Часть 2. – Ф.И.О: Хосна Хай – № Вариант: 32.
Группа: 5И 4110.

№ 8. Вычислить определённые интегралы.

$\int_1^e \frac{dx}{x\sqrt{1-(\ln x)^2}}$ ⇒ Пусть $t = \ln x$. ⇒ $dt = \frac{1}{\cancel{x}} dx$ ⇒ $\frac{dt}{\sqrt{1-t^2}}$

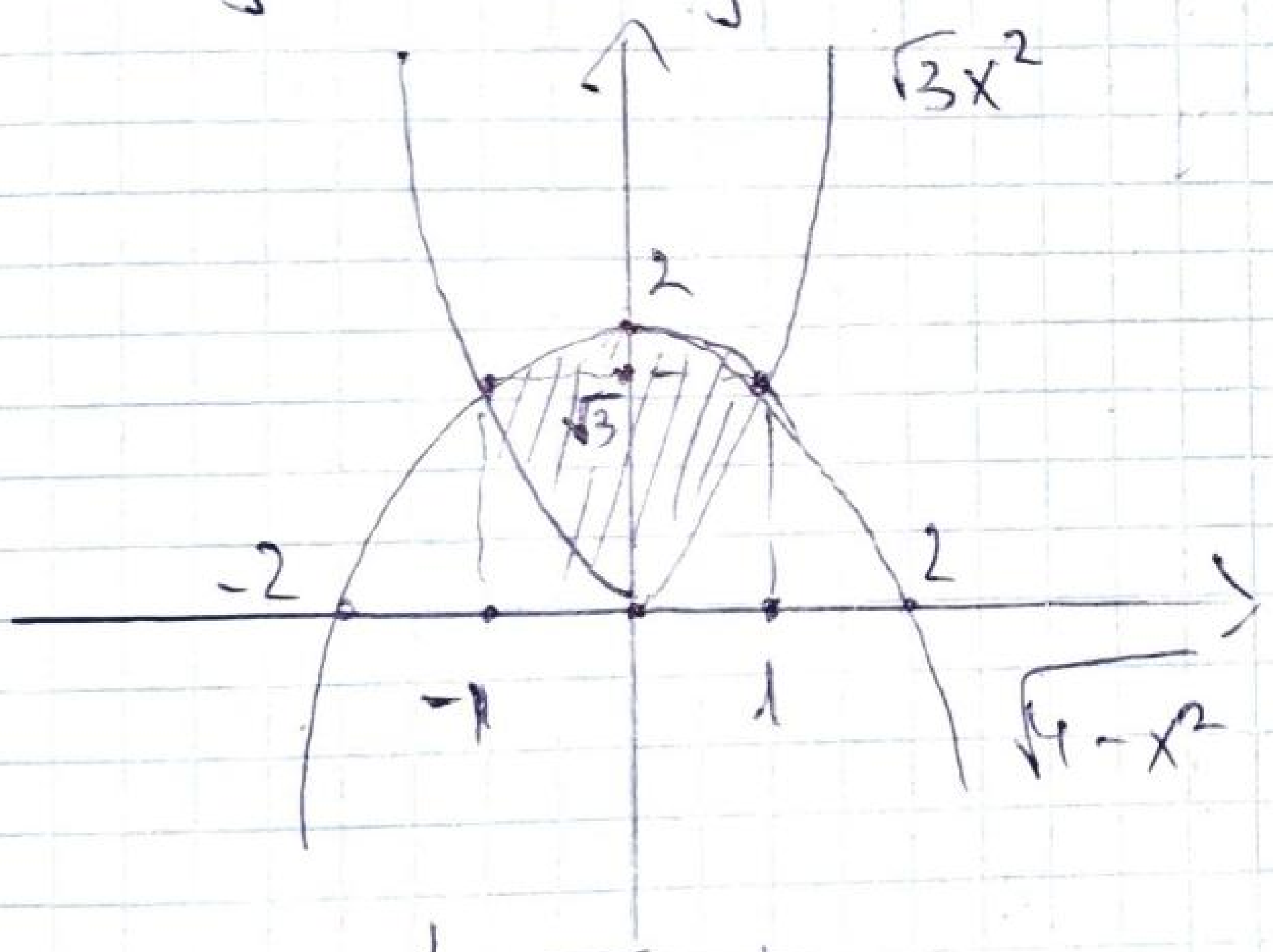
$$\Rightarrow \int_0^1 \frac{dt}{\sqrt{1-t^2}} = \arcsin t \Big|_0^1$$

