√ № 1. $f(x,y) = \dfrac{-0,6(x^2+y^2) - 6x - 5y}{x^2+y^2+10x - 5y}$

M(2; -1,5)

$f(x,y) = -0,6 \left(\dfrac{x^2+y^2+10x - 5y + \cancel{??} \cancel{?} \, 40/3 \, y}{x^2+y^2+10x - 5y} \right)$

$= -0,6 - \dfrac{\cancel{??} \, 8y}{x^2+y^2+10x-5y}$

$f'x = \dfrac{(2x+10) \cdot \cancel{??} 8y}{(x^2+y^2+10x-5y)^2}$
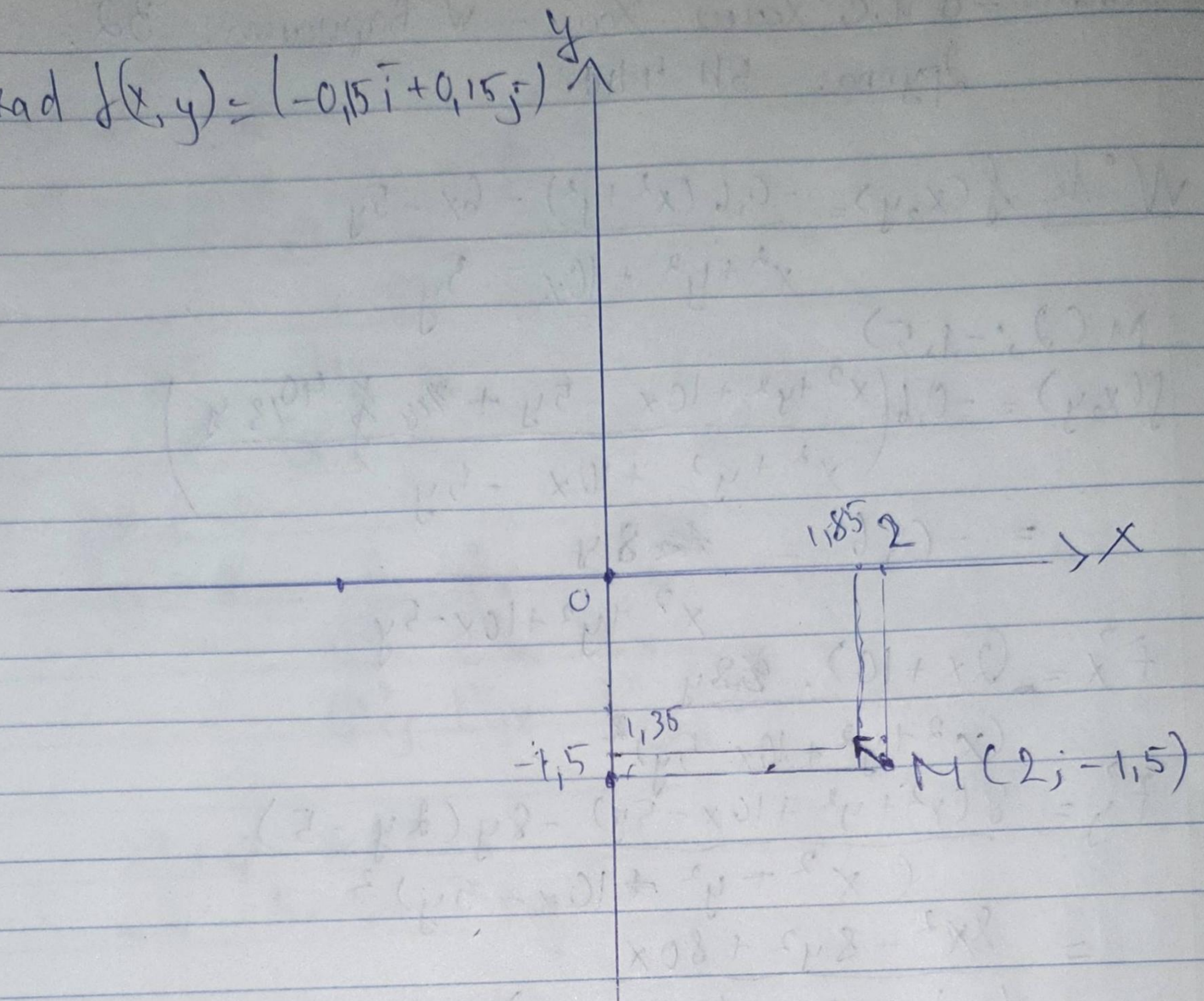
$F'y = \dfrac{8(x^2+y^2+10x-5y) - 8y(2y-5)}{(x^2+y^2+10x-5y)^2}$

$= \dfrac{8x^2 - 8y^2 + 80x}{(x^2+y^2+10x-5y)^2}$

grad $f(x,y) = (F'x|_M; F'y|_M)$ или $(F'x|_M \cdot \bar{i} + F'y|_M \cdot \bar{j})$

