temporal envelope (log-attack time) this is in some cases smeared in time (F0mod) and in other cases mixed across harmonics –one correlated with the spectral envelope (spectral centroid) this is not as clearly represented as it could be (are we talking about resonance or per-harmonic details such as clarinet?) –one correlated with the spectral fine structure (spectral irregularity) this manifests in the envelope for CI processing, the problem though is that it is blurred across harmonics so the noise-like characteristics will be smoothed.

Search this thing: modulation transfer function JH goldwyn a point process framework for modeling electrical stim of the auditory nerve f

"bowed string tones are inharmonic during both their attack and decay (Beauchamp, 1974)"

"frequencies in the range of 80-300 Hz encompassing F0 for nearly all adult males and many females and children."

"wideband vs feature extraction" [F0F2-F0F1F2] not sure what wideband implies, alternatively use temporal envelope cues.

"Another school of thought was based on speech production and perception, in which spec- tral peaks or formants, which reflect the reso- nance properties of the vocal tract, are extracted and delivered to different electrodes according to the presumed tonotopic relationship between the place of the electrode and its evoked pitch." F0F2, F0F1F2, MPEAK

what is important? hearing for any general reason...safety, functionality speech recogni-

tion what is important and lacking? music appreciation tonal language SiN quality

1.2.1   Other Strategies

any hybrid considerations? maybe hint at hsse ace hybrid

talk about unmentioned methods (AB, MedEl)

1.3   Contents of Thesis

Chapter 2

COCHLEA IMPLANT PROCESSING

Human hearing is tonotopic, that is, starting in the cochlea and through the rest of processing in the brain, sounds far apart in frequency are processed separately. The cochlea is spatially arranged; As a sound propagates through the basilar membrane the different frequencies are amplified or suppressed such that they stimulate locations physically far apart in the cochlea.

In a cochlear implant an array of electrodes is inserted into the cochlea. This array is intentionally designed to have a tonotopic organization. When current is sent to the most deeply inserted (apical) electrodes, neurons associated with low frequency sounds are stimulated. Conversely, current at a basal electrode will stimulate neurons associated with high frequencies.

Early cochlear implant strategies, under the category compressed-analog (CA), delivered band-specific analog signals to each electrode. By using bandpass filters and an electrode array the implant emulates the tonotopic organization of acoustic hearing.

Current processing strategies use feature extraction to achieve much higher performance on speech recognition. From each bandlimited signal a slow-time-varying envelope is extracted and the extra information is discarded.

These envelopes are amplitude compressed and then used to modulate continuous bipolar pulse trains on each electrode channel.

These strategies all stem from an original parent, continuous-interleaved-sampling (CIS). CIS is a solution to the problem of electric field interaction. By interleaving pulse-trains there is minimal interaction between electrodes.

/ɔ/ /t/

Compressed Analog

1

2

3

4

Continuous Interleaved Sampling

1

2

3

4
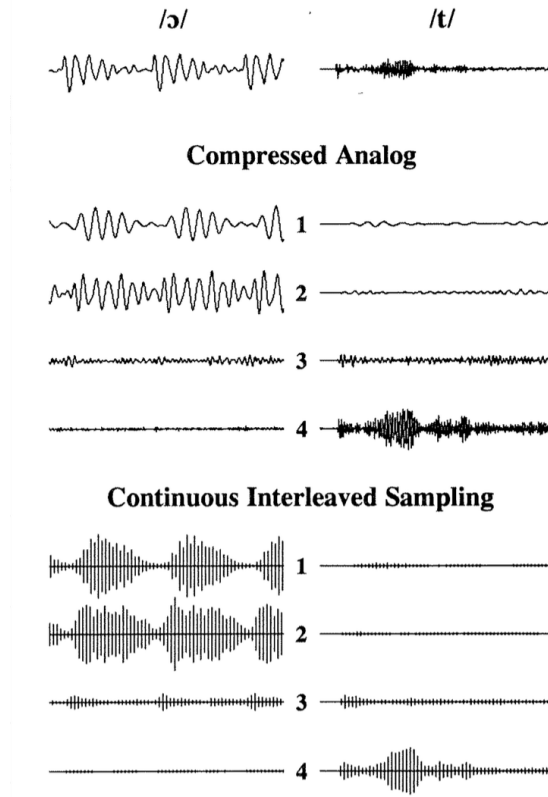
Figure 2.1: CA vs CIS

2.0.1 Sum-of-Products Model

We have now laid out enough background information to introduce a mathematical model for audio signals called the sum-of-products model.

Our digitally sampled audio signal $x[n]$, ($n \epsilon \mathbb{Z}$), is composed of bandpass components $x_k[n]$. In each bandpass component a slow-time-varying envelope $m_k[n]$ multiplies a quickly-oscillating carrier $c_k[n]$.

$$x[n] = \sum_k x_k[n] = \sum_k m_k[n]c_k[n] \tag{2.1}$$

Although there are infinite ways to decompose a signal into a sum-of-products, the model stems from real-word signals. To gain some intuition consider, for example, a voiced vowel.

The vocal tract can be thought of as the carriers, $c_k[n]$. Without changing the position of the mouth, one can change the pitch of a note. The mouth then acts as the envelope, $m_k[n]$. As the mouth changes shapes it changes the formant structure structure. Equivalently, it changes the relative amplitude of each bandpass component $x_k[n]$.

As another example we may consider musical instruments. The pitch is characterized by the carriers but the timbre which is predominantly characterized by the attack time and spectral centroid [kong 2011] will be encoded by the rise time and relative amplitude of the envelopes.

2.0.2   Why Envelopes?

One of the motivations for this approach is the limited ability to perceive temporal modulations in electric hearing. In acoustic hearing modulations up to a few kHz may be perceptible, however cochlear implant envelope extraction techniques are designed to limit modulations, typically to around 160 to 320 Hz, which is closer to the range perceptible in electric hearing.

Modulation rates are also limited by pulse rate. Although there isn't a quantitative value analogous to Nyquist rate, modulations at rates higher than a certain percentage of the constant pulse rate will not be represented accurately by the modulated pulse train. That being said, cochlear implants today support modulations typically upwards of 2000pps (pulses per second), which should be sufficient provided modulations limited to 320Hz.

2.0.3   The Channel Vocoder

To gain some intuition as to how and why CIS processing works we consider a closely related system, the channel vocoder. Vocoding is a method of signal analysis and synthesis initially designed for audio data compression in telecommunication. As of the mid 70's the vocoder has gained widespread familiarity via the music industry as a funky voice effect. It is most well known for the signature robot voice heard in hits such as Kraftwerk's song "The Robots" or Styx's "Mr. Roboto". In its application to music, the vocoder extracts the bandlimited envelopes of one source (typically vocal) and applies them to each bandlimited

components of a second source.

What's interesting is that this second source can be essentially any arbitrary broadband signal and yet we still understand speech from the first source. In this way the vocoder acts as a form of lossy data compression; the low data-rate envelopes are extracted and they may be later applied to, for example, white-noise.

This tells us that speech information is predominantly contained in the bandlimited envelopes, and thanks to the incredible robustness of speech to distortion, an estimated envelope is sufficient for speech comprehension.
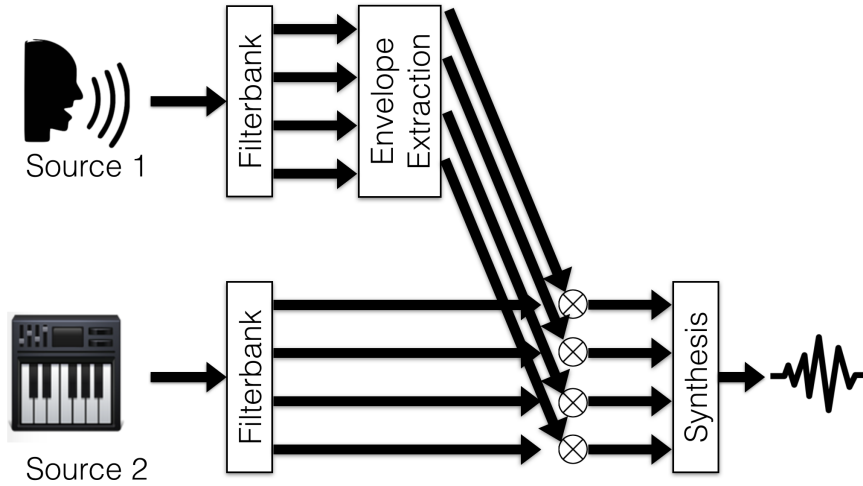
Figure 2.2: Channel Vocoder Processing

It should be noted that the second source is typically chosen to be a broadband stationary signal. If the signal is non-stationary it will have time-varying envelopes of it's own which will interact with the envelopes of the first source. Referencing back to our sum-of-products model the second source acts as a combination of carriers, $c_k[n]$, and the first source's envelopes, $m_k[n]$, are applied.

$$s^1[n] = \sum_k s_k^1[n] = \sum_k m_k^1[n]c_k^1[n] \tag{2.2}$$

$$s^2[n] = \sum_k s_k^2[n] = \sum_k m_k^2[n]c_k^2[n] \tag{2.3}$$

$$y[n] = \sum_k m_k^1[n]m_k^2[n]c_k^2[n] \tag{2.4}$$

Linking back, cochlear implant envelope extraction strategies do the same thing as vocoder signal analysis, as seen in figure 2.2, however rather than using a second source to synthesize a new sound, the envelopes directly modulate electrical pulse trains.

2.0.4 Temporal Fine Structure

The major drawback to this method of encoding is the loss of temporal fine structure. Referencing back to our sum-of-products model, we are extracting the envelopes and discarding carrier information.

When using a vocoder, vocals sung at different pitches general roughly the same output, $y[n]$. Similarly in cochlear implants temporal fine structure that encodes pitch, as well as other signal characteristics, is lost in processing.

"In most existing clinical sound processors, fine structure in the input acoustic signal is discarded, and only envelope information is preserved. "

2.0.5 Processing Blocks

Let's consider the processing blocks of a cochlear implant. The main stages to processing in cochlear implants are visualized in figure 2.3 below. While at every stage adjustments can be made, for the purpose of comparing DSP algorithms, all other stages will be assumed constant throughout this work unless otherwise specified.

In this document, the output of the DSP stage will be a strictly positive signal used to amplitude modulate a constant bipolar pulse train. T/C Level Mapping refers to a logarithmically-compressed mapping from amplitude to current level.

Figure 2.3: Signal Flow in CI
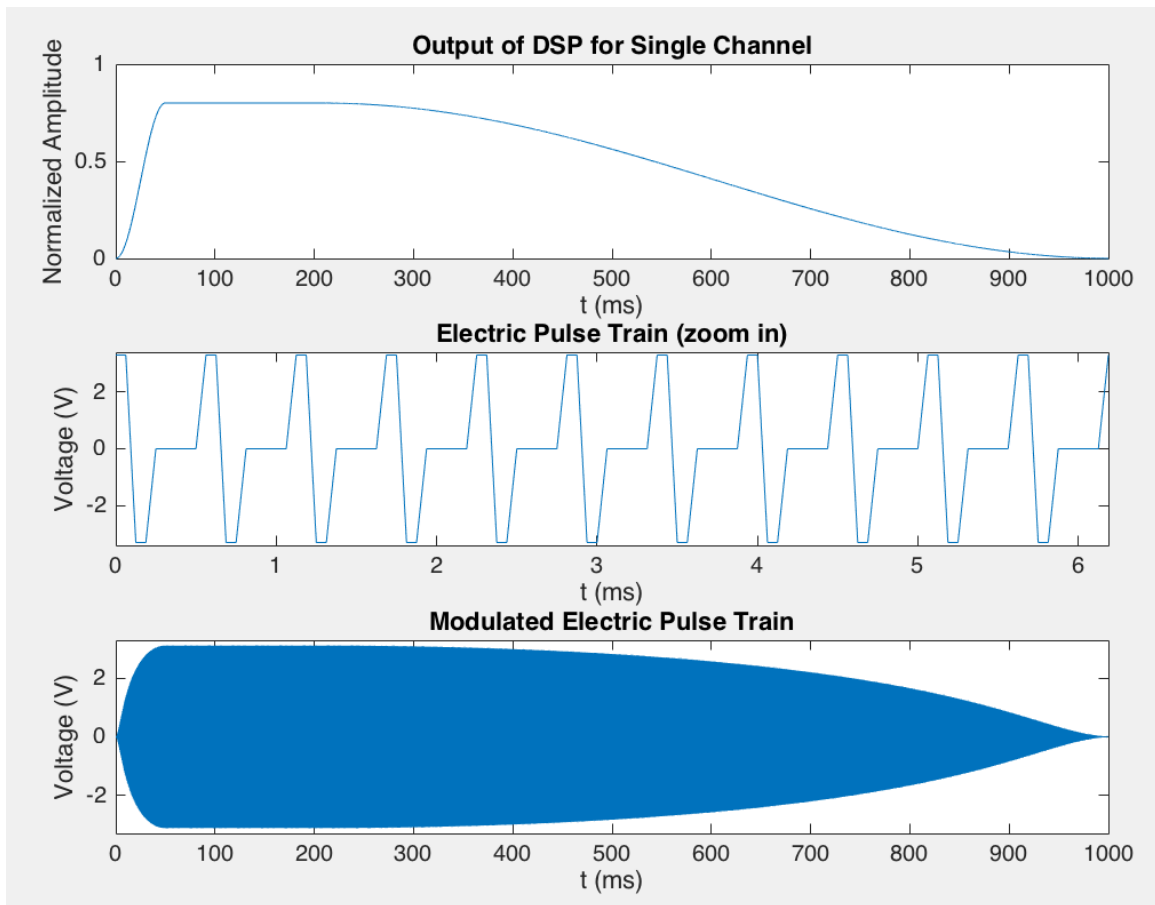
Figure 2.4: Tranformation from DSP output to Electrical Signal

2.0.6 Recap

To conclude up to this point, cochlear implants use an array of tonotopically orgranized electrodes. On each electrode a electric pulse train is transmitted and that pulse train is modulated by an envelope corresponding to a bandpass signal component

2.1 DSP Algorithms

To gain insight into how we can encode harmonic signals, in this section we will look inside the "DSP Algorithm" box; we will compare three specific strategies, ACE, F0mod, and HSSE with the goals of evaluating the pros and cons of each and considering how to optimize performance for harmonic encoding.

2.1.1 ACE

The simplest of the considered strategies is the Advanced Combination Encoder (ACE). ACE has become a clinical standard for CI processing and is used in a vast number of users.

ACE is Cochlear Ltd's instance of the auditory community's generalized category of N-of-M strategies. In these strategies, K envelopes are first extracted then allocated to M channels corresponding uniquely to electrodes. During each processing frame a subset N-of-M channel envelopes is selected for stimulation on the internal implant. In the case that more than one envelope is allocated to a channel, the allocation stage must make a decision to select or combine envelopes in some way.