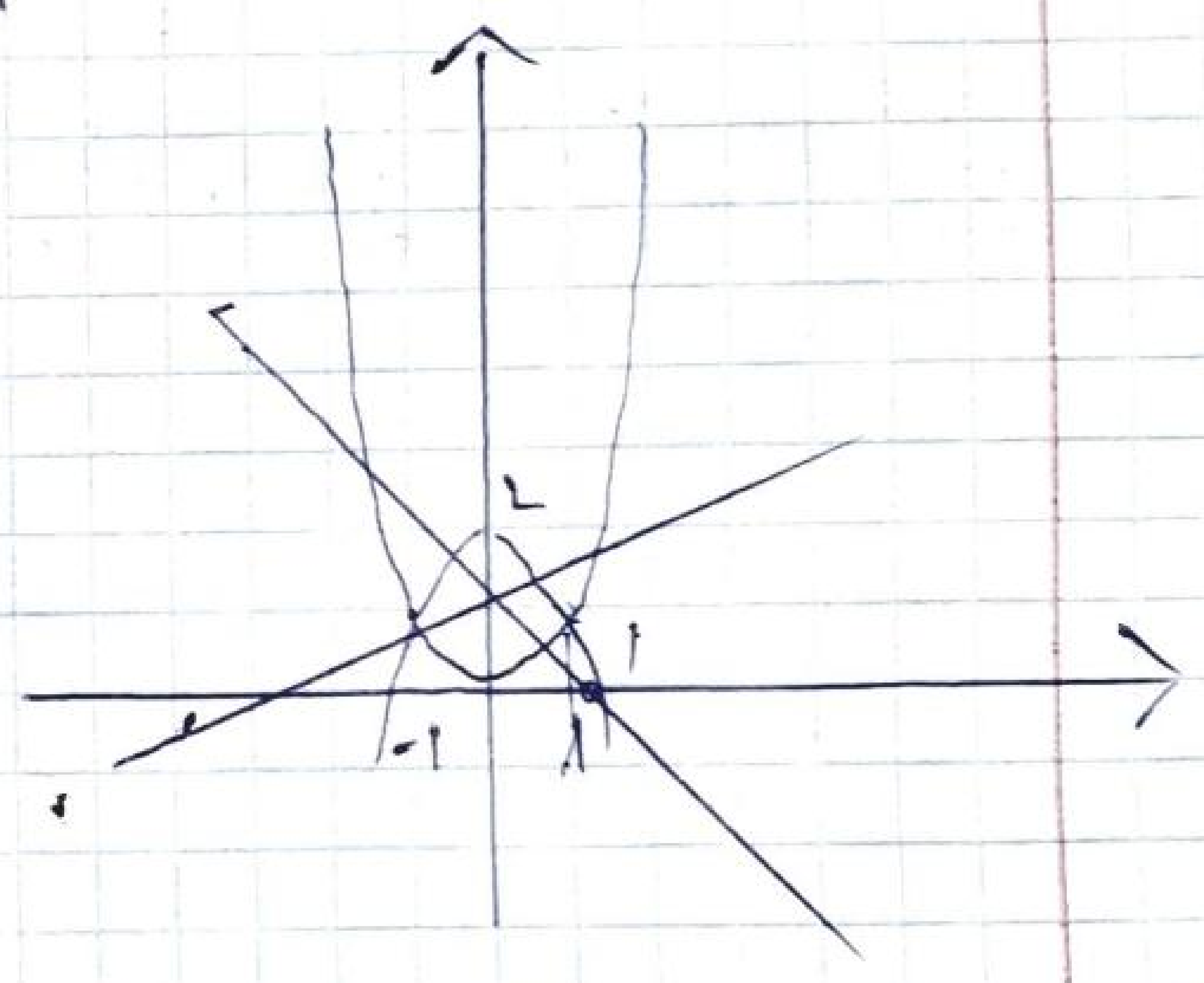
$$= \arcsin 1 - \arcsin 0.$$

$$= \frac{\pi}{2} - \pi = \frac{-\pi}{2}.$$

№ 9. Найти площадь фигуры, ограниченной линиями.