$F'x|_M = \dfrac{(4+10) \cdot 8 \cdot (-1,5)}{(4+9/4+20+\frac{15}{2})^2} = \dfrac{-896}{6075} \approx -0,15$

$F'y|_M = \dfrac{32 - 18 + 160}{(4+9/4+20+\frac{15}{2})^2} = \dfrac{928}{6075} \approx 0,15$

grad $f(x,y) = (-0,15\bar{i} + 0,15\bar{j})$

$M(2; -1,5)$

№2: $z = x^2 - xy + 3y^2 + 2x + 3y + 2$

$z'_x = 2x - y + 2$; $z'_y = -x + 6y + 3$

$\begin{cases} 2x - y + 2 = 0 \\ -x + 6y + 3 = 0 \end{cases}$ (=) $\begin{cases} y = 2x + 2 \\ -x + 12x + 12 + 3 = 0 \end{cases}$

(=) $\begin{cases} y = 2x + 2 \\ +11x = -15 \end{cases}$ (=) $\begin{cases} x = -15/11 \\ y = -8/11 \end{cases}$

$z''_{xx} = 2$; $z''_{yy} = 6$; $z''_{xy} = -1$

$\Delta = \begin{vmatrix} 2 & -1 \\ -1 & 6 \end{vmatrix} = 12 - 1 = 11 > 0 ; A = 2 > 0$

$\Rightarrow \left(\dfrac{-15}{11} ; \dfrac{-8}{11} \right) \rightarrow \min.$

N°3. $z = 10 + 4x + y$; $3x^2 - y^2 = 2$

$L = 10 + 4x + y + \lambda(3x^2 - y^2 - 2)$

$L'_x = 4 + 6\lambda x$

$L'_y = 1 - 2\lambda y$

$\Rightarrow \begin{cases} 4 + 6\lambda x = 0 \\ 1 - 2\lambda y = 0 \\ 3x^2 - y^2 = 2 \end{cases}$ $\Rightarrow \begin{cases} x = -4/6\lambda \\ y = 1/2\lambda \\ 3 \cdot x^2 - y^2 = 2 \quad ① \end{cases}$

$① \Rightarrow 3 \cdot \left(\dfrac{-4}{6\lambda}\right)^2 - \left(\dfrac{1}{2\lambda}\right)^2 - 2 = 0$

$\Rightarrow \dfrac{4}{3\lambda^2} - \dfrac{1}{4\lambda^2} = 2$

$\Rightarrow \dfrac{13}{12\lambda^2} = 2 \Rightarrow \lambda = \pm\sqrt{\dfrac{13}{24}}$

$①\ \lambda = \sqrt{\dfrac{13}{24}} \Rightarrow \begin{cases} x = -\dfrac{4\sqrt{78}}{39} \\ y = \dfrac{\sqrt{78}}{13} \end{cases} \Rightarrow M_1\left(-\dfrac{4\sqrt{78}}{39}, \dfrac{\sqrt{78}}{13}\right)$

$②\ \lambda = -\sqrt{\dfrac{13}{24}} \Rightarrow \begin{cases} x = \dfrac{4\sqrt{78}}{39} \\ y = -\dfrac{\sqrt{78}}{13} \end{cases} \Rightarrow M_2\left(\dfrac{4\sqrt{78}}{39}, -\dfrac{\sqrt{78}}{13}\right)$

$\Phi'_x = (3x^2 - y^2 - 2)'x = 6x$

$\Phi'_y = (3x^2 - y^2 - 2)'y = -2y.$

$L''_{xx} = 6\lambda$; $L''_{xy} = 0$

$L''_{yy} = -2\lambda$; $L''_{yx} = 0$

$$H = \begin{vmatrix} 0 & 6x & -2y \\ 6x & 6\lambda & 0 \\ -2y & 0 & -2\lambda \end{vmatrix}$$

$$= -6x \begin{vmatrix} 6x & 0 \\ -2y & -2\lambda \end{vmatrix} - 2y \begin{vmatrix} 6x & 6\lambda \\ -2y & 0 \end{vmatrix}$$

$$\cancel{= -6x(-6x - 12\lambda x) - 2\lambda y(-12\lambda y)}$$

$$= (-6x)(-12\lambda x) - 2y(12\lambda y)$$

$$= 72\lambda x^2 - 24\lambda y^2$$

$$= 72\lambda \cdot \frac{16}{36\lambda^2} - 24 \cdot \lambda \cdot \frac{1}{4\lambda^2}$$

$$= \frac{32}{\lambda} - \frac{6}{\lambda} \quad \cancel{2\sqrt{78}} \quad \circledast$$

+) $\lambda = \sqrt{\dfrac{13}{24}} \Rightarrow \circledast = 4\sqrt{78} > 0 \Rightarrow M_1 \max$

+) $\lambda = -\sqrt{\dfrac{13}{24}} \Rightarrow \circledast = -4\sqrt{78} < 0 \Rightarrow M_2 \min.$

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