K - number of envelopes per frame

M - number of electrode channels

N - number of electrodes stimulated per frame

$$K \geq M \geq N$$
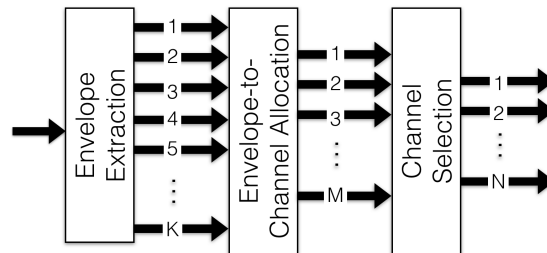
Figure 2.5: ACE Flow Diagram

The following figure is simply a condensed version of the previous flow diagram. This condensed notation will be carried through to the other strategies analyzed.

Figure 2.6: condensed ACE Flow Diagram

While ACE does a sufficient job for many CI users in speech recognition tasks, a large gap remains between normal hearing and cochlear implants in many tasks such as pitch discrimination. This is largely attributed to the lack of temporal fine structure information in this envelope encoding strategy.

ACE does, however, provide limited temporal modulations via beat frequencies. Through intentional processing artifacts, beat-frequencies will be induced in the processing of harmonic signals at a rate of the difference between the two harmonic frequencies, i.e. F_0. Typically these modulations are not full depth [ref?] and are usually limited to under 250Hz [ref?].

In this document will will refer to these artifact based modulations as induced modulations. Looking at the flow diagram of figure 2.6 it is not apparent that temporal modulations are contained in the processing path, however these modulations are encoded in the envelope itself. We can think of this as our extracted envelope containing some information about the carriers, $c_k[n]$.

Induced modulations are complementary to explicit modulation, used in F0mod and HSSE. Explicit modulations are those extracted from the signal separate from envelopes, and later applied to the final outputs.

2.1.2   F0mod

To get at the problem of pitch discrimination, (Laneau et al 2006) developed a new research strategy, F0mod. F0mod provides the same processing as ACE with one important change, explicit carrier modulation. It achieves this by adding a pitch estimator into the processing.

Once a fundamental frequency (F_0) is acquired, all output envelopes are modulated by a raised sinusoid at a rate of F_0. F_0 is used because high modulation rates (typically above 300Hz) are not noticeable with a CI.
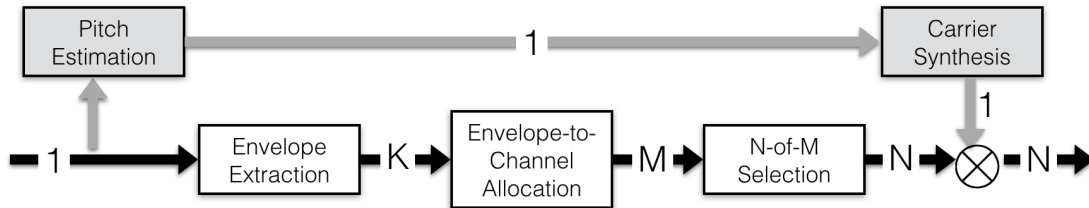
Figure 2.7: F0mod Flow Diagram

This raised sinusoid is constant modulation depth, (full dynamic range), and same across channels, (phase aligned). An example comparing this to induced modulations is shown in figure 2.8.

F0mod has shown promising results in acute tests for pitch discrimination. It has also inspired other processing strategies such as eTone, which uses a more sophisticated harmonic sieve pitch estimator as well as soft decisions to overcome the problem of encoding both harmonic and inharmonic sounds as well as those that fall somewhere in between.

2.1.3   HSSE

Looking for a novel approach to improved pitch perception and more broadly music perception, (Li, Atlas, Nie) came up with Harmonic Single Sideband Encoder (HSSE).

There are two different versions of HSSE. We will start with the version most similar to F0mod.
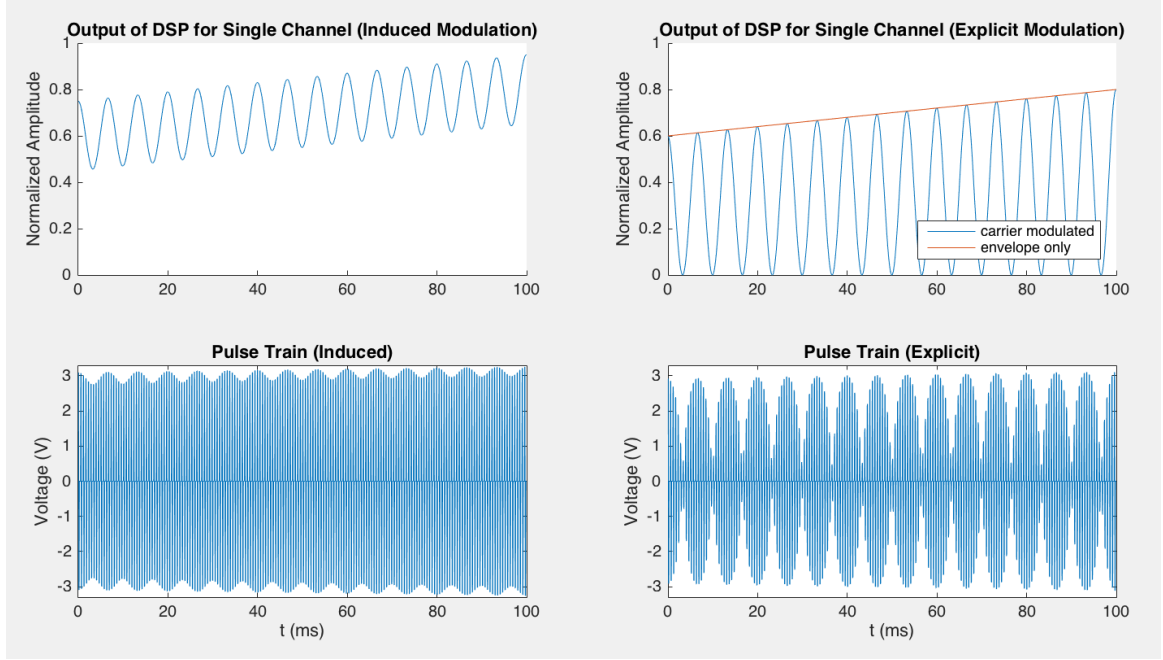
Figure 2.8: Induced vs Explicit Temporal Modulations

In this version, coherent demodulation extracts harmonic envelopes. These harmonic envelopes are then combined into channels based on the harmonic index and F_0. Just as in F0mod a subset is selected for stimulation and then these envelopes are combined with carrier modulators.

$$K, M \geq N$$
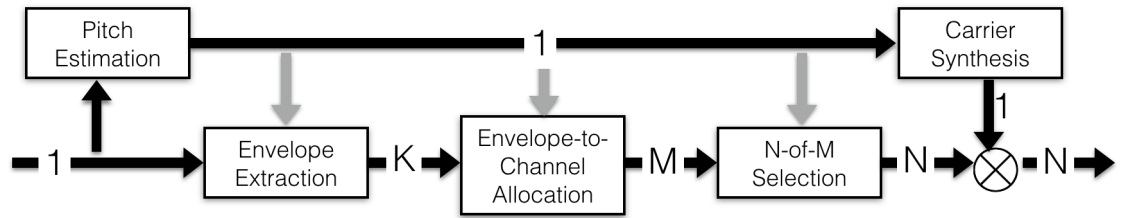
Figure 2.9: HSSE Flow Diagram

The key differences between this and F0mod can be summarized quite simply: every stage of typical ACE processing is now done coherently using F_0 information.

It should be noted that it is not necessarily true that $K \geq M$. In the case that no envelopes are allocated to a channel we may simply rule out that channel during the selection stage.

In the second version, more information about the carriers is retained. This put's some restrictions on the type of carrier than can be used, however it encodes time varying phase information which is unique to each envelope.

Because of the unique characteristics of each carrier, the carrier synthesis stage must be moved to an earlier point in the processing stage. First, complex envelopes containing phase information are extracted. These envelopes are then combined with a common carrier at a rate of F_0 however each output, which we will call a modulator, will be unique and time-varying in both magnitude and phase.
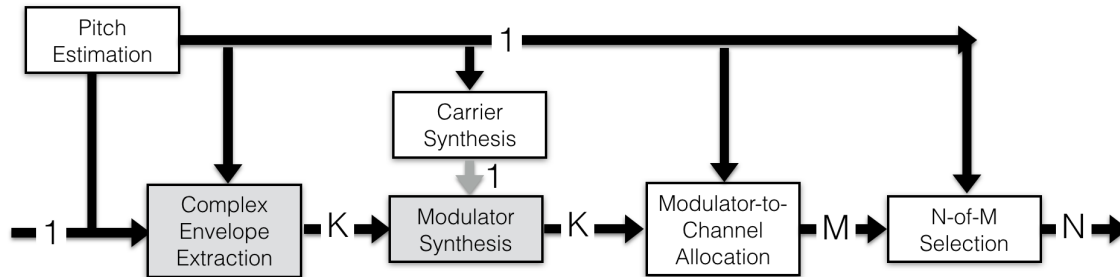
Figure 2.10: HSSE (with phase) Flow Diagram

2.1.4   Summary

Comparing these strategies, the differences may be summarized as:

    1) Envelope Extraction Method

    - not discussed yet

    2) Temporal Encoding Method

    a) induced vs explicit

b) phase preservation (explicit only)

c) modulation waveform (explicit only)

3) Envelope-to-Channel Allocation and Channel Selection

We will start by investigating 1 and 2(a,b). Some considerations for 2(c) and 3 will be brought up upon concluding this document, however the primary focus will be on 1 and 2(a,b).

The chapter 3 we will discuss mathematical methods to envelope extraction as well as phase preservation since phase is extracted at the same time. As a result we will generalize 1 and we will answer 2(b).

In chapter 4 we will evaluate design considerations for 1 and in doing so we will answer 2(a).

When we conclude we will briefly discuss 2(c) and 3.

Chapter 3

ENVELOPE EXTRACTION METHODS

In this chapter we will define the specific mathematical operations used to extract bandlimited time-varying envelopes. These methods fall under a general signal processing category of analysis-synthesis systems. In these methods a signal is decomposed into it's envelopes and carriers. Then the envelopes and/or carriers are manipulated individually before recombination.

One of the major focuses of research in this area is the evaluating the amount of distortion induced by the system. For example, Ghitza's test is a way of measuring the out-of-band distortion of a modulation filtering system. [REF]

Cochlear implant processing in unique in that the final output is not an audio signal. What this means is we only do the first half of the processing, the analysis step. This is critical to understand when considering methods, as all of the considerations related to synthesis or full-system distortion are no longer relevant.

This chapter is organized as follows. We will first introduce the envelope extraction methods to be considered. These methods are broken into to categories: incoherent and coherent. We will then take a quick detour to consider the efficacy of coherent angle encoding. Finally we will compare the methods and for a generalization of envelope extraction.

3.1 Incoherent Methods

The difference between incoherent and coherent is actually quite simple. Consider a system $H_k\cdot$. If this system is time-invariant then it is incoherent. If it is time-varying and the way in which it varies is a function of the input, we call this a coherent system. This is visualized in figure 3.1. In coherent methods the input not only passes through the system, it changes the system.

In all considered methods, the input is a real digitally sampled audio waveform, $x[n]$
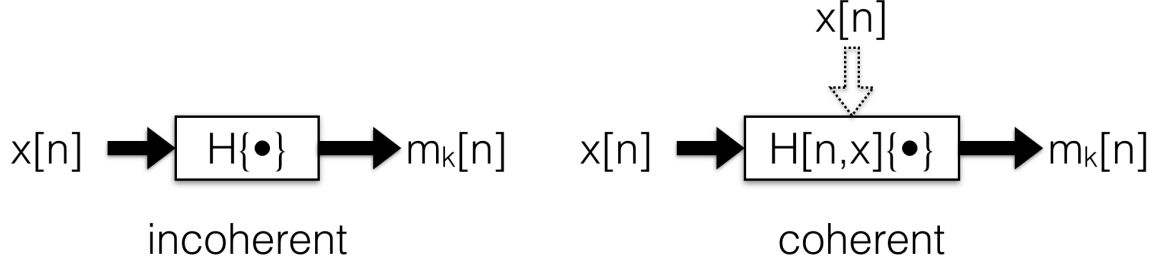
Figure 3.1: Incoherent vs Coherent Processing

bounded in the normalized range of $[-1, 1]$. In the case of incoherent methods, the output will be K real digital waveforms, $m_k[n]$, in the range $[0, 1]$. All filters considered will be finite impulse response (FIR).

3.1.1   Continuous Interleaved Sampling (CIS)

This is method specifically implemented by the CIS strategy. We first bandpass filter the signal, where h_k is a bandpass filter and k has arbitrary limits. Then we full-wave rectify (take the magnitude) and finally lowpass filter that output. $h_{lp}[n]$ is a lowpass filter, typically with a cutoff around 200-400Hz.

$$m_{k,CIS}[n] = \left| x[n] * h_k[n] \right| * h_{lp}[n] \tag{3.1}$$

3.1.2   Hilbert Envelope

The Hilbert Envelope is method of decomposition applied far more broadly than the field of cochlear implants. Despite only retaining the envelope, we look at the carrier just to gain insight into how the signal $x[n]$ is represented in the decomposition. We first acquire the bandpass signal, $x_k[n]$. We then define our envelope as the magnitude of the analytic signal, acquire via a Hilbert transform, denoted $\mathcal{H}\{\cdot\}$.

$$x_k[n] = x[n] * h_k[n] \tag{3.2}$$

$$\widehat{x}_k[n] = x_k[n] + j\mathcal{H}\{x_k[n]\} \tag{3.3}$$

$$m_{k,hilbert}[n] = \left|\widehat{x}_k[n]\right| \tag{3.4}$$

$$c_{k,hilbert}[n] = cos(\angle\widehat{x}_k[n]) \tag{3.5}$$

We can see intuitively that if the filterbank $[h_1[n], h_2[n], ...]$ has a flat total response, that all of the information of the original signal is contained in the envelopes and carriers, and thus we should be able to reconstruct the input from these components.

3.1.3 Short Time Fourier Transform (STFT)

The short-time Fourier transform (STFT) has many applications not associated with envelope extraction, however through analysis we will see that it fits the sum-of-products model we are looking for.

The STFT has two classic interpretations: a series of windowed Fourier transforms (each at a different time instant) or a collection of uniform bandpass filters (each at a different center frequency). For our purposes we will be using the later.