$y = \sqrt{3}x^2$; $y = \sqrt{4-x^2}$.





+) $I = \int_{-1}^1 \sqrt{4-x^2}\, dx$

$$\begin{cases} u = \sqrt{4-x^2} \\ dv = dx \end{cases} \Rightarrow \begin{cases} du = \frac{-x\,dx}{\sqrt{4-x^2}} \\ v = x \end{cases}$$

$$\Rightarrow I = \sqrt{4-x^2}\cdot x \Big|_{-1}^1 - \int_{-1}^1 \frac{-x^2 dx}{\sqrt{4-x^2}} = \sqrt{3}+\sqrt{3} - \int_{-1}^1 \frac{4-x^2}{\sqrt{4-x^2}} dx + \int_{-1}^1 \frac{4dx}{\sqrt{4-x^2}}$$

$$\Leftrightarrow I = 2\sqrt{3} - \int_{-1}^1 \sqrt{4-x^2}\, dx - 4\arccos\frac{x}{2}\Big|_{-1}^1$$

$$\Leftrightarrow I = 2\sqrt{3} - I - 4\arccos\frac{1}{2} + 4\arccos\frac{-1}{2}.$$

$$(\Leftrightarrow)\ 2I = 2\sqrt{3} - 4\frac{\pi}{3} + 4\cdot\frac{2\pi}{3}.$$

$$= 2\sqrt{3} - \frac{4\pi}{3}. \Rightarrow I = \sqrt{3} - \frac{2\pi}{3}.$$

$$\int_{-1}^{1} \left(\sqrt{4 - x^2} - \sqrt{3}\,x^2\right)dx.$$

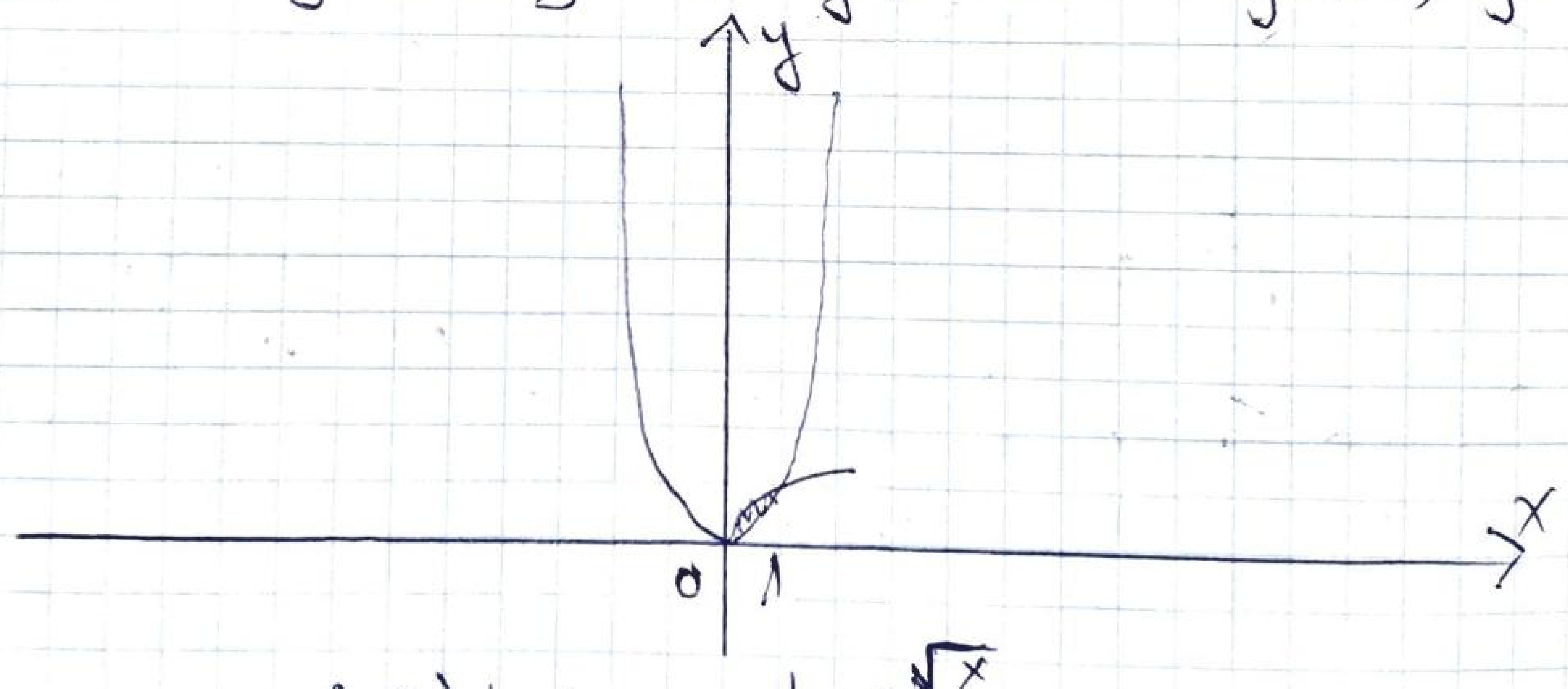
$$= \int_{-1}^{1} \sqrt{4-x^2}\,dx - \int_{-1}^{1} \sqrt{3}\,x^2\,dx.$$