An STFT bin at discrete time n and discrete frequency k is defined as

$$X[n,k] = \sum_{r=-\infty}^{\infty} x[r]w[r-n]e^{-j\frac{2\pi}{N}kr}, \qquad 0 \leq k < N \tag{3.6}$$

where N is the FFT order. Defining a new variable $r' = r - n$ and defining our window such that $w[n] = 0$ for $n < 0$ or $N \leq n$,

$$\begin{aligned} X[n,k] &= \sum_{r'=0}^{N-1} x[n+r']w[r']e^{-j\frac{2\pi}{N}k(n+r')} \\ &= e^{-j\frac{2\pi}{N}kn} \sum_{r'=0}^{N-1} x[n+r']w[r']e^{-j\frac{2\pi}{N}kr'} \end{aligned} \tag{3.7}$$

Let $X[n,k]$ be represented in polar form as the following

$$X[n,k] = |X[n,k]|e^{j\angle X[n,k]} \tag{3.8}$$

If we assume that the window $w[n] \neq 0$ for $0 \leq n \leq N-1$ then we have the inverse

$$x[n+r'] = \frac{1}{Nw[r']} \sum_{k=0}^{N-1} X[n,k]e^{j\frac{2\pi}{N}k(n+r')}$$

$$= \frac{1}{Nw[r']} \sum_{k=0}^{N-1} |X[n,k]|e^{j(\frac{2\pi}{N}k(n+r')+\angle X[n,k])} \tag{3.9}$$

$$x[n] = \sum_{k=0}^{N-1} \frac{1}{Nw[0]} |X[n,k]|e^{j(\frac{2\pi}{N}kn+\angle X[n,k])} \tag{3.10}$$

Without loss of generality we can use a STFT hop-factor of one sample. In the case of a greater hop factor we would need to compute $x[n]$ from equation 3.9 [REF?] for some samples. Of course, if the hop factor is greater than N we cannot fully reconstruct the signal. This is especially noted because we will be recurrently the factor $w[0]$.

We can now clearly see our sum-of-products model

$$m_{k,STFT}[n] = \frac{1}{Nw[0]} |X[n,k]| \tag{3.11}$$

$$c_{k,STFT}[n] = e^{j(\frac{2\pi}{N}kn+\angle X[n,k])} \tag{3.12}$$

We can think of the STFT as a series of N LTI systems that each downshift the input signal, then lowpass filter. This can be seen mathematically if we rewrite equation 3.6 as

$$X[n,k] = \sum_{r=-\infty}^{\infty} x[r]e^{-j\frac{2\pi}{N}kr}w[-(n-r)]$$

$$= x[n]e^{-j\frac{2\pi}{N}kn} * w[-n] \tag{3.13}$$

We can now look at the STFT envelope in a similar form to the other methods by plugging **??** into 3.11.

$$m_{k,STFT}[n] = \frac{1}{Nw[0]} \left| x[n]e^{-j\frac{2\pi}{N}kn} * w[-n] \right|, \qquad 0 \leq k \leq \frac{N}{2} \tag{3.14}$$

Also note that due to symmetry of the Fourier transform, envelopes are only valid for indicies between 0 and $\frac{N}{2}$.

3.2 Coherent Methods

Due to their LTI nature, incoherent methods fail to explicitly represent time varying characteristics like fundamental frequency or formant structure. [2] Alternatively, coherent methods will adapt to represent some specific characteristic.

3.2.1 Spectral Center-of-Gravity

One coherent method is the spectral center-of-gravity (COG). Similar to the previously described incoherent methods, spectral COG uses a fixed number of filters. The key difference lies in the center frequency of each of these filters which adapt over time as a function of the spectral distribution within predefined band limits.

Spectral COG certainly has some advantages of better representation of the signal in comparison to incoherent methods, however it still doesn't escape the limitation of fixed and pre-determined band limits that each filter operates within. We won't be investigating this method further.

3.2.2 Harmonic

To escape this, [Atlas and Others] proposed a harmonic method which uses knowledge of the structure of common audio signals to decompose the signal in a less arbitrary way. The first step is to get a pitch estimate $F_0[n]$ of the signal. We then define k complex carriers where there is a hard limit as a function of Nyquist sampling rate, $k \leq \lfloor \frac{F_s}{2F_0} \rfloor$

$$c_{k,harmonic}[n] = e^{jk\phi_0[n]} \tag{3.15}$$

where

$$\phi_0[n] = \frac{2\pi}{F_s} \sum_{p=0}^{n} F_0[p]$$

$$= \phi_0[n-1] + 2\pi \frac{F_0[n]}{F_s} \tag{3.16}$$

$$\phi_0[-1] = 0$$

[modulation toolbox]

As mentioned earlier there are two versions of HSSE, the first uses a real non-negative envelope, the other uses a complex envelope.

we then define our first envelope

$$m_{k,harmonic}^1[n] = \left| x[n] c_{k,harmonic}^*[n] * h\big[n, F_0[n]\big] \right|$$

$$= \left| x[n] e^{-jk\phi_0[n]} * h\big[n, F_0[n]\big] \right| \tag{3.17}$$

where $h\big[n, F_0[n]\big]$ is a lowpass filter that may vary as a function of $F_0[n]$. Note that we could have a different LPF for each k however since our carriers are linearly spaced it is natural to keep $h\big[n, F_0[n]\big]$ consistent over k.

Our second, complex envelope is the same as the first but without the final magnitude operation.

$$m_{k,harmonic}^2[n] = x[n] e^{-jk\phi_0[n]} * h\big[n, F_0[n]\big] \tag{3.18}$$

3.3 Coherent Angle Encoding

As mentioned earlier, the final DSP output is a set of real non-negative signals. We take a short aside to compare the two coherent harmonic methods, one of which, due to it's complex output, cannot be considered in a envelope-only sense.

The two alternative versions are visualized in figure 3.2. For the case of magnitude only, we can think of this as a restriction on our carrier. Since the envelope is already real non-negative the $Re\cdot$ and half-wave rectification stages don't change anything. We could pass

complex exponential through these two operations before multiplying the envelope. This is equivalent to saying our carrier is a half-wave rectified sinusoid and thus we have the same general processing blocks as a single envelope of 2.9
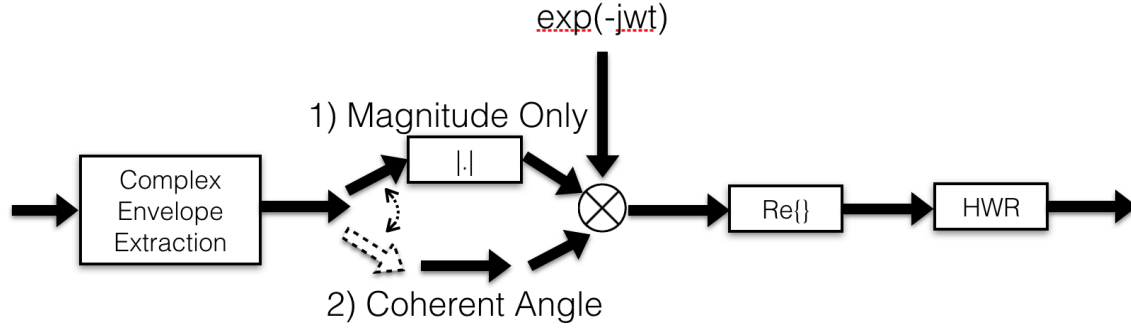
Figure 3.2: Magnitude Only VS Coherent Angle Encoding Block Diagrams

3.3.1   Two Methods

Let's consider a signal where our kth bandpass component represents the kth harmonic and is of the form

$$x_k[n] = A_k[n]cos(2\pi k F_0 n + \phi_k[n]) \qquad (3.19)$$

$$BW \leq F_0$$

where $A_k[n]$ represents a real nonnegative amplitude. And BW is the bandpass signal's bandwidth. We may assume $F_0[n] = F_0$ is constant without loss of generality so long as $F_0[n]$ is roughly constant within each processing frame.

Let's assume our filter is an ideal brick-wall filter.

$$h[n] \Longleftrightarrow H(e^{j2\pi f}) \tag{3.20}$$

$$H(e^{j2\pi f}) = 1, \quad |f| < \frac{F_0}{2} \tag{3.21}$$

$$= 0, \quad else$$

Our coherent harmonic envelopes for each method will be

$$m_{k,harmonic}^1[n] = A_k[n] \tag{3.22}$$

$$m_{k,harmonic}^2[n] = A_k[n]e^{j\phi_k[n]} \tag{3.23}$$

Let us define $Rect\{y_k[n]\}$ as the half-wave rectified carrier-modulator signal which is our end goal. Using our first harmonic method

$$y_k^1[n] = m_{k,harmonic}^1[n]cos(2\pi F_0 n) \tag{3.24}$$

$$= A_k[n]cos(2\pi F_0 n)$$

Alternatively, with our second method we get

$$y_k^2[n] = Re\{2m_{k,harmonic}^2[n]e^{j2\pi F_0 n}\} \tag{3.25}$$

$$= Re\{2A_k[n]e^{j(2\pi F_0 n + \phi_k[n])}\}$$

$$= A_k[n]cos(2\pi F_0 n + \phi_k[n])$$

It is clear that the difference between $y_k^1[n]$ and $y_k^2[n]$ is simply the extra term, $\phi_k[n]$. What this means may be best shown by example.

In figure 3.3 we see that when taking the magnitude, we force symmetry about 0. We see that the green much better represents the blue than the red does by preserving the spectral asymmetries that manifest themselves in the angle, not magnitude. It is unnatural and certainly won't happen in real world scenarios that a subband signal will be symmetric about the downshift frequency, however magnitude only methods force this to be true.
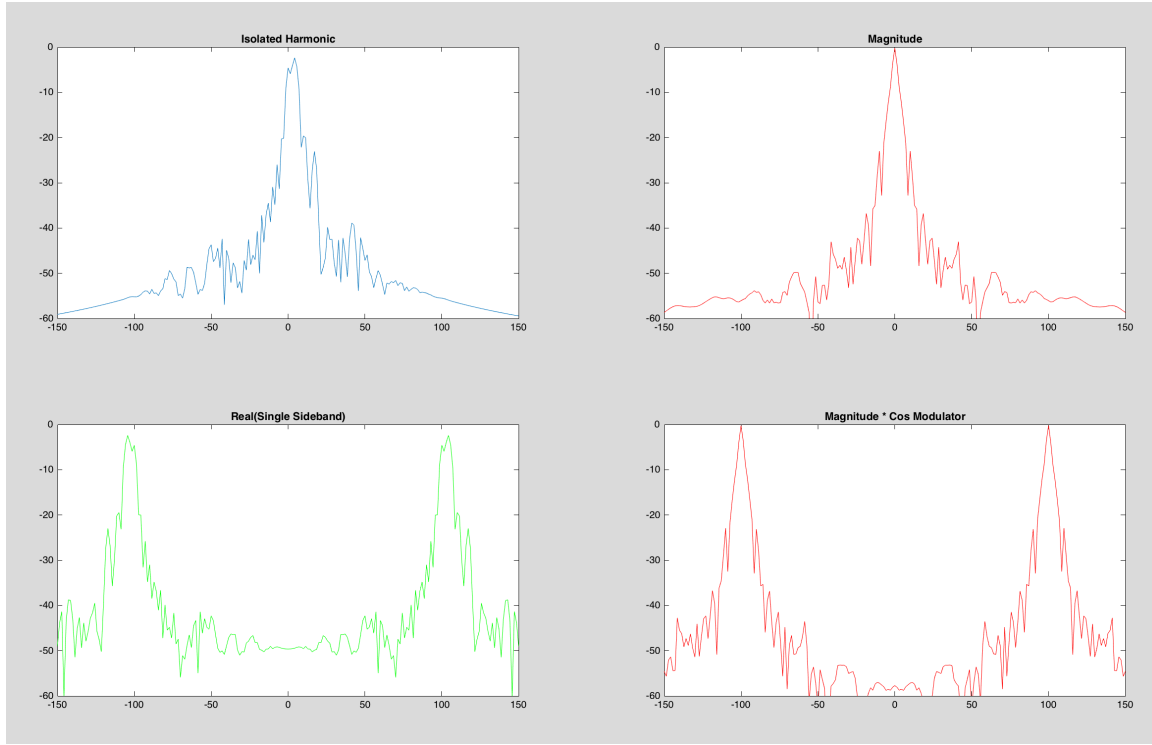
Figure 3.3: Cello Example

3.3.2 Appropriate Scaling

Despite better representing the signal, there is still an issue with $y_k^2[n]$. A more correct method is actually

$$m_{k,harmonic}^3[n] = A_k[n]e^{j\frac{1}{k}unwrap(\phi_k[n])} \tag{3.26}$$

$$y_k^3[n] = Re\{2m_{k,harmonic}^3[n]e^{j2\pi F_0 n}\} \tag{3.27}$$

$$= A_k[n]cos\Big(2\pi F_0 n + \frac{1}{k}unwrap(\phi_k[n])\Big)$$

Why do we need the $\frac{1}{k}$ term? Let's consider an example where our true pitch estimate is actually $F_{0,groundtruth} = F_0 + F_{err}$. So,

$$x_k[n] = A_k[n]cos\Big(2\pi k(F_0 + F_{err})n + \phi_k[n]\Big) \tag{3.28}$$

In this case

$$y_k^2[n] = A_k[n]cos\left(2\pi(F_0 + kF_{err})n + \frac{1}{k}unwrap(\phi_k[n])\right) \qquad (3.29)$$

$$y_k^3[n] = A_k[n]cos\left(2\pi(F_0 + F_{err})n + \frac{1}{k}unwrap(\phi_k[n])\right) \qquad (3.30)$$

Essentially the term $\phi_k[n]$ may be thought of as the deviation from kF_0. If we downshift the signal such that kF_0 is scaled to F_0 then it is appropriate that we scale $\phi_k[n]$ similarly.

3.3.3 Efficacy

Let us now consider the efficacy of 3.22 versus 3.26.

One hypothesis is that $\phi_k[n]$ may encode the noise-like characteristics of a signal, in which case it would remain constant for a pure sinusoid and fluctuate randomly for noise. Put to test, the harmonic phase preservation did little to affect the signal and this was confirmed by testing varying filter bandwidths as well. In comparison of a toy experiment, the choice of filter bandwidth dominated noise-like qualities, with wider bandwidth capturing more of the variations.

Since the term $\phi_k[n]$ does not distinguish noise-like signals from narrowband sinusoidal signals, it is only really preserving phase alignment. But this begs the question, what does it mean to preserve the phase of a harmonic when downshifted to F_0? It is questionable as to whether this even has any logical meaning.

Furthermore, it has been suggested in [REF F0mod and kaibao??] that phase alignment is important for pitch perception in CIs. By using a magnitude-only method we guarantee alignment across channels.

Having considered this option as not a path worth further investigating, for the rest of this document we will consider envelope and carrier separately with each being a real nonnegative signal at the final output.

3.4 The Relationships

All of our methods are summarized in table 3.1. We will now consider the relationships between each of these.

Method	$m_k[n] =$
CIS	$\left\vert x[n] * h_k[n] \right\vert * h_{lp}[n]$
Hilbert	$\left\vert \widehat{x}_k[n] \right\vert = \left\vert x[n] * h_k[n] + j\mathcal{H}\{x[n] * h_k[n]\} \right\vert$
STFT	$\frac{1}{Nw[0]}\left\vert x[n]e^{-j\frac{2\pi}{N}kn} * w[-n] \right\vert$
Harmonic Coherent	$\left\vert x[n]e^{-jk\phi_0[n]} * h\left[n, F_0[n]\right]\right\vert, \qquad \phi_0[n] = \frac{2\pi}{F_s}\sum_{p=0}^{n} F_0[p]$

Table 3.1: Envelope Extraction Methods

3.4.1 Hilbert VS STFT

Let us start by comparing the Hilbert and STFT methods. Since "the Hilbert transform of a convolution is the convolution of the Hilbert transform on either factor" [wikipedia] we have

$$
\begin{aligned}
\widehat{x}_k[n] =& x_k[n] + jH\{x_k[n]\} \\
=& x[n] * h_k[n] + jH\{x[n] * h_k[n]\} \\
=& x[n] * h_k[n] + x[n] * jH\{h_k[n]\} \\
=& x[n] * [h_k[n] + jH\{h_k[n]\}]
\end{aligned}
\tag{3.31}
$$

Now let us define our filter specifically as

$$
h_k[n] = \frac{1}{Nw[0]}w[-n]cos(\frac{2\pi}{N}kn)
\tag{3.32}
$$

If we assume the sidelobes of $w[n]$ roll-off sufficiently fast in relation to the center-frequency $\frac{2\pi k}{N}$, we may approximate

$$
\begin{aligned}
\mathcal{H}\{h_k[n]\} \approx& \frac{1}{Nw[0]}w[-n]H\{cos(\frac{2\pi}{N}kn)\} \\
=& \frac{1}{Nw[0]}w[-n]sin(\frac{2\pi}{N}kn)
\end{aligned}
\tag{3.33}
$$

Plugging our filter 3.32 into 3.31

$$\widehat{x}_k[n] \approx x[n] * \frac{1}{Nw[0]}w[-n]e^{j\frac{2\pi}{N}kn}$$

$$= \frac{1}{Nw[0]} \sum_{r=-\infty}^{\infty} x[n-r]w[-r]e^{j\frac{2\pi}{N}kr}$$

Let $\quad r' = -r$

$$= \frac{1}{Nw[0]} \sum_{r'=0}^{N-1} x[n+r']w[r']e^{-j\frac{2\pi}{N}kr'}$$

$$= \frac{1}{Nw[0]} \left[e^{-j\frac{2\pi}{N}kn} \sum_{r'=0}^{N-1} x[n+r']w[r']e^{-j\frac{2\pi}{N}kr'} \right] e^{j\frac{2\pi}{N}kn}$$

$$= \frac{1}{Nw[0]} X[n,i]e^{j\frac{2\pi}{N}kn}$$

$$= \left(\frac{1}{Nw[0]} x[n]e^{-j\frac{2\pi}{N}kn} * w[-n] \right) e^{j\frac{2\pi}{N}kn} \tag{3.34}$$

$$\tag{3.35}$$

When we take the magnitude the complex exponential term goes to 1 and we are left with the STFT envelope. We come to the conclusion that under the assumption of fast sidelobe rolloff we may define a filter bank of $\frac{N}{2} + 1$ filters

$$h_k[n] = w[-n]cos(\frac{2\pi}{N}kn), \qquad 0 \le k \le \frac{N}{2} \tag{3.36}$$

such that

$$m_{k,hilbert}[n] \approx m_{k,STFT}[n] \tag{3.37}$$

What this tells us is that the Hilbert decomposition may be viewed as a superset of the STFT method that is not constrained to uniform bandwidth linearly spaced filters.

3.4.2   STFT vs Harmonic

Let us now consider the relationship between STFT and harmonic coherent. We may choose our filter to be time-invariant and define it as

$$h\big[n, F_0[n]\big] = \frac{1}{Nw[0]} w[-n] \tag{3.38}$$

where $w[n]$ is a lowpass filter and

$$w[n] \neq 0, \qquad 0 \le n < N$$
$$= 0, \qquad else \tag{3.39}$$

In this case,

$$m_{k,harmonic}[n] = \left| x[n] e^{-jk\phi_0[n]} * \frac{1}{Nw[0]} w[-n] \right|$$
$$= \frac{1}{Nw[0]} \left| x[n] e^{-jk\phi_0[n]} * w[-n] \right| \tag{3.40}$$

This bears striking resemblance to equation 3.14. We can see that in the case that $F_0[n] = \frac{F_s}{N}$,

$$m_{k,harmonic}[n] = m_{k,STFT}[n] \tag{3.41}$$

More generally, for any window of time n to $n + N - 1$ where $F_0[n]$ is constant

$$m_{k,harmonic}[n] = \frac{1}{Nw[0]} \left| X\left[n, \frac{N}{1} \frac{F_0[n]}{F_s} k\right) \right|$$
$$= \frac{1}{Nw[0]} \left| X\left[n, \lambda[n]k\right) \right| \tag{3.42}$$

where $\lambda[n] = \frac{N}{1} \frac{F_0[n]}{F_s}$. The ")" is to denote that the frequency term is not necessarily an integer.

It is important to note that in practice $\lambda[n]$ is not a continuous variable. It is constrained by the quantization of the implemented pitch tracker. Provided this quantization we may compute any term $X[n, \lambda[n]k)$ by, leaving all else the same, zero-padding our FFT.

What this tells us is that in practice, we may approximate $m_{k,harmonic}[n]$ using $F_0[n]$ and a zero-padded STFT under the assumptions:

1) $F_0[n]$ is quantized

2) $F_0[n]$ is roughly constant withing a time window of $\frac{N}{Fs}$ seconds

and the restriction:

3) $h\left[n, F_0[n]\right]$ is time-invariant, i.e. $h\left[n, F_0[n]\right] = h[n]$

3.4.3   CIS VS Hilbert

Provided our envelope definitions

$$m_{k,CIS}[n] = \left| x_k[n] \right| * h_{lp}[n]$$

$$m_{k,Hilbert}[n] = \left| \widehat{x}_k[n] \right|$$

We define an ideal brick-wall filter as

$$H_k(f) = \mathcal{F}\left\{ h_k[n] \right\} \tag{3.43}$$

$$H_k(f) = 1, \quad f_k - \frac{1}{2}f_{bw} < |f| < f_k + \frac{1}{2}f_{bw} \tag{3.44}$$

$$= 0, \quad \text{else} \tag{3.45}$$

$$X(f) = \mathcal{F}\left\{ x[n] \right\} \tag{3.46}$$

$$X_k(f) = \mathcal{F}\left\{ x_k[n] \right\} \tag{3.47}$$

$$\widehat{X}_k(f) = \mathcal{F}\left\{ \widehat{x}_k[n] \right\} \tag{3.48}$$

$$X_k(f) = X(f), \quad f_k - \frac{1}{2}f_{bw} < |f| < f_k + \frac{1}{2}f_{bw} \tag{3.49}$$

$$= 0, \quad \text{else} \tag{3.50}$$

$$\widehat{X}_k(f) = X(f), \quad f_k - \frac{1}{2}f_{bw} < f < f_k + \frac{1}{2}f_{bw} \tag{3.51}$$

$$= 0, \quad \text{else} \tag{3.52}$$

$$Y_k^1(f) = \mathcal{F}\left\{\left|\widehat{x}_k[n]\right|^2\right\} \tag{3.53}$$

$$Y_k^2(f) = \mathcal{F}\left\{\left|x_k[n]\right|^2\right\} \tag{3.54}$$

$$Y_k^1(f) = \widehat{X}_k(f) * \widehat{X}_k^*(-f) \tag{3.55}$$

$$= \int_{-\infty}^{\infty} \widehat{X}_k(f-r)\widehat{X}_k^*(-r)dr \tag{3.56}$$

$$= \int_{-\infty}^{\infty} \widehat{X}_k(r+f)\widehat{X}_k^*(r)dr \tag{3.57}$$

We can narrow the integration bounds provided the restrictions

$$\widehat{X}_k^*(r) \neq 0 \Rightarrow f_k - \frac{1}{2}f_{bw} < r < f_k + \frac{1}{2}f_{bw} \tag{3.58}$$

$$\widehat{X}_k(r+f) \neq 0 \Rightarrow f_k - \frac{1}{2}f_{bw} - f < r < f_k + \frac{1}{2}f_{bw} - f \tag{3.59}$$

$$a = max\left(f_k - \frac{1}{2}f_{bw}, f_k - \frac{1}{2}f_{bw} - f\right) \tag{3.60}$$

$$b = min\left(f_k + \frac{1}{2}f_{bw}, f_k + \frac{1}{2}f_{bw} - f\right) \tag{3.61}$$

$$Y_k^1(f) = \int_a^b \widehat{X}_k(r+f)\widehat{X}_k^*(r)dr, \quad -f_{bw} < f < f_{bw} \tag{3.62}$$

$$= 0, \quad \text{else} \tag{3.63}$$

$$Y_k^2(f) = X_k(f) * X_k^*(-f) \tag{3.64}$$

$$= \int_{-\infty}^{\infty} X_k(r+f)X_k^*(r)dr \tag{3.65}$$

Case 1: $-2f_k - f_{bw} < f < -2f_k + f_{bw}$

$$Y_k^2(f) = \int_a^b X_k(r+f)X_k^*(r)dr \tag{3.66}$$

$$a = max\left(f_k - \frac{1}{2}f_{bw}, f_k - \frac{1}{2}f_{bw} - f\right) \tag{3.67}$$

$$b = min\left(f_k + \frac{1}{2}f_{bw}, f_k + \frac{1}{2}f_{bw} - f\right) \tag{3.68}$$

Case 2: $2f_k - f_{bw} < f < 2f_k + f_{bw}$

$$Y_k^2(f) = \int_a^b X_k(r+f)X_k^*(r)dr \tag{3.69}$$

$$a = max\left(-f_k - \frac{1}{2}f_{bw}, -f_k - \frac{1}{2}f_{bw} - f\right) \tag{3.70}$$

$$b = min\left(-f_k + \frac{1}{2}f_{bw}, -f_k + \frac{1}{2}f_{bw} - f\right) \tag{3.71}$$

Case 3: $-f_{bw} < f < f_{bw}$

This case is unique because there are two points of intersection. We can break up the integral into a sum. The first integral is exactly the same as in $Y_k^1(f)$.

$$Y_k^2(f) = \int_{a_1}^{b_1} X_k(r+f)X_k^*(r)dr + \int_{a_2}^{b_2} X_k(r+f)X_k^*(r)dr \tag{3.72}$$

$$a_1 = max\left(f_k - \frac{1}{2}f_{bw}, f_k - \frac{1}{2}f_{bw} - f\right) \tag{3.73}$$

$$b_1 = min\left(f_k + \frac{1}{2}f_{bw}, f_k + \frac{1}{2}f_{bw} - f\right) \tag{3.74}$$

$$a_2 = max\left(-f_k - \frac{1}{2}f_{bw}, -f_k - \frac{1}{2}f_{bw} - f\right) \tag{3.75}$$

$$b_2 = min\left(-f_k + \frac{1}{2}f_{bw}, -f_k + \frac{1}{2}f_{bw} - f\right) \tag{3.76}$$

Using the Hermitian symmetry of the real-valued $x[n]$,

$$Y_k^2(f) = \int\limits_{a_1}^{b_1} X_k(r+f)X_k^*(r)dr + \int\limits_{a_2}^{b_2} X_k^*(-r-f)X_k(-r)dr \qquad (3.77)$$

$$r' = -r - f$$

$$Y_k^2(f) = \int\limits_{a_1}^{b_1} X_k(r+f)X_k^*(r)dr + \int\limits_{a_1}^{b_1} X_k^*(r')X_k(r'+f)dr' \qquad (3.78)$$

$$= 2\int\limits_{a_1}^{b_1} X_k(r+f)X_k^*(r)dr \qquad (3.79)$$

$$= 2Y_k^1(f) \qquad (3.80)$$

If we lowpass filter $Y_k^2(f)$ with a filter defined

$$H_{lp}(f) = \frac{1}{2}, \quad f_{bw} < |f| < f_{bw} \qquad (3.81)$$

$$= 0, \quad 2f_k - f_{bw} < |f| < 2f_k + f_{bw} \qquad (3.82)$$

then

$$Y_k^2(f) = Y_k^1(f) \quad \forall f \qquad (3.83)$$

We conclude that

$$\left| x_k[n] \right|^2 * h_{lp}[n] \approx \left| \widehat{x}_k[n] \right|^2 \qquad (3.84)$$

Things to consider are delay and non-deal filters, however provided the distance between baseband and the $\pm 2f_k$ terms a sufficient filter is practical in practice.

Now the relationship between $m_{k,CIS}[n]$ and $m_{k,Hilbert}[n]$ is muddled by the nonlinear square root operation, however the nonlinearities induced won't be noticeably distorted by $h_{lp}[n]$. In practice, there only noticeable difference will be the added delay from the final lowpass filter in the CIS method.