$$= \sqrt{3} - \frac{2\pi}{3} - \frac{\sqrt{3}}{3}x^3 \Big|_{-1}^{1}$$

$$= \sqrt{3} - \frac{2\pi}{3} - \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{3} = \frac{\sqrt{3} - 2\pi}{3}.$$

$$\Rightarrow S = \frac{\sqrt{3} - 2\pi}{3}.$$

N° 10: $\iint (3xy - 2x^3y^3)\,dx\,dy$; D: $y = x^4$; $y = \sqrt{x}$.



$$\iint (3xy - 3x^3y^3)\,dx\,dy = \int_0^1 dx \int_{x^4}^{\sqrt{x}} (3xy - 3x^3y^3)\,dy$$

$$= \int_0^1 dx \left(3x\frac{y^2}{2} - 3x^3\frac{y^4}{4}\right)\Big|_{x^4}^{\sqrt{x}}$$

$$= \int_0^1 \left(3x\frac{x}{2} - 3x^3\frac{x^2}{4} - \frac{3x^8}{2}\cdot x + \frac{3x^{19}}{4}\right)dx$$

$$= \int_0^1 \left(\frac{3}{2}x^2 - \frac{3}{4}x^5 - \frac{3x^9}{2} + \frac{3x^{19}}{4}\right)dx$$

$$= \frac{3}{2}\frac{x^3}{3} - \frac{3}{4}\cdot\frac{x^6}{6} - \frac{3}{2}\cdot\frac{x^{10}}{10} + \frac{3x^{20}}{4\cdot 20}\Big|_0^1$$

$$= \frac{1}{2} - \frac{1}{8} - \frac{3}{20} + \frac{3}{80} = \frac{21}{80}.$$

№7. Вычислить определённый интеграл:

$$\cancel{32}. \int_3^6 \frac{\sqrt{x^2-9}}{x^4}\,dx.$$

Пусть ~~x = 3sec(u)~~. $x = \frac{3}{\cos(u)}$.

$$\Rightarrow \int_0^{\pi/3} \frac{\tan^2(u)}{9\left(\frac{1}{\cos(u)}\right)^3}\,du = \frac{1}{9}\int_0^{\frac{\pi}{3}} \frac{\left(\frac{\sin u}{\cos u}\right)^2}{\left(\frac{1}{\cos(u)}\right)^3}\,du$$

$$= \frac{1}{9} \cdot \int_0^{\pi/3} \sin^2(u) \cdot \cos(u)\,du.$$

Пусть $v = \sin(u)$

$$= \frac{1}{9}\int_0^{\frac{\sqrt{3}}{2}} v^2\,dv. = \frac{1}{9} \cdot \frac{v^3}{3}\Big|_0^{\sqrt{3}/2}.$$

$$= \frac{1}{9} \cdot \frac{3\sqrt{3}}{8 \cdot 3} = \frac{\sqrt{3}}{72}.$$

Scanned by TapScanner