3.4.4 Abstract Interpretation

One of the easier ways to interpret these methods is through a frequency domain analysis. Figure 3.4 shows an abstract view of the methods. The input is a magnitude spectrum of a signal with two harmonics. For mathematical convenience the output is actually the squared envelope. At each step a new operation is applied. This abstract analysis ignores scale factors that can always be modified by scaling filter coefficients.
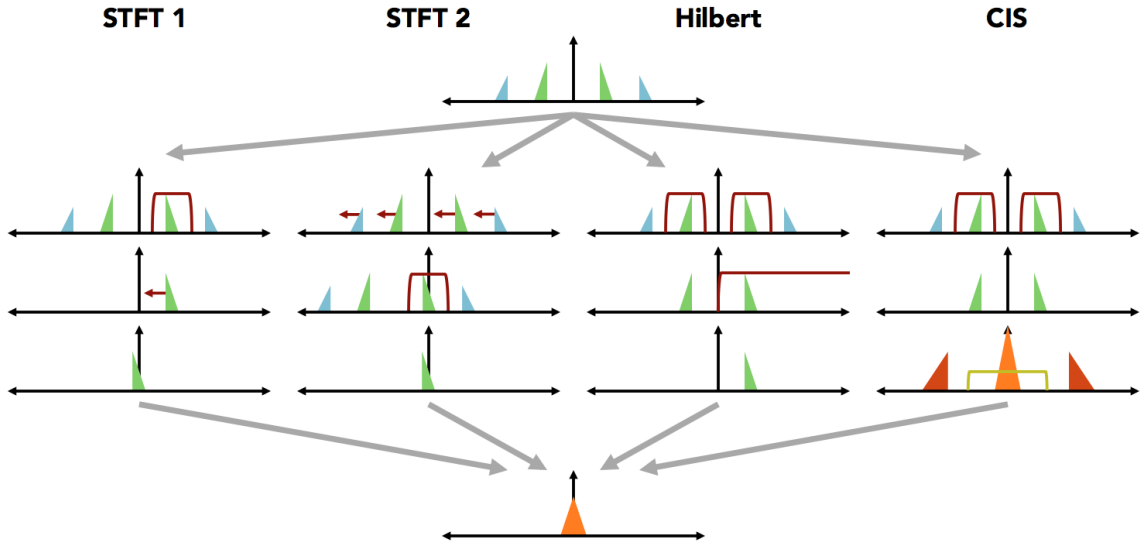
Figure 3.4: Method Comparison: magnitude spectrum at each step

First note that there are two paths for STFT. This is because there is an ambiguity in the order of operations. This can be seen mathematically in 3.85.

$$e^{-j\frac{2\pi}{N}kn}\left(x[n] * \left(w[-n]e^{j\frac{2\pi}{N}kn}\right)\right) = \left(x[n]e^{-j\frac{2\pi}{N}kn}\right) * w[-n] \tag{3.85}$$

The left side of 3.85 corresponds to the STFT 1 path. First an analytic bandpass filter centered at radian frequency $\frac{2\pi k}{N}$ is applied. The output of that is then downshifted to baseband.

The right side of 3.85 corresponds to the STFT 2 path. We first downshift by radian frequency $\frac{2\pi k}{N}$, then lowpass filter.

For both STFT 1 and STFT 2 the final operation is a magnitude.

The harmonic coherent method is missing from figure 3.4. This is because ignoring exact details of downshift frequency and filter coefficients, it is actually the same as the STFT method: downshift followed by lowpass filter.

Let's move on the the Hilbert envelope. In figure 3.4 we first bandpass filter, then acquire the analytic signal, which is equivalent to setting the negative frequencies to zero. The final operation is to take the magnitude squared, which is invariant to frequency shifts. From this abstract view, we should expect the same result as the STFT method.

In CIS, we see that taking the magnitude squared of the real bandpass signal causes double frequency terms, and the baseband term is scaled by a factor of 2. The final filter operation rescales the baseband term and eliminates the double frequency terms.

3.5   Summary

So what are the differences? To come to the conclusions made, some assumptions had to be made. We found that the Hilbert and CIS methods are approximately the same. STFT decomposition is a subset of the Hilbert method where the filterbank is comprised of uniform-bandwidth linearly spaced filters. Coherent Harmonic is a expansion of STFT decomposition using the fundamental frequency of a signal to adaptively change downshift frequency and filter bandwidth.

In 3.86 we generalize the considered methods. $h_k\left[n, F_0[n]\right]$ is a function of k allowing for non-uniform bandwidths and a function of $F_0[n]$, allowing for coherent filter adaptation. Similarly, $\omega_k\left[F_0[n]\right]$ is a function of $F_0[n]$, allowing for coherent downshift frequencies.

$$m_k[n] = \left| x[n] e^{-j\omega_k\left[F_0[n]\right]n} * h_k\left[n, F_0[n]\right] \right| \tag{3.86}$$

In the next chapter we will investigate encoding harmonics in cochlear implants using our generalized envelope extraction equation, 3.86.

Chapter 4

HARMONIC ENVELOPES

¡our hypothesis is that coherent envelopes are the best representation of harmonic signals, that being said what is important for harmonic envelopes? what numbers matter?¿

We model our harmonic signal with a sum-of-products model as:

$$x[n] = \sum_k x_k[n] = \sum_k m_k[n]c_k[n] \tag{4.1}$$

our extracted envelope can be generally defined as:

$$\tilde{m}_k[n] = \left| \widehat{x}[n]e^{-j\omega_k[x]n} * h_k[n, x] \right| \tag{4.2}$$

This is a good generalization of any envelope extraction (harmonic or not). The design can be summarized by two things:

- downshift frequency, $\omega_k[x]$

- lowpass filter, $h_k[n, x]$

If $w_k[\cdot]$ and $h_k[\cdot]$ are functions of $x[n]$ we have coherent envelope extraction. If they are time-invariant, we have incoherent extraction.

4.0.1   harmonic signals

Since harmonic signals have a specific structure, we can define our carriers from equation 4.1 as centered at multiples of F_0. In this representation $x_0[n]$ is the fundamental centered at F_0, $x_1[n]$ is the 1st harmonic centered at $2F_0$, etc.

$$\theta_k[n] = 2\pi(k+1)\frac{F_0}{F_s}n + \phi_k[n] \tag{4.3}$$

$$x[n] = \sum_{k=0}^{K} m_k[n]cos(\theta_k[n]) \tag{4.4}$$

$$\widehat{x}[n] = \sum_{k=0}^{K} m_k[n]e^{j\theta_k[n]} \tag{4.5}$$

4.1 Steady-State Analysis

We start with the simplest scenario, where $x[n]$ is a steady-state signal. The conditions we require for this are:

- constant pitch: $F_0[n] = F_0$

- narrowband modulator: $m_k[n] \approx constant$ over short periods of time

- constant phase term: $\phi_k[n] = \phi_k$, we choose $\phi_k[n] = 0$ for cleaner equations however this is not necessary

4.1.1 3 Harmonic Example: Desired Envelope

We visualize the frequency domain for a signal with three harmonics ($K = 2$) in figure 4.1. For this example we consider the 1st harmonic ($k = 1$), centered at $2F_0$.

Figure 4.1(d) is the spectrum of the squared envelope, $|\mathcal{F}\{m_1^2[n]\}|$. We see this relationship in equation 4.9

$$(a) \quad \widehat{x}[n] \Longleftrightarrow \widehat{X}[n, f] \tag{4.6}$$

$$(b) \quad \widehat{x}_1[n] \Longleftrightarrow \widehat{X}_1[n, f] \tag{4.7}$$

$$(c) \quad \widehat{x}_1^*[n] \Longleftrightarrow \widehat{X}_1^*[n, -f] \tag{4.8}$$

$$(d) \quad m_1^2[n] = \widehat{x}_1[n]\widehat{x}_1^*[n] \Longleftrightarrow \widehat{X}_1[n, f] * \widehat{X}_1^*[n, -f] \tag{4.9}$$
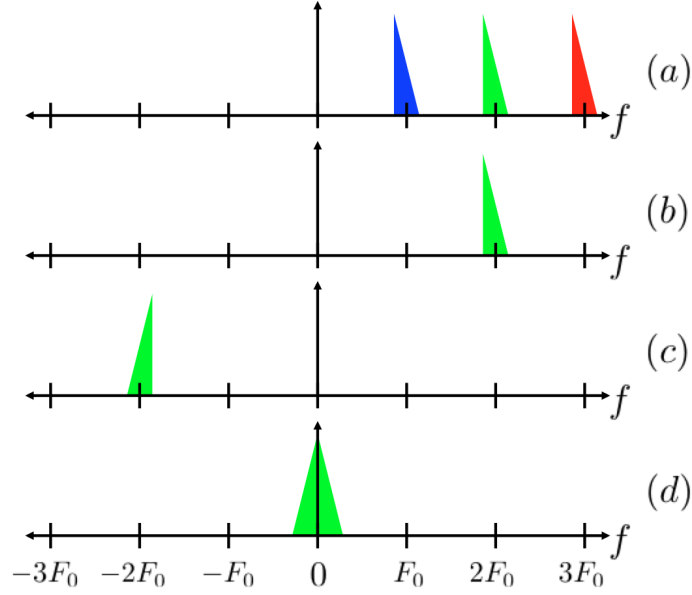
Figure 4.1: Magnitude of spectrum for equations 4.6 - 4.9

The envelope can always be acquired from the squared envelope by a final square root operation. This operation introduces nonlinearities at multiples of F_0 that are difficult to analyze. For mathematical convenience, during our analysis we can consider the squared envelope. This final square root operation will remain constant across all examples which allows us to not consider it.

$$m_1[n] = \left| \widehat{x}_1[n] \right| = \left[\widehat{x}_1[n] \widehat{x}_1^*[n] \right]^{\frac{1}{2}} \tag{4.10}$$

4.1.2 Estimated Envelope

Let's now evaluate our estimate, using equation 4.2. As stated above, we consider the squared envelope.

$$\tilde{m}_k^2[n] = \left| \widehat{x}[n] e^{-j\omega_k n} * h_k[n]] \right|^2$$

$$= \left| \sum_{l=0}^{K} m_l[n] e^{j(\theta_l[n] - \omega_k[n])} * h_k[n] \right|^2$$

$$\approx \left| \sum_{l=0}^{K} m_l[n] \left(e^{j(\theta_l[n] - \omega_k[n])} * h_k[n] \right) \right|^2$$

$$= \left| \sum_{l=0}^{K} m_l[n] e^{j\omega_{k,l} n} H_k\left(e^{j\omega_{k,l}}\right) \right|^2 \tag{4.11}$$

$$\omega_{k,l} = 2\pi \frac{(l+1)F_0 - F_{ds,k}}{F_s} \tag{4.12}$$

$$h_k[n] \Longleftrightarrow H_k\left(e^{j\omega}\right) \tag{4.13}$$

$\omega_{k,l}$ is the downshifted center frequency of the l'th harmonic for the estimate of the k'th envelope. $H_k\left(e^{j\omega}\right)$ is the discrete Fourier transform (DFT) of $h_k[n]$.

Expanding equation 4.11 we get:

$$\tilde{m}_k^2[n] = \sum_{l=0}^{K} \sum_{i=0}^{K} m_l[n] m_i^*[n] e^{j(l-i)F_0} H_k\left(e^{j\omega_{k,l}}\right) H_k^*\left(e^{j\omega_{k,i}}\right) \tag{4.14}$$

$$= \sum_{l=0}^{K} \left| m_l[n] \right|^2 \left| H_k\left(e^{j\omega_{k,l}}\right) \right|^2$$

$$+ e^{-j2\pi \frac{F_0}{F_s} n} \sum_{l=0}^{K-1} m_l[n] m_{l+1}^*[n] H_k\left(e^{j\omega_{k,l}}\right) H_k^*\left(e^{j\omega_{k,l+1}}\right)$$

$$+ e^{j2\pi \frac{F_0}{F_s} n} \sum_{l=1}^{K} m_l[n] m_{l-1}^*[n] H_k\left(e^{j\omega_{k,l}}\right) H_k^*\left(e^{j\omega_{k,l-1}}\right)$$

$$+ e^{-j2\pi \frac{2F_0}{F_s} n} \sum_{l=0}^{K-2} m_l[n] m_{l+2}^*[n] H_k\left(e^{j\omega_{k,l}}\right) H_k^*\left(e^{j\omega_{k,l+2}}\right)$$

$$+ e^{j2\pi \frac{2F_0}{F_s} n} \sum_{l=2}^{K} m_l[n] m_{l-2}^*[n] H_k\left(e^{j\omega_{k,l}}\right) H_k^*\left(e^{j\omega_{k,l-2}}\right)$$

$$+ \dots$$

$$+ e^{-j2\pi \frac{KF_0}{F_s} n} m_0[n] m_K^*[n] H_k\left(e^{j\omega_{k,0}}\right) H_k^*\left(e^{j\omega_{k,K}}\right)$$

$$+ e^{j2\pi \frac{KF_0}{F_s} n} m_K[n] m_0^*[n] H_k\left(e^{j\omega_{k,K}}\right) H_k^*\left(e^{j\omega_{k,0}}\right) \tag{4.15}$$

We can now think of $\tilde{m}_k[n]$ as a combination of terms each centered at iF_0 where the magnitude of each term is:

$$\left|\tilde{m}_{k,iF_0}[n]\right| = \left[\sum_{l=0}^{K-|i|} \left|m_l[n]\right|\left|m_{l+i}[n]\right|\left|H_k\left(e^{j\omega_{k,i}}\right)\right|\left|H_k\left(e^{j\omega_{k,l+i}}\right)\right|\right]^{\frac{1}{2}}, \quad -K \leq i \leq K \quad (4.16)$$

Evaluated at DC:

$$\left|\tilde{m}_{k,0F_0}[n]\right| = \left[\sum_{l=0}^{K} \left|m_l[n]\right|^2\left|H_k\left(e^{j\omega_{k,l}}\right)\right|^2\right]^{\frac{1}{2}} \quad (4.17)$$

4.1.3   3 Harmonic Example: Estimated Envelope

Let's go back to our three harmonic example. We are again trying to acquire the 1st harmonic, $m_1[n]$ (green). We define $\omega_1 = 2F_0$.

We can see the relationships

$$\widehat{x}[n] \Longleftrightarrow \widehat{X}[n,f] \quad (4.18)$$

$$\widehat{x}[n]e^{-j2\pi\frac{2F_0}{F_s}n} \Longleftrightarrow \widehat{X}[n,f-2F_0] \quad (4.19)$$

$$\widehat{x}[n]e^{-j2\pi\frac{2F_0}{F_s}n} * h_2[n] \Longleftrightarrow \widehat{X}[n,f-2F_0]H_1(f) \quad (4.20)$$

$$\tilde{m}_1^2[n] \Longleftrightarrow \widehat{X}[n,f-2F_0]H_1(f) * \widehat{X}^*[n,-f+2F_0]H_1^*(-f) \quad (4.21)$$

Equations 4.18 -4.21 are visualized in figure 4.2. The interesting part of figure 4.2 is (f). We see our green component that we were looking for, however there are a whole lot of other things that we didn't want.

Figure 4.1(d) is equivalent to the green component of figure 4.2(f) if our filter $|H_1(f)| = 1$ when $f \approx 0$.

The other components come from interactions with the unwanted harmonics that we failed to completely filter out. For clarity the convolution is visualized in figures 4.3, 4.4, 4.5. Positive and negative components are mirror images so the positive components are not explicitly visualized.
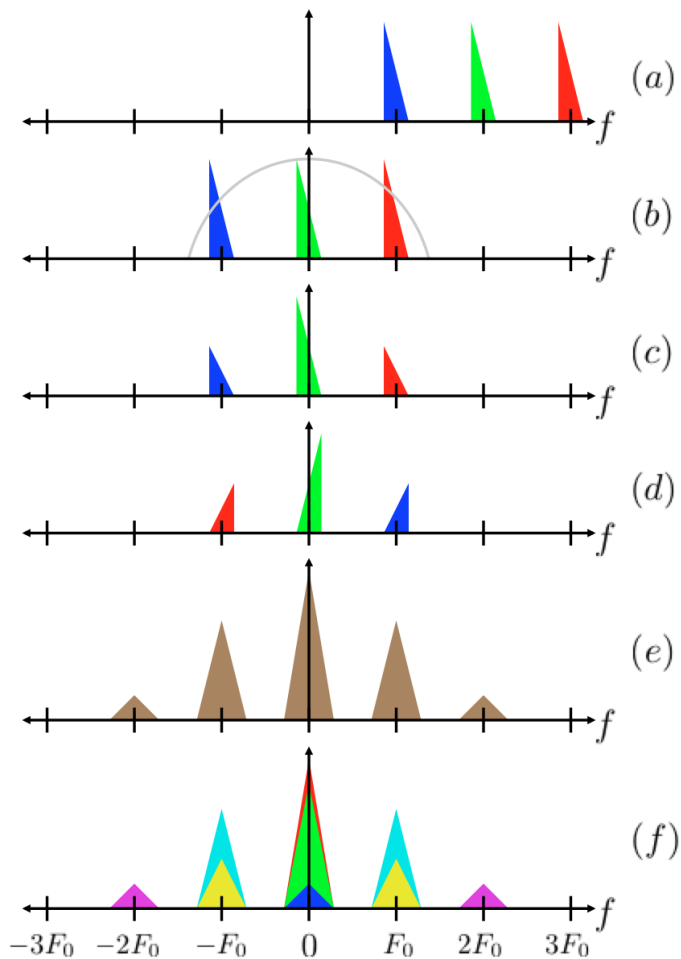
Figure 4.2: $(a)|\widehat{X}[n, f]|$ $(b)|\widehat{X}[n, f - 2F_0)|$ $(c)|\widehat{X}[n, f - 2F_0)||H_1(f)|$ $(d)|\widehat{X}^*[n, -f + 2F_0)||H_1(-f)|$ $(e)|\mathcal{F}\{\tilde{m}_1^2[n]\}|$ (f) contributions of separate components of (e)

4.2 Steady-State Metrics

In considering how well our envelope $\tilde{m}_k[n]$ estimates $m_k[n]$ there are three important metrics. We will now discuss each in detail.

4.2.1 Coherent Gain

Coherent gain is defined as the gain of the harmonic of interest, k.
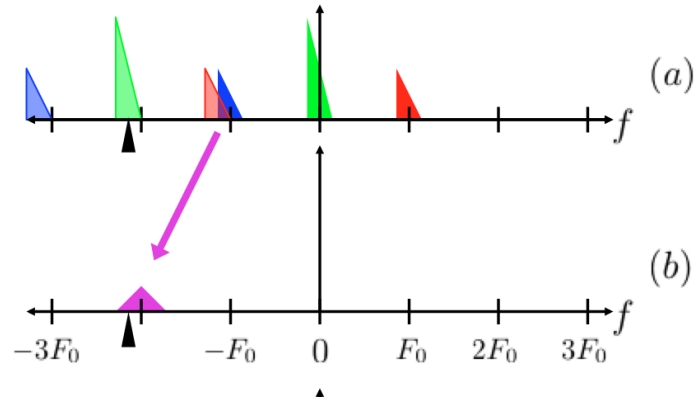
Figure 4.3: Envelope Estimate $-2F_0$ Component
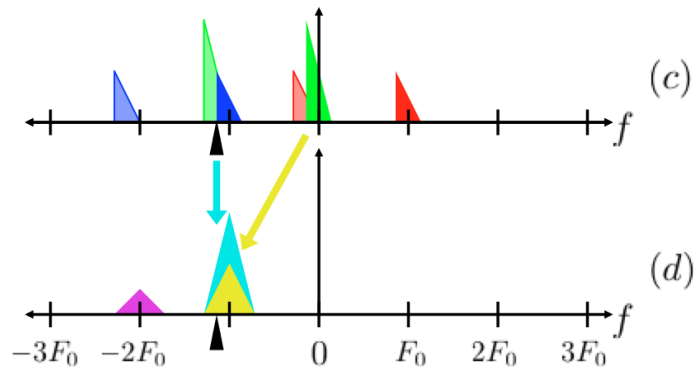
Figure 4.4: Envelope Estimate $-F_0$ Component
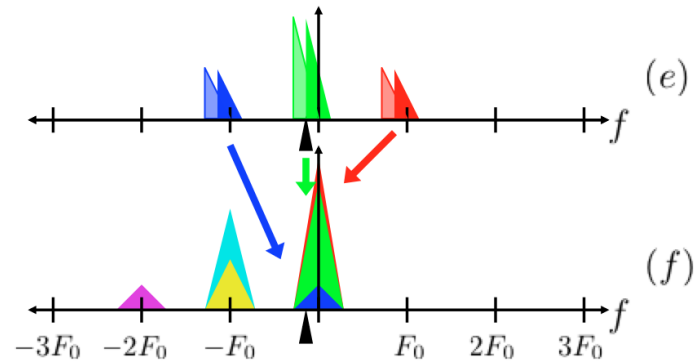
Figure 4.5: Envelope Estimate Baseband Component

$$G_k = \left| H_k\!\left(e^{j\omega_{k,k}}\right) \right| \tag{4.22}$$

Recalling equation 4.12, if $F_{ds,k} = (k+1)F_0$ then, $w_{k,k} = 0$ and the coherent gain is simply the DC gain of the filter.

$$G_k = \left| H_k(0) \right| = \sum_n h_k[n] \tag{4.23}$$

We may further simplify this by normalizing our filter such that $\left| H_k(0) \right| = 1$. Of course, our downshift frequency won't be ideal in real systems. Factors to consider include the quantization of $F_{ds,k}$ and the accuracy of F_0 estimation.

A similar metric, brought up in [REF] is termed scalloping loss, or picket-fence effect. This is the effect of the harmonic falling in between filter centers.

4.2.2   scalloping loss

CONTINUEHERE1

[windows for harmonic analysis]

scalloping loss or picket-fence effect, ratio of coherent gain for tone located half a bin from DFT sample point to coherent gain for tone located exactly at sample point

$$scallopingloss = \frac{|H(\frac{1}{2}\frac{F_s}{N})|}{H(0)} \tag{4.24}$$

"althought scalloping loss is useful, it's not entirely informative. if the scalloping loss if high, then this relates to a sharp cutoff which is actually good for increasing purity of the harmonic"

worst case processing loss = scalloping loss * PL where PL is reduced gain of window (which i have been canceling out) **where does worst case processing loss fit in?**

4.2.3   Harmonic SIR

Continuing our focus on the baseband, another question is: what is the contribution of the target harmonic versus the others? The baseband component is contributed to by spectral leakage due to non-ideal filters. This is visualized as the red and blue in figure $4.5(f)$. The harmonic signal-to-interference-ratio (SIR) quantifies the ratio of target harmonic to spectral leakage.

$$SIR_k = \frac{\left| H_k\big(e^{j\omega_{k,k}}\big) \right|}{\left[\sum_{l=0}^{K} \left| H_k\big(e^{j\omega_{k,l}}\big) \right|^2 \right]^{\frac{1}{2}}} \tag{4.25}$$

The terms will roll off as the harmonic center frequencies get further away from $F_{ds,k}$, so typically SIR_k is sufficiently described by only one or two harmonics on either side of the k'th, i.e. $k - 2 \leq l \leq k + 2$.

Harmonic SIR does not describe the true signal-dependent SIR, as varying envelope magnitudes across harmonics will change this, however it does provide an objective measure of the quality of our system to arbitrary harmonic inputs.

4.2.4   Modulation Depth

Finally, we consider the magnitude of each bandpass component relative to baseband. These terms appear in our envelope estimate as modulations at rates that are multiples of F_0. Because of the forced symmetry of the real envelope we only need to consider positive frequencies, iF_0.

$$D_{k,i} = \frac{\left[\sum_{l=0}^{K-i} \left| H_k\big(e^{j\omega_{k,l}}\big) \right| \left| H_k\big(e^{j\omega_{k,l+i}}\big) \right| \right]^{\frac{1}{2}}}{\left[\sum_{l=0}^{K} \left| H_k\big(e^{j\omega_{k,l}}\big) \right|^2 \right]^{\frac{1}{2}}}, \quad 1 \leq i \leq K \tag{4.26}$$

The largest value and, for that reason, most important value is $D_{k,1}$, the modulation depth at F_0.

4.3 Explicit Temporal Modulation

So our three metrics are coherent gain, harmonic SIR and modulation depth. We aim for a coherent gain of $G_k = 1$, maximum possible SIR and one would think minimum modulation depth.

Interestingly, some current CI processing strategies such as ACE intentionally allow for induced modulations from non-isolated harmonics. This provides a temporal cue to the user which plays into pitch percept.

The alternative is to use narrow enough filter cutoffs to eliminate these modulations, and then explicitly modulate the signal. In this option we need further processing such as a pitch estimator to determine the modulation rate.

In this document we argue that the latter, explicit modulation option is better. The reasoning is best shown by a motivational example.

Let's consider a single note played by two different instruments: clarinet and saxophone. In this example $F_0 = 261Hz$. The clarinet is interesting in that it only has energy at odd harmonics.

We attempt to estimate the 3rd harmonic, $m_3[n]$. We first downshift by $-3F_0$, then lowpass filter. The spectrum of each signal at this stage is visualized in figure 4.6. The top panel shows the output of a sufficiently narrow filter where the 3rd harmonic is isolated. The bottom panel shows a different filter design that intentionally allows the two adjacent harmonics to pass through. Here we start to see the problem, that despite the wide bandwidth filter, there is (almost) no energy around $\pm F_0$ for the clarinet because of the harmonic structure. (There is something present however it's down 30dB.)

Figure 4.7 shows the time-domain envelopes resulting from this processing. The input signals were normalized such that the top panel shows the same signal power for both instruments.

The problem is clearly represented in the bottom panel, were we have a very large F_0 modulation in the saxophone envelope but little to no change in the clarinet. The result is that we have a much stronger temporal pitch cue as well as louder overall volume to the saxophone.
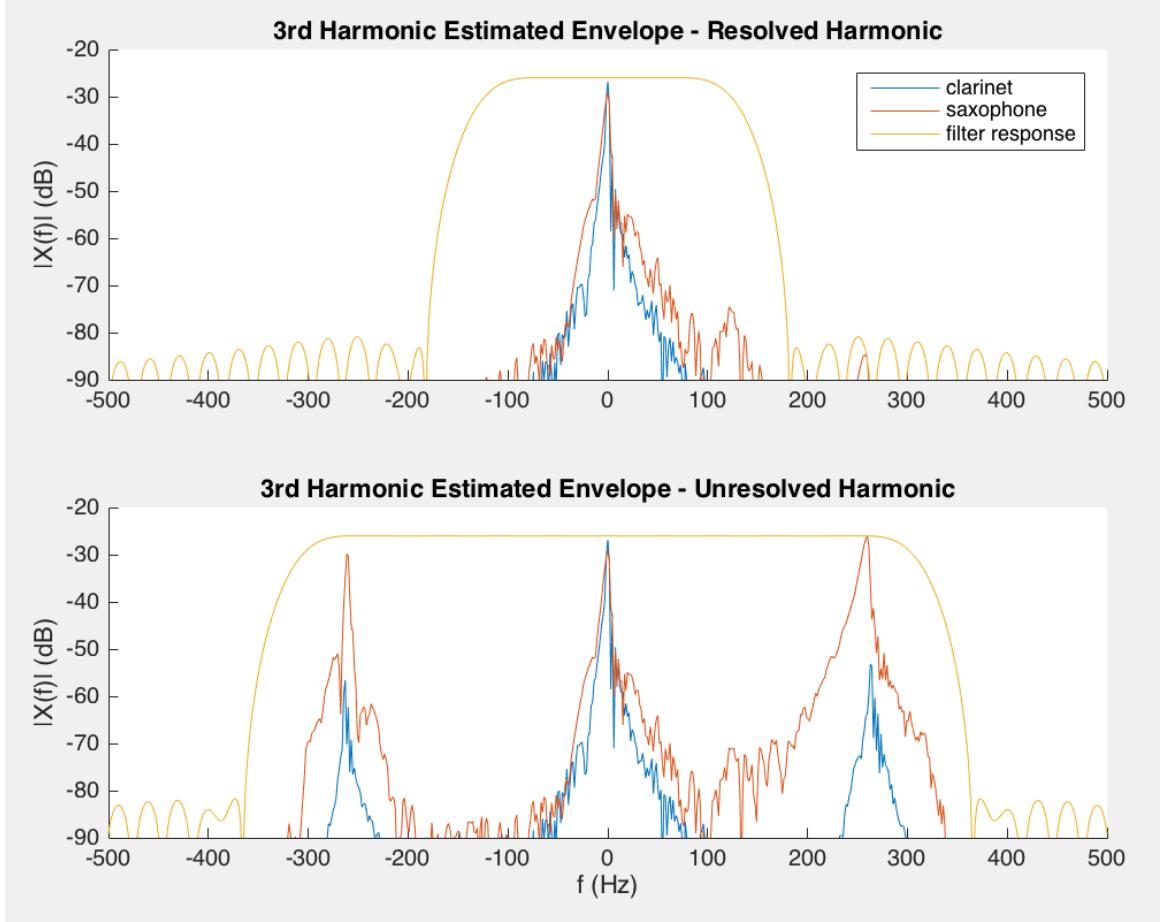
Figure 4.6: Clarinet vs Saxophone Harmonic Components

Spectral leakage into other harmonic envelopes is not natural. It forces the envelope to modulate as a function of the adjacent harmonics which, as we just saw, is signal dependent. Furthermore, if we have uniform bandwidth filters, (as ACE does), the harmonic resolution will not behave as it does in the cochlea.

Beyond this example, explicit modulation decouples F_0 and modulation depth. This way we have much more control over modulation depth while still making optimal design decisions for envelope extraction. We can decide modulation depth as a function of how harmonic the signal is. eTone [REF] uses a harmonic probability metric to do just that.
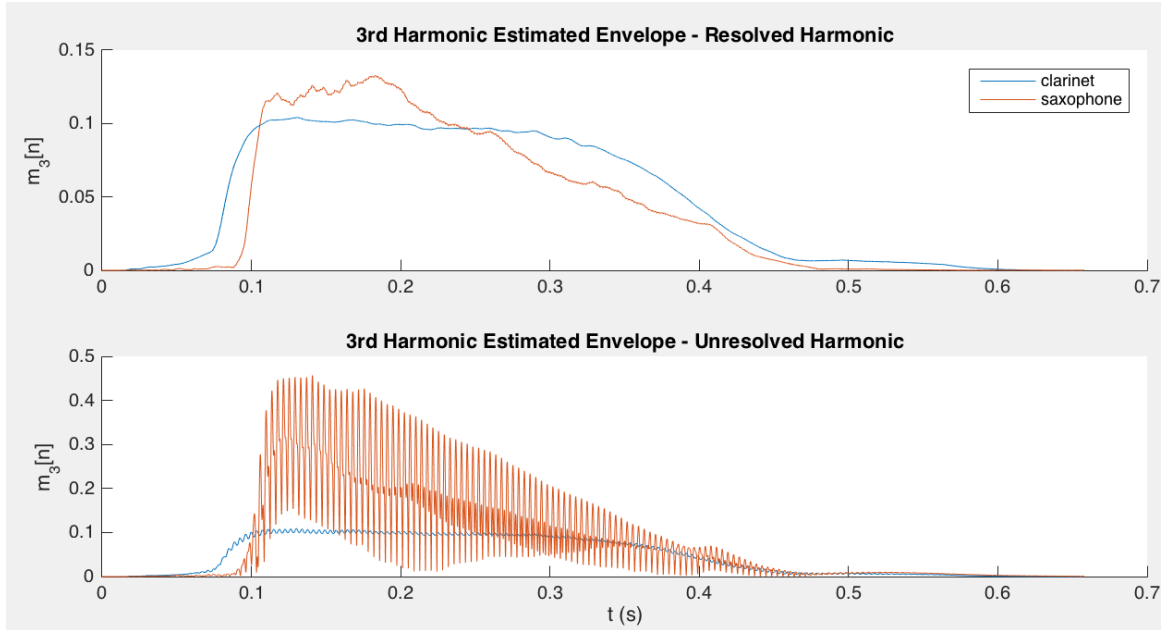
Figure 4.7: Clarinet vs Saxophone Envelope Estimates

4.3.1   Followup Filter

Another thing to note is that regardless of downshift frequency, our harmonic envelope will always have it's energy centered at baseband and multiples of F_0. An alternative way of eliminating induced modulations is to add a lowpass filter to the end of the processing chain.

There are a handful of research strategies [REF?] that have used this additional filter. eTone's envelope follower is an example of this.

The main improvement to a followup filter is that we can guarantee to eliminate modulations. This could also be achieved by designing a sufficiently narrow filter, $h_k[n]$ however this brings about a tradeoff, where the narrower our filter is the more susceptible we are to error in downshift frequency.

In terms of our three metrics, the followup filter will provide us with a robust coherent gain and guaranteed low modulation depth at the cost of lower harmonic SIR.

Another point to consider is the cost of adding an extra processing stage. The additional stage means more memory, clock cycles and processing delay.

4.4 Changing F0

dips due to quantization

CONTINUEHERE2

maybe move the plot here?

4.5 Transients

"almostall of the hearingaidstestedhaveattack timeslessthan or equalto 10 ms. A little more than half of the hearingaidshad releasetimesof 50 ms or less.The rangeof the attack times varied from 1 to 23 ms" [attack and release times of AGC hearing aids]

quantify for best case and worst case where worst case is the fastest transient relevant to music (this should also hinge on CI limitations)

maximum onset dynamic range "90ms - 10ms = 80ms" and ratio of filter smeared range to max range rinse and repeat for CI's

4.6 Steady-State Evaluation of Strategies

chosen parameter values to compare: order 128, 256, 512 quantize 1, 31, 62, 125 cutoff, rectangular, hanning, adaptive butterworth

4.6.1 Design Parameters

As stated above the design can be summarized by downshift frequency and lowpass filter.

For our steady-state scenario the the ideal downshift frequency is simply $(k+1)F_0$. The question is what degree of quantization is sufficient to estimate our signal.

For filter design we need to consider bandwidth as a function of filter order and filter/window type. Ideally our cutoff is somewhere below F_0 but high enough to incorporate the narrow bandwidth of $m_k[n]$.

The filters can be different as a function of k. This is a natural path to pursue if we consider the critical bands of the cochlea. This will be discussed in more detail later in this document however for now we will assume $h_k[n] = h[n]$. This is natural for harmonic envelopes as harmonics are linearly spaced.

4.6.2 Coherent Gain

Not overly interesting, just talk about scalloping loss and mention that number of dips is proportional to harmonic index k.

The interesting part here is the relationship between harmonics. Where the fundamental will be at a minimum, the 1st harmonic is at a maxima, example:

$F_q = 125$

$F_0 = 182.5 = 1.5F_q$ (right in between bins)

$2F_0 = 375 = 3F_q$ (right on a bin)

¡CONTINUE HERE¿

[windows for harmonic analysis]

scalloping loss or picket-fence effect, ratio of coherent gain for tone located half a bin from DFT sample point to coherent gain for tone located exactly at sample point

$$scallopingloss = \frac{|H(\frac{1}{2}\frac{F_s}{N})|}{H(0)} \tag{4.27}$$

"althought scalloping loss is useful, it's not entirely informative. if the scalloping loss if high, then this relates to a sharp cutoff which is actually good for increasing purity of the harmonic"

worst case processing loss = scalloping loss * PL where PL is reduced gain of window (which i have been canceling out) **where does worst case processing loss fit in?**

4.6.3 Harmonic SIR

Just note that major difference in filter order

4.6.4 Modulation Depth

Again...filter order

BW = F0/2 downshift = exp(kF0)

downshift quantization, bandwidth as function of F, F0?

modulation depth (kind of another SIR) as a function of downshift quantization and filter

4.6.5 figures

4.6.6 Envelope Bandwidth Tradeoff

incorporating multiple harmonic components VS eliminating transient characteristics

4.7 ALL METRICS

ENBW (accumulated noise) PL (gain at DC) PG (same as PL?) scalloping loss (downshift quantization worst case) worst case PL = PL*SL

harmonic SIR harmonic gain (maybe not overly relevant due to AGC, etc) (how is it affected in a relative sense? worst vs best) modulation depth (spectral leakage) transients changing F0 (scalloping loss dip only, and harmonic SIR)

4.7.1 Harmonic Bandwidth

what is it? look at some waveforms and read some stuff. maybe do this at the same time as transient times of instruments

Talk about filter bandwidth, what is optimal? F0/2? F0/4?

4.8 Adaptive Filters

consider all metrics for adaptive filters, order 128, 256, 512

Chapter 5

IMPLEMENTATION CONSIDERATIONS

5.1   Efficient Interpolation Algorithm

FFT with changeable window, and interpolate

Can this be done with different filter as function of F0? We probably need to design the filters such that they pass reconstruction requirements

Is the actual equation just a sinc function times a phase shift?!

READ THIS: [An Intelligent FFT-Analyzer with Harmonic Interference Effect Correction and Uncertainty Evaluation]

5.2   Mapping and Selection

Chapter 6

_____

Chapter 7

HARMONIC ENVELOPES

7.1 Pitch Estimation

Fundamental Frequency Modulation is a key prospect in temporal encoding, shared by F0mod and HSSE, but not ACE. In both HSSE and F0mod an autocorrelation followed by peak finding is implemented.

In this method...

There are various ways to estimate pitch with trade-offs for each. We are going to assume the pitch estimator is the same when analyzing F0mod and HSSE.

[people] in [ref]

talk about F0 estimator and alternatives...

our (shared) technique e-tone? harmonic sieve, etc. latency, accuracy, octave errors and range restrictions, quantization

7.2 NOTE

mixture

considerations: F0 range F0 error Harmonic Index F0 error mean/variance

downshift freq BW (as function of F0?)

7.3 Envelope Extraction

As previously mentioned only envelope information is used in CI strategies. In ACE and F0mod we use the STFT method 3.11. In HSSE we use the magnitude of the harmonic method which may be derived from B.2 as

$$m_k[n] = \frac{1}{Lw[0]}|X_L[n, \lambda[n]k]| \tag{7.1}$$

keeping in mind that this is an approximation under the assumption that $F_0[n]$ remains roughly constant over any window of time n to $n + L - 1$. L denotes FFT order.

We may generalize all three methods to 7.1 by stating $\lambda[n] = 1$ for the case of ACE/F0mod.

Despite none of these methods implementing a Hilbert envelope it is useful to also consider 3.4 for the potential benefits of nonuniform filter bandwidths.

We are left with two design components: downshift frequency and lowpass filter.

7.3.1   limbo

Let's consider an example?

switched-capacitor filterbanks consisted of only 20 filters [swanson thesis] must have been non-uniform...

"From the continuum of possible frequencies, only those which coincide with the basis will project onto a single basis vector; all other frequencies will exhibit non zero projections on the entire basis set. This is often referred to as spectral leakage and is the result of processing finite-duration records."

7.3.2   Downshift Frequency

For STFT envelopes, the downshift frequencies are fixed. For HSSE the downshift frequencies are a function of $F_0[n]$. Let us now consider the benefits of using $F_0[n]$.

$$\widehat{x}[n] = a_0[n]e^{j2\pi \frac{F_0}{F_s}n} + a_1[n]e^{j2\pi \frac{2F_0}{F_s}n} \tag{7.2}$$

$$m_1[n] = |\widehat{x}[n]e^{-j2\pi \frac{F_{DS}}{F_s}n} * w[-n]| \tag{7.3}$$

$$= \left| \left[a_0[n]e^{-j2\pi \frac{F_0 - F_{DS}}{F_s}n} + a_1[n]e^{-j2\pi \frac{2F_0 - F_{DS}}{F_s}n} \right] * w[-n] \right| \tag{7.4}$$

$$\tag{7.5}$$

...ungh I'm doing a terrible job at this, the goal is to say that the downshift frequency determines the ratio of energy of harmonics. If we downshift at exactly F_0 then we have maximum ratio of x_0 to other harmonics, relating to $|W[F_{after-downshift}]|$ for each harmonic.

The optimization of the magnitude ratio is determined by the downshift frequency, whereas the degree of mixture as well as amount of frequency beating at multiples of F_0 is determined by filter rolloff.

Maybe add $|W[f]|$ to some equation like the following. Maybe do it in frequency domain, and maybe figures?!

Two things to consider, magintude of each component at baseband, and magnitude of each component at kF_0.

$$
\begin{aligned}
m_k^2[n] =& |x_k[n]e^{-j2\pi F_{DS}n} + x_{k+1}[n]e^{-j2\pi F_{DS}n}|^2 \\
=& |x_k[n]|^2 + |x_{k+1}[n]|^2 + x_k[n]x_{k+1}^*[n] + x_k^*[n]x_{k+1}[n]
\end{aligned}
\tag{7.6}
$$

Okay, maybe I finally have it:

$$
\widehat{x}[n] = \sum_k a_k[n]e^{j2\pi \frac{kF_0}{F_s}n}
\tag{7.7}
$$

$$
m_k[n] = |a_k[n]|
\tag{7.8}
$$

$$
\begin{aligned}
\tilde{m}_k^2[n] =& |\widehat{x}[n]e^{-j2\pi \frac{F_{DS}}{F_s}n} * w[-n]|^2 \\
=& \left| \sum_{k'} a_{k'}[n]e^{j2\pi \frac{k'F_0 - F_{DS}}{F_s}n} * w[-n] \right|^2 \\
\approx& \left| \sum_{k'} a_{k'}[n]\left(e^{j2\pi \frac{k'F_0 - F_{DS}}{F_s}n} * w[-n] \right) \right|^2 \\
=& \left| \sum_{k'} a_{k'}[n]\left(e^{j2\pi \frac{k'F_0 - F_{DS}}{F_s}n} W(-k'F_0 + F_{DS}) \right) \right|^2 \\
=& \sum_{k'} |a_{k'}[n]|^2 |W(-k'F_0 + F_{DS})|^2
\end{aligned}
\tag{7.9}
$$

$$
\begin{aligned}
&+ a_k[n]a_{k-1}^*[n]W(-kF_0 + F_{DS})W^*(-(k-1)F_0 + F_{DS})e^{j2\pi \frac{F_0}{F_s}n} \\
&+ a_k^*[n]a_{k-1}[n]W^*(-kF_0 + F_{DS})W(-(k-1)F_0 + F_{DS})e^{-j2\pi \frac{F_0}{F_s}n} \\
&+ a_k[n]a_{k+1}^*[n]W(-kF_0 + F_{DS})W^*(-(k+1)F_0 + F_{DS})e^{-j2\pi \frac{F_0}{F_s}n} \\
&+ a_k^*[n]a_{k+1}[n]W^*(-kF_0 + F_{DS})W(-(k+1)F_0 + F_{DS})e^{j2\pi \frac{F_0}{F_s}n} \\
&+ ...
\end{aligned}
\tag{7.10}
$$

There will continue to be more terms at higher multiples of F_0. The first thing to point out is that these terms will become more and more negligible as one or both of the terms moves away from $|W(0)|$. The second thing is that this motivates secondary filter if we do not want modulations since we are explicitly inducing a modulation at F_0 with controlled modulation depth. The eTone strategy is an example of this, using an envelope follower...

Now lets look closer at the important term (3.17)

$$\tilde{m}_k[n] = \left[\sum_{k'} |a_{k'}[n]|^2 |W(-k'F_0 + F_{DS})|^2 \right]^{\frac{1}{2}} \tag{7.11}$$

$$SNR = \frac{|W(-kF_0 + F_{DS})|}{\left[\sum_{k'} |W(-k'F_0 + F_{DS})|^2 \right]^{\frac{1}{2}}} \tag{7.12}$$

Things that should be considered when making these decisions:

-beat frequencies (good for ACE, bad for other) harmonic isolation -anatomy (critical bands) -transient/noise preservation

downshift frequency could be important at low harmonics, however for high harmonics it fails due to accuracy of F0 estimate as well as the physical process. Humans don't resolve high harmonics, why should our system?

AN IDEA: What about using the CIS-style last stage filter to ensure we are only getting baseband information?

The human ear has much better resolution than the cochlear implant sound processor when decomposing a signal into frequency bands. The artifacts of this can be clearly demonstrated by example. In case1, the energy of the signal falls directly on the center frequency of an FFT bin. In case2 the signal falls in between two bins. In this case, neither bin represents the true energy of the signal.

We only have so many to work with in a CI. By using knowledge about the signal we can better design our filters to circumvent this limitation.

HOWEVER:

Coherent is the Same (mathematically) as Hilbert, as ACE, as CIS except...for the downshift frequency. This leads to a minimal (-1.6dB max) loss of gain for the desired

frequency however it may lead to lower SNRs when desired frequency is further from center of filter and noise is closer to center of filter simultaneously.

downshift frequencies are quantized to same as FFT (256 frequencies spaced 30Hz apart) doesn't matter though, gain is same... (¡ -1.4dB dip) NOT TRUE!!! Roll-off is not linear in dB, so since signal is not pure tone, components will roll off at faster or slower rates

show plots as well as math :D

how much does bin alignment matter? it's probably a function of F0, what about unvoiced signals? filter bandwidth? F0/2, narrower to reduce noise interference

7.3.3 Filter Design

CONTINUE HERE

filter bandwidth is a tricky one. We could have: constant function of -F0 -Fcenter

From the theoretical standpoint, envelope extraction is exactly the same in ACE and F0mod. In implementation ACE typically uses a lower order FFT. In [laneau] the authors consider 128-point for ACE and 512-point for F0mod and both will be considered here.

with respect to bandwidth we actually have to different things, filter bandwidth and effective information bandwidth. The former is obvious, the later refers to what frequencies are encoded on a electrode channel. If multiple narrowband filters are somehow combined on the same channel, they may have the same information bandwidth as one wideband filter.

Woah...come back to CIS vs ACE etc for this!

ACE currently uses modulations due to harmonic artifacts and low-order FFT. This is horrible! Let me explain why...it has nothing to do with the harmonic of interest and everything to do with the one harmonic below and one harmonic above the harmonic of interest. Because this demodulation is done incoherently the modulation depths are not directly related to the harmonic of interest. Furthermore, the cutoff is fixed and decided by parameters of the FFT and sampling rate which have nothing to do with the signal itself. This makes the modulation even further unrelated to the signal. (Could this also theoretically be a problem for F0mod? Case: F_0 is very low and the harmonic lands right

between two bins. A small modulation could come about, probably not)

An important detail to note is that of low-order-FFT induced modulations mentioned for ACE. Laneau explicitly describes two different methods as ACE128 and ACE512 corresponding to different FFT orders. F0mod uses ACE512 which keeps FFT bin modulations below roughly 60Hz in contrast to ACE128's 240Hz. This sharper cutoff keeps envelope modulations out of the carrier frequency range, isolating this component and leaving the role of carrier modulation to the explicit modulator at F_0.

This segregation allows for easier relation to the modulation model of sounds. Furthermore, F0mod is not prone to the modulation artifacts present in ACE128 and discussed in section 2.?.?

7.3.4   Unvoiced Signals

I really hope!!! This is well handled by two factors.

1) automatically choose high F0 when no good estimate exists. This allows for higher frequencies (more important and more likely to be present in unvoiced) to be acquired.

2) If filters are adaptive bandwidth, the wide-bandwidth filters will preserve more high-frequency noise-like modulations.

- Still no concrete solution for unvoiced signals, best answer so far is to have automatic high-F0 estimate during unvoiced sections (make it more stable than if bouncing between high and low)

7.3.5   Takeaway

- Phase Preservation doesn't matter (shh vs saw)

- center frequency also doesn't matter (-1.6dB)

- HSSE may be viewed as a different way of combining FFT bin magnitudes. I would argue that we do this using F0 for low frequencies, and fixed for high. (critical bands!!!)

7.4 Channel Allocation

7.4.1 Envelope Combination

now that we have considered phase and magnitude, this component of HSSE can essentially be considered as a different combination of FFT bin magnitudes when compared to ACE.

as mentioned above hsse takes F0 into account and avoid bin alignment issues, however, inaccuracies in F0 estimate can lead to loosing high energy harmonics with narrowband filters. likely need to just combine unless F0 estimator can be significantly improved

This is is where the critical band concepts come into play, would this mess up speech in noise goals? probably...but what can be done if we can't get a good pitch estimate? filtering F0 could help this a bit but it introduces further delay

updating only 9 samples of downshift per frame rather than grabbing complete complex exponential could help however once the channels are combined it shouldn't matter

7.5 N-of-M Selection 1

The key to HSSE here, is that we have isolated individual harmonics. Harmonics are mapped to associated fixed channels due to the limitations of a fixed number of channels and fixed locations in the cochlea. Because we have isolated individual harmonic envelopes there is no issue of signal energy falling in between channels.

7.5.1 Regularizer Heuristic

Another bonus to HSSE is that we may add a simple heuristic to maintain channel mapping stability. For example, if F0 has not varied significantly since the previous frame, we can allocate to the same channels to avoid unnecessary switching between channels induced by vibrato or inaccuracies in pitch estimation.

7.5.2 Multiple Harmonics Per Channel

As far as having multiple harmonics in a single channel, there are a few solutions

1) Choose highest energy harmonic.

suffers from stability issues, what about gain?

2) Choose First

suffers from missing important harmonics in channel as well as misrepresenting unvoiced signals

3) Combine

How? via sum of squares?

does a gain factor need to be applied to each channel? how was this determined for ACE?

7.5.3 Takeaway

Low Frequencies: stability heuristic keeps from jumping channels when on edge.

High Frequencies: not really relevant if critical bands are used

- gains? maybe just use same as ACE since this should be pretty similar

7.6 N-of-M Selection 2

Two general solutions

1) Adaptive (select loudest)

similar to ACE, we can choose the loudest channels. This suffers from stability issues. We can apply another heuristic to stabilize the decision based on consistency of signal energy and fundamental frequency

2) Fixed

stable, each option suffers from missing key harmonics to the signal

lowest channels will imply no high frequency energy, which could be bad for unvoiced signals

other relationships such as odd harmonics or prime numbered harmonics could miss harmonics critical to timbre perception.

What if we did F0mod with same channel selections as HSSE? What would happen?

7.6.1 N-of-M Selection HSSE

Various ideas have been proposed including N-largest and lowest-N. Fixed Greenwood bands are determined offline, corresponding each electrode with a bandwidth. The N envelopes are then mapped to electrodes by finding the greenwood bands each harmonic falls within.

7.6.2 Takeaway

- Fixed VS MaximaSelect: this is still up in the air, Fixed is complicated by not necessarily having harmonic envelopes

- for maxima select heuristics can be used to choose same if energy and F0 have not changed significantly

N-of-M, It is important to note that this is the same case for F0mod. The carrier modulation is the same on each envelope and thus does not affect the selection process.

7.7 Carrier Synthesis

talk about modulator types briefly

F0mod does raised

$$c_{ch}(t) = 0.5 + 0.5cos(2\pi F_0 t)$$

We consider a few...cite paper

Let's not really go into detail about this, just mention and cite some things. Probably put this up higher in the document?

[4 wave paper]

Swanson thesis: "A high-rate pulse train, modulated on and off at frequency F0, had a higher pitch than a train of pulses at the rate of F0. If amplitude modulation of high-rate pulse trains is to be used to convey pitch, then the shape of the modulating waveform is important: a half-wave shape is better than a square-wave (on-off) shape."

7.8 Conclusion

Chapter 8

HHE

come up with a better name!!!

MOTIVATION

8.0.1 HSSE vs F0mod Differences

harmonics are resolved

- how do we deal with should-be-unresolved harmonics?

channel combination

- further considerations are needed

- what does sum of squares mean? is it constant energy within the channel? does it cause a gain or just average the channels? look further into the gain component to ACE it's just a ¡1 gain for multiple bins in one channel

- can harmonics be combined? (higher harmonics) what does it mean to combine channel phase information?

channel selection

- further considerations are needed

- What if we did F0mod with same channel selections as HSSE? What would happen?

8.1 HSSE vs F0mod More Differences

recitified modulator (likely not too important)

also, pitch tilts

how can all of this be applied to soft decisions?

how can this all be done in real-time?

how are we accounting for non-linearities: AGC and sensitivity

8.2   Alternative Coherent Envelope Calculation using FFT bins

This could all be achieved by zero padding, but not as efficiently?

$$\beta = \frac{F_0}{F_s}N - \left\lfloor \frac{F_0}{F_s}N \right\rfloor \tag{8.1}$$

$$0 \leq \beta < 1$$

$$Z[k] = X[k + \beta] = X[k] * \delta[k + \beta]$$

We can design a filter:

$$
\begin{aligned}
h_\beta[k] =& \delta[k + \beta], \qquad 0 \leq k < N \\
=& IFFT\{e^{j\frac{2\pi}{N}\beta n}\}, \qquad 0 \leq n < N \\
=& \frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k+\beta)n} \tag{8.2}
\end{aligned}
$$

maybe be specific about circular convolution, non-infinite bounds?

$$
\begin{aligned}
Z[k] =& X[k] \circledast h_\beta[k] \\
\approx& X[k] \circledast h_\beta[k]w[k] \tag{8.3}
\end{aligned}
$$

We have an approximation where $w[k]$ is a window and $w[k] = 0, |k| > l$. We can then compute an approximate shift using $2l+1$ complex multiplies and additions. The nice thing about this is that as $|k|$ increases $h_\beta[k] \to 0$ very rapidly , so we only need a very low number l to approximate with good accuracy.

Incredibly frustrating...but do we even need this? What about just choosing the nearest FFT bin.

Another consideration:

8.3   Critical Bands

talk about filter design in F0mod and HSSE and why non-uniform is better

8.3.1 HSSE vs ACE vs Human Ear

In this subsection I will discuss the general differences in critical bandwidth:

1) how HSSE is too fine of a resolution note: HSSE originaly had BW = F0/2, however hard to implement and still not like ear

2) how ACE is overall a poorer resolution

What about doing a hybrid? This would further justify alternative HSUM in it's improved efficiency! If summing together anyway, does it matter if harmonic envelopes are used or incoherent envelopes are used?

How about specifying the bandwidth at each electrode as apposed to the frequency boundaries

Bro, you need to look into Xing's method with multiple harmonics modulated at multiples of F0...

8.3.2 Resolution Simulated by Adaptive Envelopes

The human ear has orders of magnitude more filters than ACE, (roughly 1500/22 I think).

HSSE could simulate this higher resolution by choosing different filter center frequencies based on the input signal

8.3.3 Channel Selection Analysis

ACE is like HSSE but for fixed FoI's. We extract an envelope at the FoI and then transmit it to the associated electrode.

1) this goes back to what are the implications of ACE512 vs ACE128 vs coherent-envelope if we are summing anyway

2) can HSSE be reanalyzed in these terms to better justify wide-bandwidth filters for high frequencies?

Could channel selection concepts in HSSE be important? Reflect on this in hindsight to recent discoveries. By this I mean using memory to not switch channels excessively and other decisions that were brought into account.

8.4 Other Important Components

Most everything so far has assumed the signal has an F_0, what if it doesn't? What if it is well outside the boundaries of F_0? What about polyphonic music? What about SNRs below what is needed for accurate F_0 estimation. What other flaws do these strategies have? Mention eTone and other possible solutions, or why we justify not considering these problems.

8.5 Algorithm

1) Filter Center Frequency 2) Filter BW 3) Effective Channel Information BW

8.6 Freedom details

Chapter 9

SUBJECT TESTS

initial results are...

it was important not to change other 12 HWR strategy take-home study 224 aspects of the strategy, in particular, stimulation rate. It would not be a fair comparison to trial HWR at 1800 pps against ACE at 900 pps, as the increased stimulation rate in itself could affect performance. A higher rate could potentially represent amplitude modulation cues more faithfully (McKay et al. 1994). Conversely, there is evidence that sensitivity to temporal modulation is worse at higher rates (Galvin and Fu 2005). [swanson thesis]

9.0.1   simulated real-time

9.0.2   mandarin tones pitch tilt

9.0.3   freedom processor

speech recognition... timbre recognition... other...

Chapter 10

LESS THEORETICAL STUFF

About this chappy

10.1   Engineering Decisions for Real-time

1) 8 harmonics this assumes we are dealing with musical instruments, speech is going to have characteristics well above the 8th harmonic. A hope is that with inharmonic signals the estimate will automatically bounce to $\max(F_0$ estimate) which will thus hit the highest frequencies. This also goes back to the hybrid idea

2) F_0 estimation downsampling details, ooOOooo, so impressive!

10.2 F_0 tilt, exageration

mention the point that this was already done in Xing's paper, albeit $F_0/2$ without affine shift is more more likely to hit boundaries

10.3   assembly implementation

maybe show flow diagram or talk about 128-pt fft limitations

Chapter 11

CONCLUSION

11.1 Summary

11.2 Future Work

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[2] Blake S Wilson, Charles C Finley, Dewey T Lawson, Robert D Wolford, and Mariangeli Zerbi. Design and evaluation of a continuous interleaved sampling (cis) processing strategy for multichannel cochlear implants. *Journal of rehabilitation research and development*, 30:110–110, 1993.

Appendix A

WHERE TO FIND THE FILES

The uwthesis class file, `uwthesis.cls`, contains the parameter settings, macro definitions, and other TeXnical commands which allow LaTeX to format a thesis. The source to the document you are reading, `uwthesis.tex`, contains many formatting examples which you may find useful. The bibliography database, `uwthesis.bib`, contains instructions to BibTeX to create and format the bibliography. You can find the latest of these files on:

- My page.

  `http://staff.washington.edu/fox/tex/uwthesis.html`

- CTAN

  `http://tug.ctan.org/tex-archive/macros/latex/contrib/uwthesis/`

  (not always as up-to-date as my site)

Appendix B

DERIVATIONS

$$\phi_0[n+r] = \phi_0[n] + 2\pi \frac{F_0[n]}{F_s} r, \quad 0 \le r < N \tag{B.1}$$

$$m_{k,harmonic}[n] = \left| x[n]e^{-jk\phi_0[n]} * \frac{1}{Nw[0]} w[-n] \right|$$

$$= \frac{1}{Nw[0]} \left| \sum_{r=-\infty}^{\infty} x[n-r]e^{-jk\phi_0[n-r]} w[-r] \right|$$

Let $\quad r' = -r$

$$= \frac{1}{Nw[0]} \left| \sum_{r'=0}^{N-1} x[n+r']e^{-jk\phi_0[n+r']} w[r'] \right|$$

$$= \frac{1}{Nw[0]} \left| e^{-jk\phi_0[n]} \sum_{r'=0}^{N-1} x[n+r']e^{-j\frac{2\pi F_0[n]}{F_s} kr'} w[r'] \right|$$

$$= \frac{1}{Nw[0]} \left| e^{-jk\left(\phi_0[n] - \frac{2\pi F_0[n]}{F_s} n\right)} \left[e^{-j\frac{2\pi F_0[n]}{F_s} kn} \sum_{r'=0}^{N-1} x[n+r']w[r']e^{-j\frac{2\pi F_0[n]}{F_s} kr'} \right] \right|$$

$$= \frac{1}{Nw[0]} \left| X\left[n, \frac{N}{1} \frac{F_0[n]}{F_s} k\right) \right|$$

$$= \frac{1}{Nw[0]} \left| X\left[n, \lambda[n]k\right) \right| \tag{B.2}$$

VITA

Jim Fox is a Software Engineer with UW Information Technology at the University of Washington. His duties do not include maintaining this package. That is rather an avocation which he enjoys as time and circumstance allow.

He welcomes your comments to `fox@uw.edu`.